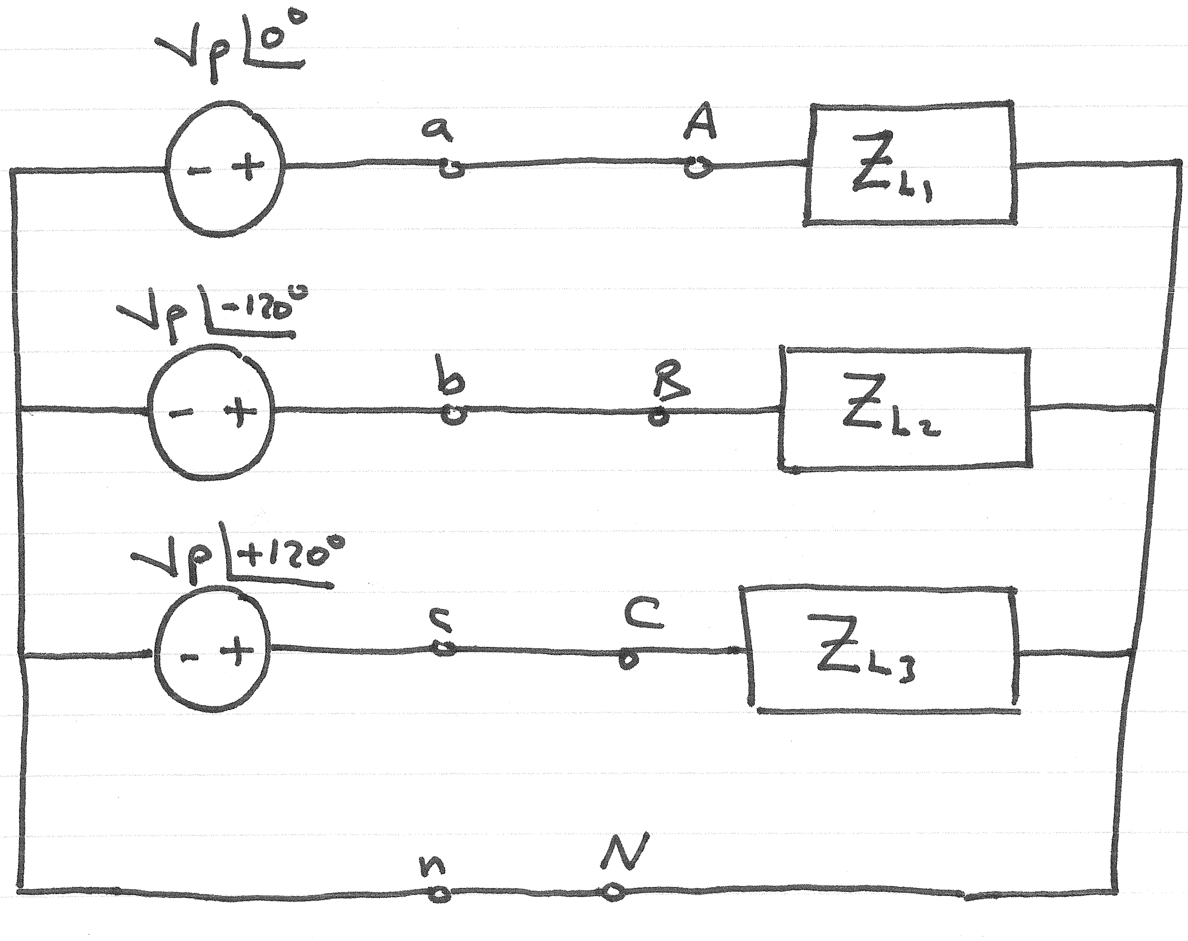
Balanced Three-phase Circuits

What is a Three-phase Circuit?

It is a system produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°.



Advantages :

1. Almost all the electric power is generated and distributed in three-phase.

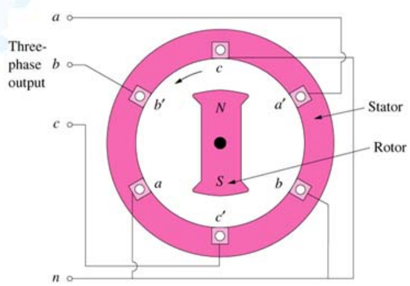
2. The instantaneous power in a three-phase system is constant
   ∴ There is less vibration in the rotating machinery which in turn performs more efficiently.

3. The amount of power loss in the three-phase system is only half the power loss in the cables for the single phase system.

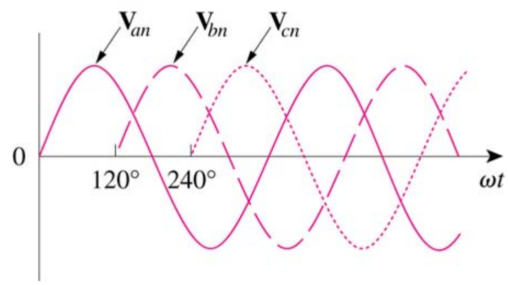
4. Thinner conductors can be used to transmit the same KVA at the same voltage.

Balanced Three – Phase Generator

A three-phase generator consists of a rotating magnet (rotor) surrounded by a stationary winding (stator).



A three-phase generator



The generated voltages

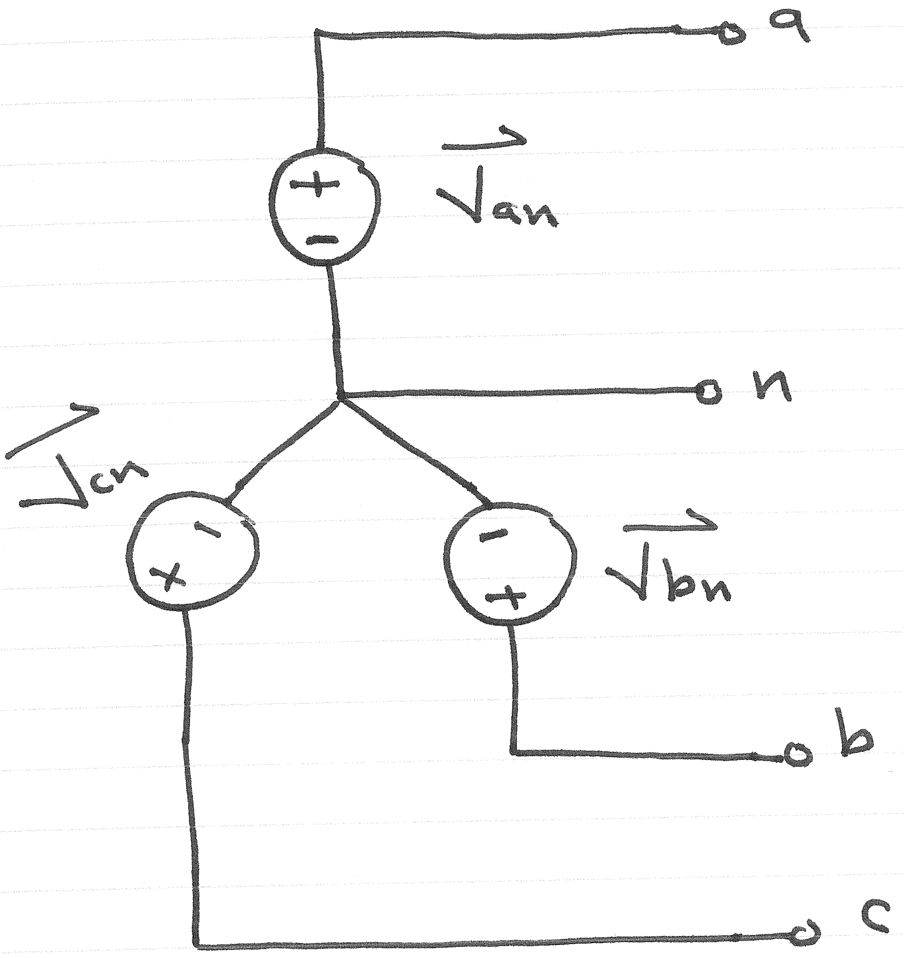
The Three – Phase Generator :

