Nodal and Mesh Analysis

- As Circuits get more Complicated, we need an organized method of applying KVL, KCL, and Ohm's

- Nodal analysis assigns voltages to each node then we apply KCL

- Mesh analysis assigns Currents to each mesh, and then we apply KVL

Nodal Analysis

Steps to Determine **Node Voltages**:

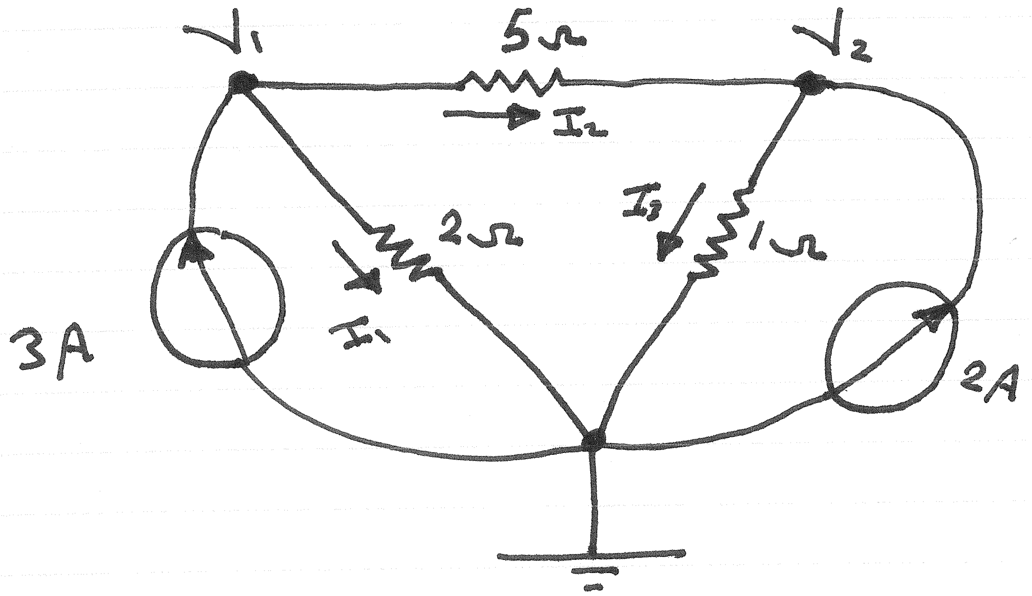
1. Select a node as the **reference node**. Assign voltage $\boldsymbol{V1, V2, \ldots V_{n\text{-}1}}$ to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.

2. Apply **KCL** to each of the $\boldsymbol{n}$**-1 nonreference nodes**. Use Ohm's law to express the branch currents in terms of node voltages.

3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

The Nodal Analysis Method

assign voltages to every node relative to a reference node.



