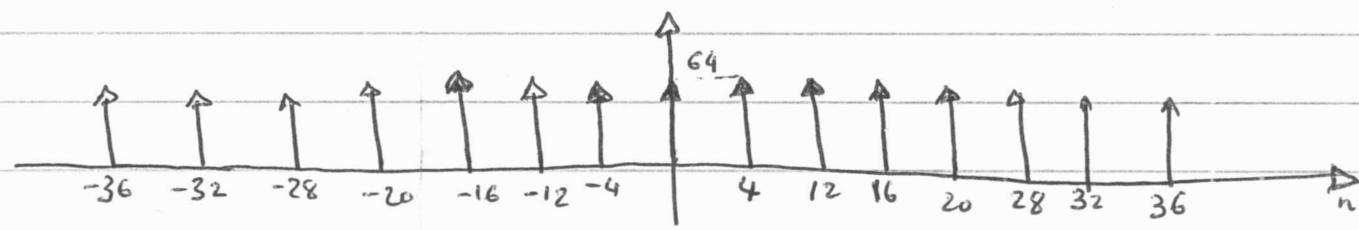
chapter 8 :

(8-1) $X(f) = 4\delta(f) + 4\delta(f-4) + 4\delta(f+4)$

$$X_s(f) = f_s \sum_{n=-\infty}^{+\infty} 4\delta(f-nf_s) + 4\delta(f-4-nf_s) + 4\delta(f+4-nf_s)$$

$f_s = 16$ sample/second. $\Longrightarrow$

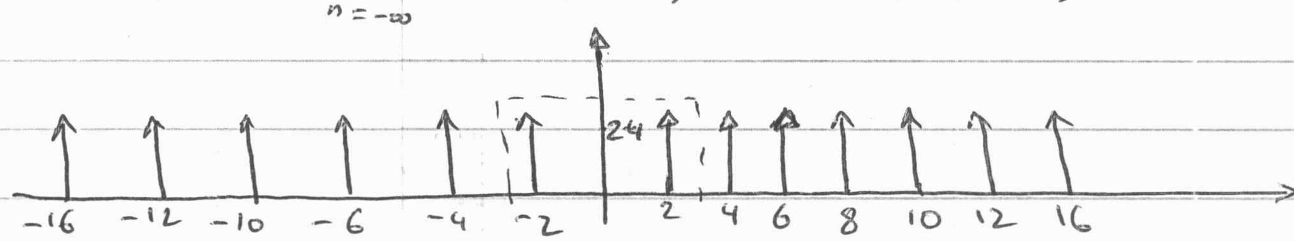
$$X_s(f) = \sum_{n=-\infty}^{+\infty} 64\,\delta(f-16n) + 64\,\delta(f-4-16n) + 64\,\delta(f+4-16n)$$



the reconstruction filter $\begin{cases} \frac{1}{16} & |f| < 8 \\ 0 & \text{otherwise} \end{cases}$

reconstruct the signal correctly

(8-2) $$X_s(f) = \sum_{n=-\infty}^{+\infty} 24\,\delta(f-6n) + 24\,\delta(f-4-6n) + 24\,\delta(f+4-6n)$$



the reconstruction can not be don correctly for the presence of the frequency component $f = \mp 2$ result of the Aliasing phenomena.

(8−18) We must sample the function with sampling frequency $f_s = 25$ sample/second $\rightarrow T_s = 0.04$ sec. so the sampled function in the range $0 \le n \le 8$ is

$$\left\{ 5\sin\left[(0.04)(20)\pi n\right] + 2\,\Pi\left(\frac{0.04n - 0.14}{0.16}\right)\right\}_{n=0,1,\dots,8}$$