a) Has three induction coils.
b) Placed 120 a part on the rotor.
c) The three coils have an equal number of turns .
d) The voltage induced across each coil will have the same peak value, shape and frequency.

Balanced Three-phase Sources

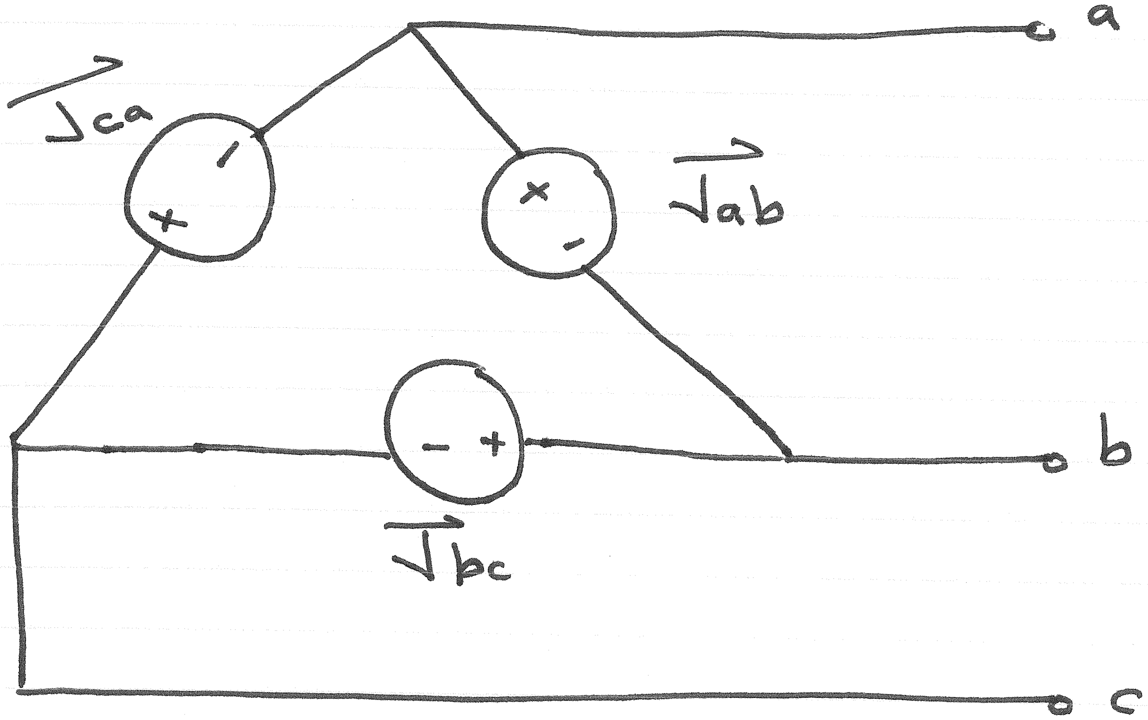
Two possible Configurations

1. The Y-connected Source



$\vec{V}_{an}$, $\vec{V}_{bn}$, and $\vec{V}_{cn}$ are Called the phase Voltages.

2. The Δ - Connected source



The phase Sequence

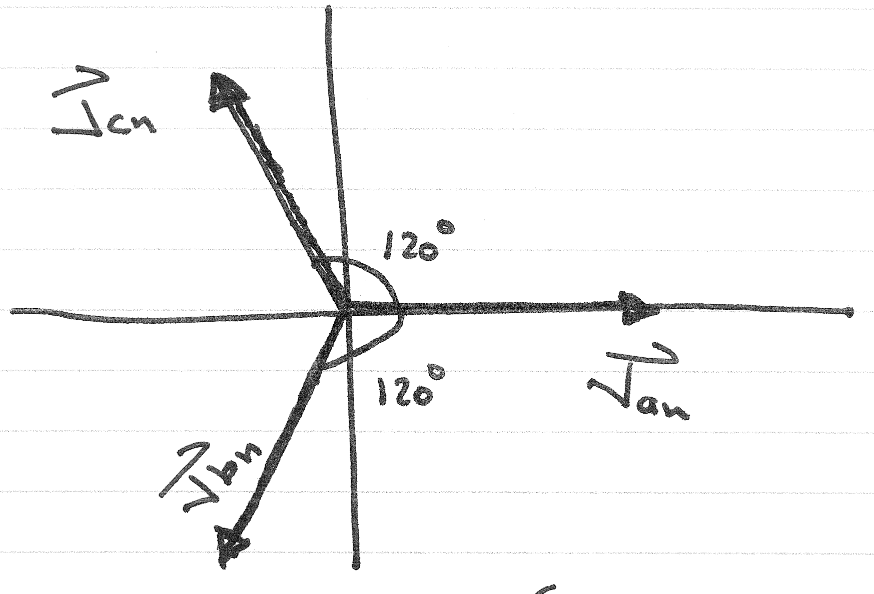
The phase sequence is the time order in which the voltages pass through their respective maximum values

1. abc sequence (positive sequence)

