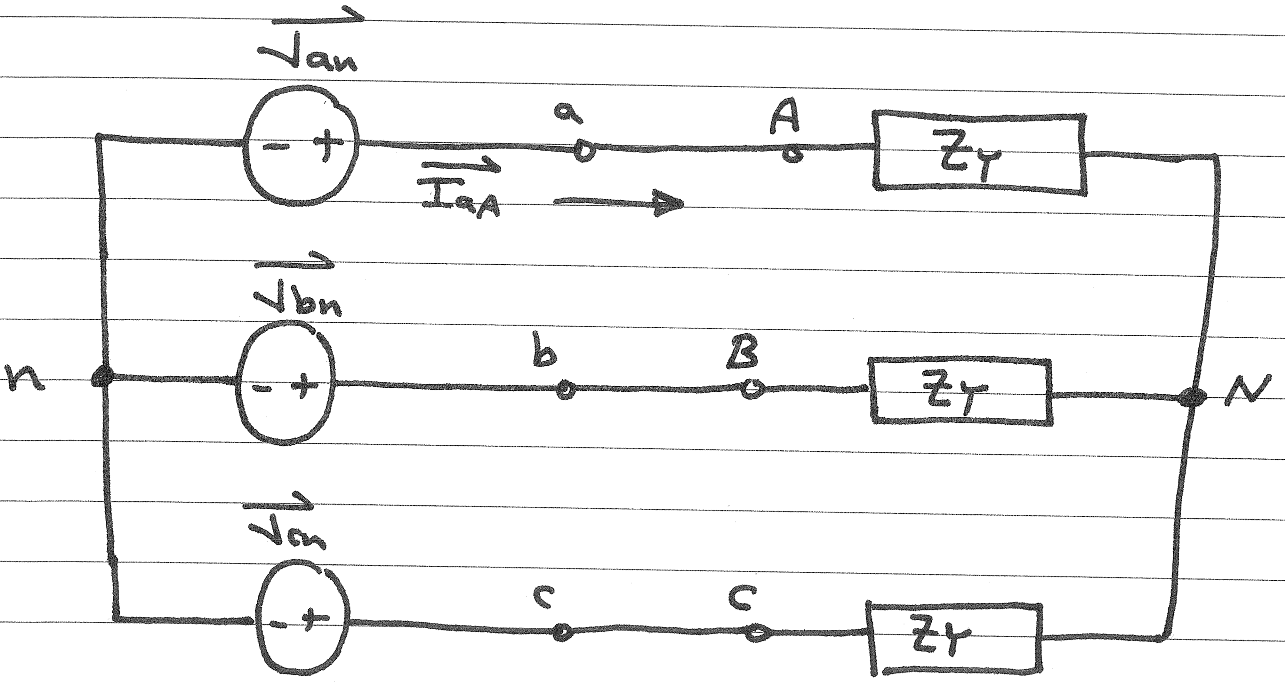
Power in a Balanced System



* The total instantaneous power in a balanced three phase system is Constant

$$v_{AN}(t) = \sqrt{2}\, V_p \cos \omega t$$

$$v_{BN}(t) = \sqrt{2}\, V_p \cos(\omega t - 120^\circ)$$

$$v_{CN}(t) = \sqrt{2}\, V_p \cos(\omega t + 120^\circ)$$

$$i_{aA}(t) = \sqrt{2}\, I_p \cos(\omega t - \theta)$$

$$i_{bB}(t) = \sqrt{2}\, I_p \cos(\omega t - \theta - 120^\circ)$$

$$i_{cC}(t) = \sqrt{2}\, I_p \cos(\omega t - \theta + 120^\circ)$$

$$P(t) = P_a(t) + P_b(t) + P_c(t)$$

$$P_a(t) = 2 V_p I_p \cos \omega t \cos(\omega t - \theta)$$

$$P_b(t) = 2 V_p I_p \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ)$$

$$P_c(t) = 2 V_p I_p \cos(\omega t + 120^\circ) \cos(\omega t - \theta + 120^\circ)$$

