UNIVERSITY OF TORONTO
Faculty of Applied Science and Engineering
ECE 310 F — Signals and Systems
Final Examination Dec, 8, 2003

Examination Type A: Closed Book
Calculator Type 3: Non-programmable calculators allowed
Aid Sheet: One double-sided 8in×11in aid sheet allowed
Instructors: Ravi Adve, Stephen Davies, Stark Draper, Wei Yu
Duration — 2.5 hours

Last Name: ______________________________

First Name: ______________________________

Student Number: |_|_|_|_|_|_|_|_|_|

*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below.)

This exam consists of 4 questions on 15 pages (including this one). *When you receive the signal to start, please make sure that your copy of the examination is complete.*

Answer all questions. Unless otherwise stated, for full credit, solutions must show your reasoning. The questions are NOT in increasing order of difficulty.

Write your student number at the bottom of pages 2-15 of this test.

# 1:	______ / 25
# 2:	______ / 25
# 3:	______ / 25
# 4:	______ / 25
TOTAL:	______ /100

Good Luck!

Question 1. [25 MARKS]

All parts of this question consider the discrete-time linear time-invariant system with impulse response:

$$h[n] = 2^{-n}u[n]$$

Part (a) [4 MARKS]

Is the LTI system stable? Causal? Give brief explanations.

Part (b) [6 MARKS]

Suppose that the input to the system is $x_1[n] = u[n] - u[n-2]$. Sketch and label the output $y_1[n]$.

Part (c) [10 MARKS]

Construct an input signal $x_2[n]$ such that the output $y_2[n] = \delta[n]$.

Part (d) [5 MARKS]

Construct an input signal $x_3[n]$ such that the output $y_3[n] = u[n]$.

Question 2. [25 MARKS]

Note: In some classes, the frequency variable in the discrete time Fourier transform was written as Ω, not ω. Use the protocol you are comfortable with. In this question, it is written as ω.

Part (a) [4 MARKS]

A discrete time low pass filter is designed such that, in the region $\omega \in (-\pi, \pi)$, its transfer function (frequency response) $H(e^{j\omega})$ is defined by

$$H(e^{j\omega}) = \begin{cases} 1 - \frac{2|\omega|}{\pi} & \text{if} \quad |\omega| \le \pi/2 \\ 0 & \text{if} \quad |\omega| > \pi/2 \end{cases}$$

Sketch the transfer function (frequency response) in the region $\omega \in (-5\pi, 5\pi)$.

Part (b) [3 MARKS]

Is the system described in part (a) causal, i.e., $h[n] = 0$ for $n < 0$? Briefly explain your answer.

Part (c) [10 MARKS]

The figure below illustrates a discrete time system composed of two individual systems. The output, $y[n]$, is the sum of the two individual outputs, i.e. $y[n] = y_1[n] + y_2[n]$.

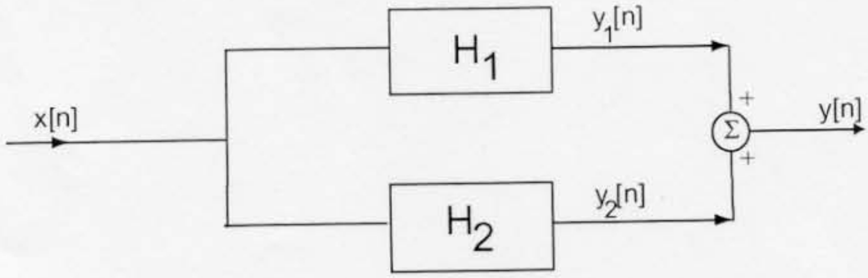


Figure 1: Block diagram for Question 2, Part (c).

The two individual system transfer functions are:

$$\begin{aligned} H_1(e^{j\omega}) &= \frac{1}{1 - 0.5e^{-j(\omega+\pi/3)}} \\ H_2(e^{j\omega}) &= \frac{1}{1 - 0.5e^{-j(\omega-\pi/3)}} \end{aligned}$$

Find the linear constant-coefficient difference equation relating the input $x[n]$ and the <u>*overall*</u> output $y[n]$. *The coefficients are real and must be written as such. Note:* $e^{j\pi/3} = \cos(\pi/3) + j\sin(\pi/3) = 0.5 + j\sqrt{3}/2$.

Part (d) [8 MARKS]

What is the impulse response $h[n]$ for the overall system in part (c)?

