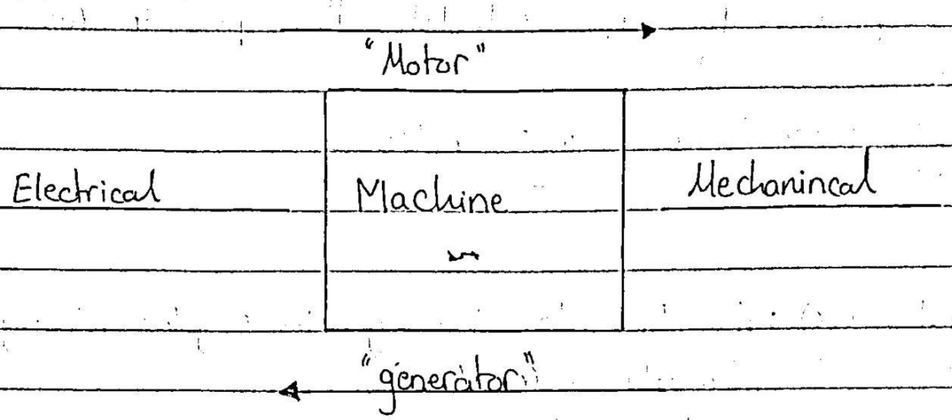
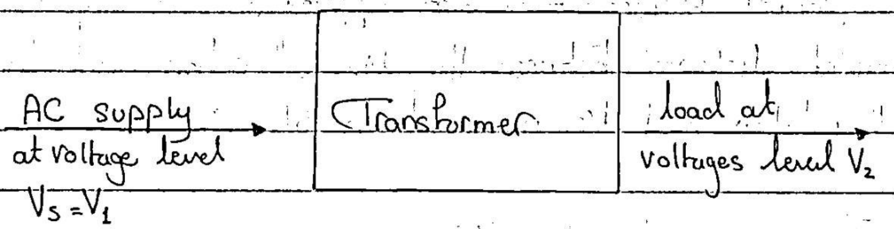
Electrical Machines

→ An electrical machine is a device that converts either electrical energy to mechanical energy. Or mechanical energy to electrical energy through the action of magnatic field.



→ Transformer is a device that converts the AC electrical energy at a voltage level to AC electric energy at another voltage level.



if $V_1 > V_2 \Rightarrow$ step down
$V_2 > V_1 \Rightarrow$ step up

→ Rotational motion :

① angular position (θ) : it's measured in (rad).

② angular velocity (ω) : it's the rate of change of angular position and it's measured by (rad/sec)

$$\omega = \frac{d\theta}{dt}$$

Another unit for ω is the (rpm) = revelution per minute.

The rpm can be converted to ω and vise versa by the relation:

$$n = \frac{60}{2\pi}\,\omega$$

where n = is the speed in (rpm).

Example: If the speed is 1000 rpm calculate the speed in rad/sec?

$n = 1000 \longrightarrow \omega = ??$

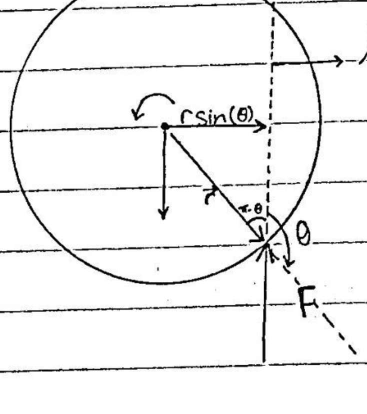
$$\omega = \frac{2\pi}{\cancel{60}_{\,3}}\cdot 100\cancel{0} \longrightarrow \omega = \frac{100\pi}{3}\ \text{rad/sec}.$$

③ angular acceleration (α) : it's the rate of change in angular velocity and measured in (rad/sec^2).

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}.$$

④ Torque (T) : It's the product between the force applied on the object and the smallest distance between the line of action and the axis of rotation and it's measured in ($N.m$).

$$T = \vec{r} \times \vec{F}$$
$$= rF\sin\theta.$$ where θ is the angle between the vectors $\vec{r}$ and $\vec{F}$.



→ line of action. الطاقة والقدرة الميكانيكية تعتمد على حساب الـ (T).

The axis of rotation is about the center.

→ Newton's second law :

In linear motion → $\underset{\text{net force}}{\Sigma F} = ma = m\frac{dv}{dt}$

In Rotational motion → $\underset{\text{net torque}}{\Sigma T} = J\alpha = J\frac{d\omega}{dt}$

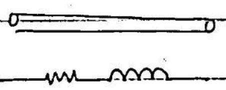
where J is called the moment of inertia and measured in $(kg.m^2)$.

اذا السرعة ثابتة الـ T صفر.
اذا تسارع الـ T موجب.
" تباطؤ الـ T سالب.

القدرة تعتمد على الـ T والسرعة.

اذا ΣT أو ΣF تساوي صفر لا يعني أن الجسم ثابت بل يمكن أن يكون قد سار بسرعة ثابتة.

→ High voltage cable :



$P_{Loss} = 3\ I^2R$ [in three phase].

$S = VI$ = constant value.

$\boxed{\uparrow V} \downarrow I$ = constant

Then $P_{Loss} \downarrow$.

→ Work (W) :

$W = \int F\,dl$ in linear motion.

$W = \int T\,d\theta$ in angular motion.

if T is constant then $W = T\theta$ (steady state).

→ Power (P) : is the rate of change of work done

$P = \frac{dW}{dt}$ => if τ is constant then $P = \tau \frac{d\theta}{dt} = \tau\omega$.

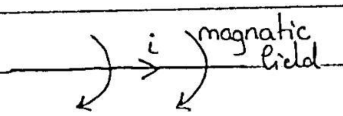
From $P = \tau\omega$ we will find or calculate the torque.

Power in (rpm) => $P = \tau \cdot \frac{2\pi n}{60}$

→ Magnatic field : It's the fundamental mechanism of energy conversion in transformers and electric machines.

The Basic principles of the magnatic field :

① A current - carrying wire produces a magnatic field around it.



② A time - changing magnatic field produces a voltage across the coil when it passes through it. (transformer) قانون فاراداي.

← إذا انقطع الملف خطوط مجال مغناطيسي متغيرة مع الزمن رح ينشأ فولت على أطراف الملف.

③ A current - carrying wire (in the presence of magnatic field) has a force induced on it.

$F_{in} = i\, l \times B$ "Motor"

$= i\, l B \sin\theta$

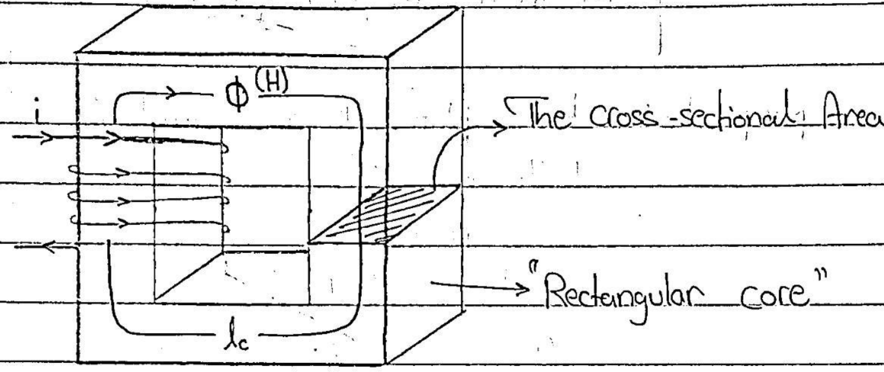
④ A moving wire in the presence of magnatic field has induced voltage across it "Generator".

$e_{in} = (V \times B) \cdot l$

$= V B l \sin\theta \cos\alpha$

→ Ampere's law : The basic law governing the production of a magnatic field by a current.

$\oint H\,dl = I_{(net)}$ ∴ H: is the magnatic field intensity produced by the current $I_{(net)}$.
dl: The differential element of length along the path of integration.
$I_{(net)}$: the current in the loop [$I_{(enc)}$].



$H l_c = N i \rightarrow H = \frac{N i}{l_c}$ ∴ H is measured by [(Amp. turns)/meter]

→ Magnatic field density (B):

$B = \mu H$

where μ is called the permability of the material which represented the relative ease of establishing a magnatic field in a given material and its unit is (H/m).

Def: Magnatic field intensity (H): The effort required to induce magnatic field.

The unit of (B) is (weber/m²) or (Tesla).

The permability of free space is called μ_o and its value is:
$\mu_o = 4\pi \times 10^{-7}$ H/m.

μ can be also represented by :

$\mu = \mu_o \mu_r$, where μ_o is the permability of free space and μ_r is the relative permability.

μ_r of the steel is located between (2000 - 6000)

→ Magnatic flux : (ϕ)

It's measured in weber (wb).

$$\phi = \int_A B \cdot dA$$

μ_r = (2000 - 6000) means that: 2000-6000 times more flux is established in a piece of steel than in a corresponding area of air.

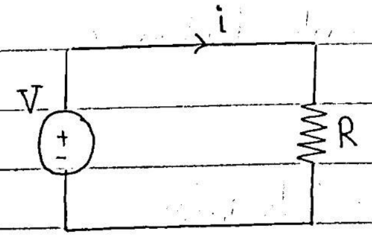
if the flux density vector is perpendecular to the plane of area, and it's constant then,

$$\phi = BA$$

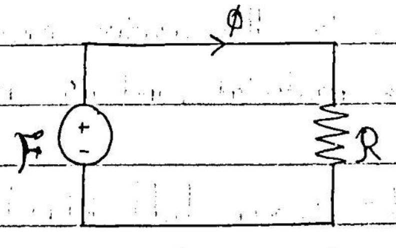
Now we have → $B = \mu H = \frac{\mu_o \cdot \mu_r \cdot N \cdot i}{l_c}$

So $\phi = \frac{\mu_o \mu_r N i A}{l_c}$ or $\phi = \frac{\mu N i A}{l_c}$

→ Magnatic circuits :-



electrical circuit



magnatic circuit

$I = \frac{V}{R}$ [ohm's law]

$\phi = \mathcal{F}/\mathcal{R}$

$\mathcal{F} = Ni$

$\mathcal{F}$: is called the magnetomotive force

ϕ : is the flux

$\mathcal{R}$: is called the Reluctance

: Note: $\frac{1}{\mathcal{R}}$ is called the permeance we know that the flux from the core is equal to :

$$\phi = \frac{\mu N i A}{l_c}$$

and it's also equal to $\phi = Ni/R$ from the magnatic circuit, compining the two resolts gives us :-

$$R = \frac{l_c}{\mu A}$$

Reluctance in magnatic circuit behave the same as the resistance in electrical circuit.

① The equivalent reluctance of series :

$R_{eq} = R_1 + R_2 + R_3 \ldots + R_n$

② Similarly, reluctances in parallel combine as :

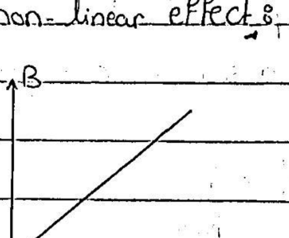
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \text{----} + \frac{1}{R_n}.$$

→ Flux calculations in a core by using the magnatic circuit ~~compon~~ concepts are always approximated within about 5 % error

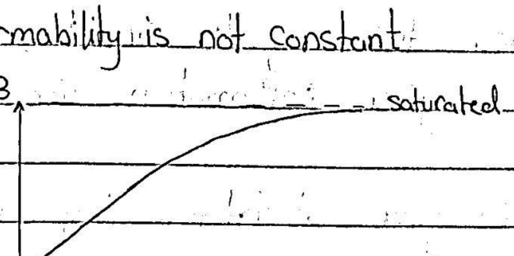
Reasons of inaccuracy of flux calculations :

① leakage flux : A small amount of magnatic flux escapes from the core into the surrounding low-permeability air

② non-linear effect : In reality the permability is not constant



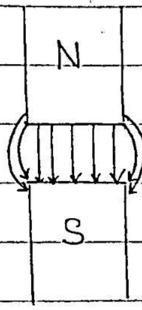
Ideal



real

③ Errors in measurring the cross-sectional area (A) and the mean path length (l_c)

④ Fringing effect : If there are air gaps in the flux path in a core, the effective cross-sectional area of the air gap will be larger than the cross-sectional area of the iron core



* See Example 1-1 page 15.

How to think :-

① determine the number of reluctances in the magnatic circuit by : determining the following :

Since $R = \frac{l}{\mu A}$

any change in : 1) the mean path length. 2) the material (permability) 3) Area → "Geometry"

Note :- Be aware while calculating the mean-path length.
All the units should be standard.
$\mu = \mu_o \mu_r$ ⚠ Don't forget.

② Draw the magnatic circuit, and label each part of it with the given information. Also determine if it's a series or a parallel.

③ make your calculations by :- ① $F = Ni$; ② $\Phi = \frac{F}{R_{eq}}$.

* See example 1-2 page 17.

How to think:-

① Determine the two reluctances in the example, Notice that each reluctance will be different by changing l, μ or A.

② Be notice that the 5% increase in the air gap area will be calculated as: $A + 0.05A =$ "$1.05A$".

③ Memorizing all the formulas will be the only way to pass this example:

1) $\phi = BA$ since B is constant.

2) $\phi = \frac{F}{R}$

3) $Ni = BA \cdot \frac{l}{\mu A} \rightarrow i = \frac{Bl}{\mu N}$

* See example 1-3 page 19:

How to think:-

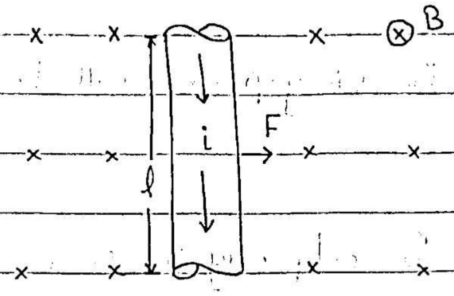
① Notice that every change in l, μ or A will leads to a new reluctance. 'Here we have 4 reluctances'.

② Using the previous formulas find the flux, then the flux density in the gap.

→ Note: All the core & the gaps & the rotor have the same flux.

→ Production of induced force on a wire:

A major effect of a magnatic field on its surronding is that it induces a force on a current-carrying wire within the field.



$F = i(l \times B)$

where the direction of l is in the direction of current flow.

$F = ilB\sin\theta$, where θ is the angle between the wire & the f. density.

تحديد اتجاه F :-

أصابعك باتجاه l باتجاه والكف باتجاه B

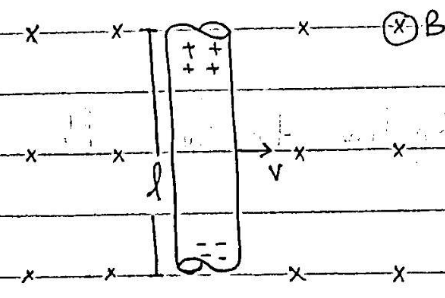
الابهام يؤشر لموقع القوة F

* see example 1-7 page 33 :-

Note:- The induction of a force in a wire by a current in the presence of a magnatic field is the basis of "Motor action".

→ Induced voltage on a conductor moving in a magnatic field:

If a wire with proper orientation moves through a magnatig field a voltage is induced in it.



$e_{ind} = (V \times B) \cdot l$

V: is the velocity.

* l: length of conductor, it points along the direction of the wire toward the end making the smallest angle with the vector $(V \times B)$.

* The voltage in the wire will be built up so that the positive end is in the direction of $(V \times B)$.

* See example 1-8 page 34 :-

How to think :-

① lets write the formula $e_{ind} = (V \times B) \cdot l$ in its expansion formula :

$e_{ind} = VB \sin\theta \, l \cos\alpha$, What are θ & α ??

→ θ : is the angle between the velocity direction & the f. density direction.

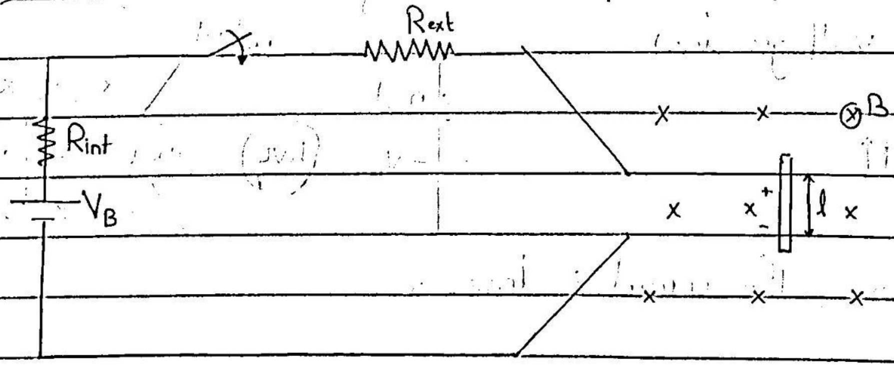
(The $(V \times B)$ can be optained by the right hand law :
الأصابع باتجاه السرعة (اتجاه الـ V) الكف باتجاه الـ B الإبهام بأشّر على (VxB))

→ α : is the angle between $(V \times B)$ and l "the smallest angle".

* Also see example 1-9 page 34 :

Note: "The induction of voltages in a wire moving in a magnatic field is fundamental to the operation of all types of "Generators".
→ it's called the generator action.

→ The linear DC machine - a simple example :



→ Rint : The internal resistence of the DC battery.

Rext : An external resistence which is used to limit the starting current.

How is this simple DC Machine works:

When the switch is closed the current will flow through the conducting bar. This current is initialy given by:

$i = \frac{V_{DC}}{R_{eq}}$; $R_{eq} = R_{ext} + R_{int}$

Since the current passes through the bar in the presence of the magnatic field; an induced force will be applied on the bar:

$F_{ind} = i(l \times B) = ilB$ since $\theta = 90°$

The bar will accelarate according to Newton's second law:

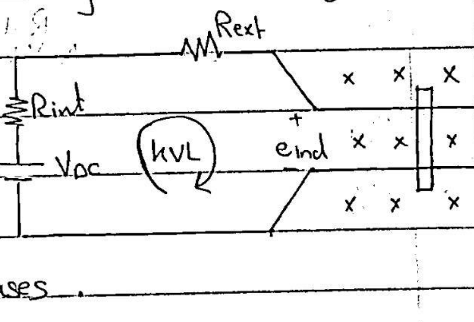
$\Sigma \vec{F} = m\vec{a} = F_{ind}$

The bar now is moving in the presence of the magnatic field (B) and an induced voltage will appear across it:

$e_{ind} = (V \times B) \cdot l = VBl$ since $\theta = 90°$ & $\alpha = 0°$

From here: $e_{ind}\uparrow = V\uparrow B\, l$ $\boxed{\text{the emf is increasing}}$.

The voltage now reduces the current flowing in the bar, since by kirchhoff's voltage law



$i\downarrow = \frac{V_{DC} - e_{ind}\uparrow}{R_{eq}}$

As eind increases, the current i decreases.

When the net force on the bar equals zero then the bar will reach the steady-state and it will move with constant speed.