Apply KCL to node ①

$$3 = I_1 + I_2$$

$$3 = \frac{V_1}{2} + \frac{V_1 - V_2}{5}$$

$$3 = 0.7V_1 - 0.2V_2 \quad \text{——} ①$$

Apply KCL to node ②

$$2 + I_2 = I_3$$

$$2 = I_3 - I_2$$

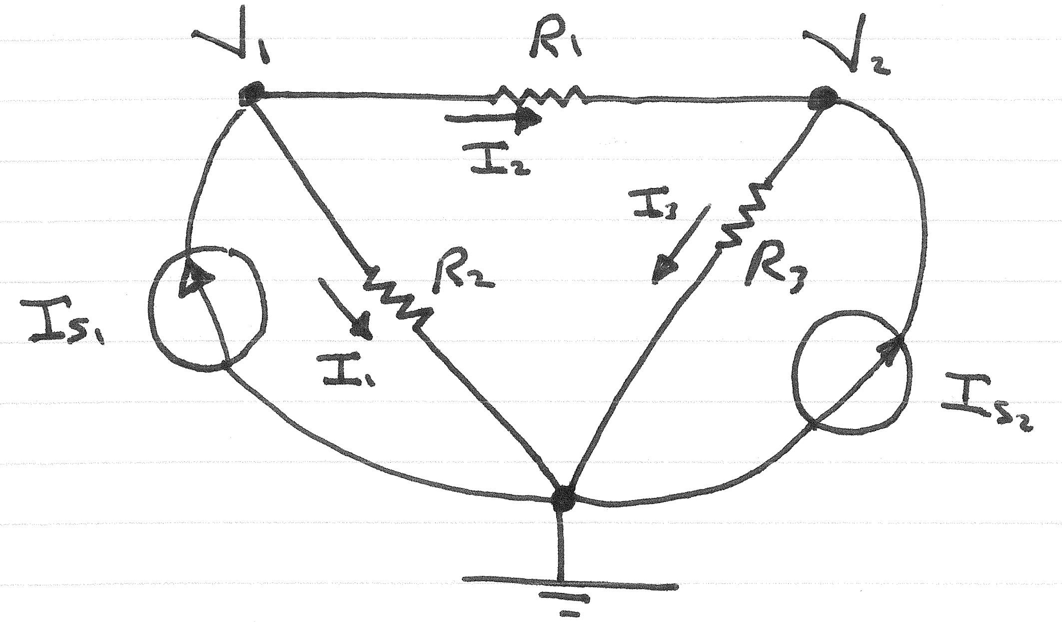
$$2 = \frac{V_2}{1} - \frac{V_1 - V_2}{5}$$

