CHAPTER 6: Induction Motors.

* The distinguishing feature of induction motor is that no DC field current is required to run the machine.

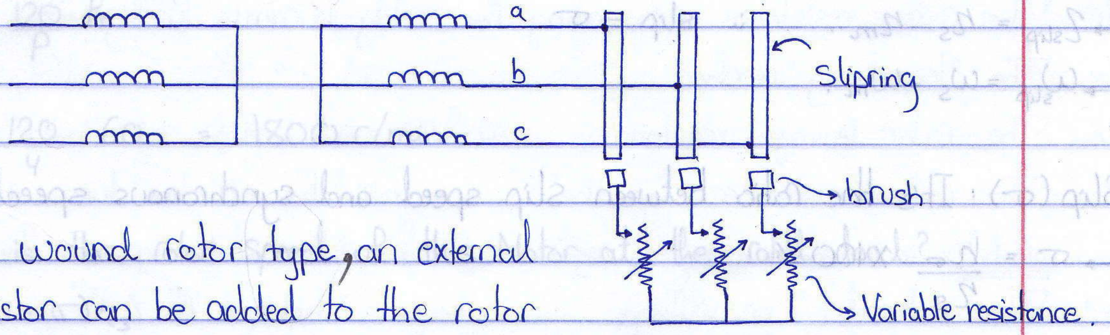
→ The construction of the induction motor

- It has the same stator construction of synchronous motor with different rotor construction.
- The induction motor has two rotor types:
  1) The squirrel cage rotor.
  2) The wound rotor.

→ Explanation of the two rotor types:

① The cage rotor:
It consists of series conducting bars laid into slots carved in the face of the rotor and shorted at either end by large shorting rings.

② The wound rotor:
It consists of a set of three-phase windings, that are mirror images of the windings of the stator. The windings are usually Y-connected. The terminals a, b & c are tied to the slip-rings on the rotor's shaft. The rotor windings are shorted via brushes riding on the sliprings.



Note: ① In wound rotor type, an external resistor can be added to the rotor circuit to modify the torque-speed characteristics.

② The wound rotor is more expensive and requires much more maintenance.

→ Operating principle of induction motor:

- A set of 3-Φ voltages is applied to the stator windings and a set of 3-ϕ stator currents is flowing
- The currents will produce a rotating magnetic field which rotats at synchronous speed (n_s) where n_s is given by:

  $n_s = \frac{120}{P} f_e$. ∴ f_e: is the system freq in hertz.
- The rotating magnetic field will pass over the rotor conducting bars and induce voltages on them by the relation:

  $e_{ind} = (V \times B) \cdot l$

  ∴ V: is the velocity of the bar <u>relative</u> to the magnetic field <u>B_s</u>.
- The induced voltages will produce a rotor current $\vec{I_r}$.
- The rotor current $\vec{I_r}$ produces a rotor magnetic field $\vec{B_r}$.
- Now the rotor magnetic field $\vec{B_r}$ interacts with the stator magnetic field $\vec{B_s}$ to produce the induced torque in the machine:

  $\tau_{ind} = k\, \vec{B_r} \times \vec{B_s}$.

Note: The rotor can speed-up to synchronous speed but it can never reach it.

→ The Concept of Rotor Slip:

- Two terms are commonly used to define the relative motion of the rotor and the magnetic fields:

① Slip speed: It's the difference between the synchronous speed & rotor speed.

→ $n_{slip} = n_s - n_m$. ∴ slip $\equiv \sigma$

→ $\omega_{slip} = \omega_s - \omega_m$.

② Slip (σ): It's the ratio between slip speed and synchronous speed.

→ $\sigma = \frac{n_\sigma}{n_s} \times 100\,\%$.

→ $\sigma = \frac{n_s - n_m}{n_s} \times 100\,\%$

→ $\sigma = \frac{\omega_s - \omega_m}{\omega_s} \times 100\,\%$

Solving the last equation for η_m:

→ $\eta_m = (1-\sigma)\eta_s$.

→ $\omega_m = (1-\sigma)\omega_s$.

Notice that is the rotor is rotating at η_s then $(\sigma = 0)$, while if it is locked $(\eta_m = 0)$ then $(\sigma = 1)$.

→ The Electric frequency on the rotor:

- If the rotor is locked $(\eta_m = 0) \Leftrightarrow (\sigma = 1)$, then it will have the same frequency as the stator. $(f_r = f_s = f_e)$

On the other hand, when the rotor turns at synchronous speed $(\eta_m = \eta_s) \Leftrightarrow (\sigma = 0)$, then the rotor frequency will be zero $(f_r = 0)$.

- At any other speed between 0 & η_s the rotor's frequency is directly proportional to the slip (σ).

→ $f_r = \sigma f_e$.

→ $f_r = \frac{P}{120}(\eta_s - \eta_m)$.

→ Example:

A 208 V, 10-hp, 4-pole, 60-Hz, Y-connected IM has a full load slip of 5%.

a) What is the synchronous speed of this Motor?

$\eta_s = \frac{120 f_e}{P}$

$= \frac{120}{4} \cdot 60 = 1800 \text{ r/min}$.

b) What is the rotor speed of this Motor at the rated load?

$\eta_m = (1-\sigma)\eta_s$

$= (1 - 0.05)(1800)$

$= 1710$ rpm.

c) What is the rotor frequency of this motor at the rated load?

$f_r = \sigma f_e$

$= 0.05\,(60)$

$= 3\ Hz$.

d) What is the shaft torque of this motor at the rated load?

$P_{out} = T_{load}\,.\,\omega_m$

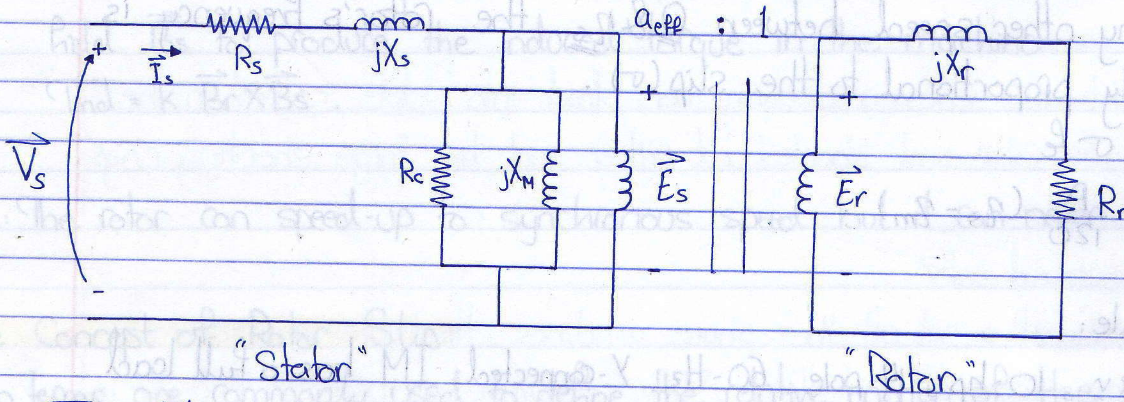
$T_{load} = P_{out} / \omega_m$

$= (10\,hp)(746) / (1710).(2\pi/60)$

$= 41.7\ N.m$

→ The Equivalent Circuit of an Induction Motor:

It is very similar to the per-phase equivalent circuit of the transformer.



∴ R_s: The stator resistance.

X_s: The stator leakage reactance.

R_c: The core resistance accounting for eddy currents & hysteresis.

X_M: The Magnetizing reactance.

X_r: The rotor leakage reactance.

R_r: The rotor resistance, for the windings in (wound), bars in (cage).

E_s: The internal stator voltage.

E_r: The induced rotor voltage.

I_s: The stator (source) current.

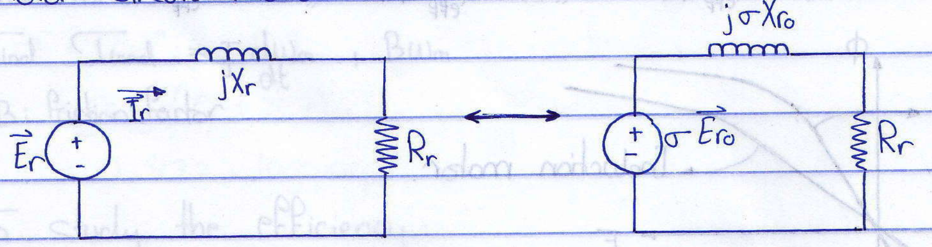
I_r: The rotor current.

V_s: The stator (source) phase voltage.

* a_{eff}: The effective turns ratio of the motor which couples between $\vec{E_s}$ & $\vec{E_r}$.

Note: a_{eff} is fairly easy to determine for the wound-rotor motor.

- Rotor circuit model:



- In general, the greater the relative motion between the rotor & the stator magnetic fields, the greater the resulting rotor voltage & rotor frequency.
- The largest relative motion occurs when the rotor is locked,

  $\sigma = 1$ and $\vec{E_r} = 1.\vec{E_{ro}}$ "The largest rotor Voltage".

  $f_r = 1.f_e$ "The largest rotor frequency".
- When the rotor moves at the same speed as the stator magnetic field, resulting in no relative motion,

  $\sigma = 0$ and $\vec{E_r} = 0$ "The smallest rotor Voltage".

  $f_r = 0$ "The smallest rotor frequency".
- At any other speed, the voltage and frequency of the rotor is directly proportional to the slip of the rotor.

  $\vec{E_r} = \sigma\vec{E_{ro}}$

  $f_r = \sigma f_e$.
- The reactance of an induction motor rotor depends on the inductance of the rotor and its voltage frequency:

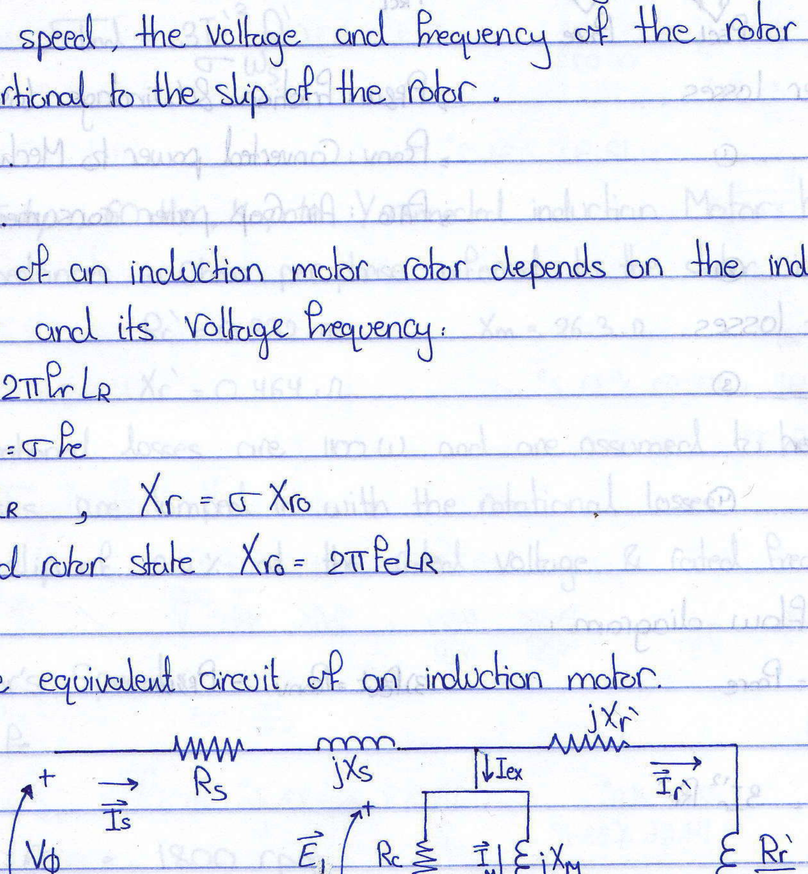
  $X_r = \omega_r L_R = 2\pi f_r L_R$

  So, since $f_r = \sigma f_e$

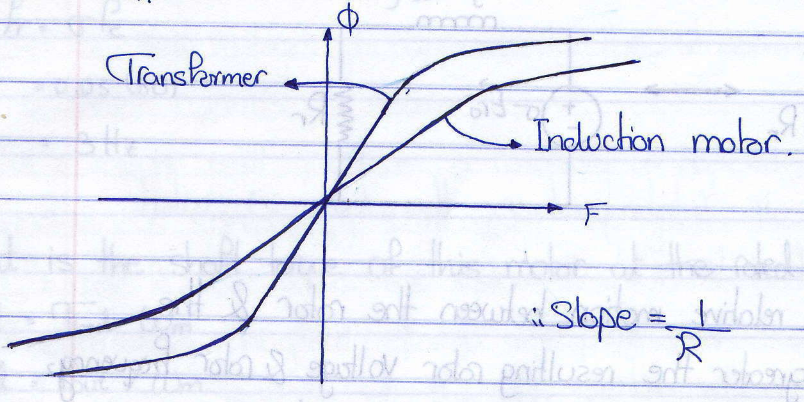
  $X_r = 2\pi(\sigma f_e)L_R$, $X_r = \sigma X_{ro}$

  at the locked rotor state $X_{ro} = 2\pi f_e L_R$

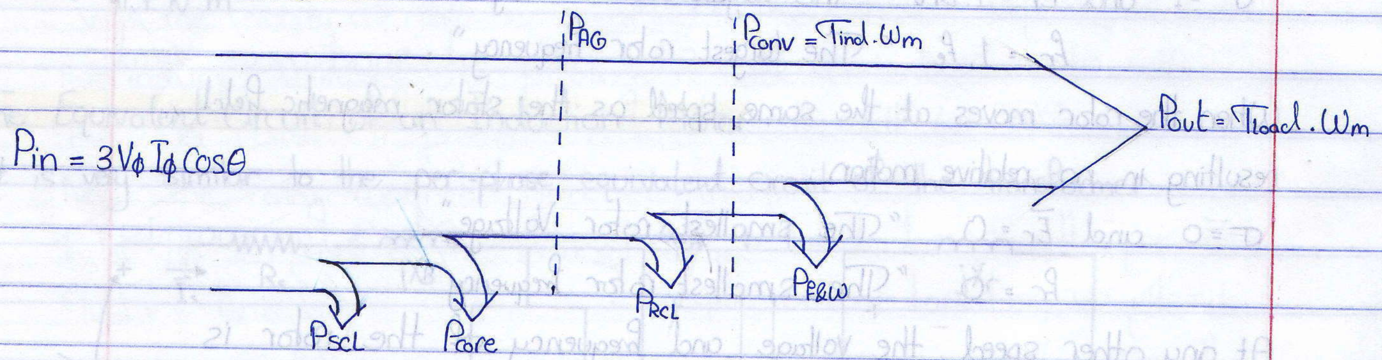
→ The per-phase equivalent circuit of an induction motor.



$\therefore \vec{I_r'} = \frac{1}{a_{eff}} \vec{I_r}$ $\therefore X_r' = a_{eff}^2 X_r$ $\therefore R_r' = a_{eff}^2 R_r$ $\therefore \vec{E_1} = a_{eff} \vec{E_{r0}}$



→ **Power Calculations:**



→ P_{SCL}: Stator copper losses

$= 3 I_s^2 R_s$ ---- ①

→ P_{core}: Core losses.

$= 3 E_1^2 / R_c$ ---- ②

→ P_{RCL}: Rotor copper losses.

$= 3 I_r'^2 R_r'$ ---- ③

→ P_{AG}: Air Gap power

$= 3 I_r'^2 \frac{R_r'}{\sigma}$ ④

→ $P_{F\&W}$: Friction & Windage losses.

→ P_{conv}: Converted power to Mechanical.

→ P_{AG}: Air Gap power consumed in "$\frac{R_r'}{\sigma}$".

From the power flow diagram:

1) $P_{AG} = P_{in} - P_{SCL} - P_{core}$

2) $P_{conv} = P_{AG} - P_{RCL}$

$= 3 I_r'^2 \frac{R_r'}{\sigma} - 3 I_r'^2 R_r'$

3) $P_{out} = P_{conv} - P_{F\&W}$

$P_{conv} = 3 I_r'^2 R_r' \left(\frac{1-\sigma}{\sigma} \right) \longrightarrow = 3 I_r'^2 \frac{R_r'}{\sigma} (1-\sigma) \longrightarrow = P_{AG}(1-\sigma)$.

- Now From Newton's second law:

$$T_{ind} - T_{load} = J\frac{d\omega_m}{dt} + B\omega_m$$

$\because B$: friction factor.

- To study the efficiency:

① Rotor efficiency:

$$\eta_r = \frac{P_{conv}}{P_{AG}} = (1-\sigma)$$

② Motor efficiency:

$$\eta_M = \frac{P_{out}}{P_{in}}$$

- Induced Torque:

$P_{conv} = T_{ind}\,\omega_m$

$\because \omega_m = (1-\sigma)\omega_s$

$P_{conv} = (1-\sigma)P_{AG}$

$$T_{ind} = \frac{P_{AG}}{\omega_s} \longrightarrow T_{ind} = \frac{3I_r'^2 R_r'}{\sigma\,\omega_s}$$

Example:

A 460-V, 25-hp, 60-Hz, 4-pole, Y-connected induction Motor has the following impedances in Ohms per phase referred to the stator circuit:

- $R_s = 0.641\,\Omega$ - $R_r' = 0.332\,\Omega$ - $X_m = 26.3\,\Omega$

$X_s = 1.106\,\Omega$ $X_r' = 0.464\,\Omega$

The total rotational losses are 1100 W and are assumed to be constant. The core losses are lumped in with the rotational losses.

For a rotor slip of 2.2% at the rated voltage & rated frequency: Find:

a) The Motor's Speed:

$$n_s = \frac{120}{P} f_e$$

$$= \frac{120}{4} \cdot 60 = 1800 \text{ rpm.}$$

$$\omega_s = n_s\left(\frac{\pi}{30}\right) = 188.5 \text{ rad/sec.}$$

* To Find the Motor's speed (η_m)

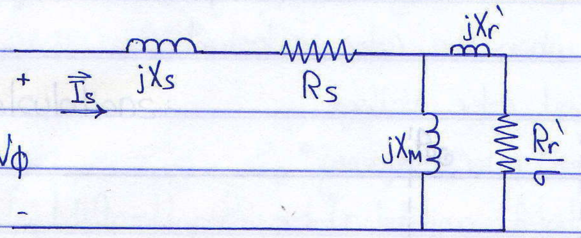
$\eta_m = (1-\sigma)\,\eta_s$

$= (1-0.022)(1800)$

$= 1760$ rpm.

b) The Motor's stator current:

To find the stator current we should get the equivalent impedance of the circuit:



To Find Req:

1) $jX_M \,//\, (jX_r' + \frac{R_r'}{\sigma})$

$\therefore jX_M = 26.3j$

$jX_r' + \frac{R_r'}{\sigma} = 0.464j + \frac{0.332}{0.022} \Rightarrow 15.1 \measuredangle 1.76°$

$\rightarrow (26.3j \,//\, (15.1\measuredangle 1.76°))$

$Req(1) \rightarrow \dfrac{1}{\frac{1}{26.3j} + \frac{1}{15.1\measuredangle 1.76°}} = 12.92 \angle 31.2°$

2) $jX_s + R_s + Req_{(1)}$:

$= 1.106j + 0.641 + 12.92 \measuredangle 31.2°$

$= 14.06 \measuredangle 33.7°$.

Then Find V_ϕ:

$V_L = \sqrt{3}\,V_\phi \rightarrow V_\phi = \frac{V_L}{\sqrt{3}} = \cancel{\frac{208}{\sqrt{3}}}\ \frac{460}{\sqrt{3}} = 266 \measuredangle 0°\ V$.

Now the current:

$\vec{I_s} = \frac{V_\phi \measuredangle 0}{Z_{eq}} \rightarrow \frac{266 \measuredangle 0°}{14.06 \measuredangle 33.7°} = 18.88 \measuredangle -33.7°\ A$

c) The Motor's PF:

$PF = \cos^{-1}(\theta_v - \theta_i)$

$= \cos^{-1}(0 + 33.7)$

$= 0.832$ lagging

d) The Motor's P_{conv} & P_{out}:

i. $P_{conv} = (1-\sigma) P_{AG}$, so we need to find P_{AG}:

$P_{AG} = P_{in} - P_{SCL}$

$\therefore P_{in} = 3 V_\phi I_\phi \cos\theta$

$= 3(266)(18.88)(0.832)$

$= 12.53$ KW.

$P_{SCL} = 3 I_s^2 R_s$

$= 3(18.88)^2(0.641)$

$= 0.685$ KW

$P_{AG} = (12.53k - 0.685k)$ W

$= 11.845$ KW

$P_{conv} = (1 - 0.022)(11.845k)$

$= 11.585$ KW

ii. $P_{out} = P_{conv} - P_{F\&W}$

$= (11.585k - 1.1k)$ W

$= 10.485$ kW

e) τ_{ind} & τ_{load} for the Motor:

i. $\tau_{ind} = \frac{P_{conv}}{\omega_s} \Rightarrow \frac{11585}{1800 \cdot \frac{\pi}{30}} = 62.8$ N.m

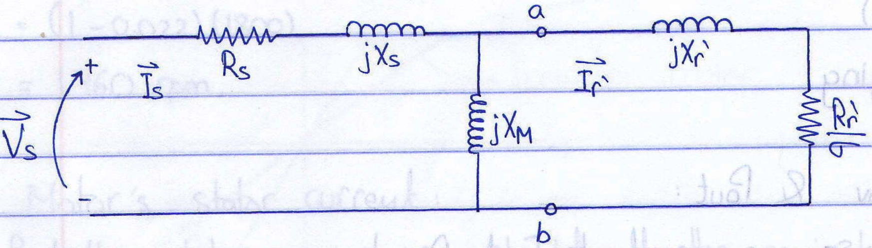
ii. $\tau_{load} = \frac{P_{out}}{\omega_m} \Rightarrow \frac{10.485k}{1760(\frac{\pi}{30})} = 56.9$ N.m

f) The Motor's efficiency:

$\eta_s = \frac{P_{out}}{P_{in}} \times 100\%$

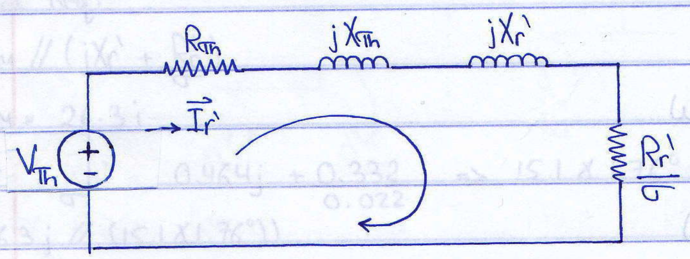
$= \frac{10485}{12530} \times 100\% = 83.7\%$

→ Derivation of induced torque equation:



$$\tau_{ind} = \frac{P_{AG}}{\omega_s} = \frac{3 I_r'^2 R_r'}{\sigma\, \omega_s}$$ ∴ I_r' is the magnitude of the current.

