CHAPTER 6: Induction Motors.

* The distinguishing feature of induction motor is that no DC field current is required to run the machine.

→ The construction of the induction motor

- It has the same stator construction of synchronous motor with different rotor construction.
- The induction motor has two rotor types:
  1) The squirrel cage rotor.
  2) The wound rotor.

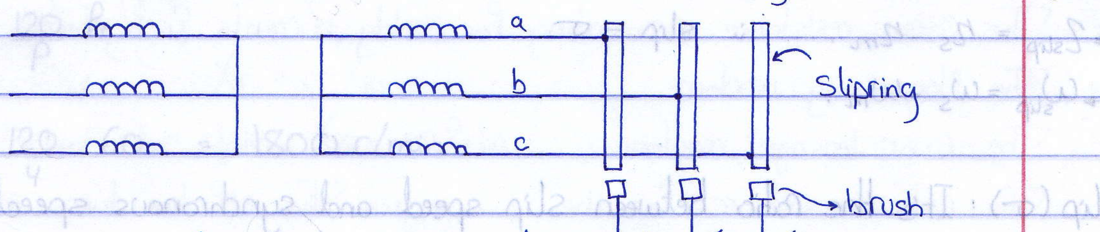
→ Explanation of the two rotor types:

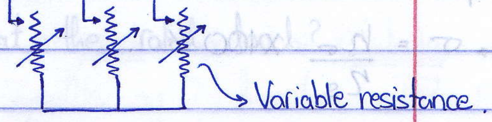
① The cage rotor:

It consists of series conducting bars laid into slots carved in the face of the rotor and shorted at either end by large shorting rings.

② The wound rotor:

It consists of a set of three-phase windings, that are mirror images of the windings of the stator. The windings are usually Y-connected. The terminals a, b & c are tied to the slip-rings on the rotor's shaft. The rotor windings are shorted via brushes riding on the sliprings.





Note: ① In wound rotor type, an external resistor can be added to the rotor circuit to modify the torque-speed characteristics.

② The wound rotor is more expensive and requires much more maintenance.

→ Operating principle of induction motor:

- A set of 3-ϕ voltages is applied to the stator windings and a set of 3-ϕ stator currents is flowing
- The currents will produce a rotating magnetic field which rotats at synchronous speed (n_s) where n_s is given by:

$n_s = \frac{120}{P} f_e$.   ∴ f_e: is the system freq in hertz.

- The rotating magnetic field will pass over the rotor conducting bars and induce voltages on them by the relation:

$e_{ind} = (V \times B) \cdot l$

∴ V: is the velocity of the bar <u>relative</u> to the magnetic field <u>B_s</u>.

- The induced voltages will produce a rotor current $\vec{I_r}$.
- The rotor current $\vec{I_r}$ produces a rotor magnetic field $\vec{B_r}$.
- Now the rotor magnetic field $\vec{B_r}$ interacts with the stator magnetic field $\vec{B_s}$ to produce the induced torque in the machine.

$\tau_{ind} = k\, \vec{B_r} \times \vec{B_s}$.

Note: The rotor can speed-up to synchronous speed but it can never reach it.

→ The Concept of Rotor Slip:

- Two terms are commonly used to define the relative motion of the rotor and the magnetic fields:

① Slip speed: It's the difference between the synchronous speed & rotor speed.

→ $n_{slip} = n_s - n_m$.   ∴ slip $\equiv \sigma$

→ $\omega_{slip} = \omega_s - \omega_m$.

② Slip (σ): It's the ratio between slip speed and synchronous speed.

→ $\sigma = \frac{n_\sigma}{n_s} \times 100\%$.

→ $\sigma = \frac{n_s - n_m}{n_s} \times 100\%$

→ $\sigma = \frac{\omega_s - \omega_m}{\omega_s} \times 100\%$

Solving the last equation for η_m:

→ $\eta_m = (1-\sigma)\eta_s$.

→ $\omega_m = (1-\sigma)\omega_s$.

Notice that is the rotor is rotating at η_s then $(\sigma = 0)$, while if it is locked $(\eta_m = 0)$ then $(\sigma = 1)$.

→ The Electric Frequency on the rotor:

- If the rotor is locked $(\eta_m = 0) \Leftrightarrow (\sigma = 1)$, then it will have the same frequency as the stator. $(f_r = f_s = f_e)$

On the other hand, when the rotor turns at synchronous speed $(\eta_m = \eta_s) \Leftrightarrow (\sigma = 0)$, then the rotor frequency will be zero $(f_r = 0)$.

- At any other speed between 0 & η_s the rotor's frequency is directly proportional to the slip (σ).

→ $f_r = \sigma f_e$.

→ $f_r = \frac{P}{120}(\eta_s - \eta_m)$.

→ Example:

A 208 V, 10-hp, 4-pole, 60-Hz, Y-connected IM has a full load slip of 5%.

a) What is the synchronous speed of this Motor?

$\eta_s = \frac{120 f_e}{P}$

$= \frac{120}{4} \cdot 60 = 1800$ r/min.

b) What is the rotor speed of this Motor at the rated load?

$\eta_m = (1-\sigma)\eta_s$

$= (1 - 0.05)(1800)$

$= 1710$ rpm.

c) What is the rotor frequency of this motor at the rated load?

$f_r = \sigma f_e$

$= 0.05\,(60)$

$= 3\ Hz$

d) What is the shaft torque of this motor at the rated load?

$P_{out} = \tau_{load}\,\omega_m$

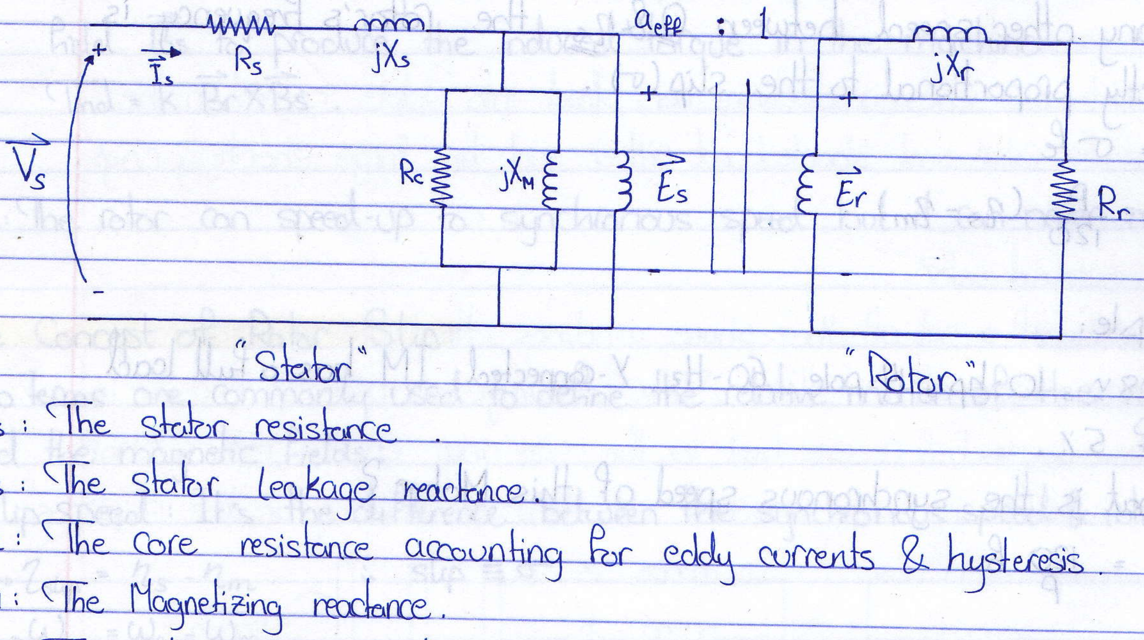
$\tau_{load} = P_{out}/\omega_m$

$= (10hp)(746)/(1710)\cdot(2\pi/60)$

$= 41.7\ N.m$

→ The Equivalent Circuit of an Induction Motor:

It is very similar to the per-phase equivalent circuit of the transformer.



∴ R_s: The stator resistance.

X_s: The stator leakage reactance.

R_c: The core resistance accounting for eddy currents & hysteresis.

X_M: The Magnetizing reactance.

X_r: The rotor leakage reactance.

R_r: The rotor resistance, for the windings in (wound), bars in (cage).

E_s: The internal stator voltage.

E_r: The induced rotor voltage.

I_s: The stator (source) current.

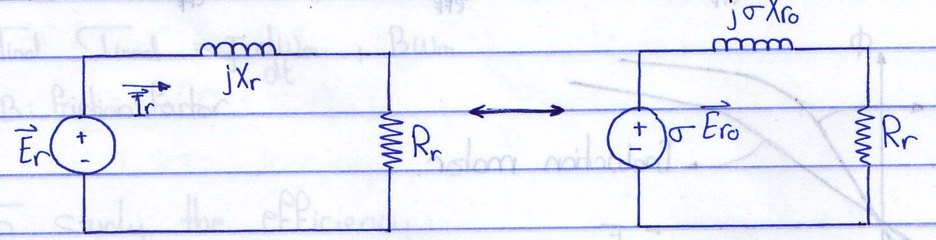
I_r: The rotor current.

