EE 261 The Fourier Transform and its Applications
Fall 2005
Midterm Exam

- There are four questions for a total of 90 points.
- Please write your answers in the exam booklet provided, and make sure that your answers stand out.
- Don't forget to write your name on your exam book!

1. (25 points)*Finding Fourier transforms:* The following three questions are independent.

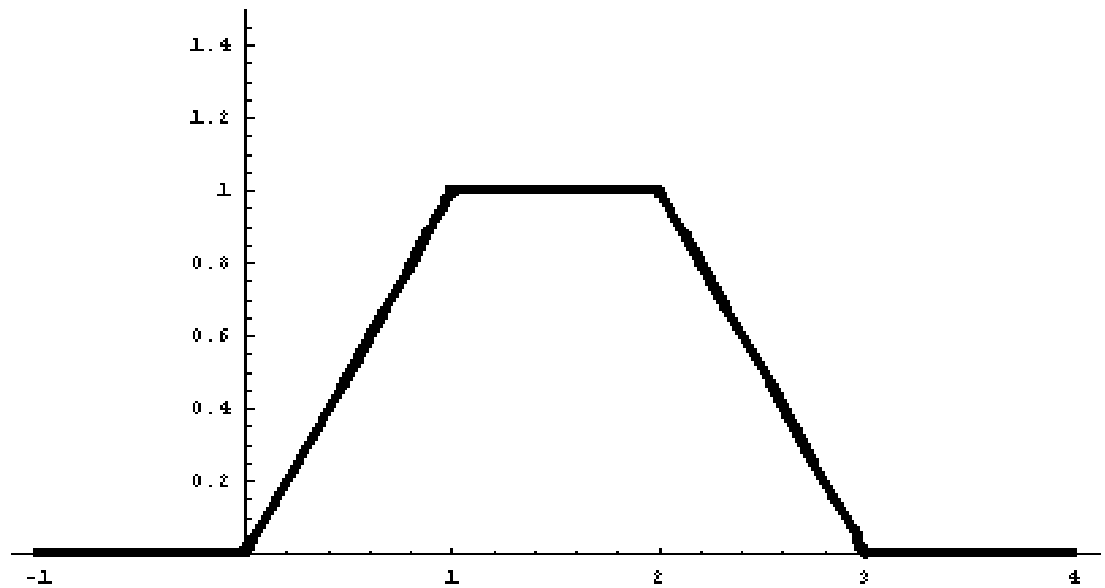
(a)(5) In communications theory the *analytic signal* $f_a(t)$ of a signal $f(t)$ is defined, via the Fourier transform, by

$$\mathcal{F}f_a(s) = \begin{cases} \mathcal{F}f(s), & s \geq 0, \\ 0, & \text{otherwise}. \end{cases}$$

For a real-valued signal $f(t)$, that is not identically zero, could the corresponding analytic signal $f_a(t)$ also be real? Why or why not?

(b)(10) Compute the Fourier transform of $f(x) = \cos(\pi x)\Pi(x)$, which is a half-cycle of a cosine.

(c)(10) Use the derivative theorem for Fourier transforms to find the Fourier transform of the function sketched below.



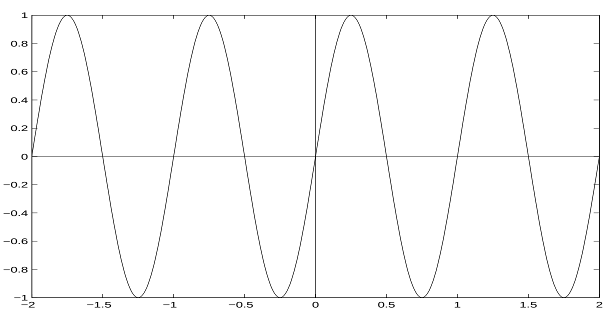
2. (30 points) *Finding Fourier series: Sometimes one is enough.*

(a)(5) Let $f(t)$ be periodic of period 1 with Fourier coefficients $\hat{f}(n)$, $n = 0, \pm 1, \pm 2, \ldots$. Let $g(t) = f(t - a)$. Express $\hat{g}(n)$ in terms of $\hat{f}(n)$.

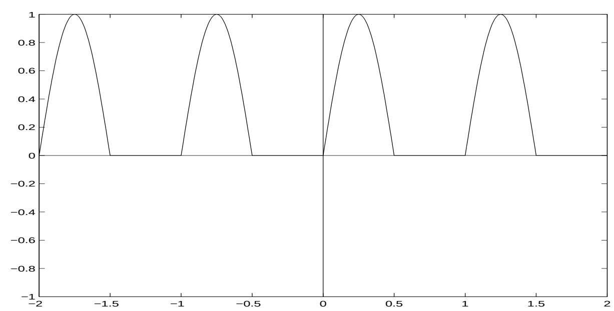
(b)(25) Signals $f_1(t), \ldots, f_6(t)$, together with their graphs, are as below. Your task is to find the Fourier series for each of these six signals, *but* you are allowed to compute *only one* Fourier series and you must deduce the others. For this, you choose one signal to work with and you express the remaining five signals in terms of the one you chose. Carry this out, explaining how the Fourier coefficients of each of the five remaining signals are related to the Fourier coefficients of the one you chose. (You will need part (a).)

