BIRZEIT UNIVERSITY

Faculty of Engineering

Electrical and Computer Engineering Department

SIGNALS AND SYSTEMS, ENEE2302

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First Exam

Date: Sunday, 15/07/2018 | Time: 70 minutes

Name: [redacted] [redacted] | Student #:

Problem 1 (5+6+4+5=20 pts):

a) Is the signal $x(t) = cos\left(2\pi t + \frac{\pi}{3}\right) + 2sin(8\pi t)$ periodic, if so, find its period.

b) Draw the amplitude and phase Double sided spectrum of the signal in (a).

c) Compute the following integral: $\int_0^3 (t^2+2)\delta(2t-2)dt$

d) Is the signal, $x(t) = \cos(0.25\pi t)\pi(\frac{t-6}{4})$ power signal, energy signal, or neither? If it is power, then find its power. If it is energy, then find its energy.

Problem 2 (9+11=20 pts):

a) Determine if the following system $y(t) = (t)x(2t+1)$ is linear, time invariant and/or causal :

b) Determine the impulse response of the system modeled by

δ

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \dot{x}(t-3)$$

Where y(t): system response , x(t): system input

Problem 3 (15 +5=20 pts):

a. Plot the following signal using the elementary signals

$$x(t) = r\left(\frac{4-t}{4}\right) + \Pi\left(\frac{t-12}{4}\right) - \Pi\left(\frac{t-1}{8}\right) - 2u(1-t)$$

2. Given the signal in Fig. 1:

$$x(t) = \begin{Bmatrix} -1 & -0.5 \le t \le 0 \\ 1 & 0 \le t \le 0.5 \end{Bmatrix}$$

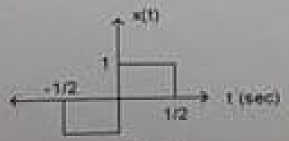


Fig.1

Plot the signal $y(t) = \sum_{n=-\infty}^{\infty} x(t-2n)$

problem 1:

a) $x(t) = \cos(2\pi t - \frac{\pi}{3}) + 2\sin(8\pi t)$

$2\pi f_1 = 2\pi \qquad 2\pi f_2 = 8\pi$

$f_1 = 1 \qquad f_2 = 4$

$\frac{f_1 = n_1 f_0}{f_2 = n_2 f_0} \Rightarrow \frac{1 = n_1 \cancel{f_0}}{4 = n_2 \cancel{f_0}} \Rightarrow n_2 = 4 n_1$

let $n_1 = 1 \Rightarrow n_2 = 4$

$f_0 = 1\ Hz$ $\quad$ ∴ The signal is periodic and it period is $\frac{1}{f_0} = 1$ sec

b) $x(t) = \cos(2\pi t - \frac{\pi}{3}) + 2\sin(8\pi t)$

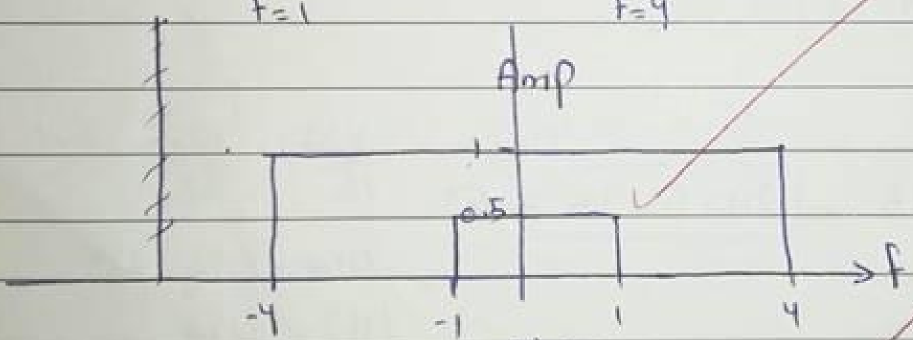
$\sin(8\pi t) = \cos(8\pi t \pm \frac{\pi}{2})$ ∴

