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Ass #2.

Q1 Determine if the following system
~~y(t) =~~ fixed, dynamic, causal, linear.
$y(t) = \sqrt{x(t^2)}$

① to check it's fixed or not.

delay time
delay function of time.

$y_1(t-t_0) = \sqrt{x(t-t_0)^2}$

$y_2(t-t_0) = \sqrt{x(t^2-t)}$

since $y_1 \neq y_2$ the system is variant.

② causal: suppose $t = 2$.

$y(2) = \sqrt{x(4)}$ since $4 > 2$

the system non causal.

③ dynamic.
↳ because output depends on past or future values of the input in addition to present time.

④ $\alpha_1 y_1(t) = \alpha_1 \sqrt{x_1(t^2)}$

$\alpha_2 y_2(t) = \alpha_2 \sqrt{x_2(t^2)}$

$\alpha_1 y_1(t) + \alpha_2 y_2(t) = \alpha_1 \sqrt{x_1(t^2)} + \alpha_2 \sqrt{x_2(t^2)}$ — ①

Since $\alpha_3 y_3(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$

$\alpha_3 x_3(t^2) = \alpha_1 x_1(t^2) + \alpha_2 X_2(t^2)$

$\alpha_3 y_3(t) = \alpha_3 \sqrt{x_3(t^2)}$

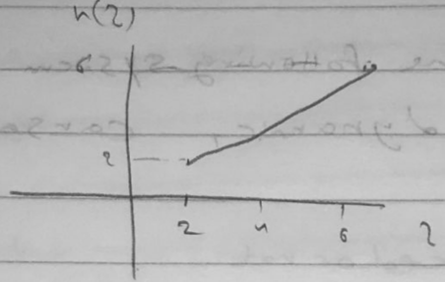
$\alpha_1 y_1(t) + \alpha_2 y_2(t) = \sqrt{\alpha_1 x_1(t^2) + \alpha_2 x_2(t^2)}$ — ②

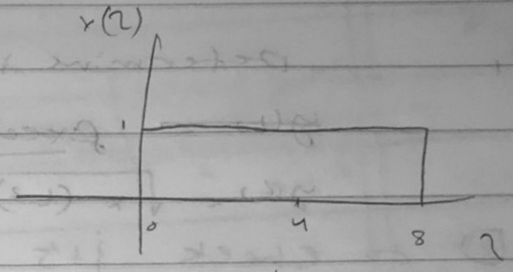
$1 \neq 2$ → the system non linear.

Problem # 2: ~~Refer~~

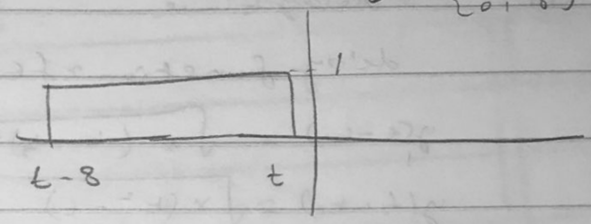
$h(t) = t\,\Pi\left(\frac{t-4}{4}\right) \qquad x(t) = \Pi\left(\frac{t-4}{8}\right)$

