

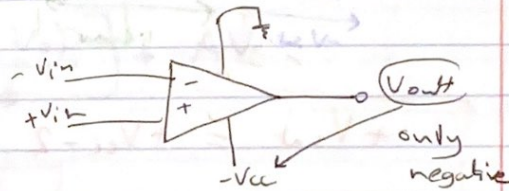
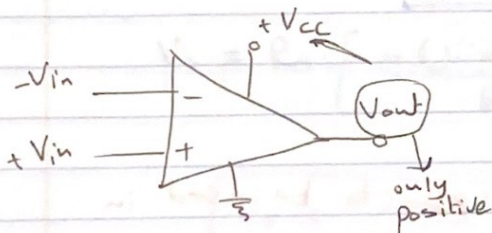
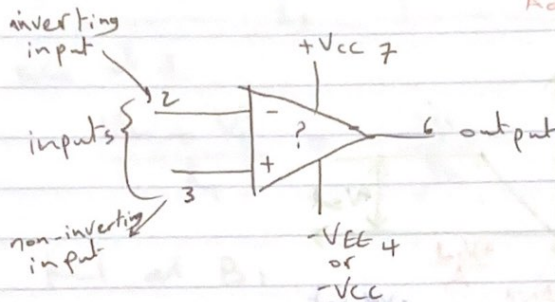
T9: OpAmps

Very High Voltage gain: 200 000

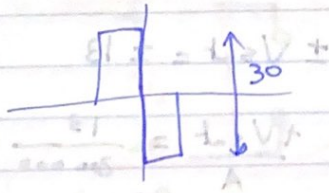
Very High Input Impedance: 10 M ohm

Very small output Impedance: 75 ohm

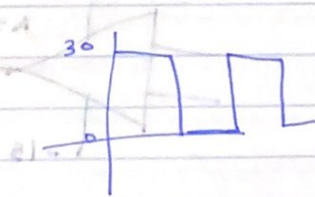
$$V_o = \begin{cases} +V_{sat} & V_d > \frac{V_{sat}}{A_d} \\ A_d V_d & \frac{V_{sat}}{A_d} > V_d > -\frac{V_{sat}}{A_d} \\ -V_{sat} & V_d < -\frac{V_{sat}}{A_d} \end{cases}$$



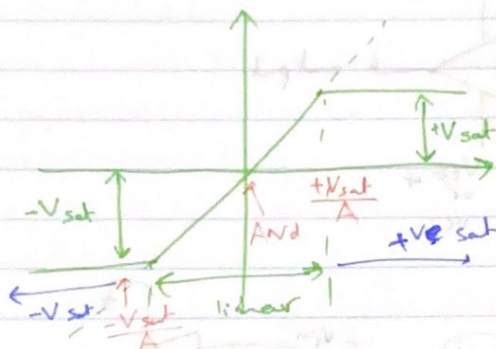
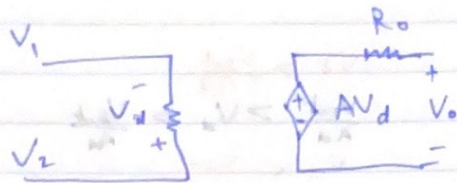
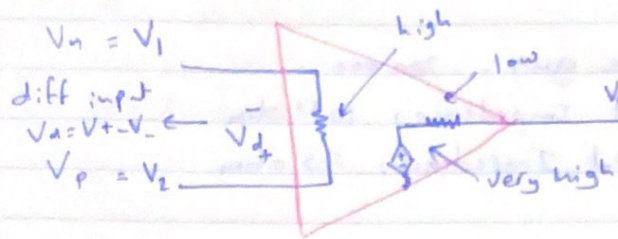
dual:



single:



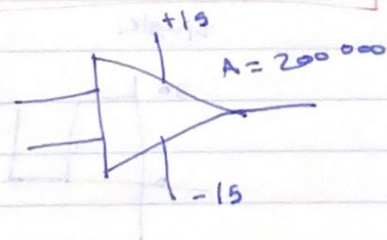
$$V_{out} = \frac{A_d V_{in}}{1 + A_d R_f} = \frac{10^5 V_{in}}{1 + 10^5 \cdot 10^3} = \frac{10^5 V_{in}}{10^8} = \frac{1}{1000} V_{in}$$



$$+V_{sat} \approx +V_{cc} - 2$$

$$-V_{sat} \approx -V_{cc} + 2$$

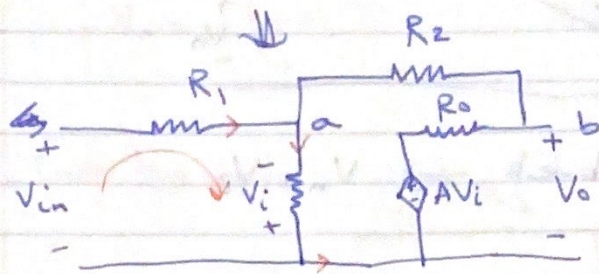
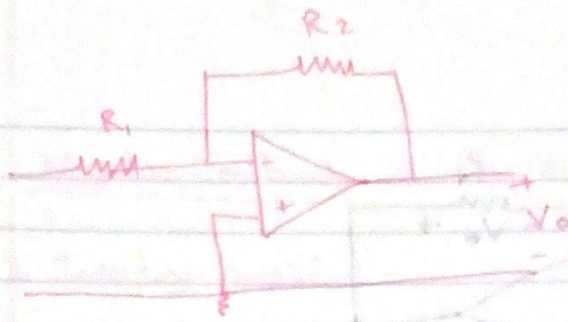
to be used in this course