_ From the equation it's clear that we need $\underline{\underline{I_r'}}$ to calculate the τ_{ind}, but in fact there is no access on the rotor so its difficult to find it:
The solution is to get the thevinin of the stator part from points (a, b).



① $\vec{V_{Th}} = \vec{V_{oc}} = \frac{jX_M}{R_s + j(X_s + X_M)} \vec{V_s}$

∴ $X_M + X_s \gg R_s$

$\vec{V_{Th}} = \frac{X_M}{(X_s + X_M)} \vec{V_s}$ ________ ①

② $Z_{Th} = (R_s + jX_s) // jX_M$

$= \frac{(R_s + jX_s)(jX_M)}{R_s + j(X_s + X_M)}$

∴ $X_M + X_s \gg R_s$

$Z_{Th} = \frac{(R_s + jX_s)(X_M)}{(X_s + X_M)}$

i

$Z_{Th} = R_s \frac{X_M}{X_s + X_M} + j\, X_s \frac{X_M}{X_s + X_M}$

$R_{Th} = R_s \frac{X_M}{X_s + X_M}$ ____ ②

$X_{Th} = X_s \frac{X_M}{X_s + X_M}$ ____ ③

ii

To find I_r':

$\vec{I_r'} = \dfrac{\vec{V_{Th}}}{(R_{Th} + \frac{R_r'}{\sigma}) + j(X_{Th} + X_r')}$, but to find the τ_{ind} we need the magnitude.

$$|\vec{I_r'}| = \frac{|\vec{V_{Th}}|}{\sqrt{(R_{Th} + \frac{R_r'}{\sigma})^2 + (X_{Th} + X_r')^2}}$$

Now: $\tau_{ind} = \dfrac{3 I_r'^2 R_r'}{\sigma\, \omega_s}$, substitute I_r' in τ_{ind}.

$$\tau_{ind} = \frac{3 V_{Th}^2}{\omega_s \left([R_{Th} + \frac{R_r'}{\sigma}]^2 + [X_{Th} + X_r']^2 \right)} \cdot \frac{R_r'}{\sigma}$$

* Note: In Calculations we can make an approximation in:

$\dfrac{X_M}{X_s + X_M} \simeq 1$ since $X_M \gg X_s$.

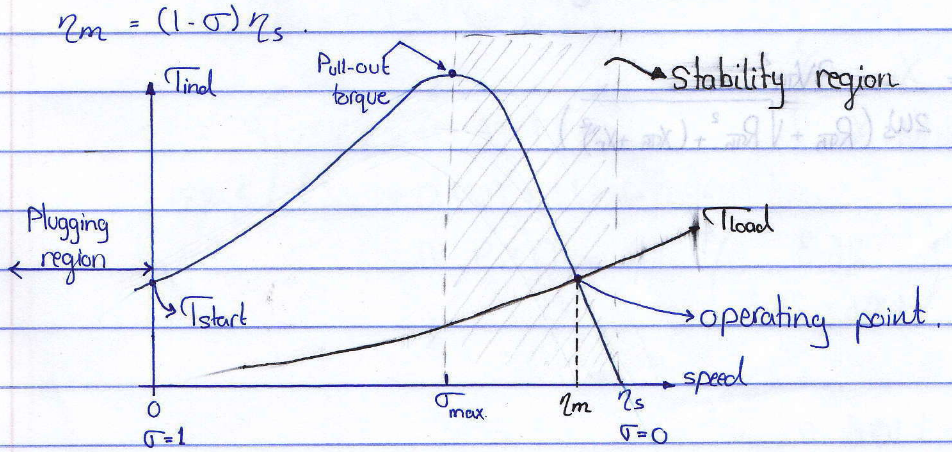
Therefore, $\left.\begin{array}{l} R_{Th} \simeq R_s \\ X_{Th} \simeq X_s \\ V_{Th} \simeq V_s \end{array}\right\} \longrightarrow \tau_{ind} = \dfrac{3 V_s^2}{\omega_s \left([R_s + \frac{R_r'}{\sigma}]^2 + [X_{Th} + X_r']^2 \right)} \cdot \dfrac{R_r'}{\sigma}$

- Induction Motor Torque - Speed characteristics:

From the τ_{ind} equation we can see that the torque is a function of slip, and since the slip is changing by n_m so it's also a function of n_m.

$\tau_{ind} = f(\sigma) = g(n_m)$

$n_m = (1 - \sigma) n_s$.



Notes on the curve :

① When the slip range is very small, the induced torque is approximately proportional to the slip (linear relationship).

$$\tau_{ind} \simeq \frac{3V_{Th}^2}{\omega_s \left(\frac{R_r'}{\sigma}\right)^{\cancel{2}}} \cdot \frac{R_r'^{\cancel{}}}{\cancel{\sigma}} \longrightarrow \tau_{ind} \simeq \frac{3V_{Th}^2}{\omega_s R_r'} \cdot \sigma \longrightarrow \tau_{ind} = K\sigma .$$

② Under steady state (speed is constant)
From Newton's second law :

$$\tau_{ind} - \tau_{load} = J \frac{d\omega}{dt} \longrightarrow \text{So } \tau_{ind} = \tau_{load} .$$

- Plugging Region :
It's used to stop the motor rapidly by switching any two of the three phases.

- Starting torque (τ_{start}) :
It's slightly higher than the motor rated torque.

- $\tau_{pull\text{-}out} = \tau_{max}$: The maximum torque.

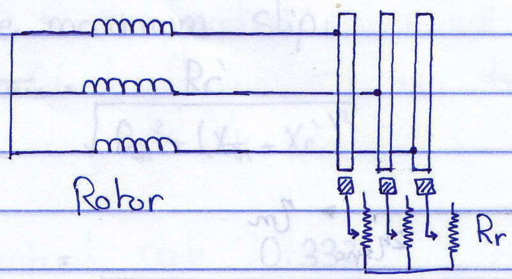
$$\tau_{max} = (2-3)\,\tau_{rated}$$

→ To find the maximum torque :

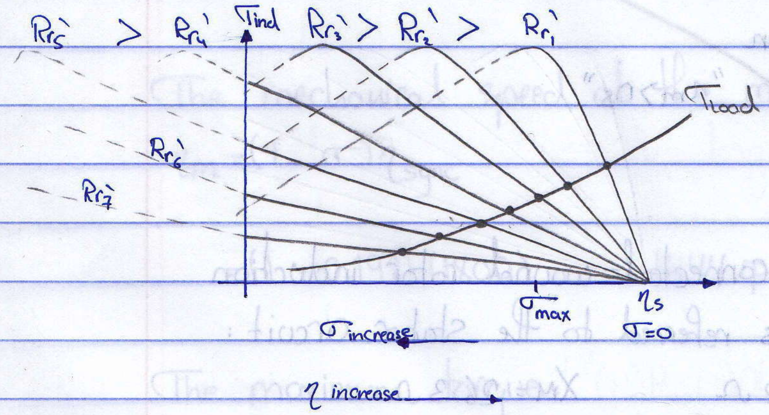
$$\frac{\partial \tau_{ind}}{\partial \sigma} = 0 \longrightarrow \sigma_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}}$$

$$\tau_{max} = \frac{3V_{Th}^2}{2\omega_s \left(R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2} \right)}$$

→ In Wound Rotor Induction Motor:



* only in the wound rotor we can change "Rr"



When $R_{r_1}' \le R_r' \le R_{r_4}'$ → Then $R_r' \uparrow$ the $\sigma_{max} \uparrow$ → $\sigma \uparrow$ → $\eta \downarrow$

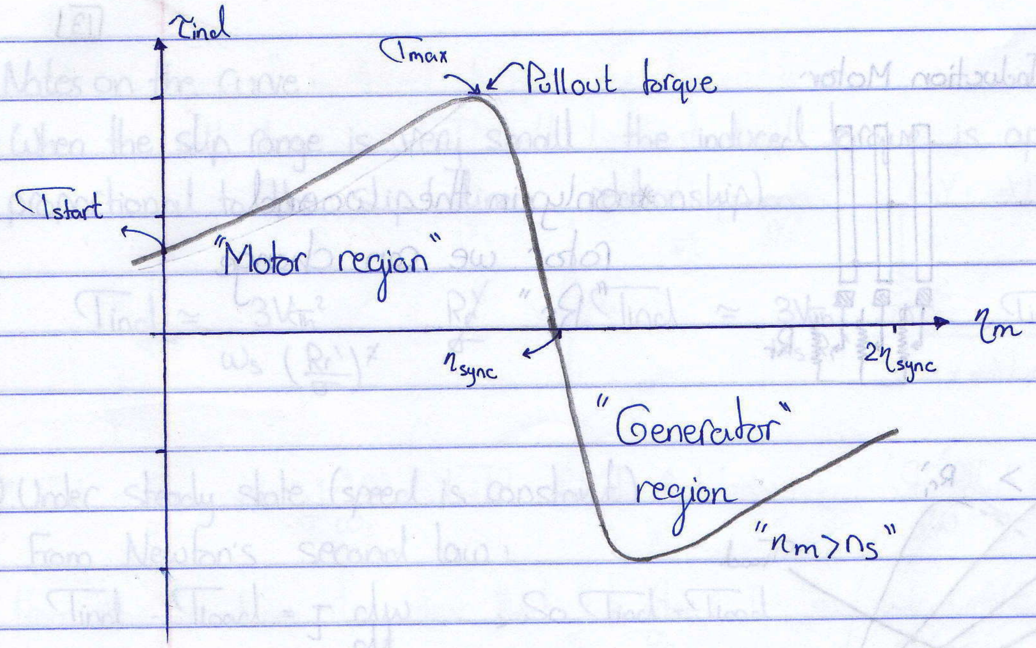
So $\eta_r = (1-\sigma)$ [Rotor efficiency] ↓

and $\eta \downarrow$ → [Motor efficiency].

and $T_{start} \uparrow$

When $R_r' > R_{r_4}'$ → $T_{start} \downarrow$ ⇒ η (efficiency) ↓

The Rotor resistance in the wound Rotor Induction Motor is a method to control the <u>speed</u> & the <u>starting torque</u>.



Example: A 460-volt, 25-hp, 4-pole, Y-connected wound rotor induction motor has the following impedances referred to the stator circuit:

$R_s = 0.641\,\Omega$ $\quad R_r' = 0.332\,\Omega$ $\quad X_M = 26.3\,\Omega$

$X_s = 1.106\,\Omega$ $\quad X_r' = 0.464\,\Omega$

a) What is the maximum torque of this motor? At what speed & slip does it occur?