Question 3. [25 MARKS]

As sketched in Figure 2, the signal $x(t)$ has a continuous-time Fourier transform

$$X(j\omega) = \begin{cases} 1 - \frac{|\omega|}{2\omega_M} & \text{if } |\omega| < \omega_M \\ 0 & \text{if } |\omega| \geq \omega_M \end{cases},$$

where $\omega_M = 2\pi \times 10^4$ radians/second.

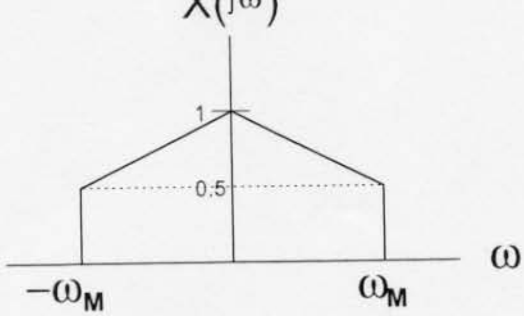


Figure 2: The spectrum of $x(t)$.

As shown in Figure 3, the signal $x(t)$ is sampled by the impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$, producing $x_p(t) = x(t)p(t)$.

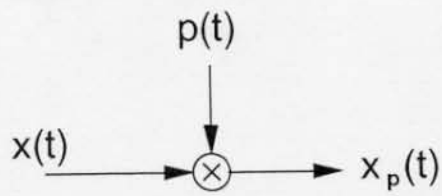


Figure 3: The impulse train sampler producing $x_p(t) = x(t)p(t)$.

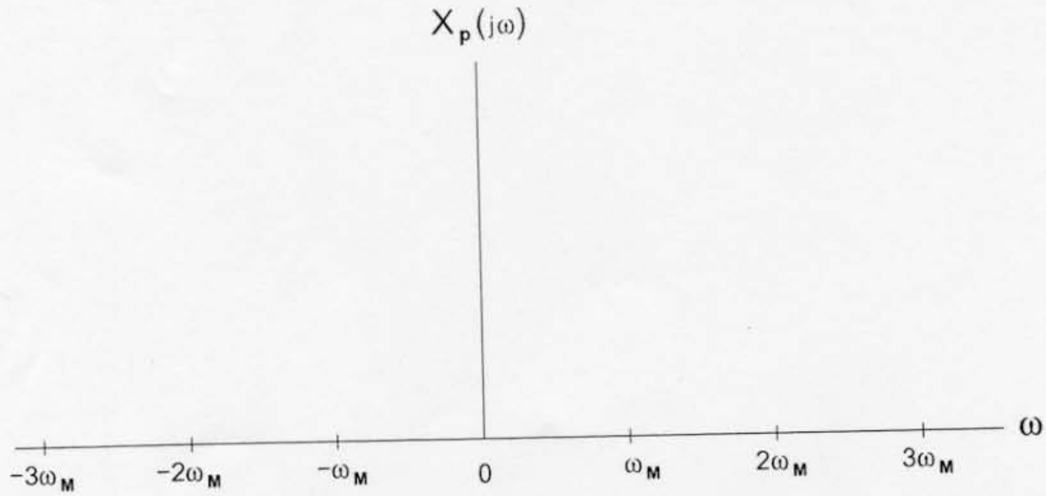
Part (a) [4 MARKS]

In terms of seconds, indicate the range of T for which aliasing occurs.

Range of T:

Part (b) [6 MARKS]

Make a detailed and labelled plot of $X_p(j\omega)$ for $-3\omega_M \leq \omega \leq 3\omega_M$, when $T = (2/3) \times 10^{-4}$ seconds.



In this part of the question, you will design a discrete-time filter $h_{\text{comp}}[n]$ to compensate for using a non-ideal reconstruction filter $h(t)$. The frequency responses of the two filters are, respectively, $H_{\text{comp}}(e^{j\Omega})$ and $H(j\omega)$, where we distiguish between the discrete-time and continuous-time frequency variables by using Ω in discrete-time and ω in continuous-time.

Figure 4 shows the overall system. The input is sampled by an impulse train, giving $x_p(t) = x(t)p(t)$ where $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$. The first converter outputs $x_d[n] = x(nT)$. This is the input to the compensation filter, giving output $y_d[n] = x_d[n] * h_{\text{comp}}[n]$. The output of the second converter is $y_p(t) = \sum_{n=-\infty}^{\infty} y_d[n]\delta(t - nT)$, which is processed by the reconstruction filter $h(t)$, giving $x_r(t) = y_p(t) * h(t)$.