$$\vec{V}_{an} = V_p\angle 0^\circ$$

$$\vec{V}_{bn} = V_p\angle -120^\circ$$

$$\vec{V}_{cn} = V_p\angle +120^\circ$$

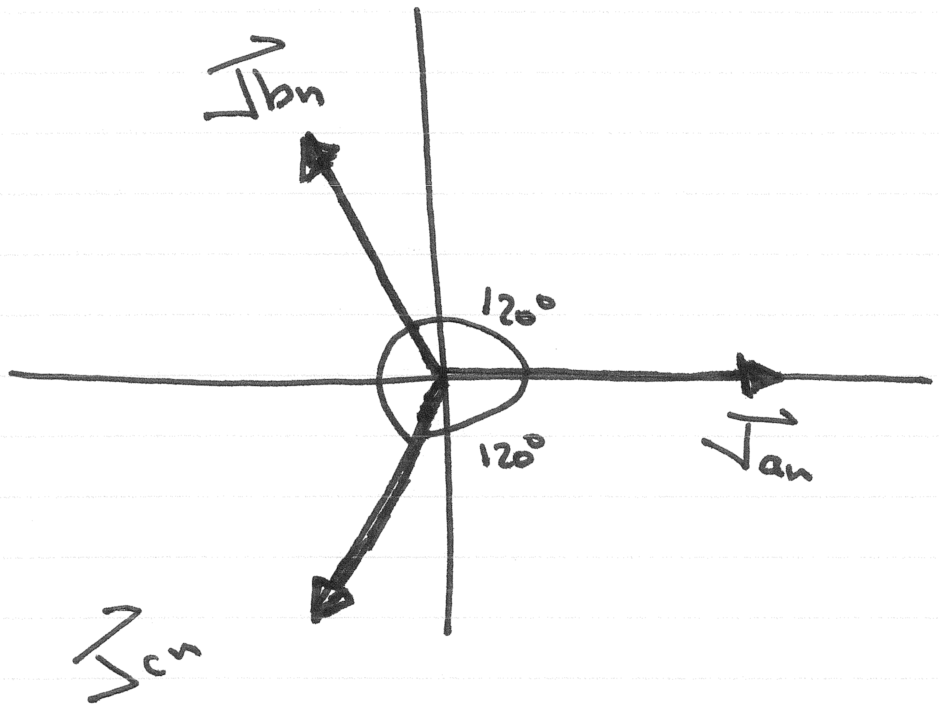


2. acb sequence (negative sequence)

$$\vec{V}_{an} = V_p \angle 0^\circ$$

$$\vec{V}_{bn} = V_p \angle +120^\circ$$

$$\vec{V}_{cn} = V_p \angle -120^\circ$$



$$\vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn} = 0$$

$$\vec{V}_{an} = V_p \angle 0^\circ$$

$$\vec{V}_{bn} = V_p \angle -120^\circ$$

$$\vec{V}_{cn} = V_p \angle +120^\circ$$

$$\vec{V}_{an} = V_p$$

$$\vec{V}_{bn} = V_p \cos(-120^\circ) + jV_p \sin(-120^\circ)$$

$$\vec{V}_{bn} = V_p \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$\vec{V}_{cn} = V_p \cos(+120^\circ) + j\, V_p \sin(+120^\circ)$$

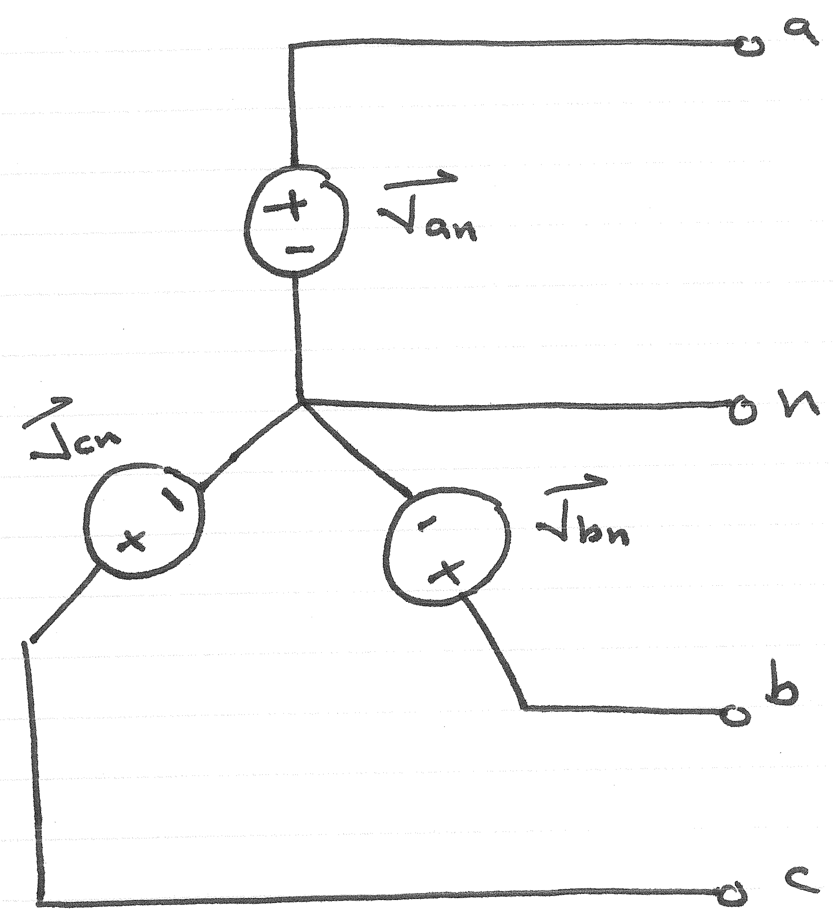
$$\vec{V}_{cn} = V_p \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$\therefore \vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn} = 0$$

$$\therefore v_{an}(t) + v_{bn}(t) + v_{cn}(t) = 0$$

Balanced Set

Line-to Line Voltages



$\vec{V}_{ab}$, $\vec{V}_{bc}$, $\vec{V}_{ca}$ are called the line-to-line voltages let $\vec{V}_{an} = V_p \angle 0^\circ$ ✓

$\vec{V}_{bn} = V_p \angle -120^\circ$ ✓

$\vec{V}_{cn} = V_p \angle +120^\circ$ ✓

$$\vec{V}_{ab} = \vec{V}_{an} + \vec{V}_{nb}$$

$$\vec{V}_{ab} = \vec{V}_{an} - \vec{V}_{bn}$$

$$\vec{V}_{ab} = V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$\vec{V}_{ab} = V_p - V_p \left(\cos(-120^\circ) + j \sin(-120^\circ) \right)$$

$$\vec{V}_{ab} = V_p - V_p \left(-\frac{1}{2} - j \frac{\sqrt{3}}{2} \right)$$

$$\vec{V}_{ab} = V_p \left(1 + \frac{1}{2} + j \frac{\sqrt{3}}{2} \right)$$

$$\vec{V}_{ab} = V_p \left(\frac{3}{2} + j \frac{\sqrt{3}}{2} \right)$$

$$\vec{V}_{ab} = V_p \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \angle \tan^{-1} \frac{\frac{\sqrt{3}}{2}}{3/2}$$