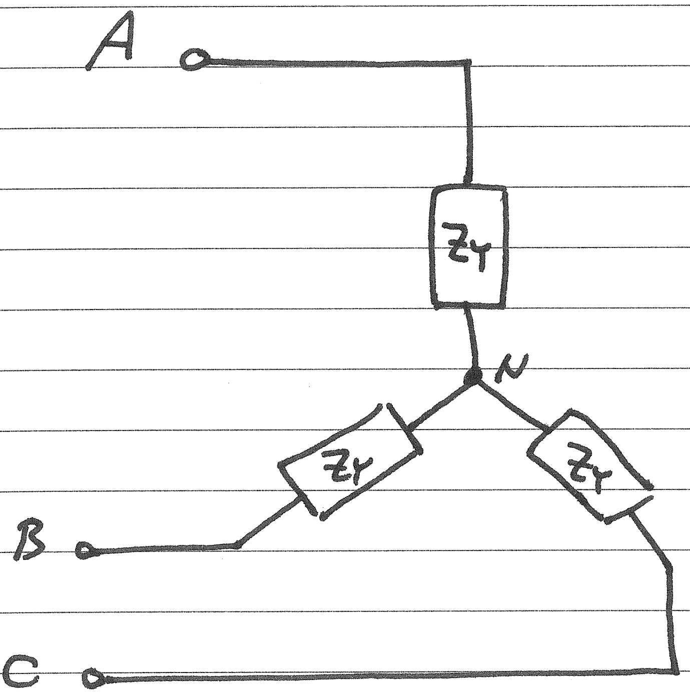
using $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$$P(t) = V_p I_p \left[3 \cos \theta\right]$$