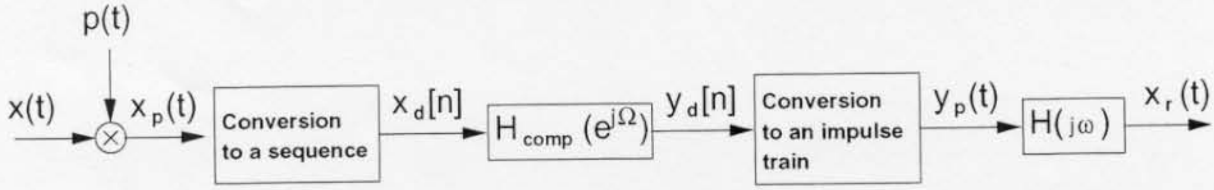
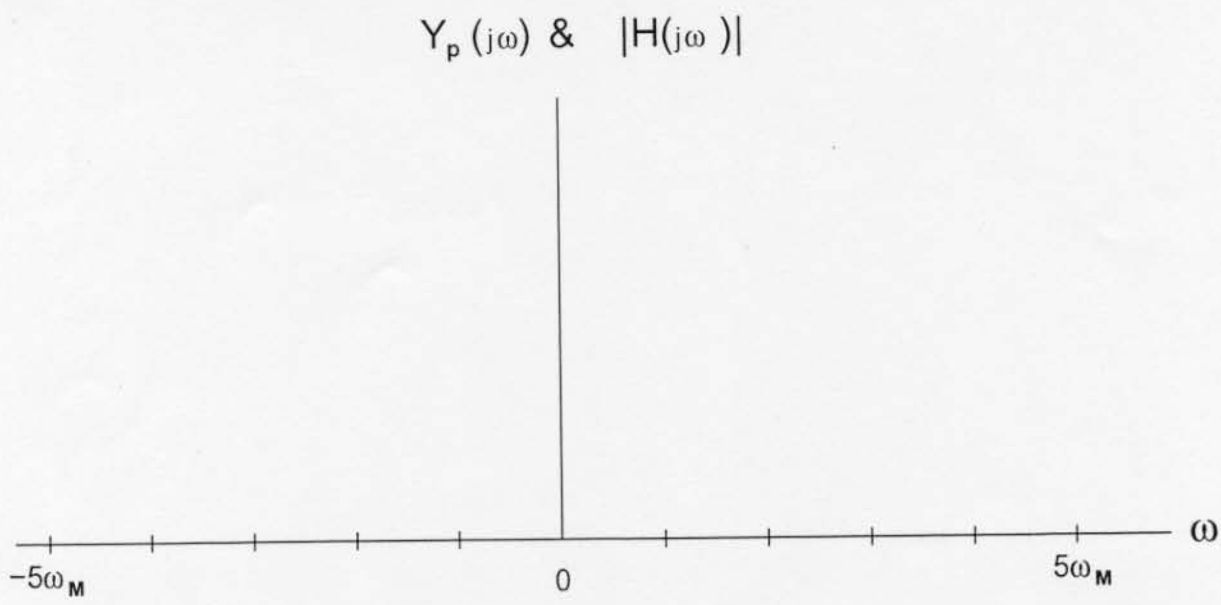


Figure 4: The system considered in this problem.

The input signal is bandlimited so that $X(j\omega) = 0$ if $|\omega| > \omega_M$, and the reconstruction filter $H(j\omega)$ has an impulse response $h(t) = 5\omega_M e^{-5\omega_M t}u(t)$.

Part (c) [7 MARKS]

First, consider the case when $h_{\text{comp}}[n] = \delta[n]$. Given a sampling time $T = \pi/(2\omega_M)$, and using the spectrum of $x(t)$ given in Figure 2, sketch $Y_p(j\omega)$ for $-5\omega_M < \omega < 5\omega_M$ on the axes provided below. On the same axes sketch the magnitude of the Fourier transform $|H(j\omega)|$ of the reconstruction filter. Label your sketches, and show any supporting work on the next page.



(Put your work supporting your answer to part (c) on this page.)

Part (d) [5 MARKS]

For the sampling time $T = \pi/(2\omega_M)$, specify a frequency response $H_{\text{comp}}(e^{j\Omega})$ so that $X_r(j\omega) = X(j\omega)$ for $|\omega| \leq \omega_M$.