V_s: The stator (source) phase voltage.

* a_{eff}: The effective turns ratio of the motor which couples between $\vec{E_s}$ & $\vec{E_r}$.

Note: a_{eff} is fairly easy to determine for the wound-rotor motor.

- Rotor circuit model:



- In general, the greater the relative motion between the rotor & the stator magnetic fields, the greater the resulting rotor voltage & rotor frequency.
- The largest relative motion occurs when the rotor is locked,

  $\sigma = 1$ and $\vec{E}_r = 1.\vec{E}_{ro}$ "The largest rotor Voltage".

  $f_r = 1.f_e$ "The largest rotor frequency".
- When the rotor moves at the same speed as the stator magnetic field, resulting in no relative motion,

  $\sigma = 0$ and $\vec{E}_r = 0$ "The smallest rotor Voltage".

  $f_r = 0$ "The smallest rotor frequency".
- At any other speed, the voltage and frequency of the rotor is directly proportional to the slip of the rotor.

  $\vec{E}_r = \sigma \vec{E}_{ro}$

  $f_r = \sigma f_e$.
- The reactance of an induction motor rotor depends on the inductance of the rotor and its voltage frequency:

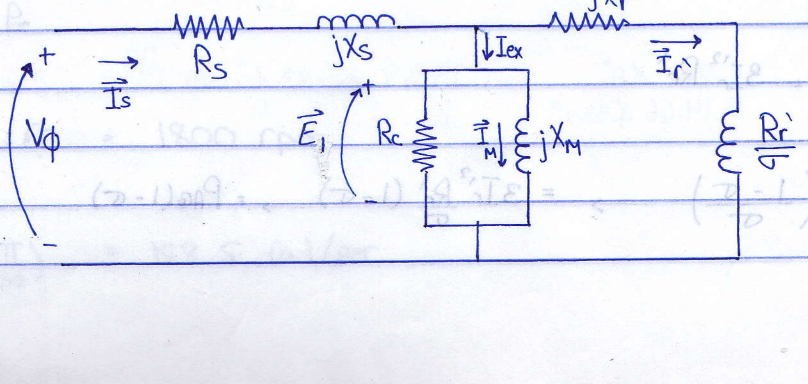
  $X_r = \omega_r L_R = 2\pi f_r L_R$

  So, since $f_r = \sigma f_e$

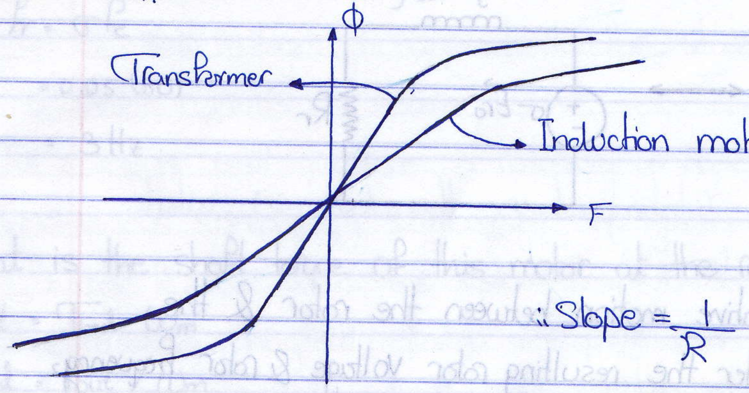
  $X_r = 2\pi(\sigma f_e)L_R$, $X_r = \sigma X_{ro}$

  at the locked rotor state $X_{ro} = 2\pi f_e L_R$

→ The per-phase equivalent circuit of an induction motor.

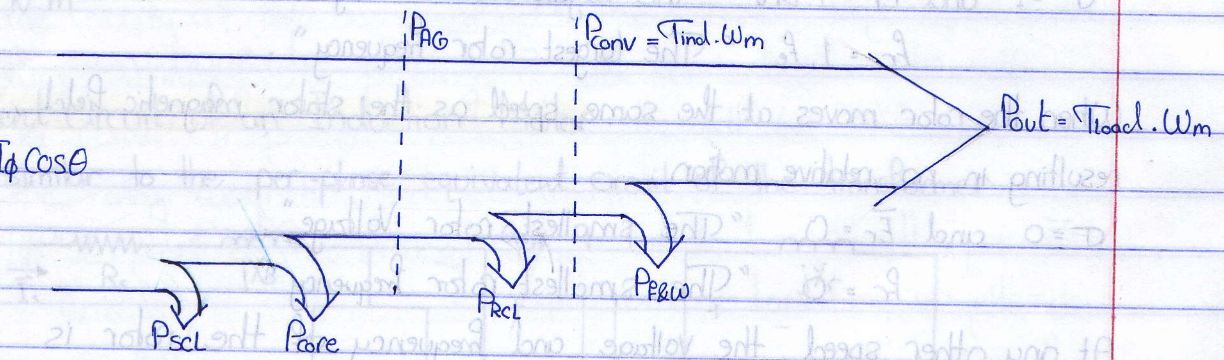


$\therefore \vec{I_r}' = \frac{1}{a_{eff}} \vec{I_r}$ $\quad \therefore X_r' = a_{eff}^2 X_r$ $\quad \therefore R_r' = a_{eff}^2 R_r$ $\quad \therefore \vec{E_1} = a_{eff} \vec{E_{ro}}$



$\therefore$ Slope $= \frac{1}{R}$

→ Power Calculations:



→ P_{SCL} : Stator copper losses.
$= 3 I_s^2 R_s$ ---- ①

→ P_{core} : Core losses.
$= 3 E_1^2 / R_C$ ---- ②

→ P_{RCL} : Rotor copper losses.
$= 3 I_r'^2 R_r'$ ---- ③

→ P_{AG} : Air Gap power
$= 3 I_r'^2 \frac{R_r'}{\sigma}$ ---- ④

→ $P_{F\&W}$: Friction & Windage losses.

→ P_{conv} : Converted power to Mechanical.

→ P_{AG} : Air Gap power consumed in "$\frac{R_r'}{\sigma}$".

From the power Flow diagram:

1) $P_{AG} = P_{in} - P_{SCL} - P_{core}$

2) $P_{conv} = P_{AG} - P_{RCL}$

$= 3 I_r'^2 \frac{R_r'}{\sigma} - 3 I_r'^2 R_r'$

3) $P_{out} = P_{conv} - P_{F\&W}$

$P_{conv} = 3 I_r'^2 R_r' \left(\frac{1-\sigma}{\sigma} \right) \longrightarrow = 3 I_r'^2 \frac{R_r'}{\sigma} (1-\sigma) \longrightarrow = P_{AG}(1-\sigma).$

- Now from Newton's second law:

$$T_{ind} - T_{load} = J\frac{d\omega_m}{dt} + B\omega_m$$

$\therefore B$: friction factor.

- To study the efficiency:

① Rotor efficiency:

$$\eta_r = \frac{P_{conv}}{P_{AG}} = (1-\sigma)$$

② Motor efficiency:

$$\eta_M = \frac{P_{out}}{P_{in}}$$

- Induced Torque:

$P_{conv} = T_{ind}\,\omega_m$

$\therefore \omega_m = (1-\sigma)\omega_s$

$P_{conv} = (1-\sigma)P_{AG}$

$$T_{ind} = \frac{P_{AG}}{\omega_s} \longrightarrow T_{ind} = \frac{3I_r'^2 R_r'}{\sigma\,\omega_s}$$

Example:

A 460-V, 25-hp, 60-Hz, 4-pole, Y-connected induction Motor has the following impedances in Ohms per phase referred to the stator circuit:

- $R_s = 0.641\,\Omega$ - $R_r' = 0.332\,\Omega$ - $X_m = 26.3\,\Omega$

$X_s = 1.106\,\Omega$ $X_r' = 0.464\,\Omega$

The total rotational losses are 1100 W and are assumed to be constant. The core losses are lumped in with the rotational losses.