$$2 = -0.2V_1 + 1.2V_2 \quad \text{———} \quad (2)$$

Solving equations (1) and (2), we get

$$V_1 = 5V$$

$$V_2 = 2.5V$$



Applying KCL to node ①

$$I_{s1} = I_1 + I_2$$

$$I_{s1} = \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_1}$$

$$I_{s1} = \left(\frac{1}{R_2} + \frac{1}{R_1}\right) V_1 - \frac{1}{R_1} V_2$$

$$I_{s1} = (G_2 + G_1) V_1 - G_1 V_2$$

Self Conductance = $G_2 + G_1$

mutual Conductance = $- G_1$

Applying KCL to node ②

$$I_{s2} + I_2 = I_3$$

$$I_{s2} = I_3 - I_2$$

$$I_{s2} = \frac{V_2}{R_3} - \frac{V_1 - V_2}{R_1}$$

$$I_{s2} = -\frac{1}{R_1} V_1 + \left(\frac{1}{R_3} + \frac{1}{R_1}\right) V_2$$

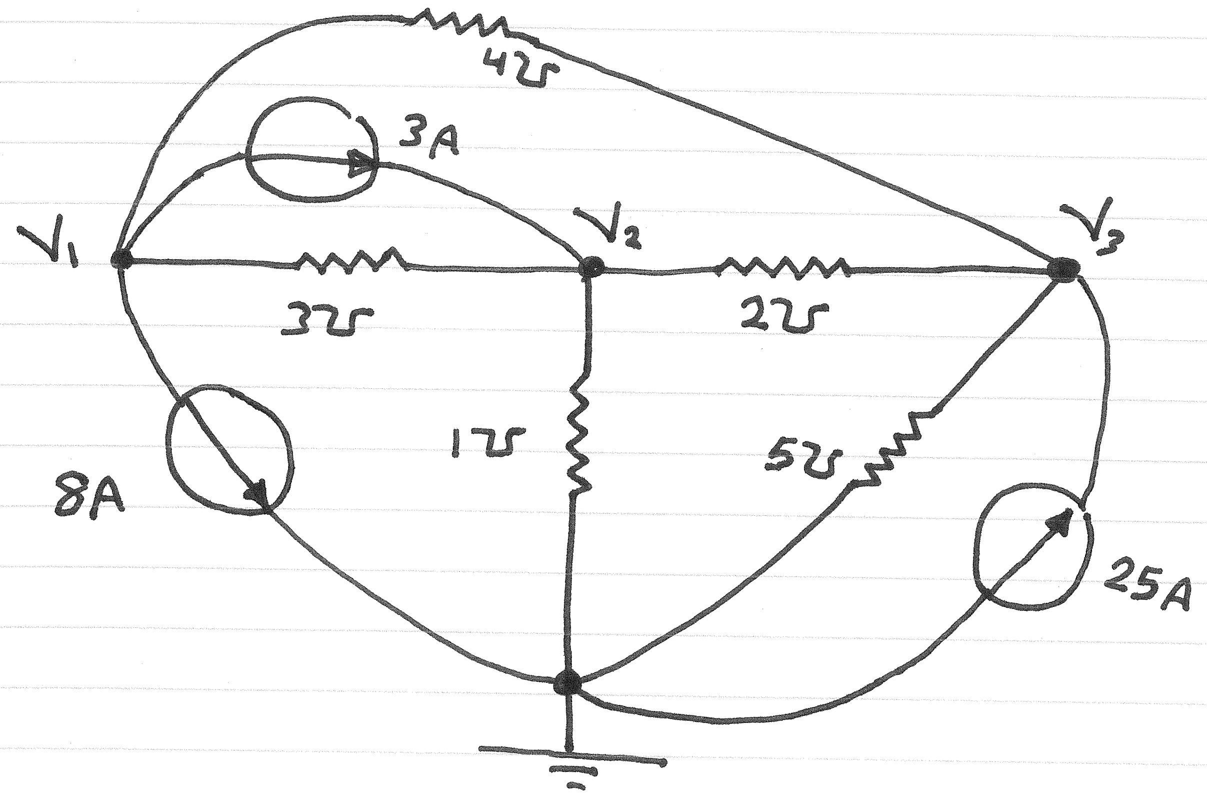
$$I_{s2} = -G_1 V_1 + (G_3 + G_1) V_2$$

Self Conductance of node ② $= (G_3 + G_1)$

mutual Conductance between nodes ① and ②

$= -G_1$

Writing Nodal equations by inspection



KCL at node ①:

$$7V_1 - 3V_2 - 4V_3 = -11$$

KCL at node ②:

$$-3V_1 + 6V_2 - 2V_3 = 3$$

KCL at node ③:

$$-4V_1 - 2V_2 + 11V_3 = 25$$

Solving:

$V_1 = 1\,\underline{V}$; $V_2 = 2\,\underline{V}$; $V_3 = 3\,\underline{V}$

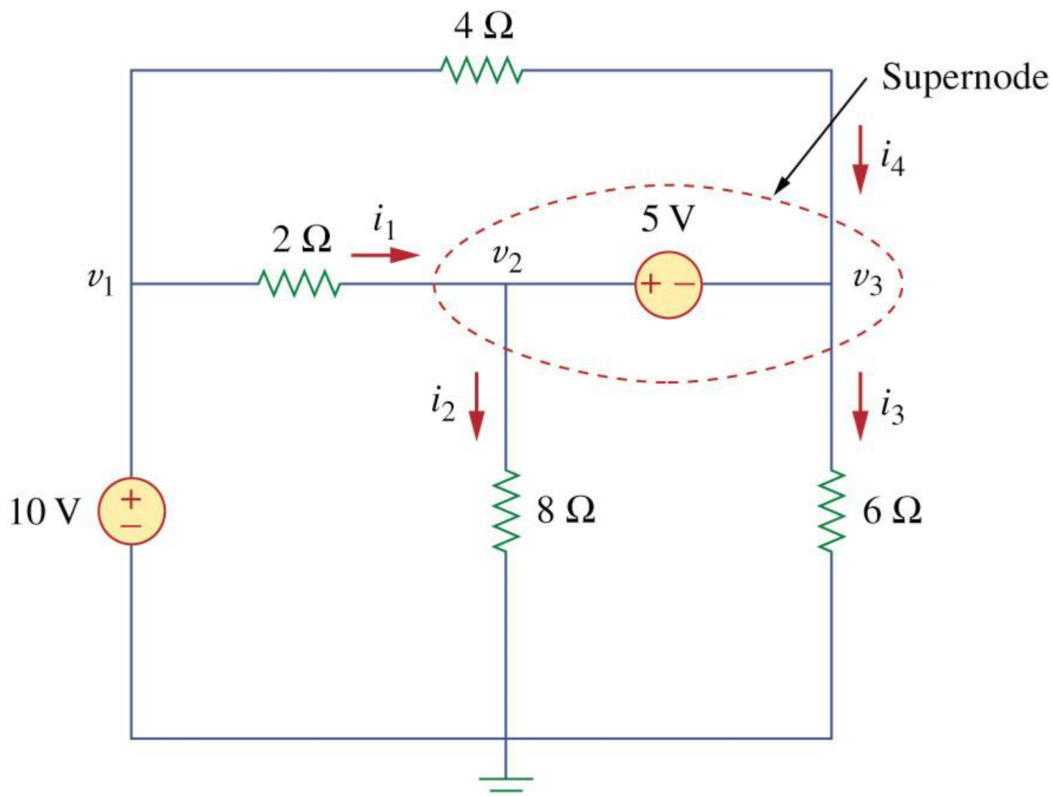
Nodal Analysis with Voltage Sources

Case 1: The voltage source is connected between a non reference node and the reference node: The non reference node voltage is equal to the magnitude of voltage source and the number of unknown non reference nodes is reduced by one.

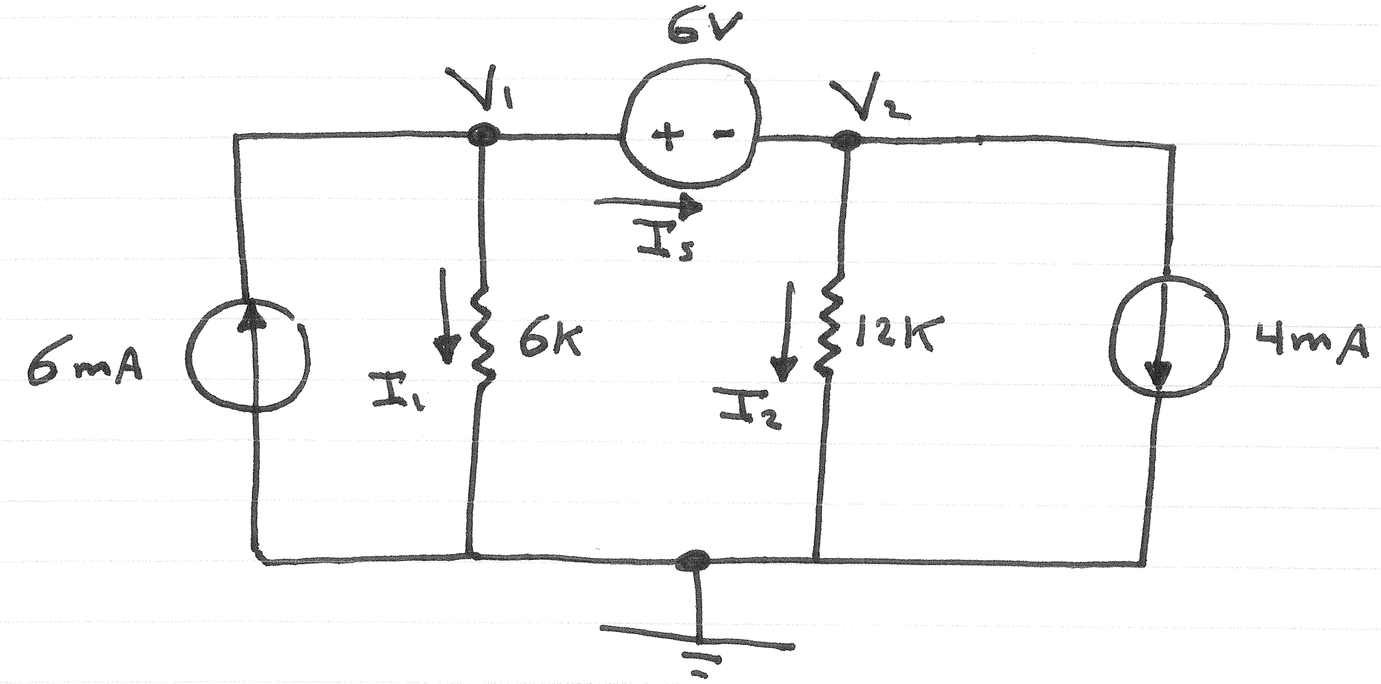
Case 2: The voltage source is connected between two non referenced nodes: a generalized node (**supernode**) is formed.

Nodal Analysis with Voltage Sources

A circuit with a **supernode**.