$$P(t) = 3 V_p I_p \cos \theta$$

Power Calculation in a Balanced 3ϕ systems

1) Average power in Balanced Y-Load



$$P_A = V_{AN} I_{AN} \cos(\theta_{vA} - \phi_{iA})$$

$$P_B = V_{BN} I_{BN} \cos(\theta_{vB} - \phi_{iB})$$

$$P_C = V_{CN} I_{CN} \cos(\theta_{vC} - \phi_{iC})$$

$$V_{AN} = V_{BN} = V_{CN} = V\phi$$

$$I_{AN} = I_{BN} = I_{CN} = I\phi$$

$$\theta_{vA} - \phi_{iA} = \theta_{vB} - \phi_{iB} = \theta_{vC} - \phi_{iC}$$

$$\therefore P_A = P_C = P_C = V_\phi I_\phi \cos\theta$$

$$P_T = 3 V_\phi I_\phi \cos\theta$$

But $V_\phi = \dfrac{V_L}{\sqrt{3}}$

$$I_\phi = I_L$$

$$\therefore P_T = \sqrt{3}\, V_L I_L \cos\theta$$

2) Reactive Power in Balanced Y- load

$$Q_A = Q_B = Q_C = V_\phi I_\phi \sin\theta$$

$$Q_T = 3 V_\phi I_\phi \sin\theta$$

$$Q_T = \sqrt{3}\, V_L I_L \sin\theta$$

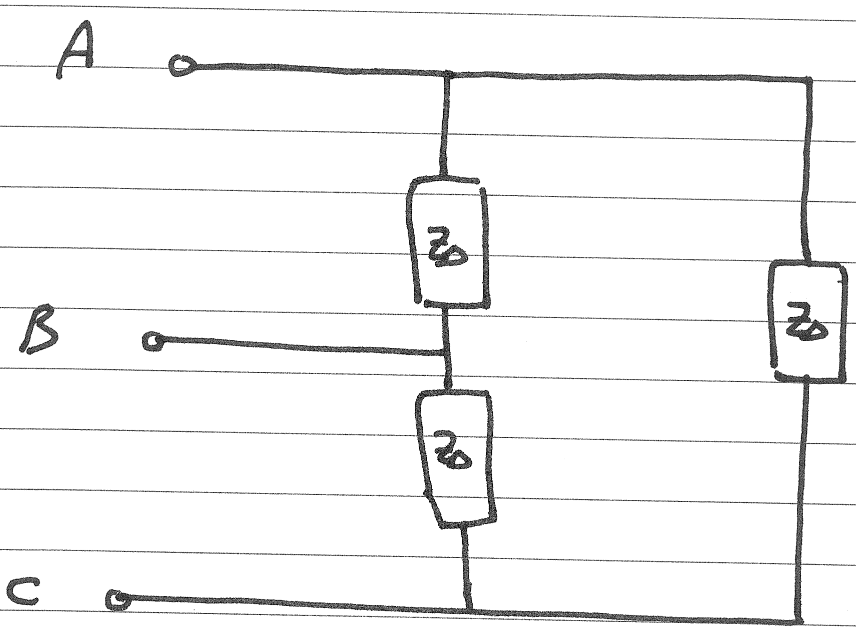
3) Complex Power in Balanced Y- load

$$\vec{S_A} = \vec{S_B} = \vec{S_C} = \vec{V_\phi} \cdot \vec{I_\phi^*} = P_\phi + j Q_\phi$$

$$\vec{S_T} = 3\vec{S_\phi} = 3\vec{V_\phi} \cdot \vec{I_\phi^*}$$

$$\vec{S_T} = \sqrt{3}\, V_L I_L \angle\theta$$

Power Calculation in Balanced Δ - load



$$P_A = V_{AB} I_{AB} \cos(\theta_{v_{AB}} - \phi_{i_{AB}})$$

$$P_B = V_{BC} I_{BC} \cos(\theta_{v_{BC}} - \phi_{i_{BC}})$$

$$P_C = V_{CA} I_{CA} \cos(\theta_{v_{CA}} - \phi_{i_{CA}})$$

$$V_{AB} = V_{BC} = V_{CA}$$

$$I_{AB} = I_{BC} = I_{CA}$$

$$\theta_{v_{AB}} - \phi_{i_{AB}} = \theta_{v_{BC}} - \phi_{i_{BC}} = \theta_{v_{CA}} - \phi_{i_{CA}}$$

$$\therefore \quad P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta$$

$$P_T = 3 P_\phi = 3 V_\phi I_\phi \cos\theta$$

$$V_\phi = V_L$$

$$I_\phi = \frac{I_L}{\sqrt{3}}$$

$$\therefore P_T = \sqrt{3}\, V_L I_L \cos\theta$$

$$Q_\phi = V_\phi I_\phi \sin\theta$$

$$Q_T = 3 Q_\phi = 3 V_\phi I_\phi \sin\theta$$

$$Q_T = \sqrt{3}\, V_L I_L \sin\theta$$

$$\vec{S_T} = 3 \vec{S_\phi}$$

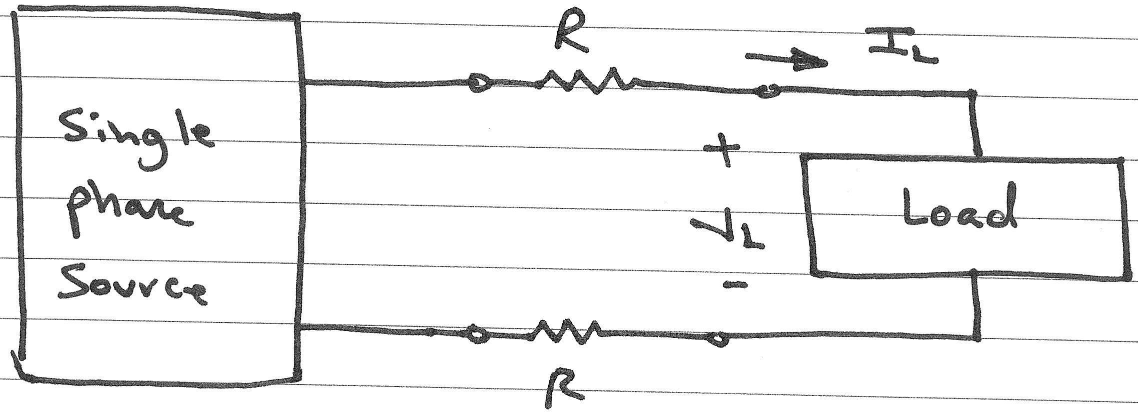
$$\vec{S_\phi} = \vec{V_\phi}\, \vec{I_\phi^{*}} = P_\phi + j Q_\phi$$

