

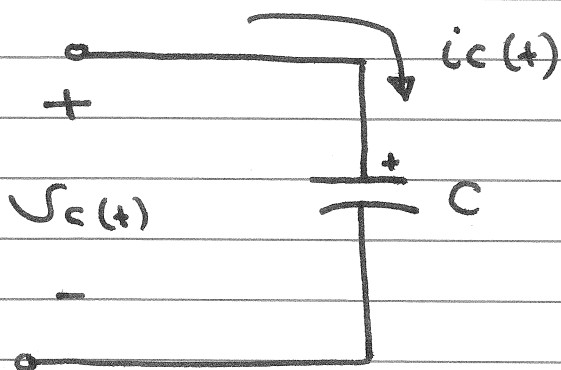
Chapter 7

'Response of First-Order RL and RC Circuits''

Capacitors and Inductors

- Resistors are passive elements which dissipate energy only.
- Two important passive Linear Circuit elements : Capacitor, and Inductor
- Capacitors and Inductors do not dissipate but store energy, which can be retrieved at a later time
- Capacitors and Inductors are called Storage elements.
- $W_C(t) = \frac{1}{2} C v_C^2(t)$
- $W_L(t) = \frac{1}{2} L i_L^2(t)$

Some of the important characteristics of a Capacitor



$$i_C(t) = C \frac{dV_C(t)}{dt}$$

$$V_C(t) = V_C(0) + \frac{1}{C} \int_0^t i_C(t) dt, \text{ for } t \geq 0$$

1. The current through a capacitor is zero if the voltage across it is not changing with time.

A capacitor is therefore an open circuit to dc.

2. A finite amount of energy can be stored in a capacitor even if the current through the capacitor is zero.

3. The capacitor never dissipate energy, but only store it.

$$V_c(t) = V_c(\bar{0}) + \frac{1}{C} \int_{0^-}^t i_c(t) dt, \text{ for } t \geq 0$$

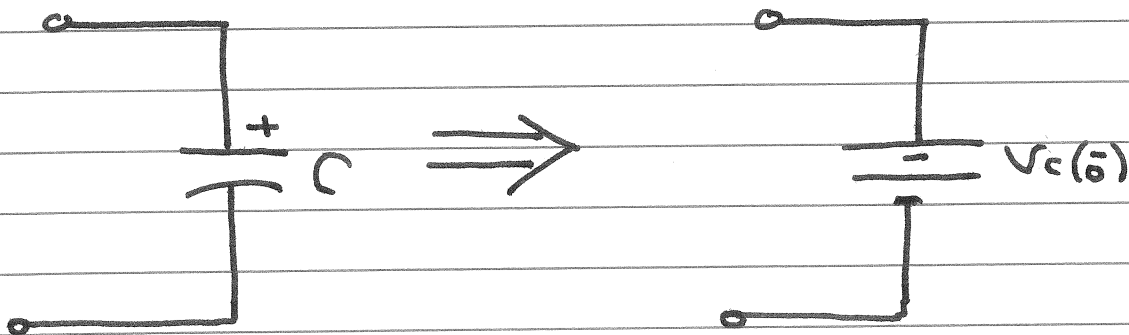
at $t = 0^+$

$$V_c(0^+) = V_c(\bar{0}) + \frac{1}{C} \int_{0^-}^{0^+} i_c(t) dt$$

$$V_c(0^+) = V_c(\bar{0})$$

4. It is impossible to change the voltage across a capacitor by a finite amount in zero time, for this requires an infinite current through the capacitor

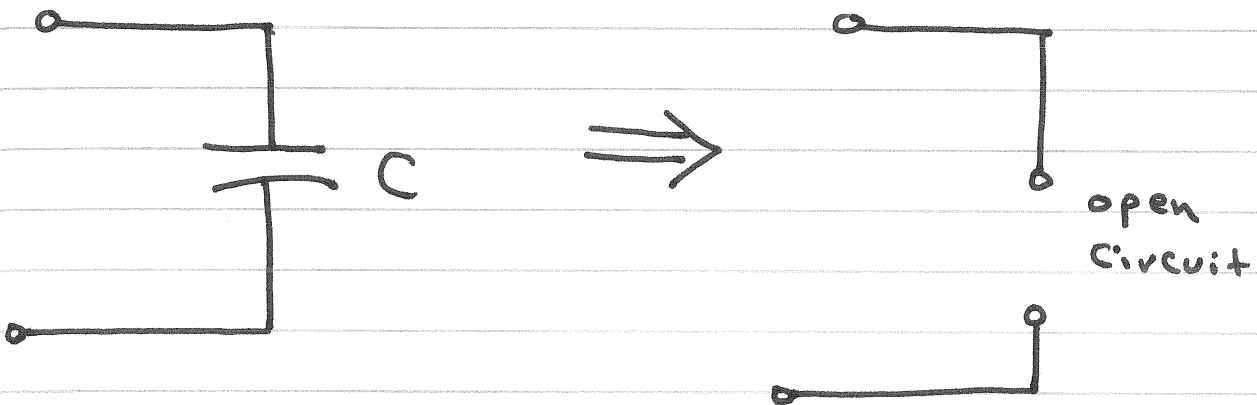
5. At $t = 0^+$



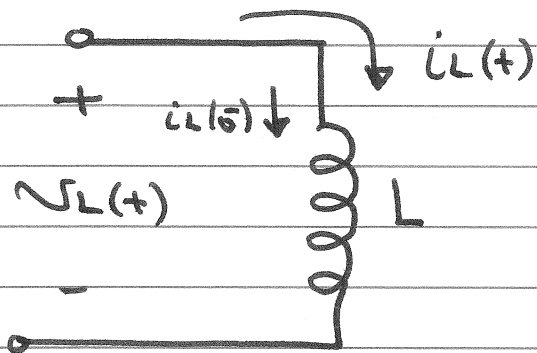
$$i_C(t) = C \frac{dv_C(t)}{dt}$$

A capacitor is therefore an open circuit to dc

At $t = 0^-$, and $t = \infty$ (After the change)



Some of the important characteristics of an inductor



$$v_L(t) = L \frac{di_L(t)}{dt}$$

$$i_L(t) = i_L(0^-) + \frac{1}{L} \int_{0^-}^t v_L(t) dt ; \text{ for } t \geq 0$$

1. There is no voltage across an inductor if the current through it is not changing with time.

An inductor is therefore a short circuit to dc

2. A finite amount of energy can be stored in an inductor even if the voltage across the inductor is zero.

3. The inductor never dissipate energy, but only

Store it

$$4. \quad i_L(t) = i_L(0^-) + \frac{1}{L} \int_{0^-}^t v_L(t) dt \quad ; \text{ for } t \geq 0$$

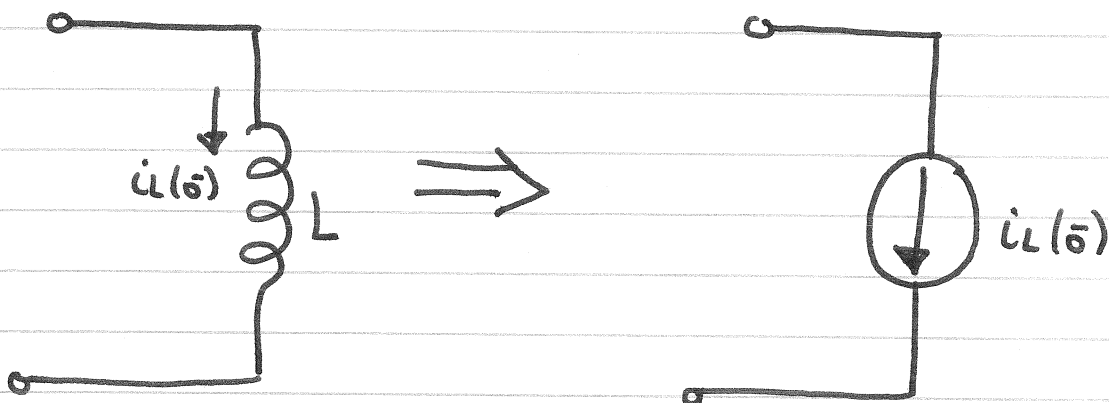
at $t = 0^+$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} v_L(t) dt$$

$$\therefore i_L(0^+) = i_L(0^-)$$

It is impossible to change the current through an inductor by a finite amount in zero time, for this requires an infinite voltage across the inductor.

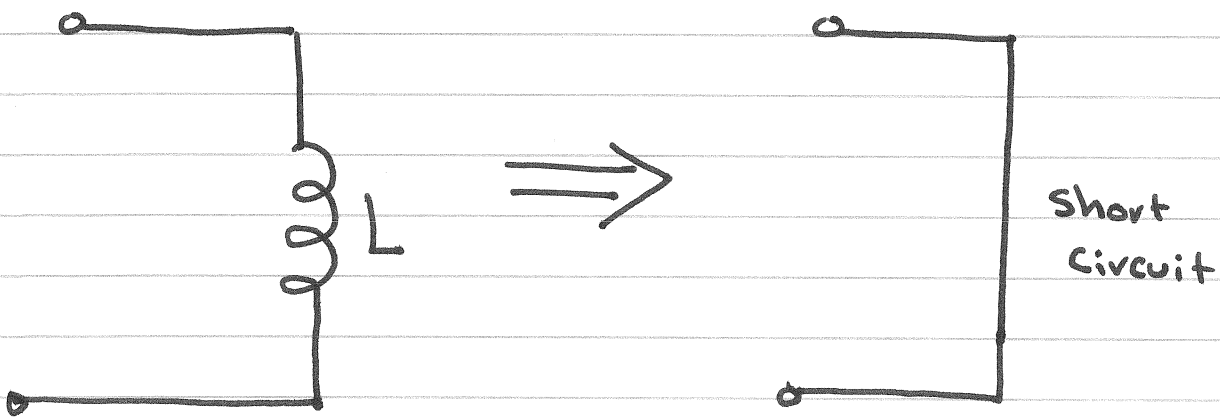
At $t = 0^+$



$$V_L(t) = L \frac{di_L(t)}{dt}$$

An inductor is therefore a short circuit
to dc

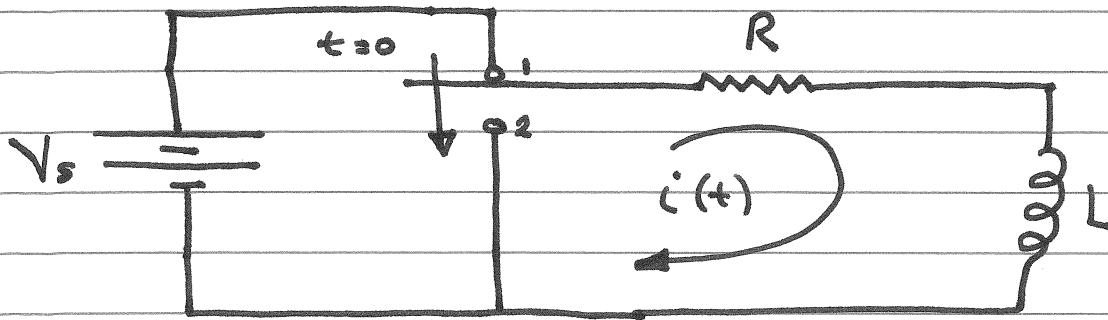
At $t = 0^-$, and $t = \infty$ (After the change)



First order Circuit

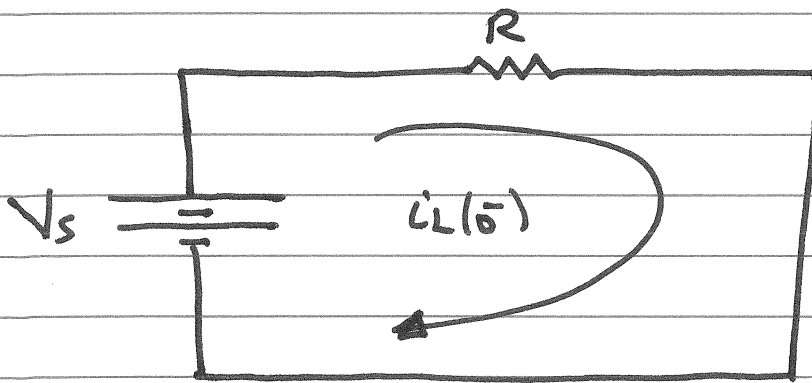
- A first-order circuit can only contain one energy storage element (a capacitor or an inductor) or a combination of capacitors or inductors that can be reduced to one capacitor or inductor
- The circuit will also contain one or more resistances
- A first-order circuit is characterized by a first-order differential equation

Natural Response of First-Order Circuit



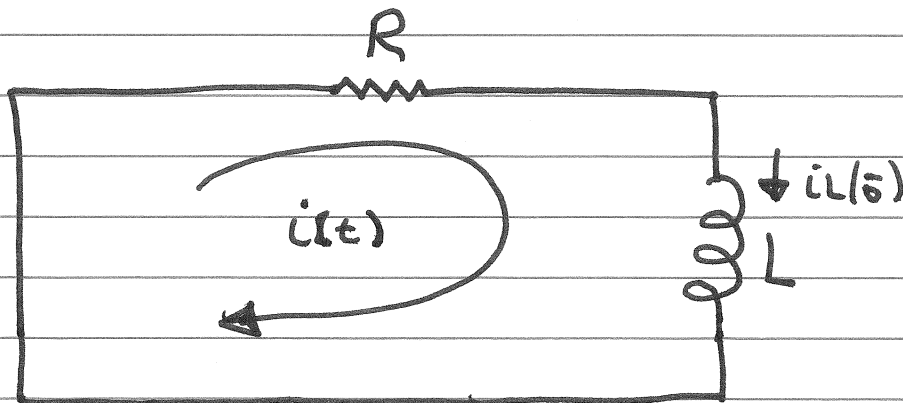
Find $i(t)$ for $t > 0$

1) For $t < 0$; $t = 0^-$

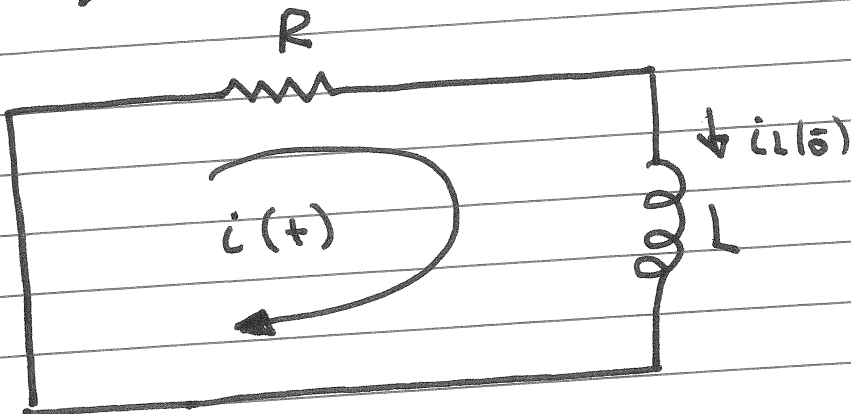


$$i_L(0^-) = \frac{V_s}{R}$$

2) For $t > 0$



For $t > 0$



KVL :

$$Ri(t) + L \frac{di(t)}{dt} = 0$$

homogeneous first order differential equation

$$i(t) = A e^{st} \quad \text{for } t > 0$$

$$RA e^{st} + LA s e^{st} = 0$$

$$A e^{st} (R + Ls) = 0$$

$$\therefore s = -\frac{R}{L}$$

To find A :

$$i(t) = A e^{st} \quad t > 0$$

$$i(0^+) = A$$

$$i(0^+) = i_L(0^+) = i_L(0^-)$$

$$\therefore A = i_L(0^-) = \frac{V_s}{R}$$

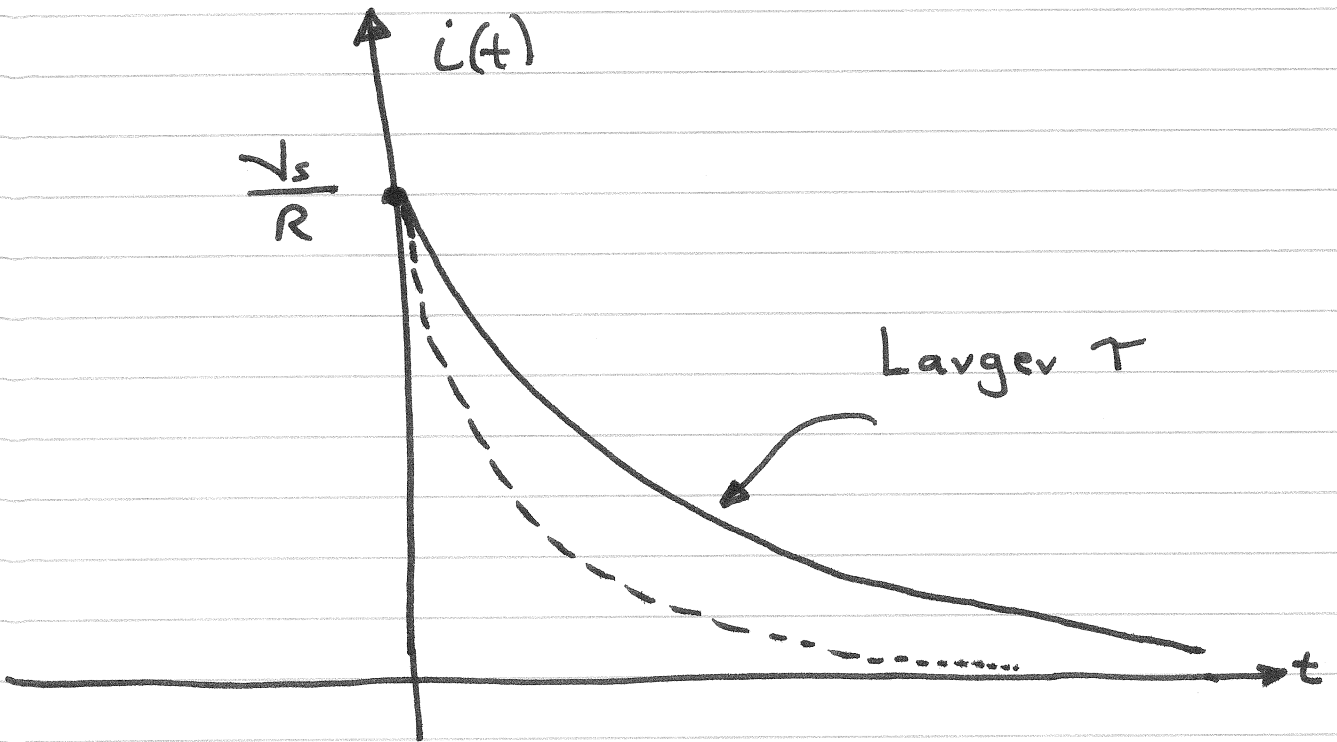
$$\therefore i(t) = \frac{V_s}{R} e^{-\frac{R}{L}t} \quad t > 0$$