Voltage Sources and the supernode



Constrain equation :

$$V_1 - V_2 = 6 \qquad \text{——— (1)}$$

KCL at node (1) :

$$6mA = I_1 + I_s$$

$$6mA = \frac{V_1}{6K} + I_s \qquad \text{——— (2)}$$

KCL at node (2) :

$$I_s = I_2 + 4mA$$

$$4mA = I_s - I_2$$

$$4\,mA = I_S - \frac{V_2}{12K} \quad \text{———} \quad (3)$$

Subtracting (3) from (2)

$$2\,mA = \frac{V_1}{6K} + \frac{V_2}{12K} \quad \text{———} \quad (4)$$

This is the supernode equation

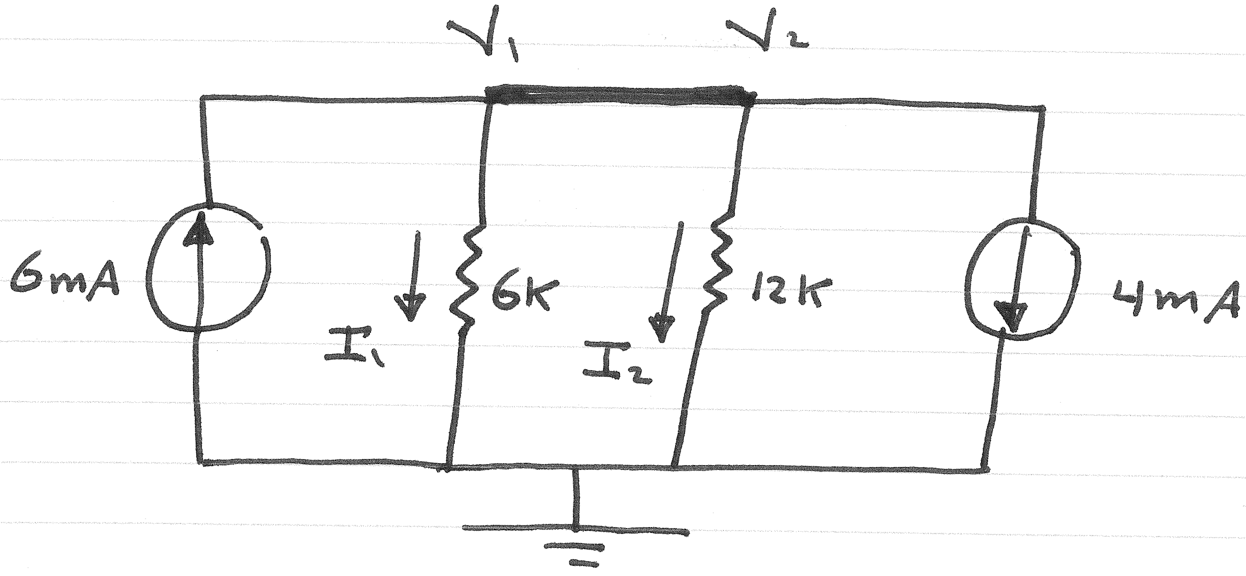
Solving (1) and (4)