$F_{ind} = 0$ when $e_{ind} = V_{DC}$ since i will be zero:

$i = \frac{V_{DC} - e_{ind}}{R_{eq}} = 0$

So $\Sigma \vec{F} = 0 = m\vec{a}$ $\rightarrow$ So $\vec{a} = 0$ $\rightarrow$ the bar is moving at constent speed. $= (V_s)$

Vs is given by:

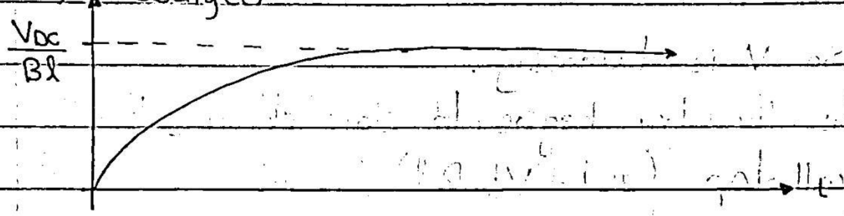
$e_{ind} = V_{DC} = \underbrace{V}_{\text{velocity}} B l \rightarrow V$ is now $V_s \rightarrow \boxed{V_s = \frac{V_{DC}}{Bl}}$

The bar will continue to move with this speed unless some external force disturbs it.

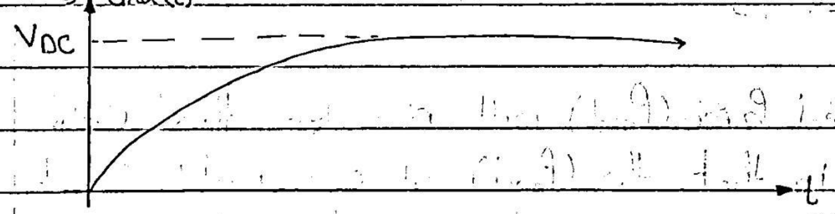
→ Charts & Graphs:

When the motor is started then:-

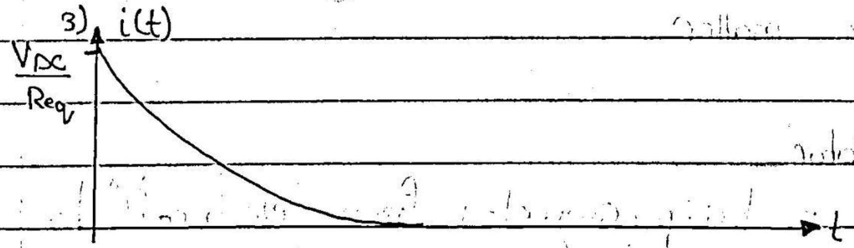
1) Velocity(t)



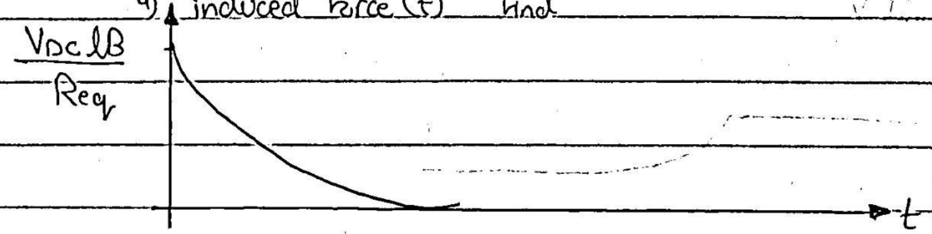
2) $e_{ind}(t)$



3) i(t)



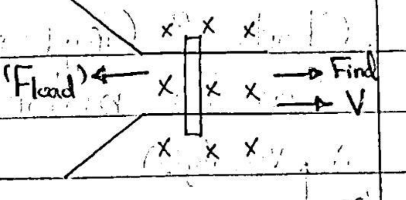
4) induced force (t) "F_{ind}"



→ The linear DC Machine as a "Motor":

Assume that the machine is initially running under the steady state condition with no load (force) acting on it. Then the speed (V_s) is given by: $V_s = \frac{V_{DC}}{Bl}$

Now, Assume that a load force (F_{load}) is applied with opposite direction of motion,



$\Sigma F = F_{ind} - F_{load}$, since F_{ind} is zero initially

then $\Sigma F = -F_{load} = m\vec{a}$

$= a < 0$, So V is decreasing

But just as soon as the bar begins to slow down, the induced voltage (e_{ind}) will drop ($e_{ind} = V\downarrow Bl$).

So, the current flow in the bar rises:

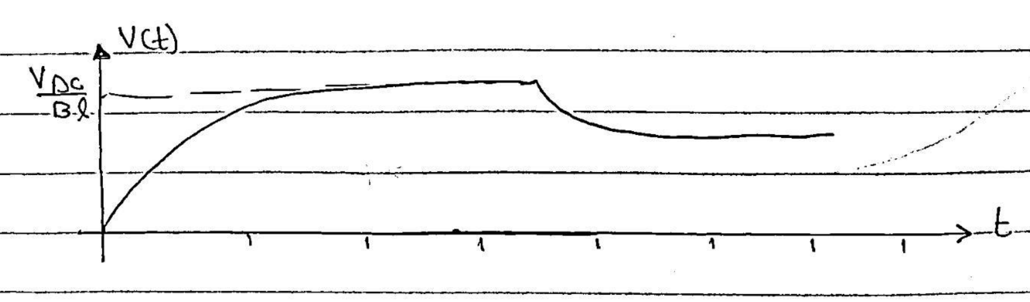
$$i\uparrow = \frac{V_{DC} - e_{ind}\downarrow}{R_{eq}},$$

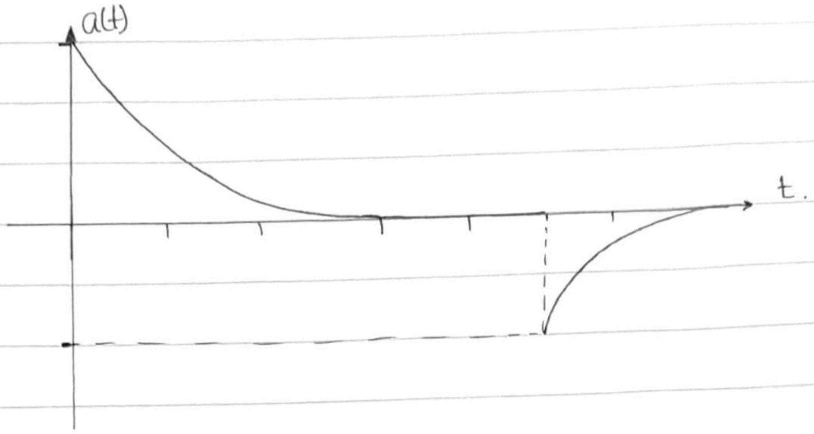
Therefor the induced force (F_{ind}) will rise too, this chain of events will result in that the (F_{ind}) will rise until the it equals the (F_{load}) [The bar will reach a steady-state again]

But now V_s will be smaller

The power now is being converted from "electrical" to "Mechanical". And the amount of it is equal to:

$P_{conv} = e_{ind}\, i = F_{ind}\, V$



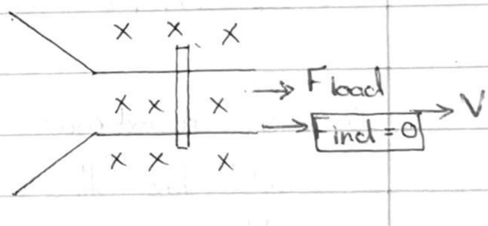


Note: In the Power conversion formula we use (e_{ind}) not (V_{DC}); because some power has been lost in the resistors.

The linear DC Machine as a Generator:

Assume that the Machine is running under steady-state with no load.
The speed in this state is: $V_s = \frac{V_{DC}}{Bl}$.

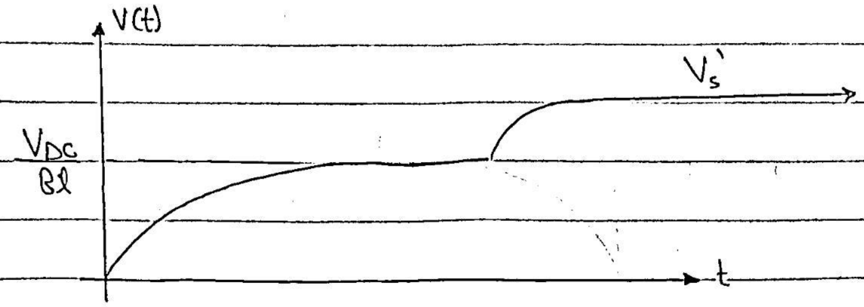
Assume now that an external force is applied on the bar in the direction of motion.
The idea here is to make $e_{ind} > V_{DC}$ So $\underline{\underline{V}}$ increases.

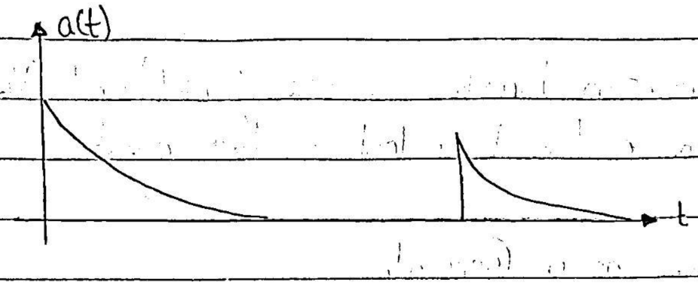


→ $\sum F = F_{ind} + F_{load}$, $F_{ind} = 0$ initially.
$\sum F = F_{load} = m\vec{a}$.
$\sum F > 0 \rightarrow a > 0 \rightarrow$ V increases.

→ $e_{ind}\uparrow = V\uparrow Bl$, also *i: will increase but in the negative direction.

→ $|F_{ind}|\uparrow = ilB$ in the opposite direction. It will rises until it reaches the F_{load}, Then: كيف طبقهم معاها. ?
$\sum F = 0 \rightarrow m\vec{a} = 0 \rightarrow V_s'$ is the constant velocity.
Such that $V_s' > V_s$





- See example 1-10 page 43 :

How to think :-

1) In the steady state condition $V_{DC} = e_{ind}$.

2) if a force acting in an opposite direction of the original (F_{ind}) then it will act like a motor. However, if the force act in a direction of the original (F_{ind}) then it will act like a generator.

3) When an external force is applied, the following procedures should be followed :-

i - determine the F_{ind} in the steady-state, $F_{ind} = F_{applied} = i\,lB$.

ii - Then a KVL over the mesh to find e_{ind}, but before that you should find i.

iii - Now $V_{ss}' = e_{ind}/Bl$.

iv - You can be verify from your answer if it's true, by :

1- if it's a motor V_{ss}' should be less than V_{ss}.

2- if it's a generator V_{ss}' should be more than V_{ss}. ?

4) The power can be studied as follows:

i. in the Motor:

1. The bar is consuming power. $P = e_{ind} \cdot V_{ss}$

2. The battery is producing power. $P = V_{DC} \cdot i$

ii. in the Generator:

1. The bar is producing power.

2. The battery is consuming power

5) Notes for (e):-

$V_{DC,r}$: Rated (Nominal) DC Voltage of the machine,

هي القيمة مصمم عليها الجهاز ع شان يشتغل للأبد

zero → $V_{DC,r}$ → ∞

→ in this period you can control "B"

→ in this period you can control the "voltage" in order to get higher velocity.

→ $V_s = \frac{e_{ind}}{B l}$ at s.s "e_{ind} & l" are consants So $V_s \propto \frac{1}{B}$

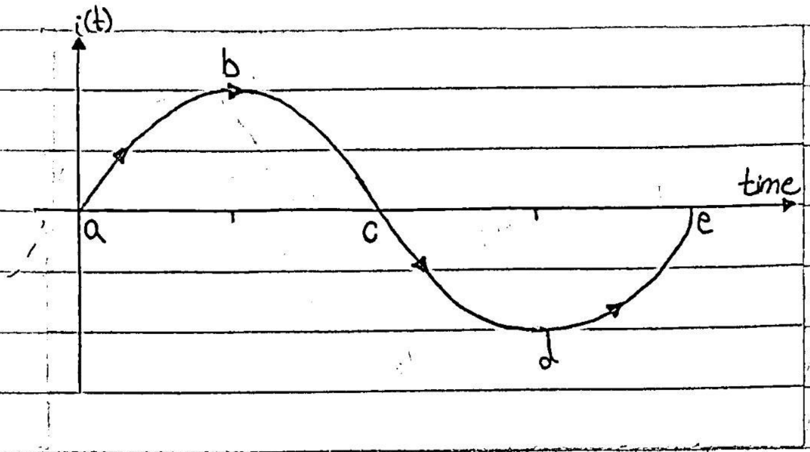
from $F_{ind} = i l \times B$ → $F_{ind} \propto \frac{1}{V_s} \propto B$

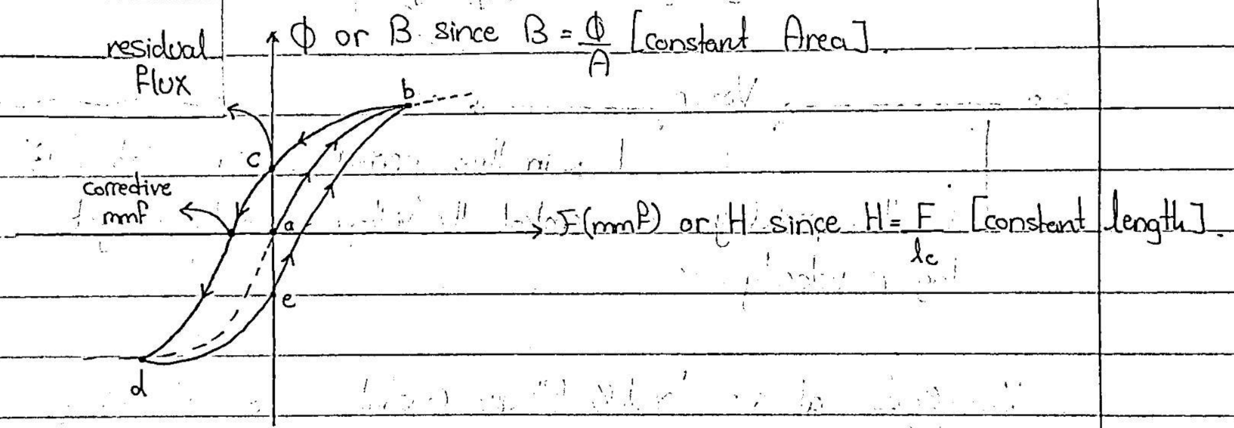
⇒ Power $= F_{ind} \cdot V_s$ → Power $= K \frac{1}{V_s} \cdot V_s = k$ a "constant" when $V_s > V_{s,r}$

∴ $V_{s,r}$ is the "rated speed".

Energy losses in the Ferromagnatic Material:-

Assume that an AC current is applied to the windings of the core, and the flux is initially zero.





The above figures represents: The hysteresis loop traced out by the flux in the core when the current is applied to it.

Notes:- The ab part represents the saturation curve.

The flux depends not only on the current but also on the previous history of the flux in the core. This dependancy is called "Hysteresis".

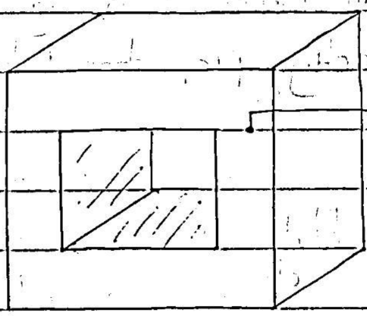
$\Phi = f(i) + \Phi_i$

When a large magnumotive force is applied to the core and then removed, the path of the flux is abc. The flux doesn't go to zero when mmf is removed. Therefor a corrective mmf is applied in the opposite direction to force the flux to go to zero.

Residual flux: it is the flux left in the core when the mmf is removed.

Corrective mmf: it is the mmf that should be applied in the opposite direction to force the flux to go to zero in the core.

The structure of Ferromagnatic material:



A point on the core:
The ferromagnatic material has domains where each domain consists of set of atoms (Nickel, cobult, Iron).

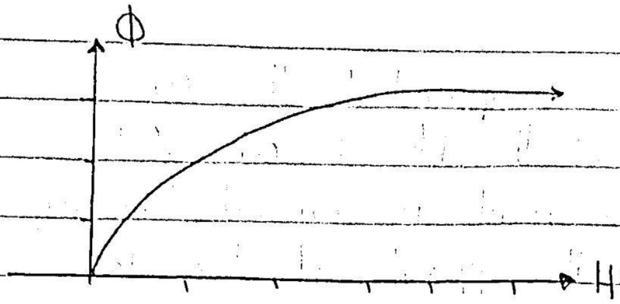
Domain

✗	↗	×	←
→	←	↗	·
↓	↗	×	↑
↙	×	↗	·

"Magnatig domains oriented randomly"

→	→	→	→
→	→	→	↗
→	→	↗	→
→	→	→	→

"Magnatig domains lined up in the presence of an external magnatig field"



"The Magnatizing Curve".

→ Hysteresis loss:
it is the energy required to accomplish reorientation of magnetic domains per each cycle of applied AC curve. This energy is converted to heat in the core.

→ Faraday's law:-
It is the fundamental property of magnetic field involved in Transformer operation.

If a flux passed through a coil, it will induce a voltage across the coil. The voltage is directly proportional to the rate of change of the flux:

$$e_{ind} \propto \frac{d\Phi}{dt} \rightarrow \text{For N turns: } e_{ind} = N\frac{d\Phi}{dt}$$

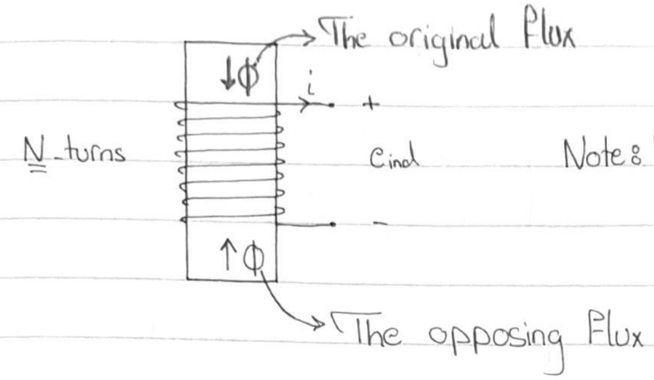
When Leakage flux is considered:
if there is a leakage flux out-side the core (in the air), each turn might have a slightly different flux.
The e_{ind} ith term: $e_i = \frac{d\Phi_i}{dt}$; $i = 1, 2, 3, 4, \ldots$

The e_{ind} across the N turns is:

$$e_{ind} = \sum_{i=1}^{N} e_i$$

$$= \sum_{1}^{N} \frac{d\Phi_i}{dt} = \frac{d \sum_{1}^{N}\Phi_i}{dt}$$ where $\sum_{1}^{N}\Phi_i$ is the "flux linkage"

$$e_{ind} = \frac{d\lambda}{dt}$$, where $\lambda = \sum_{1}^{N}\Phi_i$ (wb.turns).

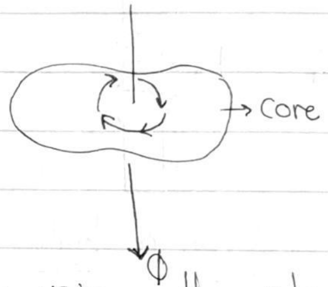


Note: The polarity of e_{ind} is determined by Lenz law.

Eddy currents: The time-changing flux $\frac{d\phi}{dt}$ induced a voltage within the core. The induced voltage causes eddy currents to flow in the core. The eddy currents flow through the ~~core~~ resistive material of the core which causes energy loss that heats up the core.

$$i_e \propto \frac{d\phi}{dt}$$

eddy current (i_e)



The paths of eddy currents are broken using the steel laminations with high permability [to transfer flux] and low conductivity. The sheets are isolated from each other using insulating material.