For a rotor slip of 2.2% at the rated voltage & rated frequency:

Find:

a) The Motor's Speed:

$$\eta_s = \frac{120}{P} f_e$$

$$= \frac{120}{4} \cdot 60 = 1800 \text{ rpm}.$$

$$\omega_s = \eta_s\left(\frac{\pi}{30}\right) = 188.5 \text{ rad/sec}.$$

* To Find the Motor's speed (η_m)

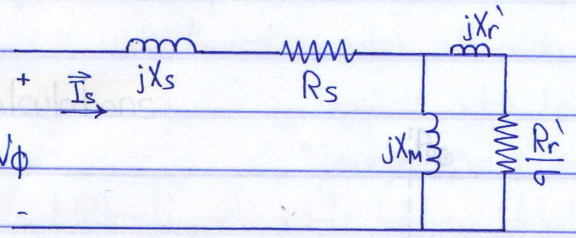
$\eta_m = (1-\sigma)\,\eta_s$

$= (1-0.022)(1800)$

$= 1760$ rpm.

b) The Motor's stator current:

To find the stator current we should get the equivalent impedance of the circuit:



To Find Req:

1) $jX_M \,//\, (jX_r' + \frac{R_r'}{\sigma})$

$\therefore jX_M = 26.3j$

$jX_r' + \frac{R_r'}{\sigma} = 0.464j + \frac{0.332}{0.022} \Rightarrow 15.1 \measuredangle 1.76°$

$\rightarrow (26.3j \,//\, (15.1\measuredangle 1.76°))$

$Req_{(1)} \rightarrow \dfrac{1}{\frac{1}{26.3j} + \frac{1}{15.1\measuredangle 1.76°}} = 12.92 \angle 31.2°$

2) $jX_s + R_s + Req_{(1)}$:

$= 1.106j + 0.641 + 12.92 \measuredangle 31.2°$

$= 14.06 \measuredangle 33.7°$.

Then Find $V\phi$:

$V_L = \sqrt{3}\,V\phi \rightarrow V\phi = \frac{V_L}{\sqrt{3}} = \cancel{\frac{208}{\sqrt{3}}}\ \frac{460}{\sqrt{3}} = 266 \measuredangle 0°\ V$.

Now the current:

$\vec{I_s} = \dfrac{V\phi \measuredangle 0}{Zeq} \rightarrow \dfrac{266\measuredangle 0°}{14.06 \measuredangle 33.7°} = 18.88 \measuredangle -33.7°\ A$

c) The Motor's PF:

$PF = \cos^{-1}(\theta_v - \theta_i)$

$= \cos^{-1}(0 + 33.7)$

$= 0.832$ lagging

d) The Motor's P_{conv} & P_{out}:

i. $P_{conv} = (1-\sigma) P_{AG}$, so we need to find P_{AG}:

$P_{AG} = P_{in} - P_{SCL}$

$\therefore P_{in} = 3 V_\phi I_\phi \cos\theta$

$= 3(266)(18.88)(0.832)$

$= 12.53$ KW

$P_{scl} = 3 I_s^2 R_s$

$= 3(18.88)^2(0.641)$

$= 0.685$ kW

$P_{AG} = (12.53k - 0.685k)$ W

$= 11.845$ kW

$P_{conv} = (1 - 0.022)(11.845k)$

$= 11.585$ KW

ii. $P_{out} = P_{conv} - P_{F\&W}$

$= (11.585k - 1.1k)$ W

$= 10.485$ kW

e) τ_{ind} & τ_{load} for the Motor:

i. $\tau_{ind} = \dfrac{P_{conv}}{\omega_s} \Rightarrow \dfrac{11585}{1800 \cdot \frac{\pi}{30}} = 62.8$ N.m

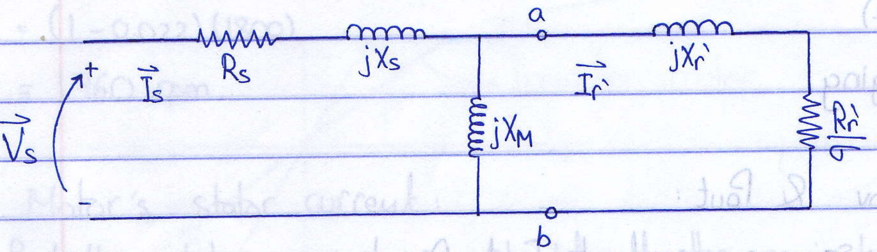
ii. $\tau_{load} = \dfrac{P_{out}}{\omega_m} \Rightarrow \dfrac{10.485k}{1760\left(\frac{\pi}{30}\right)} = 56.9$ N.m

f) The Motor's efficiency:

$\eta_s = \dfrac{P_{out}}{P_{in}} \times 100\%$

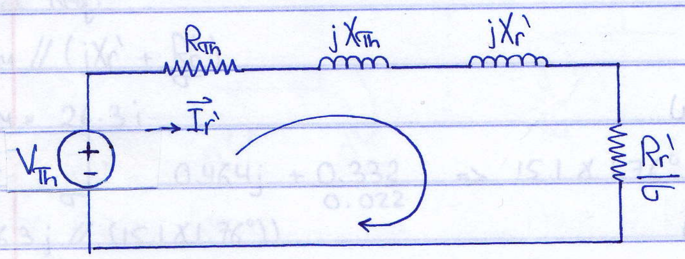
$= \dfrac{10485}{12530} \times 100\% = 83.7\%$

→ Derivation of induced torque equation:



$\tau_{ind} = \frac{P_{AG}}{\omega_s} = \frac{3 I_r'^2 R_r'}{\sigma \omega_s}$ ∴ I_r' is the magnitude of the current.

- From the equation it's clear that we need $\underline{I_r'}$ to calculate the τ_{ind}, but in fact there is no access on the rotor so its difficult to find it:
The solution is to get the thevinin of the stator part from points (a, b).



① $\vec{V_{Th}} = \vec{V_{oc}} = \frac{jX_M}{R_s + j(X_s + X_M)} \vec{V_s}$

∴ $X_M + X_s \gg R_s$

$\vec{V_{Th}} = \frac{X_M}{(X_s + X_M)} \vec{V_s}$ ------- ①

② $Z_{Th} = (R_s + jX_s) // jX_M$

$= \frac{(R_s + jX_s)(jX_M)}{R_s + j(X_s + X_M)}$

∴ $X_M + X_s \gg R_s$

$Z_{Th} = \frac{(R_s + jX_s)(X_M)}{(X_s + X_M)}$

i

$Z_{Th} = R_s \frac{X_M}{X_s + X_M} + j X_s \frac{X_M}{X_s + X_M}$

$R_{Th} = R_s \frac{X_M}{X_s + X_M}$ ---- ②

$X_{Th} = X_s \frac{X_M}{X_s + X_M}$ ---- ③

ii

To find I_r':

$\vec{I_r'} = \dfrac{\vec{V_{Th}}}{(R_{Th} + \frac{R_r'}{\sigma}) + j(X_{Th} + X_r')}$, but to find the τ_{ind} we need the magnitude.

$$|\vec{I_r'}| = \frac{|\vec{V_{Th}}|}{\sqrt{(R_{Th} + \frac{R_r'}{\sigma})^2 + (X_{Th} + X_r')^2}}$$

Now: $\tau_{ind} = \dfrac{3 I_r'^2 R_r'}{\sigma\, \omega_s}$, substitute I_r' in τ_{ind}.

$$\tau_{ind} = \frac{3V_{Th}^2}{\omega_s\left([R_{Th} + \frac{R_r'}{\sigma}]^2 + [X_{Th} + X_r']^2\right)} \cdot \frac{R_r'}{\sigma}$$

* Note: In Calculations we can make an approximation in:

$\dfrac{X_M}{X_S + X_M} \approx 1$ since $X_M \gg X_S$.

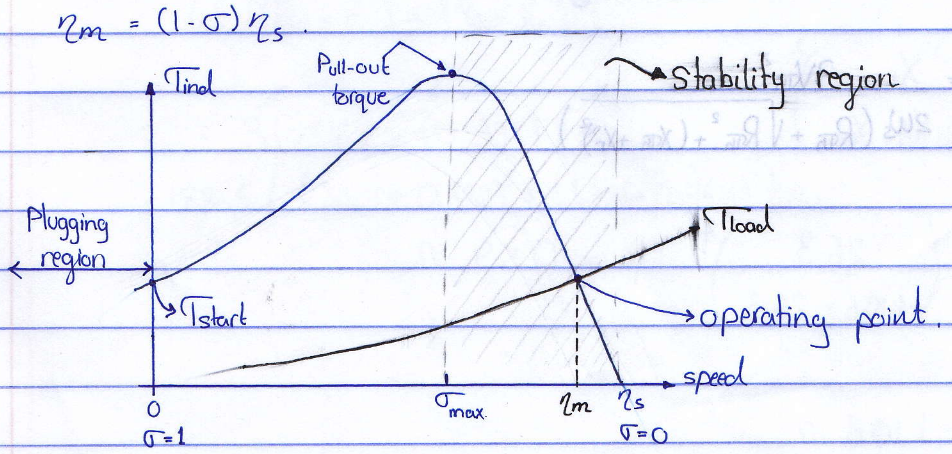
Therefore, $\left.\begin{aligned} R_{Th} &\simeq R_s \\ X_{Th} &\simeq X_s \\ V_{Th} &\simeq V_s \end{aligned}\right\} \longrightarrow \tau_{ind} = \dfrac{3V_s^2}{\omega_s\left([R_s + \frac{R_r'}{\sigma}]^2 + [X_{Th} + X_r']^2\right)} \cdot \dfrac{R_r'}{\sigma}$

- Induction Motor Torque-Speed characteristics:

From the τ_{ind} equation we can see that the torque is a function of slip, and since the slip is changing by n_m so it's also a function of n_m.

