Birzeit University-Faculty Of Engineering
Electrical Engineering Department
EE334-Signals& Systems
Final Exam

Inst.: Jamal Siam — Summer semester 2009

Question I(5+5+(4+6)+6):

1. Determine if the difference of $x(t) = 15\sin 18\pi t$ and $y(t) = 27\cos(72\pi t)$ is periodic and determine the fundamental frequency if exists
2. Compute the folowing integral $m(t) = \int_{-15}^{20} 10e^{2|t|}\,\dot{\delta}\,(t-10)dt$
3. Consider the following signal $s(t) = \sum_{k=-\infty}^{\infty} 10r(t-2-8k)u(5-t-8k)$ and
   - Plot the signal
   - Determine if it is an energy signal ,power signal or not then compute the energy or the power of the signal if exists
4. Consider the following system $y(t) = e^{[x(\cos(t))]}$ and determine if it is linear , time invariant.

Question II (10+10+5 +5 points):

1. Given the impulse response of the system M : $h(t) = 10\,r(t-4)\pi(\frac{t-7}{2})$, find using the convolution integral the response of the system to the input $x(t) = 2\pi(\frac{t-2}{2})$
2. Consider $p(t) = e^{-.05t}\cos(100\pi t)u(t)$
   - Find the Fourier transform of p(t)
   - Determine the energy spectral density function of p(t)
   - Determine the band(80%) that is the frequency range for which the energy equals 80% of the total energy.

Question III(5+5+8+5 points):

1. The signal $b(t) = 15\cos(25\pi t) + 10\cos(32\pi t)$ is sampled with a sampling frequency of 50 Hz
   - Plot the spectra of the sampled signal and determine if the signal has aliasing and determine the frequency components that prohibit the signal reconstruction with ideal L.P.F.
   - Determine the minimum frequency of sampling (if exists) so that the signal can be reconstructed and determine the impulse response of the ideal reconstruction filter
2. Consider the discrete system with impulse response $h(n) = \left(\frac{1}{4}\right)^n u(n) - \left(\frac{1}{8}\right)^n u(n-4)$
   - Determine using the convolution sum the response y(n) of the system to the input $x(n) = (2)^n u(n)$
   - Find the transfer function of the system in the z-form

$\frac{Y(n)}{X(n)} = H(n)$

Question IV(10+3+3+5 points):

Consider the discrete system with transfer function $H(z) = \frac{z(3z+2)}{z^2+4z+5}$ and

1. Determine the impulse response of the system
2. Determine if the system is IIR or FIR system.
3. Study the stability of the system not stable
4. Implement the system in direct form

Good Luck

①

$x(t) \cdot y(t) = 15 \sin 18\pi t - 27 \cos(72\pi t)$

$f_1 = 9 \Rightarrow T_1 = \frac{1}{9}$

$f_2 = 31 \Rightarrow T_2 = \frac{1}{36}$

$\frac{T_2}{T_1} = \frac{\frac{1}{31}}{\frac{1}{9}} \cdot \frac{9}{36} = \frac{1}{4} = \frac{N}{M}$

$N T_1 = M T_2 = T_0$

$1 \cdot \frac{1}{9} = T_0 = \frac{1}{9} \Rightarrow f_0 = 9$

②

$m(t) = \int_{-15}^{20} 10\, e^{2t t}\, \dot{\delta}(t-10)\, dt$

$\Rightarrow \frac{d}{dt} 10 e^{2t} \Big|_{t=10}$

$= \frac{20 e^{2t}}{2} \Big|_{t=10} \quad \frac{20\, e^{20}}{2} = 10e$

③

$s(t) = \sum_{k=-\infty}^{\infty} 10\, r(t-2-8k)\, u(5-t-8k)$

(9) $y(t) = e^{x\cos(t)}$.

$x(t) = a_1 x_1(t) + a_2 x_2(t)$.

$y_1(t) = e^{x_1(\cos(t))}$

$y_2(t) = e^{x_2(\cos(t))}$

$y(t) = a_1 y_1 + a_2 y_2$

$= a_1 e^{x_1(\cos(t))} + a_2 e^{x_2(\cos(t))}$.

$= a_1 e^{x_1(\cos(t))} + a_2 e^{x_2(\cos(t))} \neq e^{x\cos t}$

= non linear

time invariant ??

$x(t-t_0) = e^{x(t-t_0)}$.

$y(t-t_0) = e^{x(t-t_0)}$.

time invariant

(9) $y(t) = e^{x\cos(t)}$.

$x(t) = a_1 x_1(t) + a_2 x_2(t)$.