→ We can see that energy loss comes from two major things:

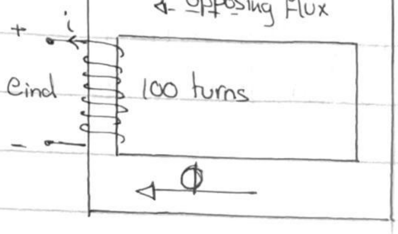
$$\text{Energy loss} = \text{Hysteresis}_{loss} + \text{Eddy currents}_{loss}.$$

Example:- A coil of wire wrapped around an iron core. The flux in the core is: $\phi = 0.05 \sin 377t$ Wb. There are 100 turns on the core

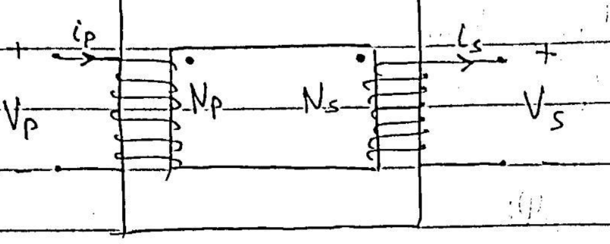
Ans:

$$e_{ind} = N\frac{d\phi}{dt} \rightarrow e_{ind} = 100\,[0.05 \cdot 377 \cos 377t]$$

$$= 188.5 \cos 377t \;\; V$$



ideal transformer:



$\rightarrow \frac{V_p}{V_s} = \frac{N_p}{N_s} = a$ [turns ratio]

$\rightarrow \frac{i_p}{i_s} = \frac{N_s}{N_p} = \frac{1}{a}$

Dot convention:

→ If i_p flows into the doted end and i_s flows out the dotted end.

→ if V_p is positive at the dotted end and V_s is positive at the dotted end.

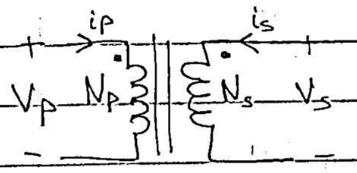
input power: $V_p I_p \cos(\theta_p)$ ∴ $\theta_p = \theta_{v_p} - \theta_{i_p}$

output power: $V_s I_s \cos(\theta_s)$ ∴ $\theta_s = \theta_{v_s} - \theta_{i_s}$

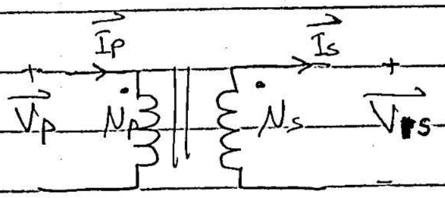
in ideal transformer: $\theta_p = \theta_s$

$P_{in} = P_{out}$.

Model of ideal transformer:



t-domain

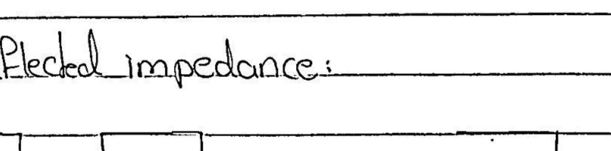


phasor-domain: نهتم بحل المقدار والزاوية

$x(t) = X_m \cos(\omega t + \phi)$

$\vec{x}(t) = X_m \angle \theta$ "steady-state".

Reflected impedance:



$\rightarrow Z_{Th} = a^2 Z_L$

Theory of operation of 1-Φ Real transformer :

The indused voltage is given by "Faraday's law" :

$e_{ind} = \frac{d\lambda}{dt}$: λ : linkage flux , $\lambda = \sum_{i}^{N} \phi_i$

The average flux $\bar{\Phi} = \frac{\sum_{i}^{N} \Phi_i}{N}$ $\rightarrow$ $N\bar{\Phi} = \sum_{i}^{N} \Phi_i$

induced voltage $e_{ind} = N \frac{d\bar{\Phi}}{dt}$.

The average flux $\bar{\Phi}$ has two components :-

$\bar{\Phi} = \Phi_M + \Phi_L$

= Mutual flux + leakage flux

Primary flux = $\bar{\Phi}_p = \Phi_M + \Phi_{LP}$

secondary flux = $\bar{\Phi}_s = \Phi_M + \Phi_{LS}$

By appling Farady's law we get :-

$$V_P = N_P \frac{d\bar{\Phi}_P}{dt} = N_P \frac{d\Phi_M}{dt} + N_P \frac{d\Phi_{LP}}{dt} = e_P + e_{PL}$$

$$V_S = N_S \frac{d\bar{\Phi}_S}{dt} = N_S \frac{d\Phi_M}{dt} + N_S \frac{d\Phi_{LS}}{dt} = e_S + e_{SL}$$

Now : $\boxed{\frac{e_P}{e_S} = \frac{N_P}{N_S}}$.

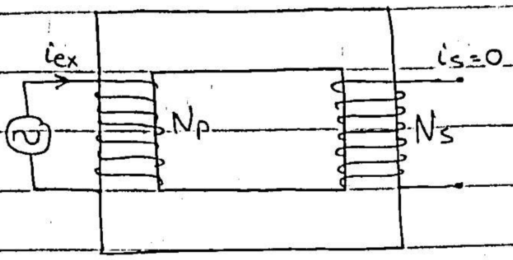
When design transformer :

$\rightarrow$ $\Phi_M \gg \Phi_{PL}$ & $\Phi_M \gg \Phi_{SL}$

So $\rightarrow$ $V_P \simeq e_P$ & $V_S \simeq e_S$

$\rightarrow$ $V_P / V_S = N_P / N_S$ close to ideal transformer.

The excitation current in real transformer:



unloaded [open circuit].

Assume, the primary is connected to an AC source, secondary is unloaded.

The primary unloaded current is called "excitation current" (i_{ex})
This current has two components:
1) Magnatizing current (i_m): it produces a flux in the core "Mutual flux"

2) Core losses current (i_{e+H}): it makes up the eddy current and Hystresis losses.

→ Magnatizing Current (i_m):
The primary voltage is given by:

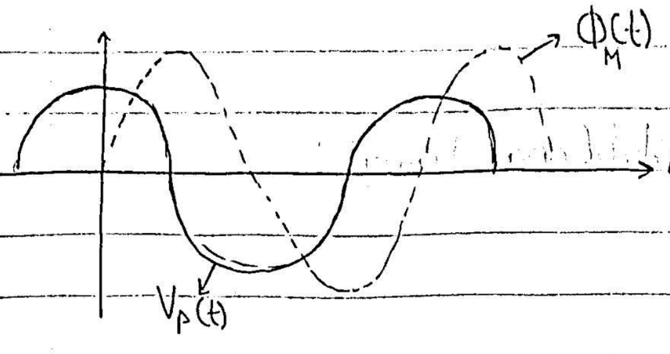
$$V_p = N_p \frac{d\Phi_p}{dt} \simeq N_p \frac{d\Phi_M}{dt}$$

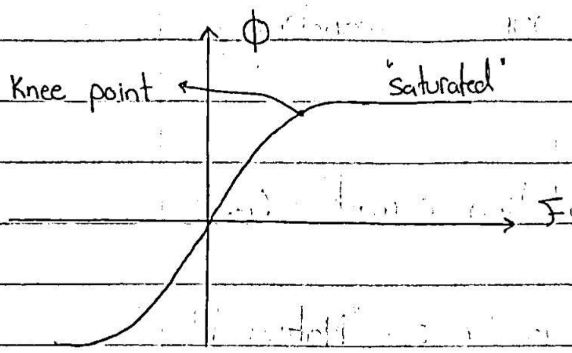
$$\Phi_M = \frac{1}{N_p} \int V_p \, dt$$

Assume that $V_p = V_m \cos(\omega t)$
Then $\Phi_M = \frac{1}{N_p} \frac{V_m}{\omega} \sin(\omega t)$

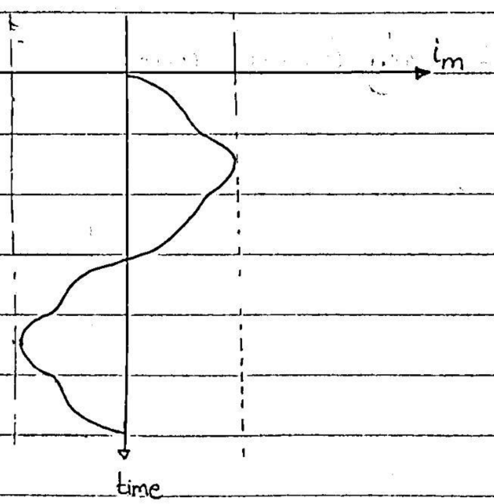
عشان تحافظ على ال flux لازم تحافظ على نسبة $\frac{V_m}{\omega}$

Charts :-





Note: see page (82)



Notes:

① The magnatizing current (i_m) in the transformer is not sinusoidal. The higher frequency components [the peaks] in the curve are due to magnetic saturation in the core.

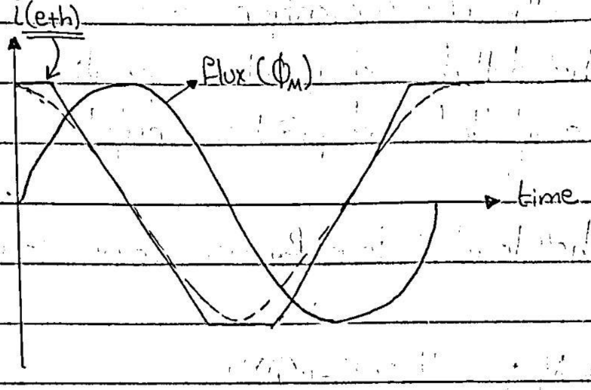
② When the flux reaches it's saturation level in the core, a small increase in the flux (Φ_M) requires a very large increase in the magnetizing current.

③ The fundamental component of the magnetizing current (i_M) lags the applied voltage (V_P) by 90°

④ The higher frequency components in the magnetizing current (i_m) can be large compared to the fundamental component

2) Core-loss current (i_{e+h}):

By assuming the flux in the core is sinusoidal. Since the eddy currents in the core are proportional to $d\Phi/dt$, then the eddy currents are largest when the flux in the core is passing through 0 Wb. Therefore, the core-loss current (i_{e+h}) is gratest when the flux passes through zero.



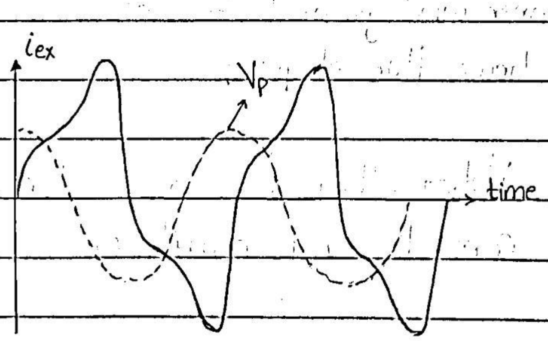
Notes:

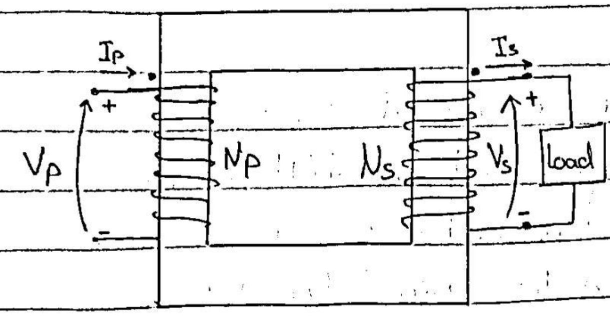
① i(e+h) is non-linear because of the non-linear effects of hysteresis.

② The fundamental component of the core-loss current (i(e+h)) is in phase with the voltage applied to the core (V_p).

Recalls: The total no-load current in the core is called the "excitation current"

$$i_{ex} = i_m + i_{(e+h)}$$





The current ratio on a transformer and the dot convention:

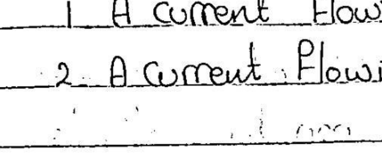
Assume that a load is connected to the secondary of the transformer.

The physical significance of the dot convention is:

1 A current flowing into the dotted end produces positive mmf.

2 A current flowing out the dotted end produces negative mmf.

The magnetic circuit of the loaded transformer core is:



$\therefore F_{net} = N_p i_p - N_s i_s = \phi R$

if $R = 0$ [very small (nearly zero)]

Then, $N_p i_p - N_s i_s \simeq 0 \rightarrow \dfrac{i_p}{i_s} = \dfrac{N_s}{N_p}$

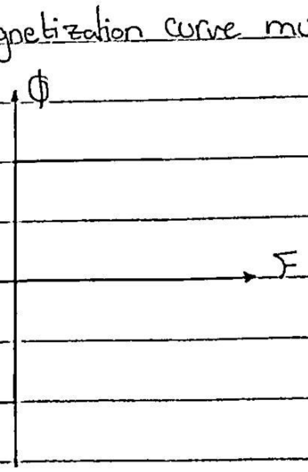
Assumptions to convert a real transformer to ideal transformer:

① core-loss current must be zero ($i_{(e+h)} = 0$).

② The leakage flux in the core must be zero, $\phi_{LP} = \phi_{LS} = 0$

③ The resistance of the transformer windings must be zero.

④ The magnetization curve must have this shape:



Notice that for an unsaturated core the net mmf = 0

→ The equivalent circuit of a transformer :

The losses that occur in real transformers :

1) Copper losses (I^2R) : 1) primary circuit → Resistance R_p

2) secondary circuit → Resistance R_s

2) leakage flux: These escaped fluxes produce a leakage inductance in the primary and secondary coils.

$\therefore V_p = e_{Lp} + e_p$ [due to leakage flux + due to mutual flux]

$e_{Lp} = N_p \frac{d\phi_{Lp}}{dt} = N_p \frac{d(N_p i_p)}{dt \, R_p} = \frac{N_p^2}{R_p} \frac{di_p}{dt}$

From this equation we can see that the leakage flux is represented by an inductor.

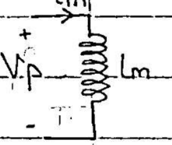
The same steps for $V_s = e_{Ls} + e_s$, we get ⇒ $e_{Ls} = \frac{N_s^2}{R_s} \frac{di_s}{dt}$

also we can represent it with an inductor

3) Magnetizing current (i_m): i_m is proportional to V_p,

i_m lags V_p by $90°$

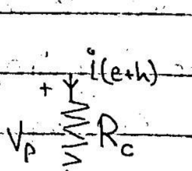
it's modeled by an inductance (L_m) [magnetizing inductance]



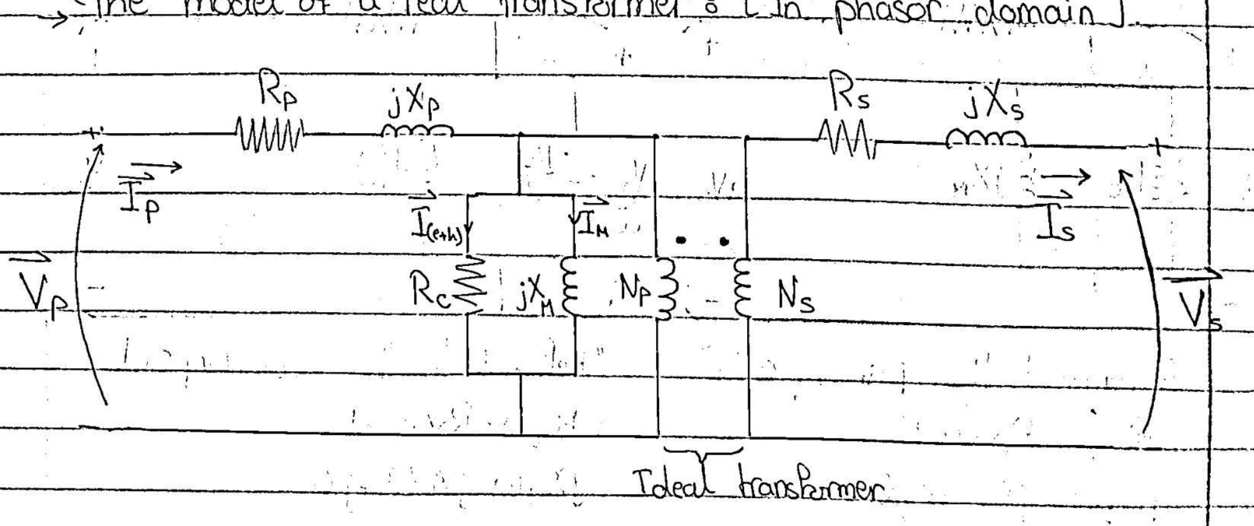
4) Core-loss current (i_{e+h}): is proportional to V_p

is in phase with V_p

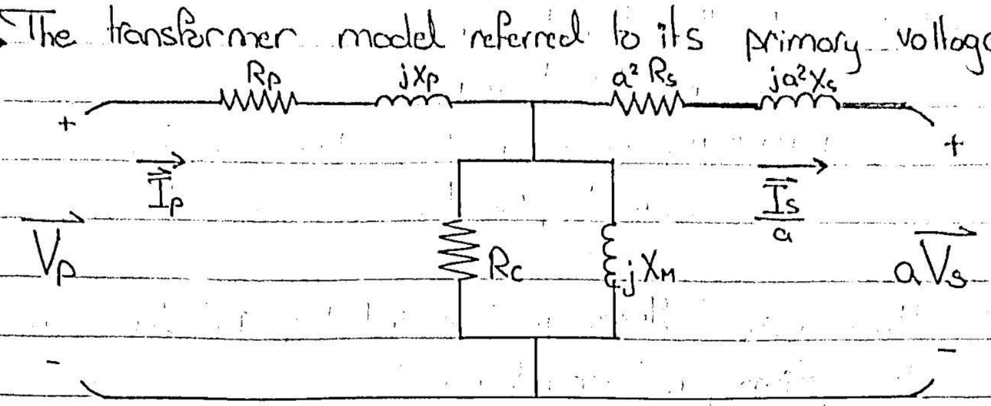
it's represented as a resistance : [core resistance]



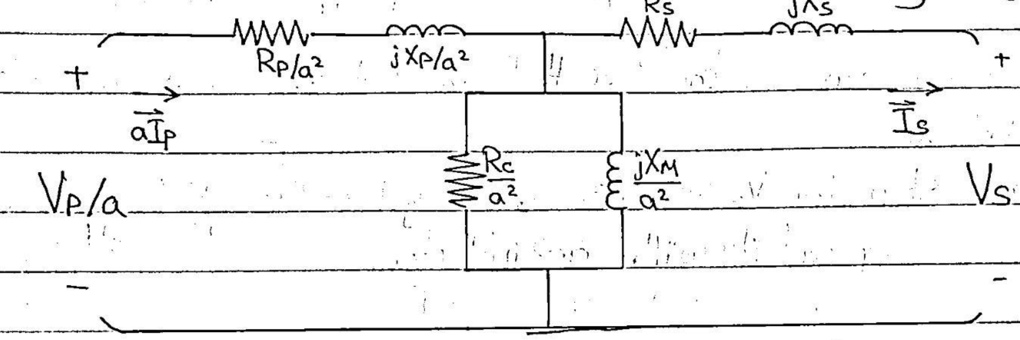
→ The model of a real transformer : [In phasor domain]



→ The transformer model referred to its primary voltage level:

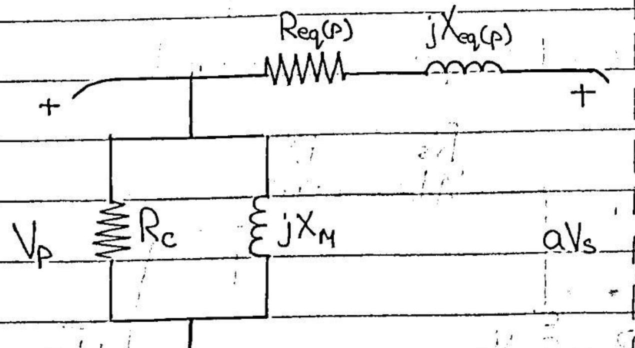


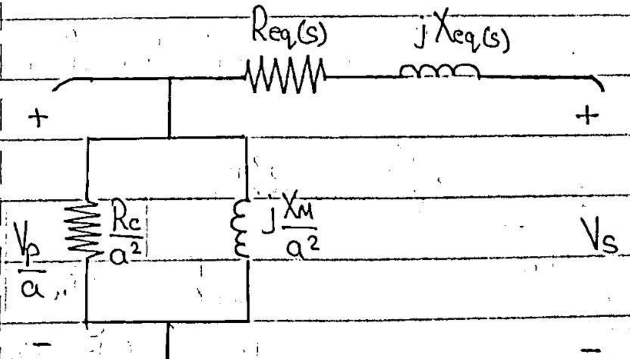
→ The transformer model referred to its secondary voltage level:



→ Approximate Equivalent circuits of a transformer:

The excitation branch has a very small current compared to the load current of the transformer. The excitation branch is simply moved to the front of the transformer and the primary and secondary impedances are left in series with other. This will leave us creating the approximate equivalent circuit showing below:





"Referred to the primary side"

$\therefore R_{eq(p)} = R_p + a^2R_s$

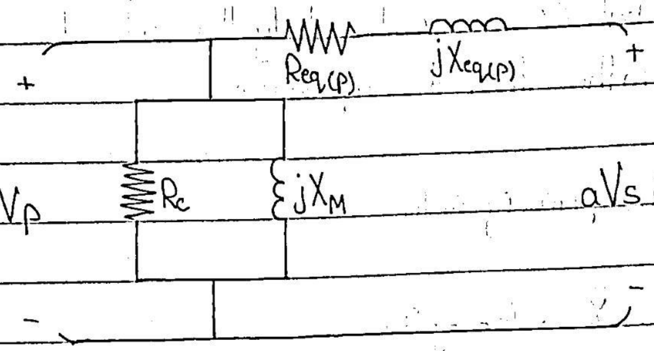
$jX_{eq(p)} = jX_p + a^2jX_s$

"Referred to the secondary side"

$\therefore R_{eq(s)} = R_p/a^2 + R_s$

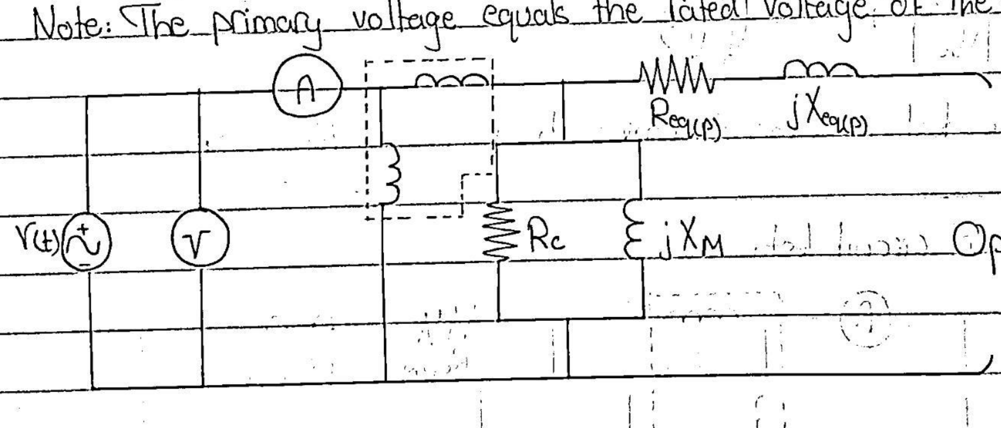
$jX_{eq(s)} = jX_p/a^2 + jX_s$

→ Determining the values of components in the transformer Model:
[The parameter of the transformer].



An adequate approximation of [R_c, jX_M, $R_{eq(p)}$ & $jX_{eq(p)}$] can be obtained with only two tests, the open circuit and the short circuit:

1) The open circuit test:
→ The secondary is opened, the primary is connected to full rated line voltage [Ac voltage source].
Note: The primary voltage equals the rated voltage of the transformer



→ The series elements R_p & X_p are too small in comparison to R_c and X_M to cause a significant voltage drop, so all the input voltage is dropped across the excitation branch.

→ After a full line voltage is applied to one side of the transformer we can measure (read) the input voltage (V_{oc}), the input current (I_{oc}) and the input power (P_{oc}).

→ The impedance seen by the primary side:

$\overrightarrow{Z_{oc}} = \dfrac{\overrightarrow{V_{oc}}}{\overrightarrow{I_{oc}}} = R_c \,//\, jX_M$, but it's easier to study the addmitance rather than the impedance.

→ The addmitance seen by the primary side:

$\overrightarrow{Y_{oc}} = \dfrac{1}{\overrightarrow{Z_{oc}}} = \dfrac{1}{R_c} + \dfrac{1}{jX_M} \Rightarrow \overrightarrow{Y_{oc}} = G_c - jB_M$

∴ $G_c = \dfrac{1}{R_c}$ & $B_M = \dfrac{1}{X_M}$ [The conductance and susceptance].

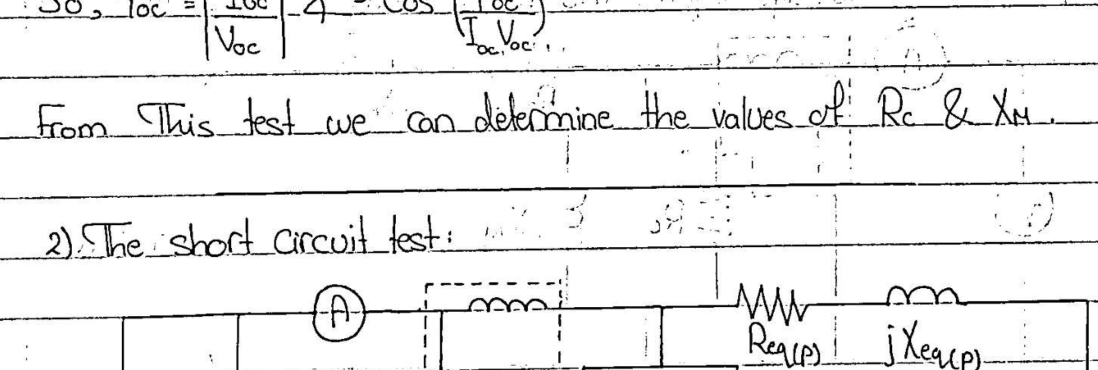
$\overrightarrow{Y_{oc}} = \dfrac{\overrightarrow{I_{oc}}}{\overrightarrow{V_{oc}}} = \left|\dfrac{I_{oc}}{V_{oc}}\right| \angle (\theta_{ioc} - \theta_{voc}) = \left|\dfrac{I_{oc}}{V_{oc}}\right| \angle -(\theta_{voc} - \theta_{ioc})$

we know that the $PF = \cos(\theta_{voc} - \theta_{ioc}) = \dfrac{P_{oc}}{V_{oc} I_{oc}}$

So, $\overrightarrow{Y_{oc}} = \left|\dfrac{I_{oc}}{V_{oc}}\right| \angle -\cos^{-1}\left(\dfrac{P_{oc}}{I_{oc} V_{oc}}\right)$

From This test we can determine the values of R_c & X_M.

2) The short circuit test:



→ The secondary is shorted, and the input AC voltage source is set initially to zero then it's increased gradually until the current in the short circuit is equal to it's rated value.
Note: Be sure to keep the primary voltage at a safe level. If not you will burn the transformer's windings while you test it.

Since the input voltage is low then the load current $\gg i_{ex}$. Then all the voltage drop in the transformer can be attributed to the series elements [$R_{eq(P)}$ & $X_{eq(P)}$].

→ The impedance seen by the primary side:

$$\vec{Z}_{SC} = \frac{\vec{V}_{SC}}{\vec{I}_{SC}} = \left[(R_{eq(P)} + jX_{eq(P)}) \,//\, (R_C \,//\, jX_M)\right] \simeq R_{eq(P)} + jX_{eq(P)}$$

$$\vec{Z}_{SC} = \frac{\vec{V}_{SC}}{\vec{I}_{SC}} = \left|\frac{V_{SC}}{I_{SC}}\right| \measuredangle (\theta_{VSC} - \theta_{ISC}) \quad \therefore PF = \cos(\theta_{VSC} - \theta_{ISC})$$

$$\vec{Z}_{SC} = \left|\frac{V_{SC}}{I_{SC}}\right| \measuredangle \cos^{-1}\left(\frac{P_{SC}}{V_{SC} I_{SC}}\right)$$

→ From this test we can determine the values of $R_{eq(P)}$ & $X_{eq(P)}$.

* See example 2-2 page 92-93. [4th & 5th]

→ The per-unit system (PU):

In this system each electrical quantity is not measured in it's SI unit, but it is measured in decimal fraction of some base value.

The per-unit quantity $= \dfrac{\text{Actual quantity}}{\text{base quantity}}$

Voltage (V), Current (I), Power (Q, P, $\underline{S}$), impedance (X, R, $\underline{Z}$)

→ S_b and V_b are selected.
Z_b and I_b are calculated.

$$I_b(\text{magnitude}) = \frac{S_b}{V_b} \quad \& \quad Z_b = \frac{V_b}{I_b} \text{ or } \frac{V_b^2}{S_b}$$

∴ S_b [base value of Power, the base value of P, S, Q].
Z_b [base value of impedance, the base value of X, R, Z]
V_b [base value of voltage].
I_b [base value of current]

Note: In Power systems, S_b is always constant [since power in equals power out], and V_b changes according to transformer turns ratio.

→ Converge from one base to another base:

① $(P,Q,S)_{actual} = (P,Q,S)_{PU1} \cdot S_{b_1} = (P,Q,S)_{PU_2} \cdot S_{b_2}$

$$\Rightarrow (S,P,Q)_{PU2} = (S,P,Q)_{PU1} \cdot \frac{S_{b1}}{S_{b2}}$$

② $V_{actual} = V_{PU1} \cdot V_{b1} = V_{PU2} \cdot V_{b2}$

$$\Rightarrow V_{PU2} = V_{PU1} \cdot \frac{V_{b1}}{V_{b2}}$$

③ $I_{actual} = I_{PU1} \cdot I_{b1} = I_{PU2} \cdot I_{b2} \rightarrow I_{PU_2} = I_{PU1} \cdot \frac{I_{b1}}{I_{b2}}$

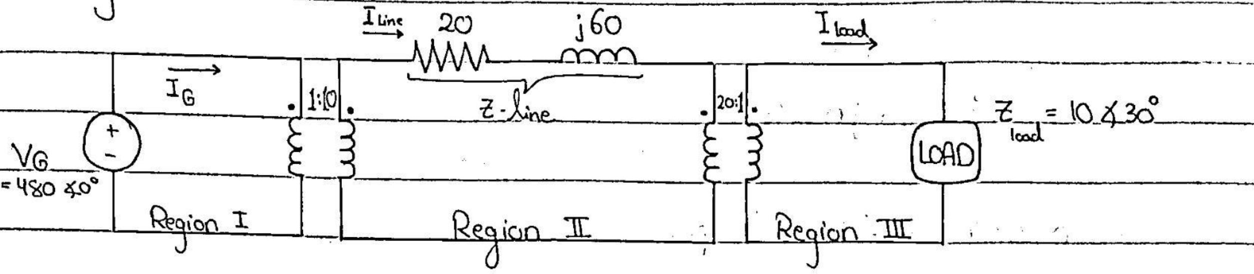
$$\Rightarrow I_{PU2} = I_{PU1} \cdot \frac{S_{b1}}{S_{b2}} \cdot \frac{V_{b2}}{V_{b1}}$$

④ $(R,X,Z)_{actual} = (R,X,Z)_{PU1} \cdot Z_{b1} = (R,X,Z)_{PU2} \cdot Z_{b2}$

$\rightarrow (R,X,Z)_{PU2} = (R,X,Z)_{PU1} \cdot \frac{Z_{b1}}{Z_{b2}} =$

$$\Rightarrow (R,X,Z)_{PU2} = (R,X,Z)_{PU1} \cdot \left(\frac{V_{b1}}{V_{b2}}\right)^2 \cdot \frac{S_{b2}}{S_{b1}}$$

Example: A simple power system contains a 480 V generator connected to an ideal 1:10 step-up transformer, a transmission line, an ideal 20:1 step-down transformer and a load. The impedance of the transmission line is $20+60j\ \Omega$ and the impedance of the load is $10\angle 30°\ \Omega$. The base values for this system are chosen to be 480 V and 10KVA at the generator:



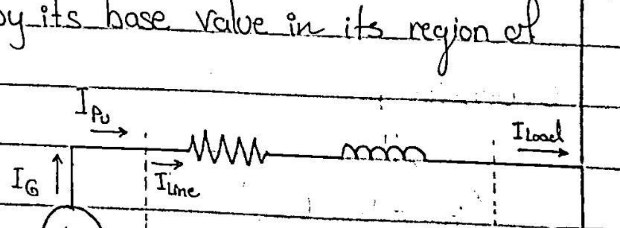
a) Find the base Voltage, current, impedance and apparent power at every point in the power system:

Region I	Region II	Region III
1) $S_b = 10{,}000$ VA	1) $S_b = 10{,}000$ VA	1) $S_b = 10{,}000$ VA.
2) $V_b = 480$ V	2) $V_b = 4800$ [step-up]	2) $V_b = 240$ [step-down]
3) $I_b = \frac{10{,}000}{480} = 20.83$ A	3) $I_b = \frac{10{,}000}{4800} = 2.083$ A	3) $I_b = \frac{10{,}000}{240} = 41.67$
4) $Z_b = \frac{480}{20.83} = 23.04\ \Omega$	4) $Z_b = \frac{4800}{2.083} = 2304\ \Omega$	4) $Z_b = \frac{240}{41.67} = 5.76\ \Omega$

b) Convert this system to its per-unit equivalent circuit:

→ Each component must be divided by its base value in its region of the system:

1) $V_{G_{pu}} = \frac{480\angle 0°}{480} = 1\ pu\ \angle 0°$



2) $Z_{Line_{pu}} = \frac{20+j60}{2304} = 0.0087 + j0.026\ pu$

3) $Z_{load_{pu}} = \frac{10\angle 30°}{5.76} = 1.736\angle 30°\ pu$

Note: $I_{G_{pu}} = I_{Line_{pu}} = I_{load_{pu}} = I_{pu}$

c) Find the power supplied to the load:

⚠ The power only dissipated in the resistance ⚠

→ The current flowing in this pu system is:

$$I_{pu} = \frac{V_{pu}}{Z_{pu}} = \frac{1 \angle 0^\circ}{(1.736 \angle 30^\circ) + 0.0087 + j0.026}$$

$$= \frac{1 \angle 0^\circ}{1.512 + j0.894}$$

$$= 0.569 \angle -30.6^\circ \text{ pu}$$

The per-unit power ⇒ $P_{load\,pu} = (I_{pu})^2 \cdot R_{load\,pu}$

$= (0.569)^2 \, (1.736 \cos(30))$

$= 0.487$ pu

The power in Watt ⇒ $P_{load} = P_{load\,pu} \cdot S_b$

$= 0.487 \cdot 10{,}000$

$= 4870$ watt.

d) Find the power lost in the transmission line:

$P_{line\,pu} = (I_{pu})^2 \cdot R_{line\,pu}$

$= (0.569)^2 \cdot 0.0087$

$= 0.00282$ pu

The power in watt:

$P_{line} = P_{line\,pu}$ "Power base (S_b)"

$= 0.00282 \cdot 10{,}000$

$= 28.2$ watt.

Note: We can find the generated power by:

① $P_{line} + P_{load}$.

② $P = VI \cos(\theta_v - \theta_i)$

→ The voltage Regulation of transformer:
it's given by:

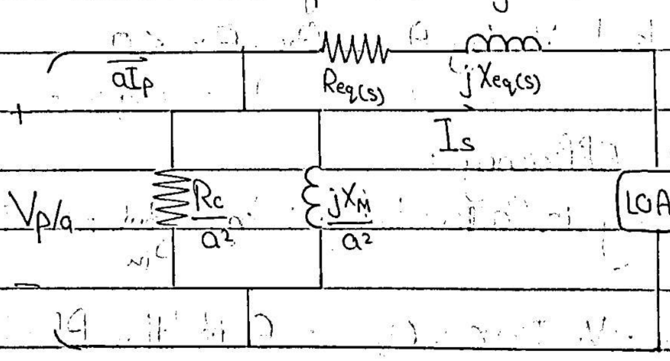
$$VR = \frac{V_{s,nl} - V_{s,fl}}{V_{s,fl}} \times 100\% \quad \rightarrow \text{but } V_{s,nl} = V_p/a \rightarrow \frac{V_p/a - V_{s,fl}}{V_{s,fl}} \times 100\%$$

∴ $V_{s,nl}$: is the output voltage at no load, [No current is used].
$V_{s,fl}$: is the output voltage at full load, [rated current is used].

For the ideal transformer $VR = 0\%$

→ The Transformer phasor Diagram:
To determine the voltage regulations of a transformer, it is necessary to understand the voltage drop within it.
The easiest way to determine the effect of the impedances and the current phase angles is to examine a phasor diagram.

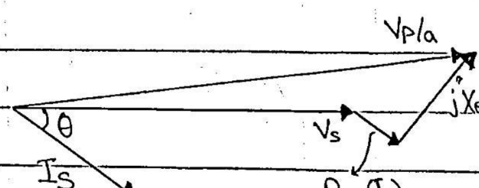
Applying KVL to find the primary voltage:



$$\text{"}\frac{V_P}{a} = V_s + R_{eq} I_s + j X_{eq} I_s\text{"}$$

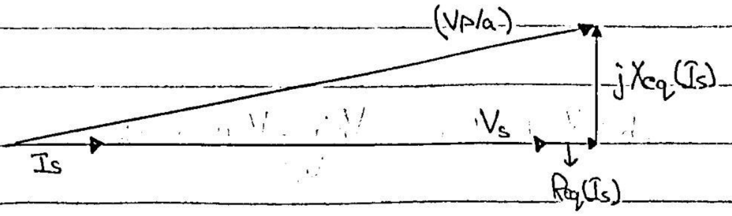
The phasor voltage V_s is assumed to be at angle 0°.
→ A Transformer phasor diagram is a visual representation of the KVL equation.