$\tau_{ind} = f(\sigma) = g(n_m)$

$n_m = (1-\sigma)\, n_s$.



Notes on the curve:

① When the slip range is very small, the induced torque is approximately proportional to the slip (linear relationship).

$$\tau_{ind} \simeq \frac{3V_{Th}^2}{\omega_s \left(\frac{R_r'}{\sigma}\right)^{\cancel{2}}} \cdot \frac{\cancel{R_r'}}{\cancel{\sigma}} \longrightarrow \tau_{ind} \simeq \frac{3V_{Th}^2}{\omega_s R_r'} \cdot \sigma \longrightarrow \tau_{ind} = K\sigma .$$

② Under steady state (speed is constant)

From Newton's second law:

$$\tau_{ind} - \tau_{load} = J\frac{d\omega}{dt} \longrightarrow \text{So } \tau_{ind} = \tau_{load} .$$

- Plugging Region:

It's used to stop the motor rapidly by switching any two of the three phases.

- Starting torque (τ_{start}):

It's slightly higher than the motor rated torque.

- $\tau_{pull\text{-}out} = \tau_{max}$: The maximum torque.

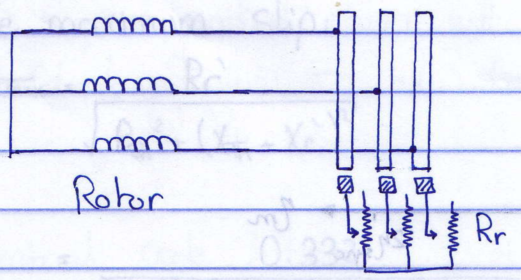
$\tau_{max} = (2-3)\ \tau_{rated}$

→ To find the maximum torque:

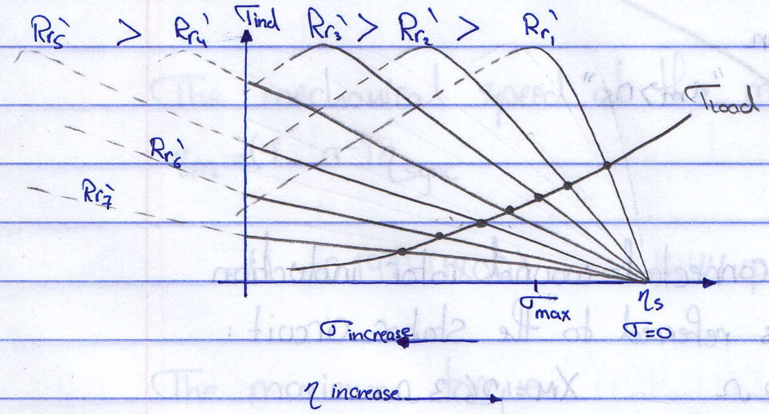
$$\frac{\partial \tau_{ind}}{\partial \sigma} = 0 \longrightarrow \sigma_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}}$$

$$\tau_{max} = \frac{3V_{Th}^2}{2\omega_s \left(R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}\right)}$$

→ In Wound Rotor Induction Motor:



* only in the wound rotor we can change "Rr"



When $R_{r1}' \leq R_r' \leq R_{r4}'$ → Then $R_r' \uparrow$ the $\sigma_{max} \uparrow \rightarrow \sigma \uparrow \rightarrow \eta \downarrow$

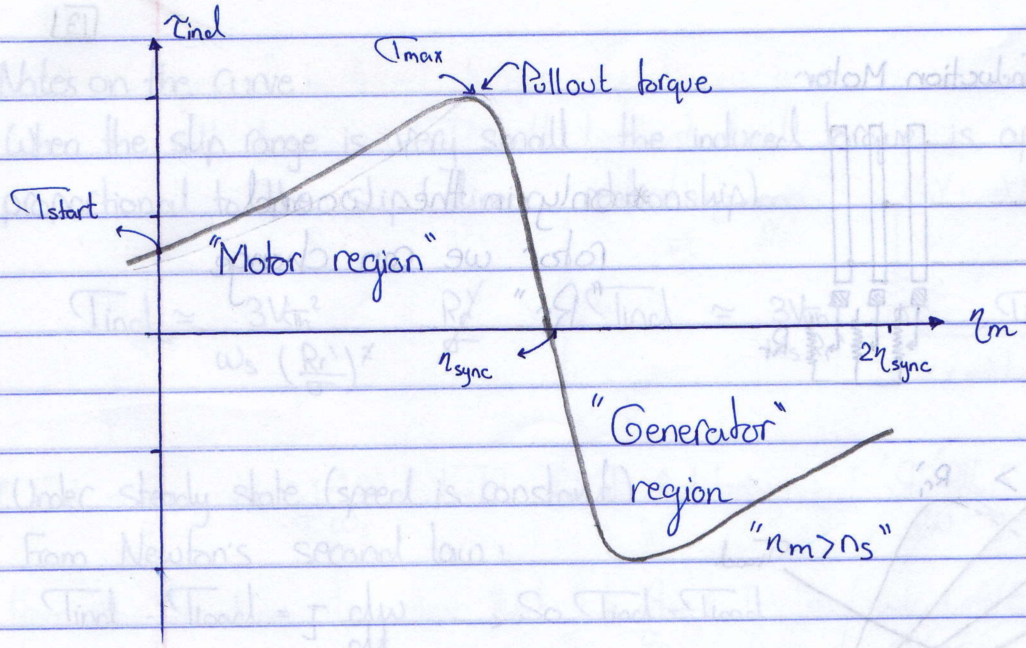
So $\eta_r = (1-\sigma)$ [Rotor efficiency] ↓

and $\eta \downarrow$ → [Motor efficiency].

and $\tau_{start} \uparrow$

when $R_r' > R_{r4}'$ → $\tau_{start} \downarrow \Rightarrow \eta$ (efficiency) ↓

The Rotor resistance in the wound Rotor Induction Motor is a method to control the <u>speed</u> & the <u>starting torque</u>.



Example: A 460-volt, 25-hp, 4-pole, Y-connected wound rotor induction motor has the following impedances referred to the stator circuit:

$R_s = 0.641\,\Omega$ $\quad R_r' = 0.332\,\Omega$ $\quad X_M = 26.3\,\Omega$

$X_s = 1.106\,\Omega$ $\quad X_r' = 0.464\,\Omega$

a) What is the maximum torque of this motor? At what speed & slip does it occur?

[1] Find $V_{Th} = V_\phi \dfrac{X_M}{\sqrt{R_s^2 + (X_s + X_M)^2}}$

$$= 266 \frac{26.3}{\sqrt{(0.641)^2 + (1.106+26.3)^2}} = 255.2\ V$$

[2] Find $R_{Th} = R_s \left(\dfrac{X_M}{X_s + X_M}\right)^2$

$$= 0.641 \left(\frac{26.3}{1.106 + 26.3}\right)^2 = 0.59\ \Omega$$

[3] Find $X_{Th} = X_r' = 1.106\ \Omega.$

Now:

The maximum slip: *Note: $n_s = \frac{120}{4} \cdot 60 = 1800$ rpm.

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}}$$

$$= \frac{0.332}{\sqrt{(0.59)^2 + (1.106 + 0.464)^2}} = 0.198\ \Omega$$

The mechanical speed at the maximum slip:

$$n_m = (1 - \sigma) n_{sync}$$

$$= (1 - 0.198)(1800) = 1444 \text{ rpm}$$

The maximum torque: *Note: $\omega_s = n_s \cdot \frac{2\pi}{60}$

$$T_{max} = \frac{3 V_{Th}^2}{2\omega_s [R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}}$$

$$= \frac{3(255.2)^2}{2(188.5)[0.59 + \sqrt{0.59^2 + (1.106 + 0.464)^2}} = 229 \text{ N.m.}$$

b) What is the starting torque of the motor?
In the torque equation set the σ to $\underline{1}$ & solve it as T_{start}:

$$T_{start} = \frac{3 V_{Th}^2 R_r'}{\omega_s [(R_{Th} + R_r')^2 + (X_{Th} + X_r')^2]}$$

$$= \frac{3(255.2)^2 (0.332)}{188.5[(0.59 + 0.332)^2 + (1.106 + 0.464)^2]} = 104 \text{ N.m.}$$

c) When the Rotor resistance is doubled, What is the speed at which the max torque ocurred? What is the new Starting torque?

→ if we double the rotor resistance then the σ will be doubled so

$\sigma_{max} = 0.396$

→ The speed at which this slip occures is:

$n_m = (1-\sigma) n_s$

$= (1-0.396)(1800)$

$= 1087 rpm$.