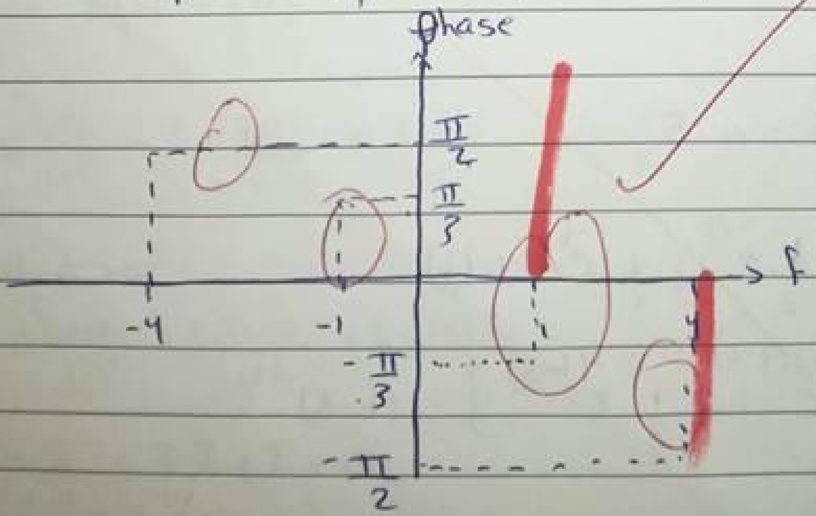
$\cos(8\pi t - \frac{\pi}{2}) = \cos 8\pi t \cancel{\cos\frac{\pi}{2}} + \sin 8\pi t \underbrace{\sin\frac{\pi}{2}}_{1} = \sin 8\pi t$ ✓

∴ $x(t) = \cos(2\pi t - \frac{\pi}{3}) + 2\cos(8\pi t - \frac{\pi}{2})$

$f = 1 \qquad f = 4$

Amp = even
phase = odd





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c) $\int_0^3 (t^2+2)\,\delta(2t-2)\,dt$

assume $T = 2t$

$\frac{dT}{dt} = 2 \Rightarrow dt = \frac{dT}{2}$

$\cancel{\frac{dT}{dt}}$

when $t = 0 \rightarrow T = 0$

$t = 3 \rightarrow T = 6$

$$\int_0^6 \frac{1}{2}\left(\left(\frac{T}{2}\right)^2 + 2\right) \delta(T-2)\, dT$$

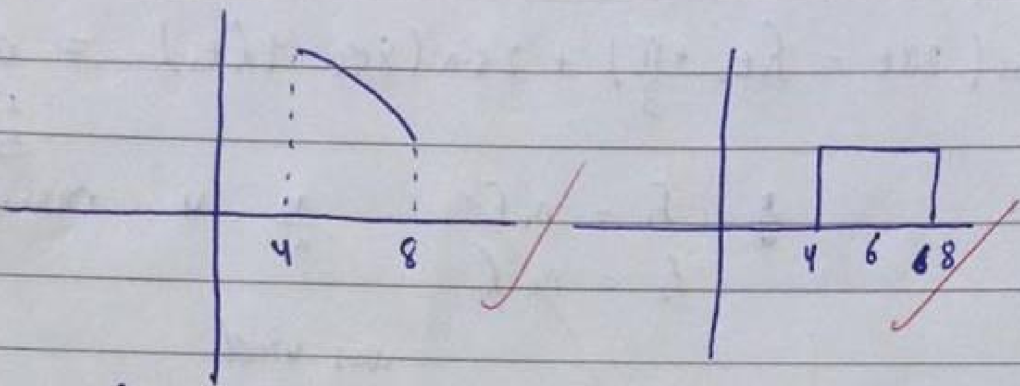
$0 < 2 < 6$ so $\rightarrow$

$X(T) = \frac{1}{2}\left(\left(\frac{T}{2}\right)^2 + 2\right)$

$X(2) = \frac{1}{2}(1+2)$

$= \boxed{\frac{3}{2}}$

(d) $x(t) = \cos(0.25\pi t)\,\Pi\left(\frac{t-6}{4}\right) \Rightarrow \cos(0.25\pi t)\,\Pi\left(\frac{1}{4}(t-6)\right)$



$$\Rightarrow \int_4^8 \cos^2(0.25\pi t)\,dt = \cancel{\sin(0.25\pi t)\Big|_4^8}$$

as the signal is bounded $\Rightarrow$ Energy signal

$$\cancel{-\sin(0.25\pi * 8) + \sin(0.25\pi * 4)}$$
$$\cancel{= \sin(2\pi) + \sin\pi}$$

$$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos^2\theta = \frac{1}{2}\cos 2\theta + \frac{1}{2}$$

$$\int_4^8 \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right)dt = \int_4^8 \left[\frac{1}{2} + \frac{1}{2}\cos(0.5\pi t)\right]dt$$