$\vec{V}_{ab} = V_p \sqrt{3} \angle +30^\circ$ ✓

$\vec{V}_{ab} = \sqrt{3}\, \vec{V}_{an} \angle +30^\circ$ ✓

$\vec{V}_{bc} = \sqrt{3}\, \vec{V}_{bn} \angle +30^\circ$ ✓

$\vec{V}_{ca} = \sqrt{3}\, \vec{V}_{cn} \angle +30^\circ$ ✓

For negative sequence :

$$\vec{V}_{an} = V_p \angle 0^\circ \quad V$$

$$\vec{V}_{bn} = V_p \angle +120^\circ \quad V$$

$$\vec{V}_{cn} = V_p \angle -120^\circ \quad V$$

$$\vec{V}_{ab} = V_p \sqrt{3} \angle -30^\circ \quad V$$

$$\therefore \vec{V}_{ab} = \sqrt{3}\, \vec{V}_{an} \angle -30^\circ \quad V$$

$$\therefore \vec{V}_{bc} = V_p \sqrt{3} \angle +90^\circ \quad V$$

$$\therefore \vec{V}_{bc} = \sqrt{3}\, \vec{V}_{bn} \angle -30^\circ \quad V$$

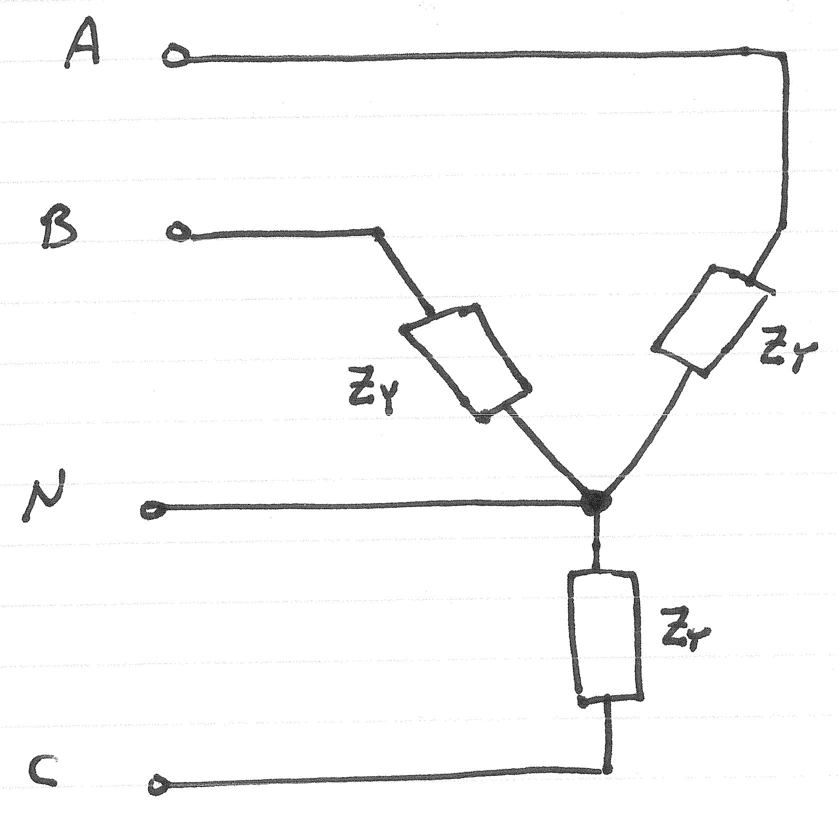
$$\therefore \vec{V}_{ca} = V_p \sqrt{3} \angle -150^\circ \quad V$$

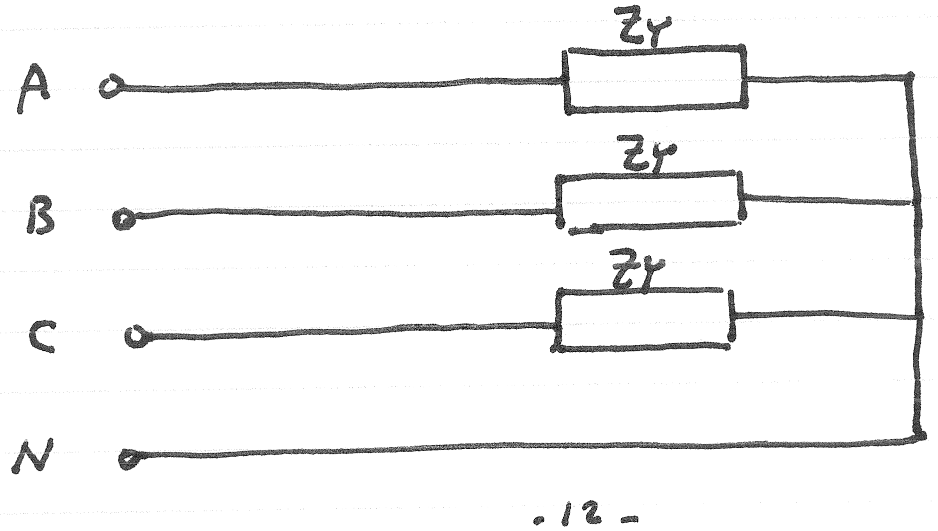
$$\therefore \vec{V}_{ca} = \sqrt{3}\, \vec{V}_{cn} \angle -30^\circ \quad V$$

Balanced Three phase Loads

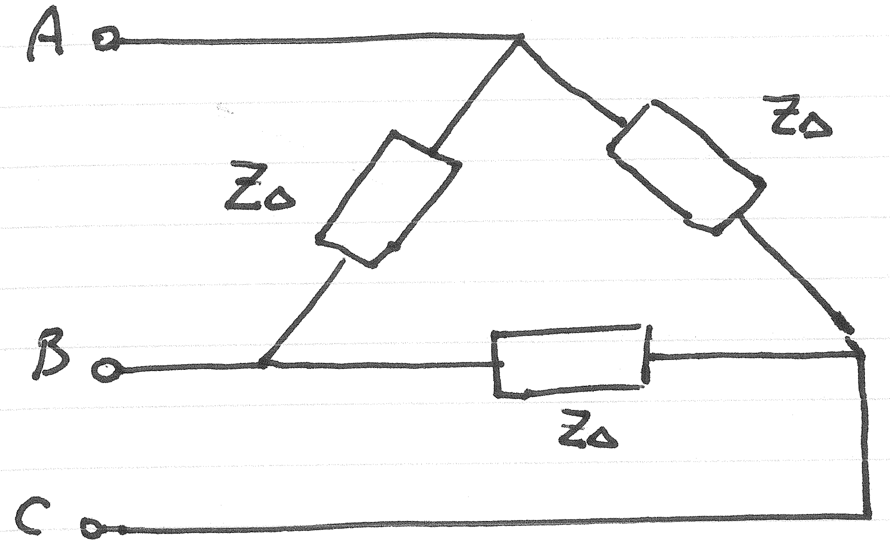
A balanced Load has equal impedances on all the phases.

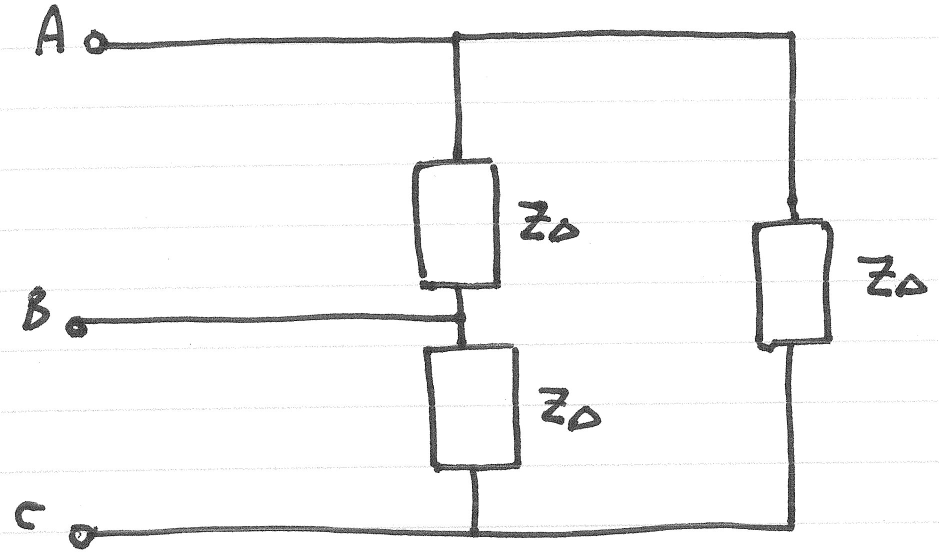
1) Y- Connected Load





2) Δ - Connected Load





$$Z_Y = \frac{Z_\Delta}{3}$$

$$Z_\Delta = 3 Z_Y$$

Three phase Connections

Both the three phase source and the three phase load can be connected either Wye or Delta

∴ We have 4 possible connection types.