$$\pm V_{sat} = \pm 13$$

$$\frac{+V_{sat}}{A} = \frac{13}{200,000} = 65 \mu V$$

$$\frac{-V_{sat}}{A} = \frac{-13}{200,000} = -65 \mu V$$



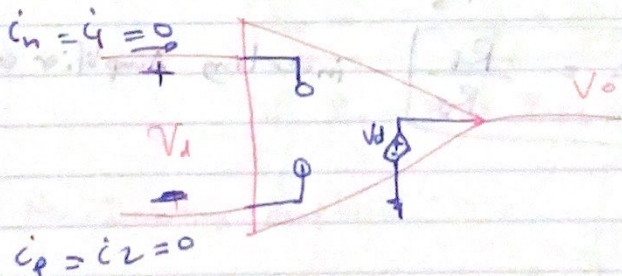
KCL at A:

$$\frac{V_{in} + V_i}{R_1} = -\frac{V_i}{R} - \frac{V_i + V_0}{R}$$

KCL at B:

$$V_0 = R_0 \left[-\frac{(V_i + V_0)}{R_2} \right] + AV_i$$

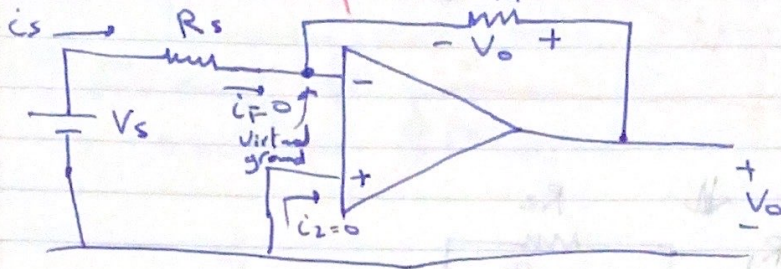
$$V_0 = -3.99 V_{in}$$



$$i_1 = i_2 = 0$$

$$V_1 \approx V_2$$

① Inverting Amplifier



negative feedback $\Rightarrow V_n = V_p$

$$\text{but } V_p = 0$$

$$\therefore V_n = 0$$

$$\text{KCL: } i_s = i_1 + i_f$$

$$i_s = i_f$$

$$i_s = \frac{V_s - V_n}{R_s} = \frac{V_s}{R_s} = i_f$$

$$V_o = -i_f R_f$$

$$V_o = -\frac{V_s}{R_s} R_f$$

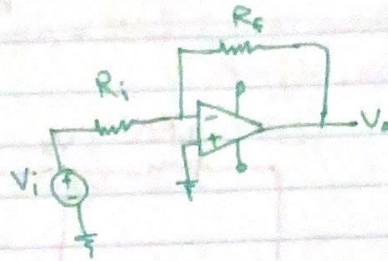
$$\boxed{\frac{V_o}{V_s} = -\frac{R_f}{R_s}}$$