[1] Find $V_{TH} = V_\phi \dfrac{X_M}{\sqrt{R_s^2 + (X_s + X_M)^2}}$

$$= 266 \frac{26.3}{\sqrt{(0.641)^2 + (1.106 + 26.3)^2}} = 255.2\ V$$

[2] Find $R_{TH} = R_s \left(\dfrac{X_M}{X_s + X_M}\right)^2$

$$= 0.641 \left(\frac{26.3}{1.106 + 26.3}\right)^2 = 0.59\,\Omega$$

[3] Find $X_{TH} = X_r' = 1.106\,\Omega$.

Now:

The maximum slip: *Note: $n_s = \frac{120}{4} \cdot 60 = 1800$ rpm.

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}}$$

$$= \frac{0.332}{\sqrt{(0.59)^2 + (1.106 + 0.464)^2}} = 0.198\ \Omega$$

The mechanical speed at the maximum slip:

$$n_m = (1 - \sigma)\, n_{sync}$$

$$= (1 - 0.198)(1800) = 1444 \text{ rpm}$$

The maximum torque: *Note: $\omega_s = n_s \cdot \frac{2\pi}{60}$

$$T_{max} = \frac{3V_{Th}^2}{2\omega_s \left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}\right]}$$

$$= \frac{3(255.2)^2}{2(188.5)\left[0.59 + \sqrt{0.59^2 + (1.106 + 0.464)^2}\right]} = 229 \text{ N.m.}$$

b) What is the starting torque of the motor?

In the torque equation set the σ to $\underline{\underline{1}}$ & solve it as T_{start}:

$$T_{start} = \frac{3V_{Th}^2 R_r}{\omega_s \left[(R_{Th} + R_r')^2 + (X_{Th} + X_r')^2\right]}$$

$$= \frac{3(255.2)^2(0.332)}{188.5\left[(0.59 + 0.332)^2 + (1.106 + 0.464)^2\right]} = 104 \text{ N.m.}$$

c) When the Rotor resistance is doubled. What is the speed at which the max torque ocurred? What is the new Starting torque?

→ if we double the rotor resistance then the σ will be doubled so

$\sigma_{max} = 0.396$

→ The speed at which this slip occures is:

$$n_m = (1-\sigma)n_s$$
$$= (1-0.396)(1800)$$
$$= 1087 \text{ rpm}.$$

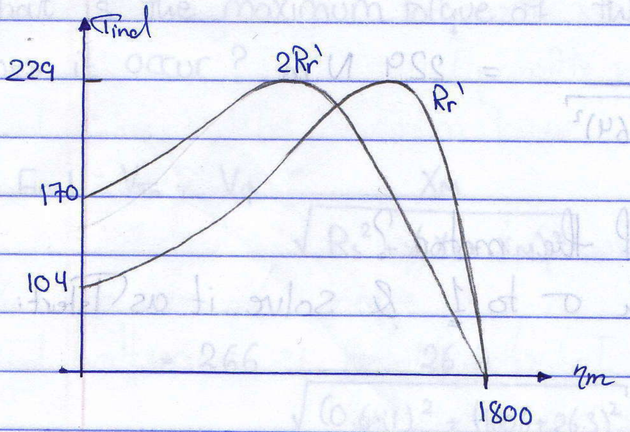
→ The max torque will not be changed.

$\tau_{max} = 229$ N.m.

→ The starting torque will change:

$$\tau_{start} = \frac{3(255.2)^2(0.664)}{188.5\left[(0.59+0.664)^2+(1.106+0.464)^2\right]}$$

$$= 170 \text{ N.m}$$



→ Calculations of Motor Starting current:

$I_{L\,start} = \dfrac{S_{start}}{\sqrt{3}\,V_L}$ ∴ S_{start} : Rated power (hp) * "code letter Factor"

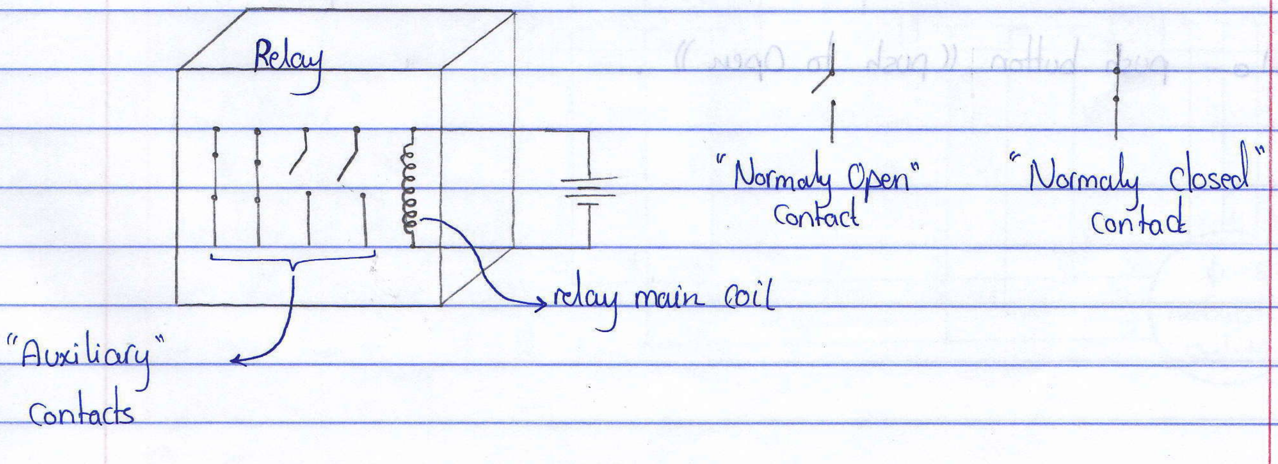
Nominal code letter	Locked Rotor [KVA/hp]
A	0 - 3.15
B	3.15 - 3.55
C	3.55 - 4.00
D	4.00 - 4.50
E	4.50 - 5.00
F	5.00 - 5.60
⋮	⋮

Example:
What's the starting current of a 15 hp, 208 v, code letter F 3-Φ induction Motor.

$I_{L\,start} = \dfrac{S_{start}}{\sqrt{3}\,V_L} = \dfrac{15\,(5.6)\,k}{\sqrt{3}\,(208)} = 233$ Amp.

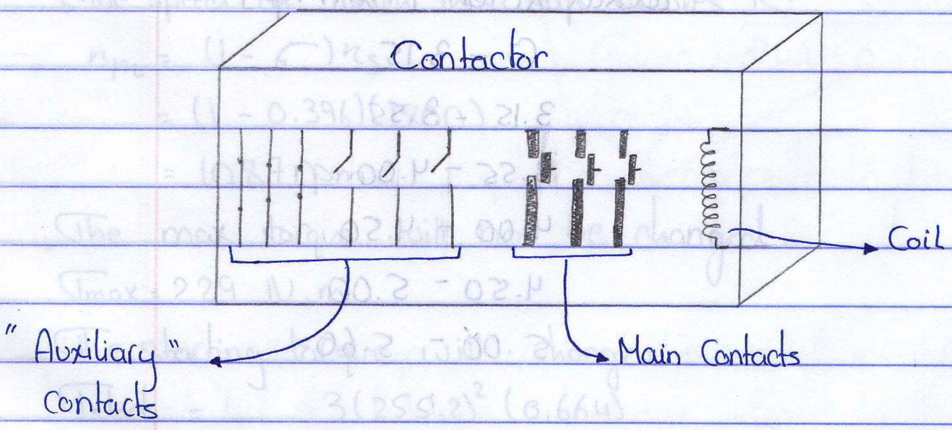
→ Relay logic control of 3-Φ induction Motor: (ON-OFF control).

Relay: It is an electromagnetic switch that has a coil and a set of auxiliary contacts.
The contacts can be either normally open or normally closed



When the main coil is energized, the normally open contact will close & the normally close contact will open.

Contactor: It is an electromagnetic switch that has three main contacts with a set of auxiliary contacts.



The auxiliary contacts can be either normally open or normally closed.

Symbols:

① —○— or —( )— coil of relay or contactor.

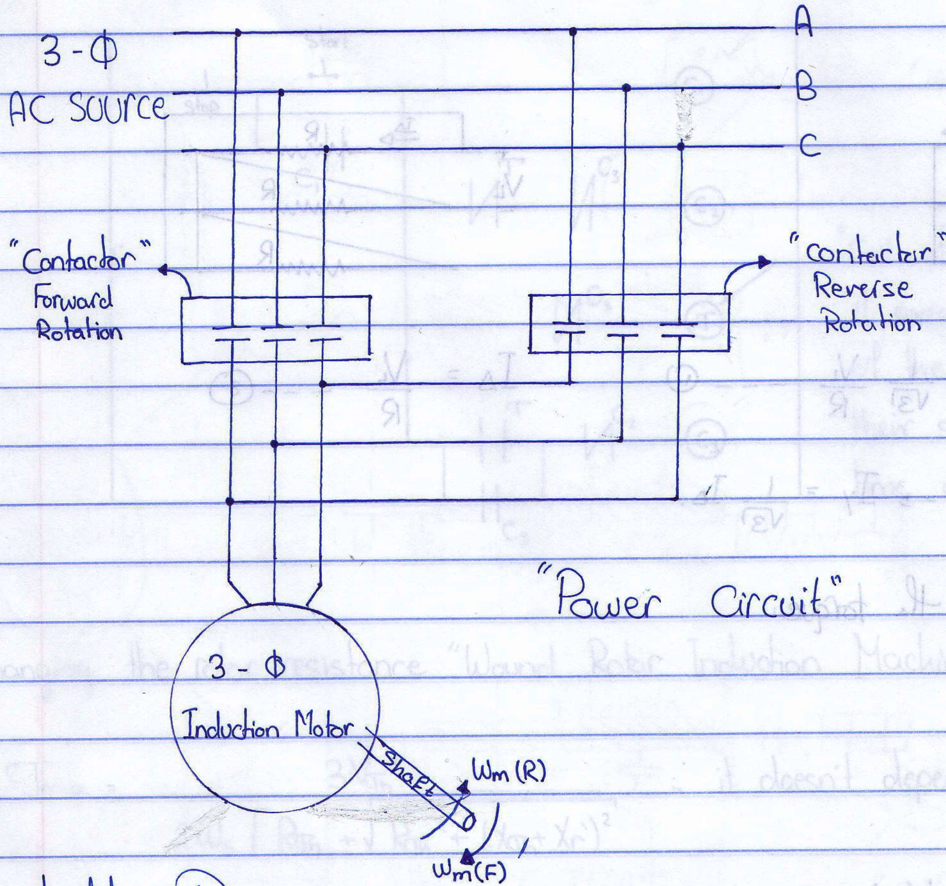
② —| |— Normally Open.

③ —|/|— Normally closed.

④ push button ((push to close)).

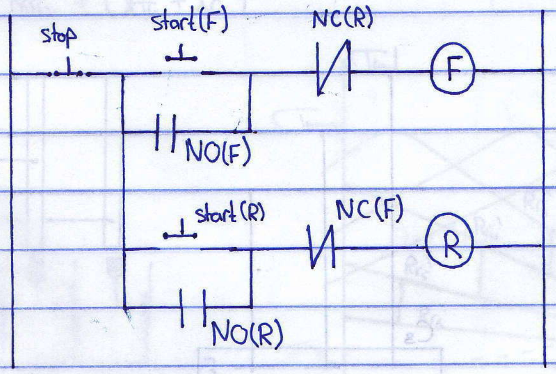
⑤ push button ((push to Open)).