we get

$$V_1 = 10\,\underline{V}$$

$$V_2 = 4\,\underline{V}$$

Supernode equation by inspection

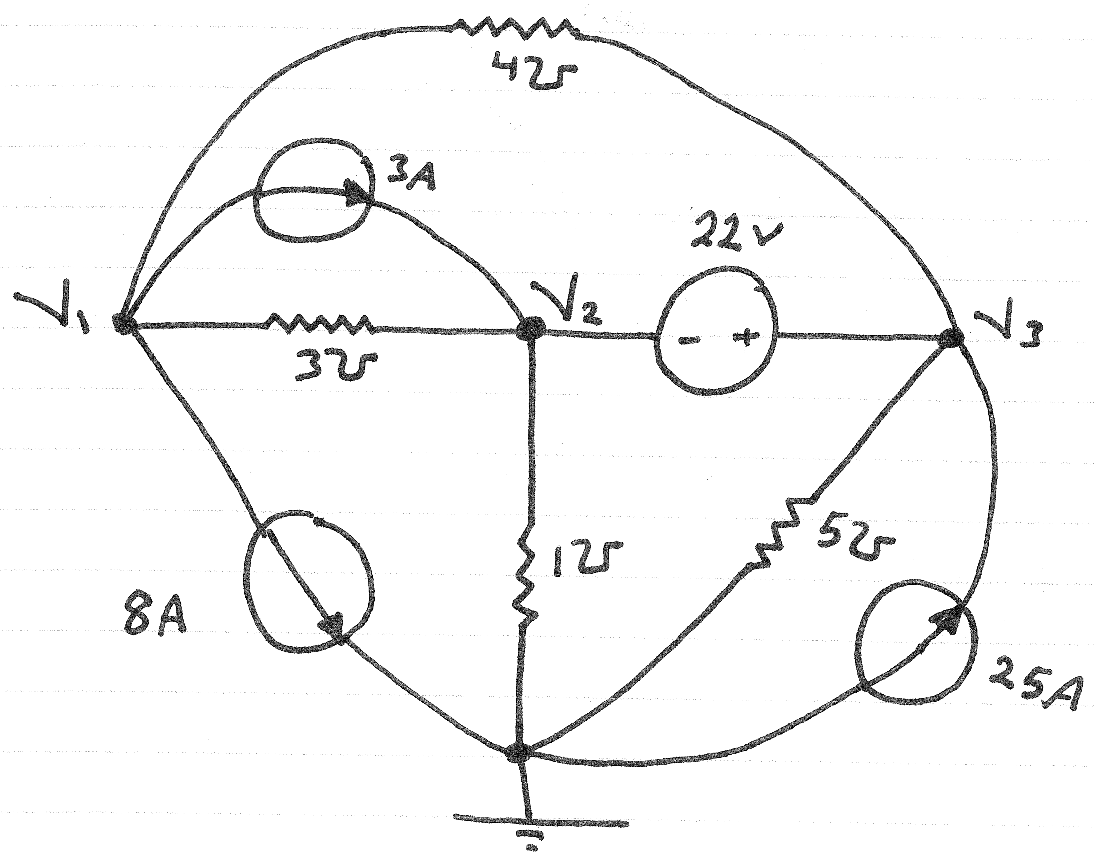


$$6mA = I_1 + I_2 + 4mA$$

$$2mA = I_1 + I_2$$

$$2mA = \frac{V_1}{6K} + \frac{V_2}{12K}$$

Voltage Sources and the Supernode



Constrain equation :

$$V_3 - V_2 = 22$$

KCL at node ① :

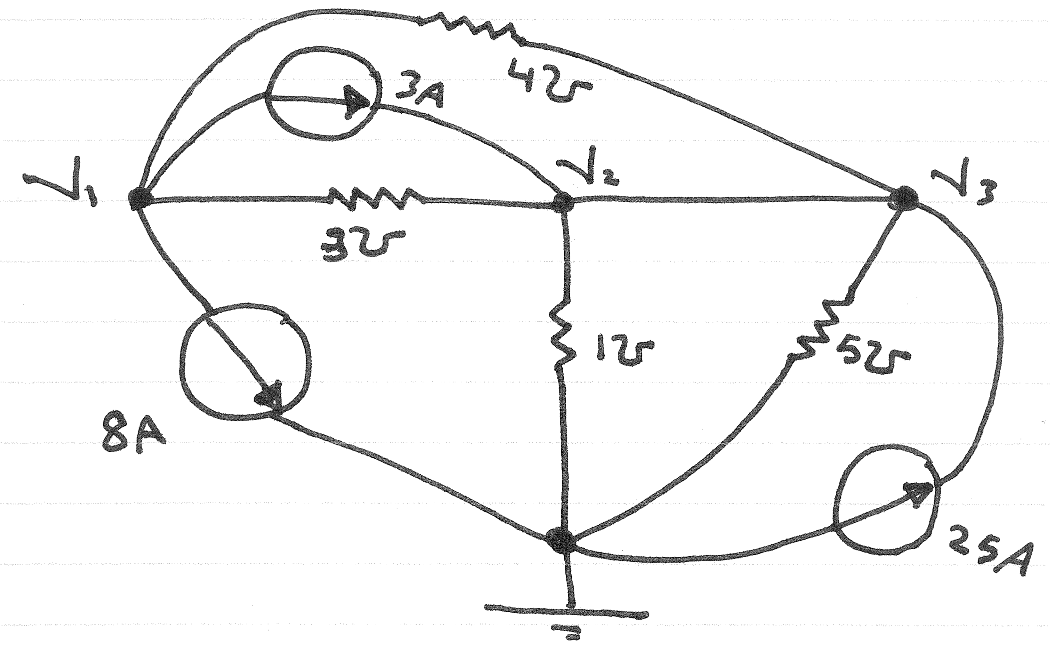
$$7V_1 - 3V_2 - 4V_3 = -11$$

Supernode equation :