[1] A Transformer operating at a lagging power factor:



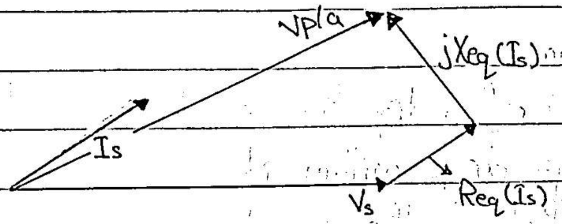
1) $V_p/a > V_s$ → VR is positive ≥ 0
2) losses are small → R is small
So, $R_{eq}(\vec{I_s})$ is smaller than $\vec{I_s}$.
3) i_s lags V_s by θ
$\theta = \theta_{V_s} - \theta_{i_s} > 0$

[2] The unity power factor: $\theta_{vs} = \theta_{is} \rightarrow \theta = 0$



① $V_p/a > V_s \rightarrow VR$ is positive >0
② VR is smaller than VR in the lagging case.

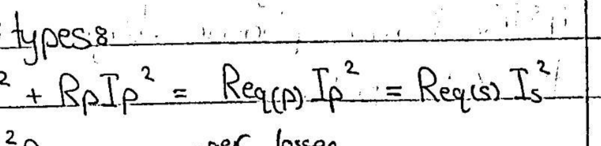
[3] The leading power factor:



① $V_p/a < V_s \rightarrow V_s$ is negative
② is leads Vs by $\theta \rightarrow \theta_{vs} - \theta_{is} < 0$.

→ Transformer Efficiency:
The efficiency is calculated as: $\eta = \frac{P_{out}}{P_{in}} \times 100\%$

where: $P_{out} = V_s I_s \cos\theta_s$ ∴ θ is the PF angle
$P_{in} = P_{losses} + P_{out}$
∴ P_{losses} are presented in three types:
1) Copper losses $(I^2R) \Rightarrow R_s I_s^2 + R_p I_p^2 = R_{eq(p)} I_p^2 = R_{eq(s)} I_s^2$
2) Hysteresis losses
3) Eddy current losses $\} \rightarrow [i_{(e+h)}]^2 R_c$



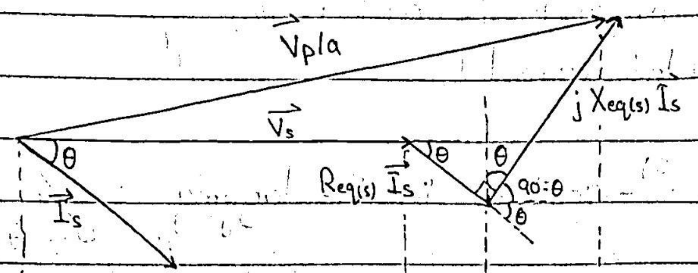
Then the efficiency can be expressed as:

$$\eta = \frac{V_s I_s \cos\theta_s}{P_{copper} + P_{core} + V_s I_s \cos\theta_s} \times 100\%$$

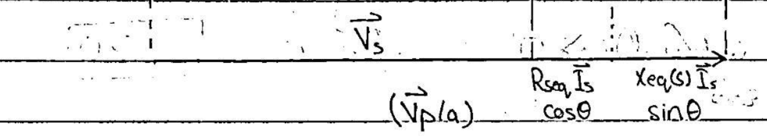
Simplified VR calculations:

For lagging PF operation, the vertical components of $R_{s\,eq}$ + $X_{s\,eq}$ will partially cancel each other. Therefore, the angle of $\vec{V_p}/a$ will small

* $\vec{V_p}/a$ can be assumed horizontal



→ $\vec{V_p}/a$ can be replaced by a horizontal vector:



since the angle between $\vec{V_s}$ & $\vec{V_p}/a$ is too small

$$\left[\ \frac{V_p}{a} = V_s + R_{eq(s)} I_s \cos\theta + X_{eq(s)} I_s \sin\theta\ \right] \text{as magnitudes.}$$

* See example 2-5 page 103 4th & page 102 5th

How to think:

① Note: Notice the data, is it taken from the primary or the secondary?

② To find the equivalent circuit: i- Find Y_{oc} from the open circuit test.
ii- Find Z_{sc} from the short circuit test.

since the data is on the primary we can draw the equivalent circuit refered to the primary

we can simply find the equivalent refered to the secondary [low voltage] by changing: ① $V_p \to V_p/a$, $Z_p \to Z_p/a^2$, $I_p \to aI_p$.

② $aV_s \to V_s$, $I_s/a \to I_s$.

③ In VR calculations:

① write the equation: $\vec{V}_{P/a} = (R_{s\,eq} + jX_{s\,eq})\vec{I}_s + \vec{V}_s$

② you have $(R_{eq(s)}$ & $jX_{eq(s)})$, $(V_{s\,FL})$ so you need to find only $\underbrace{\vec{I}_s}$:-

↳ it has a magnitude & angle:

Magnitude: $\frac{S_{rated}}{V_{rated}}$ [rated current].

To find the angle: a) $-\cos^{-1}(PF)$ in the lagging ∴ $\cancel{\theta_v} - \theta_i > 0$

$0 - \theta_i > 0 \rightarrow \theta_i > \theta$

$\boxed{\theta_i < 0}$

b) $\cos^{-1}(PF)$ in the leading

∴ $\underbrace{\cancel{\theta_v}}_{Zero} - \theta_i < 0 \rightarrow -\theta_i < 0 \rightarrow \boxed{\theta_i > 0}$

③ $VR = \frac{V_{P/a} - V_{S,F}}{V_{S,F}} \times 100\%$

Notes:- in the unity PF $\rightarrow \boxed{\theta = 0} \rightarrow \theta_v - \theta_i = 0 \rightarrow \theta_i = 0$

* We can find $V_{p/a}$ in the lagging part in the simplified calculations:

$|V_p/a| = R_{eq(s)}|I_s|\cos(\theta) + X_{eq(s)}|I_s|\sin(\theta) + |V_s|$

④ The efficiency: $\eta = \frac{P_{out}}{P_{out} + P_{co} + P_{core}}$, find all these powers.

→ The problem of the Magnetic Inrush current:

This problem is related to the voltage level in the transformer at the starting.

Inrush current: It's a huge current that will flow in the primary windings when the secondary is first connected to the power line. At this moment the primary voltage is passing through zero.

Assume that the primary voltage $V_p(t) = V_m \sin(\omega t + \theta)$ volt, and by assuming (ω) the frequency is constant then by "Farady's Law":

$$V_p(t) = N_p \frac{d\Phi}{dt}$$

Then the flux in the core $\Phi(t)$ is:

$$\Phi(t) = \frac{1}{N_p} \int V_p(t)\, dt$$

$$= \frac{1}{N_p} \int V_m \sin(\omega t + \theta)\, dt$$

$$= \frac{-V_m}{\omega N_p} \cos(\omega t + \theta) + A \qquad \therefore \frac{V_m}{\omega N_p} \text{ is the max flux } \Phi_m$$

$$\Phi(t) = -\Phi_m \cos(\omega t + \theta) + A$$

At the starting moment ($t = 0$), there is a flux lefted in the core:

$$\Phi(0) = \Phi_R \text{ [Residual flux]} = -\Phi_m \cos(\theta) + A$$

So, $A = \Phi_R + \Phi_m \cos(\theta)$

Then, $\Phi(t) = -\Phi_m \cos(\omega t + \theta) + \Phi_R + \Phi_m \cos(\theta)$

Assuming that $\phi_R = 0$, Then the flux is given by:

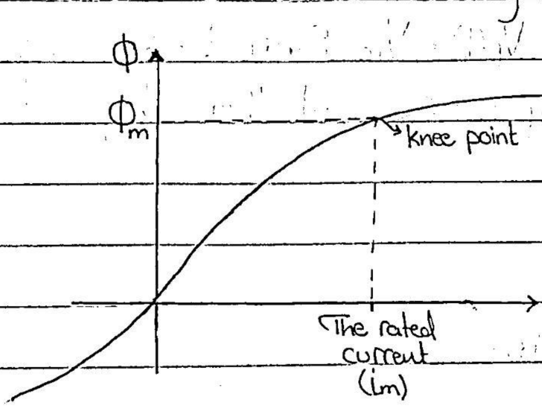
$$\phi(t) = -\phi_m \cos(\omega t + \theta) + \phi_m \cos(\theta).$$

Another Assumption that $\theta = 0°$ & $\omega t = \pi$, then the flux is:

$$\phi(t) = -\phi_m \cos(\pi) + \phi_m \rightarrow \phi(t) = 2\phi_m \quad ; t = \frac{\pi}{\omega} = \frac{1}{2f}$$

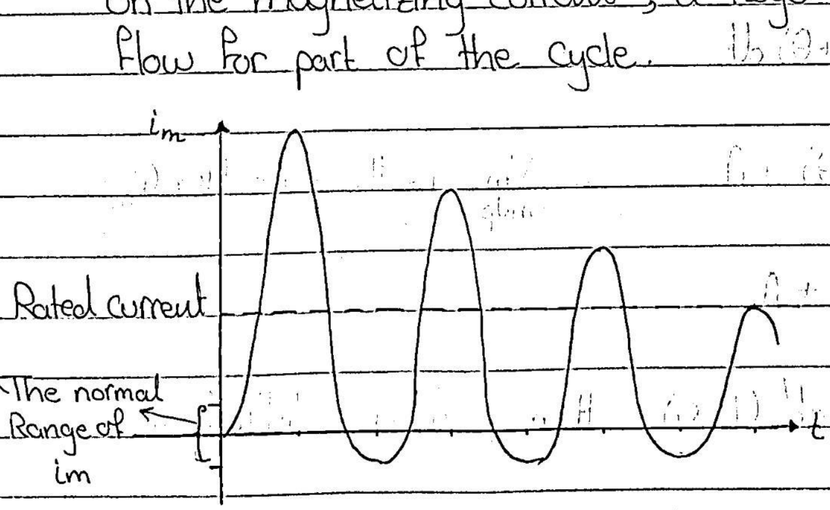
→ The worst case: $[\theta = 0 \ \& \ \omega t = \pi \rightarrow t = \frac{1}{2f}]$

In this case the maximum value of the flux will be twice the normal steady-state peak value of the flux.



From the curve it is easy to notice that doubling ϕ_m will result in an enormous magnetization current.

So, since the core is designed to operate at the knee point on the magnetizing current, a huge starting current will flow for part of the cycle.



→ The transformer must be able to withstand the starting inrush current.

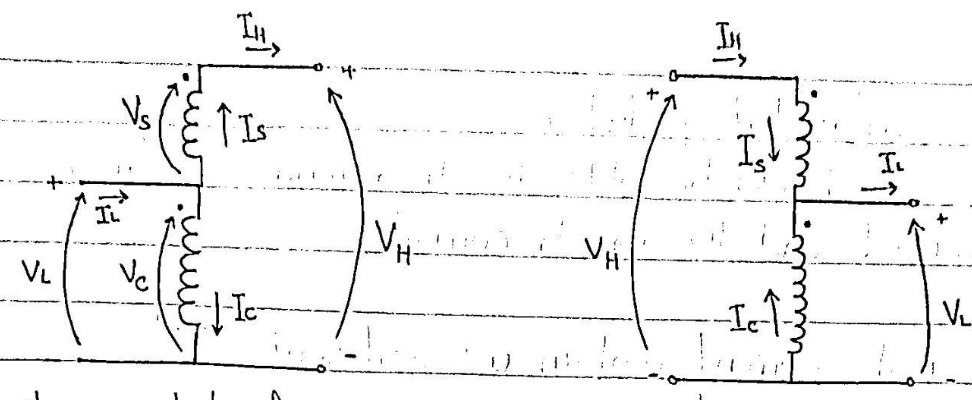
→ Transformer differential protection:
The transformer should be able to discriminate between the fault current and the inrush current.

Note: The inrush current contain a significant 2nd harmonic component.
There are two ways to avoid this:
1) The 2nd harmonic component protection
2) Gap protection technique.

→ The Autotransformer:
Some-times it's needed to change voltage levels by a small amount, such as, increasing the voltage drops that occur in power systems a long way from the generator, but it's expensive to wind a two windings transformer So, a special purpose transformer called "Autotransformer" is used.

→ It converts a fixed AC voltage to variable AC voltage:
It consists of two windings:
a) The common winding:
i. it's common between input and output parts
ii. it has Nc turns, Vc voltage, Ic current

b) The series winding:
i. it's connected in series with the common coil & smaller
ii. it has Ns turns, Vs voltage, Is current.



step-up auto transformer

step-down transformer

From the step-up autotransformer figure:

① $V_S = N_S \dfrac{d\Phi}{dt}$ & $V_C = N_C \dfrac{d\Phi}{dt}$

$\rightarrow \dfrac{V_C}{V_S} = \dfrac{N_C}{N_S}$ ①

$\left.\begin{aligned} * \; V_H &= V_C + V_S \\ V_L &= V_C \end{aligned}\right\} \rightarrow \dfrac{V_H}{V_L} = \dfrac{V_S}{V_C} + 1$ OR $\dfrac{V_L}{V_H} = \dfrac{1}{\frac{V_S}{V_C} + 1}$

* in terms of the turns:

$\dfrac{V_L}{V_H} = \dfrac{N_C}{N_S + N_C}$ ∴ Note V_H is Fixed.

$\rightarrow N_C I_C = N_S I_S$ ②

$\left.\begin{aligned} * \; I_L &= I_C + I_S \\ I_H &= I_S \end{aligned}\right\} \rightarrow \dfrac{I_L}{I_H} = \dfrac{I_C}{I_S} + 1$

* in terms of the turns:

$\dfrac{I_L}{I_H} = \dfrac{N_C + N_S}{N_C}$

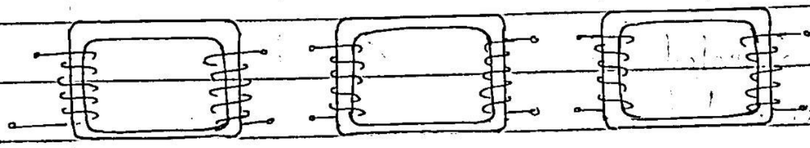
② Apparent power

The input apparent power [low voltage side] equals the output apparent power [high voltage side]

$$S_{in} = S_{out}$$

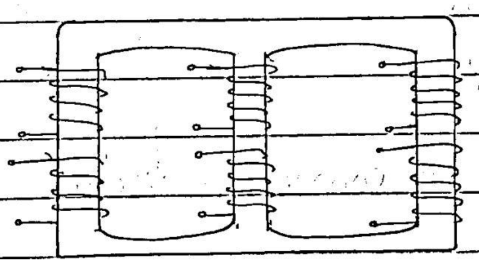
$$V_L I_L = V_H I_H$$

→ Three-phase transformers:
Transformers for three-phase circuits can be constructed in two ways:
① A Bank composed of three single-phase transformers.
This type has an advantage in:
→ That each unit in the bank could be replaced individually in the event of trouble
And has some disadvantages in:
→ That it's heavy and expensive.



phase (a) phase (b) phase (c)

② A common ferromagnatic core with 3-Φ transformers windings wrapped on it:
This type has some advantages in:
→ That it's smaller, cheaper, lighter and more efficient
And has some disadvantages in:
→ That any problem in the core will affect the total work of the transformer.



phase(a) phase(b) phase(c)

→ There are four possible connections for a 3-Φ transformer:
① Wye - Wye [Y-Y]
② Wye - delta [Y-Δ]
③ delta - Wye [Δ-Y]
④ delta - delta [Δ-Δ].

→ Winding connection and Vector Diagram:

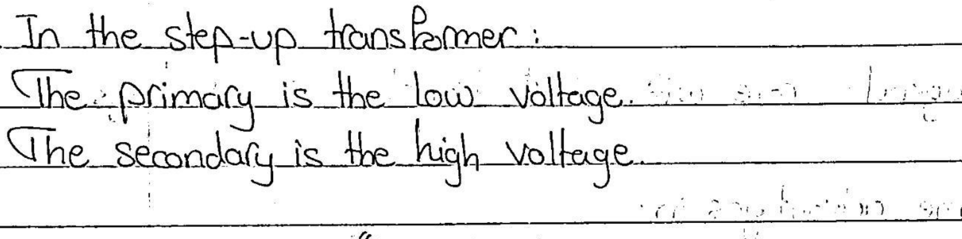
Example: Yd 11 : Y is the connection of the high voltage side.
d is the connection of the low voltage side.
11 is the rotation from Y to d in hours.

Notes: Y: High voltage side Y-connected
D: High voltage side Δ-connected

y: low voltage side Y-connected
d: low voltage side Δ-connected

→ In the step-up transformer:
The primary is the low voltage
The secondary is the high voltage

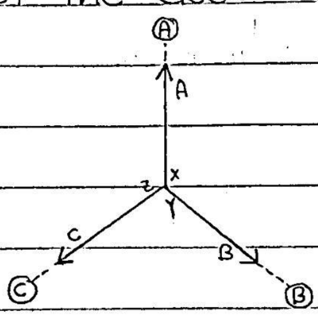
Again looking into "Yd 11" example:



Y-connected "high voltage"   Δ-connected "low voltage"

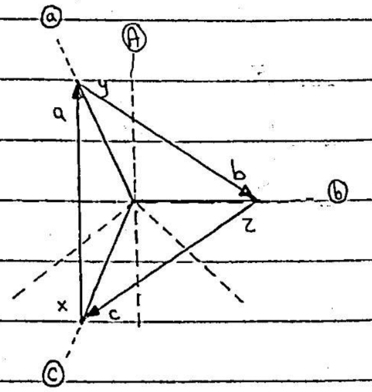
The question is: How can we determine the Δ-connection vectors or directions?

① Draw the Y-connected side as the reference, let it take the shape of the clock [12 hours]



Rotate the abc sequence clock wise.

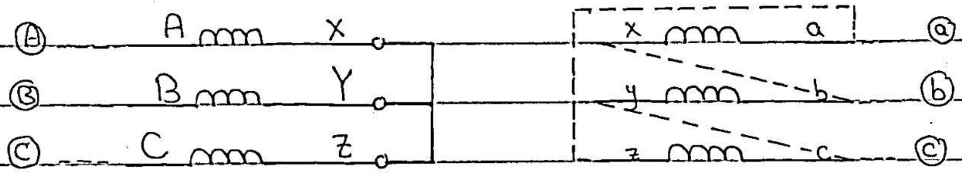
② Draw the delta connection, be aware of the rotation ⚠
Here it is 11 hours, which mean that you should rotate the high voltage diagram 11 hours to get the starting point of the low voltage diagram.



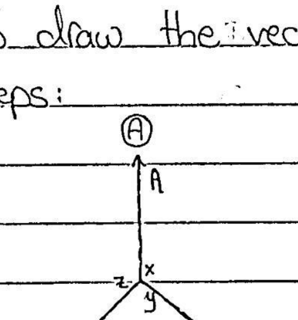
studying this diagram leads to:
① The $\overrightarrow{xa}$ vector is in the direction of the $\overrightarrow{XA}$ vector in the Y-connection
② The rotation has been done clockwise
③ (a) & (y) are both connected at ⓐ terminal
(b) & (z) " connected at ⓑ terminal
(c) & (x) are connected at ⓒ terminal

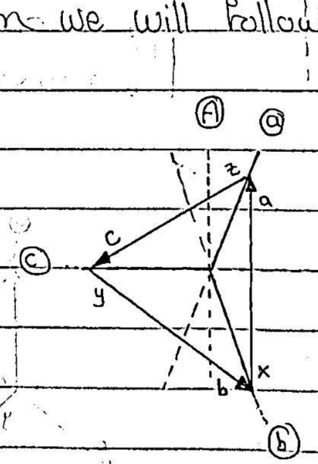
these three connections have determined the Δ-connection shape.