Example: Control & power circuits to reverse the direction of motor.



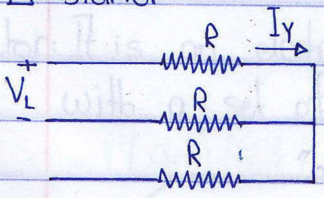
"Power Circuit"

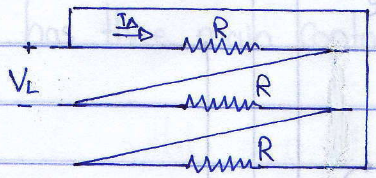
Ladder Diagram:
For the control circuit:



→ Methods to reduce the starting current of the 3-Φ induction Motor:

1 Y-Δ startor:





$$I_Y = \frac{V_L/\sqrt{3}}{R} = \frac{1}{\sqrt{3}}\,\frac{V_L}{R} \quad --- ①$$

$$I_\Delta = \frac{V_L}{R} \quad --- ②$$

→ From ① & ② → $I_Y = \frac{1}{\sqrt{3}} I_\Delta$.

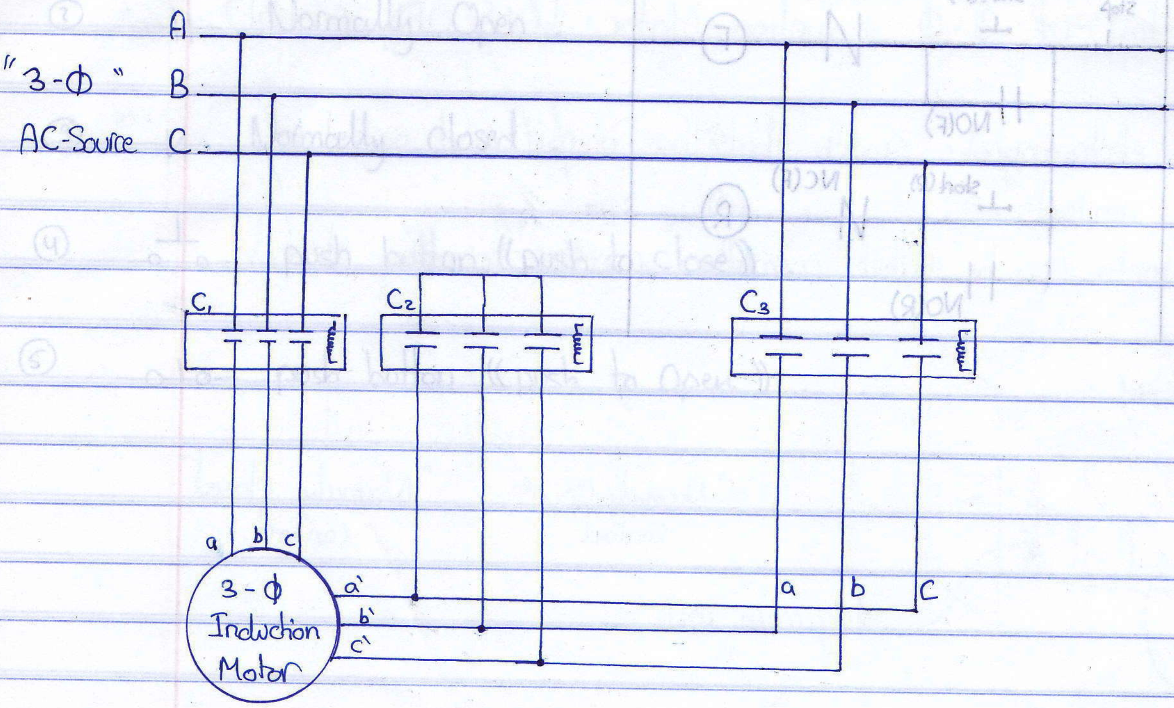
We can also study the th torque:

$$\tau_{ind} = \frac{3 I_r'^2 R_r'}{\sigma\,\omega_s}$$

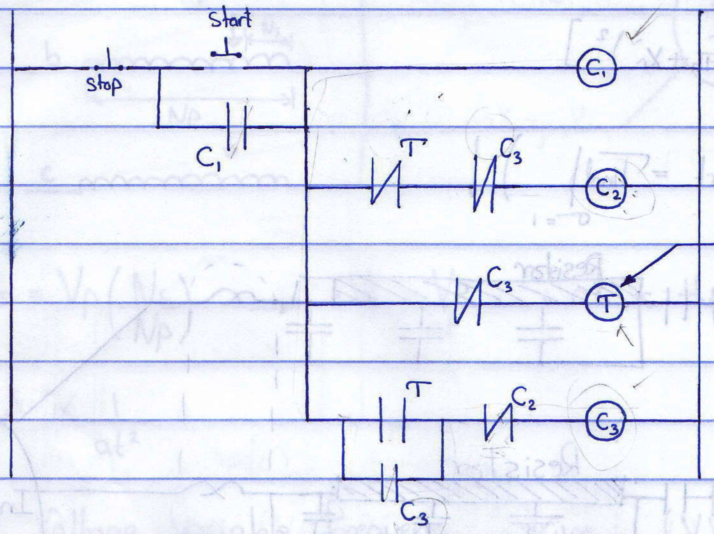
$$\tau_{\substack{start\\induced}} = \frac{3 I_r'^2 R_r'}{\omega_s}$$

$$\tau_{ind,start}(Y) = \frac{1}{3}\,\tau_{ind,start}(\Delta)$$

The Power Circuit:



The control circuit: For Y-Δ starter.

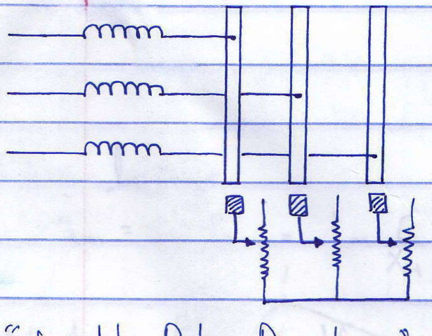


"Coil of timer, when it energized the contactor of the timer will change their state after present time or time delay"

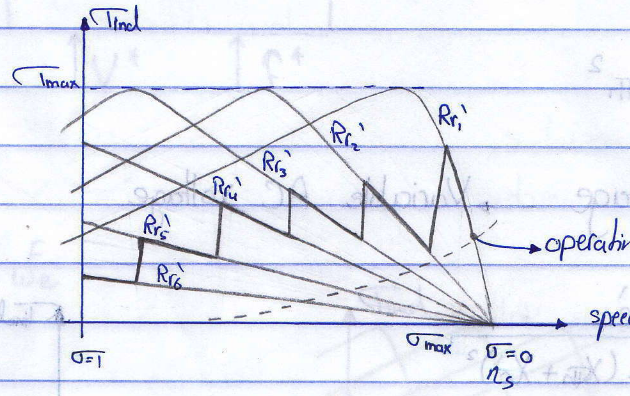
[2] Changing the rotor resistance "Wound Rotor Induction Machine":

$$T_{max} = \frac{3V_{Th}^2}{2W_s\left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}\right]} \quad \therefore \text{ it doesn't depend on } R_r'$$

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}} \quad \therefore \text{ if } R_r' \uparrow \Rightarrow \sigma_{max} \uparrow$$



"Variable Rotor Resistance"
"Potential Meter"



Note: n_s is constant for R_r' values since $n_s = \frac{120}{P} f_e$.

$R_{r6}' > R_{r5}' > R_{r4}' > R_{r3}' > R_{r2}' > R_{r1}'$

The disadvantage of this method:

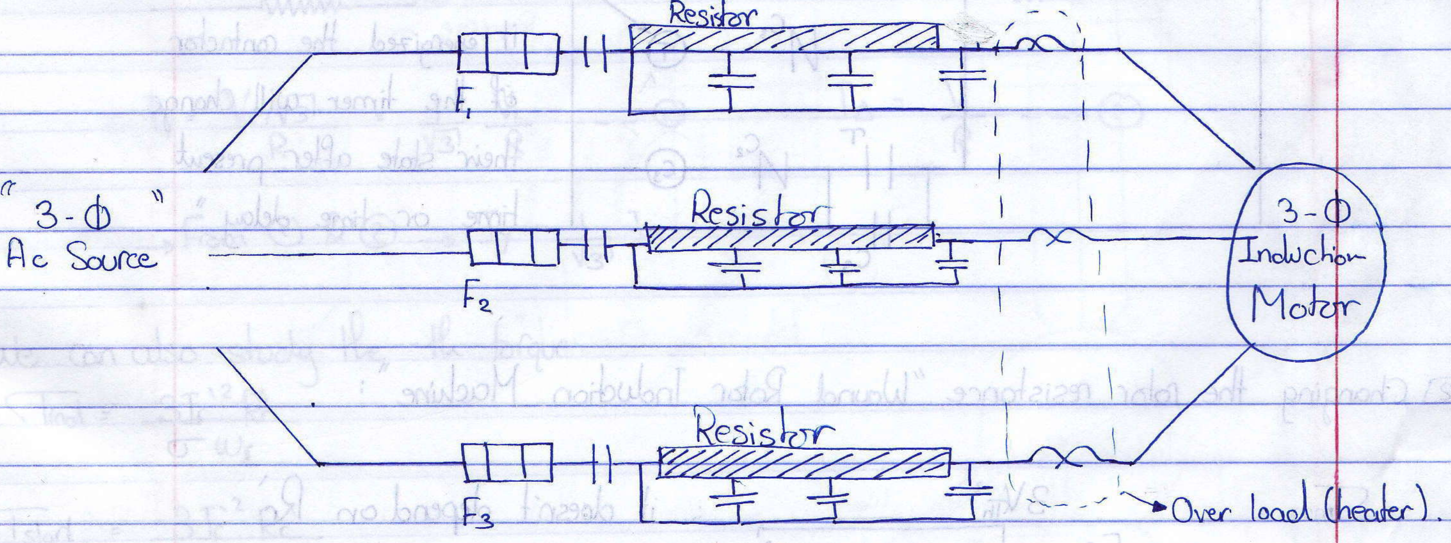
It increases the copper losses so reducing the efficiency

$R_r' \uparrow \rightarrow P_{RCL} \uparrow \Rightarrow \eta_{motor} \downarrow \;\&\; n_r = (1-\sigma) \downarrow$

[3] Stator resistance or inductance stator

$$T_{ind} = \frac{3V_{Th}^2 R_r'}{\sigma \omega_s \left[\left(\frac{R_r'}{\sigma} + R_{Th}\right)^2 + (X_{Th} + X_r')^2 \right]}$$

$$\left(R_{Th} \simeq \frac{X_m}{X_m + X_s} \cdot R_s \uparrow \right) \uparrow \Rightarrow \left(T_{start} = T_{ind}\Big|_{\sigma=1} \right) \downarrow$$



[4] Auto-transformer "VARIC"