inverting Amplifier Gain

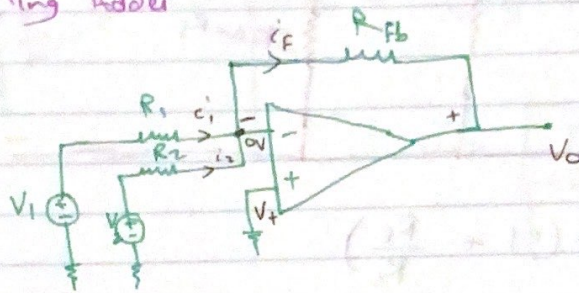
Negative Feedback

1. Inverting Amplifier

$$V_o = -\frac{R_F}{R_i} V_i$$



. Inverting Adder



$$V_+ = 0, V_- = V_+ = 0$$

$$i_1 = \frac{V_1}{R_1}, i_2 = \frac{V_2}{R_2}$$

$$i_f = i_1 + i_2$$

$$V_o = -i_f R_{fb} = -(i_1 + i_2) R_{fb}$$

$$V_o = -\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right) R_{fb}$$

$$\text{if } R_1 = R_2 = R_{fb} \Rightarrow V_o = -(V_1 + V_2) *$$

2. Non-Inverting Amplifier

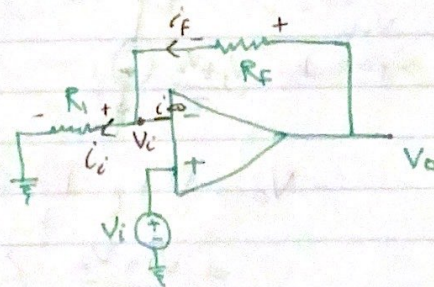
$$V_+ = V_- \text{ but } V_+ = V_i$$

$$\therefore V_- = V_i$$

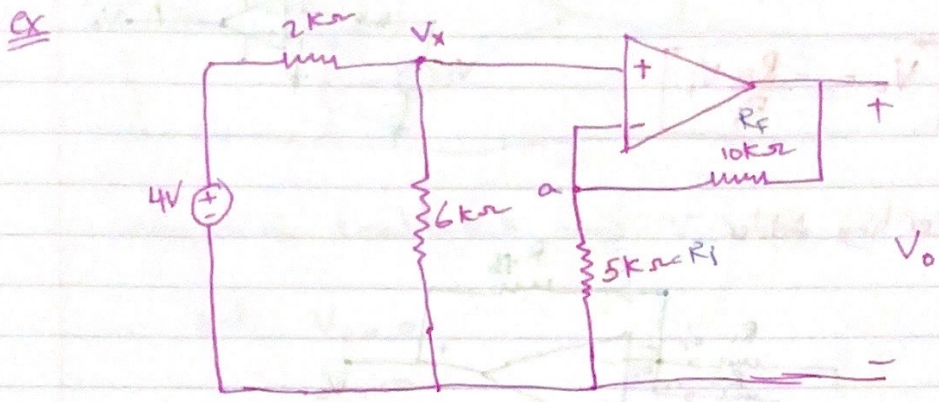
$$i_i = \frac{V_i}{R_i} = i_f$$

$$V_o = i_f R_f + i_i R_i$$

$$= i_f (R_f + R_i) = \frac{V_i}{R_i} (R_f + R_i) = V_i \left(1 + \frac{R_f}{R_i}\right) = V_o$$



$$V_o = V_t \left(1 + \frac{R_f}{R_i} \right)$$



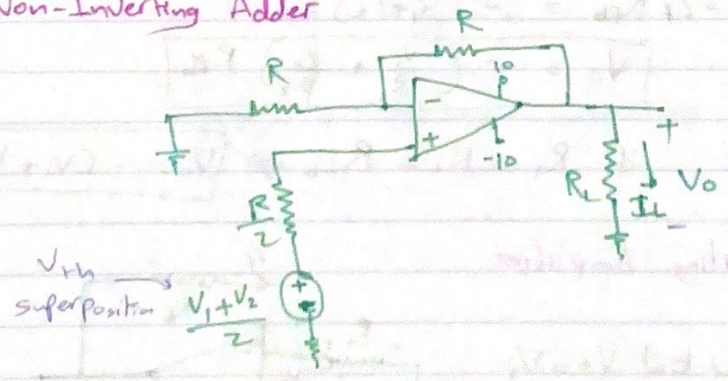
$$V_o = V_t \left(1 + \frac{R_f}{R_i} \right)$$

$$V_t = V_x = \frac{6k}{6k+2k} \times 4V = 3V$$

$$V_o = 3 \left(1 + \frac{10k}{5k} \right) = 9V$$

$-V_{sat} < V_o < +V_{sat}$

• Non-Inverting Adder



$$V_o = V_t \left(1 + \frac{R}{R} \right)$$

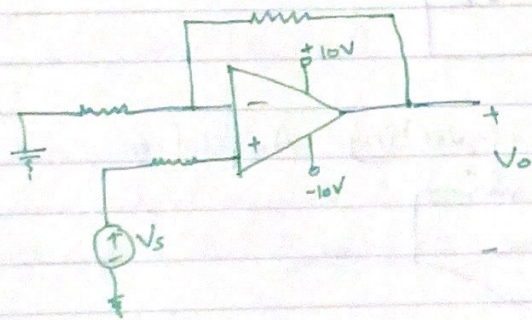
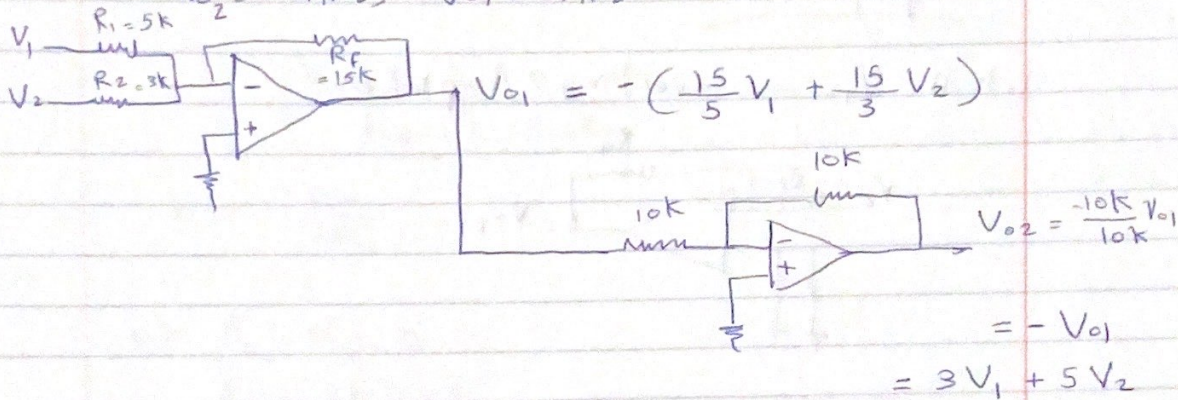
$$= \frac{V_1 + V_2}{2} \left(1 + \frac{R}{R} \right) = \boxed{V_1 + V_2 = V_o}$$