$$\text{let } \tau = \frac{L}{R}$$

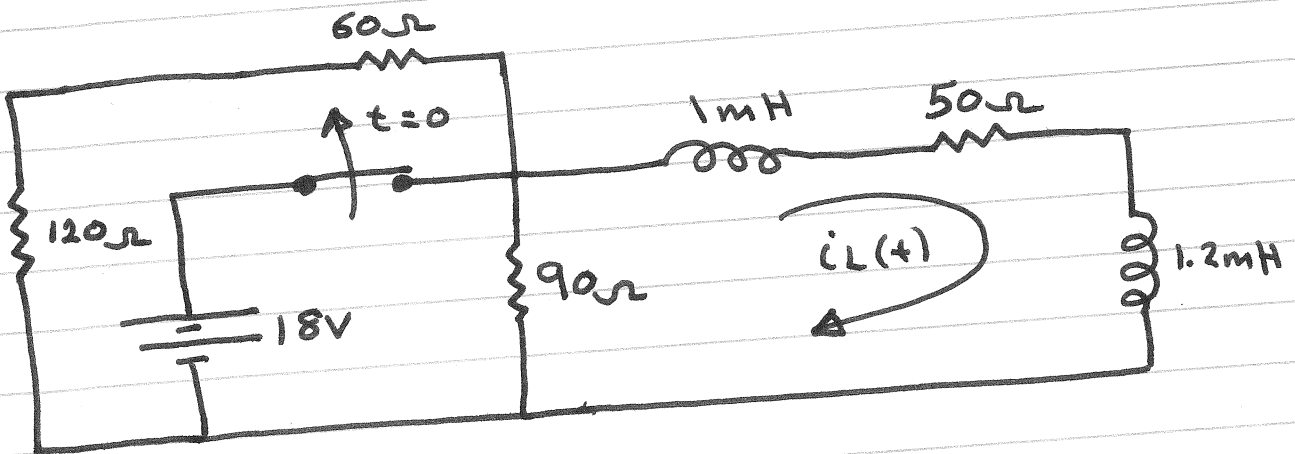
$\tau \equiv$ Time Constant

$$\therefore i(t) = \frac{V_s}{R} e^{-\frac{t}{\tau}} \quad t > 0$$

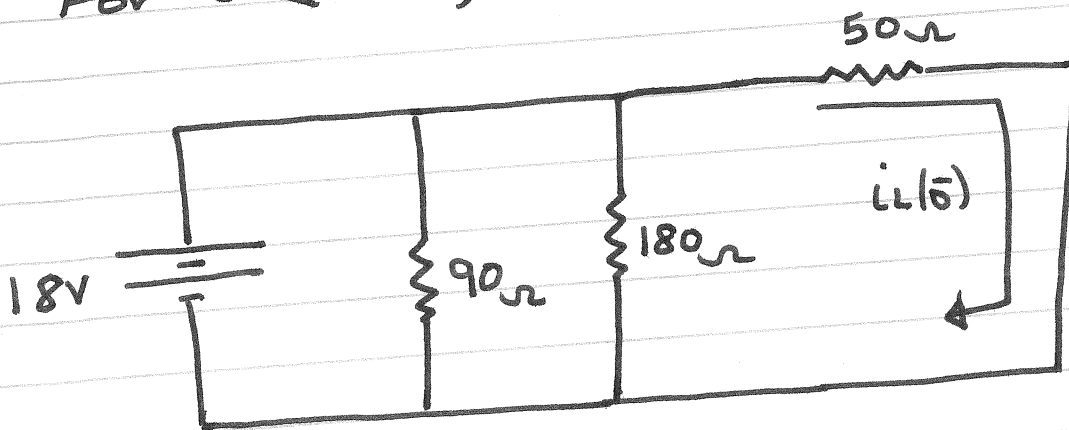
t	$i(t)$
0	$\frac{V_s}{R}$
τ	$0.37 \frac{V_s}{R}$
2τ	$0.14 \frac{V_s}{R}$
3τ	$0.05 \frac{V_s}{R}$
4τ	$0.018 \frac{V_s}{R}$
5τ	$0.0067 \frac{V_s}{R}$



Find $i_L(t)$ for $t > 0$

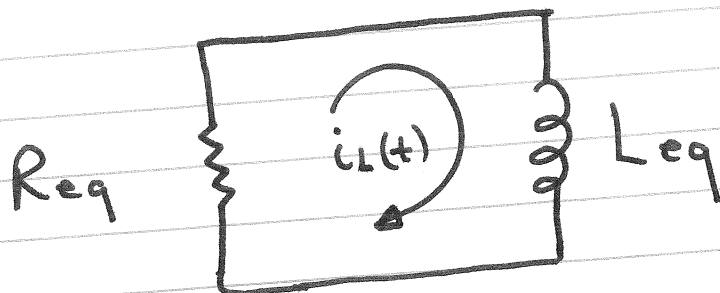


1) For $t < 0$; $t = 0^-$



$$i_L(0^-) = \frac{18V}{50\Omega} = 0.36A$$

2) For $t > 0$



$$R_{eq} = 90\Omega \parallel 180\Omega + 50\Omega = 110\Omega$$

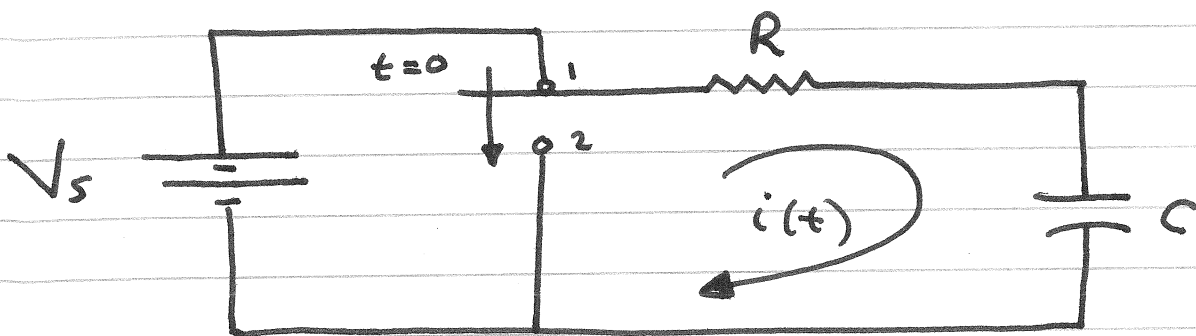
$$L_{eq} = 1\text{mH} + 1.2\text{mH} = 2.2\text{mH}$$

$$\tau = \frac{L_{eq}}{R_{eq}} = 20\mu\text{s}$$

$$\therefore i_L(t) = A e^{-t/\tau} \quad \text{for } t > 0$$

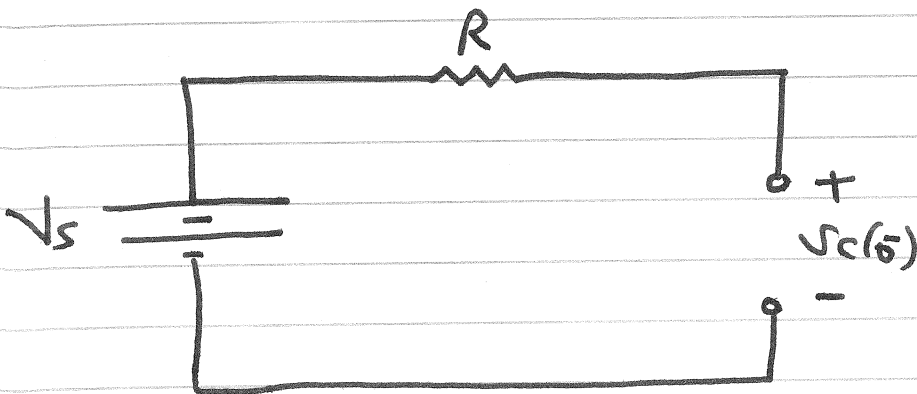
$$i_L(t) = 0.36 e^{-50,000t} \quad \text{for } t > 0$$

RC Circuit



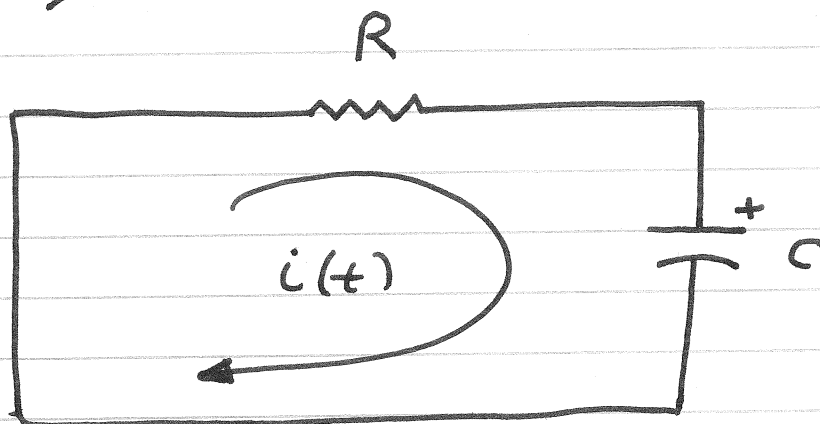
Find $i(t)$ for $t > 0$

1) For $t < 0$; $t = 0^-$

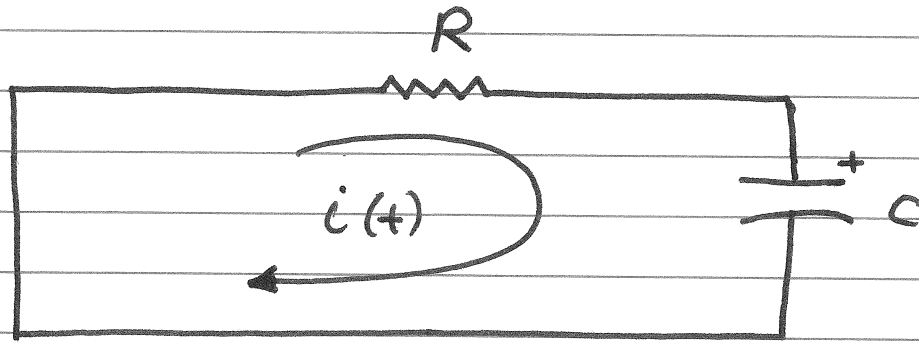


$$V_c(0^-) = V_s$$

2) For $t > 0$



For $t > 0$



KVL :

$$Ri(t) + V_c(0) + \frac{1}{C} \int_0^t i(t) dt \quad t > 0$$

$$R \frac{di(t)}{dt} + \frac{1}{C} i(t) = 0$$

homogeneous first order differential equation

$$\therefore i(t) = A e^{st} \quad t > 0$$

$$R A s e^{st} + \frac{1}{C} A e^{st} = 0$$

$$A e^{st} \left(R s + \frac{1}{C} \right) = 0$$

$$\therefore s = -\frac{1}{RC}$$

$$\therefore i(t) = A e^{-\frac{t}{RC}} \quad t > 0$$

let $\tau = RC = \text{time constant}$

$$\therefore i(t) = A e^{-t/\tau} \quad t > 0$$

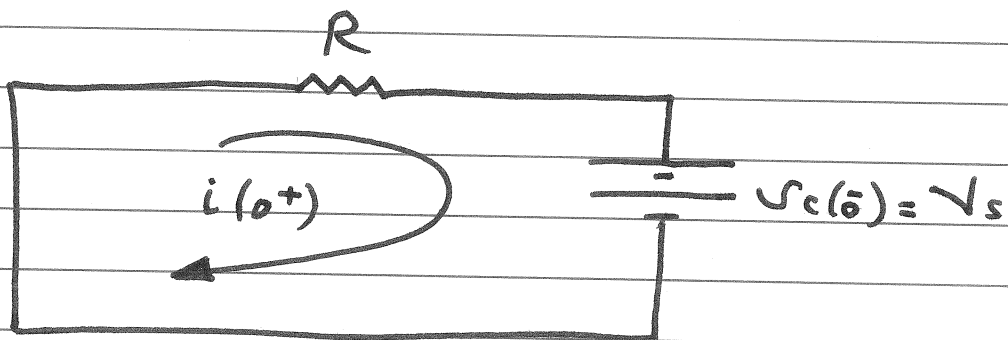
To find A

$$i(t) = A e^{-t/\tau} \quad t > 0$$

$$i(0^+) = A$$

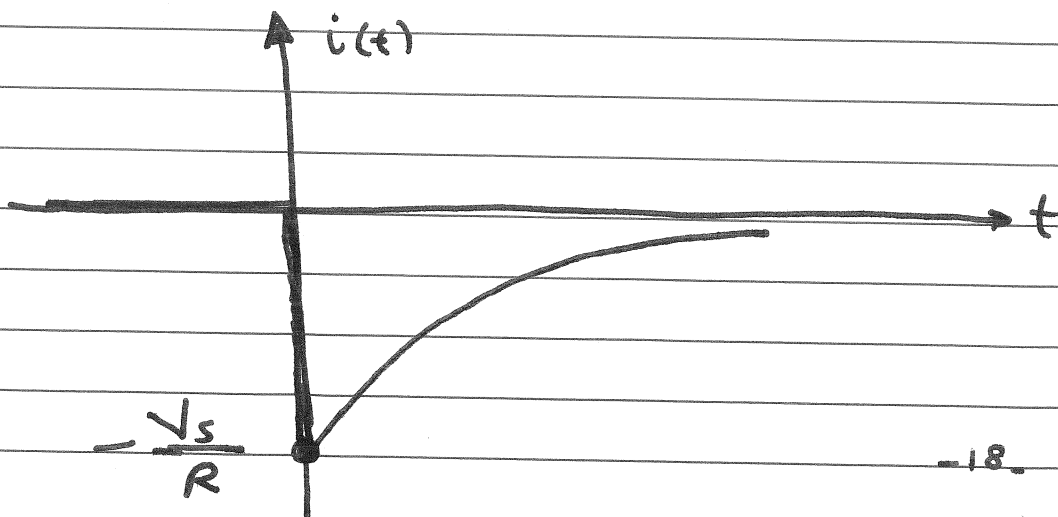
To find $i(0^+)$

at $t=0^+$

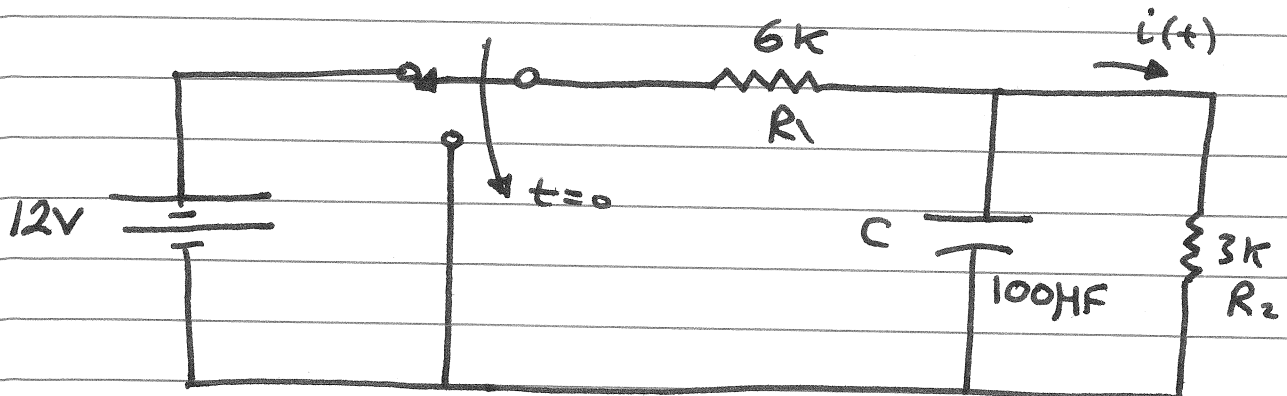


$$i(0^+) = -\frac{V_c(0)}{R} = -\frac{V_s}{R}$$

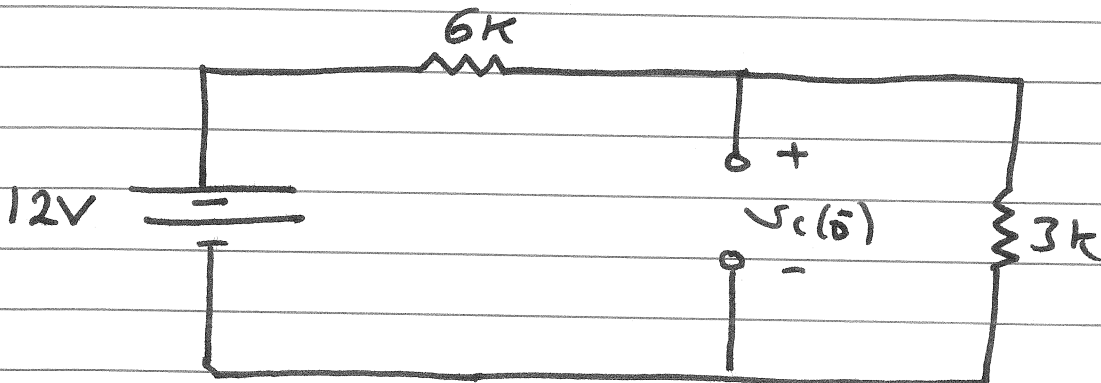
$$\therefore i(t) = -\frac{V_s}{R} e^{-t/\tau}$$



