1. (a) $X(e^{j0})$

$$X(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n] = 0$$

(b) $arg\{X(e^{j\Omega})\}$

$x[n]$ is a real and odd function shifted by 2 to the right, i.e. $x[n] = x_o[n-2]$.
Since $x_o[n]$ is real and odd, $X_o(e^{j\Omega})$ is purely imaginary, thus

$$X(e^{j\Omega}) = |X_o(e^{j\Omega})|e^{-j2\Omega}e^{j\frac{\pi}{2}}$$

which means,

$$\arg\{X(e^{j\Omega})\} = \frac{\pi}{2} - 2\Omega$$

(c)

$$\int_{-\pi}^{\pi} |X(e^{j\Omega})|^2 d\Omega = 2\pi \sum_{n=-\infty}^{\infty} |x[n]|^2 = 28\pi$$

(d)

$$\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j3\Omega} d\Omega = 2\pi x(3) = -2\pi$$

(e)

$$y[n] \;\leftrightarrow\; Re\{e^{j2\Omega}X(e^{j\Omega})\} \;\text{ by }\; DTFT.$$

2.

$$x_s(t) = x(t)p(t) = x(t)\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

But since $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ is periodic, it can be represented as a Fourier Series:

$$p(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn2\pi f_s t},$$

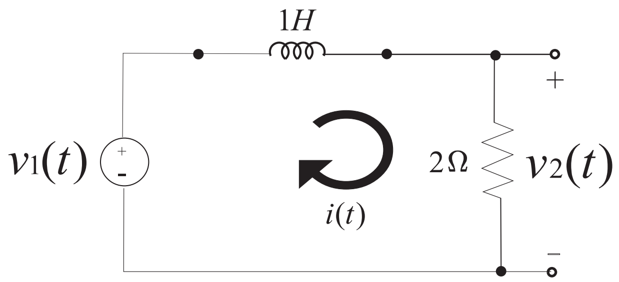
where

$$\begin{aligned} C_n &= \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} p(t)e^{-jn2\pi f_s t}dt \\ &= \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} \delta(t)e^{-jn2\pi f_s t}dt = \frac{1}{T_s} = f_s \end{aligned}$$

$$\therefore p(t) = \sum_{n=-\infty}^{\infty} f_s e^{jn2\pi f_s t} \text{ and } x_s(t) = x(t) \sum_{n=-\infty}^{\infty} f_s e^{jn2\pi f_s t}$$

Then

$$\begin{aligned} X_s(f) &= \int_{-\infty}^{\infty} x_s(t) e^{-j2\pi f t} dt = \int_{-\infty}^{\infty} f_s \cdot x(t) \sum_{n=-\infty}^{\infty} f_s e^{jn2\pi f_s t} e^{-j2\pi f t} dt \\ &= \sum_{n=-\infty}^{\infty} f_s \int_{-\infty}^{\infty} x(t) e^{-j2\pi(f-nf_s)t} dt \\ &= \sum_{n=-\infty}^{\infty} f_s X(f - nf_s) \end{aligned}$$



3. (a)

$$\begin{aligned} & L\frac{di}{dt} + Ri = v_1 \Rightarrow 1 \cdot \frac{di}{dt} + 2 \cdot i = v_1 \\ & sI(s) + 2I(s) = V_1(s) \quad (\text{assuming} \;\; i(0) = 0) \\ & I(s) = \frac{1}{s+2} V_1(s) \\ & V_2(s) = 2I(s) = \frac{2}{s+2} V_1(s) \\ & H_a(s) = \frac{V_2(s)}{V_1(s)}|_{i(0)=0} \\ & = \frac{2}{s+2} = \frac{1}{1+\frac{s}{2}} \end{aligned}$$

(b)

$$H_a(j\omega) = \frac{1}{1 + \frac{j\omega}{2}}$$

$\therefore \omega_{3dB} = 2$ is the 3-dB dropping frequency.