Ex Design an amplifier that have 2-input voltages V_1 & V_2 such that the output $V_o = 3V_1 + 5V_2$. (use inverting Amplifiers).

with inverting Amplifier $V_{o1} = -(3V_1 + 5V_2)$

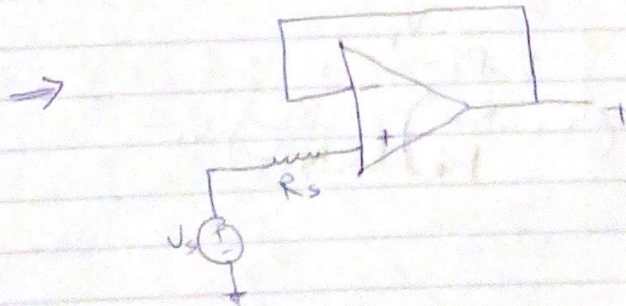
⇒ We can have a second stage with

$$V_{o2} = -V_{in2}, \quad V_{o1} = V_{in2}$$

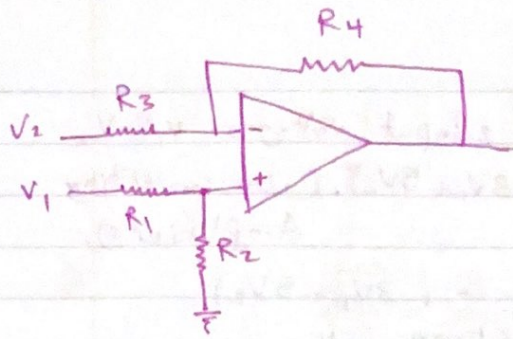


$$V_o = V_+ \left(1 + \frac{R_F}{R_i}\right)$$

$$V_o = V_s \left(1 + \frac{R_F}{R_i}\right) \rightarrow = 0 \quad \begin{matrix} R_F = 0 \\ R_i = \infty \end{matrix}$$



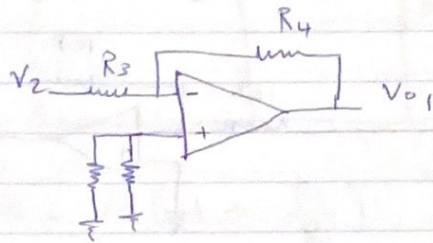
Difference Amplifier



mix of inverting & non-inverting Amplifier

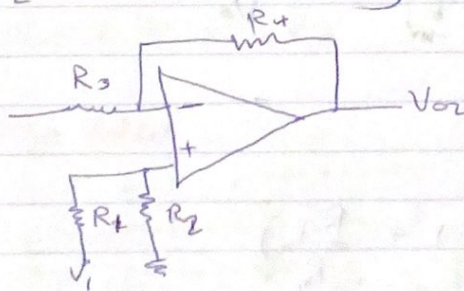
Superposition

1. Kill $V_1 \rightarrow$ inverting Amplifier



$$V_{01} = \frac{-R_4}{R_3} \cdot V_2$$

2. Kill $V_2 \rightarrow$ non-inverting Amplifier



$$V_{02} = \left(1 + \frac{R_4}{R_3}\right) V_+$$

$$V_+ = \frac{R_2}{R_1 + R_2} \cdot V_1$$

$$V_{02} = \left(1 + \frac{R_4}{R_3}\right) \left(\frac{R_2}{R_1 + R_2}\right) V_1$$

$$V_o = V_{o1} + V_{o2}$$

$$V_o = \left(\frac{R_3 + R_4}{R_3} \right) \left(\frac{R_2}{R_1 + R_2} \right) V_1 - \frac{R_4}{R_3} V_2$$

$$V_o = a V_1 - b V_2$$

$$a = \left(\frac{R_3 + R_4}{R_3} \right) \left(\frac{R_2}{R_1 + R_2} \right)$$

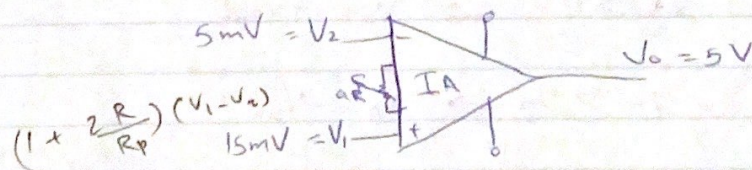
$$b = \frac{R_4}{R_3}$$

$$\text{let } R_1 = R_3 = R$$

$$R_2 = R_4 = mR$$

$$V_o = m (V_1 - V_2)$$

ex Use an instrumentation amplifier in order to get an output $V_o = 5V$ if $V_1 = 15mV$ (sensor 1), $V_2 = 5mV$ (sensor 2) & given that internal IA resistance = $20k\Omega$



$$V_o = \left(1 + \frac{2R}{a} \right) (V_1 - V_2)$$

$$5 = \left(1 + \frac{2}{a} \right) (15m - 5m)$$

$$5 = \left(1 + \frac{2}{a} \right) (10mV)$$

$$1 + \frac{2}{a} = \frac{5V}{10mV} = \frac{5000mV}{10mV} = 500$$

$$500 = 1 + \frac{2}{a} \Rightarrow \frac{2}{a} = 499 \Rightarrow a = \frac{2}{499}$$