$y_1(t) = e^{x_1(\cos(t))}$

$y_2(t) = e^{x_2(\cos(t))}$

$y(t) = a_1 y_1 + a_2 y_2$

$= a_1 e^{x_1(\cos(t))} + a_2 e^{x_2(\cos(t))}$.

$= a_1 e^{x_1(\cos(t))} + a_2 e^{x_2(\cos(t))} \neq e^{x\cos(t)}$

= non linear

time invariant ??

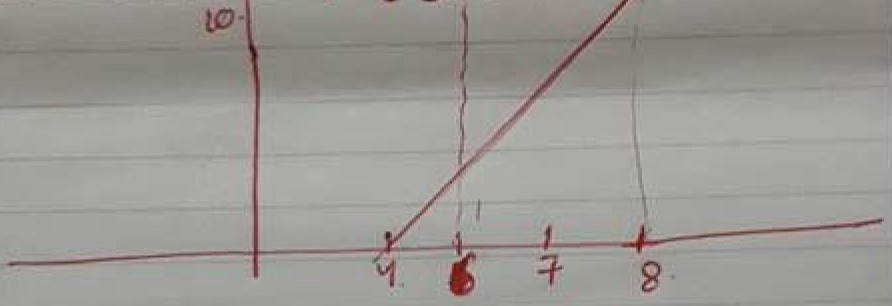
$x(t-t_0) = e^{x(t-t_0)}$.

$y(t-t_0) = e^{x(t-t_0)}$.

time invariant

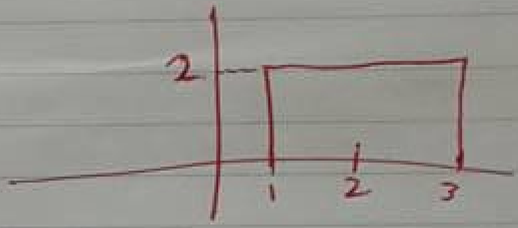
1

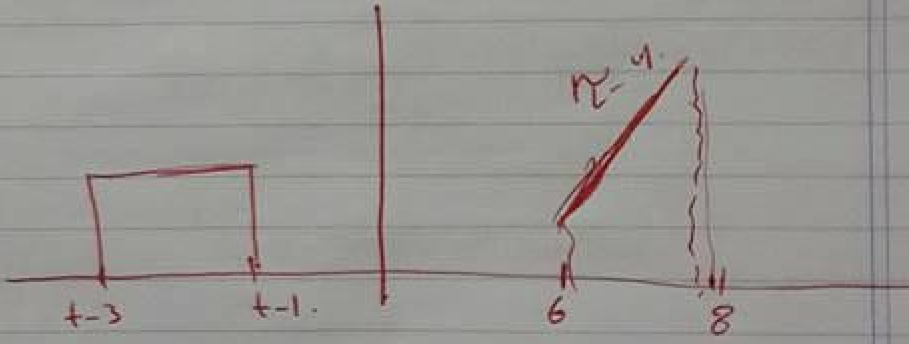
$h(t) = 10r(t-4)\,\Pi\left(\frac{t-7}{2}\right)$



$h(t) = 10(t-4)$

$x(t)$





① $\int_{6}^{t-1} 10(\tau-4)\cdot 2\, d\tau$

② $\int_{t-3}^{8} 10(\tau-4)\cdot 2\, d\tau$

③

6, 8
1, 3

7, 9, 9, 11

الفترات

$7 \rightarrow 9$
$9 \rightarrow 11$

$\boxed{2}$

$$f(t) = e^{-0.5t} \cos(100\pi t)\, u(t)$$

$$F(f) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-0.5t} \cos(100\pi t)\, e^{-j2\pi f t}\, u(t)\, dt$$

$$= \int_{0}^{\infty} \cos(100\pi t)\, e^{-[j2\pi f t + 0.5t]}\, dt$$

$$= \int_{0}^{\infty} \left[\frac{e^{100\pi t}}{2} + \frac{e^{-100\pi t}}{2} \right] e^{-[j2\pi f t + 0.5t]}\, dt$$

$$= \int_{0}^{\infty} \frac{1}{2} \left[e^{-[j2\pi f t + 0.5t - 100\pi t]} + e^{-[j2\pi f t + 0.5t + 100\pi t]} \right]$$

$$= \frac{1}{2} \left[\frac{e^{-[j2\pi f + 0.5 - 100\pi]t}}{-[j2\pi f + 0.5 - 100\pi]} + \frac{e^{-[j2\pi f + 0.5 + 100\pi]t}}{-[j2\pi f + 0.5 + 100\pi]} \right]_{0}^{\infty}$$