Calculate $V_C(t)$ and $i(t)$ for $t > 0$

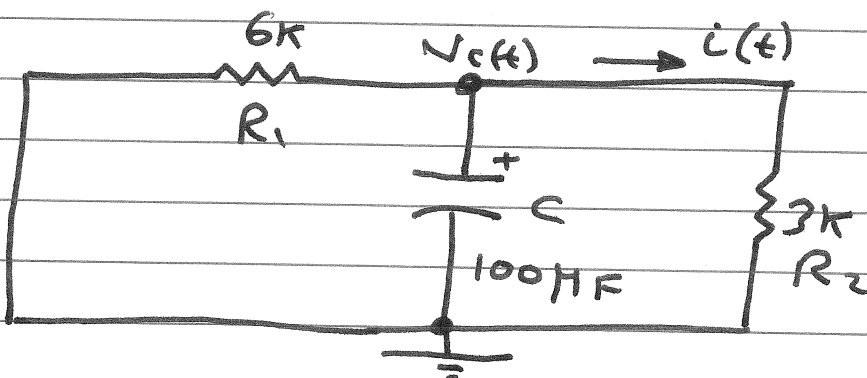


1) For $t < 0$; $t = 0^-$



$$V_C(0^-) = \frac{3k}{3k + 6k} 12V = 4V$$

2) For $t > 0$



KCL :

$$\frac{V_C(t)}{6k} + \frac{V_C(t)}{3k} + C \frac{dV_C(t)}{dt} = 0$$

$$\frac{dV_c(t)}{dt} + 5V_c(t) = 0$$

$$\therefore V_c(t) = A e^{-t/\tau} \quad t > 0$$

$$\tau = R_{eq} C$$

$$R_{eq} = 6k \parallel 3k = 2k$$

$$C = 100 \mu F$$

$$\tau = R_{eq} C = 0.2 \text{ sec}$$

$$V_c(t) = A e^{-t/0.2} \quad t > 0$$

To find A

$$V_c(0^+) = A = V_c(0) = 4V$$

$$\therefore V_c(t) = 4 e^{-5t} \quad t > 0$$

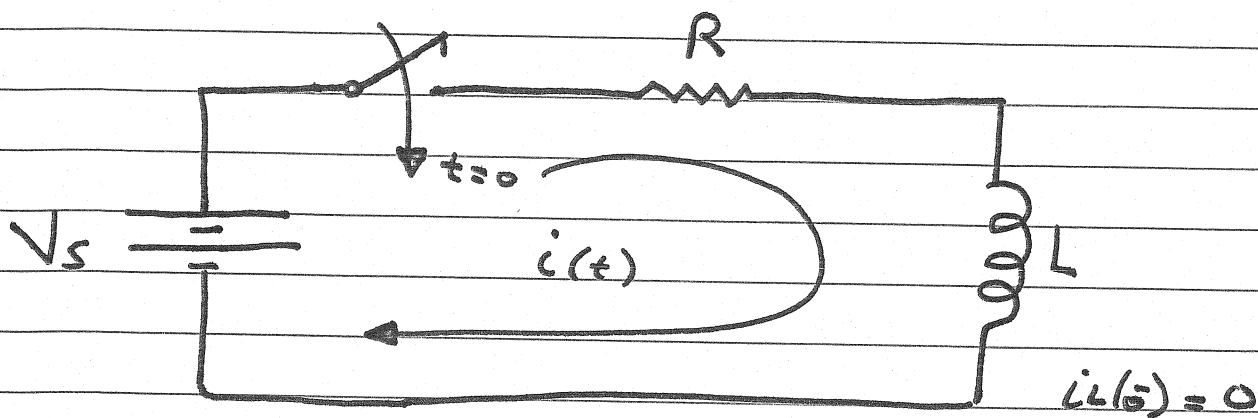
$$i(t) = \frac{V_c(t)}{R_2}$$

$$i(t) = \frac{4}{3} e^{-5t} \text{ mA} \quad t > 0$$

The step Response of RC and RL Circuits

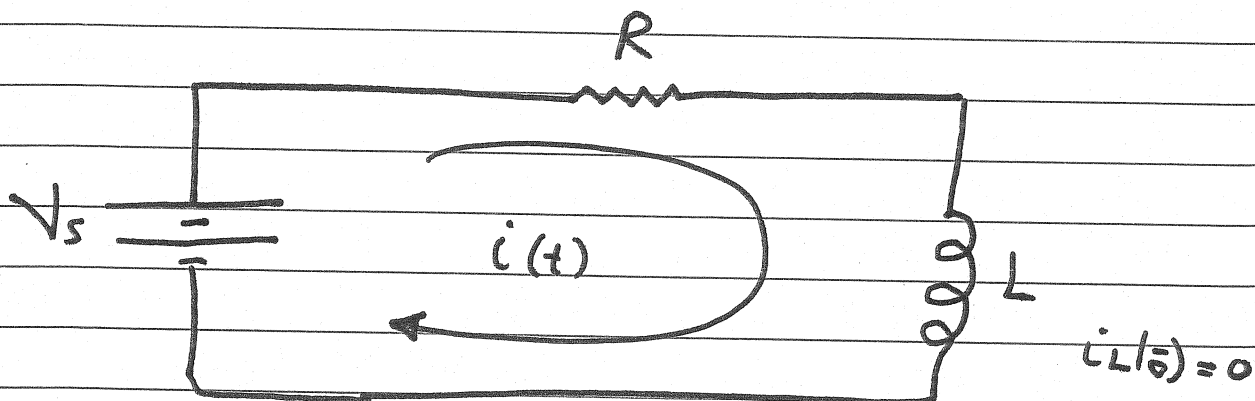
The response of a circuit to the sudden application of a constant voltage or current source is referred to as the step response of the circuit.

The step response of an RL Circuit



Find $i(t)$ for $t > 0$

For $t > 0$



KVL :

$$V_s = Ri(t) + L \frac{di(t)}{dt} \quad t > 0$$

non homogenous First order differential equation

$$\therefore i(t) = i_n(t) + i_f(t)$$

$i_n(t) \equiv$ natural response

$i_f(t) \equiv$ forced response

To find $i_f(t)$

$$\text{let } i_f(t) = K$$

$$V_s = Ri(t) + L \frac{di(t)}{dt}$$

$$V_s = RK + L(0)$$

$$V_s = RK$$

$$\therefore K = \frac{V_s}{R} = i_f(t)$$

now

$$i(t) = i_n(t) + i_f(t) \quad t > 0$$

$$i(t) = Ae^{-t/\tau} + K \quad t > 0$$

$$\tau = \frac{L}{R}$$

$$i(t) = Ae^{-t/\tau} + \frac{V_s}{R} \quad t > 0$$

To find A

$$i(t) = \frac{V_s}{R} + A e^{-t/\tau} \quad t \geq 0$$

$$i(0^+) = \frac{V_s}{R} + A$$

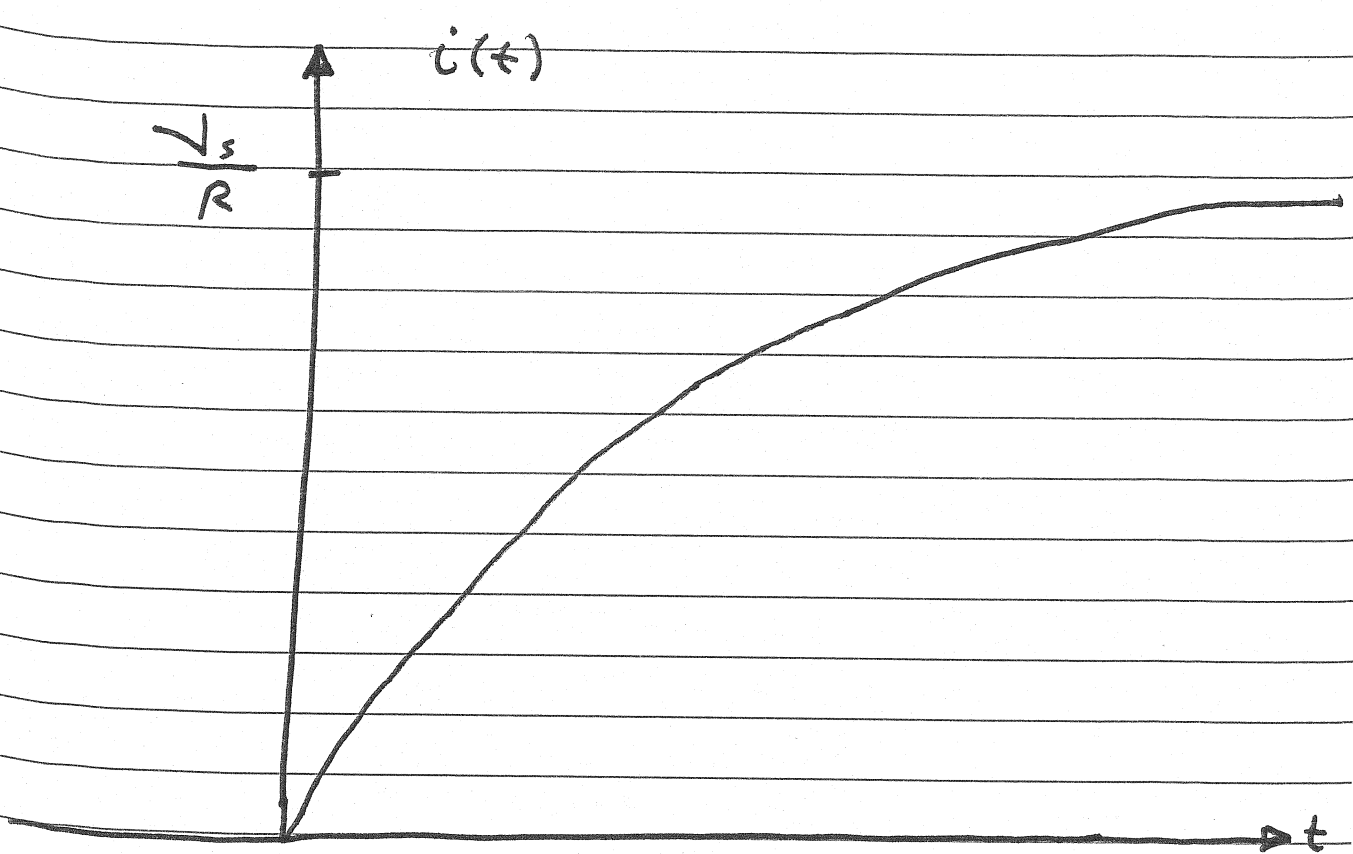
$$\text{But } i(0^+) = i_L(0^+) = i_L(0) = 0$$

$$\therefore A = -\frac{V_s}{R}$$

$$\therefore i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-t/\tau} \quad t \geq 0$$

$$i(t) = \frac{V_s}{R} \left(1 - e^{-t/\tau} \right) \quad t \geq 0$$

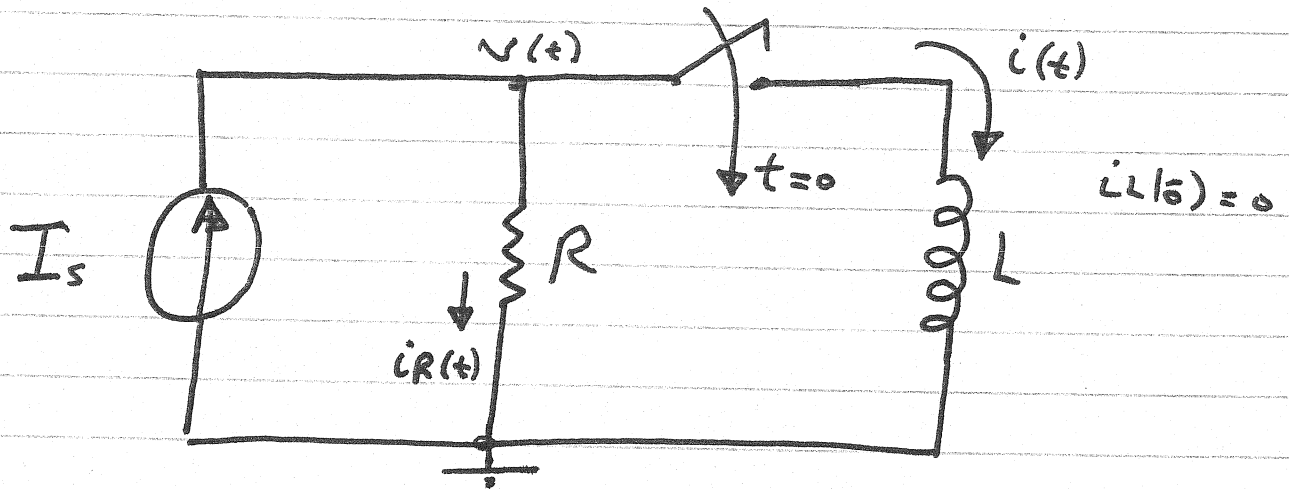
t	$i(t)$
0	0
τ	$0.63 \frac{V_s}{R}$
2τ	$0.86 \frac{V_s}{R}$
3τ	$0.95 \frac{V_s}{R}$
4τ	$0.98 \frac{V_s}{R}$
5τ	$0.99326 \frac{V_s}{R}$



$$i(t) = \frac{V_s}{R} \left(1 - e^{-t/\tau} \right)$$

$$\tau = \frac{L}{R}$$

Find $i(t)$ for $t > 0$



For $t > 0$

KCL :

$$I_s = i_R(t) + i(t) \quad t > 0$$

$$I_s = \frac{v(t)}{R} + i(t) \quad t > 0$$

$$v(t) = v_R(t) = v_L(t) = L \frac{di(t)}{dt}$$

$$I_s = \frac{L}{R} \frac{di(t)}{dt} + i(t) \quad t > 0$$

$$\therefore i(t) = i_n(t) + i_f(t)$$

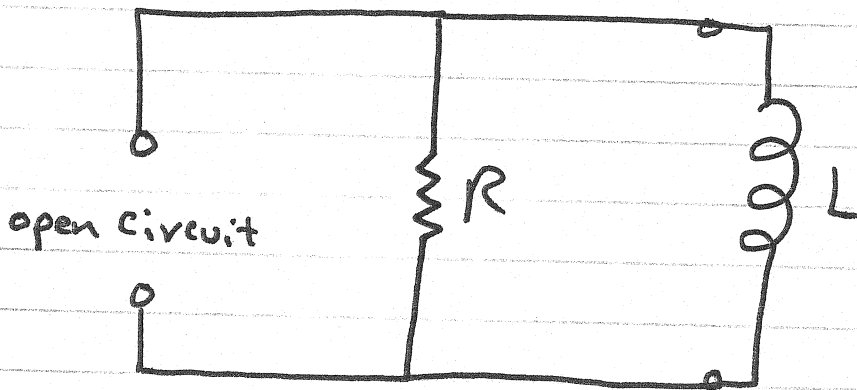
$$\text{Let } i_f(t) = K$$

$$I_s = \frac{L}{R} \cdot 0 + K$$

$$\therefore K = I_s = i_f(t)$$

$$\tau = \frac{L}{R_{eq}}$$

R_{eq} = The Thevenin resistance seen by the inductor



$$\therefore R_{eq} = R$$

$$i(t) = K + A e^{-t/\tau} \quad ; t > 0$$

$$i(t) = I_s + A e^{-t/\tau} \quad ; t > 0$$

To find A

$$i(0^+) = i_L(0^+) = i_L(0) = 0$$

$$i(0^+) = I_s + A = i_L(0) = 0$$

$$\therefore A = -I_s$$

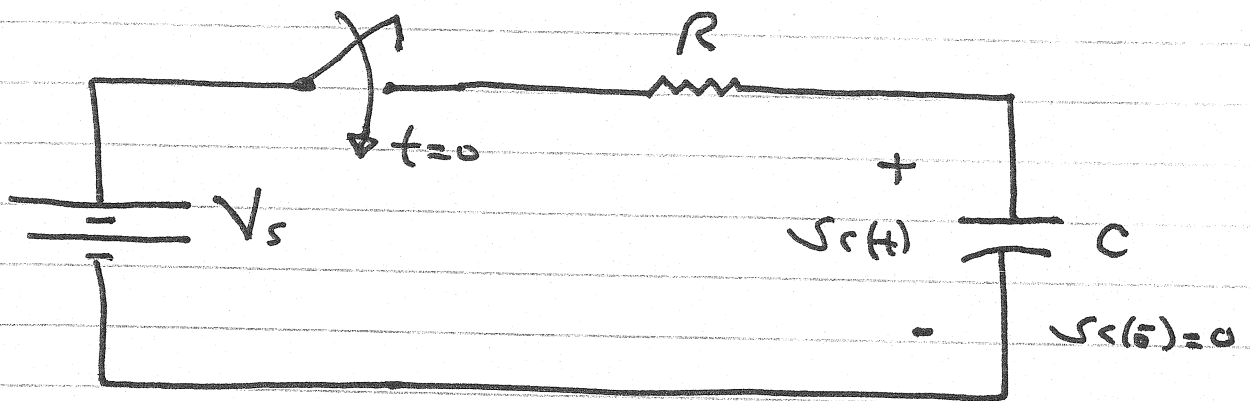
$$\therefore i(t) = i_n(t) + i_f(t) \quad ; t > 0$$