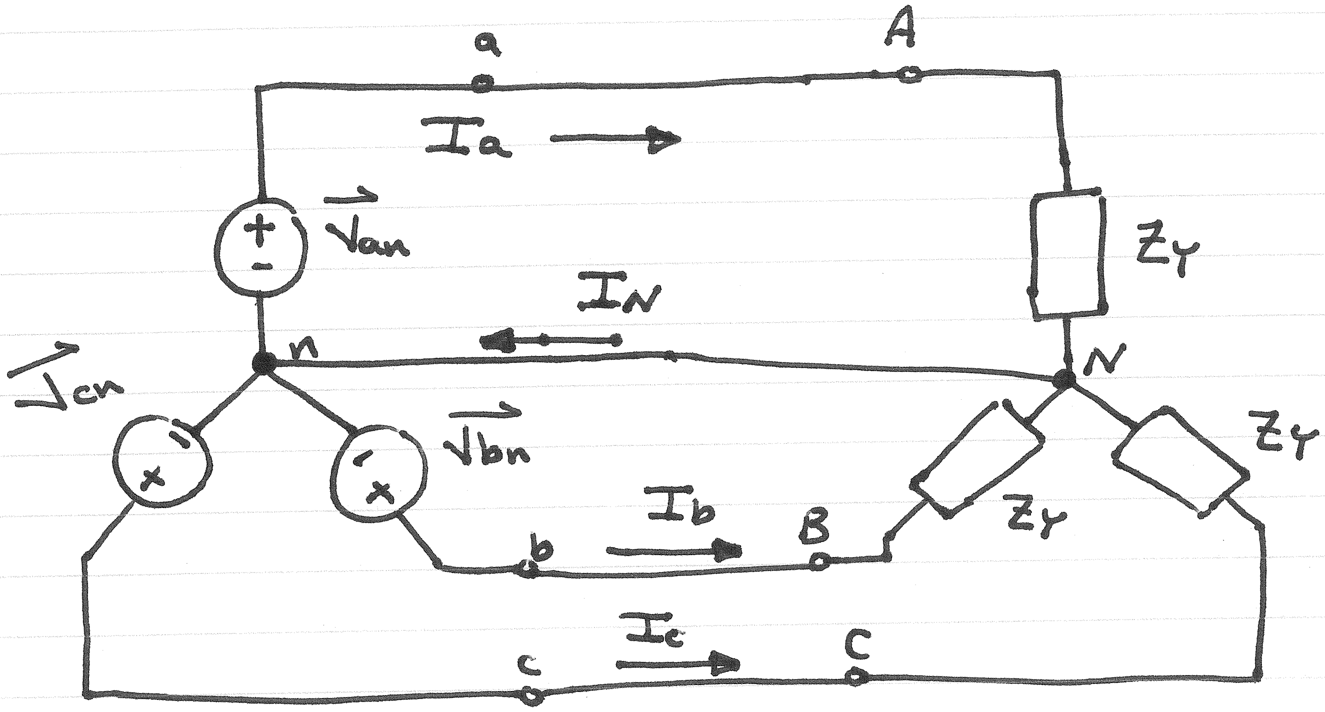
Y_Y Connection

Y_Δ Connection

Δ-Δ Connection

Δ_Y Connection

Balanced Y-Y system



$$\vec{I}_a = \frac{\vec{V}_{an}}{Z_Y}$$

$$\vec{I}_b = \frac{\vec{V}_{bn}}{Z_Y}$$

$$\vec{I}_c = \frac{\vec{V}_{cn}}{Z_Y}$$

KCL: $\vec{I}_N = \vec{I}_a + \vec{I}_b + \vec{I}_c$

$$= \frac{\vec{V}_{an}}{Z_Y} + \frac{\vec{V}_{bn}}{Z_Y} + \frac{\vec{V}_{cn}}{Z_Y}$$

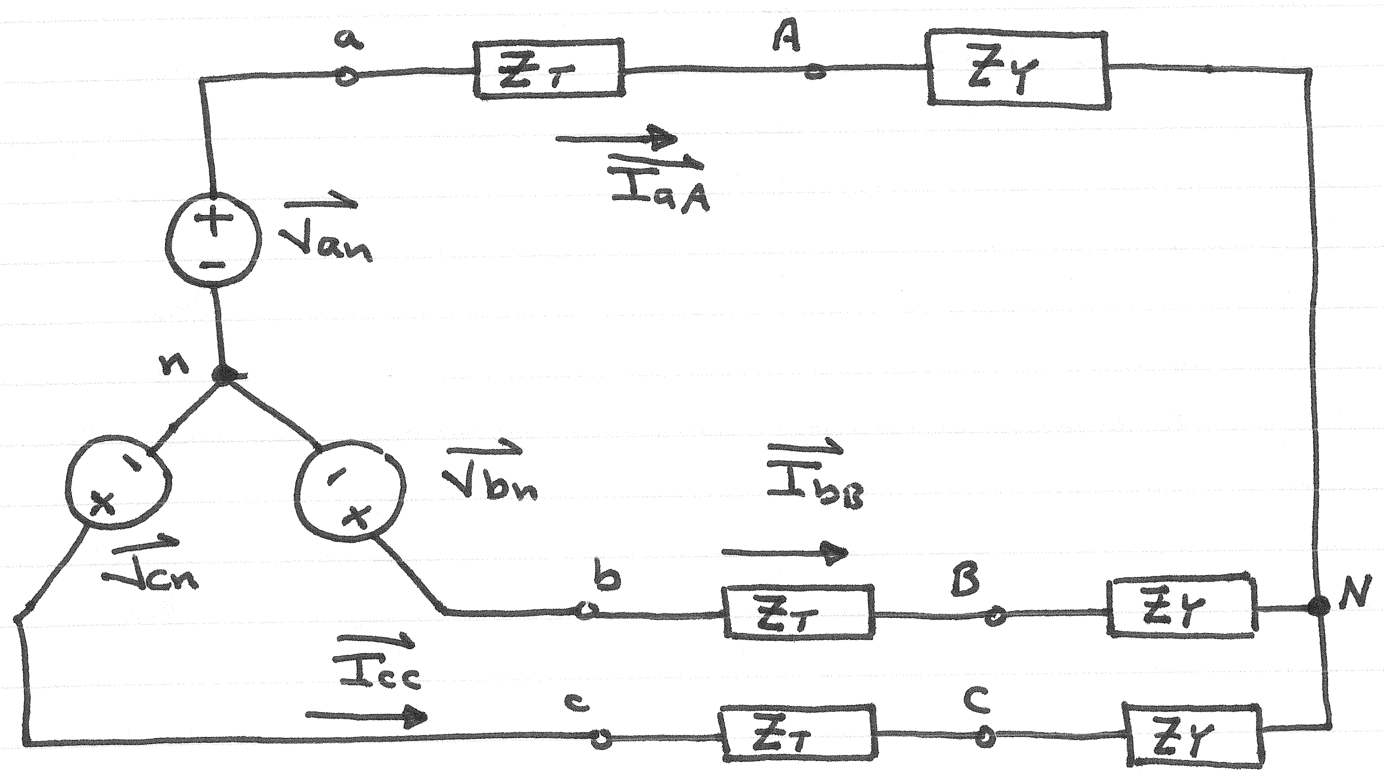
$$= \frac{1}{Z_Y}\left(\vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn}\right)$$