Another example: "Yd1"



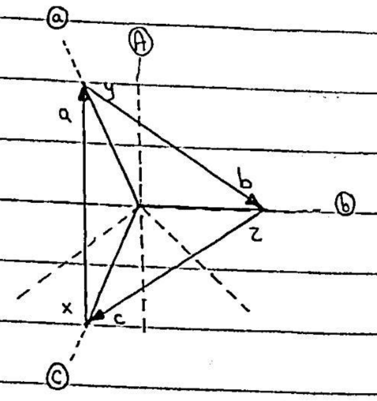
To draw the vector diagram we will follow the previous steps:





Note: ① in the Δ-connection it's not the vector's direction which is in the clockwise rotation, but it's abc sequence
② The direction of $\overrightarrow{XA}$ is the same as $\overrightarrow{xa}$
" " " $\overrightarrow{YB}$ is the same as $\overrightarrow{yb}$
" " " $\overrightarrow{ZC}$ is the same as $\overrightarrow{zc}$

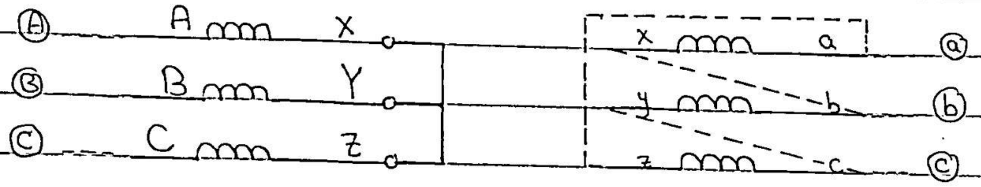
② Draw the delta connection, be aware of the rotation ⚠
Here it is 11 hours, which mean that you should rotate the high voltage diagram 11 hours to get the starting point of the low voltage diagram.



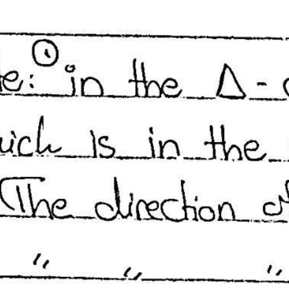
studying this diagram leads to:
① The $\vec{xa}$ vector is in the direction of the $\vec{XA}$ vector in the Y-connection.
② The rotation has been done clockwise
③ (a) & (y) are both connected at ⓐ terminal
(b) & (z) " connected at ⓑ terminal
(c) & (x) are connected at ⓒ terminal

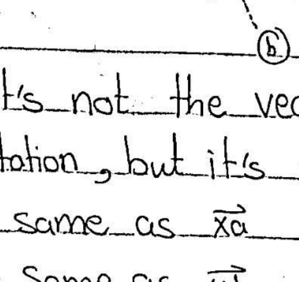
these three connections have determined the Δ-connection shape.

Another example: "Yd 1"



To draw the vector diagram we will follow the previous steps:

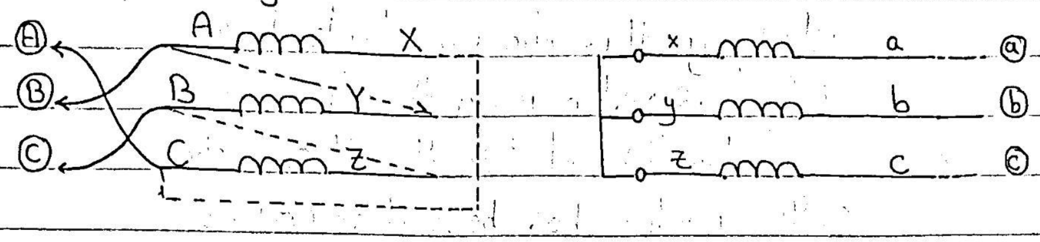




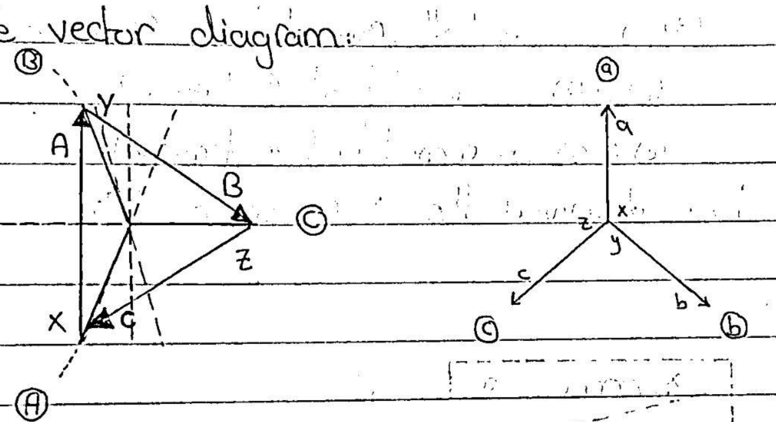
Note: ① in the Δ-connection it's not the vector's direction which is in the clockwise rotation, but it's abc sequence
② The direction of $\vec{XA}$ is the same as $\vec{xa}$
" " " $\vec{YB}$ is the same as $\vec{yb}$
" " " $\vec{ZC}$ is the same as $\vec{zc}$

The previous two examples, the high voltage was connected as Y-connection, in the next two examples the high voltage will be connected as Δ-connection!

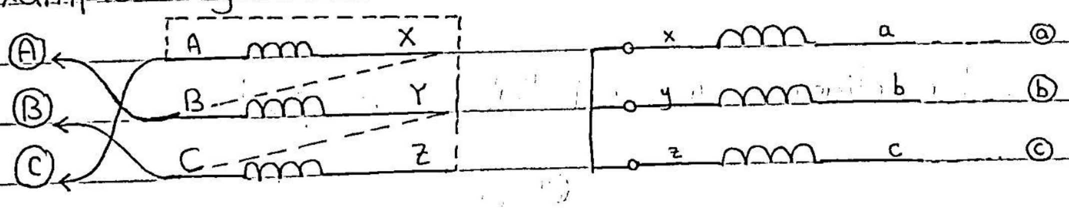
Example: "Dy 5"



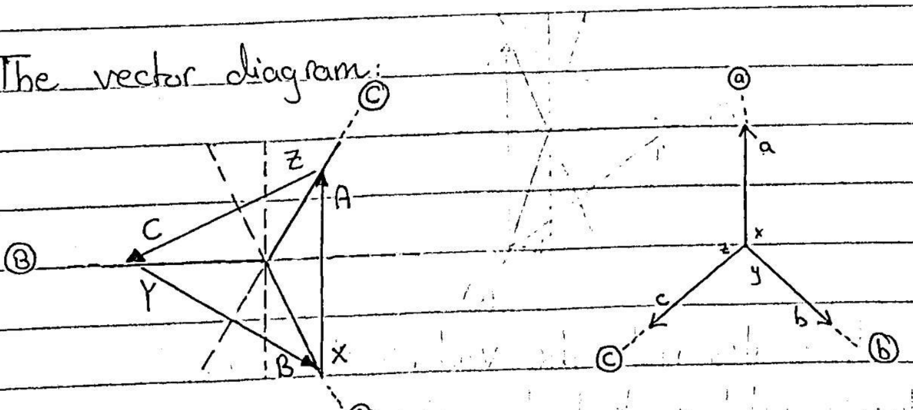
Now the vector diagram:



Example: "Dy 7"

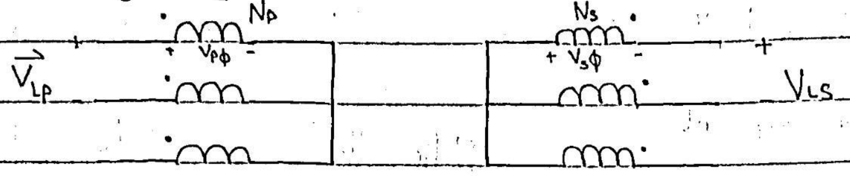


The vector diagram:



Winding connection in 3-ϕ transformer.

1 Wye-Wye connection (Y-Y):

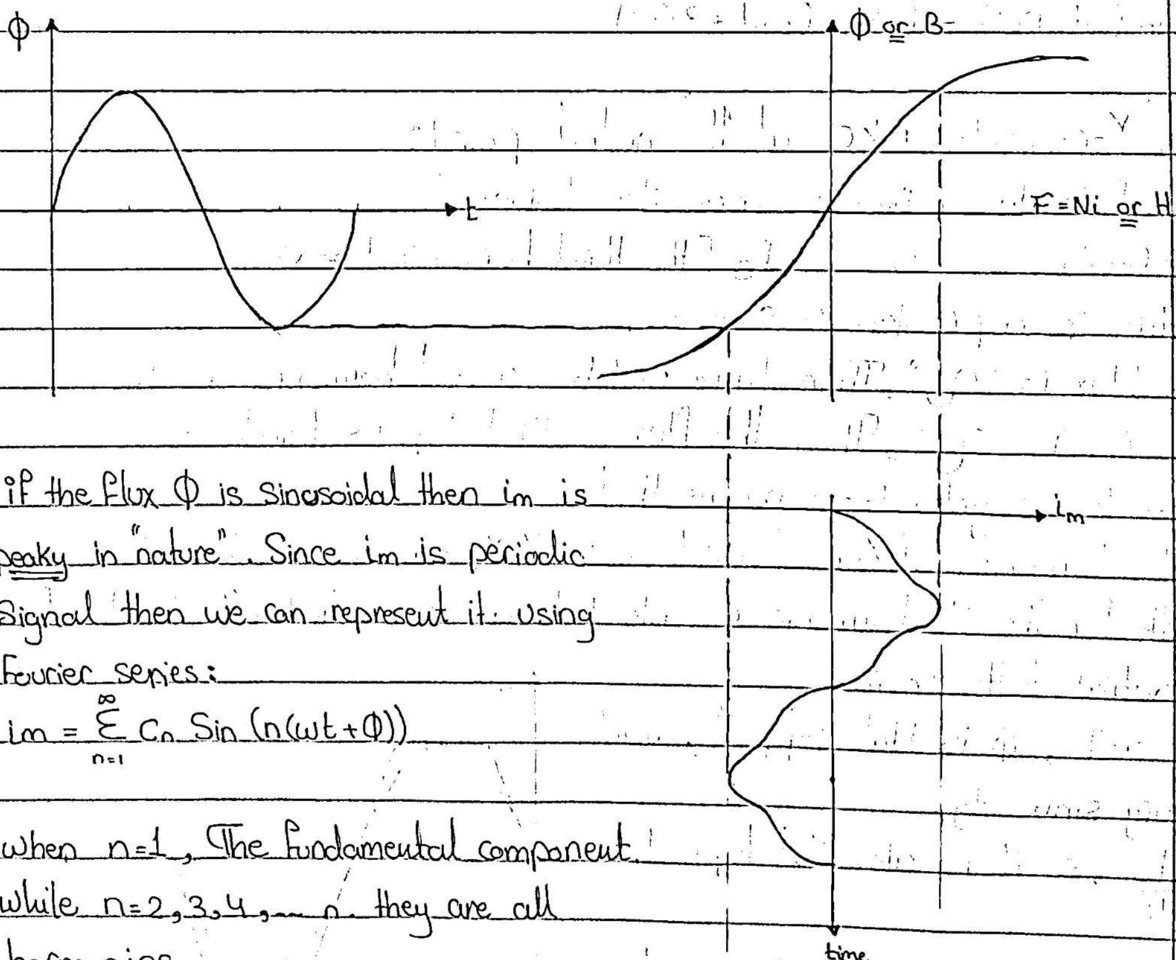


$$\frac{\vec{V}_{LP}}{\vec{V}_{LS}} = \frac{\vec{V}_{\phi P}}{\vec{V}_{\phi S}} = \frac{N_P}{N_S} = a$$

This type has two very serious problems:

① If the loads on the transformer circuit are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.

② Third-harmonic problem due to non-linearity of the core. The non-linearity because of [saturation, hystresis].

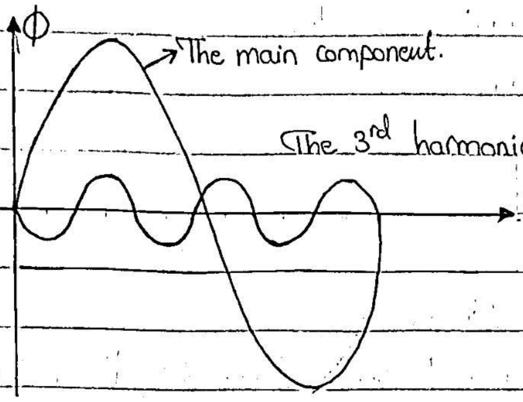


if the flux ϕ is sinusoidal then i_m is peaky in "nature". Since i_m is periodic signal then we can represent it using Fourier series:

$$i_m = \sum_{n=1}^{\infty} C_n \sin(n(\omega t + \phi))$$

when $n=1$, The fundamental component. while $n = 2, 3, 4, \ldots n$ they are all harmonics.

→ Since i_m is half-wave symetry, i_m has only odd harmonics [3, 5, ...] and the 3rd harmonic causes the peaky nature.
→ The fundamental component and the 3rd harmonic can be represented as:



* Notice that each 3rd harmonic rise the peak of the main component

* $i_m = i_{m_1} + i_{m_3} + i_{m_5} + \dots$
"The main component & the harmonics"

As we are talking about three phase, Then:

$I_a = I_1 \sin(\omega t) - I_3 \sin(3\omega t) + \dots$

$I_b = I_1 \sin(\omega t - 120°) - I_3 \sin(3\omega t - 360°) + \dots$

$I_c = I_1 \sin(\omega t + 120°) - I_3 \sin(3\omega t + 360°) + \dots$

Since it's Y-connected: KCL at the nutral point:

$I_a + I_b + I_c = I_N$ ∴ $I_N = 0$ since it's balanced.

$-3 I_3 \sin(3\omega t) + \dots = 0$ → I_3 [the third harmonic] $= 0$.

But this is a problem:- Since:

① If the flux is "∿", Then there will be a 3rd harmonic in i_m.

② If the i_m is "∿", Then the flux is flat top [saturated]

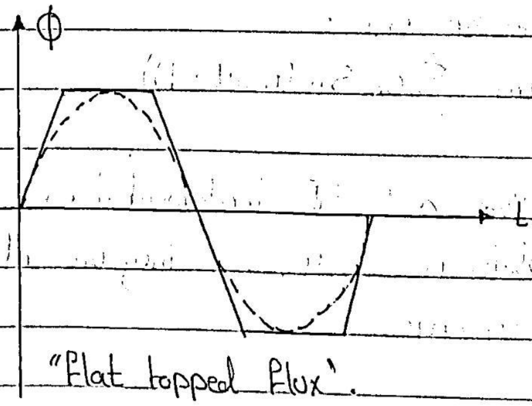
The result we get: $I_3 = 0$ means that:

No triples harmonics of current will flow in Y-Y connection if there is no return path for the 3rd harmonic currents between the nutral of the transformer and the nutral of the source.

As a result, ϕ is flat topped, which does not vary sinusoidly.

$e_{ind} \propto \frac{d\phi}{dt}$, so it is also flat topped.

*Math Note: Any three waves are 120° seperated from each other, their sum is zero.



"Flat topped flux".

→ Solution to the Y-Y problems:

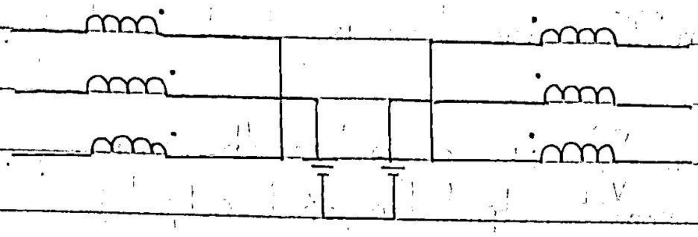
1) Solidly grounding the neutral point of the transformer especially the primary windings.
This allow the 3rd Harmonic current to flow in the neutral instead of building up large 3rd Harmonic voltages.

$I_N = 3 i_3 \neq 0 \xrightarrow{1)} i_m = i_1 + i_3$

$\xrightarrow{2)} \phi$ is sinusoidal

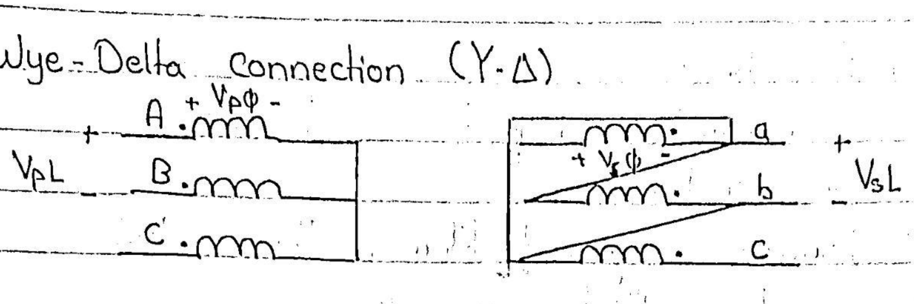
$\xrightarrow{3)} e_{ind}$ is sinusoidal.

This also provides a return path for any current imbalances in the load.



2) Adding a tertiary (trith) winding connected in Δ.
This winding allows the 3rd Harmonic current to flow as a circulating current in the Δ winding. Hence, restoring the flux to be sinusoidal again.
This winding is usually used to feed light loads in the substation [small motors].
Rated power of tertiary winding $= \frac{1}{3}$ rated power of the main winding.
In the case of unbalanced load the current which is imbalances will circulate in the Δ winding.
This tends to equalize the phase voltages and distribute the unbalances load more equally among the primary phases.

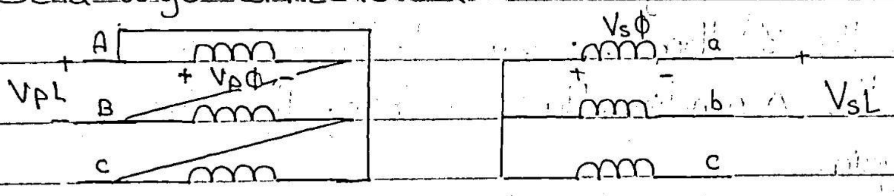
[2] Wye-Delta connection (Y-Δ)



$$\overrightarrow{V_{PL}} = \sqrt{3}\,\angle 30^\circ\,\overrightarrow{V_{P\phi}} \Longrightarrow \frac{\overrightarrow{V_{PL}}}{\overrightarrow{V_{SL}}} = \frac{N_P}{N_S}\sqrt{3}\,\angle 30^\circ = a\sqrt{3}\,\angle 30^\circ.$$
$$\overrightarrow{V_{SL}} = \overrightarrow{V_{S\phi}}$$

* $\overrightarrow{V_{SL}}$ lags V_{PL} by 30°, this can be assumed as a disadvantage.
* While the advantages are:
  1) No 3rd Harmonic problem. The connection is stable with respect to unbalanced load.
  2) Since "$\sqrt{3}$" has appeared in $\frac{\overrightarrow{V_{PL}}}{V_{SL}}$ we can reduce the primary windings and gets the same output [required output].

[3] Delta-Wye connection (Δ-Y)

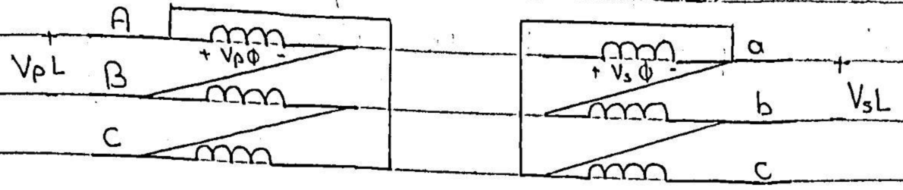


$$\overrightarrow{V_{PL}} = \overrightarrow{V_{P\phi}} \Longrightarrow \frac{V_{PL}}{V_{SL}} = \frac{a}{\sqrt{3}}\,\angle -30^\circ$$
$$\overrightarrow{V_{SL}} = \sqrt{3}\,\angle 30^\circ\,\overrightarrow{V_{S\phi}}$$

Disadvantage: $V_{LS}(t)$ leads $V_{LP}(t)$ by 30°

Advantage: The same advantages of Y-Δ connection.