$$\vec{S_T} = P_T + j Q_T$$

$$\vec{S_T} = \sqrt{3}\, V_L I_L \angle\theta$$

Comparing the Power Loss

a) a single phase system

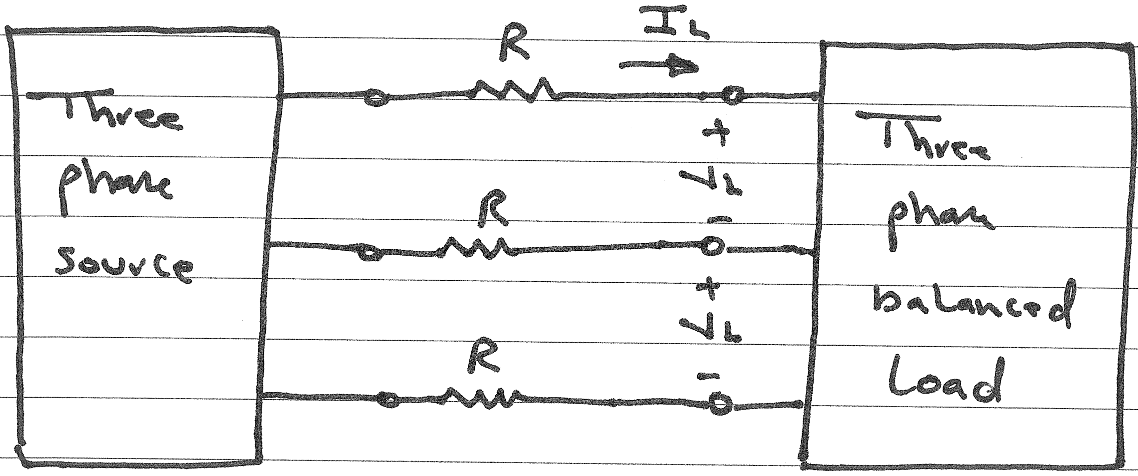


$$P_{Loss} = 2 I_L^2 \cdot R$$

$$I_L = \frac{P_L}{V_L \cdot pf}$$

$$P_{Loss} = 2 \frac{P_L^2}{V_L^2 \cdot pf^2} R$$

b) a three phase system



$$P_{Loss} = 3\, I_L^2 \cdot R$$

$$I_L = \frac{P_L}{\sqrt{3}\, V_L \cdot pf}$$

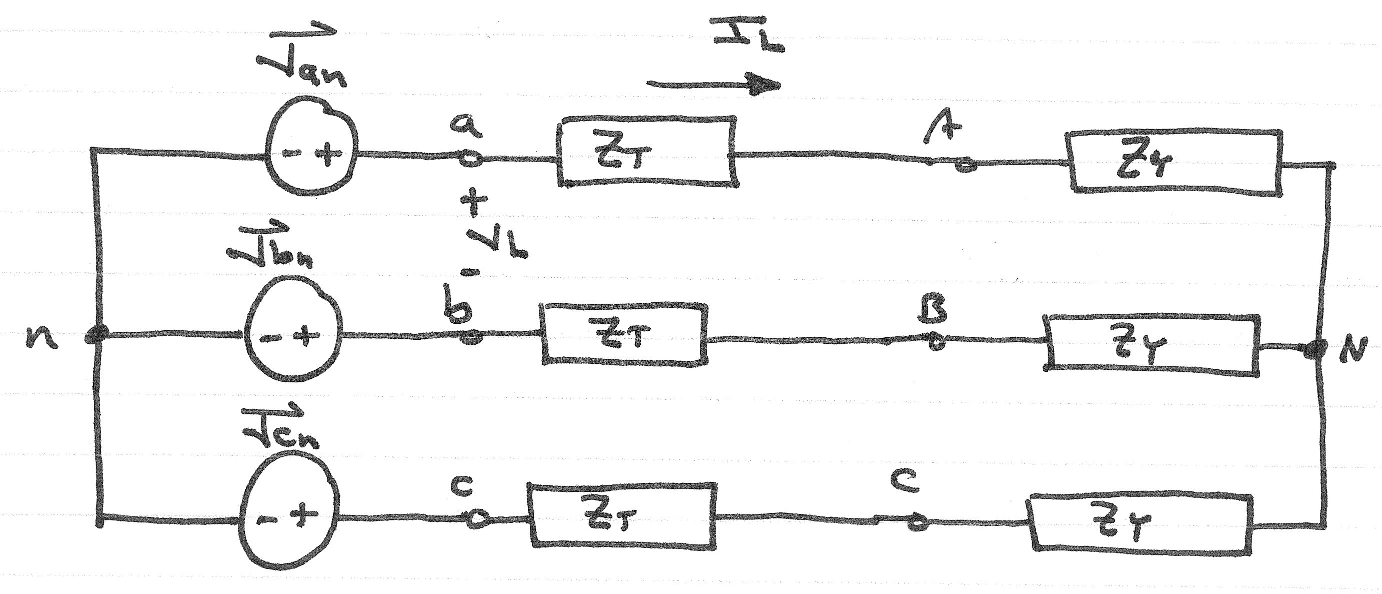
$$P_{Loss} = \frac{P_L^2}{V_L^2 \cdot pf^2} \cdot R$$