$$= \frac{1}{2} \left[0 + \frac{1}{j2\pi f + 0.5 - 100\pi} + \frac{1}{j2\pi f + 0.5 + 100\pi} \right]$$

$$G(x) = \left| P(f) \right|^2 = \frac{1}{\sqrt{(2\pi f)^2 + (0.5 - 100\pi)^2}}$$

$$+ \frac{1}{\sqrt{(2\pi f)^2 + (0.5 + 100\pi)^2}}$$

$\boxed{f_h = 16}$

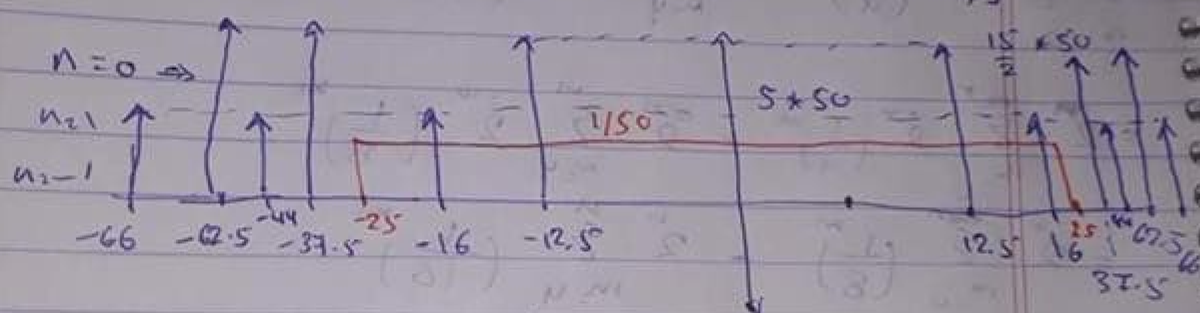
prohibit

[3] $b(t) = 15\cos(25\pi t) + 10\cos(32\pi t)$

- $b(f) = \frac{15}{2}\delta(f-12.5) + \frac{15}{2}\delta(f+12.5) + 5\,\delta(f-16) + 5\,\delta(f+16)$

$b_s(f) = f_s \sum_{n=-\infty}^{\infty} b(f - n f_s) \qquad f_s = 50$

$= 50\Big[\sum_{n=-\infty}^{\infty} \frac{15}{2}(f-12.5-50n) + \frac{15}{2}\delta(f+12.5-50n) + 5\,\delta(f-16-50n) + 5\,\delta(f+16-50n)\Big]$

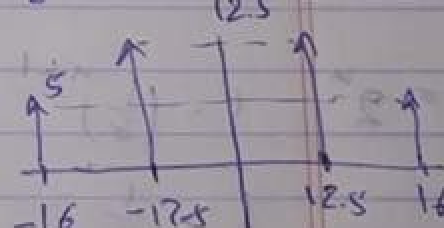


* porhibit → يمنع → $f < 2f_h$

$\boxed{f < 32}$ ← [illegible] استعادة ال signal الاصلية

* minimum frequency of sampling is $2f_h = 32$ Hz
* Impulse respons → output :-

$\cancel{\text{Filter}} = H(f) = \begin{cases} \frac{1}{50} & |f| \le 25 \\ 0 & \text{o.w} \end{cases} \Rightarrow$



$\Rightarrow \cancel{h(f)} = \boxed{h(t) = \text{sinc}(50t)}$ impulse responce

since $h(t) = \text{sinc}(f_s t)$

$$x(n) = 2^n u(n)$$

$\boxed{9.2}$ $h(n) = \left(\frac{1}{4}\right)^n u(n) - \left(\frac{1}{8}\right)^n u(n-4)$

$$\cancel{y(n) = \sum_{m=-\infty}^{\infty} \ldots}$$

$$y(n) = \sum_{m=-\infty}^{\infty} 2^{n-m} u(n-m) \left[\left(\frac{1}{4}\right)^m u(m) - \left(\frac{1}{8}\right)^m u(m-4)\right]$$

$$= \sum_{m=-\infty}^{\infty} 2^{n-m} u(n-m) \left(\frac{1}{4}\right)^m u(m) - \sum_{m=-\infty}^{\infty} 2^{n-m} u(n-m) \left(\frac{1}{8}\right)^m u(m-4)$$

$$= \sum_{m=0}^{n} 2^{n-m} \left(\frac{1}{4}\right)^m - \sum_{m=4}^{n} 2^{n-m} \left(\frac{1}{8}\right)^m$$

$$= 2^n \sum_{m=0}^{n} 2^{-m} \left(\frac{1}{4}\right)^m - 2^n \sum_{m=4}^{n} 2^{-m} \left(\frac{1}{8}\right)^m$$