(8−20) c

(8-19)

$$\sum_{n=0}^{+\infty}\left[2u(n) + \left(\frac{3}{4}\right)^n u(n-4)\right] z^{-n}$$

$$= 2\sum_{n=0}^{+\infty} z^{-n} + \sum_{n=0}^{+\infty}\left(\frac{3}{4}\right)^n u(n-4)\, z^{-n}$$

$$n-4 = m \longrightarrow n = 0 \longrightarrow m = -4$$

$$n = +\infty \longrightarrow m = +\infty$$

$$\Rightarrow\quad 2\sum_{n=0}^{+\infty} z^{-n} + \sum_{m=-4}^{+\infty}\left(\frac{3}{4}\right)^{m+4} u(m)\, z^{-(m+4)}$$

$$= 2\sum_{n=0}^{+\infty} z^{-n} + \left(\frac{3}{4}\right)^4 z^{-4}\sum_{m=0}^{+\infty}\left(\frac{3}{4}z^{-1}\right)^m$$

$$= 2\,\frac{z}{z-1} + \left(\frac{3}{4}\right)^4 z^{-4}\,\frac{1}{1-\frac{3}{4}z^{-1}}$$

$$= \frac{2z}{z-1} + \left(\frac{4}{3}z\right)^{-4}\frac{z}{z-3/4}$$

(8-21) b :-

$$X(z) = \sum_{n=0}^{+\infty} \left(\frac{2}{3}\right)^{n-4} u(n-4)\, z^{-n}$$

$n-4 = m \quad , \quad n = 0 \longrightarrow m = -4$

$n = +\infty \longrightarrow m = +\infty$

$$X(z) = \sum_{m=-4}^{+\infty} \left(\frac{2}{3}\right)^{m} u(m)\, z^{-(m+4)}$$

$$= z^{-4} \sum_{m=0}^{+\infty} \left(\frac{2}{3}\right)^{m} z^{-m} = z^{-4} \frac{1}{1-\frac{2}{3}z^{-1}}$$

$$= \frac{z^{-3}}{z-\frac{2}{3}} = \frac{1}{z^{3}(z-2/3)}$$

(8-24)

$x(n) = a^{n} \sin\left(\frac{n\pi}{2}\right) \qquad n \geq 0 \qquad a$: real const.

$$x(n) = \begin{cases} 0 & n \text{ even} \\ 1 & n = 1, 5, 9, \ldots \\ -1 & n = 3, 7, 11, \ldots \end{cases}$$

$\Downarrow$

$$x(m) = \begin{cases} 0 \ , \ x(2m) = 0 \\ a^{4m+1} - a^{4m+3} & m = 0, 1, 2, \ldots \end{cases}$$

$$X(z) = \sum_{m=0}^{+\infty} a^{4m+1} z^{-(4m+1)} - \sum_{m=0}^{+\infty} a^{4m+3} z^{-(4m+3)}$$

$$a z^{-1} \sum_{m=0}^{+\infty} \left(a^{+4} z^{-4}\right)^{m} - a^{3} z^{-3} \sum_{m=0}^{+\infty} \left(a^{4} z^{-4}\right)^{m}$$

$$= \frac{z^{-1} a}{1-a^{4}z^{-4}} - \frac{z^{-3}a^{3}}{1-a^{4}z^{-4}}$$

$$= \frac{a z^{-1} - a^{3} z^{-3}}{1-a^{4}z^{-4}} = \frac{a z^{3} - a^{3} z}{z^{4}-a^{4}}$$

(8-26) c

$$x(nT) = \left(\frac{1}{2}\right)^{n-6} u(n-6)$$

$$|x(nT)|^2 = \left[\left(\frac{1}{2}\right)^{n-6} u(n-6)\right]^2$$

$$E = \sum_{n=0}^{+\infty} \left[\left(\frac{1}{2}\right)^{n-6} u(n-6)\right]^2$$

$n-6 = m \longrightarrow n = m+6$

$n=0 \longrightarrow m = -6$

$n = \infty \longrightarrow m = +\infty$

$$E = \sum_{m=-6}^{+\infty} \left[\left(\frac{1}{2}\right)^m u(m)\right]^2 = \sum_{m=0}^{\infty} \left[\left(\frac{1}{2}\right)^m\right]^2$$

$$= \sum_{m=0}^{+\infty} \left(\frac{1}{4}\right)^m = \frac{1}{1-1/4} = 4/3 \text{ Joule.}$$