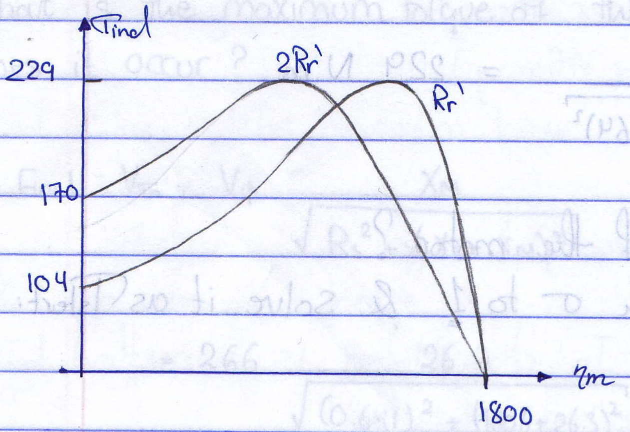
→ The max torque will not be changed.

$\tau_{max} = 229$ N.m.

→ The starting torque will change:

$$\tau_{start} = \frac{3(255.2)^2 (0.664)}{188.5\left[(0.59+0.664)^2 + (1.106+0.464)^2\right]}$$

$= 170$ N.m



→ Calculations of Motor Starting current:

$$I_L\ \text{start} = \frac{S\ \text{start}}{\sqrt{3}\ V_L}$$ ∴ S start : Rated power (hp) * "code letter Factor"

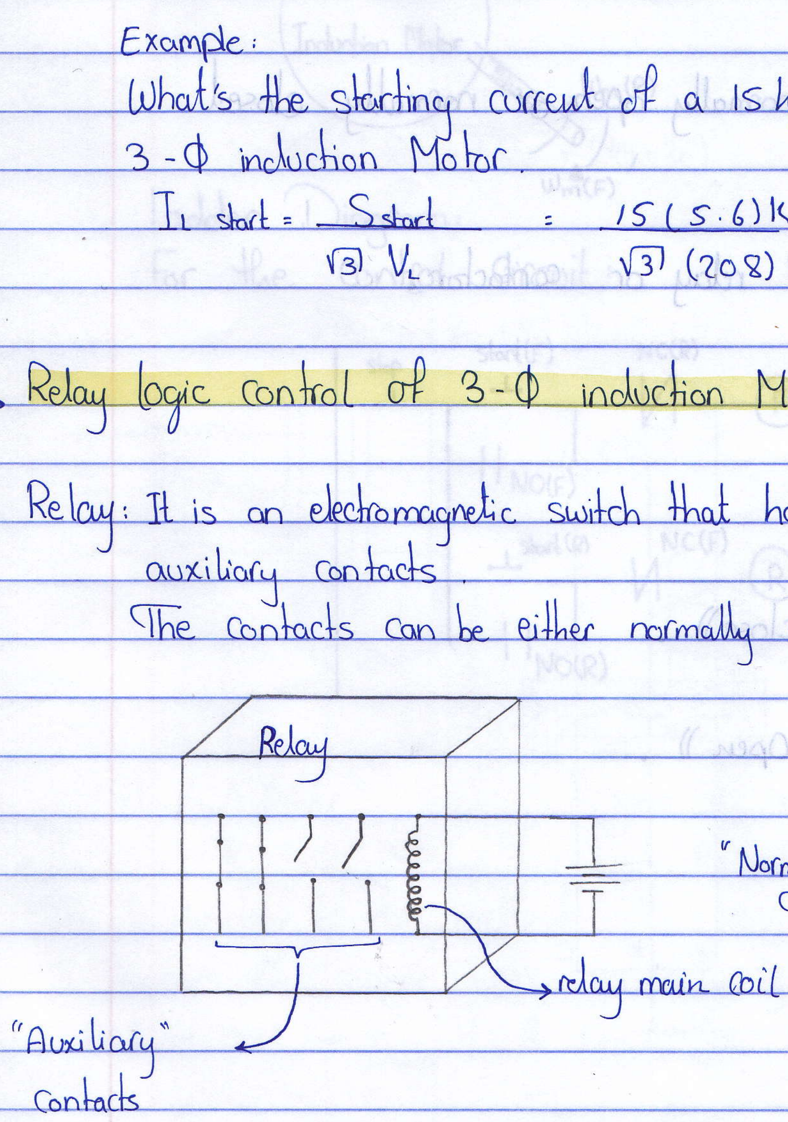
Nominal code letter	locked Rotor [KVA/hp]
A	0 – 3.15
B	3.15 – 3.55
C	3.55 – 4.00
D	4.00 – 4.50
E	4.50 – 5.00
F	5.00 – 5.60
⋮	⋮

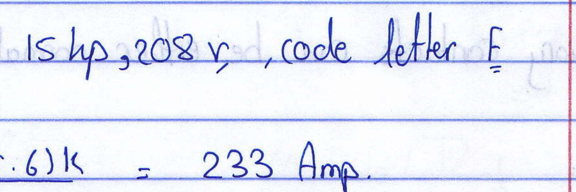
Example:
What's the starting current of a 15 hp, 208 v, code letter F 3-Φ induction Motor.

$$I_L\ \text{start} = \frac{S\ \text{start}}{\sqrt{3}\ V_L} = \frac{15\,(5.6)\,k}{\sqrt{3}\,(208)} = 233\ \text{Amp.}$$

→ Relay logic control of 3-Φ induction Motor: (ON-OFF control).

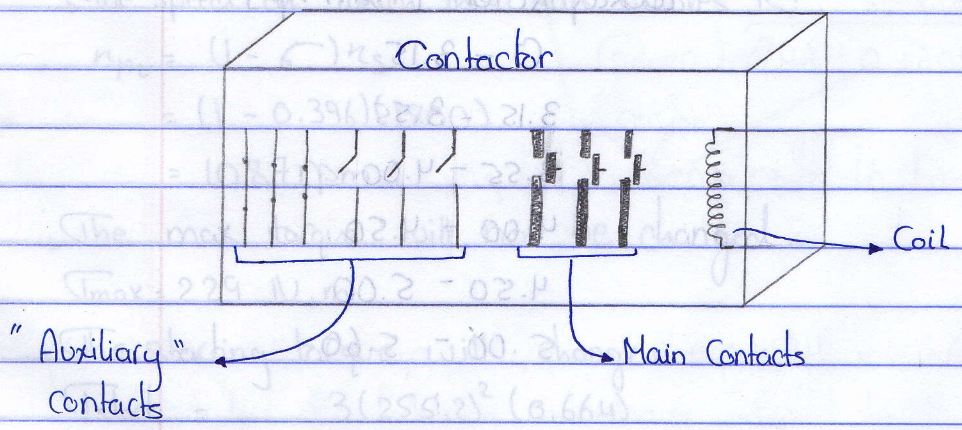
Relay: It is an electromagnetic switch that has a coil and a set of auxiliary contacts.
The contacts can be either normally open or normally closed





When the main coil is energized, the normally open contact will close & the normally close contact will open.

Contactor: It is an electromagnetic switch that has three main contacts with a set of auxiliary contacts.



The auxiliary contacts can be either normally open or normally closed.

Symbols:

① —○— or —( )— coil of relay or contactor.

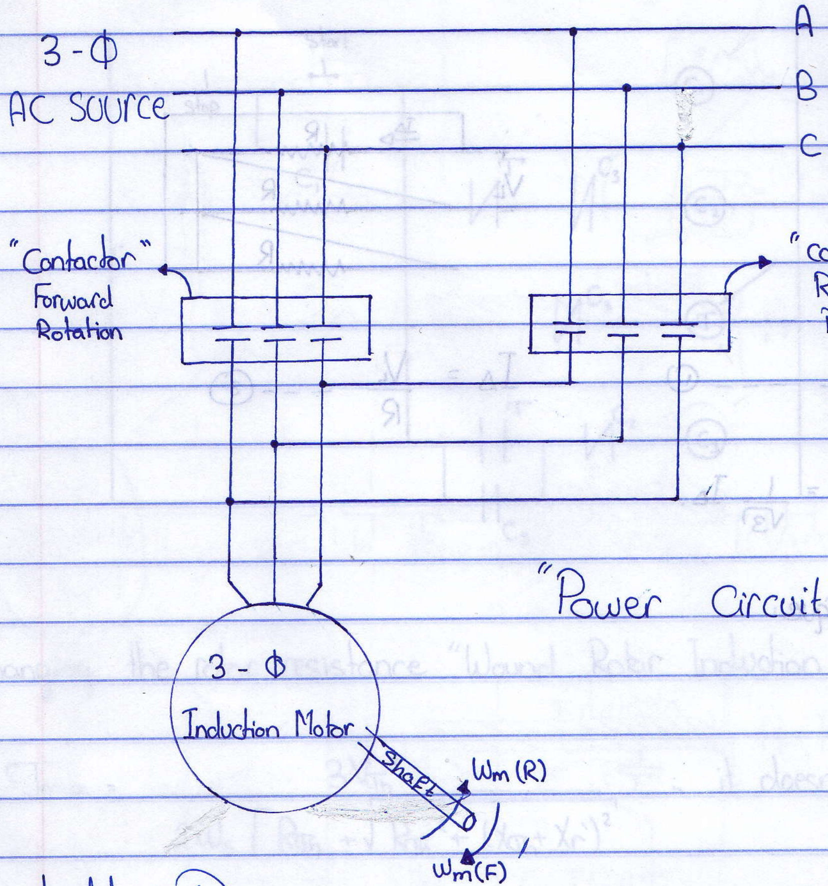
② —| |— Normally Open.