$$i(t) = A e^{-t/\tau} + I_s \quad ; t > 0$$

$$i(t) = -I_s e^{-t/\tau} + I_s \quad ; t > 0$$

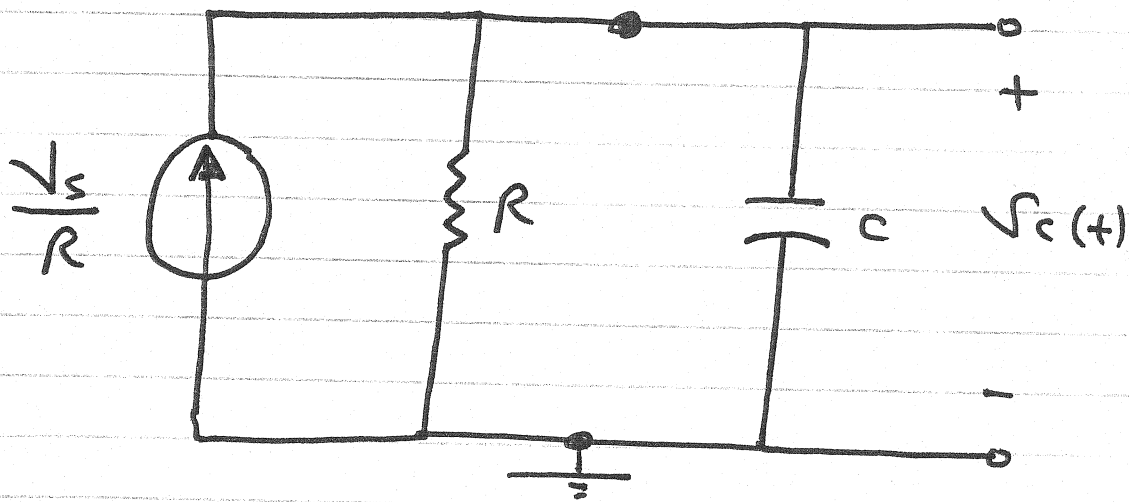
$$\therefore i(t) = I_s \left(1 - e^{-t/\tau} \right) \quad t \geq 0$$

The step response of an RC Circuit



Find $v_c(t)$ for $t > 0$

For $t > 0$



KCL :

$$\frac{V_s}{R} = i_R(t) + i_C(t) \quad t > 0$$

$$\frac{V_s}{R} = \frac{v_c(t)}{R} + C \frac{dv_c(t)}{dt} \quad t > 0$$

$$\frac{V_s}{R} = \frac{V_c(t)}{R} + C \frac{dV_c(t)}{dt}$$

First order nonhomogeneous differential equation

$$\therefore V_c(t) = V_{cn}(t) + V_{cf}(t) \quad t > 0$$

$$V_c(t) = A e^{-t/\tau} + K$$

To find K

$$\frac{V_s}{R} = \frac{K}{R} + 0$$

$$\therefore K = V_s$$

$$\therefore V_c(t) = A e^{-t/\tau} + V_s \quad t > 0$$

$$\tau = R_{eq} C$$

$$R_{eq} = R$$

To find A

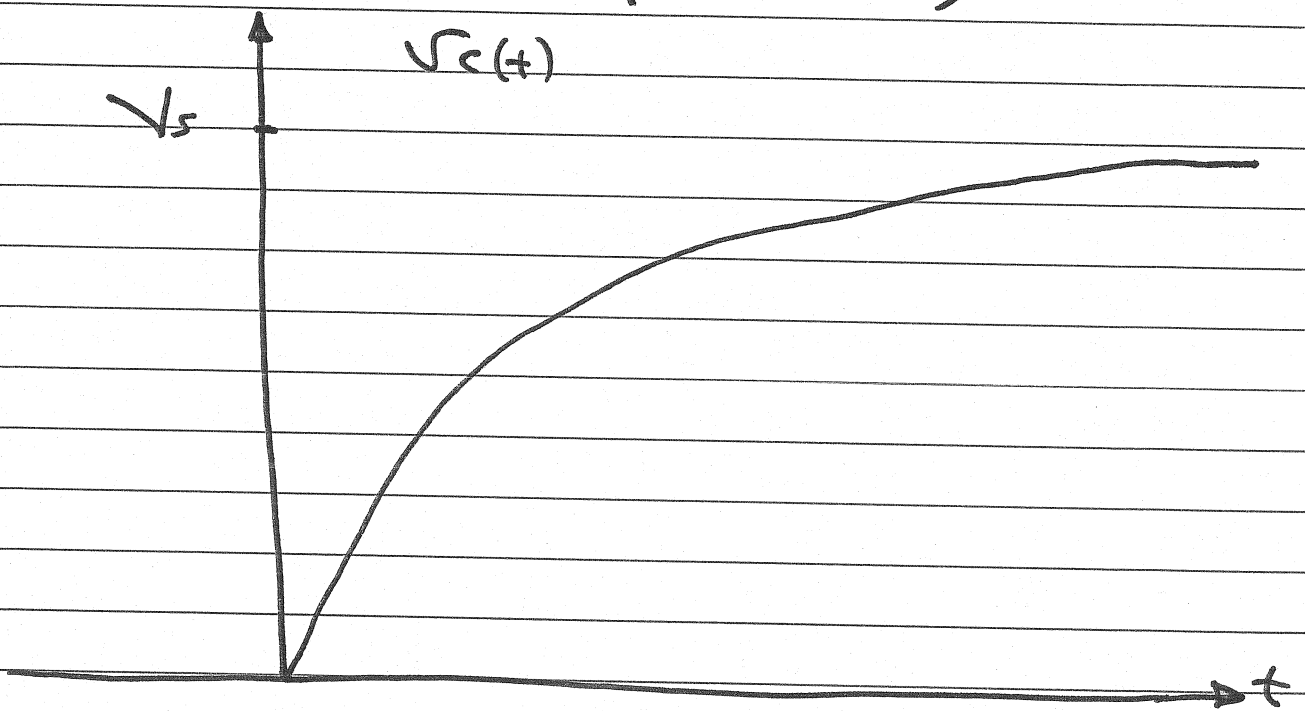
$$V_c(0^+) = V_c(0^-) = 0$$

$$V_c(0^+) = K + A = V_c(0^-) = 0$$

$$\therefore A = -K = -V_s$$

$$\therefore V_c(t) = V_s - V_s e^{-t/\tau} \quad t \geq 0$$

$$V_c(t) = V_s \left(1 - e^{-t/\tau} \right) \quad t \geq 0$$



When all independent sources are constant, the response of the first order circuit has the form

$$v(t) = v_n(t) + K \quad t > 0$$

$$v(t) = A e^{-t/\tau} + K \quad t > 0$$

$$v(\infty) = K$$

$$\therefore v(t) = A e^{-t/\tau} + v(\infty) \quad t > 0$$

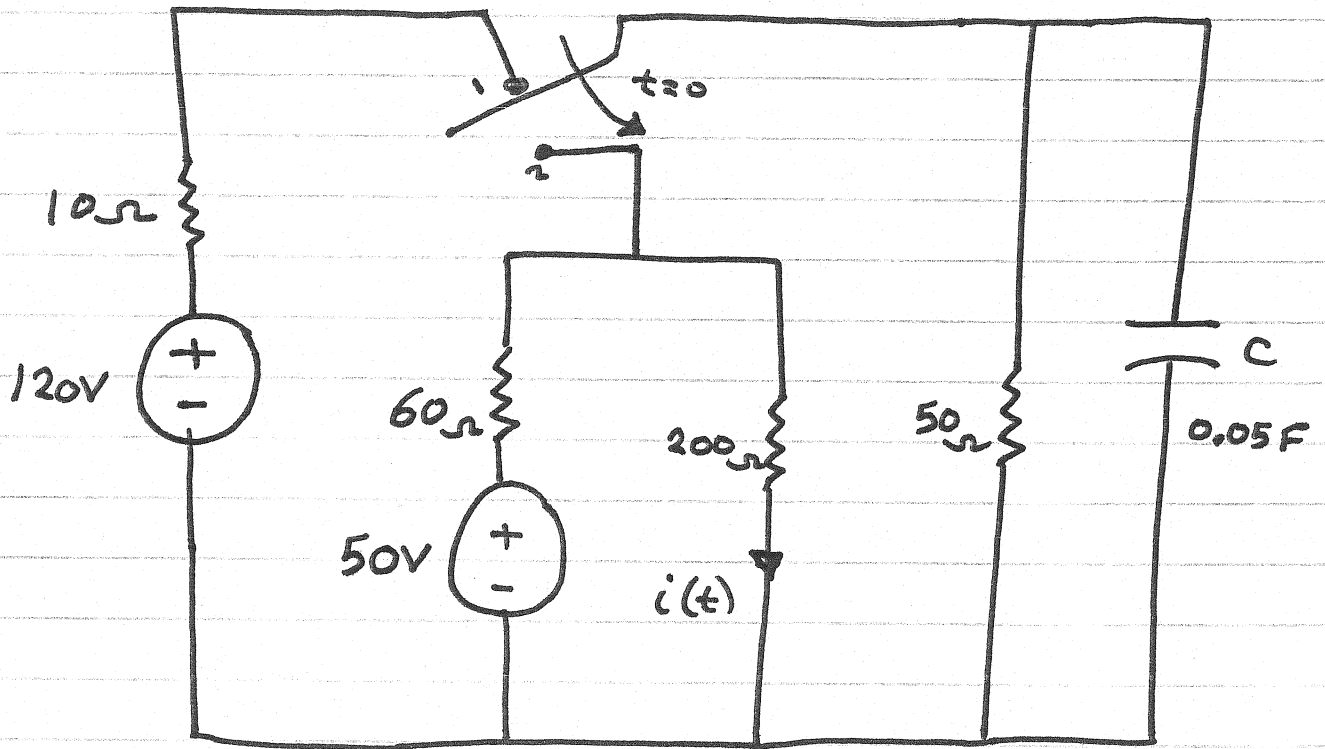
$$v(0^+) = A + v(\infty)$$

$$\therefore A = v(0^+) - v(\infty)$$

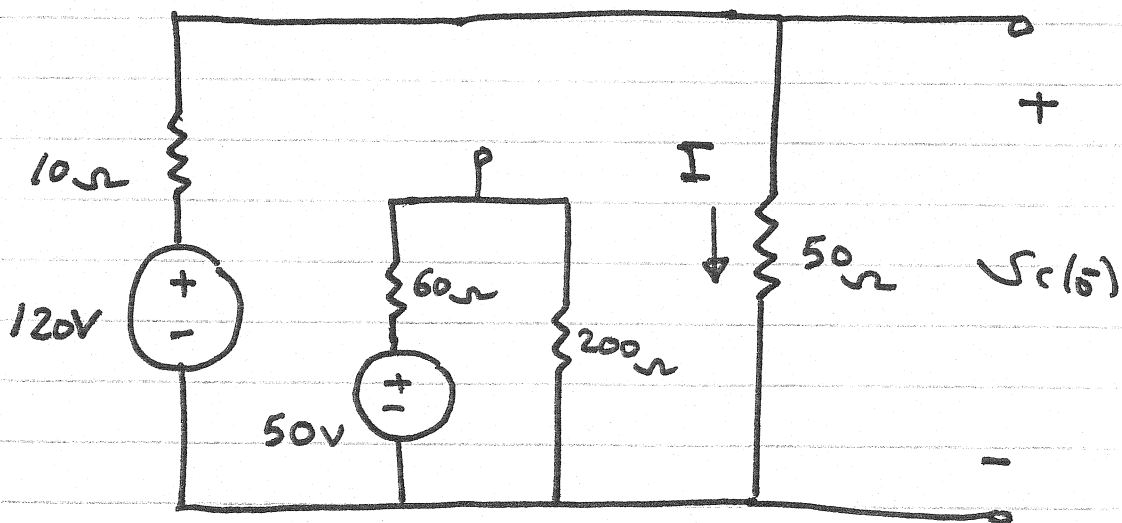
$$\therefore v(t) = v(\infty) + [v(0^+) - v(\infty)] e^{-t/\tau} \quad t > 0$$