(8-27) a

We must find the coefficients of the power series in z^{-1}

$x(n) = 1, -0.3, 0.7, 0.8, -0.3, 0 \ldots$

(8-28) a

$$X(z) = z^{-2}(1+0.3z^{-1}-0.7z^{-2}) + z^{-5}(1+0.3z^{-1}-0.7z^{-2})$$

$$= z^{-2} + 0.3z^{-3} - 0.7z^{-4} + z^{-5} + 0.3z^{-6} - 0.7z^{-7}$$

$x(n) = 0, 0, 1, 0.3, -0.7, 1, 0.3, -0.7, 0 \ldots$

(8-29) b

$$X(z) = \frac{1 - 0.7z^{-1}}{1 + z^{-1} + \frac{1}{4}z^{-2}} = \frac{z^2 - 0.7z}{z^2 + z + 0.25}$$

$$\begin{array}{r|l}
 & 1 - 1.7z^{-1} + 1.45z^{-2} - 1.025z^{-3} \\
\hline
z^2 + z + 0.25 & z^2 - 0.7z \\
 & \underline{z^2 + z + 0.25} \\
 & -1.7z - 0.25 \\
 & \underline{-1.7z - 1.7 - (0.25)(1.7)z^{-1}} \\
 & 1.45 + (0.25)(1.7)z^{-1} \\
 & \underline{1.45 + 1.45z^{-1} + (0.25)(1.45)z^{-2}} \\
 & -1.025z^{-1} + 0.36z^{-2} \\
 & \underline{-1.025z^{-1} - 1.025z^{-2} - 0.26z^{-3}} \\
 & \vdots
\end{array}$$

the coefficients are:

$1, -1.7, 1.45, -1.025$

(8-32) a

$$X(z) = \frac{2}{(1 - z^{-1})(1 - 0.2z^{-1})} = \frac{2z^2}{(z-1)(z-0.2)}$$

$$\frac{X(z)}{z} = \frac{2z}{(z-1)(z-0.2)} = \frac{A}{(z-1)} + \frac{B}{(z-0.2)}$$

$$A = \left.\frac{2z}{z-0.2}\right|_{z=1} = \frac{2}{0.8} = 5/2$$

$$B = \left.\frac{2z}{z-1}\right|_{z=0.2} = \frac{0.4}{-0.8} = -1/2 \longrightarrow$$

$$X(z) = \frac{5}{2}\frac{z}{z-1} - \frac{1}{2}\frac{z}{z-0.2}$$

$$x(n) = \frac{5}{2}u(n) - \frac{1}{2}(0.2)^n u(n)$$

(8-35)b:

$$X(z) = \frac{1}{(1-0.81z^{-1})^2} = \frac{z^2}{(z-0.81)^2}$$

$$\frac{X(z)}{z} = \frac{z}{(z-0.81)^2} = \frac{A_2}{(z-0.81)^2} + \frac{A_1}{(z-0.81)}$$

$$A_2 = z\Big|_{z=0.81} = 0.81$$

$$A_1 = \frac{d}{dz} z\Big|_{z=0.81} = 1$$

$$\frac{X(z)}{z} = \frac{0.81}{(z-0.81)^2} + \frac{1}{(z-0.81)}$$

$$X(z) = 0.81 \frac{z}{(z-0.81)^2} + \frac{z}{(z-0.81)}$$

$$x(n) = 0.81\, n\,(0.81)^n u(n) + (0.81)^n u(n)$$

→ (8-36 following page

(8-45)c

$$y(nT) - y(nT-T) + 0.16\, y(nT-2T) = x(nT)$$

$$Z\left[y(nT) - y(nT-T) + 0.16\, y(nT-2T)\right] = Z\left[x(nT)\right]$$

$$Y(z) + z^{-1} Y(z) + 0.16 z^{-2} Y(z) = X(z)$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{1+z^{-1}+0.16z^{-2}} = \frac{z^2}{z^2+z+0.16}$$