③ —|/|— Normally closed.

④ push button ((push to close)).

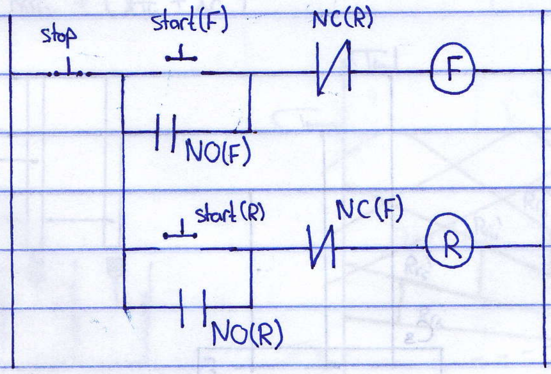
⑤ push button ((push to Open)).

Example: Control & power circuits to reverse the direction of motor.



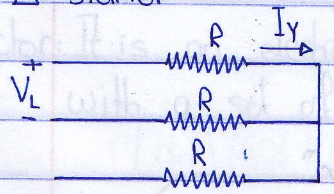
"Power Circuit"

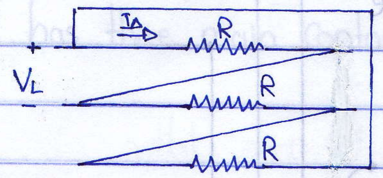
Ladder Diagram:
For the control circuit:



→ Methods to reduce the starting current of the 3-Φ induction Motor:

1 Y-Δ startor:





$$I_Y = \frac{V_L/\sqrt{3}}{R} = \frac{1}{\sqrt{3}}\frac{V_L}{R} \quad --- (1)$$

$$I_\Delta = \frac{V_L}{R} \quad --- (2)$$

→ From (1) & (2) → $I_Y = \frac{1}{\sqrt{3}} I_\Delta$.

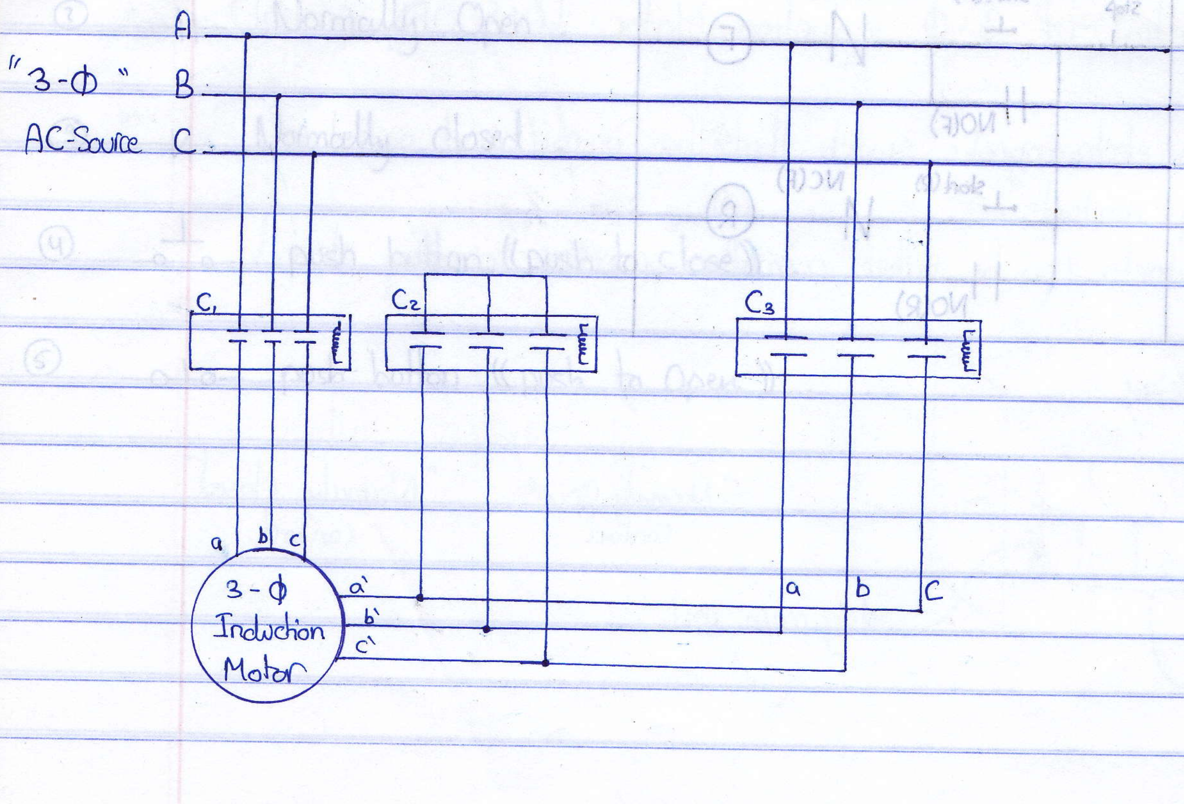
We can also study the th torque:

$$\tau_{ind} = \frac{3 I_r'^2 R_r'}{\sigma \omega_s}$$

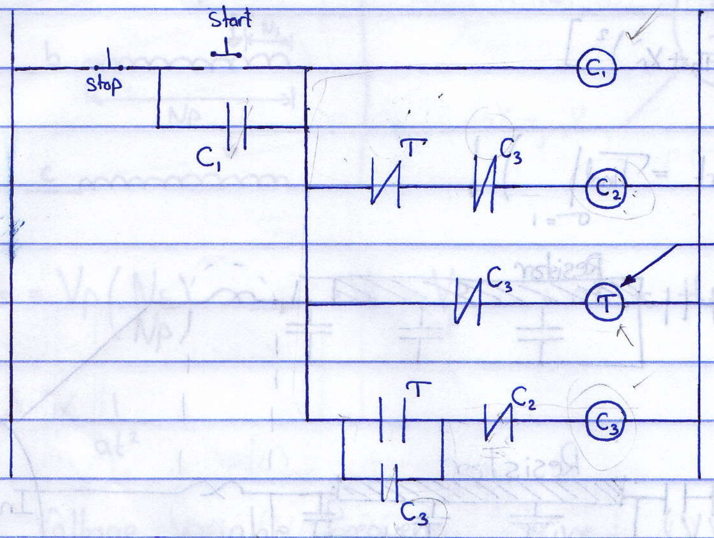
$$\tau_{\substack{start \\ induced}} = \frac{3 I_r'^2 R_r'}{\omega_s}$$

$$\tau_{ind,start(Y)} = \frac{1}{3}\tau_{ind,start(\Delta)}$$

The Power Circuit:



The control circuit: For Y-Δ starter.

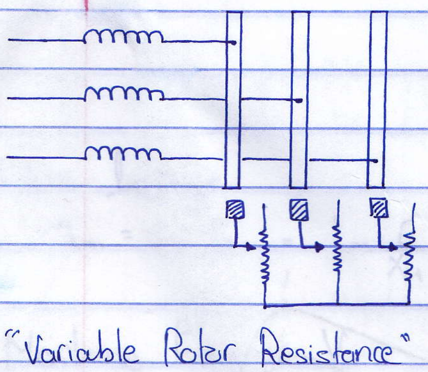


"Coil of timer, when it energized the contactor of the timer will change their state after present time or time delay"

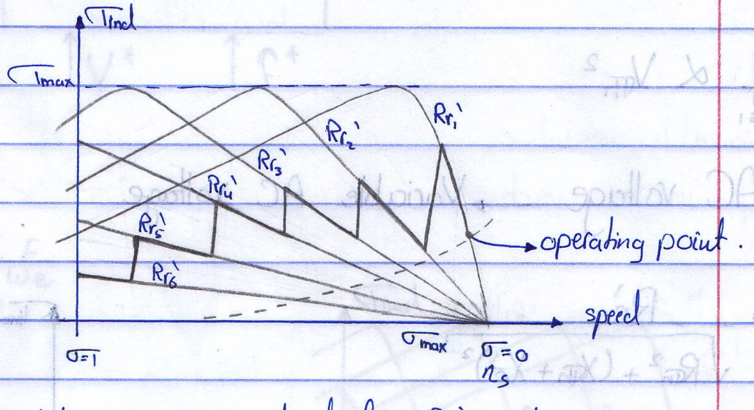
2. Changing the rotor resistance "Wound Rotor Induction Machine":

$$T_{max} = \frac{3V_{Th}^2}{2W_s\left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}\right]} \quad \therefore \text{ it doesn't depend on } R_r'$$

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}} \quad \therefore \text{ if } R_r' \uparrow \Rightarrow \sigma_{max} \uparrow$$



"Variable Rotor Resistance"
"Potential Meter"



Note: n_s is constant for R_r' values since $n_s = \frac{120}{P} f_e$.