$$\tau = \frac{L}{R_{eq}} \quad \text{or} \quad \tau = R_{eq} C$$

Find $i(t)$ for $t > 0$



For $t < 0$; $t = 0^-$

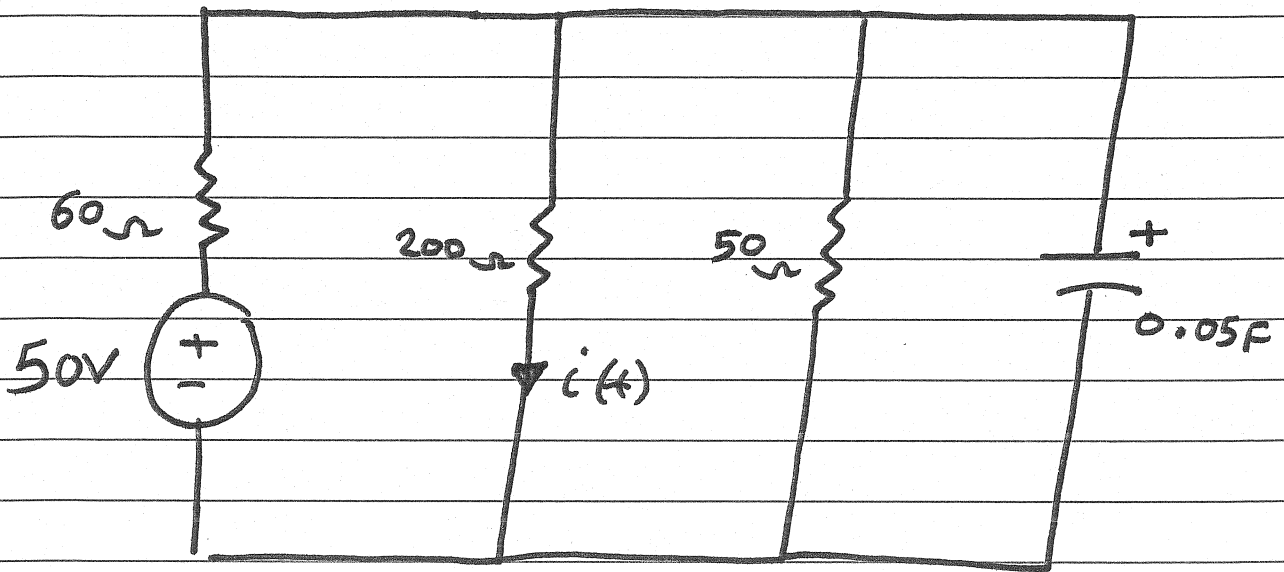


$$\text{KVL : } -120 + 10I + 50I = 0$$

$$\therefore I = 2A$$

$$N_c(s) = 50 I = 100V$$

For $t > 0$

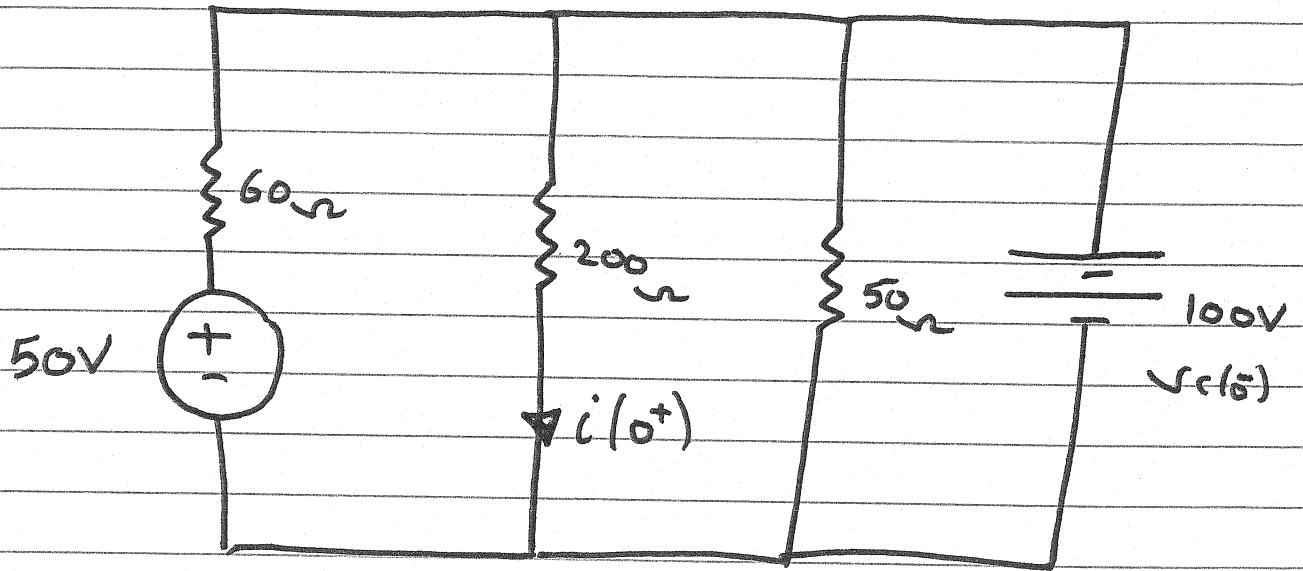


$$v(t) = v(\infty) + [v(0^+) - v(\infty)] e^{-t/\tau} \quad t > 0$$

$$i(t) = i(\infty) + [i(0^+) - i(\infty)] e^{-t/\tau} \quad t > 0$$

To find $i(0^+)$

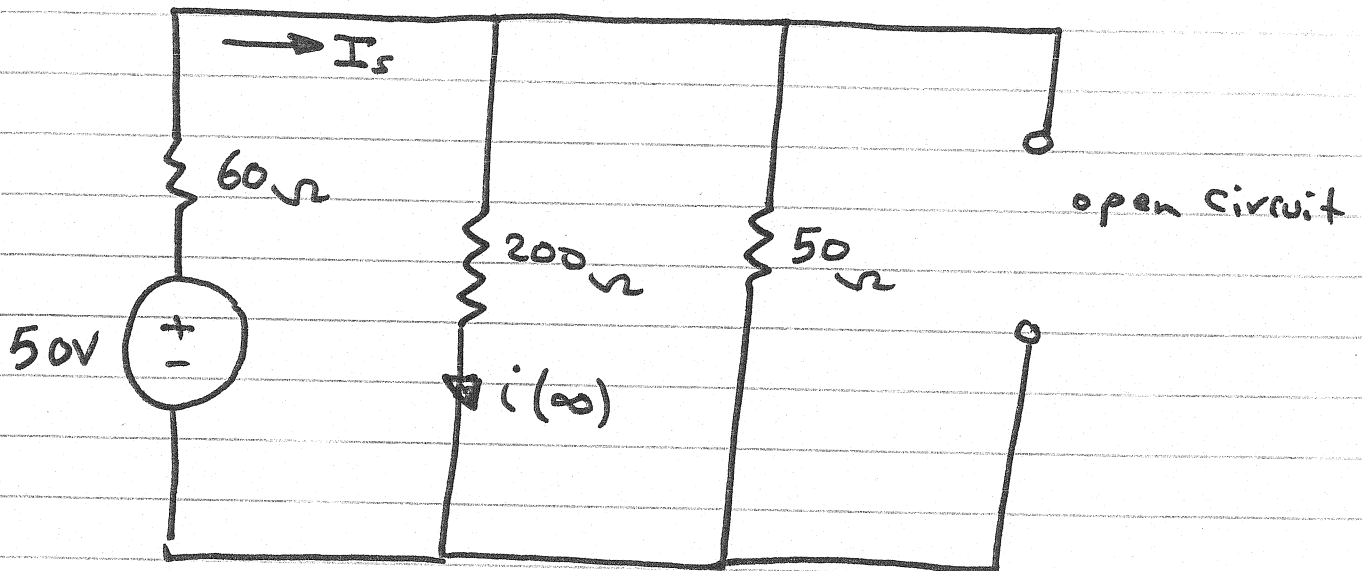
at $t = 0^+$



$$i(0^+) = \frac{v_c(t)}{200} = \frac{100}{200} = 0.5 \text{ A}$$

To find $i(\infty)$

at $t = \infty$



$$i(\infty) = \frac{50}{50+200} I_s$$

$$I_s = \frac{50}{50\Omega \parallel 200\Omega + 60\Omega} = 0.5A$$

$$\therefore i(\infty) = 0.1A$$

To find τ

for $t > 0$

$$\tau = R_{eq} C$$

$R_{eq} = R_{TH}$ seen by the Capacitor



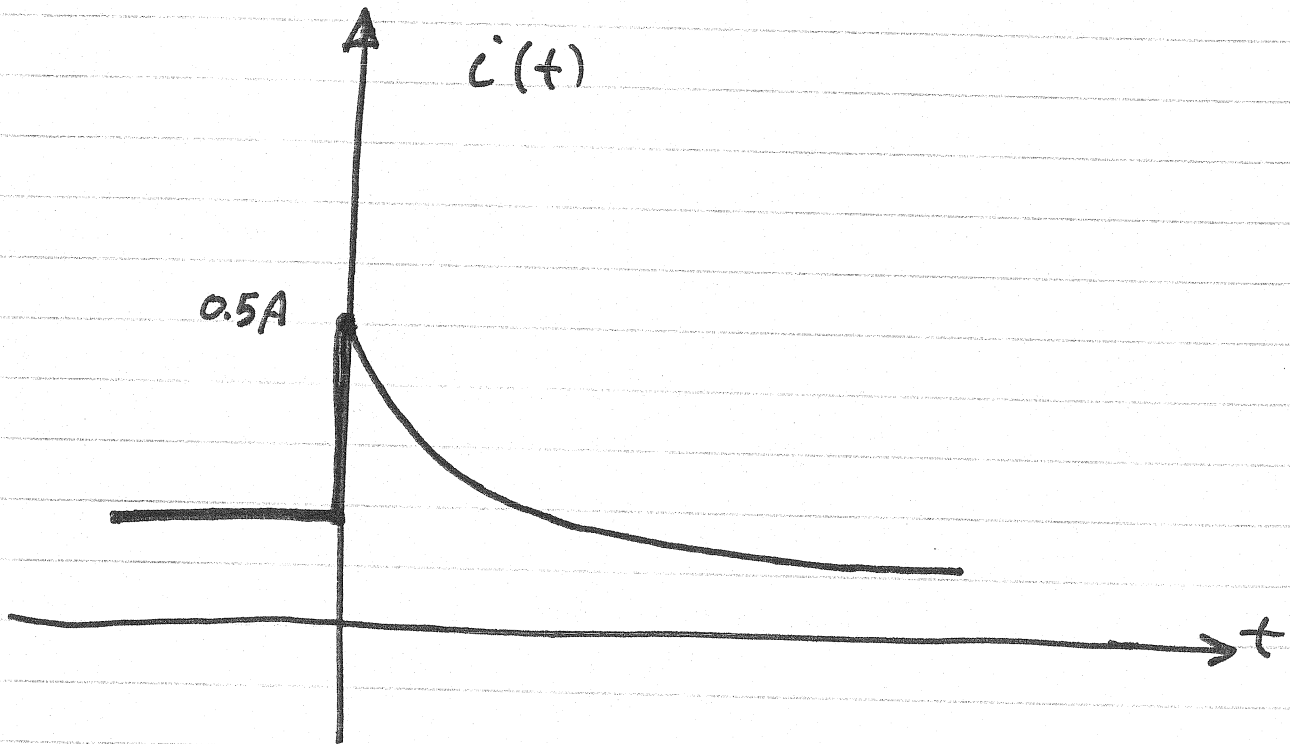
$$R_{TH} = 60\ \Omega \parallel 200\ \Omega \parallel 50\ \Omega$$

$$R_{TH} = 24\ \Omega$$

$$\tau = (0.05)(24) = 1.2\ \text{sec}$$

$$\therefore i(t) = (0.1 + 0.4 e^{-t/1.2})\ \text{A}; t > 0$$

$$i(t) = \left(0.1 + 0.4 e^{-t/12} \right) \text{ A} ; t > 0$$



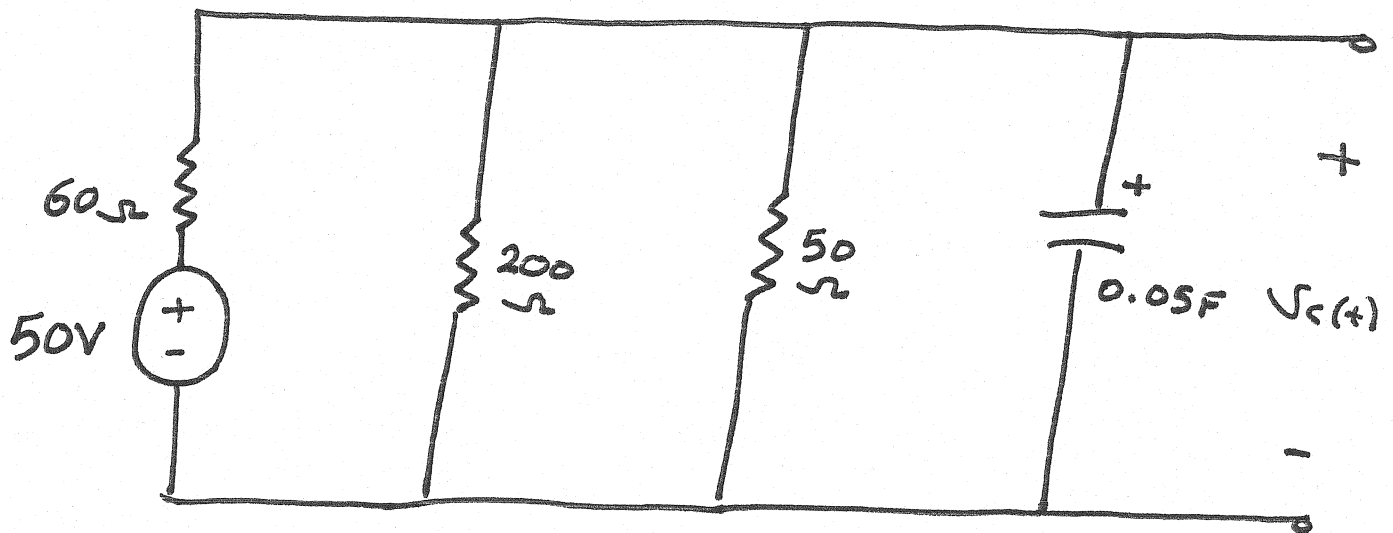
$$i(0^-) = 0.192 \text{ A}$$

$$i(0^+) = 0.5 \text{ A}$$

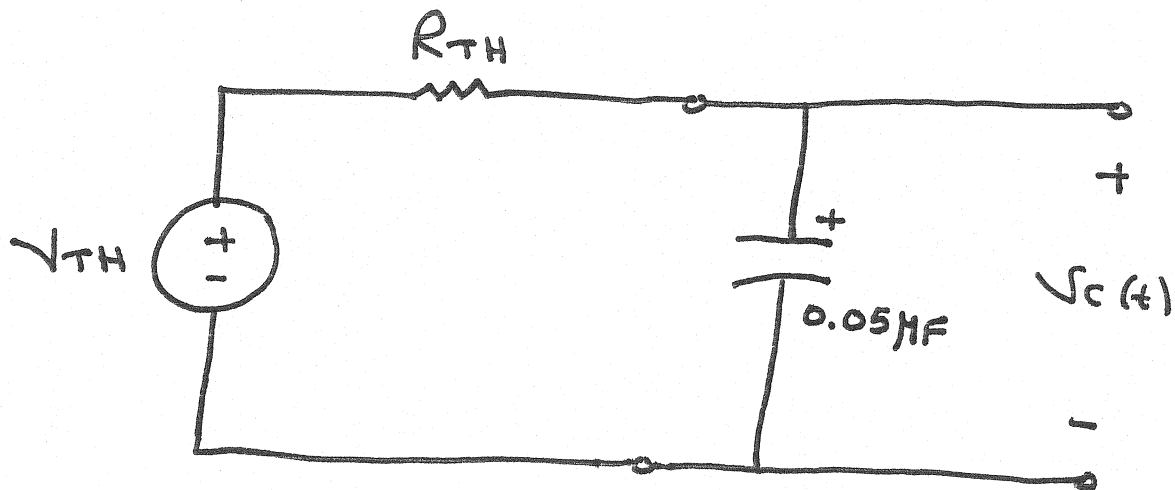
$$i(\infty) = 0.1 \text{ A}$$

To find $V_C(t)$ for $t > 0$

For $t > 0$

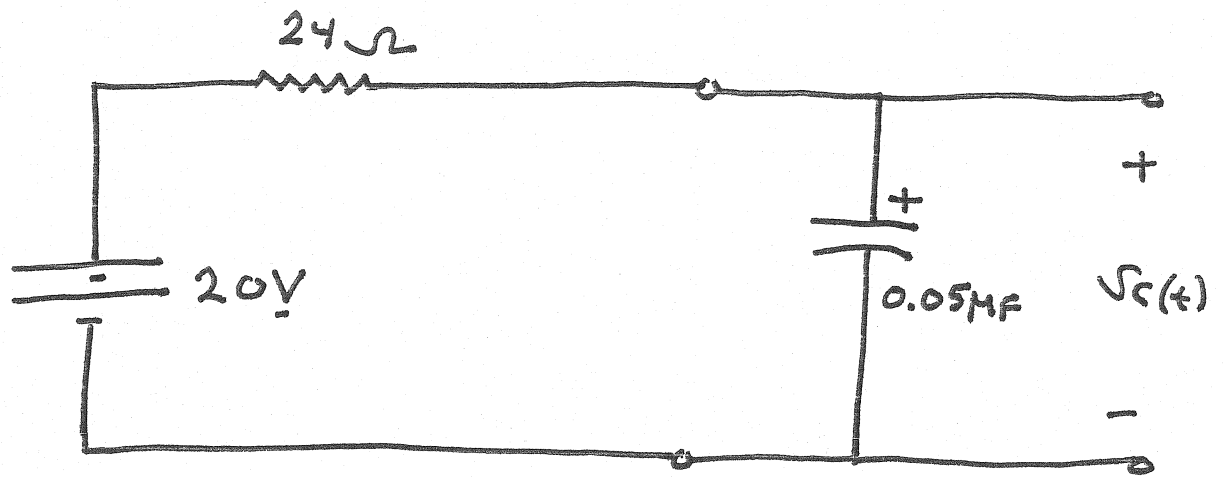


The circuit can be simplified to



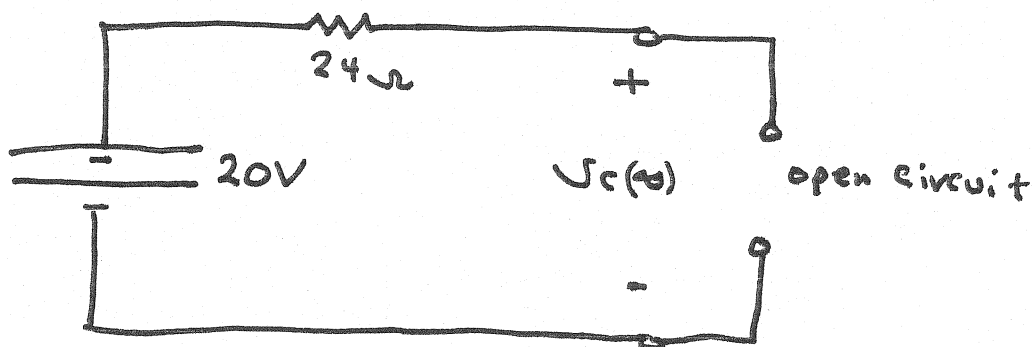
$$R_{TH} = 60\Omega \parallel 200\Omega \parallel 50\Omega = 24\Omega$$

$$V_{TH} = \frac{50\Omega \parallel 200\Omega}{50\Omega \parallel 200\Omega + 60\Omega} \cdot 50 = 20V$$