Example

A balanced 3ϕ load requires 480 KW at a lagging power factor of 0.8. The Load is fed from a Line having an impedance of $(0.005+j0.025)\ \Omega/\phi$

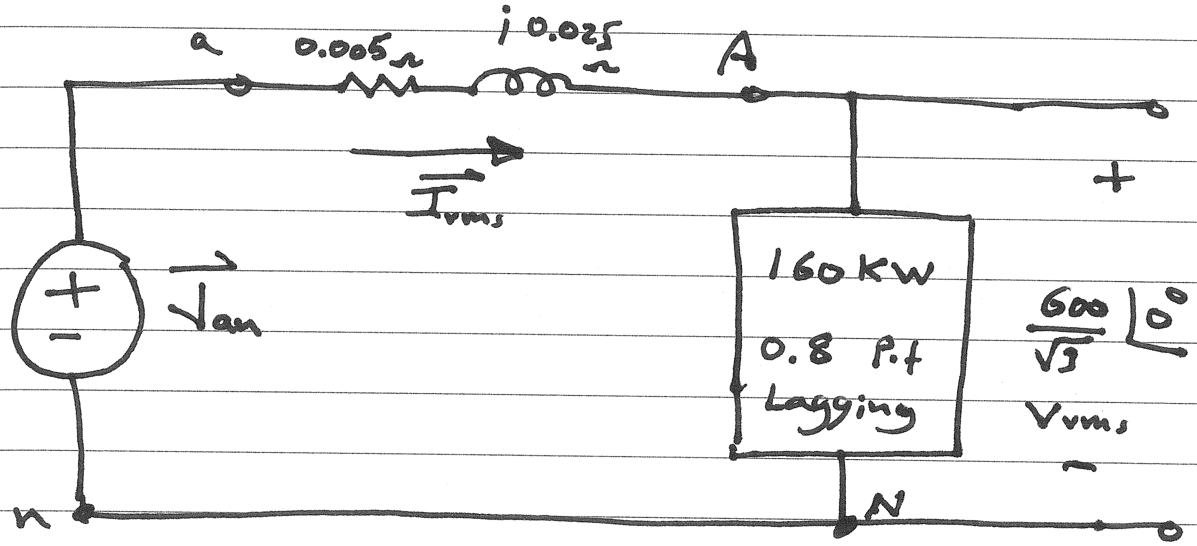
The Line voltage at the terminal of the load is 600 V rms

1) Calculate the magnitude of the line current
2) Calculate the magnitude of the Line voltage at the sending end of the Line
3) Calculate the power factor at the sending end of the Line.