$H_{\text{comp}}(e^{j\Omega}) =$

Part (e) [3 MARKS]

Is $x_r(t) = x(t)$? Make sure to justify your answer – no credit without justification.

Question 4. [25 MARKS]

Part (a) [13 MARKS]

The filter,

$$H(j\omega) = \begin{cases} j & \text{if} \quad \omega \le 0 \\ -j & \text{if} \quad \omega > 0 \end{cases}$$

is used as shown in Figure 5 to create a new signal $p(t)$ from $x(t)$. Assume that $x(t)$ is real and bandlimited to ω_M, which is much less than ω_c. Derive $P(j\omega)$ in terms of $X(j\omega)$. Show your work.

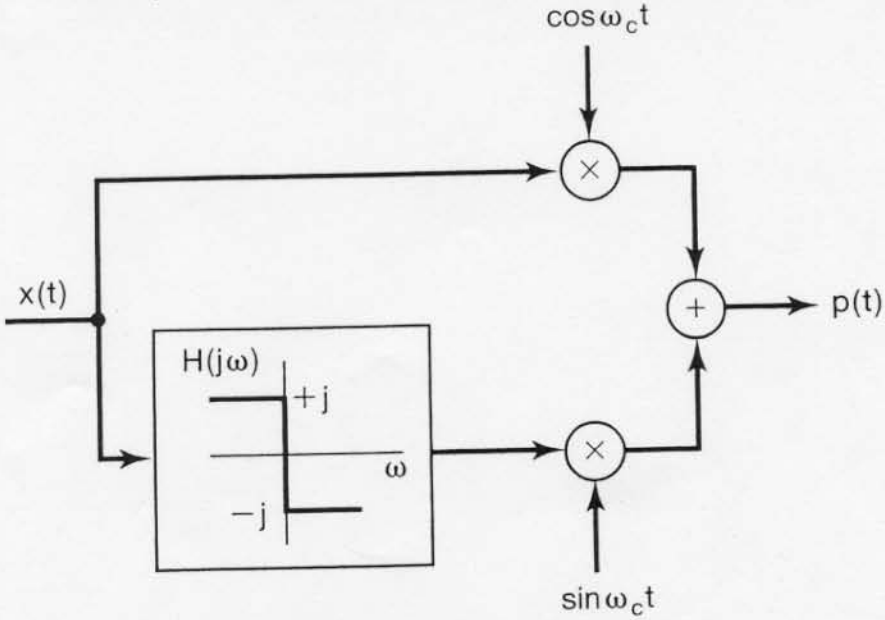


Figure 5: Block diagram for Question 4, Part (a).

Response to 4a (cont)

Part (b) [8 MARKS]

Devise a receiver to recover $X(j\omega)$ from $P(j\omega)$, i.e., a system that takes $p(t)$ as input and outputs $x(t)$. Draw a block diagram of your receiver. Clearly label key parameters of the receiver and relate them to signal parameters such as carrier frequency ω_c and bandwidth ω_M.

Part (c) [4 MARKS]

Assume that a real band-limited audio signal whose Fourier transform extends from -20 kHz to 20 kHz is to be transmitted by a radio station. Further, let that station share the radio spectrum from 10 Mhz to 20 Mhz with other stations transmitting different audio signals that are also real and band-limited with Fourier transforms extending from -20 kHz to 20 kHz. Each station employs a transmitting scheme as shown in Figure 5, but each uses a different carrier frequency ω_c. What is the maximum number of stations that you can fit into the assigned spectrum without their spectra overlapping? Justify your answer.