$$\vec{I}_N = 0$$

$\therefore$ could be replaced by open Circuit

Example :

Calculate the Line currents.



$$\vec{V}_{an} = 120 \angle 0^\circ \ \text{V rms}$$

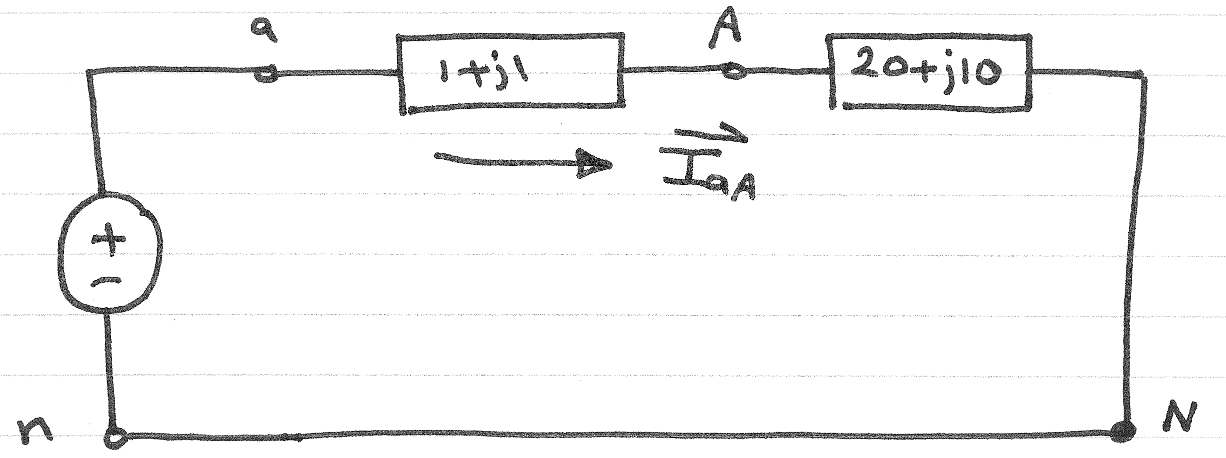
$$\vec{V}_{bn} = 120 \angle -120^\circ \ \text{V rms}$$

$$\vec{V}_{cn} = 120 \angle +120^\circ \ \text{V rms}$$

$$Z_T = (1 + j1)\ \Omega$$

$$Z_Y = (20 + j10)\ \Omega$$

Single phase representation



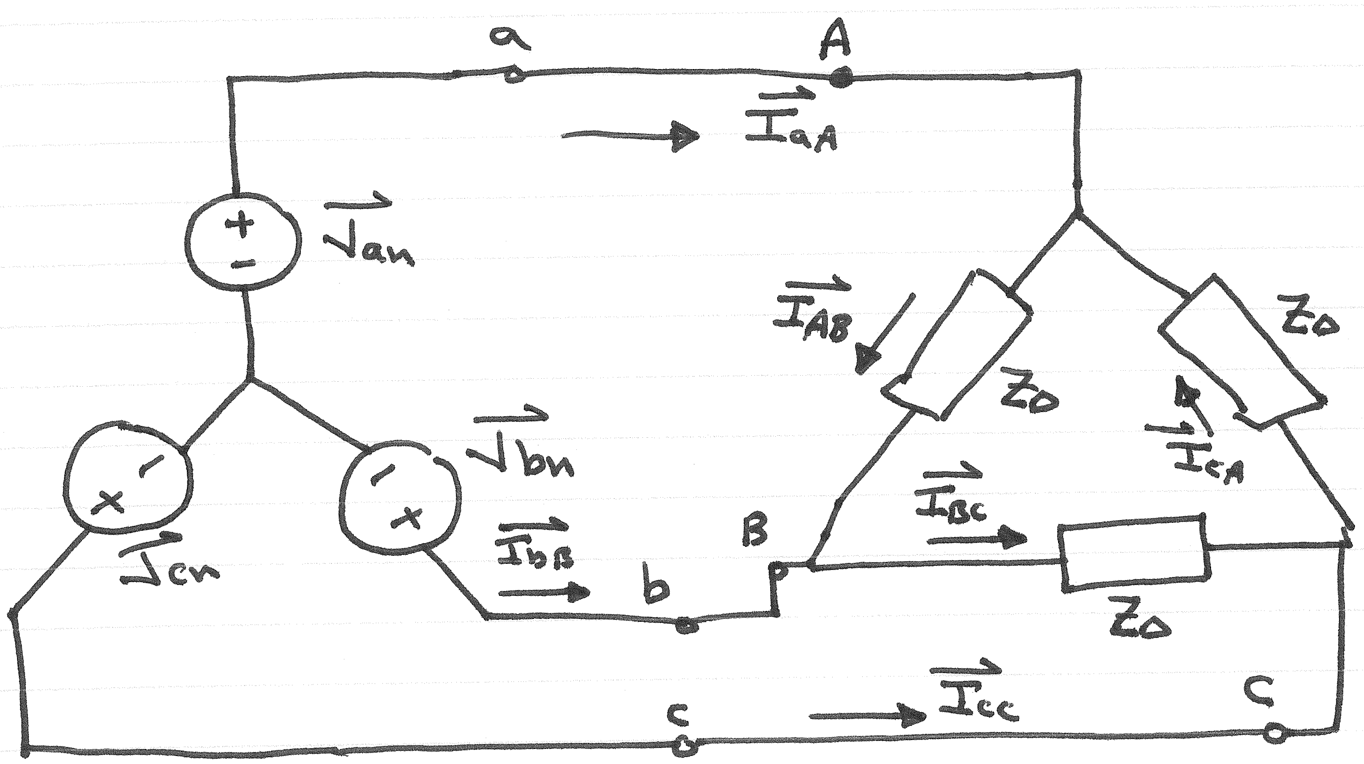
$$\vec{I}_{aA} = \frac{\vec{V}_{an}}{Z_T + Z_Y} = \frac{120\angle 0^\circ}{21 + j11}$$

$$\therefore \quad \vec{I}_{aA} = 5.06 \angle -27.65^\circ \ \text{A rms}$$

$$\therefore \quad \vec{I}_{bB} = 5.06 \angle -147.65^\circ \ \text{A rms}$$

$$\therefore \quad \vec{I}_{cC} = 5.06 \angle 92.35^\circ \ \text{A rms}$$

Balanced Y-Δ System



Example:

$\vec{V}_{an} = 120\angle 30°$ V_{rms}

$Z_\Delta = (6+j6)\ \Omega$

positive sequence

Calculate the Line Currents.

$$\vec{V}_{ab} = \vec{V}_{AB} = 120\sqrt{3}\angle 60° \quad V_{rms}$$

$$\vec{I}_{AB} = \frac{\vec{V}_{AB}}{Z_\Delta} = 24.5\angle 15° \quad A_{rms}$$

$$\therefore \vec{I}_{BC} = 24.5\angle -105° \quad A_{rms}$$

$$\therefore \vec{I}_{CA} = 24.5\angle 135° \quad A_{rms}$$

$\vec{I}_{AB}$, $\vec{I}_{BC}$, and $\vec{I}_{CA}$ are the phase currents of the Load.

KCL :

$$\vec{I}_{aA} = \vec{I}_{AB} - \vec{I}_{CA}$$

$$\vec{I}_{aA} = 24.5\angle 15^\circ - 24.5\angle 135^\circ$$

$$\vec{I}_{aA} = 42.44\angle -15^\circ \ A_{rms}$$

$$\vec{I}_{aA} = \sqrt{3}\ \vec{I}_{AB}\angle -30^\circ$$

Line current Lags the phase current by 30° only for abc sequence

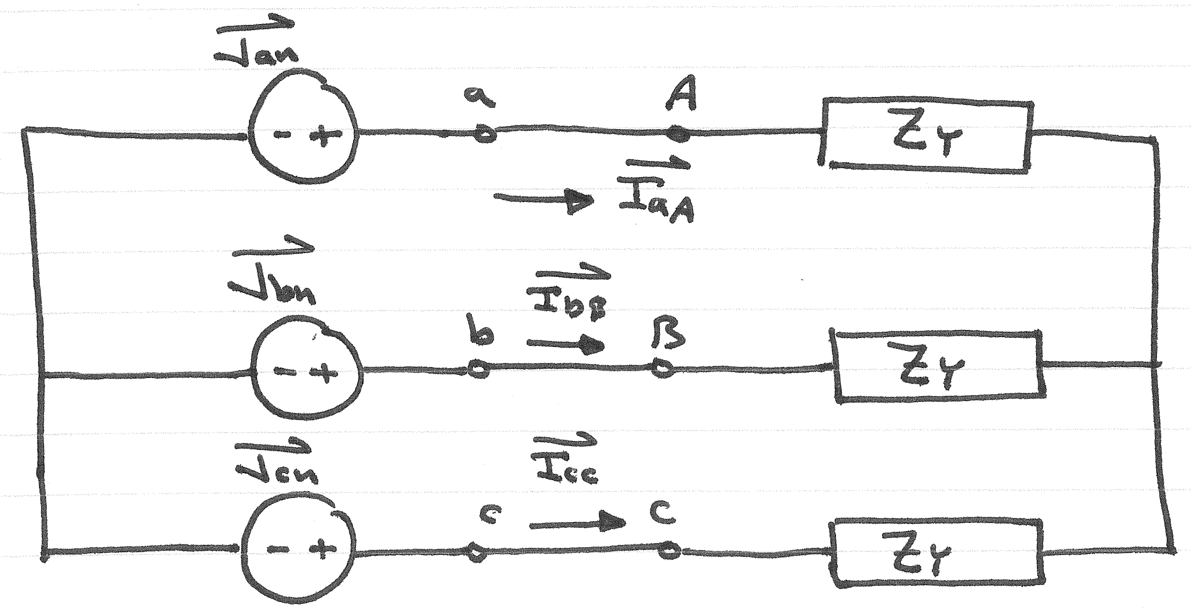
$$\therefore \vec{I}_{bB} = 42.44\angle -135^\circ \ A_{rms}$$

$$\therefore \vec{I}_{cC} = 42.44\angle 105^\circ \ A_{rms}$$

Second method

Using $\Delta - Y$ Transformation

$$Z_Y = \frac{Z_\Delta}{3}$$



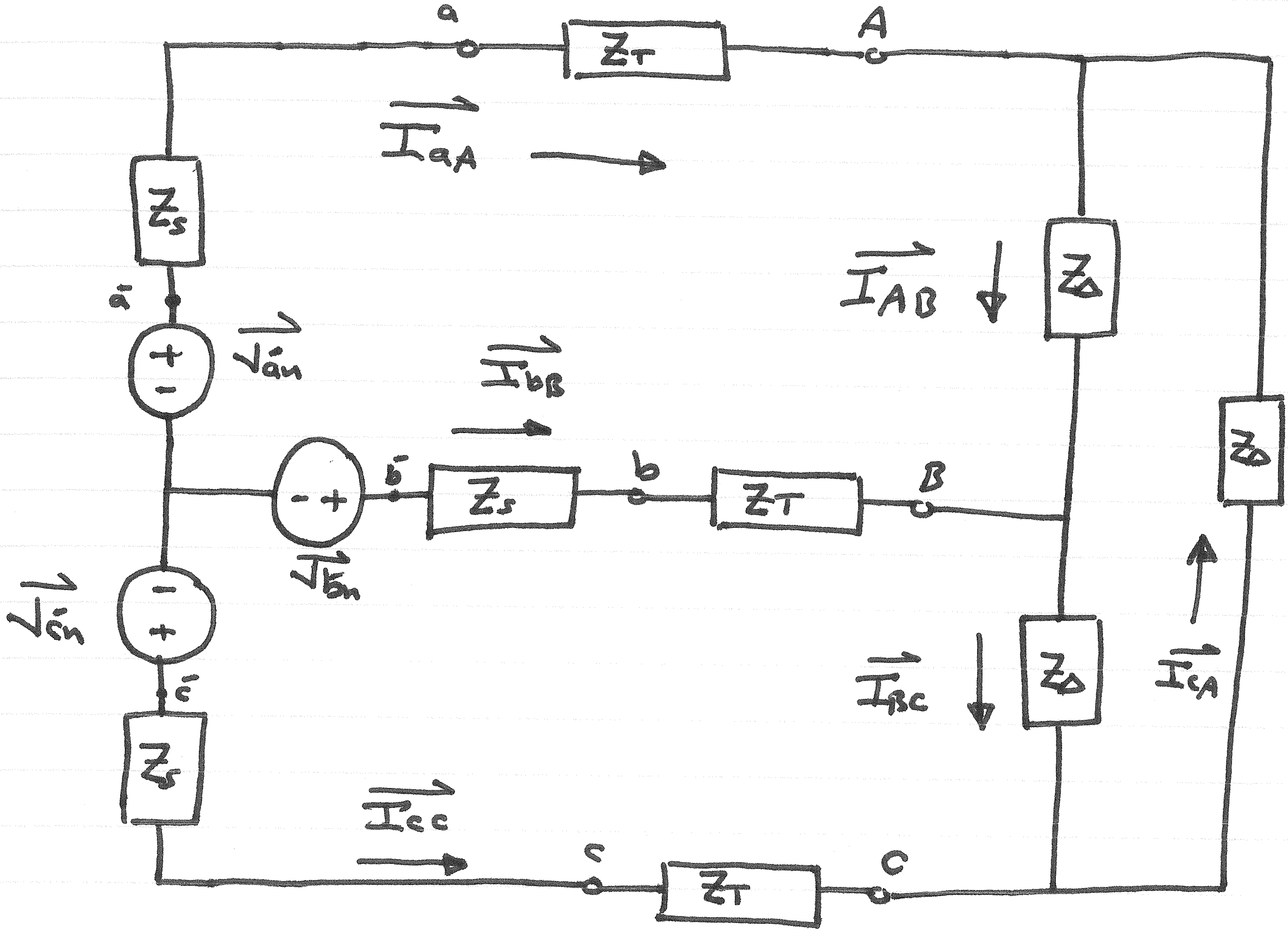
$$Z_Y = \frac{6+j6}{3} = (2+j2)\,\Omega$$