① slide





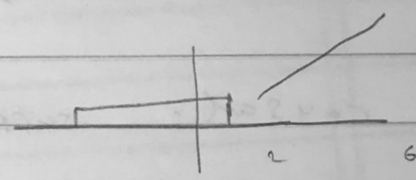
$[2, 6]$ $\qquad [0, 8]$



interval $\{2, 6, 10, 14\}$

① $-\infty < t < 2$

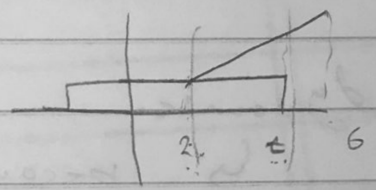
$y(t) = 0.$



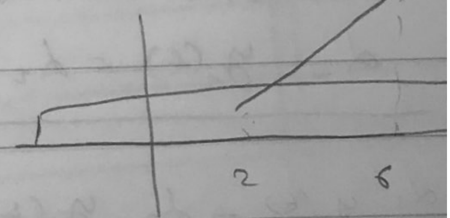
② $2 < t < 6$

$$y(t) = \int_2^t (1)(\lambda)\, d\lambda$$

$$= \frac{\lambda^2}{2}\Big|_2^t = \frac{t^2 - 4}{2}$$



③ $6 < t < 10$



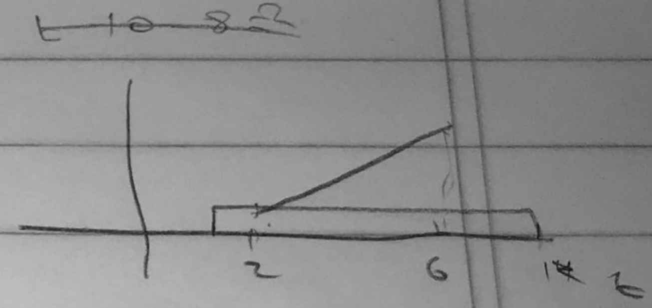
$$= \int_2^6 (1)\,\lambda\, d\lambda$$

$$= \frac{\lambda^2}{2}\Big|_2^6 = \frac{1}{2}(36 - 4)$$

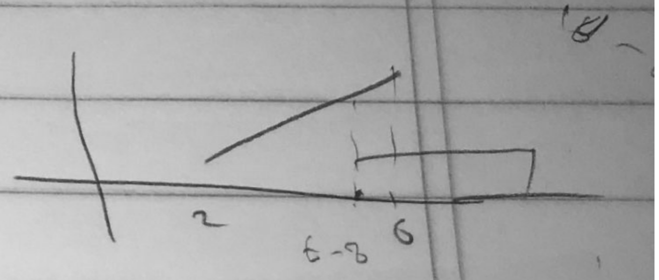
(4) $10 < t < 14$

$$y = \int_2^6 u(\lambda)\, \lambda \, d\lambda = \frac{1}{2}(36 - 4)$$

$y(t) = 16.$



(5) $14 < t$



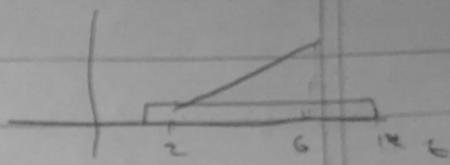
$$\int_{t-8}^{6} u(\lambda)\, \lambda \, dt.$$

$$y(t) = \left.\frac{\cancel{x}^2}{2}\right|_{t-8}^{6} = \frac{1}{2}\left(36 - (t-8)^2\right).$$

$$y(t) = -14 - \frac{t^2}{2} + 8t.$$

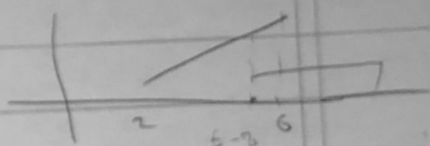
④ $10 < t < 14$

$$y = \int_2^6 u(\lambda)\, \lambda\, d\lambda = \frac{1}{2}(36 - 4)$$



$y(t) = 16$.

⑤ $14 < t$



$$\int_{t-8}^{6} u(\lambda)\, \lambda\, dt.$$

$$y(t) = \frac{\lambda^2}{2}\Big|_{t-8}^{6} = \frac{1}{2}\left(36 - (t-8)^2\right).$$

$$y(t) = -14 - \frac{t^2}{2} + 8t.$$

$$y(t) = \begin{cases} \frac{t^2}{2} - 2 & 2 < t < 6 \\ 16 & 6 < t < 10 \\ \frac{36}{2} - \frac{(t-8)^2}{2} & 10 < t < 14 \end{cases}$$

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