$$h(nT) = Z^{-1}\left[H(z)\right]$$

$$z_{1,2} = \frac{-1 \mp \sqrt{1-4*0.16}}{2} = \begin{cases} -0.8 \\ -0.2 \end{cases}$$

$$H(z) = \frac{z^2}{(z+0.8)(z+0.2)} \Rightarrow \frac{H(z)}{z} = \frac{z}{(z+0.8)(z+0.2)}$$

$$A = \frac{z}{(z+0.8)}\Big|_{z=-0.2} = \frac{-0.2}{0.6} = -\frac{1}{3}$$

$$B = \left.\frac{z}{(z+0.2)}\right|_{z=-0.8} = \frac{-0.8}{(-0.8+0.2)} = \frac{-0.8}{-0.6} = 8/6$$

$$H(z) = -\frac{1}{3}\frac{z}{(z+0.2)} + \frac{8}{6}\frac{z}{(z+0.8)}$$

$$= -\frac{1}{3}(-0.2)^n u(n) + \frac{8}{6}(-0.8)^n u(n)$$

8-36

$$X(z) = \frac{1}{1+0.81z^{-2}} = \frac{z^2}{z^2+0.81}$$

$$\frac{X(z)}{z} = \frac{z}{z^2+0.81} = \frac{A}{z-j0.9} + \frac{A^*}{z+j0.9}$$

$$A = \frac{z}{z+j0.9}\Bigg|_{z=j0.9} = \frac{j0.9}{j0.9+j0.9} = \frac{1}{2}$$

$$A^* = \frac{1}{2}$$

$$X(z) = \frac{1}{2}\frac{z}{z-j0.9} + \frac{1}{2}\frac{z}{z+j0.9}$$

$$x(n) = 2.\frac{1}{2}\, e^{+\alpha n}\cos \beta n + \theta =$$

$$= e^{+\alpha n} \cos \beta n \qquad k = 1/2, \quad \theta = 0$$

$$\alpha = \ln|P| = \ln(0.9)$$

$$\beta = \tan^{-1}\frac{0.9}{0} = \pi/2$$

$$x(n) = e^{-n\ln 0.9} \cos n\frac{\pi}{2}.$$

(8-46)a

$$Y(z)\left[1-kz^{-1}+k^2z^{-2}\right]=X(z)$$

$$\frac{Y(z)}{X(z)}=\frac{1}{1-kz^{-1}+k^2z^{-2}}=\frac{z^2}{z^2-kz+k^2}$$

$$z_{1,2}=\frac{k\mp\sqrt{k^2-4k^2}}{2}=\frac{k\mp j\sqrt{3}\,k}{2}$$

$$=k\left(\frac{1\mp j\sqrt{3}}{2}\right)$$

for the system to be stable

$|z_{1,2}|<1 \Rightarrow |k|\sqrt{\frac{1}{4}+\frac{3}{4}}<1 \Rightarrow$

$|k|<1$

(8-47)c

$$y(nT)=x(nT+T)+x(nT-T)$$

the system is linear, shift invariant and non causal.

(8-49)

$y(n)=0 \qquad n\le 5$

$y(6)=4$

$y(7)=4+4=8$

$y(8)=4+4+4=12$

$y(9)=4+4+4+4=16$

$y(10)=4+4+4+4=16$

$y(11)=4+4+4=12$

$y(12)=4+4=8$

$y(13)=4$

$y(n)=0 \qquad n\ge 14$

(8-55) Convolution

$$y(n) = \sum_{k=0}^{+\infty} x(k)\, h(n-k)$$

$$= \sum_{k=0}^{+\infty} \left(\frac{1}{4}\right)^k * \left(\frac{1}{3}\right)^{n-k}$$

$$= \left(\frac{1}{3}\right)^n \sum_{k=0}^{+\infty} \left(\frac{1}{4}\right)^k * \left(\frac{1}{3}\right)^{-k}$$

$$= \left(\frac{1}{3}\right)^n \sum_{k=0}^{+\infty} \left(\frac{3}{4}\right)^k = \left(\frac{1}{3}\right)^n \frac{1}{1-3/4} = 4\left(\frac{1}{3}\right)^n u(n)$$