$$\vec{I}_{aA} = \frac{\vec{V}_{an}}{Z_Y} = 42.44 \angle -15^\circ \text{ Arms}$$

$$\therefore \vec{I}_{bB} = 42.44 \angle -135^\circ \text{ A rms}$$

$$\therefore \vec{I}_{cC} = 42.44 \angle 105^\circ \text{ A rms}$$

Example



given $\vec{V}_{a'n} = 120\angle 0^\circ$ Vrms

abc sequence

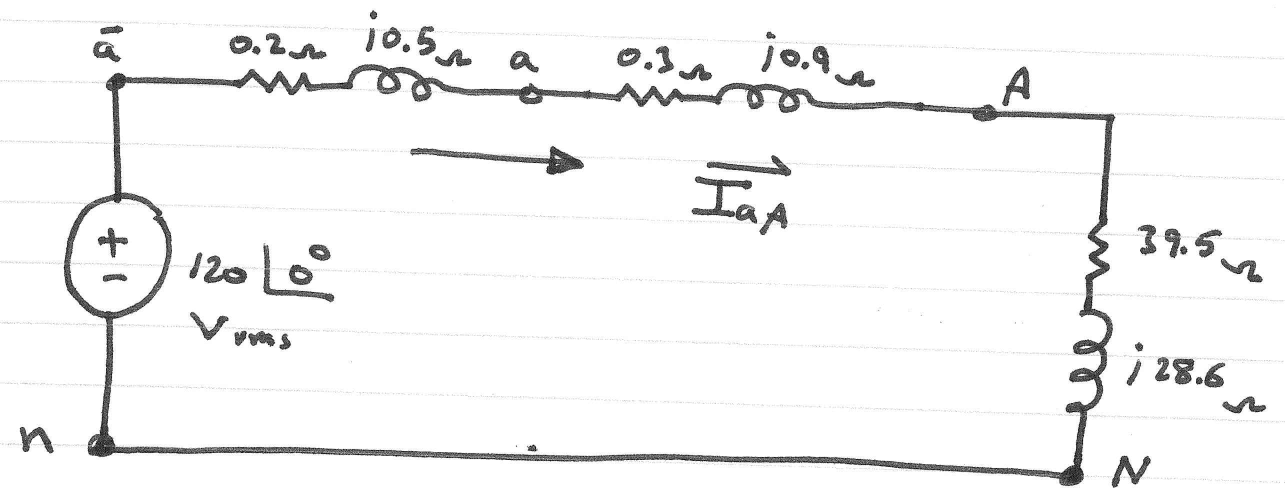
$Z_s = (0.2 + j0.5)\,\Omega$

$Z_T = (0.3 + j0.9)\,\Omega$

$Z_\Delta = (118.5 + j85.8)\,\Omega$

1) Calculate the line currents

Single phase representation



$$Z_Y = \frac{Z_\Delta}{3} = \frac{118.5 + j85.8}{3} = (39.5 + j\,28.6)\ \Omega$$

$$\overrightarrow{I_{aA}} = \frac{120\angle 0^\circ}{(0.2+j0.5)+(0.3+j0.9)+39.5+j\,28.6}$$

$$\overrightarrow{I_{aA}} = 2.4\angle -36.87^\circ \quad A_{rms}$$

$$\therefore\ \overrightarrow{I_{bB}} = 2.4\angle -156.87^\circ \quad A_{rms}$$

$$\therefore\ \overrightarrow{I_{cC}} = 2.4\angle 83.13^\circ \quad A_{rms}$$

2) Calculate the phase currents of the load

$$\overrightarrow{I_{AB}} = \frac{1}{\sqrt{3}} \angle{+30^\circ} \; \overrightarrow{I_{aA}}$$

$\therefore \quad \overrightarrow{I_{AB}} = 1.39 \angle{-6.87^\circ} \quad A_{rms}$

$\therefore \quad \overrightarrow{I_{BC}} = 1.39 \angle{-126.87^\circ} \quad A_{rms}$

$\therefore \quad \overrightarrow{I_{CA}} = 1.39 \angle{113.13^\circ} \quad A_{rms}$

3) Calculate the phase voltages at the load terminals ; $\overrightarrow{V_{AB}}$, $\overrightarrow{V_{BC}}$, and $\overrightarrow{V_{CA}}$

a) First method

$$\overrightarrow{V_{AB}} = Z_\Delta \, \overrightarrow{I_{AB}}$$

$$\overrightarrow{V_{AB}} = (118.5 + j\,85.8)\,(1.39 \angle{-6.87^\circ})$$

$$\overrightarrow{V_{AB}} = 202.72 \angle{29.04^\circ} \quad V_{rms}$$

$\therefore \quad \overrightarrow{V_{BC}} = 202.72 \angle{-90.96^\circ} \quad V_{rms}$

$\therefore \quad \overrightarrow{V_{CA}} = 202.72 \angle{149.04^\circ} \quad V_{rms}$

b) second method

From the single phase representation

$$\vec{V}_{AN} = Z_Y \, \vec{I}_{aA}$$

$$\vec{V}_{AN} = (39.5 + j\,28.6)\,(2.4\,\angle -36.87^\circ)$$

$$\vec{V}_{AN} = 117.04\,\angle -0.96^\circ \quad V\,rms$$

$$\therefore \vec{V}_{AB} = \sqrt{3}\,\angle +30^\circ \;\; \vec{V}_{AN}$$

$$\therefore \vec{V}_{AB} = 202.72\,\angle 29.04^\circ \quad V\,rms$$

$$\therefore \vec{V}_{BC} = 202.72\,\angle -90.96^\circ \quad V\,rms$$

$$\therefore \vec{V}_{CA} = 202.72\,\angle 149.04^\circ \quad V\,rms$$