$$= 2^n \sum_{m=0}^{n} \left(\frac{1}{8}\right)^m - 2^n \sum_{m=4}^{n} \left(\frac{1}{16}\right)^m$$

$$= 2^n \sum_{m=0}^{n} \left(\frac{1}{8}\right)^m - 2^n \sum_{m=0}^{n-4} \left(\frac{1}{16}\right)^{m+4}$$

$$= 2^n \sum_{m=0}^{n} \left(\frac{1}{8}\right)^m - 2^n \left(\frac{1}{16}\right)^4 \sum_{m=0}^{n-4} \left(\frac{1}{16}\right)^m$$

$$= 2^n \frac{1 - \left(\frac{1}{8}\right)^{n+1}}{1 - \frac{1}{8}} - \frac{2^n}{(16)^4} \frac{1 - \left(\frac{1}{16}\right)^{n-3}}{1 - \frac{1}{16}}$$

$$h(n) = \left(\frac{1}{4}\right)^{n} u(n) - \left(\frac{1}{8}\right)^{(n-4)+4} u(n-4)$$

$$h(Z) = \frac{1}{1 - \frac{1}{4} Z^{-1}} - Z\left[\left(\frac{1}{8}\right)^{n-4} u(n-4)\right]\left(\frac{1}{8}\right)^{4}$$

$$= \frac{1}{1 - \frac{1}{4} Z^{-1}} - \frac{1}{1 - \frac{1}{8} Z^{-1}} Z^{-4} \cdot \left(\frac{1}{8}\right)^{4}$$

Q(iv):

1) $h(n) = ?$

$$H(z) = \frac{z(3z+2)}{z^2+4z+5}.$$

$$\frac{H(z)}{z} = \frac{3z+2}{z^2+4z+5} = \frac{3z+2}{(z+2-j)(z+2+j)}$$

$$\frac{H(z)}{z} = \frac{k_1}{(z+2-j)} + \frac{k^*}{z+2+j}$$

$$k_1 = (z+2-j) \cdot \frac{H(z)}{z}\Big|_{z=-2+j} = k_1 = \frac{4-3j}{-2j}$$

$$k_1 = \frac{3}{2} - 2j = 2.3\angle{-59}$$

$$k_1^* = \frac{3}{2} + 2j = 2.3\angle{59}.$$

$$H(z) = \frac{z \cdot 2.3\angle{-59}}{z+2-j} + \frac{z \cdot 2.3\angle{59}}{z+2+j}$$

$$h(n) = \left[r|\lambda|^n \cos(\beta n + \phi) \right] u(n).$$

$$= 4.6\,(2.2)^n \cos(26n + 59)$$

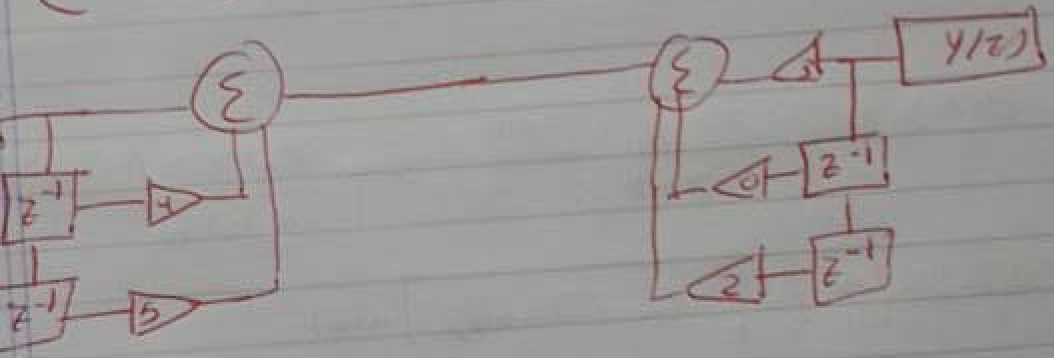
(2) IIR system

(3) not stable because $|z| > 1$

(4) Direct form:

$$\frac{\cancel{z^2}(3+2z^{-2})}{\cancel{z^2}(1+4z^{-1}+5z^{-2})}$$

$$(3+2z^{-2})Y(z) = X(z)[1+4z^{-1}+5z^{-2}]$$



5 Direct form :

$$\frac{z^2(3+2z^{-2})}{z^2(1+4z^{-1}+5z^{-2})} = \frac{Y}{X}$$

$$(3+2z^{-2})X(z) = Y(z)[1+4z^{-1}+5z^{-2}]$$