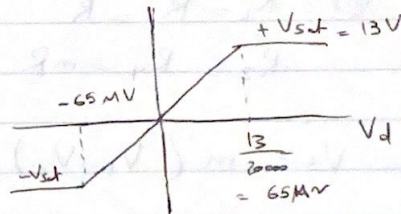
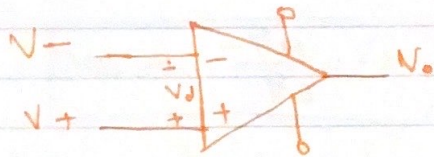
$$a R = \frac{2}{499} \times 20k = 80.16 \Omega$$

So far circuits with negative feedback

- inverting
 - non-inverting
 - Difference
 - Instrumentation
 - Voltage to current
 - Current to voltage
- } Amplifiers
- } Converters

2) Open loop \Rightarrow no feedback

Comparator



if $V_D = V_+ - V_- > 65 \mu V$

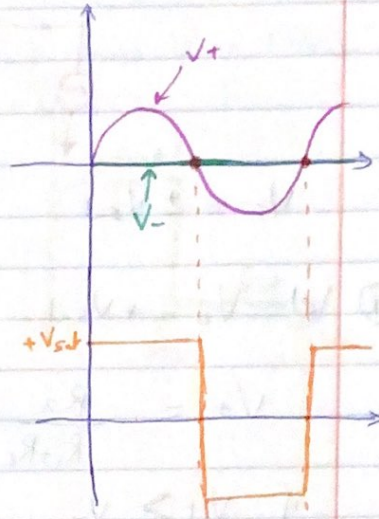
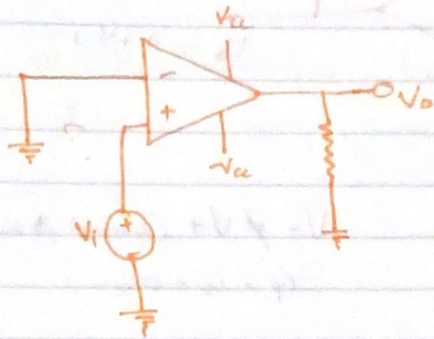
$\Rightarrow V_O = +V_{sat}$

if $V_D < -65 \mu V$

• If $V_+ > V_- \Rightarrow V_O = +V_{sat}$

• If $V_- > V_+ \Rightarrow V_O = -V_{sat}$

• Comparator, Zero-level detector



$$V_+ > V_- \Rightarrow V_o = +V_{sat}$$

$$V_- > V_+ \Rightarrow V_o = -V_{sat}$$

ex Given how an op amp functions, what do you expect V_o to be if $V_2 = 5V$ when:

1. $V_1 = 0V$?

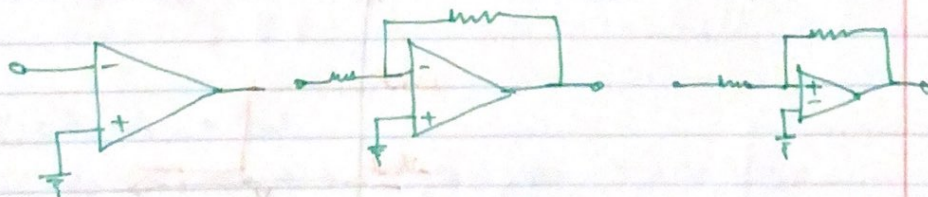
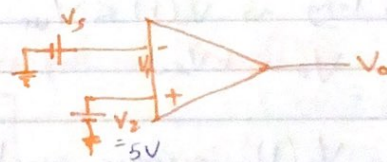
$$+V_{sat}$$

2. $V_1 = 6V$?

$$-V_{sat}$$

3. $V_1 = 5V$?

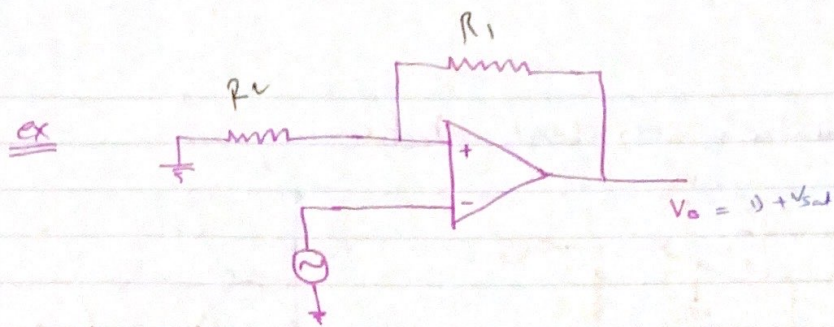
$$V_o = 0$$



(a) no feedback
open loop
comparator
circuit

(b) negative feedback

(c) positive feedback



$$V_o = \pm V_{sat}$$

$$V_- \neq V_+ \text{ (cannot be used)}$$

$$i_p = i_n = 0$$

① let $V_o = +V_{sat}$

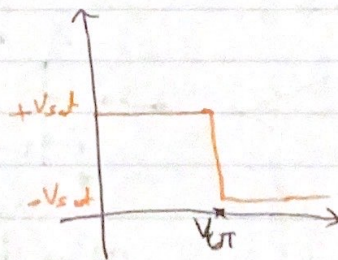
$$V_+ = \frac{R_2}{R_1 + R_2} (+V_{sat}) = V_{UT}$$