$$\Rightarrow \frac{1}{2}t - \frac{1}{2}\sin(0.5\pi t)\Big|_4^8$$

$$\left[4 - \frac{1}{2}\cancel{\sin 4\pi}^{\,0}\right] - \left[2 - \frac{1}{2}\cancel{\sin 2\pi}^{\,0}\right] = 6 \text{ Joul}$$

problem 2:

a) $y(t) = (t)\, x(2t+1)$

$y_1(t-t_0) = (t-t_0)\, x(2(t-t_0)+1)$

$y_2(t-t_0) = (t-t_0)\, x(2t-t_0+1)$

$y_1(t-t_0) \neq y_2(t-t_0)$ $\therefore$ $\boxed{\text{Time-variant}}$

$y(t)$ is $\boxed{\text{non casual}}$ because $x(t)$ depends on a future value

$\alpha_1 y_1(t) = t\, \alpha_1 x_1(2t+1)$

$\alpha_2 y_2(t) = t\, \alpha_2 x_2(2t+1)$

$\alpha_1 y_1(t) + \alpha_2 y_2(t) = t\, \alpha_1 x_1(2t+1) + t\, \alpha_2 x_2(2t+1)$

$\alpha_3 y_3(t) = t[\alpha_1 x_1(2t+1) + \alpha_2 x_2(2t+1)]$

$\alpha_3 y_3(t) = t\, \alpha_3 x_3'(2t+1)$

$\therefore$ $y(t)$ is $\boxed{\text{linear}}$

b) $\dfrac{\partial^2 y(t)}{dt^2} + \dfrac{3d\,y(t)}{dt} + 2y(t) = x'(t-3)$

let $y(t) = h(t)$

$x(t) = \delta(t)$

$\dfrac{\partial^2 h(t)}{dt^2} + 3\dfrac{d\,h(t)}{dt} + 2h(t) = \dot{\delta}(t-3)$

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at $t>0$

$\dfrac{\partial^2 h(t)}{dt^2} + 3\dfrac{dh(t)}{dt} + 2h(t) = 0$

$\lambda^2 + 3\lambda + 2 = 0$

$(\lambda+2)(\lambda+1)$

$\lambda_1 = -2 \quad , \lambda_2 = -1$

$$h(t) = \underbrace{\left[A e^{-2t} + B e^{-t}\right]}_{g(t)} u(t)$$

$$h(t) = g(t)\,u(t)$$

$$h'(t) = g(t)\,\delta(t) + g'(t)\,u(t)$$

$$= g(0)\,\delta(t) + g'(t)\,u(t)$$

$$h''(t) = g(t)\,\dot{\delta}(t) + g'(t)\,\delta(t) + g'(t)\,\delta(t) + g''(t)\,u(t)$$

$$= g(0)\,\dot{\delta}(t) + g'(0)\,\delta(t) + g'(0)\,\delta(t) + g''(t)\,u(t)$$

First I solve with $\delta(t)$ the trans it to $\delta(t-3)$

$$g(0)\,\dot{\delta}(t) + 2g'(0)\,\delta(t) + g''(t)u(t) + 3g(0)\,\delta(t) + 3g'(t)u(t) + g(t)u(t) = \dot{\delta}(t)$$

$$g(0) = 1$$

$$3g(0) + 2g'(0) = 0 \rightarrow 3(1) + 2g'(0) \Rightarrow g'(0) = \frac{-3}{2} = -1.5$$

$$g(t) = A e^{-2t} + B e^{-t}$$

$$g(0) = \boxed{A + B = 1} \ldots (1)$$

$$g'(t) = -2A e^{-2t} - B e^{-t}$$

$$g'(0) = -2A - B = -1.5 \rightarrow \boxed{2A + B = 1.5} \ldots (2)$$

From eq (1) and (2)

$$2A + B = 1.5$$

$$-\quad A + B = 1$$

$$A = 0.5 \quad , \quad B = 0.5$$

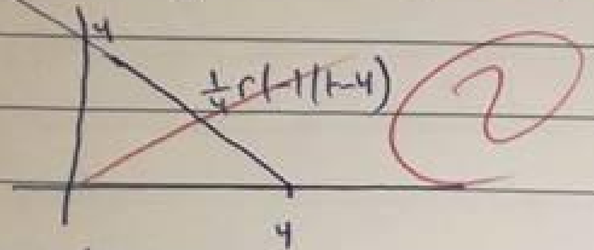
in general $h(t) = \left(\frac{1}{2} e^{-2t} + \frac{1}{2} e^{-t}\right) u(t)$

$$h(t-3) = \left[\frac{1}{2} e^{-2(t-3)} + \frac{1}{2} e^{-(t-3)}\right] u(t-3)$$