$$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)] e^{-t/\tau} ; t > 0$$

- 1) $V_c(0^+) = V_c(0^-) = 100V$
- 2) To find $V_c(\infty)$



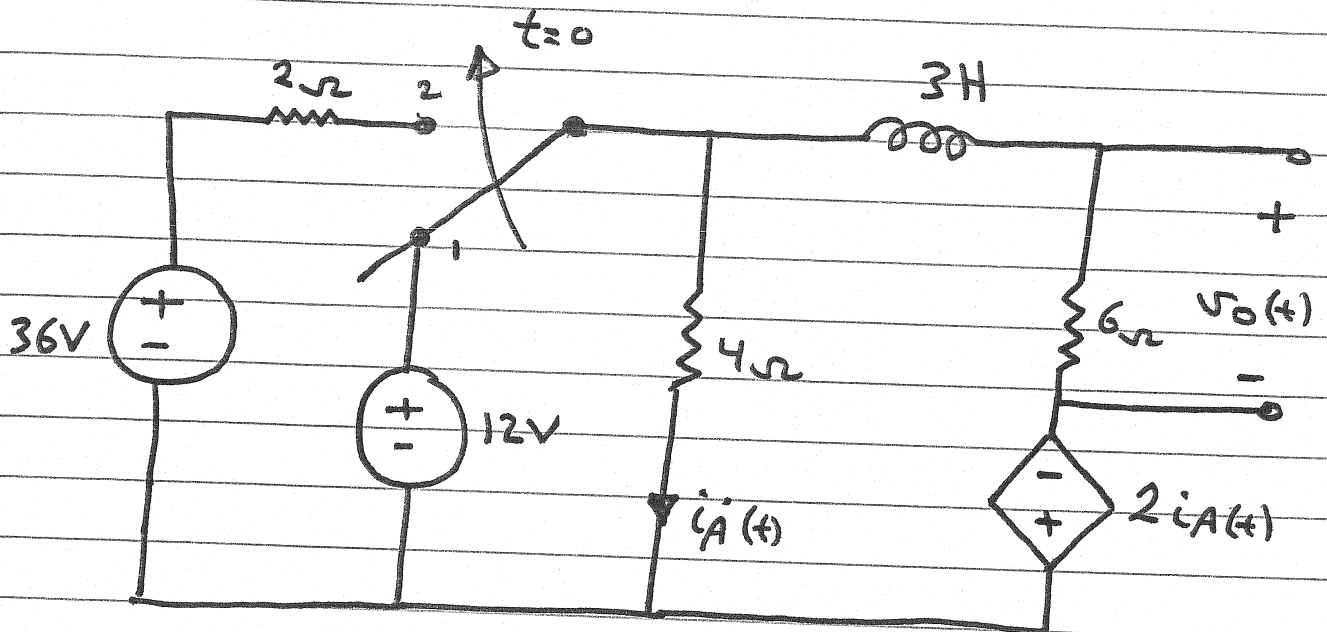
$$\therefore V_c(\infty) = 20V$$

$$3) \tau = R_{TH} C = 1.2 \text{ sec}$$

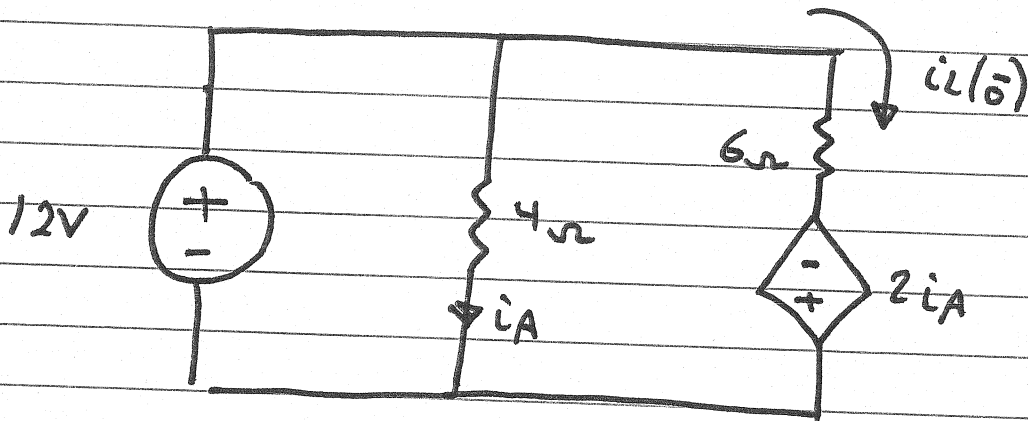
$$V_c(t) = 20 + (100 - 20) e^{-t/1.2} \text{ V}, t \geq 0$$

$$V_c(t) = 20 + 80 e^{-t/1.2} \text{ V}, t \geq 0$$

Find $v_o(t)$ for $t > 0$



For $t < 0$; $t = 0^-$



$$i_A = \frac{12}{4} = 3A$$

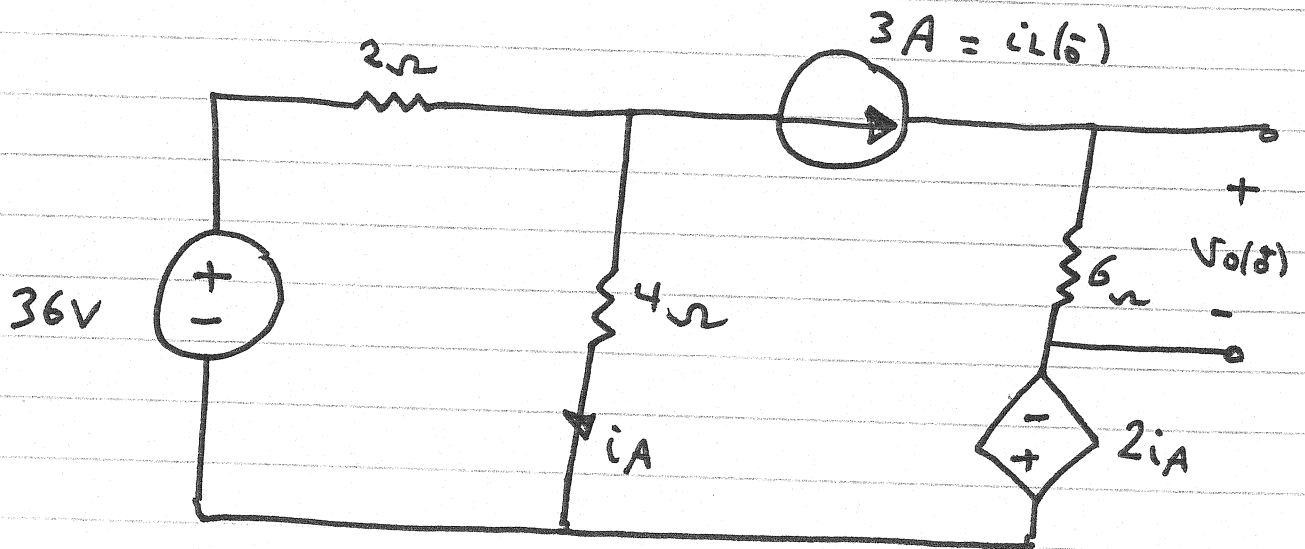
KVL :

$$-12 + 6 i_L(0^-) - 2 i_A = 0$$

$$\therefore i_L(0^-) = 3A$$

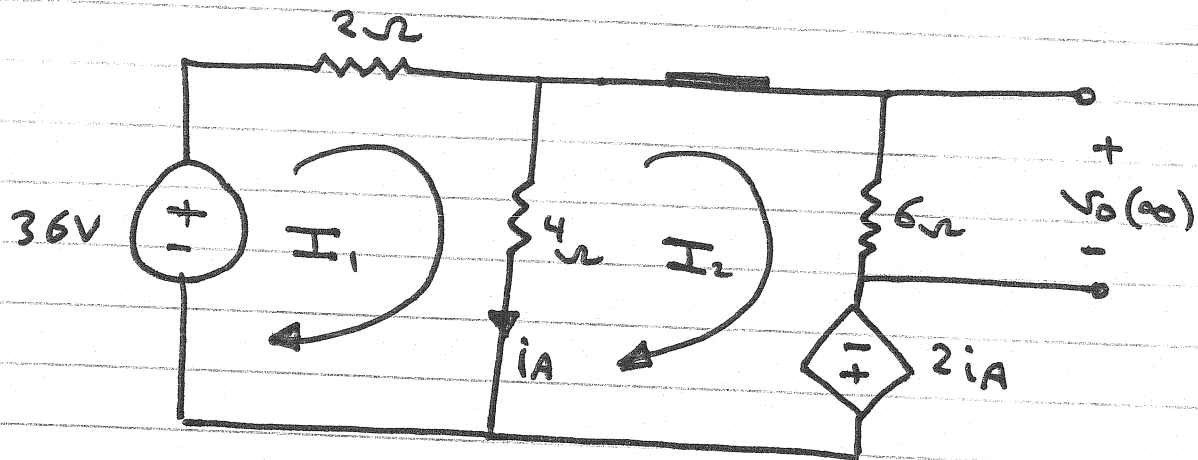
$$V_o(t) = V_o(\infty) + [V_o(0^+) - V_o(\infty)] e^{-t/\tau} \quad t > 0$$

To find $V_o(0^+)$



$$V_o(0^+) = (3A)(6\Omega) = 18V$$

To find $V_o(\infty)$



KVL for mesh ① :

$$36 = 6I_1 - 4I_2$$

KVL for mesh ② :

$$2iA = -4I_1 + 10I_2$$

$$iA = I_1 - I_2$$

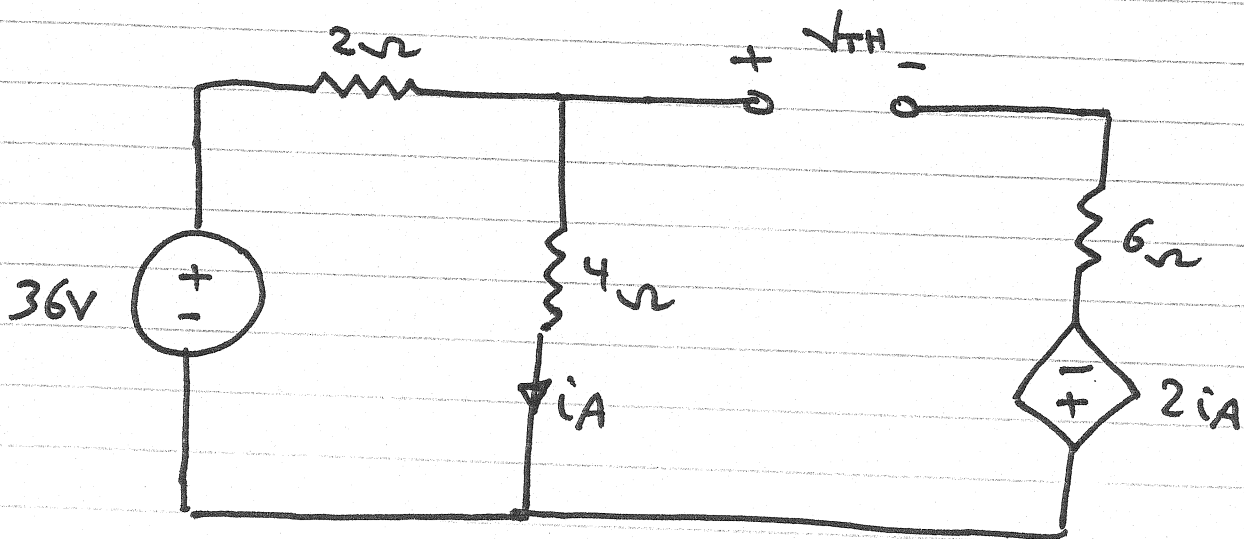
Solving for I_2 ; we get $I_2 = \frac{36}{8} A$

$$V_o(\infty) = (6\Omega) \left(\frac{36}{8} A \right) = 27V$$

To find $\tau = \frac{L}{R_{TH}}$

To find R_{TH}

$$R_{TH} = \frac{V_{TH}}{I_{sc}}$$

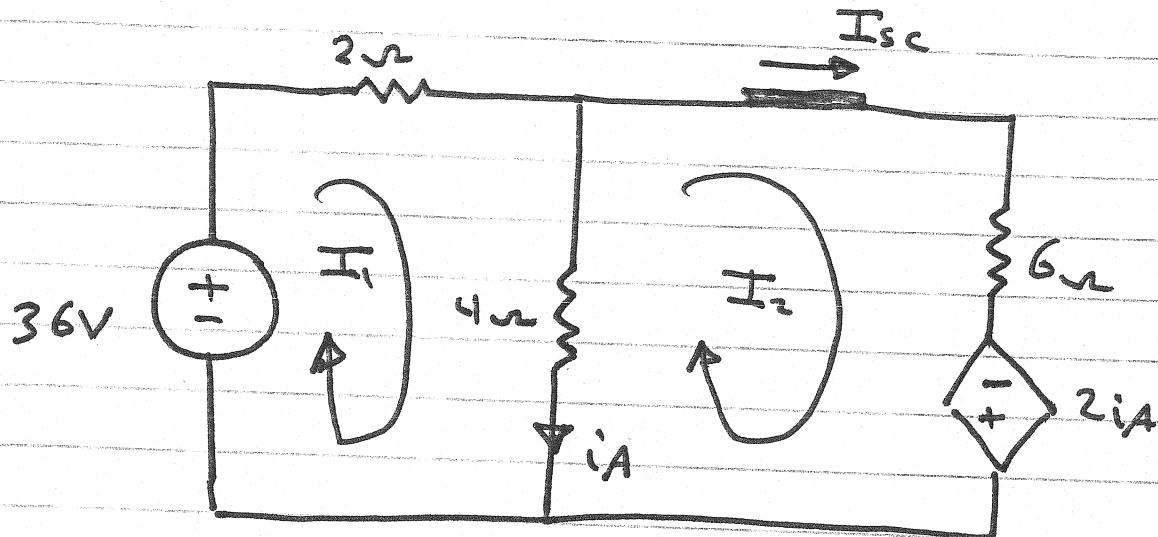


$$V_{TH} = 4\Omega i_A + 2i_A = 6i_A$$

$$i_A = \frac{36}{6} = 6A$$

$$\therefore V_{TH} = 36V$$

To find I_{sc}



KVL for mesh 1 :

$$36 = 6I_1 - 4I_2$$

KVL for mesh 2 :

$$2i_A = -4I_1 + 10I_2$$

$$i_A = I_2 - I_1$$

Solving for $I_2 = I_{sc} = \frac{36}{8} A$

$$\therefore R_{TH} = \frac{V_{TH}}{I_{sc}} = 8 \Omega$$

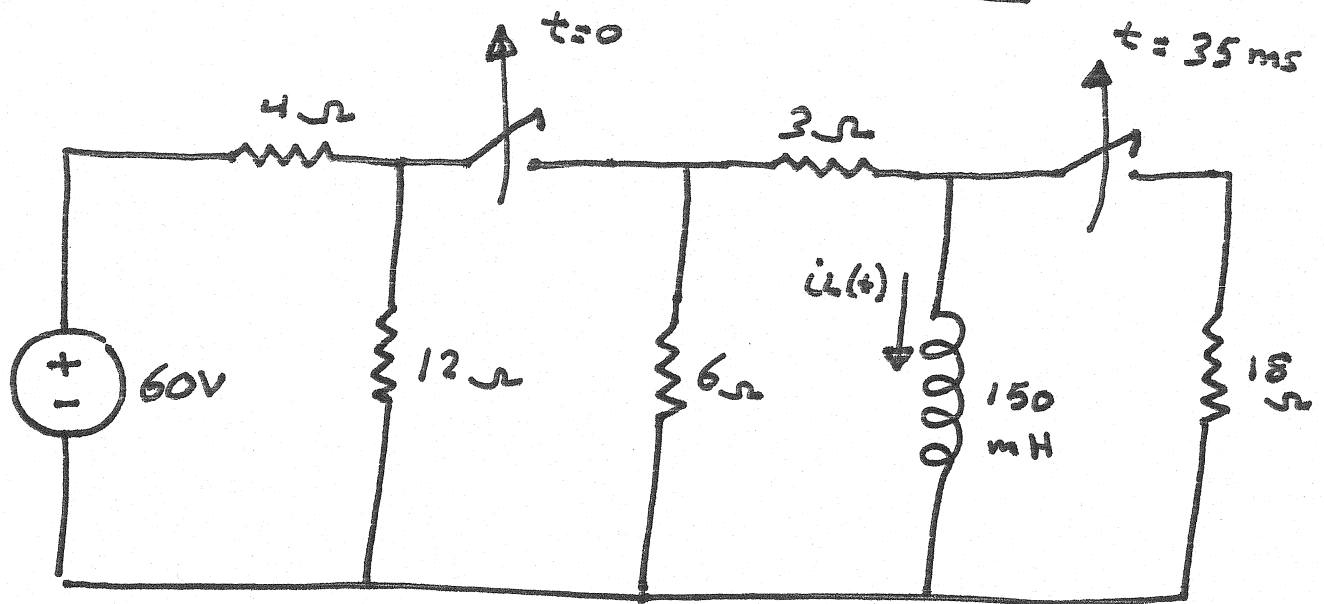
$$\therefore \tau = \frac{L}{R_{TH}} = \frac{3}{8} \text{ sec}$$

$$\therefore V_o(t) = V_o(\infty) + [V_o(0^+) - V_o(\infty)] e^{-t/\tau} \quad t > 0$$

$$\therefore V_o(t) = 27 + [18 - 27] e^{-t/\tau} \quad t > 0$$

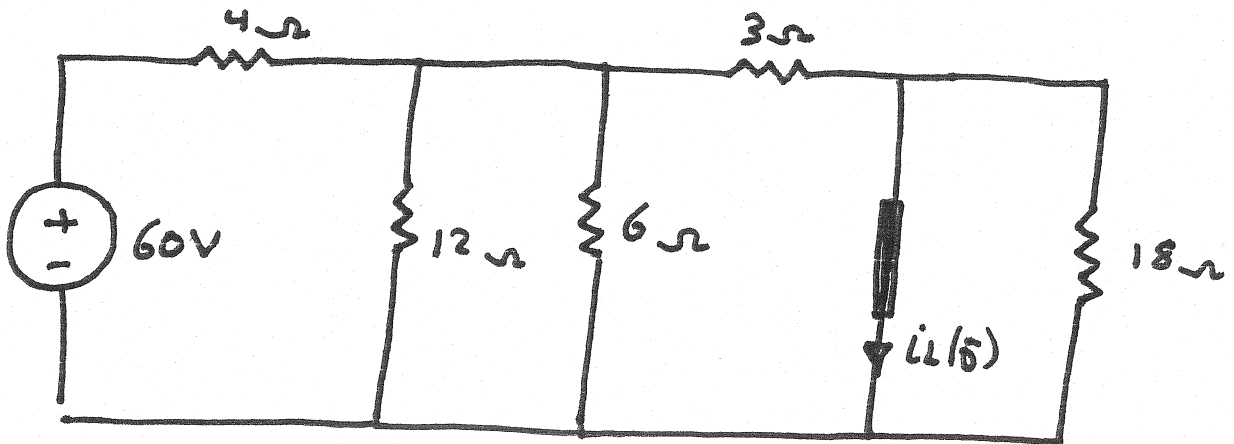
$$\therefore V_o(t) = (27 - 9 e^{-\frac{8}{3}t}) \text{ V for } t > 0$$

