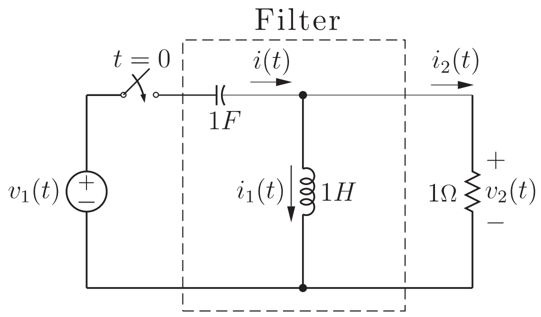
1.

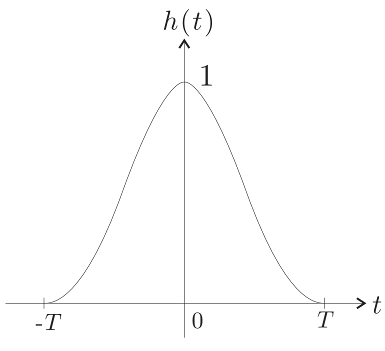


(a) (15) Assume that there is no initial energy at the capacitor and the inductor. When $v_1(t) = u(t)$ $(t \geq 0)$ (*unit-step input*), find $v_2(t)$.

(b) (15) Find $H(j\omega) = \dfrac{V_2(j\omega)}{V_1(j\omega)}$, and plot $|H(j\omega)|$ in original ω-scale.

2. (20) Assume that the sampling frequency is large compared with the Nyquist rate, i.e. $f_s \gg 2f_h$. The unit impulse response of a filter is given by

$$h(t) = \begin{cases} \frac{1}{2}\left(1 + \cos\frac{\pi t}{T}\right), & |t| < T \\ 0, & \text{otherwise} \end{cases}$$



Then using the interpolation equation

$$y(t) = \sum_{k=-\infty}^{\infty} x(kT)h(t-kT),$$

determine the interpolation formula for the output, corresponding to this filter subject to the input $\{x(nT)\}$.

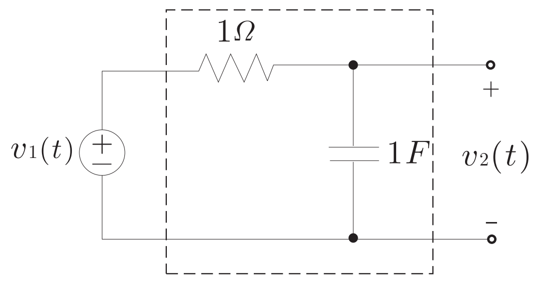
3. (a) (10) Given a time series

$$x(n) = \sum_{k=0}^{\infty} x_1(k) x_2(n-k),$$

find its z-transform $X(z)$.

(b) (10) Given $X(z) = \dfrac{2 \cdot z^{-2}}{1 + z^{-2}}$, find its time sequence $\{x(k)\}$, in closed form.

4.



(a) (5) When initial condition is assumed to be zero (i.e. initial capacitor voltage is zero), find $H_a(s) = \dfrac{V_2(s)}{V_1(s)}$.

(b) (10) Using the step-invariance synthesis procedure, find $H(z)$.

(c) (10) Using the bilinear z-transform, find $H(z)$ having the same 3-dB bandwidth as $H_a(s)$. The sampling frequency is $f_s = 10$ Hz.

(d) (5) Plot $|H_a(j\omega)|$ and $|H(e^{j\omega T})|$, where T is the sampling period. Use $H(z)$ in (c), and any scale will be O.K.