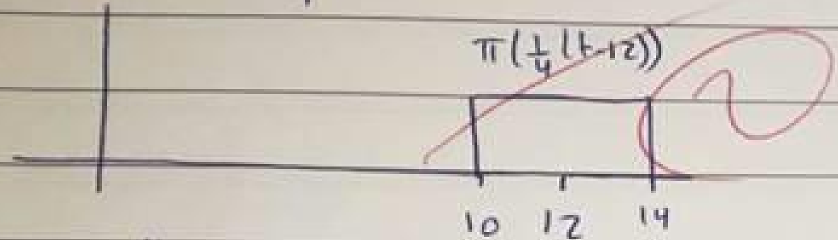
Problem 3 :

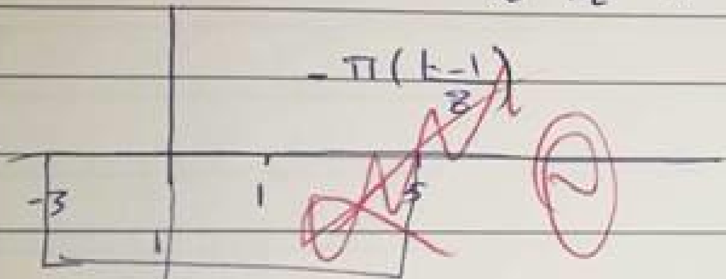
a) $x(t) = r\left(\frac{4-t}{4}\right) + \Pi\left(\frac{t-12}{4}\right) - \Pi\left(\frac{t-1}{8}\right) - 2u(1-t)$

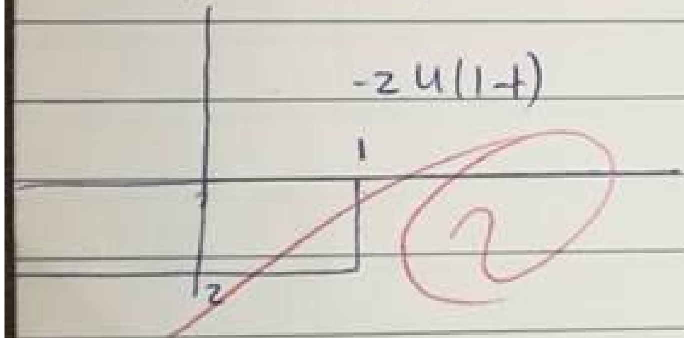
$x(t) = \frac{1}{4} r\left(-1(t-4)\right) + \Pi\left(\frac{1}{4}(t-12)\right) - \Pi\left(\frac{1}{8}(t-1)\right) - 2u(1-t)$