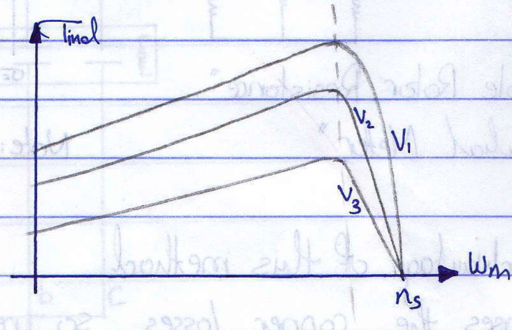
$$T_{ind} = \frac{3V_{Th}^2}{\omega_s \left[\left(\frac{R_r'}{\sigma} + R_{Th}\right)^2 + (X_{Th} + X_r')^2 \right]} \cdot \frac{R_r'}{\sigma}$$

$$T_{start}\Big|_{\sigma=1} \propto V_{Th}^2$$

Fixed AC voltage → Variable AC voltage

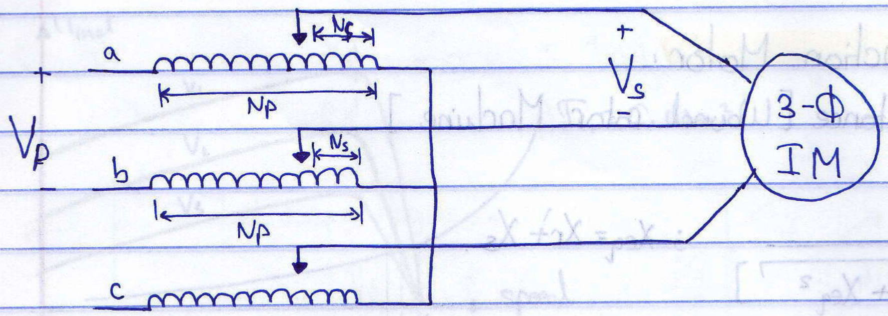
$$\sigma_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}}$$

$$T_{max} = \frac{3V_{Th}^2}{2\omega_s \left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2} \right]}$$



* $V_1 > V_2 > V_3$

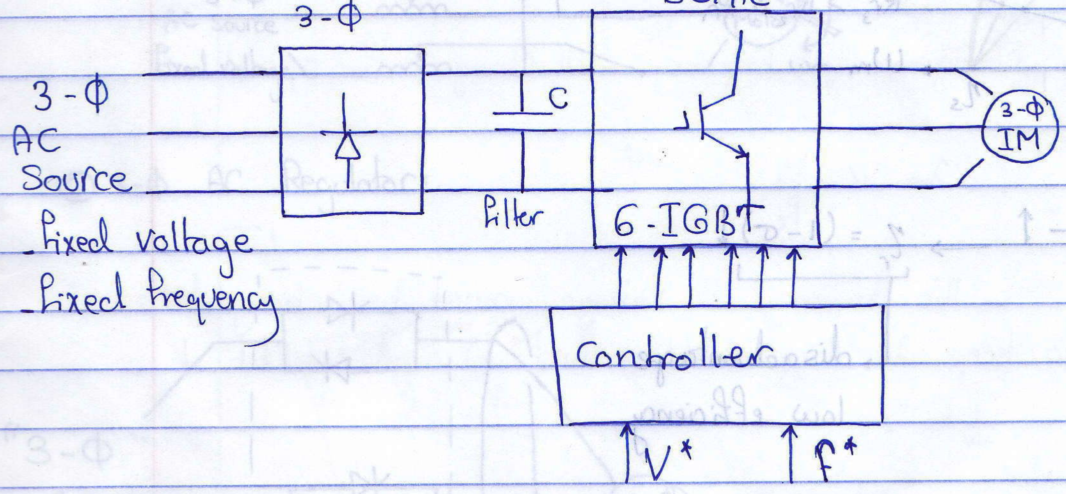
$T_{s1} > T_{s2} > T_{s3}$.



$$V_s = V_p\left(\frac{N_s}{N_p}\right) \longrightarrow V_s = \frac{V_p}{a_t}$$ ∴ at is the turns ratio.

$$T_{ind} \propto \frac{1}{a_t^2}$$

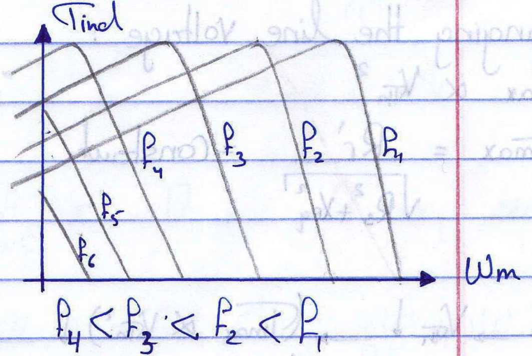
$\boxed{5}$ Variable Voltage variable Frequency Drive : (VVVF)



Flux linkage:

$$\lambda_m = L_m I_m = L_m \frac{E}{X_M} = \frac{E}{\omega_e}$$

$$\text{Machine Flux} \propto \frac{V}{f}$$



The Voltage and frequency most decrease by the Same factor to keep the Flux constant and not let it saturated.

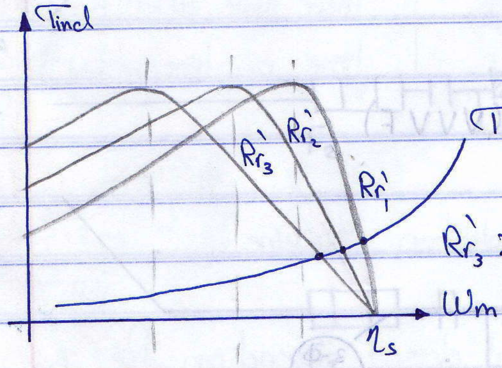
$$T_{max} = \frac{3V_{Th}^2}{2\omega_s\left(R_{Th} + \sqrt{(X_{Th} + X_r')^2 + R_{Th}^2}\right)}$$

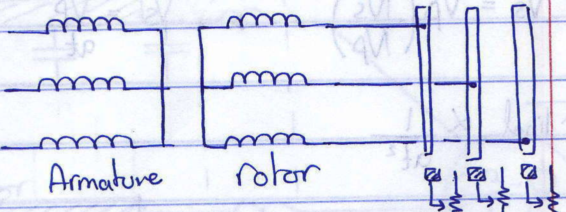
→ Speed control of induction Motor:

① Changing the rotor resistance [Wound rotor Machine]

$$T_{max} = \frac{3 V_{Th}^2}{2 \omega_s \left[R_s + \sqrt{R_s^2 + X_{eq}^2} \right]} \quad ; \; X_{eq} = X_r' + X_s .$$

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_s^2 + X_{eq}^2}}$$





T_{max} is constant.

$R_r' \uparrow \longrightarrow \underbrace{\omega_m \downarrow}_{} \longrightarrow \sigma \uparrow \longrightarrow \underbrace{\eta_r = (1-\sigma) \downarrow}_{}$

↳ speed control ↳ disadvantage low efficiency

* Using variable resistor
  Using power Convertor

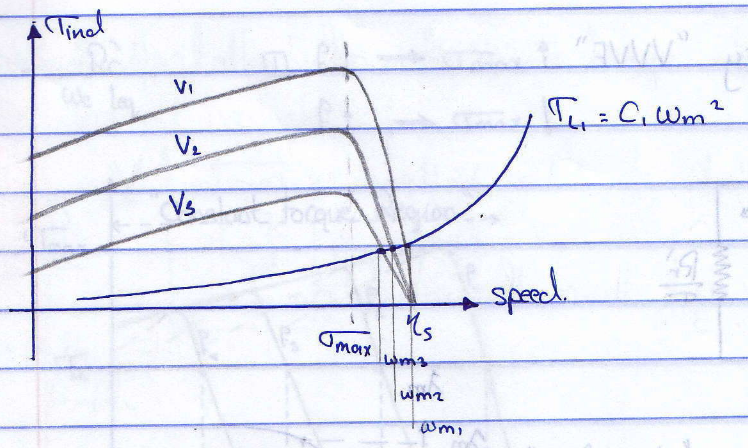
② Changing the line voltage:

$T_{max} \propto V_{Th}^2$

$$\sigma_{max} = \frac{R_r'}{\sqrt{R_s^2 + X_{eq}^2}} = \text{Constant.}$$

$V_s \downarrow \rightarrow V_{Th} \downarrow \longrightarrow \underbrace{(T_{max} \propto V_{Th}^2) \downarrow}_{} \longrightarrow \underbrace{\omega_m \downarrow}_{} \longrightarrow \sigma \uparrow \longrightarrow \underbrace{\eta_r \downarrow}_{}$

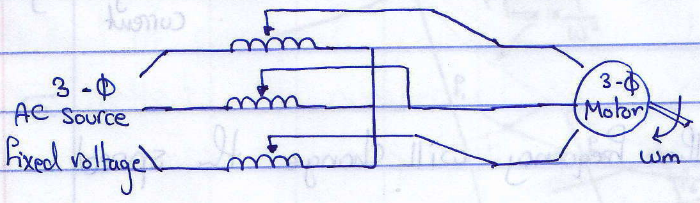
↳ disadvantage ↳ speed control. ↳ disadvantage in the efficiency



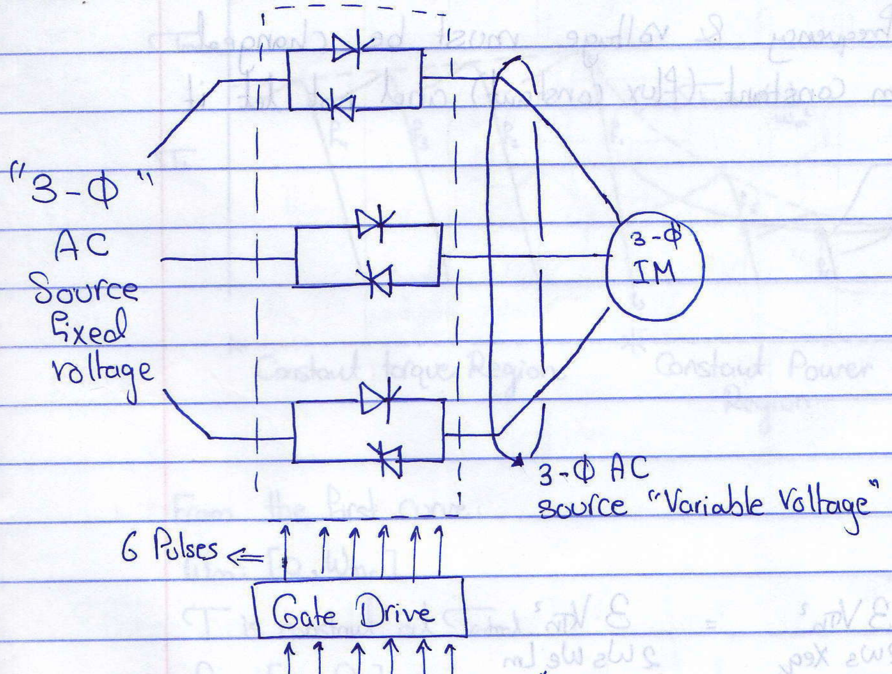
$V_1 > V_2 > V_3 \rightarrow \omega_{m1} > \omega_{m2} > \omega_{m3}$.