Sequential Switching



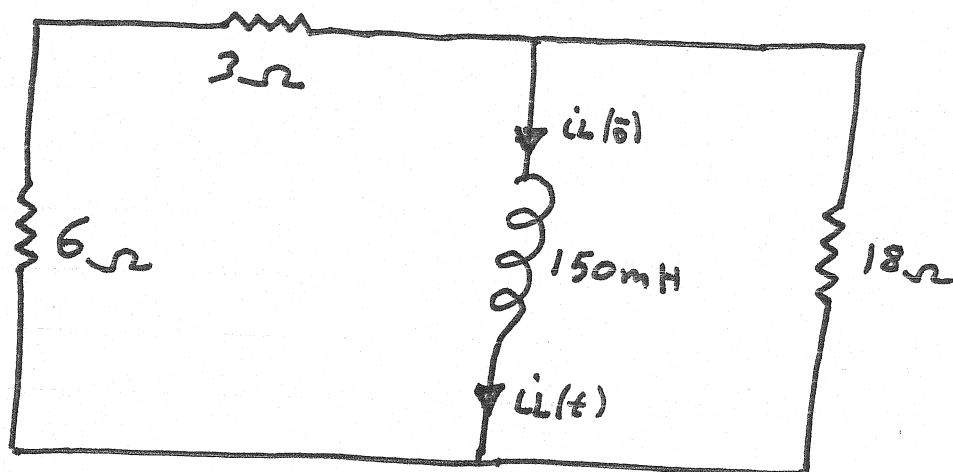
Find $i_L(t)$ for $t > 0$

1) For $t < 0$; $t = 0^-$



$$i_L(0^-) = 6\text{A}$$

2) For $35\text{ms} \geq t \geq 0$



Source-free circuit

$$\therefore i_L(t) = A e^{-t/\tau}, \quad t \geq 0$$

$$\tau_1 = \frac{L}{R_{TH}}$$

$$R_{TH} = 18\Omega \parallel (6\Omega + 3\Omega) = 6\Omega$$

$$\therefore \tau_1 = \frac{150\text{mH}}{6\Omega} = 25\text{ms}$$

To find A

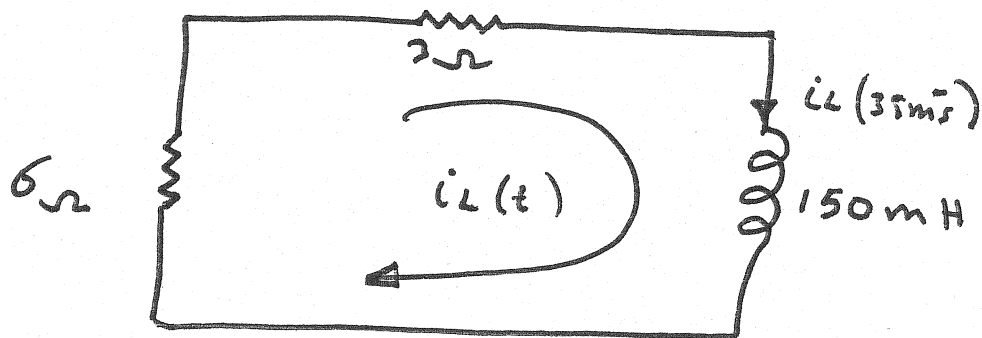
$$i(0^+) = A = i_L(0) = \underline{6\text{A}}$$

$$\therefore i_L(t) = 6 e^{-40t} \text{ A}, \quad 0 \leq t \leq 35\text{ms}$$

at $t = 35 \text{ ms}$

$$i_L(35 \text{ ms}) = 1.48 \text{ A}$$

2) For $t > 35 \text{ ms}$



Source free circuit

$$\therefore i_L(t) = A e^{-\frac{(t-35 \text{ ms})}{\tau_2}}$$

$$\tau_2 = \frac{L}{R_{TH2}}$$

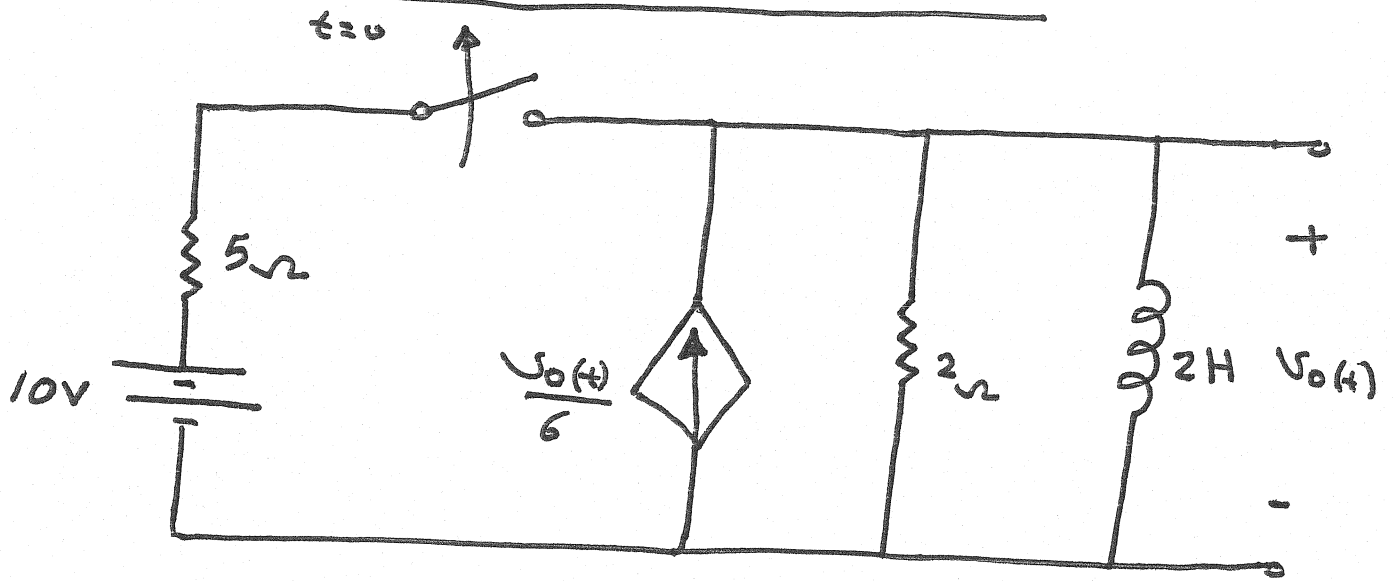
$$R_{TH2} = 6 \Omega + 3 \Omega = 9 \Omega$$

$$\therefore \tau_2 = 16.67 \text{ ms}$$

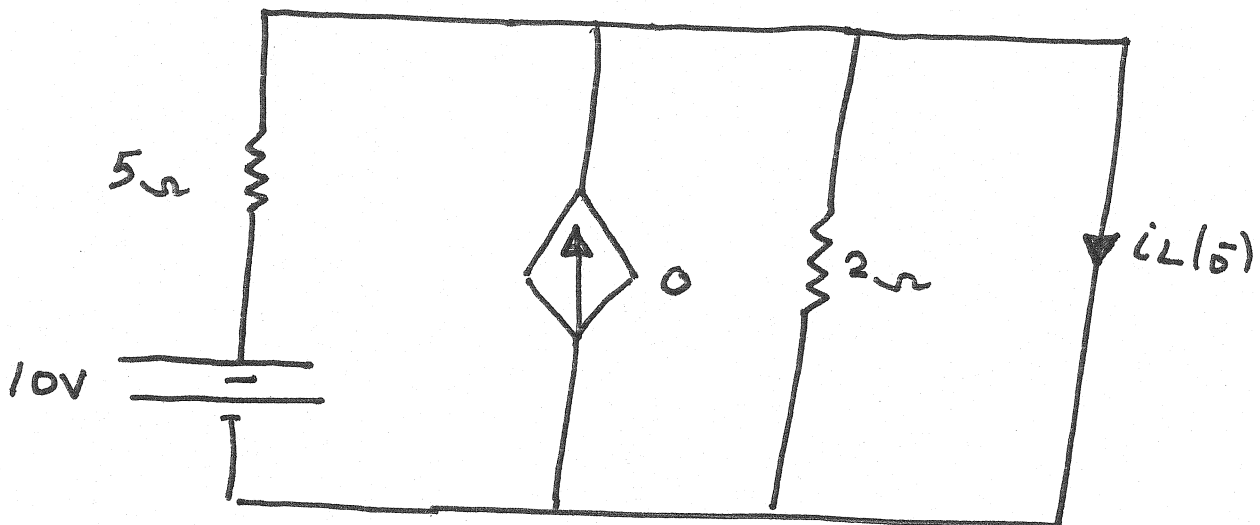
$$\therefore i_L(t) = 1.48 e^{-\frac{(t-35 \text{ ms})}{16.67 \text{ ms}}}$$

$$i_L(t) = 1.48 e^{-60(t-0.035)} \text{ A} ; t > 35 \text{ ms}$$

Circuit with dependent sources

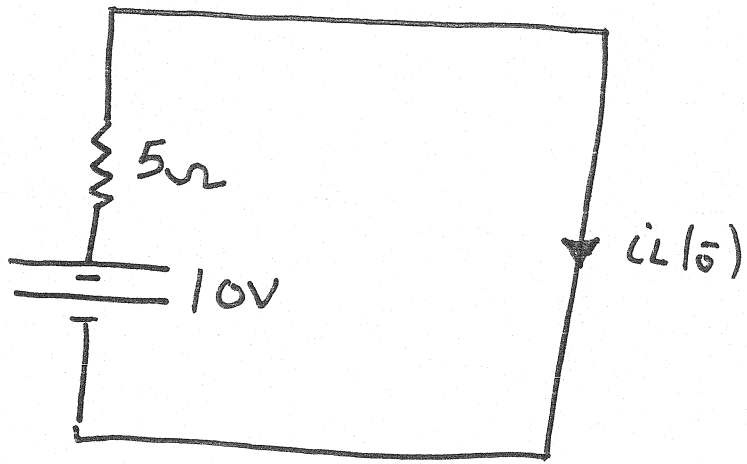


1) for $t < 0$; $t = 0^-$



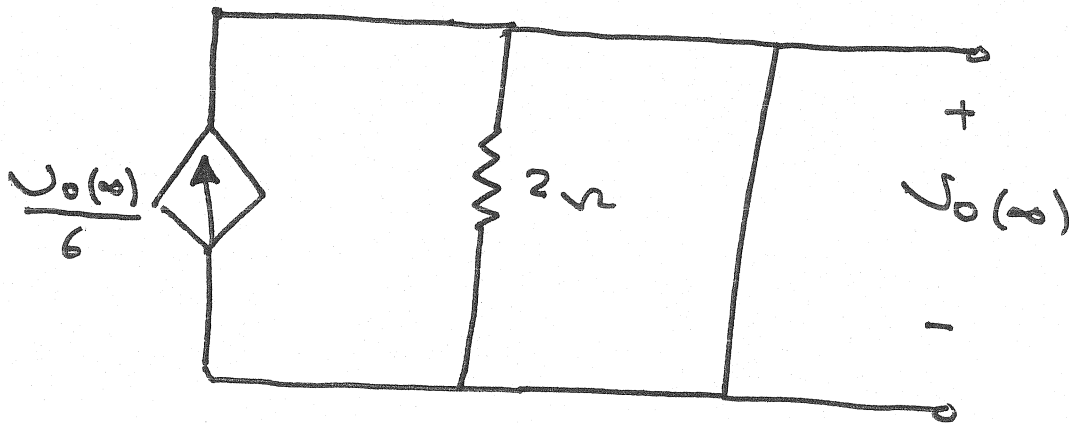
$$\frac{v_o}{6} = 0 \rightarrow \text{open circuit}$$

$$2\Omega \parallel 0 = 0 \rightarrow 2\Omega \rightarrow \text{open circuit}$$



$$\therefore i_L(0) = \underline{2A}$$

2) At $t = \infty$

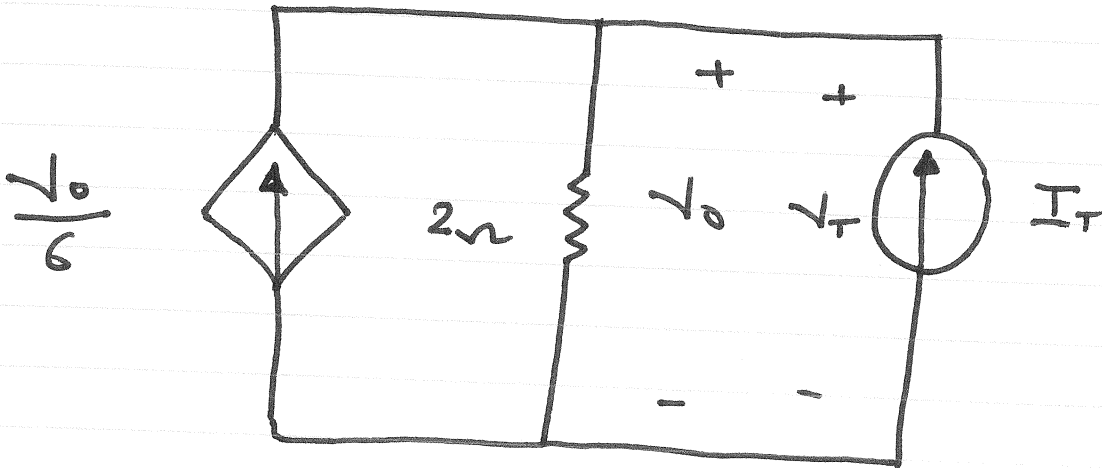


$$\therefore V_0(\infty) = 0$$

3) To find τ

$$\tau = \frac{L}{R_{TH}}$$

$$R_{TH} = \frac{V_T}{I_T}$$



KCL :

$$\frac{V_0}{6} + I_T = \frac{V_0}{2}$$

$$V_0 = V_T$$

$$\therefore \frac{V_T}{I_T} = 3 \Omega = R_{TH}$$

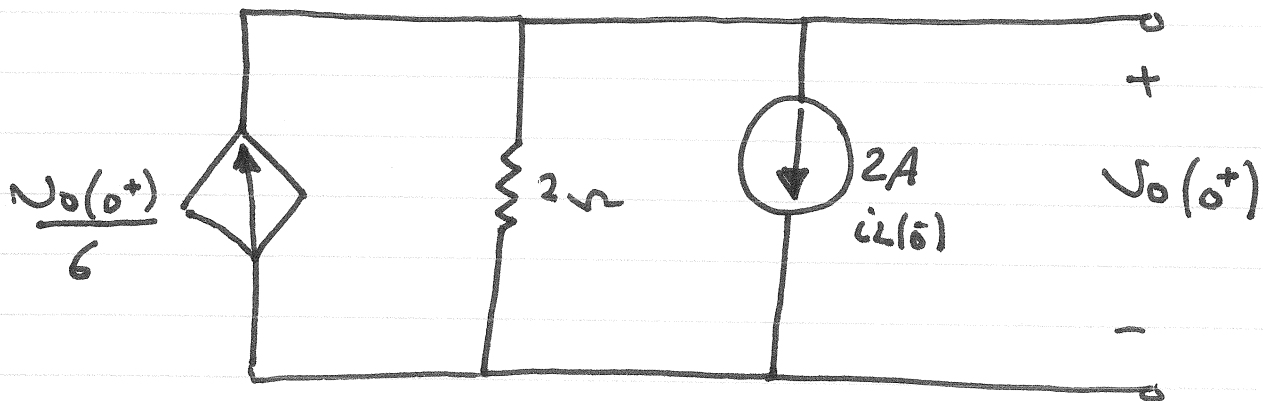
$$\therefore \tau = \frac{L}{R_{TH}} = \frac{2}{3} \text{ sec}$$

$$V_0(t) = V_0(\infty) + [V_0(0^+) - V_0(\infty)] e^{-t/\tau}, t > 0$$

$$\therefore V_0(t) = -6 e^{-1.5t} \text{ A}; t > 0$$

. 50° .