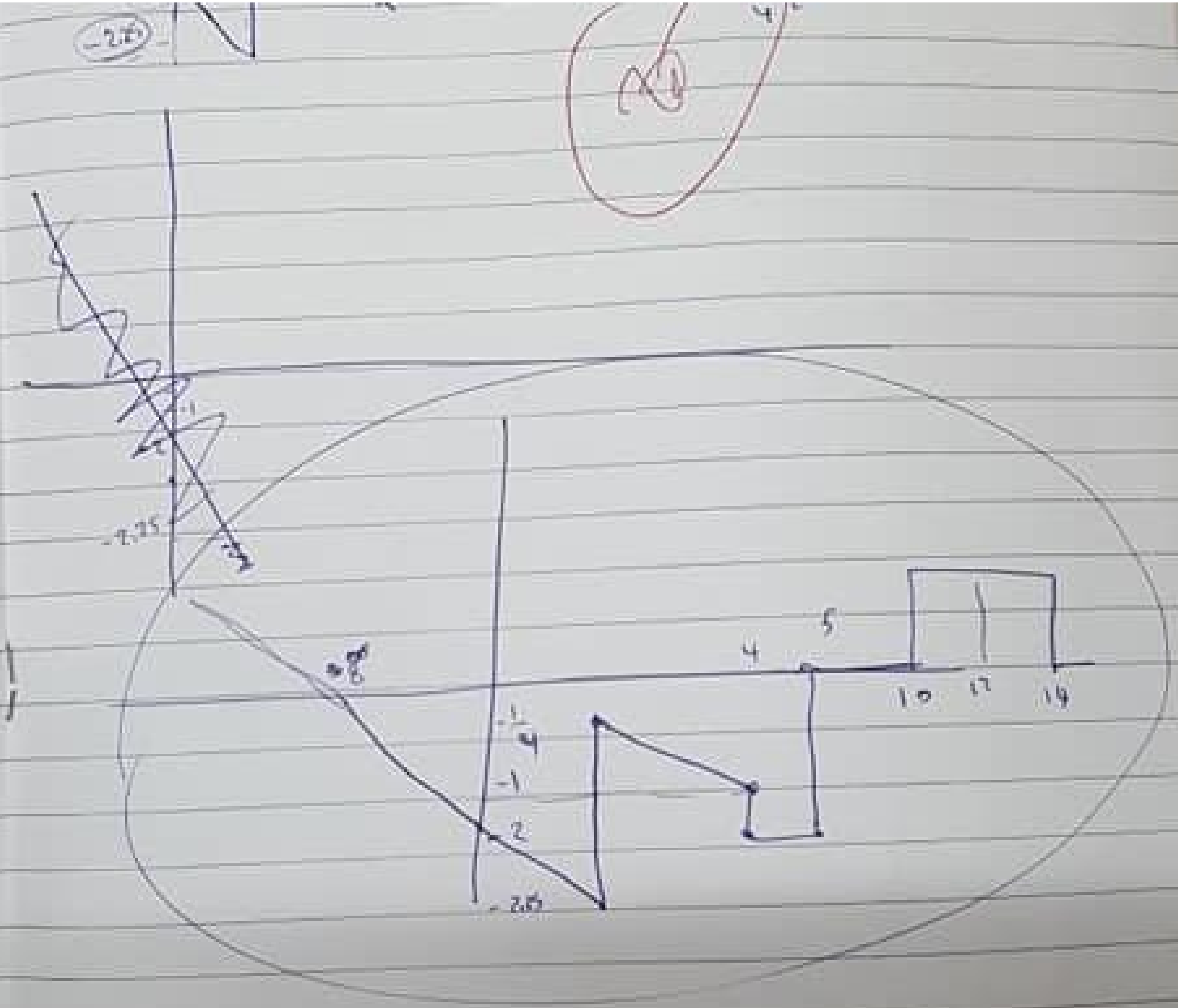


$[4 - t + t - 1] = \boxed{3}$



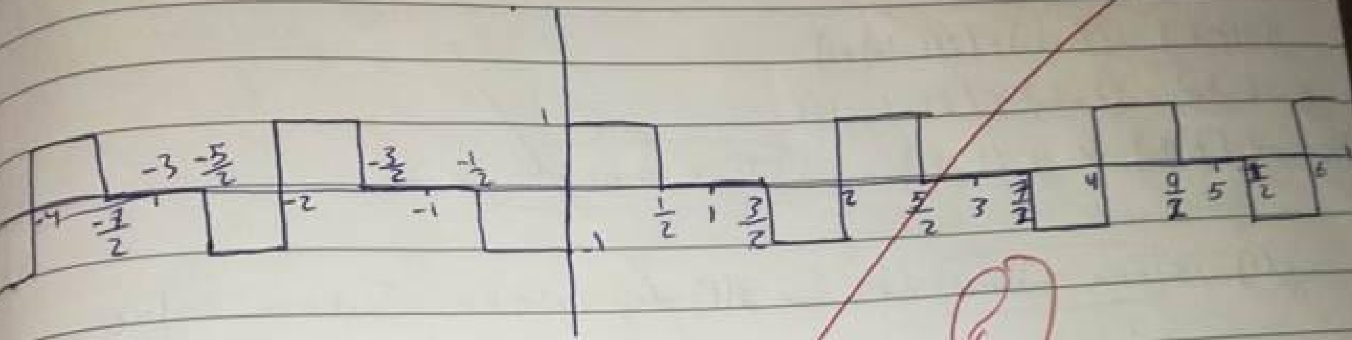






-2.25
4
5
10
12
14
-1
2
-2.25

2) $y(t) = \sum_{n=-\infty}^{\infty} x(t-2n)$ $\quad x(t) = \begin{cases} -1 & -0.5 < t < 0 \\ 1 & 0 \le t < 0.5 \end{cases}$



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