$$-7V_1 + 4V_2 + 9V_3 = 28$$

Solving for V_1 $\Rightarrow$ $V_1 = -4.5\,\underline{V}$

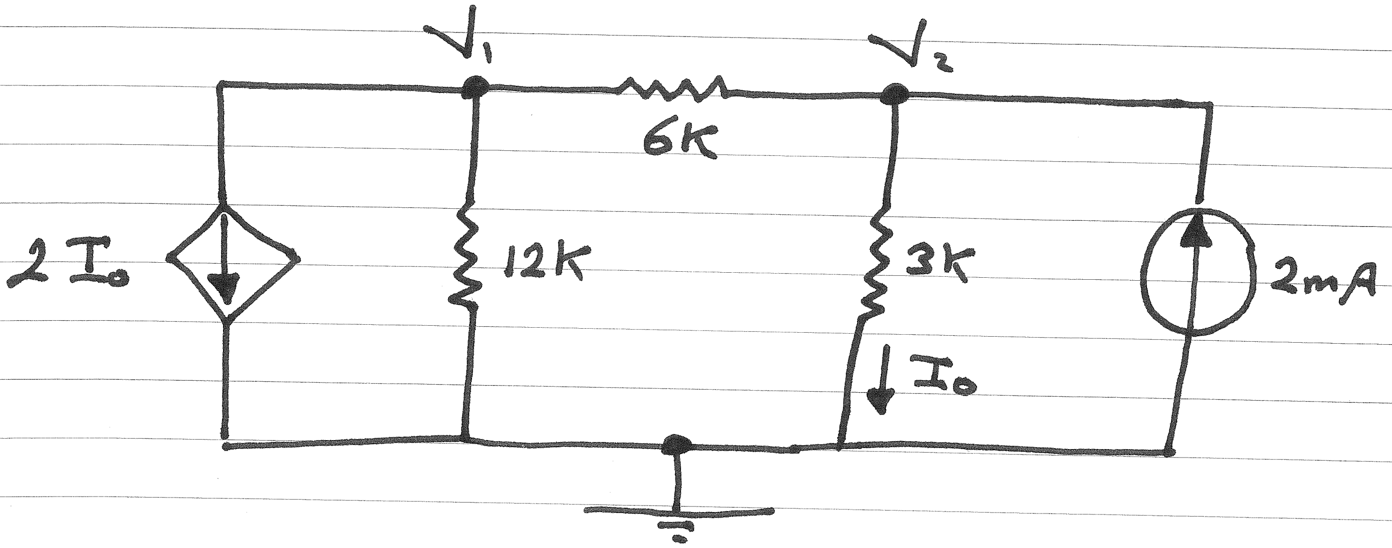
Supernode equation by inspection



$$25 + 3 = (1+3)V_2 + (5+4)V_3 - (3+4)V_1$$

$$28 = 4V_2 + 9V_3 - 7V_1$$

Circuits with dependent sources



KCL at node 1 :

$$-2I_o = \left(\frac{1}{12K} + \frac{1}{6K}\right)V_1 - \frac{1}{6K}V_2$$

$$I_o = \frac{V_2}{3K}$$

$$0 = \left(\frac{1}{12K} + \frac{1}{6K}\right)V_1 + \left(\frac{2}{3K} - \frac{1}{6K}\right)V_2$$