if $V_+ > V_-$

$$\boxed{V_{UT} > V_i(t)} \Rightarrow V_o = +V_{sat}$$

→ as long as $V_i(t) < V_{UT}$
 $V_o = +V_{sat}$

→ if $V_i(t)$ increases such that $V_i(t) > V_{UT}$
 $V_o = -V_{sat}$



② let $V_o = -V_{sat}$

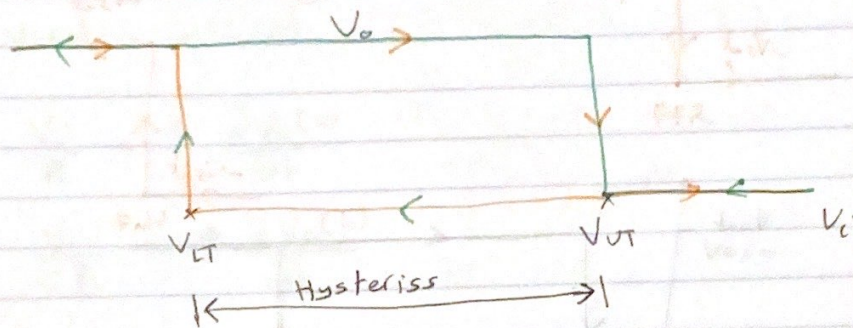
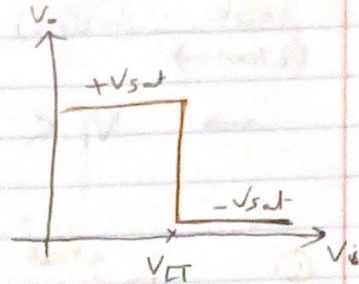
$$V_+ = \left(\frac{R_2}{R_1 + R_2} \right) (-V_{sat}) = V_{LT}$$

$$V_+ - V_- < 0 \Rightarrow V_o = -V_{sat}$$

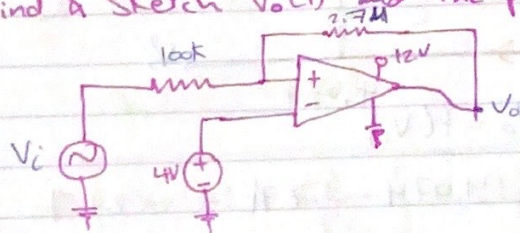
$$V_{LT} - V_i(t) < 0 \Rightarrow V_{LT} < V_i(t)$$

- as long as $V_i(t) > V_{LT} \Rightarrow V_o = -V_{sat}$

- if $V_i(t)$ decreases below $V_{LT} \Rightarrow V_o = +V_{sat}$



ex Find & sketch $V_o(t)$ and the plot of $V_o = f(V_i)$



$$+V_{sat} = 12 - 2 = 10V$$

$$-V_{sat} = 0 + 2 = 2V$$

① let $V_o = +V_{sat}$

$$V_o > 0 \Rightarrow V_+ - V_- > 0 \Rightarrow V_+ > V_-$$

$$V_- = 4V \Rightarrow V_+ > 4V$$

$$V_+ = \frac{100k}{(270k+100k)} (+V_{sat}) + \frac{270k}{(100+270)k} (V_i) \quad \text{by super position}$$

$$\frac{100k}{270k+100k} (10V) + \frac{270k}{100+270k} (V_i) > 4V$$

$$\Rightarrow V_i > \underline{\underline{3.777V}} \quad \text{when } V_i < 3.777 \Rightarrow V_o = -V_{sat}$$

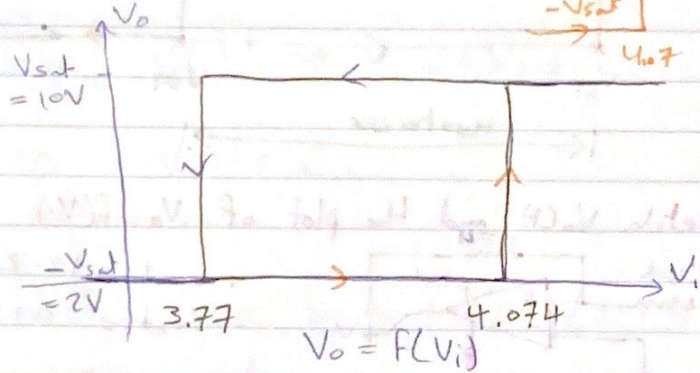
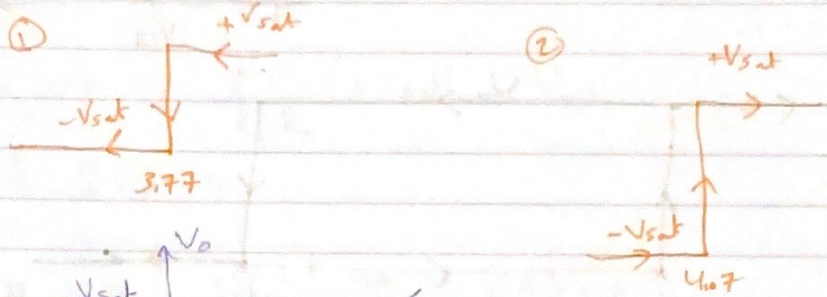
$$V_{UT}$$