→ Note: Methods to change the motor voltage:

① Auto-transformer: (VARIAC)

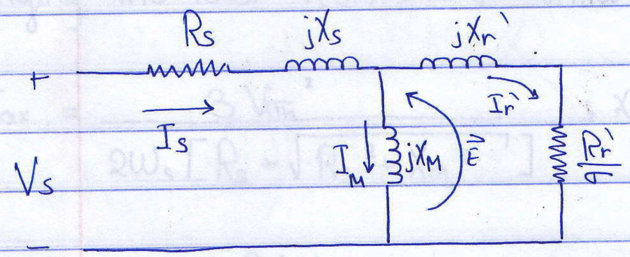


② 3-Φ AC Regulator:

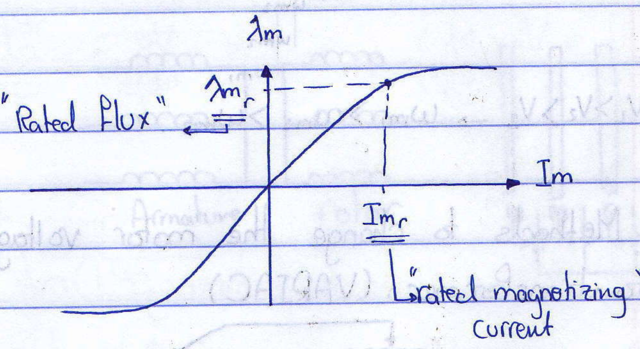


* The Note ends here

③ Variable Voltage Variable Frequency "VVVF":



Flux Linkage: $\lambda m = Lm\ Im$



$$\lambda m = Lm\left(\frac{E}{X_M}\right) = Lm\frac{E}{we\,Lm} = \frac{E}{we}$$

$$\lambda m = \frac{E}{we} = \frac{E}{2\pi f_e}$$

Recall: $\eta_s = \frac{120\ f_e}{P}$, so changing the frequency will change the speed.

$$\lambda m = \frac{E}{2\pi f_e} \simeq \frac{V_s}{2\pi f_e} \longrightarrow \lambda m \propto \frac{V_s}{f_e}$$

When ω_m < rated speed → The frequency & voltage must be changed by the same factor to keep λm constant (flux constant) and not let it saturated

$$T_{max} = \frac{3\,V_{Th}^2}{2\,\omega_s\left[R_s + \sqrt{R_s^2 + X_{eq}^2}\right]}$$

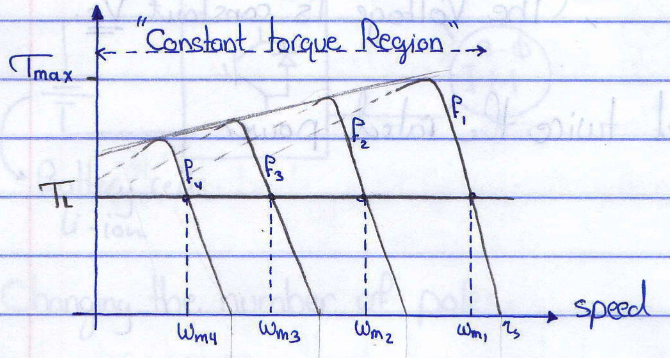
$$\sigma_{max} = \frac{R_r'}{\sqrt{R_s^2 + X_{eq}^2}}$$

If we ignor Rs $\longrightarrow T_{max} = \frac{3\,V_{Th}^2}{2\,\omega_s\,X_{eq}} = \frac{3\,V_{Th}^2}{2\,\omega_s\,we\,Lm}$

but $We = \frac{\omega_s}{2P} \longrightarrow T_{max} \propto \frac{V_{Th}^2}{\omega_s^2} \propto \frac{V^2}{f^2}$

$$\tau_{max} = \frac{R_r'}{\omega_e L_{eq}} \quad \boxed{!} \quad f\downarrow \Longrightarrow \tau_{max}\uparrow$$

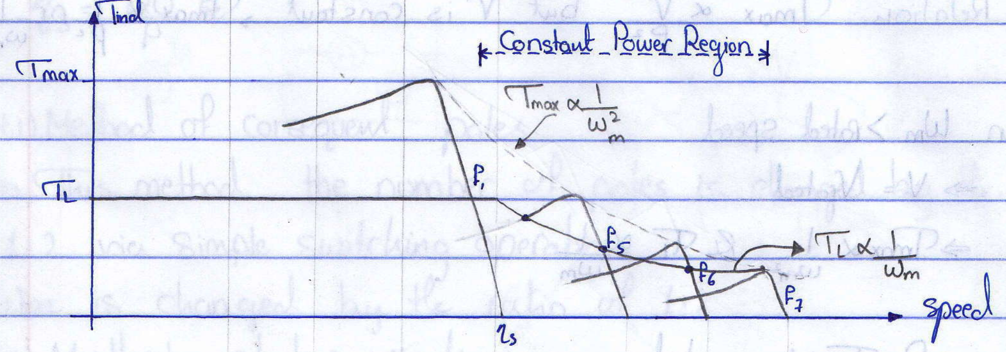
$$f\uparrow \Longrightarrow \tau_{max}\downarrow$$

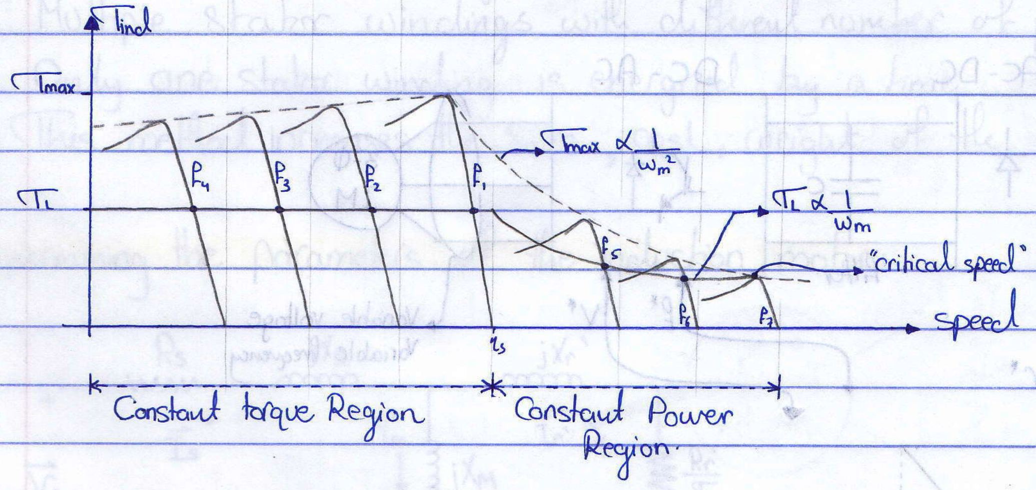


$f_1 > f_2 > f_3 > f_4$

$\omega_{m_1} > \omega_{m_2} > \omega_{m_3} > \omega_{m_4}$

$\therefore f_1$ is the rated frequency f_r.





From the first curve:

$\omega_m : [0, \omega_{m_r}]$

T is constant at T_{rated}.

$P : [0, P_r]$

$V : [0, V_r]$ ⚠ "Note we cann't have a voltage higher than the rated voltage"

From the last note, we are going to discuss what will happen if we want to increase the speed more than the rated speed.

From the second curve:

$\omega_m : [\omega_{mr}, 2\omega_{mr}]$, $T_L = T_{rated}$, The voltage is constant V_r

It's known that $P = \omega T$

So $P \overset{??}{=} 2P_r$, but we cann't get twice the rated power

The Solution is:

$T_L \propto \frac{1}{\omega_m}$ not constant

So $P = T\omega \Rightarrow$ constant.

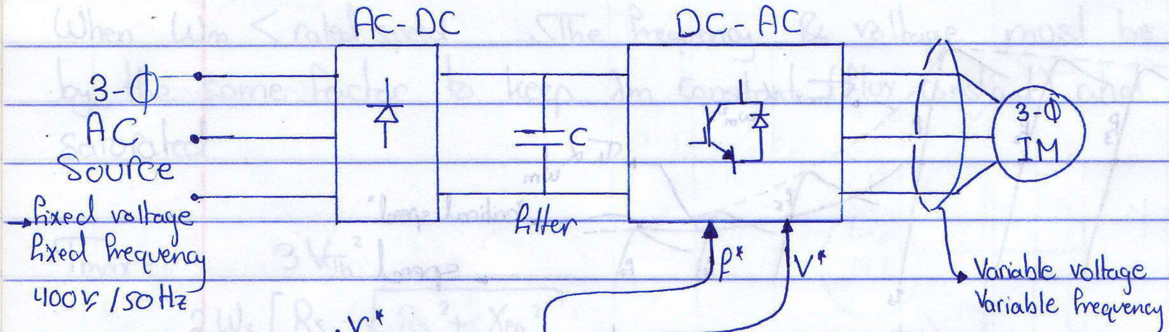
Now from the Relation $T_{max} \propto \frac{V^2}{f^2}$ but V is constant $\rightarrow T_{max} \propto \frac{1}{f^2}$ or $\frac{1}{\omega_m^2}$

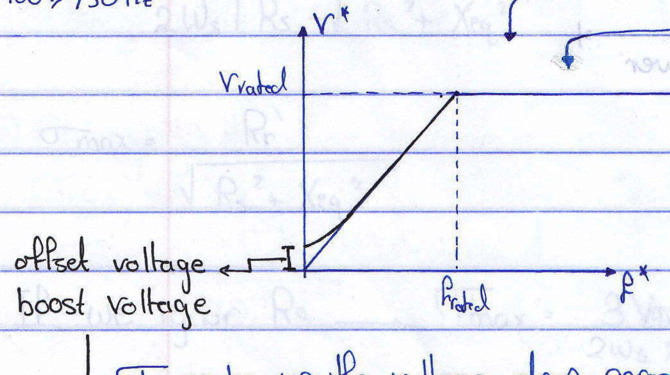
To conclude: When $\omega_m >$ rated speed

$\Rightarrow V = V_{rated}$

$\Rightarrow T_{max} \propto \frac{1}{\omega_m^2}$ & $T_L \propto \frac{1}{\omega_m}$

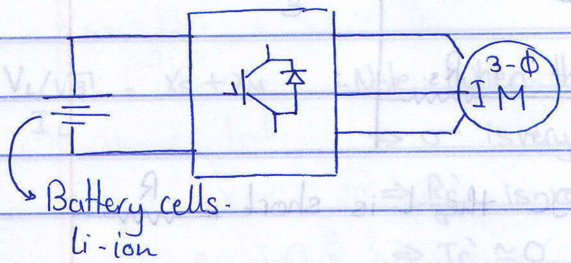
$\Rightarrow P_m = T_L \, \omega_m =$ rated power





To make up the voltage drop across R_s at low frequencies.

* Industrial Application on the VVVF:
The electrical vehicle

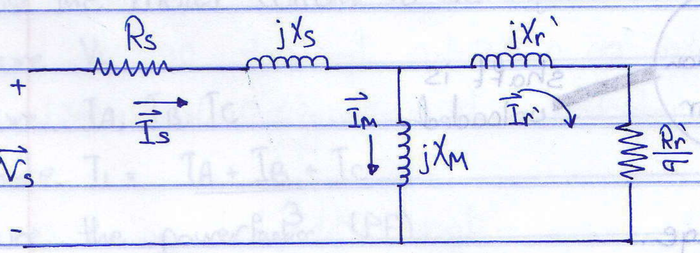


④ Changing the number of poles:

$$n_s = \frac{120\, f_e}{P}$$

4.1) Method of consequent poles:
In This method, the number of poles is changed by the ratio of 1:2 via simple switching operation.
W_m is changed by the ratio of 1:2.
4.2) Multiple stator windings:
Multiple stator windings with different number of poles.
Only one stator winding is energized by a time.
This method increases the size, cost, weight of the machine.