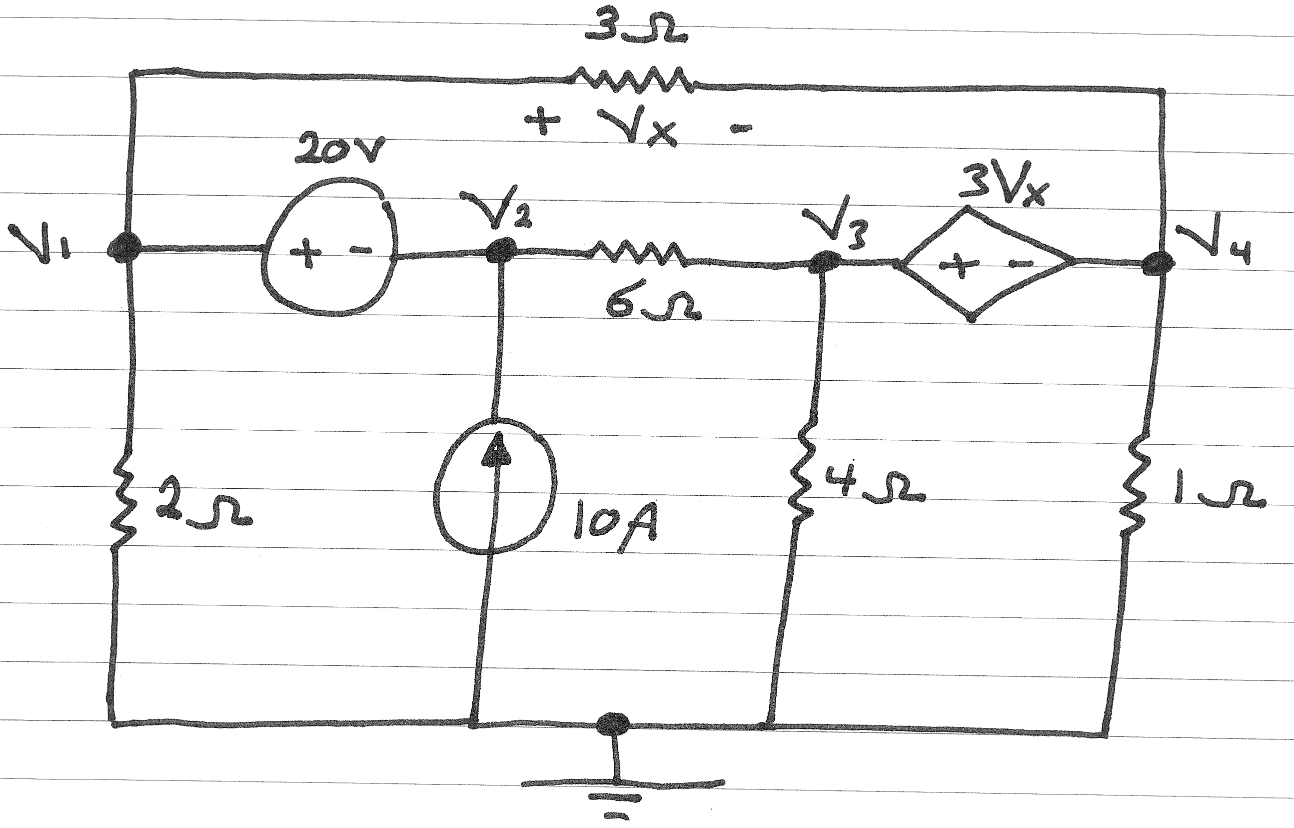
KCL at node 2 :

$$2mA = -\frac{1}{6K}V_1 + \left(\frac{1}{3K} + \frac{1}{6K}\right)V_2$$

Solving for V_1 and V_2, we get

$$V_1 = -\frac{24}{5}\,\underline{V} \quad ; \quad V_2 = \frac{12}{5}\,\underline{V}$$

Nodal Analysis : Supernode



KCL at Supernode 1,2 :

$$10 = \left(\frac{1}{2}+\frac{1}{3}\right)V_1 + \frac{1}{6}V_2 - \frac{1}{6}V_3 - \frac{1}{3}V_4$$

KCL at Supernode 3,4 :

$$0 = \left(\frac{1}{4}+\frac{1}{6}\right)V_3 + \left(\frac{1}{1}+\frac{1}{3}\right)V_4 - \frac{1}{6}V_2 - \frac{1}{3}V_1$$

Constrain equation :

$$V_1 - V_2 = 20$$

Constrain equation :

$V_3 - V_4 = 3V_x$

$V_x = V_1 - V_4$

$0 = 3V_1 - V_3 - 2V_4$

What is the current through the independent voltage source ?