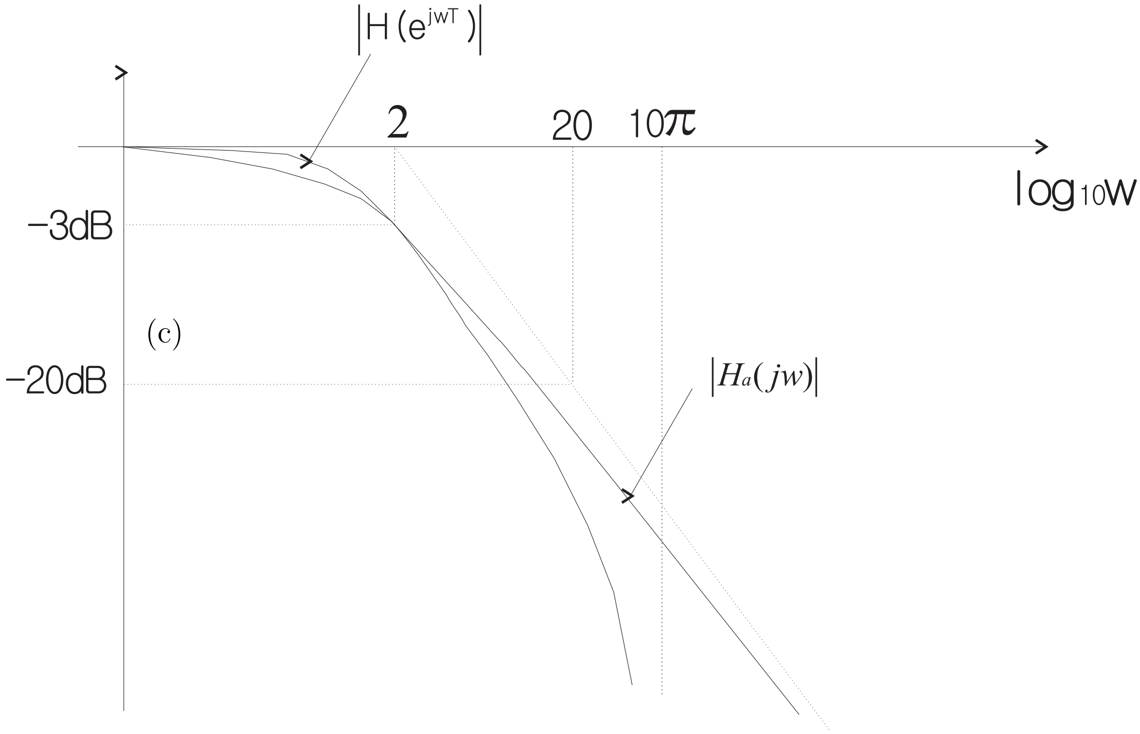
On the other hand, for bilinear z-transform,
we set

$$s = C \cdot \frac{1 - z^{-1}}{1 + z^{-1}} \quad ,$$

where C is to be determined to have the same 3-dB bandwidth.

$$\therefore c = \omega_r \cot(\frac{\omega_r T}{2}) = 2\cot(\frac{2\cdot(\frac{1}{10})}{2}) \simeq 2(\frac{1}{0.1}) = 20$$

$$\begin{aligned}\therefore H(z) &= \frac{1}{1+\frac{1}{2}(20\frac{1-z^{-1}}{1+z^{-1}})} \\ &= \frac{1}{1+(10)\frac{1-z^{-1}}{1+z^{-1}}} \\ &= \frac{1+z^{-1}}{(1+z^{-1})+(10)(1-z^{-1})} \\ &= \frac{1+z^{-1}}{11-9z^{-1}}\end{aligned}$$



(d)

$$\begin{aligned}
H(z) &= \mathscr{Z}(\mathscr{L}^{-1}\{\frac{1-e^{-sT_s}}{s}H_a(s)\}|_{t=nT_s}) \\
&= (1-z^{-1})\mathscr{Z}(\mathscr{L}^{-1}(\frac{H_a(s)}{s})|_{t=nT_S}) \\
&= (1-z^{-1})\mathscr{Z}(\mathscr{L}^{-1}(\frac{2}{s(s+2)})|_{t=nT_S}) \\
&= (1-z^{-1})\mathscr{Z}(\mathscr{L}^{-1}(\frac{1}{s}-\frac{1}{s+2})|_{t=nT_S}) \\
&= (1-z^{-1})\mathscr{Z}((u(t)-e^{-2t}u(t))|_{t=nT_S}) \\
&= (1-z^{-1})\mathscr{Z}(u(nT_s)-e^{-2nT_s}u(nT_s)) \\
&= (1-z^{-1})(\frac{1}{1-z^{-1}}-\frac{1}{1-e^{-2T_s}z^{-1}}) \\
&= 1-\frac{1-z^{-1}}{1-e^{-2T_s}z^{-1}}
\end{aligned}$$

4. (a)

$$\begin{aligned}
y(t) &= \int_{t-T_s}^{t} x(\tau)d\tau \quad \text{(by inspection)} \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= x(t) * h(t)
\end{aligned}$$

$$\text{choose} \quad h(t) = \begin{cases} 1 & 0 \le t \le T_s \\ 0 & otherwise \end{cases}$$

$$h(t-\tau) = \begin{cases} 1 & t-T_s \le \tau \le T_s \\ 0 & otherwise \end{cases}$$

$$h(t) = u(t) - u(t-T_s)$$

(b)

$$Y(j\omega) = X(j\omega)H(j\omega)$$

$$y(nT_s) \leftrightarrow \frac{1}{T_s}\sum_{k=-\infty}^{\infty} Y(j(\omega - k\frac{2\pi}{T_s})) \quad \text{(by} \quad FT)$$

$$FT\{x[n]\} = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j(\omega - k\frac{2\pi}{T_s}))H(j(\omega - k\frac{2\pi}{T_s}))$$