4 Delta-Delta connection (Δ-Δ)



$$\vec{V_{pL}} = \vec{V_{p\phi}} \qquad \Longrightarrow \frac{V_{pL}}{V_{sL}} = a$$
$$\vec{V_{sL}} = \vec{V_{s\phi}}$$

Advantages:

1) No phase shift between V_{pL} and V_{sL}
2) No third Harmonic
3) stable with respect to unbalanced load.

Disadvantage:

A detection of (line to ground) faults can not be done using Δ-Δ.

→ The Per-Unit system for Three-Phase Transformers:

1) If the total base complex power of the three-phase transformer is S_{base}, then one of the transformer's base complex power is $S_{1\phi,base}$:

$$S_{base} = S_{3\phi,base} \quad ①$$
$$S_{base} = 3S_{1\phi,base} \quad ②$$
$$S_{base} = \sqrt{3}\, V_{L,base} I_{L,base} = 3V_{\phi,base} I_{\phi,base} \quad ③$$

2) Current:

$$I_{L,base} = \frac{S_{base}}{\sqrt{3}\, V_{L,base}}$$

3) impedance:

$$Z_{,base} = \frac{V_{\phi,base}^2}{S_{1\phi,base}} = \frac{3V_{\phi,base}^2}{S_{base}}$$

* The relationship between the base Line Voltage and the base phase voltage depends on the Connection:

In delta (Δ):

1) $V_{L,base} = V_{\phi,base}$

In Wye (Y):

1) $V_{L,base} = \sqrt{3}\, V_{\phi,base}$

* The base line current:

$$I_{L,base} = \frac{S_{base}}{\sqrt{3}\, V_{L,base}}$$

Example (2-9):-

A 50-kVA, 13800/208-V, Δ-Y distribution transformer has a resistance of 0.01 & a reactance of 0.07 per-unit.

a) What is the transformer's phase impedance referred to the high voltage side?

Notes:

1) The 50-kVA is the $S_{3\phi}$

2) The 13800/208-V are the line-to-line (rms) value.

How to start solving such questions:

① High-voltage side has 13800 base line voltage and has a 50-kVA base power.

② Since it's Δ connected: $V_L = V_\phi$

③ Use the equation: $Z_{base} = \frac{3 V_{\phi,base}^2}{S_{base}} = \frac{3 \cdot (13800)^2}{50000} = 11426\ \Omega$

④ $Z_{eq}(PU) = \frac{Z_{eq}}{Z_{base}} \Rightarrow Z_{eq} = Z_{base} \cdot Z_{eq}(PU)$

$Z_{eq} = 11426 \cdot (0.01 + j0.07)$

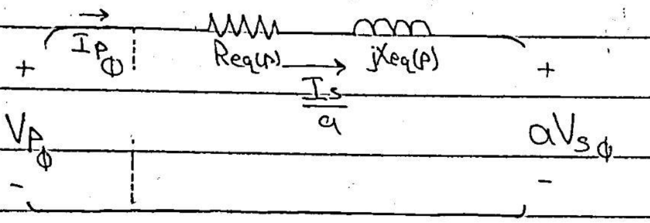
$= 114.2 + j800\ \Omega$

b) Calculate this transformer's voltage regulation at full load and 0.8 PF Lagging, use the calculated high-side impedance.

→ To calculate the voltage regulation of a 3-Φ transformer, you can determine the VR of any single phase transformer. So, The voltages on a single phase are phase voltages:-

① $VR = \dfrac{V_{\phi p} - aV_{\phi s}}{aV_{\phi s}} \times 100\%$ OR ② $VR = \dfrac{\frac{V_{\phi p}}{a} - V_{\phi s}}{V_{\phi s}} \times 100\%$

We use equation ① if the transformer model is drawn referred to the primary, and since the question ask to use the equivalent primary impedance:



$V_{\phi p} = (R_{eq(P)} + jX_{eq(P)}) I_{\phi p} + aV_{\phi s}$

∴ i) $aV_{\phi s} = \dfrac{13800}{\left(\frac{208}{\sqrt{3}}\right)} \cdot \left(\dfrac{208}{\sqrt{3}}\right) = 13800$ ∴ Note $a \neq \dfrac{13800}{208}$ because the connection is Δ-Y.

ii) $I_{\phi p} = \dfrac{S_{1\Phi}}{V_\phi} = \dfrac{50000}{3} \cdot \dfrac{1}{13800} = 1.208\ A$

$V_{\phi p} = (114.2 + j800)\, 1.208 \angle -\cos^{-1}(PF) + 13800 \angle 0°$

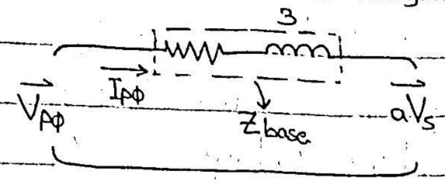
Note: The current angle is $-\cos^{-1}(PF)$ since θ is lagging.

$V_{\phi p} = 14506 \angle 2.73°$

→ $VR = \dfrac{14506 - 13800}{13800} \times 100\% = \underline{\underline{5.1}}$

c) Calculate the voltage regulation under the same conditions using the per-unit system.

→ Take $S_{base} = \frac{50000}{3}$, $V_{base} = 13800$, $I_{base} = 1.208$



1) Z_{pu} (given) $= (0.01 + j0.07)$

2) $\vec{aV_s} = 13800 \rightarrow \vec{aV_s}(pu) = \frac{\vec{aV_s}}{V_{base}} = \frac{13800}{13800} = 1\angle 0^\circ$

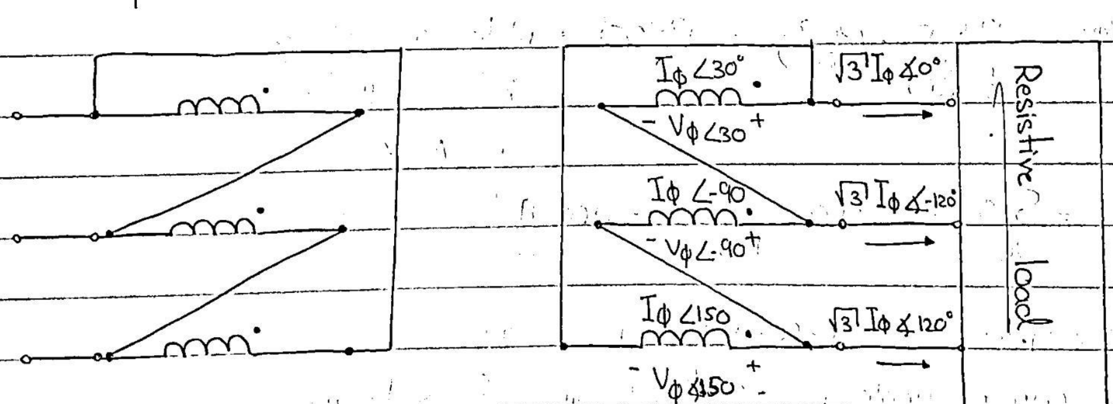
3) $\vec{I_\phi} = 1.208\angle -36.87^\circ \rightarrow \vec{I_\phi}(pu) = \frac{\vec{I_\phi}}{I_{base}} = 1\angle -36.87^\circ$

$V_{\phi P}(pu) = \vec{aV_s}(pu) + (0.01 + j0.07)\, I_\phi(pu)$

$= 1.051\angle 2.73^\circ$

$VR(pu) = \frac{1.051 - 1}{1}$ since it's $\frac{V_{\phi P} - \vec{aV_s}}{aV_s} \Rightarrow \boxed{5.1\%}$

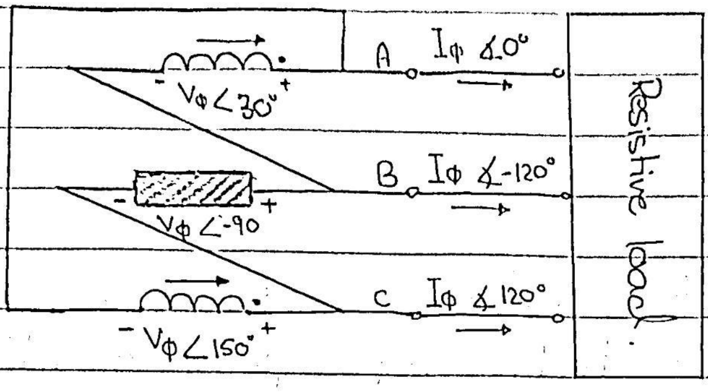
→ The Open-Δ connection



In Δ: $I_L = \sqrt{3}\, I_\phi \angle -30^\circ$

Since the load is Resistive, then $\theta = \theta_v - \theta_i = 0$.

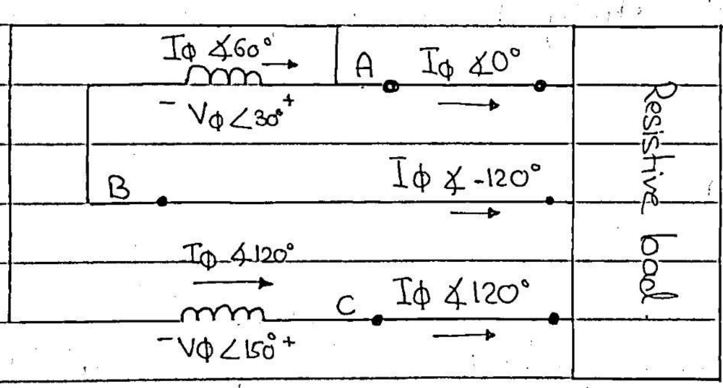
Assume that one of the phases is damaged, and hence it is removed for repair.



* It's important to note the angles on the voltages and currents in the 3-Φ transformer.

[1] Because one transformer is missing; the transmission line current is now equal to the phase current in magnitude. So "$\sqrt{3}$" is gone now.

[2] The currents and voltages in the transformer's bank differ in angle by 30°.



Note: How the angle of the phase current is 60°?

$I_\phi \measuredangle +60° = -I_\phi \measuredangle -120 \rightarrow I_\phi \measuredangle -120+180 = I_\phi \measuredangle 60°$ ※

Note: Since one transformer has gone we expect the power to be dropped to $\frac{2}{3}$ of the original power But:
Studying each remaining transformer lead to a different conclusion:

i. $P_1 = V_\phi I_\phi \cos(30-60)$

$= \frac{\sqrt{3}}{2} V_\phi I_\phi$ ——— ①

ii. $P_2 = V_\phi I_\phi \cos(150-120)$
$= \frac{\sqrt{3}}{2} V_\phi I_\phi$

iii. $P_3 = 0$ since it's gone.

The total power $= \sqrt{3} V_\phi I_\phi$.

Notice that the ratio of the output power available from the open - delta 3-ϕ transformer, to the output power available from the normal 3-ϕ transformer is:

$\frac{P_{open-\Delta}}{P_{\Delta-\Delta}} = \frac{\sqrt{3} V_\phi I_\phi}{3 V_\phi I_\phi} = 57.7\%$ which is less than $\frac{2}{3}$ as expected.

Now another question appears, where has the Power difference gone?
To understand that lets study the reactive power:

1) $Q_1 = V_\phi I_\phi \sin(30-60)$
$= -\frac{1}{2} V_\phi I_\phi$

2) $Q_2 = V_\phi I_\phi \sin(150-120)$
$= \frac{1}{2} V_\phi I_\phi$

Therefore, one transformer producing reactive power while other is consuming

Chapter 3: AC Machinary Fundamentals

AC Motor →

AC Electrical Energy	AC Machine	Mechanical Energy

← AC Generator

→ Types of AC Machine:

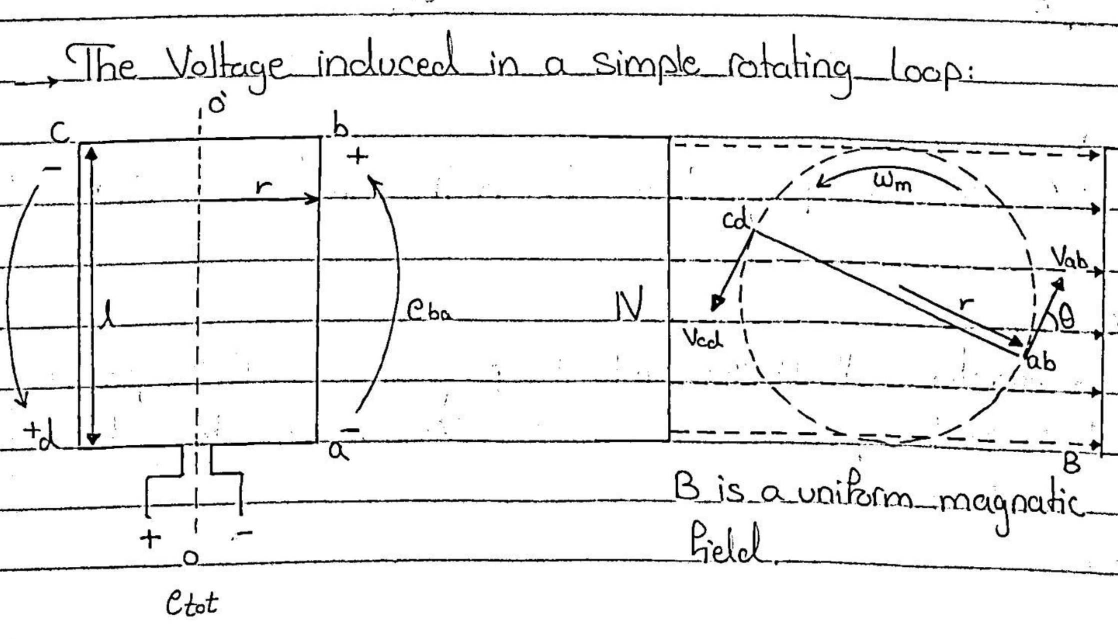
1. Synchronous Machine:
They are generator or motors whose field current is supplied by a separate [external] DC source.

2. Induction Machine:
They are generators or motors whose magnetic field current is supplied by magnatic induction [transformer action] into their field windings.

→ Construction of AC-Machine:

1) Rotor: The rotating part in the machine.
2) Stator: The stationary part [which carries a high current].

→ The Voltage induced in a simple rotating loop:



B is a uniform magnatic field.

Each segment will induce a voltage from the relation:

$e_{ind} = (V \times B) \cdot l$

∴ V is the velocity of the conductor.

B is the Magnatic field density.

l is the length of the segment.

From the previous figures:

OO` is the axis of rotation

Wm is the speed of rotation [The mechanical speed]

Some Facts:

1) To find the linear velocity of a rotating body:

$V = \omega r$, to find the direction make a tangent in the direction of ω.

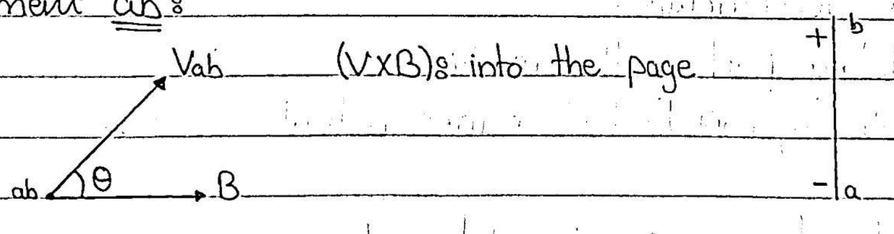
2) One loop Coil has an Area (A) $= 2rl$.

3) $e_{ind} = VB \sin\alpha \, l \cos\theta$

∴ α: the angle between $\underline{\underline{V}}$ & $\underline{\underline{B}}$, θ: the angle between $\underline{\underline{(V \times B)}}$ & $\underline{l}$

Now lets examine each segment to find the total e_{ind}:

1) Segment ab:



$e_{ab} = (V \times B) \cdot l$

$= VBl \sin\theta$ into the page since l is in the direction of $(V \times B)$

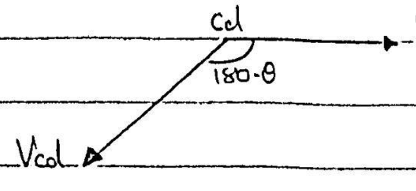
2) segment bc:

Regardless the direction of (VXB) [into the page or out of the page].

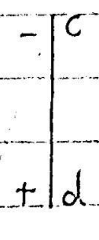
The length l is in the plane of the page, (VXB) is ⊥ to l for any direction of (VXB):

So $e_{cb} = 0$

3) Segment cd:



(VxB) is out of the page



$e_{cd} = (V \times B) \cdot l$

$= VBl\sin(180-\theta) = VBl\sin\theta$ * out of the page

since l is in the direction of $(V \times B)$ [smallest angle].

4) segment da:

Just as in segment bc, $(V \times B)$ is perpendicular to l.

So, $e_{da} = 0$

The total induced voltage on the loop "e_{ind}" is the sum of the voltages in each segment:

A KVL loop:

$e_{ind} = e_{ba} + e_{cd} + e_{dc} + e_{ad}$

$= 2VBl\sin\theta$

We can express this equation in a way that we can study a larger, real AC Machine!!

1) $\theta = \omega t$, if the loop is rotating at a constant angular velocity.

since $\theta = \int \omega \, dt$

2) $V = r\omega$

3) $A = 2rl$

So:

$e_{ind} = AB\omega_m \sin(\omega t)$

4) $\phi_{max} = AB$

$\rightarrow$ $e_{ind} = \phi_{max}\,\omega_m \sin(\omega t)$.