4) at $t = 0^+$



KCL :

$$\frac{V_0(0^+)}{6} = 2 + \frac{V_0(0^+)}{2}$$

$$\therefore V_0(0^+) = -6\text{V}$$

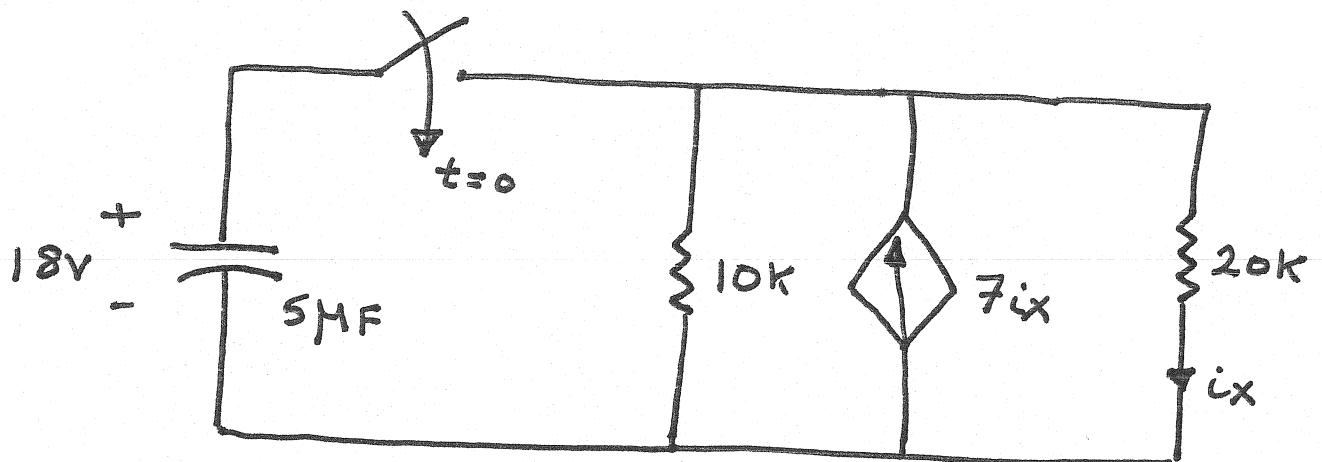
Now

$$V_0(t) = V_0(\infty) + [V_0(0^+) - V_0(\infty)] e^{-t/\tau}, \quad t > 0$$

$$\therefore V_0(t) = -6 e^{-t/\tau} \text{ A} ; t > 0$$

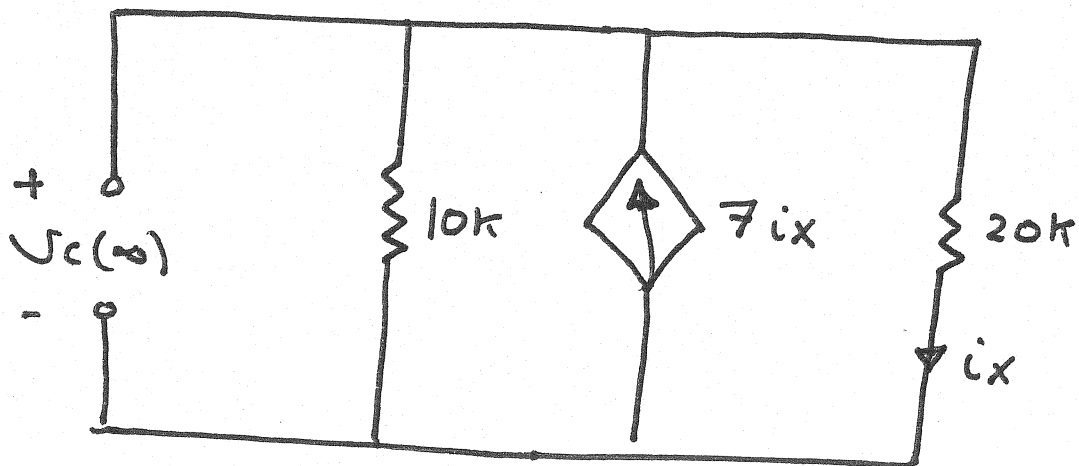
$$\therefore V_0(t) = -6 e^{-1.5t} \text{ A} ; t > 0$$

Unbounded Response



Find $V_C(t)$ for $t > 0$

- 1) $V_C(0^+) = V_C(0) = 18V$
- 2) To find $V_C(\infty)$



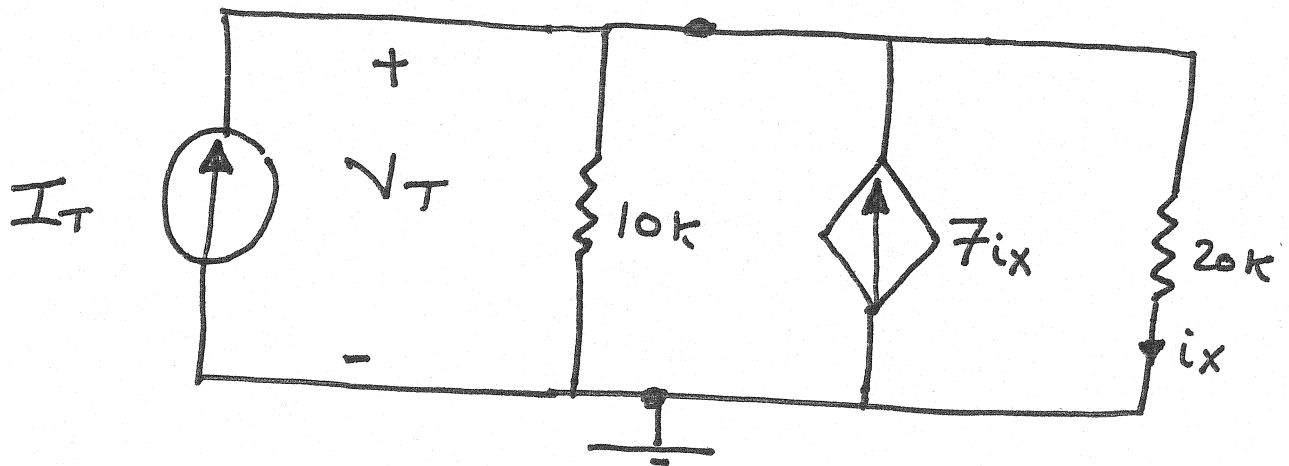
Since the circuit is dead

$$\therefore V_C(\infty) = 0$$

3) To find τ

$$\tau = R_{TH} C$$

$$R_{TH} = \frac{V_T}{I_T}$$



KCL :

$$I_T + 7i_x = \frac{V_T}{10k} + \frac{V_T}{20k}$$

$$i_x = \frac{V_T}{20k}$$

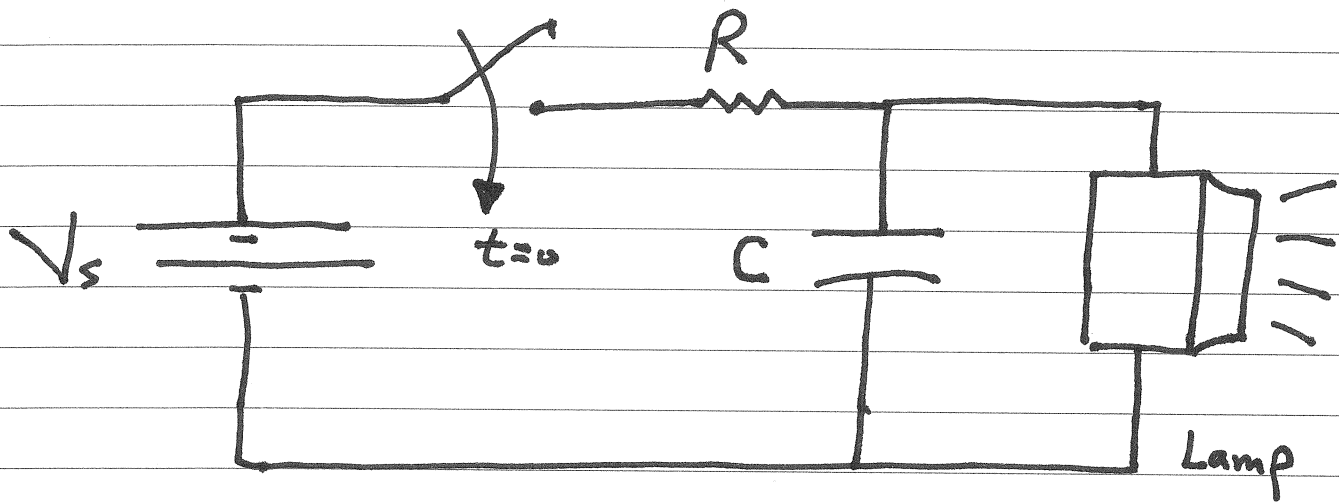
$$\therefore \frac{V_T}{I_T} = -5k = R_{TH} \quad *$$

$$\therefore \tau = R_{TH} C = -25ms$$

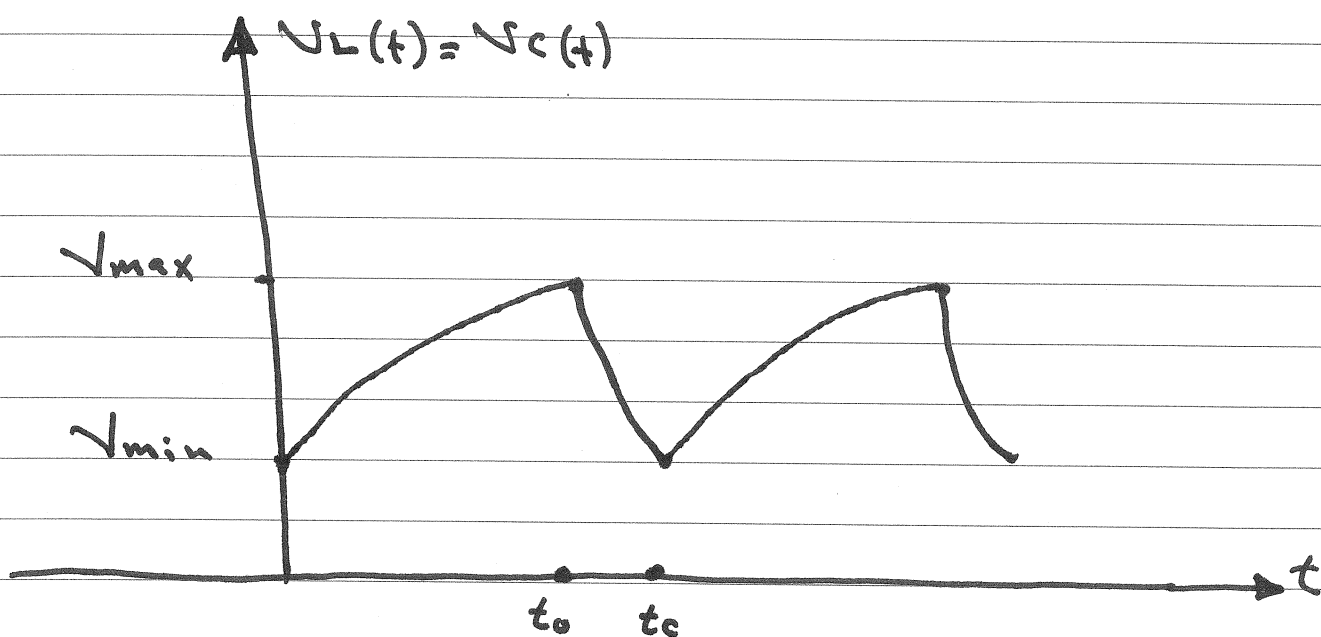
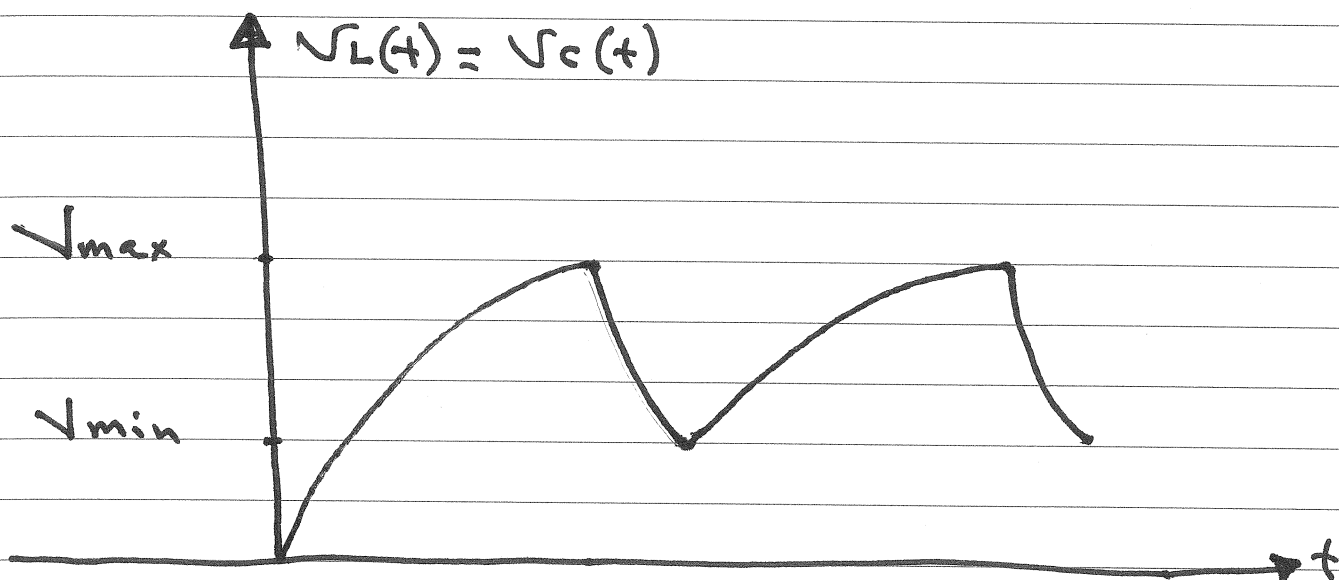
$$\therefore V_C(t) = 18 e^{+40t} \quad \forall \quad \text{for } t \geq 0$$

Practical Perspective

A Flashing Light Circuit



- The Lamp starts to Conduct whenever the Lamp voltage reaches V_{max}
- During the time the Lamp Conducts, it can be modeled as R_L
- The Lamp will Continue to Conduct until the Lamp voltage drops to V_{min}
- During the time the lamp is not conducting, it can be modeled as open circuit.



1) In the interval $t_0 > t > 0$

$$V_L(t) = V_s + (V_{\min} - V_s) e^{-t/\tau_1}$$

$$\tau_1 = RC, \quad R_L = \infty$$

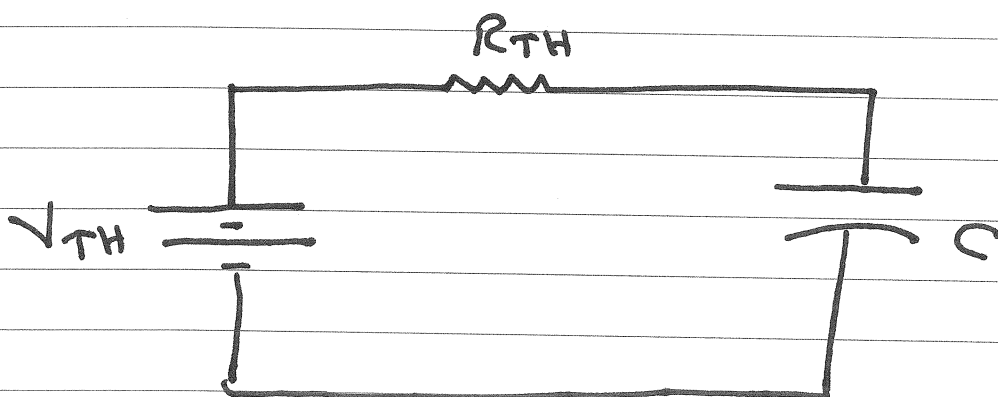
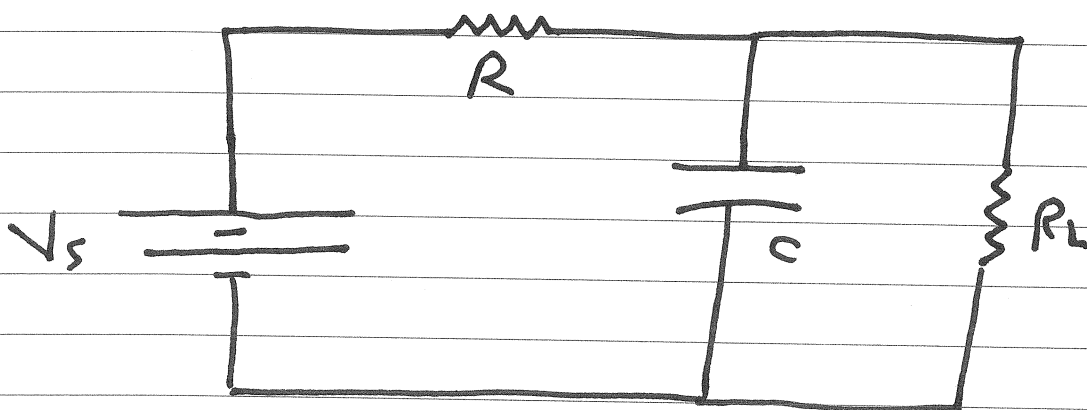
$$V_L(t) = V_s + (V_{\min} - V_s) e^{-t/\tau_1}$$

at $t = t_0$; $V_L(t_0) = V_{\max}$

$$V_{\max} = V_s + (V_{\min} - V_s) e^{-t/RC}$$

$$\therefore t_0 = RC \ln \frac{V_{\min} - V_s}{V_{\max} - V_s}$$

2) In the interval $t_c > t > t_0$



$$R_{TH} = R \parallel R_L$$

$$V_{TH} = \frac{R_L}{R_L + R_{TH}} V_s$$

$$V_L(t) = V_{TH} + (V_{max} - V_{TH}) e^{-\frac{(t-t_0)}{\tau_2}}$$

$$\tau_2 = R_{TH}C = \frac{R R_L}{R_L + R} C$$

at $t = t_c$; $V_L(t_c) = V_{min}$

$$V_{min} = V_{TH} + (V_{max} - V_{TH}) e^{-\frac{(t_c - t_0)}{\tau_2}}$$

$$\therefore t_c - t_0 = \frac{R R_L}{R + R_L} C \ln \frac{V_{max} - V_{TH}}{V_{min} - V_{TH}}$$