$R_{r6}' > R_{r5}' > R_{r4}' > R_{r3}' > R_{r2}' > R_{r1}'$

The disadvantage of this method:

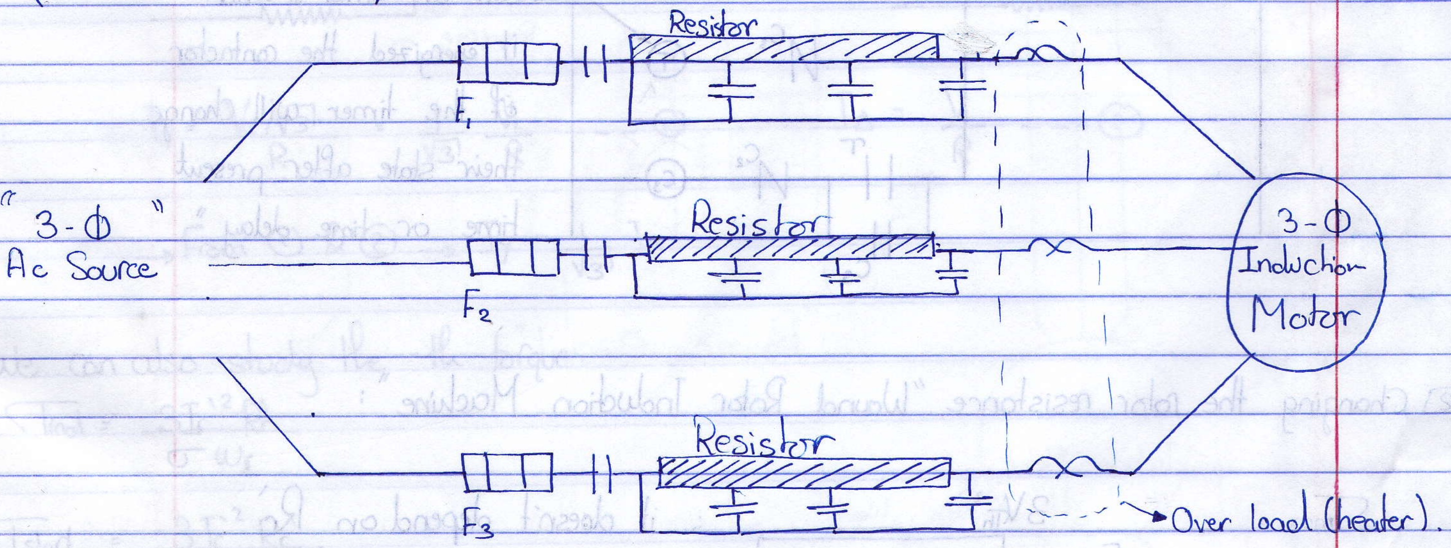
It increases the copper losses so reducing the efficiency

$R_r' \uparrow \rightarrow P_{RCL} \uparrow \Rightarrow \eta_{motor} \downarrow \;\&\; n_r = (1-\sigma) \downarrow$

[3] Stator resistance or inductance stator

$$T_{ind} = \frac{3 V_{Th}^2 R_r'}{\sigma \omega_s \left[\left(\frac{R_r'}{\sigma} + R_{Th}\right)^2 + (X_{Th} + X_r')^2 \right]}$$

$$\left(R_{Th} \simeq \frac{X_m}{X_m + X_s} \cdot R_s \uparrow \right) \uparrow \Rightarrow \left(T_{start} = T_{ind}\Big|_{\sigma=1} \right) \downarrow$$



[4] Auto-transformer "VARIC"

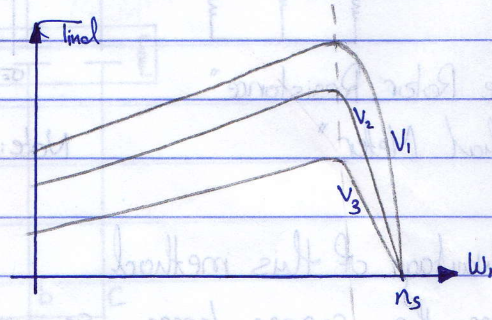
$$T_{ind} = \frac{3 V_{Th}^2}{\omega_s \left[\left(\frac{R_r'}{\sigma} + R_{Th}\right)^2 + (X_{Th} + X_r')^2 \right]} \cdot \frac{R_r'}{\sigma}$$

$$T_{start}\Big|_{\sigma=1} \propto V_{Th}^2$$

Fixed AC voltage $\rightarrow$ Variable AC voltage

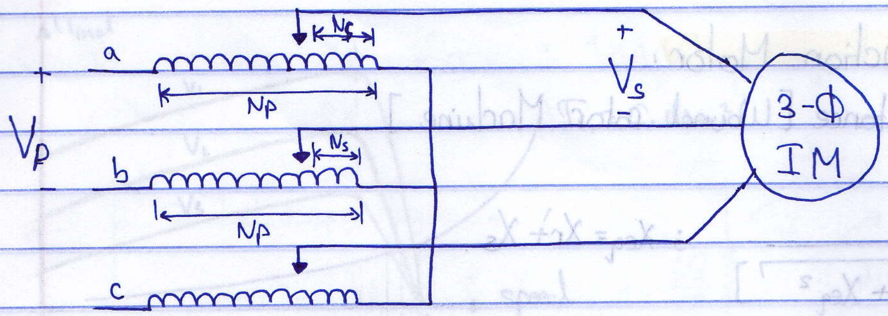
$$\sigma_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}}$$

$$T_{max} = \frac{3 V_{Th}^2}{2\omega_s \left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2} \right]}$$



* $V_1 > V_2 > V_3$

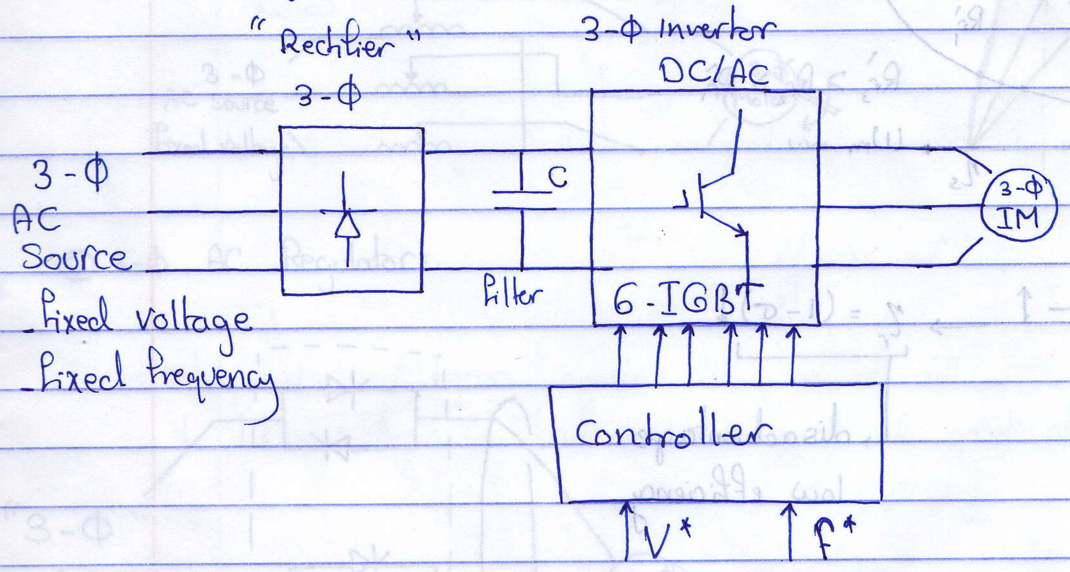
$T_{s1} > T_{s2} > T_{s3}$.



$$V_s = V_p\left(\frac{N_s}{N_p}\right) \longrightarrow V_s = \frac{V_p}{at}$$ ∴ at is the turns ratio.

$$T_{ind} \propto \frac{1}{at^2}$$

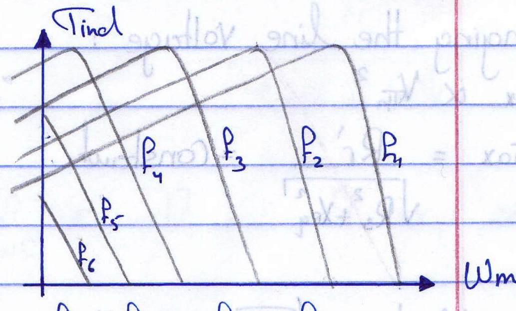
[5] Variable Voltage variable Frequency Drive: (VVVF)



Flux linkage:

$$\lambda_m = L_m I_m = L_m \frac{E}{X_M} = \frac{E}{\omega_e}$$

Machine Flux $\propto \frac{V}{f}$



The Voltage and Frequency most decrease by the Same factor to keep the flux constant and not let it saturated.