② let $V_o = -V_{sat}$

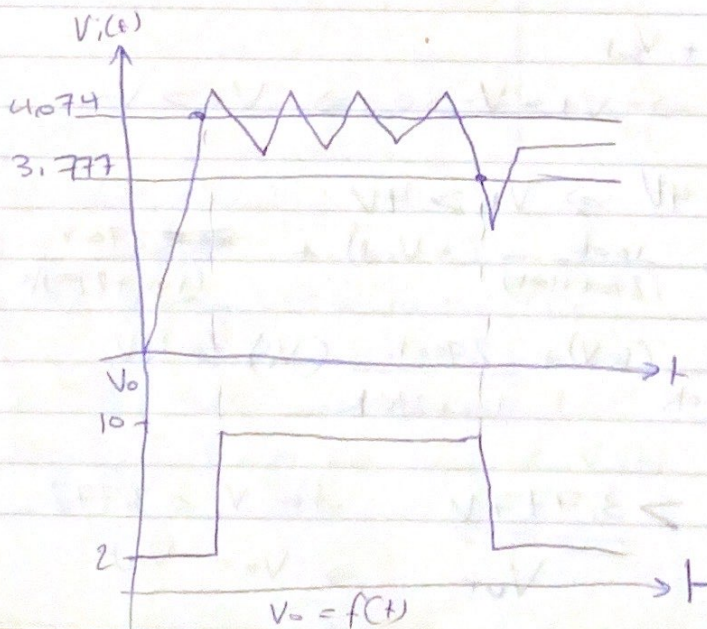
$V_d < 0 \Rightarrow V_+ < V_-$

$\frac{100k}{(2700+100k)} (-V_{sat}) + \frac{2700k}{(2700+100k)} (V_i) < 4V$

$\Rightarrow V_i < 4.074$, But when $V_i > \boxed{4.074}$ $\Rightarrow V_o = +V_{sat}$



Hysteresis = $|4.074 - 3.77| = 0.297$



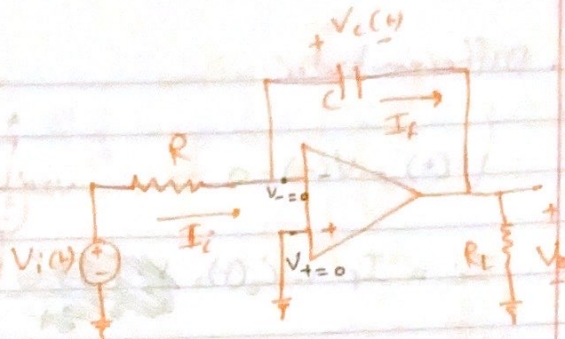
Integrator
+
Differentiator
opAmp 4
22137

• Integrator

negative feedback

$$V_+ = V_-$$

$$i_p = i_n = 0$$



$$I_i = \frac{V_i}{R} = I_f$$

$$i_c(t) = C \frac{dV_c(t)}{dt} = I_f$$

$$V_o(t) = -V_c(t)$$

$$\frac{V_i}{R} = C \frac{dV_c(t)}{dt}$$

$$\frac{V_i}{RC} = \frac{dV_c(t)}{dt}$$

$$\Rightarrow V_c(t) = \frac{1}{RC} \int V_i(t) dt$$

$$V_o(t) = -V_c(t)$$

$$\Rightarrow \boxed{V_o(t) = -\frac{1}{RC} \int V_i(t) dt}$$

Differentiator

$$V(+)=V(-)=0$$

$$I_i = I_f = i_c(t) = \dots$$

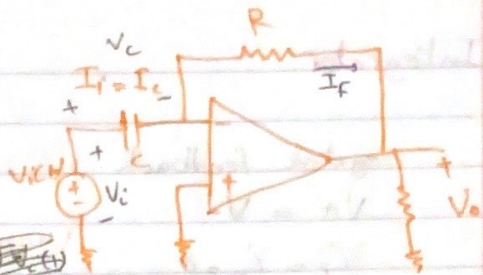
$$V_i(t) = V_c(t)$$

$$i_c = C \frac{dV_i(t)}{dt}$$

$$V_o = -i_f(t)R$$

$$V_o = -C \frac{dV_i(t)}{dt} (R)$$

$$V_o = -RC \frac{dV_i(t)}{dt}$$



Filters

LPF, HPF, BPF, BRF

Passive
R, L, C

Active

R, C, op-Amp

- no loading effect

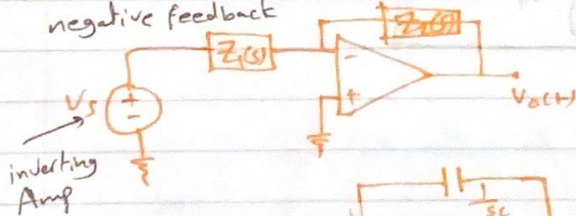
- ω_c is well defined & constant

1st order

2nd order

nth order

The active Low-Pass Filter
negative feedback



$$H = A_v = - \frac{Z_2(j\omega)}{Z_1(j\omega)}$$

$$Z_2 = \frac{R_2 \frac{1}{sC}}{R_2 + \frac{1}{sC}} = \frac{R_2 \frac{1}{j\omega}}{R_2 + \frac{1}{j\omega}}$$