$\vec{V}_{an}$
$\vec{V}_{bn}$
$\vec{V}_{cn}$
I_L
a
b
c
A
B
C
n
N
Z_T
Z_Y
+
V_L
-

Solution

Single phase representation



1) $P_{av} = 160$ KW

$Q = P_{av} \tan \cos^{-1} P.f = 120$ KVAR

$\vec{S} = P_{av} + jQ$

$\vec{S} = 160 + j\,120$ KVA

$\vec{S} = \overrightarrow{V_{rms}} \cdot \overrightarrow{I^*_{rms}}$

$\therefore\ \overrightarrow{I_{rms}} = 577.35 \angle -36.87°$ A_{rms}

$\therefore\ I_L = 577.35$ A_{rms}

2) $\overrightarrow{V_{an}} = (0.005 + j0.025)\,\overrightarrow{I_{rms}} + \frac{600}{\sqrt{3}} \angle 0°$

$\overrightarrow{V_{an}} = 357.51 \angle 1.57°$ V_{rms}

$\therefore \quad V_{an} = 357.51 \ V_{rms}$

$\therefore \quad V_L = \sqrt{3} \ V_{an}$

$\therefore \quad V_L = 619.23 \ V_{rms}$

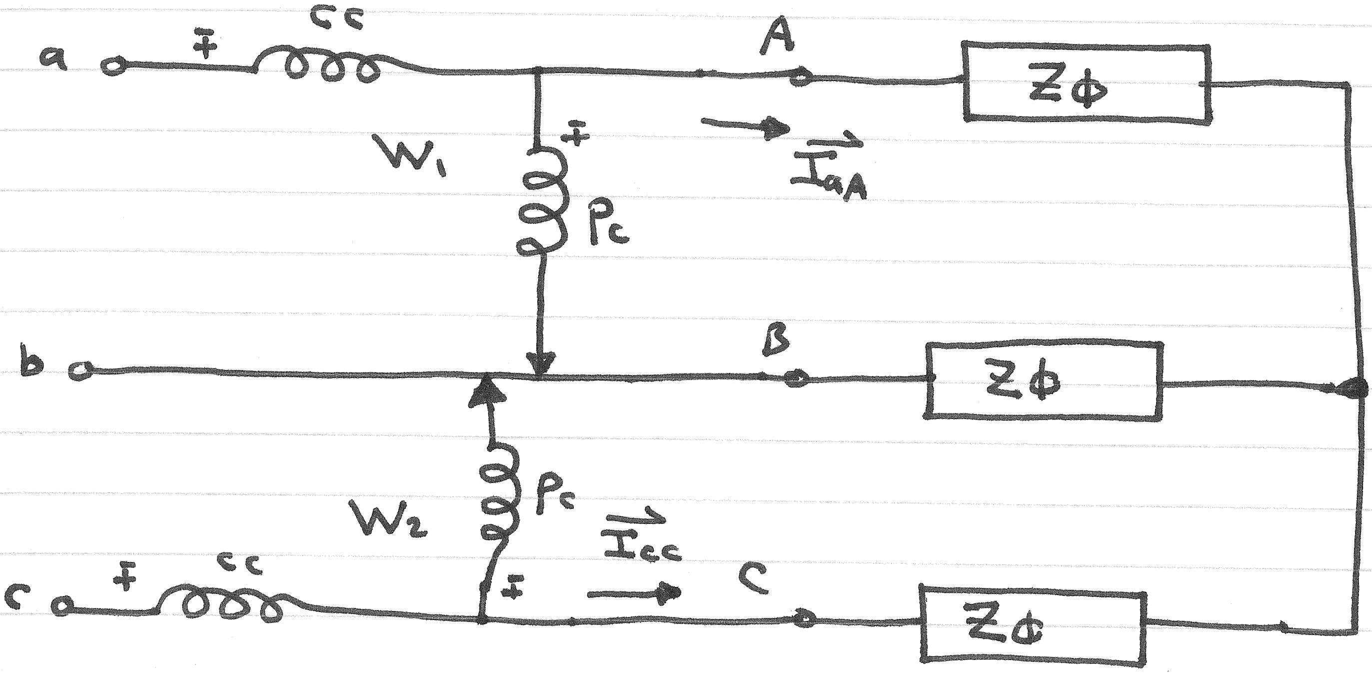
3) $Pf = \cos(\theta_v - \phi_i)$

$Pf = \cos(1.57^\circ + 36.87^\circ)$

$Pf = 0.783$ Lagging

Measuring Average power in 3ϕ system

The Two-Wattmeter method



$Z\phi = |Z| \angle \theta_z$; θ_z = impedance angle

$$W_1 = V_{AB}\, I_{aA} \cos \theta_1$$

θ_1 = The angle between $\vec{V}_{AB}$ and $\vec{I}_{aA}$

$$\theta_1 = \theta_z + 30^\circ$$

$$\vec{V}_{AB} = \vec{V}_{AN} \sqrt{3} \angle 30^\circ$$

$$\vec{I}_{aA} = \vec{I}_{AN}$$

$$\therefore \theta_1 = \theta_z + 30^\circ$$

$$\therefore W_1 = V_L\, I_L \cos(\theta_z + 30^\circ)$$

$W_2 = V_{CB} I_{cc} \cos\theta_2$

$\theta_2 \equiv$ The angle between $\vec{V}_{CB}$ and $\vec{I}_{cc}$

$\theta_2 = \theta_z - 30°$

$\vec{V}_{CB} = -\vec{V}_{BC}$

$\vec{V}_{CB} = \vec{V}_{BC} \angle 180°$

$\vec{V}_{CB} = \vec{V}_{CA} \angle -240° \angle 180°$

$\vec{V}_{CB} = \vec{V}_{CA} \angle -60°$

$\vec{V}_{CB} = \sqrt{3}\,\vec{V}_{CN} \angle +30° \angle -60°$