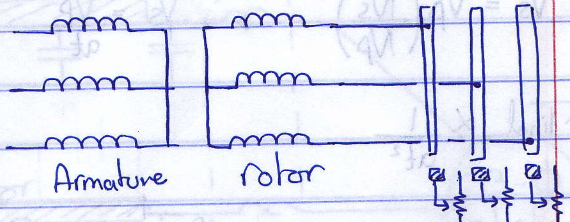
$$T_{max} = \frac{3V_{Th}^2}{2W_s\left(R_{Th} + \sqrt{(X_{Th} + X_r')^2 + R_{Th}^2}\right)}$$

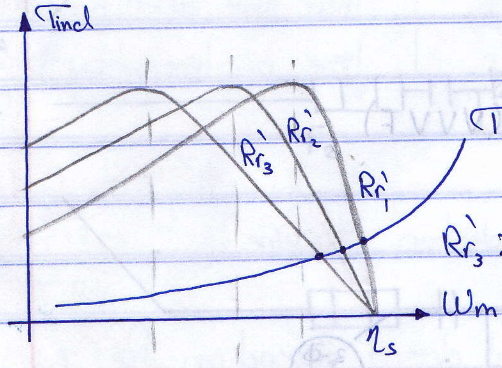
→ Speed control of induction Motor:

① Changing the rotor resistance [Wound rotor Machine]

$$\tau_{max} = \frac{3 V_{Th}^2}{2W_s\left[R_s + \sqrt{R_s^2 + X_{eq}^2}\right]} \quad ; X_{eq} = X_r' + X_s .$$

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_s^2 + X_{eq}^2}}$$





τ_{max} is constant.

$R_r' \uparrow \longrightarrow \underbrace{W_m \downarrow}_{\text{speed control}} \longrightarrow \sigma \uparrow \longrightarrow \underbrace{\eta_r = (1-\sigma)\downarrow}_{\text{disadvantage low efficiency}}$

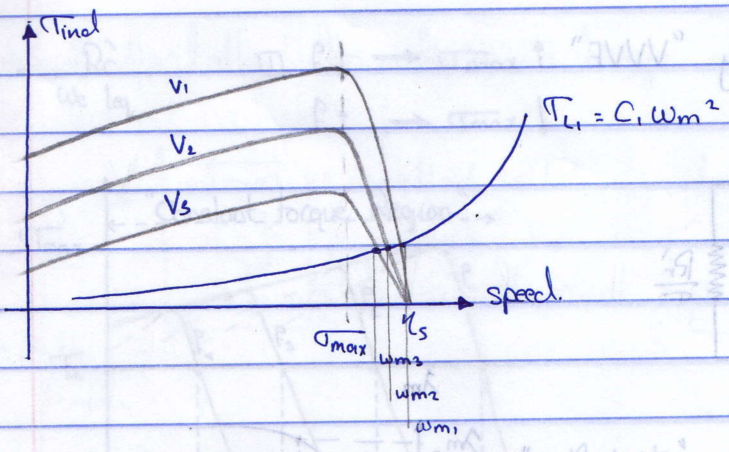
* Using variable resistor
 Using power Convertor

② Changing the line voltage:

$\tau_{max} \propto V_{Th}^2$

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_s^2 + X_{eq}^2}} = \text{Constant}.$$

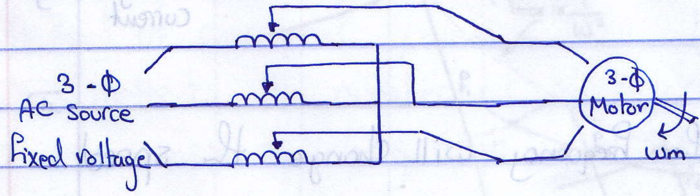
$V_s \downarrow \longrightarrow V_{Th} \downarrow \longrightarrow \underbrace{(\tau_{max} \propto V_{Th}^2)\downarrow}_{\text{disadvantage}} \longrightarrow \underbrace{W_m \downarrow}_{\text{speed control.}} \longrightarrow \sigma \uparrow \longrightarrow \underbrace{\eta_r \downarrow}_{\text{disadvantage in the efficiency}}$



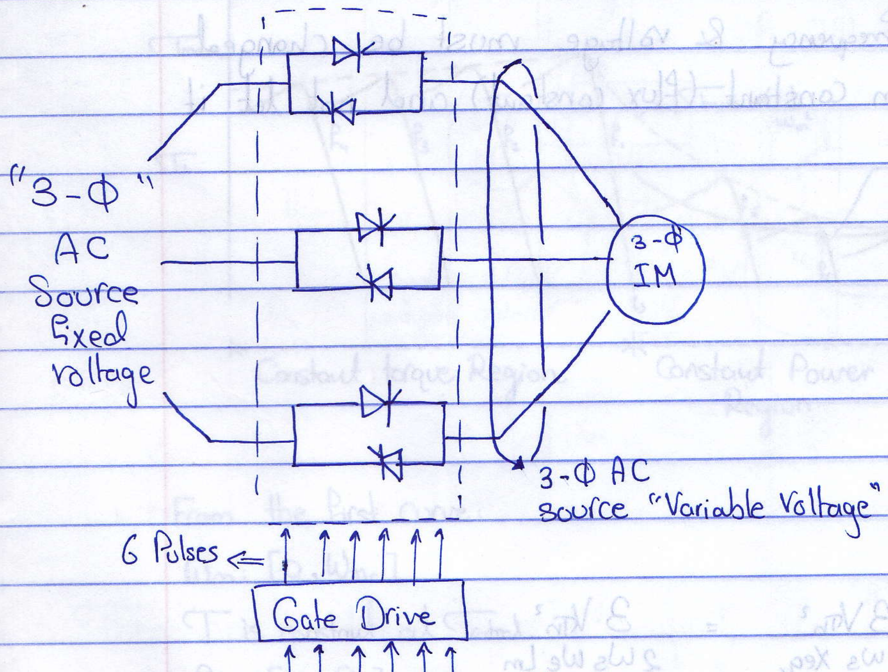
$V_1 > V_2 > V_3 \rightarrow \omega_{m1} > \omega_{m2} > \omega_{m3}$.

→ Note: Methods to change the motor voltage:

① Auto-transformer: (VARIAC)

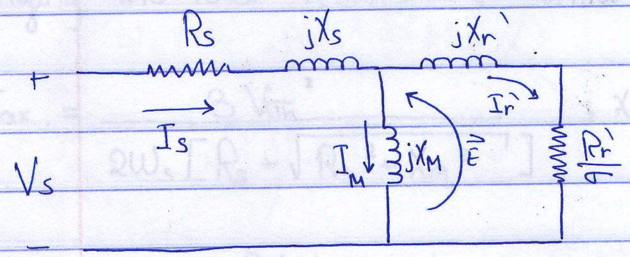


② 3-Φ AC Regulator:

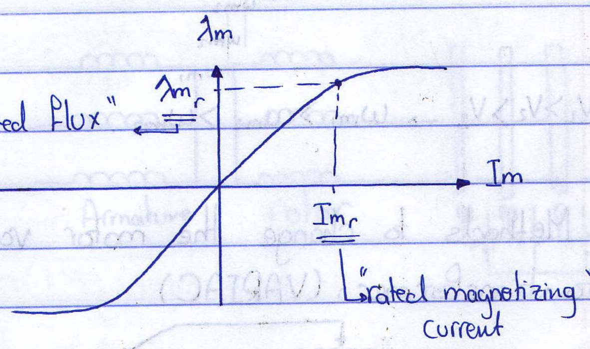


* The Note ends here

③ Variable Voltage Variable Frequency "VVVF":



Flux Linkage: $\lambda m = Lm\, Im$



$$\lambda m = Lm\left(\frac{E}{X_M}\right) = Lm\,\frac{E}{\omega e\, Lm} = \frac{E}{\omega e}$$

$$\lambda m = \frac{E}{\omega e} = \frac{E}{2\pi fe}$$

Recall: $n_s = \frac{120\, fe}{P}$, so changing the frequency will change the speed.

$$\lambda m = \frac{E}{2\pi fe} \simeq \frac{Vs}{2\pi fe} \longrightarrow \lambda m \propto \frac{Vs}{fe}$$

When ωm < rated speed → The frequency & voltage must be changed by the same factor to keep λm constant (flux constant) and not let it saturated.

$$T_{max} = \frac{3 V_{Th}^2}{2\omega_s\left[R_s + \sqrt{R_s^2 + X_{eq}^2}\right]}$$

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_s^2 + X_{eq}^2}}$$

If we ignor Rs → $T_{max} = \frac{3 V_{Th}^2}{2\omega_s X_{eq}} = \frac{3 V_{Th}^2}{2\omega_s\, \omega e\, Lm}$

but: $\omega e = \frac{\omega s}{2P}$ → $T_{max} \propto \frac{V_{Th}^2}{\omega_s^2} \propto \frac{V^2}{f^2}$