Note! You *do not* have to compute the Fourier coefficients of the signal you choose, you only have to give expressions *relating* the Fourier coefficients of the other signals to those of the one you chose.

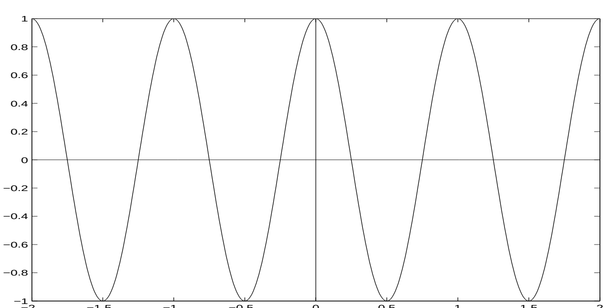
$f_1(t) = \sin(2\pi t)$



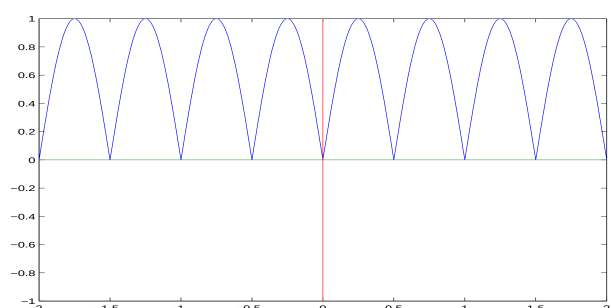
$f_2(t) = \max\{\sin(2\pi t), 0\}$



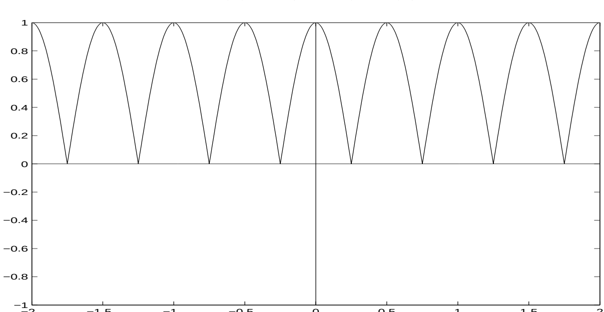
$f_3(t) = \cos(2\pi t)$



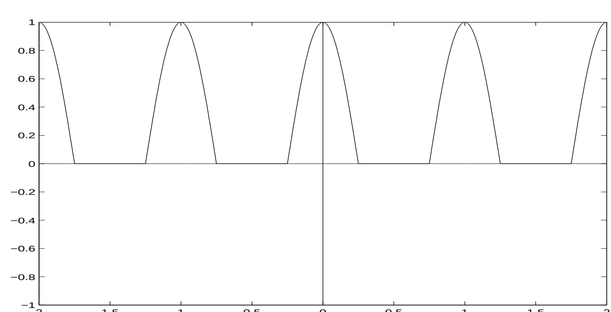
$f_4(t) = |\sin(2\pi t)|$



$f_5(t) = |\cos(2\pi t)|$



$f_6(t) = \max\{\cos(2\pi t), 0\}$



3. (15 points) *Could be convolution:* The following two questions are independent.

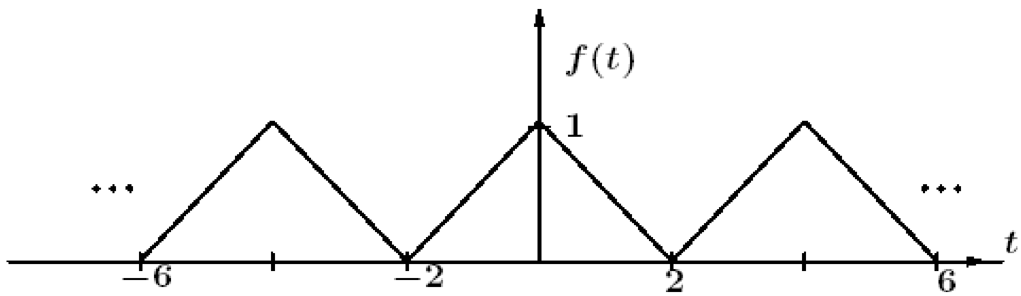
(a)(5) Find all functions $f(t)$ satisfying $(f * f)(t) = e^{-\pi t^2}$.

(b)(10) Solve the equation

$$x(t) + 4\int_{-\infty}^{\infty} e^{-|\tau|} x(t-\tau)\, d\tau = e^{-|t|}\,.$$

4. (20 points) *Periodizing and filtering: Smoothing a triangle train.*

(a)(10) Find the Fourier transform of the periodic function $f(t)$ sketched below; the period is 4 and three complete cycles are shown. Sketch $\mathcal{F}f(s)$ for $-0.5 \leq s \leq 0.5$.



(b)(10) We filter $f(t)$ by multiplying its Fourier transform $\mathcal{F}f(s)$ by the function $H(s)$ sketched below. Find the filtered version of $f(t)$; that is, find $g(t) = (h * f)(t)$ where $h = \mathcal{F}^{-1}H$.