$\vec{V}_{CB} = \sqrt{3}\,\vec{V}_{CN} \angle -30°$

$\vec{I}_{cc} = \vec{I}_{CN}$

$\therefore \theta_2 = \theta_z - 30°$

$\therefore W_2 = V_L I_L \cos(\theta_z - 30°)$

$$W_1 = V_L I_L \cos(\theta_z + 30^\circ)$$

$$W_2 = V_L I_L \cos(\theta_z - 30^\circ)$$

$$P_T = W_1 + W_2$$

$$\cos(\theta_z + 30^\circ) = \cos\theta_z \cos 30^\circ - \sin\theta_z \sin 30^\circ$$

$$\cos(\theta_z - 30^\circ) = \cos\theta_z \cos 30^\circ + \sin\theta_z \sin 30^\circ$$

$$\therefore \quad W_1 + W_2 = V_L I_L \left(2 \cos\theta_z \cos 30^\circ\right)$$

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\theta_z$$

Example

Calculate the reading of each Wattmeter if the phase voltage at the load is $120\angle 0^\circ$ Vrms

a) and $Z_\phi = (8+j6)\ \Omega$

$$\vec{I}_{aA} = \frac{\vec{V}_{AN}}{Z_\phi} = 12\angle -36.87^\circ \ A_{rms}$$

$$Z_\phi = 8+j6 = 10\angle 36.87^\circ \ \Omega$$

$$V_L = \sqrt{3}\cdot(120) \ V_{rms}$$

$$I_L = 12 \ A_{rms}$$

$$\theta_z = 36.87^\circ$$

$$W_1 = V_L I_L \cos(\theta_z + 30^\circ)$$

$$= (120\sqrt{3})(12)\cos(66.87^\circ)$$

$$W_1 = 979.75 \ \text{Watt}$$

$$W_2 = V_L I_L \cos(\theta_z - 30^\circ)$$

$$W_2 = (120\sqrt{3})(12)\cos(6.87^\circ)$$

$$W_2 = 2476.25 \ \text{Watt}$$

b) $Z\phi = 8 - j6 = 10 \angle -36.87^\circ \ \Omega$

$\therefore \theta_z = -36.87^\circ$

$W_1 = (120\sqrt{3})(12) \cos(-36.87^\circ + 30^\circ)$

$W_1 = 2476.25$ Watt

$W_2 = (120\sqrt{3})(12) \cos(-36.87^\circ - 30^\circ)$

$W_2 = 979.75$ Watt

c) $Z\phi = 5 + j5\sqrt{3} = 10 \angle 60^\circ$

$\therefore \theta_z = 60^\circ$

$W_1 = (120\sqrt{3})(12) \cos(60 + 30)$

$W = 0$

$W_2 = (120\sqrt{3})(12) \cos(60 - 30^\circ)$

$W_2 = 2160$ Watt