$$Z_2 = \frac{R_2}{j\omega C R_2 + 1}$$

$$Z_1 = R_1$$

$$A_v = - \frac{R_2}{R_1(1 + j\omega C R_2)}$$

complex number

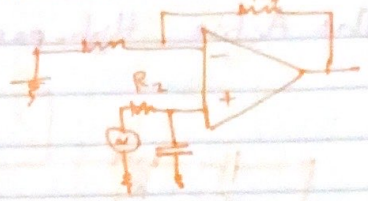
$$A_V = \frac{-K}{1 + \frac{j\omega}{\omega_c}}$$

$$K = \frac{R_2}{R_1}$$

$$\omega_c = 2\pi f_c = \frac{1}{R_2 C}$$

$$\therefore f_c = \frac{1}{2\pi R_2 C}$$

another low pass filter config.



$$\omega_c = \frac{1}{R_2 C}$$

$$K = 1 + \frac{R_2}{R_1}$$

$$|A_V| = \frac{|-K|}{|1 + \frac{j\omega}{\omega_c}|} = \frac{K}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$$

OR

$$= \frac{K}{\sqrt{1 + (\frac{f}{f_c})^2}}$$

ex $A_V = 40 \text{ dB}$, $R_{in} = 5 \text{ k}\Omega$, $f_c = 2 \text{ kHz}$

Ideal op-Amp

$$20 \log |A_V| = 40 \text{ dB}$$

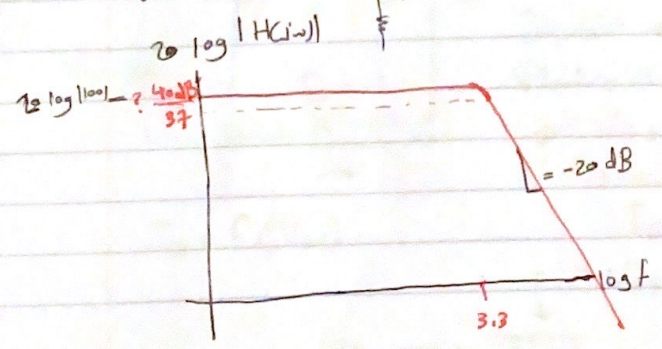
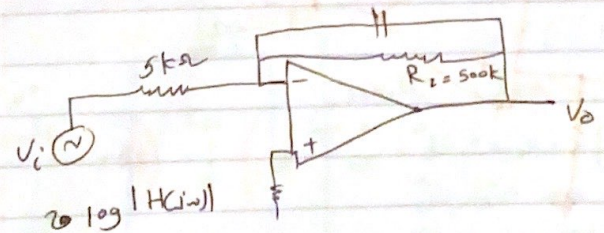
$$\log |A_V| = \frac{40}{20} = 2$$

$$|A_V| = 10^2 = 100 \Rightarrow \text{dc gain} = 100 = \left| \frac{R_2}{R_1} \right|$$

$$\Rightarrow R_2 = 500 \text{ k}\Omega$$

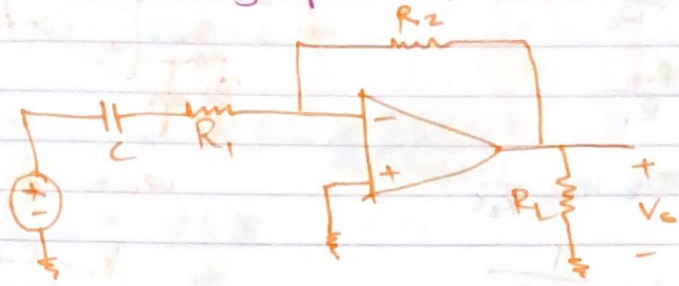
$$\omega_c = \frac{1}{R_2 C} = 2\pi f_c$$

$$\Rightarrow C = 159 \text{ pF}$$



$$\log(2000) = 3.3$$

*The Active High-pass Filter



$$A_v = -\frac{Z_2}{Z_1} = -\frac{R_2}{\left(R_1 + \frac{1}{j\omega C}\right)}$$

$$A_v = \frac{-K}{\left(1 + \frac{1}{j\omega R_1 C}\right)} \quad ; \quad K = \frac{R_2}{R_1}$$

$$A_v = \frac{-K}{1 + \frac{\omega_c}{j\omega}} \quad , \quad \omega_c = \frac{1}{R_1 C}$$

HPF

$$\Rightarrow |A_v| = \frac{K}{\sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2}} \quad \xrightarrow[\text{LPF}]{\text{compare with}}$$

$$|A_v| = \frac{K}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