Total Marks = 100

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{array}{l}x(t)\\y(t)\end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t}x(t) = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases}a_k = a^*_{-k}\\ \Re e\{a_k\} = \Re e\{a_{-k}\}\\ \Im m\{a_k\} = -\Im m\{a_{-k}\}\\ \lvert a_k\rvert = \lvert a_{-k}\rvert\\ \measuredangle a_k = -\measuredangle a_{-k}\end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases}x_e(t) = \mathcal{E}v\{x(t)\} & [x(t)\text{ real}]\\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t)\text{ real}]\end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{array}{l}x[n]\\y[n]\end{array}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{array}{l}a_k\\b_k\end{array}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n}x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases}x[n/m], & \text{if } n \text{ is a multiple of } m\\ 0, & \text{if } n \text{ is not a multiple of } m\end{cases}$ (periodic with period mN)	$\frac{1}{m}a_k$ (viewed as perio… with period m…)
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k]$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases}a_k = a^*_{-k}\\ \Re e\{a_k\} = \Re e\{a_{-k}\}\\ \Im m\{a_k\} = -\Im m\{a_{-k}\}\\ \lvert a_k\rvert = \lvert a_{-k}\rvert\\ \measuredangle a_k = -\measuredangle a_{-k}\end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases}x_e[n] = \mathcal{E}v\{x[n]\} & [x[n]\text{ real}]\\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n]\text{ real}]\end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega-\theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \mathcal{R}e\{X(j\omega)\} = \mathcal{R}e\{X(-j\omega)\} \\ \mathcal{I}m\{X(j\omega)\} = -\mathcal{I}m\{X(-j\omega)\} \\ \|X(j\omega)\| = \|X(-j\omega)\| \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\mathcal{R}e\{X(j\omega)\}$ $j\mathcal{I}m\{X(j\omega)\}$
4.3.7	Parseval's Relation for Aperiodic Signals	$\int_{-\infty}^{+\infty} \|x(t)\|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \|X(j\omega)\|^2 d\omega$	

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k\delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi\delta(\omega)$	$a_0 = 1,\ a_k = 0,\ k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \|t\| < T_1 \\ 0, & T_1 < \|t\| \leq \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2\sin k\omega_0 T_1}{k}\delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi}\operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t)\begin{cases} 1, & \|t\| < T_1 \\ 0, & \|t\| > T_1 \end{cases}$	$\frac{2\sin\omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \|\omega\| < W \\ 0, & \|\omega\| > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

TABLE 5.1 PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Section	Property	Aperiodic Signal	Fourier Transform
		$x[n]$ $y[n]$	$X(e^{j\omega})$ $Y(e^{j\omega})$ } periodic with period 2π
5.3.2	Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
5.3.3	Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
5.3.3	Frequency Shifting	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
5.3.4	Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
5.3.6	Time Reversal	$x[-n]$	$X(e^{-j\omega})$
5.3.7	Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
5.4	Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
5.5	Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
5.3.5	Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
5.3.5	Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-e^{-j\omega}}X(e^{j\omega}) + \pi X(e^{j0})\sum_{k=-\infty}^{+\infty}\delta(\omega - 2\pi k)$
5.3.8	Differentiation in Frequency	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
5.3.4	Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \mathcal{R}e\{X(e^{j\omega})\} = \mathcal{R}e\{X(e^{-j\omega})\} \\ \mathcal{I}m\{X(e^{j\omega})\} = -\mathcal{I}m\{X(e^{-j\omega})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert \\ \sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega}) \end{cases}$
5.3.4	Symmetry for Real, Even Signals	$x[n]$ real an even	$X(e^{j\omega})$ real and even
5.3.4	Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
5.3.4	Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ [$x[n]$ real] $x_o[n] = \mathcal{O}d\{x[n]\}$ [$x[n]$ real]	$\mathcal{R}e\{X(e^{j\omega})\}$ $j\mathcal{I}m\{X(e^{j\omega})\}$
5.3.9	Parseval's Relation for Aperiodic Signals	$\sum_{n=-\infty}^{+\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{2\pi}\lvert X(e^{j\omega})\rvert^2 d\omega$	

TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi\sum_{k=-\infty}^{+\infty} a_k\delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi\sum_{l=-\infty}^{+\infty}\delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos\omega_0 n$	$\pi\sum_{l=-\infty}^{+\infty}\{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin\omega_0 n$	$\frac{\pi}{j}\sum_{l=-\infty}^{+\infty}\{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \ldots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$2\pi\sum_{l=-\infty}^{+\infty}\delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & N_1 < \lvert n\rvert \le N/2 \end{cases}$ and $x[n + N] = x[n]$	$2\pi\sum_{k=-\infty}^{+\infty} a_k\delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N\sin[2\pi k/2N]}, \quad k \neq 0, \pm N, \pm 2N, \ldots$ $a_k = \frac{2N_1 + 1}{N}, \quad k = 0, \pm N, \pm 2N, \ldots$
$\sum_{k=-\infty}^{+\infty}\delta[n - kN]$	$\frac{2\pi}{N}\sum_{k=-\infty}^{+\infty}\delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & \lvert n\rvert > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \le \lvert\omega\rvert \le W \\ 0, & W < \lvert\omega\rvert \le \pi \end{cases}$ $X(\omega)$ periodic with period 2π	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty}\pi\delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n + 1)a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n + r - 1)!}{n!(r - 1)!}a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—