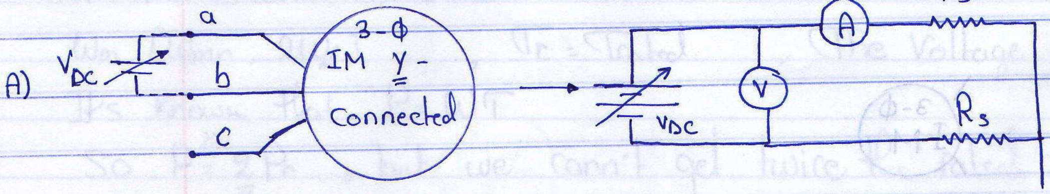
→ Determining the parameters of the induction motor:

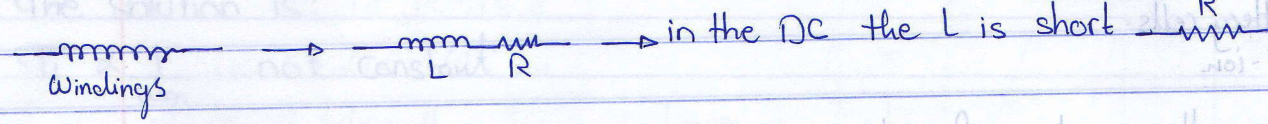


There are 3 tests for Determining the IM parameters:
① DC test.
② No load test.
③ locked rotor test.

1. DC test:

A)





Procedurcers:

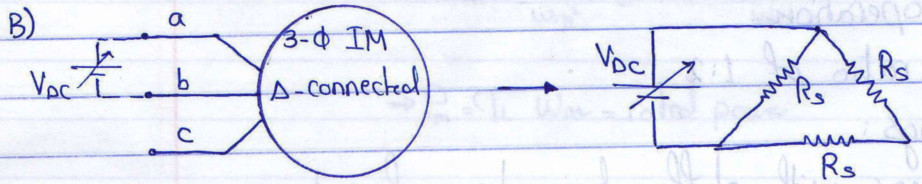
1) Adjust the current in the stator windings to be equal to the motor's rated current.

2) Measure V_{DC} & I_{DC}

3) Calculate R_s.

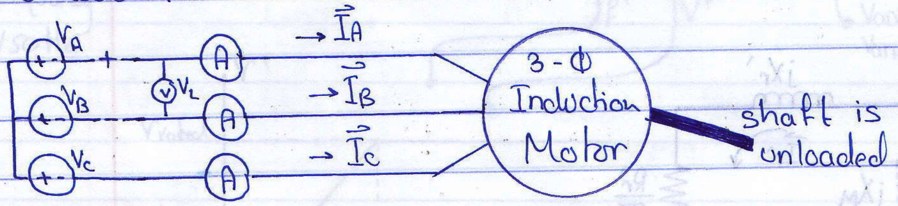
$$\rightarrow \frac{V_{DC}}{I_{DC}} = 2R_s.$$

B)



$$* \ \frac{V_{DC}}{I_{DC}} = R_{eq} \xrightarrow{*} R_{eq} = R_s // 2R_s = \frac{2}{3} R_s \longrightarrow \frac{V_{DC}}{I_{DC}} = \frac{2}{3} R_s$$

2. No load test:



$|V_A| = |V_B| = |V_B|$ = Motor's rated voltage.

Frequency: Rated Frequency.

Procedure:

① Make sure the shaft is unloaded

② Connect the motor to a 3-Φ source (V = rated voltage, f = rated voltage).

③ measure V_L

④ Measure I_A, I_B, I_C

⑤ estimate $I_L = \dfrac{I_A + I_B + I_C}{3}$

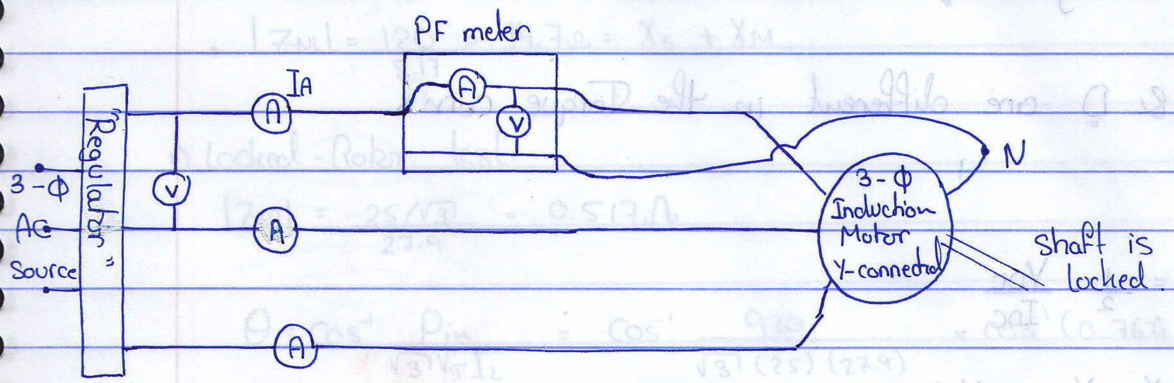
$\rightarrow \dfrac{V_L/\sqrt{3}}{I_L} = X_S + X_M$ ∴ Note: When the motor is unloaded

$\Rightarrow \sigma$ is very small

$\Rightarrow \dfrac{R_r'}{\sigma}$ is very high

$\Rightarrow I_r' \simeq 0$

[3] Locked rotor test:



Procedure:

① Connect the motor terminal to variable voltage variable frequency drive.

② Set the motor voltage initially to zero and then increase it gradually.

③ Set or Adjust the motor frequency to be equal 25% of the rated frequency

④ Adjust the motor current to be equal the motor's rated current.

⑤ Measure V_L

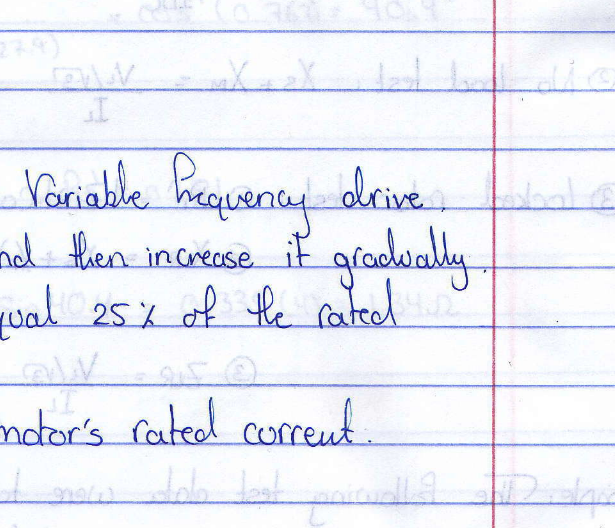
⑥ Measure I_A, I_B, I_C

⑦ estimate $I_L = \dfrac{I_A + I_B + I_C}{3}$

⑧ Measure the powerfactor (PF).

Note: Since the shaft is locked $\Rightarrow \sigma = 1$

$\dfrac{R_r'}{\sigma}$ is very small $\Rightarrow \vec{I_s} \approx I_r'$



* $\dfrac{V_L/\sqrt{3}}{I_L} = |Z_{LR}|$, $\theta_{LR} = \cos^{-1}(PF)$, $Z_{LR} = (R_s + R_r') + j\,(X_s + X_r')\,\dfrac{F_{test}}{F_{rated}}$

$|Z_{LR}| \cos\theta_{LR} = R_S + R_r'$

$|Z_{LR}| \sin\theta_{LR} = (X_S + X_r') \dfrac{f_{test}}{f_{rated}}$ $\therefore X_S + X_r' = X_{LR}$

Rotor Design: Wound Rotor $X_S = 0.5 X_{LR}$ $X_r' = 0.5 X_{LR}$

Cage Rotor :-

① Design $\underline{\underline{A}}$ $X_S = 0.5 X_{LR}$ $X_r' = 0.5 X_{LR}$

② Design $\underline{\underline{B}}$ $X_S = 0.4 X_{LR}$ $X_r' = 0.6 X_{LR}$

③ Design $\underline{\underline{C}}$ $X_S = 0.3 X_{LR}$ $X_r' = 0.7 X_{LR}$

④ Design $\underline{\underline{D}}$ $X_S = 0.5 X_{LR}$ $X_r' = 0.5 X_{LR}$

Note: Design $\underline{\underline{A}}$ & $\underline{\underline{D}}$ are different in the Torque curves.

To summarize:

① DC test: $R_S = \dfrac{1}{2} \dfrac{V_{DC}}{I_{DC}}$

② No-load test: $X_S + X_M = \dfrac{V_L/\sqrt{3}}{I_L}$

③ locked rotor test: ① $R_r' = |Z_{LR}| \cos\theta - R_S$

② $X_{LR} = X_S + X_r' = |Z_{LR}| \sin\theta \cdot \dfrac{f_{rated}}{f_{test}}$

③ $Z_{LR} = \dfrac{V_L/\sqrt{3}}{I_L}$; $\measuredangle Z_{LR} = \cos^{-1}(PF) = \theta_{LR}$.

Example: The following test data were taken on a 7.5-hp, four-pole, 208-V, 60-Hz, Design $\underline{\underline{A}}$, Y-connected induction motor having a rated current of 28 A.

Dc test:	No-load test	Locked-rotor test
① $V_{DC} = 13.6$ V	① $V_T = 208$ V	① $V_T = 25$ V
② $I_{DC} = 28.0$ A	② $f = 60$ Hz	② $f = 15$ Hz
	③ $P_{in} = 420$ W	③ $P_{in} = 920$ W
	④ $I_A = 8.12$ A	④ $I_A = 28.1$ A
	⑤ $I_B = 8.20$ A	⑤ $I_B = 28.0$ A
	⑥ $I_C = 8.18$ A	⑥ $I_C = 27.6$ A

a) Sketch the per-phase equivalent circuit for this Motor:

1) DC test:

$$R_S = \frac{13.6}{2(28)} = 0.243\,\Omega$$

2) No-load test:

$$I_{L,av} = \frac{8.12 + 8.20 + 8.18}{3} = 8.17\,A$$

$$V_{\phi_{NL}} = \frac{208}{\sqrt{3}} = 120\,V$$

$$\rightarrow |Z_{NL}| = \frac{120}{8.17} = 14.7\,\Omega = X_S + X_M$$

3) Locked-Rotor test:

$$|Z_{LR}| = \frac{25/\sqrt{3}}{27.9} = 0.517\,\Omega$$

$$\theta = \cos^{-1}\frac{P_{in}}{\sqrt{3}\,V_T I_L} = \cos^{-1}\frac{920}{\sqrt{3}\,(25)\,(27.9)} = \cos^{-1}(0.762) = 40.4^\circ$$

$$R_{LR} = 0.517\cos(40.4)$$
$$= 0.394\,\Omega \longrightarrow R_r' + R_S = 0.394 \longrightarrow R_r' = 0.151$$

$$X_{LR} = (X_S + X_r') = \frac{60}{15}(0.517)\sin 40.4 = 0.335\,(4) = 1.34\,\Omega$$

Since it's design A: $X_S = X_r' = 0.67\,\Omega$

$$\rightarrow X_M = 14.7 - 0.67 = 14.03\,\Omega.$$