For N loops:

$e_{ind} = N\phi_{max}\omega_m \sin(\omega t)$

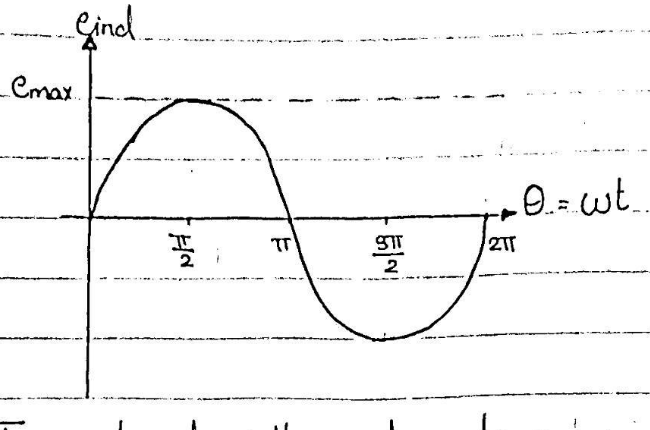
and,

In general:

$e_{ind} = KN\phi_{max}\omega_m \sin(\omega t)$

$e_{ind} = e_{max}\sin(\omega t)$

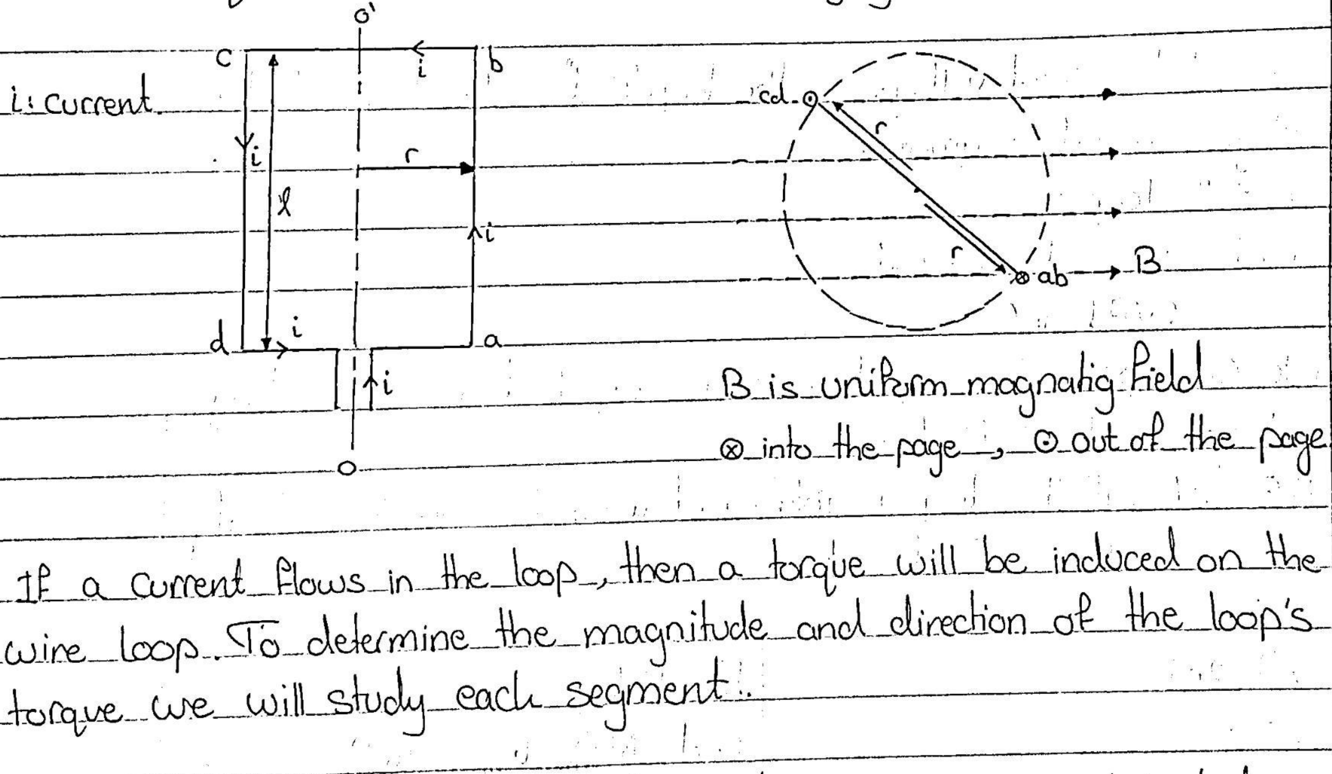
$\therefore e_{max} = KN\phi_{max}\omega_m$



→ The induced voltage depends on:

1) The Magnetic flux of the Machine.
2) The speed of rotation.
3) A constant representing the construction of the Machine.

→ The Torque Induced in a Current-Carrying loop:



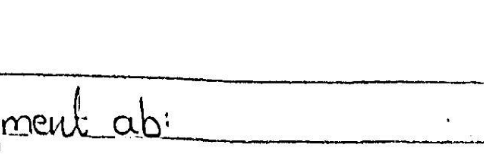
If a current flows in the loop, then a torque will be induced on the wire loop. To determine the magnitude and direction of the loop's torque we will study each segment.

But before that lets talk about what we are going to study:
Since the Torque $= r \times F_{ind}$,
we shall first study the force on each segment:

$F_{ind} = i(\vec{l} \times \vec{B})$

∴ i = Magnitude of current in the segment.
l = Length of the segment, with direction in the current flows direction.
B = Magnatic flux density vector.

1. Segment ab:

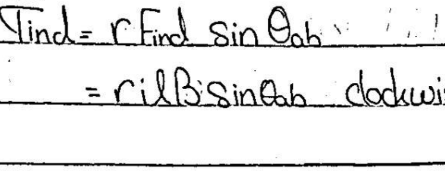


l into page
i into page

$\vec{F}_{ind} = i(l \times B)$
$= ilB$ down.

$\vec{\tau}_{ind} = r F_{ind} \sin\theta_{ab}$
$= r i l B \sin\theta_{ab}$ clockwise.

2. Segment bc:

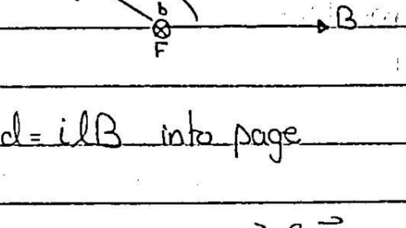


r, F into page

$\vec{F}_{ind} = ilB$ into page

$\vec{\tau}_{ind} = 0$ since $\vec{r}$ & $\vec{F}_{ind}$ are parallel.

3. Segment cd:



l out of page

$\vec{F}_{ind} = ilB$ up

$\tau_{ind} = r i l B \sin\theta_{cd}$ clockwise.

4. Segment da:
Just as segment bc
$\tau_{ind} = 0$ $\vec{r}, \vec{F}_{ind}$ out of page.

The total induced torque on the loop τ_{ind}:

$$\tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$$
$$= 2 r i l B \sin\theta$$

Noting that:

$$\left.\begin{array}{l} 2rl = A \\ AB = \Phi \end{array}\right\} \rightarrow \tau_{ind} = \Phi i \sin\theta.$$

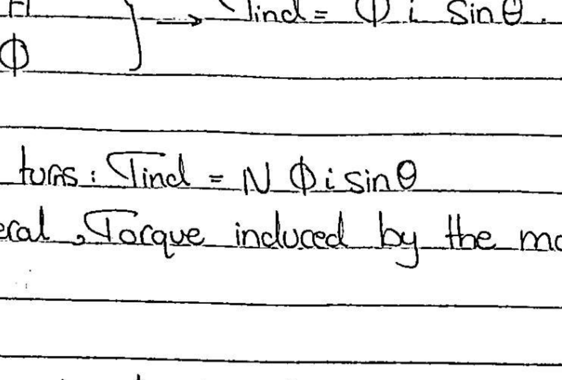
For N turns: $\tau_{ind} = N \Phi i \sin\theta$

In general, Torque induced by the machine is: $\tau_{ind} = K\Phi i \sin\theta$

$$\tau_{ind} = \tau_{max} \sin\theta$$

Torque induced depends on:

1) The flux of the machine.
2) The current in the loop.
3) A constant representing the construction of the machine such as: Number of turns, geometry, shape, et...



Note: The expression of the torque with the Area $\tau_{ind} = ABi\sin\theta$ lead us to the following conclusion:

1) Since the area vector is perpendicular to the loop, then The torque is maximum when the plane of the loop is parallel to the magnetic field.
2) The torque is zero when the plane is perpendicular to the magnetic field.

An Alternative way to express τ_{ind}:
Any current flows in the loop will generate the magnetic field [B_r or B_{loop}], where B_r is the rotor magnetic flux density

From Ampere's law: $\oint H_r . dl = I_{enc}$
where H_r is the rotor magnetic field intensity.

$H_r G = i$, where G is a factor that depends on the geometry of the loop.
Knowing that $H_r = \frac{B_r}{\mu}$, and by equalizing the two equations:

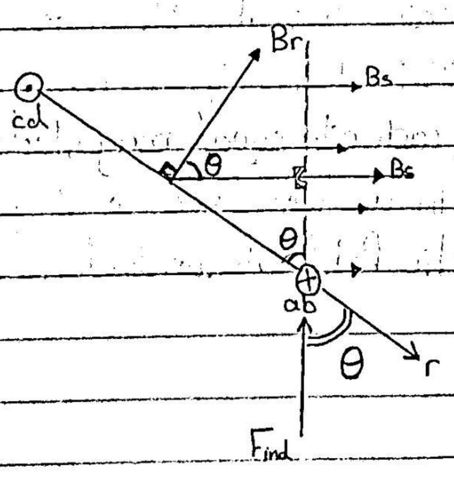
The magnitude of the B_r will equal: $B_r = \frac{\mu i}{G}$, then

$i = \frac{G B_r}{\mu}$

Now, recalling the induced torque equation from the stator:
$\tau_{ind} = A i B_s \sin\theta$, B_s: is the stator magnetic field.

Substituting i from the first & put it in the second equation:
$\tau_{ind} = \frac{AG}{\mu} B_r B_s \sin\theta$, where $\frac{AG}{\mu}$ are constants

$= K B_r B_s \sin\theta$



a) The current in the loop produces a magnetic flux density B_r perpendicular to the plane of the loop.

From the previous figure it's seen that θ is the angle between the Force and length vectors, but it's also the angle between $\vec{B_r}$ & $\vec{B_s}$, this will lead to the following equation:

$\tau_{ind} = k \vec{B_r} \times \vec{B_s}$,

From this equation Both the magnitude and direction of the τ_{ind} can be determined.

In general, the Torque in any real machine will depend on four factors:

1. The strength of the rotor magnetic field.
2. The strength of the external magnetic field.
3. The Sin of the angle between $\vec{B_r}$ & $\vec{B_s}$
4. A constant representing the construction of the machine.

→ The Rotating Magnetic field:

Note: If two Magnetic fields are present in a Machine, then the torque will be created which will tend to line up (align) the two magnetic fields.

From this, The torque induced in the rotor by $\vec{B_r}$ will cause the rotor to <u>turn</u> and <u>align</u> it-self with $\vec{B_s}$ from the stator.

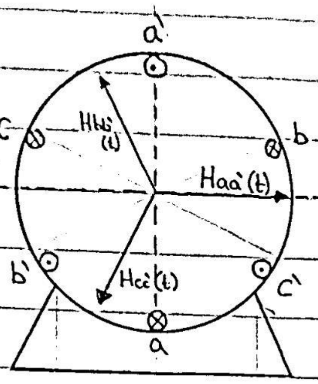
From this concept, if we can make $\vec{B_s}$ rotating then the rotor will constantly chase it around in a circle.

* The question Now is How to make a stator Magnetic field?

The fundamental principle of AC Machine Operation:-

If a three-phase set of currents, each of equal magnitude and different in phase by 120°, flows in a three-phase winding, then it will produce a rotating magnetic field of constant magnitude.

Assume that we have the windings shifted from each other by 120° :



A simple three-phase stator.

Assume that the phase currents applied to the coil are.

1. $i_{aa'}(t) = I_m \sin(\omega_e t)$ A
2. $i_{bb'}(t) = I_m \sin(\omega_e t - 120°)$ A
3. $i_{cc'}(t) = I_m \sin(\omega_e t + 120°)$ A

$\therefore \omega_e$: is the electric radian frequency.

Each current in a coil will produce a magnetic field intensity :

1. $H_{aa'}(t) = H_m \sin(\omega_e t) \angle 0°$ A.turns /m
2. $H_{bb'}(t) = H_m \sin(\omega_e t - 120) \angle +120°$ A.turns /m.
3. $H_{cc'}(t) = H_m \sin(\omega_e t + 120) \angle -120°$ A.turns /m.

Each intensity will produce a Magnetic field density:

1. $B_{aa'}(t) = B_m \sin(\omega_e t) \angle 0°$ T
2. $B_{bb'}(t) = B_m \sin(\omega_e t - 120°) \angle 120°$ T
3. $B_{cc'}(t) = B_m \sin(\omega_e t + 120°) \angle -120°$ T

Now the currents and their corresponding flux densities will be examined at specific times to determine the resulting net magnetic field in the stator:

$\vec{B}_{s(net)} = \vec{B}_{aa'} + \vec{B}_{bb'} + \vec{B}_{cc'}$

1. When $\omega_e t = 0$:-

i. $\vec{B}_{aa'} = 0$

ii. $\vec{B}_{bb'} = \frac{-\sqrt{3}}{2} B_m \angle 120°$

iii. $\vec{B}_{cc'} = \frac{\sqrt{3}}{2} B_m \angle -120$

$\vec{B}_{s(net)} = -j\frac{3}{2} B_m = \frac{3}{2} B_m \angle -90°$

$B_{s(net)}$ when $\omega_e t = 0°$

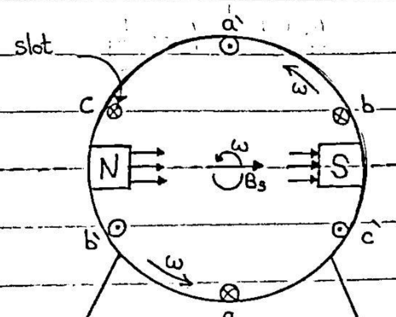
In general, the net magnetic density can be given by:

$$B_{net}(t) = 1.5\, B_m \sin(\omega_e t)\,\hat{x} - 1.5\, B_m \cos(\omega_e t)\,\hat{y}$$

For the proof see pages 163-164 in the book.

The following table represent some values to $\omega_e t$:

$\omega_e t$	$0°$	$90°$	$180°$	$270°$
$\vec{B}_{(s)net}$	$-1.5 B_m \hat{y}$	$1.5 B_m \hat{x}$	$1.5 B_m \hat{y}$	$-1.5 B_m \hat{x}$



The rotating magnetic field in a stator represented as "moving north and south stator poles.

Relationship between Electrical frequency and the speed of Magnetic field rotation:

We can represent the rotating magnetic field in the stator as a north pole and a south pole.

Before we start studying the relation we should know the Winding Sequence; [The sequence of the stator windings]:

From the figure, the counter clock-wise sequence is:

"a c` b a` c b`"

For this sequence, we have [2-poles] or [1 pair of poles]. These magnetic poles complete one mechanical rotation around the stator surface for each electrical cycle of the applied current.

Therefore:

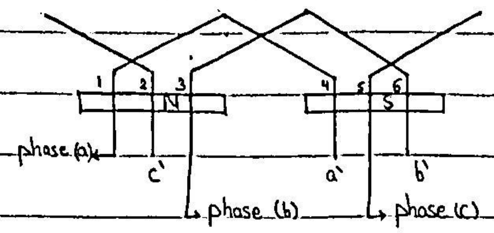
(1) $\theta_e = \theta_m$ → The mechanical angular position.

↳ The electrical angular position

[2] $\omega_e = \omega_m$ → The mechanical speed of rotation [radians per second]
↳ The electrical radian frequency [radian per seconds]

[3] $f_e = f_m$ → [revolutions per second]
↳ [Hertz]

The winding diagram of : [6 slots, 3Φ, 2 poles AC MACHINE]



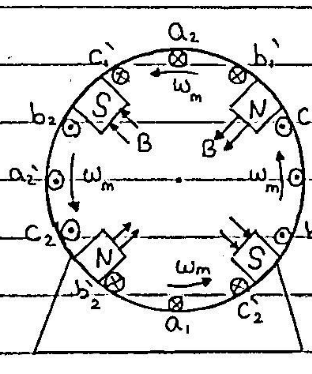
To calculate the phase difference [The slots # between the phases].

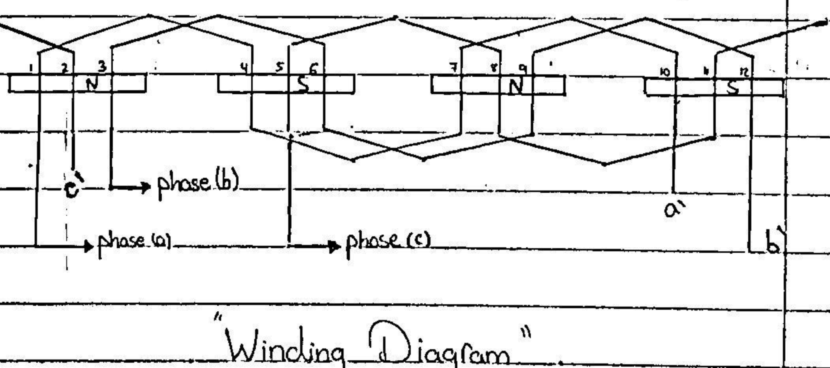
$$\text{phase difference} = \frac{120^\circ}{360} * \frac{\#\text{slots}}{\#\text{Pole-Pair}}$$

$$= \frac{1}{3} * \frac{6}{1} = \boxed{2}$$

If phase a at slot #1
Then phase b at 1+2 = #3
and phase C at 3+2 = #5

What if the winding sequence was repeated?
Then we will get a 4-poler AC Machine
The sequence of the stator winding is :
"ac'ba'cb' ac'ba'cb'"





"Winding Diagram"

In this sequence the net magnetic flux density field will move half a cycle per each electrical cycle.

[1] $\theta_m = \frac{\theta_e}{2}$ [3] $f_m = \frac{f_e}{2}$

[2] $\omega_m = \frac{\omega_e}{2}$

The phase difference = $\frac{120}{360} \cdot \frac{12}{2} = \boxed{2}$

a → 1
b → 1+2 = 3
c → 3+2 = 5

In general the relationship between ω_m & ω_e is given by:

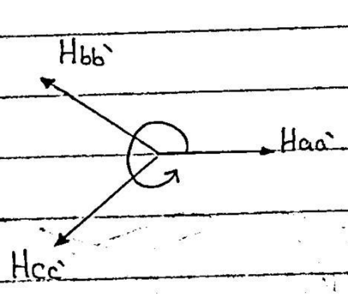
$\omega_m = \frac{\omega_e}{\text{\# pole-pair}} = \frac{\omega_e}{\text{\# poles}/2}$

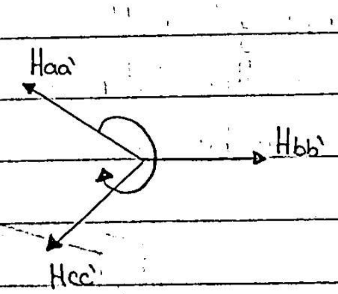
$= \frac{2\omega_e}{\text{\# poles}} = \frac{4\pi f_e}{\text{\# poles}}$

and since: $n_m = \omega_m \frac{60}{2\pi}$

$n_m = \frac{120 f_e}{\text{\# poles}}$

→ The direction of the rotating magnetic field is reversed by: Swapping the phase current in any two coils.





Induced Voltage in a three-phase set of coil:

Asoume that the rotor is rotating at angular speed of ω_m and the magnetic flux density is uniform, then the magnetic flux density in the air gap between the rotor and stator varies sinousoidaly with mechanical angle (α):

$B = B_M \cos(\alpha)$

If N_c is the number of turns per each coil, then the induced voltage across the coil of phase (a) is:

$e_{aa'} = N_c \Phi \omega \sin(\omega t)$

and in phase (b) & (c) are:

$e_{bb'} = N_c \Phi \omega \sin(\omega t - 120^\circ)$

$e_{cc'} = N_c \Phi \omega \sin(\omega t + 120^\circ)$

The peak value of induced voltage is:

$e_{peak} = N_c \Phi \omega = 2\pi N_c \Phi f$

The rms value of the induced voltage:

$e_{rms} = e_{peak} / \sqrt{2}$

$\quad = \sqrt{2} \pi N_c \Phi f$

The line to line rms value of the induced voltage:

$$e_{L-L} \begin{cases} \sqrt{3} e_{rms} & \text{Y-Connected generator} \\ e_{rms} & \Delta\text{-Connected generator} \end{cases}$$

Torque induced in AC Machine: "Motor"

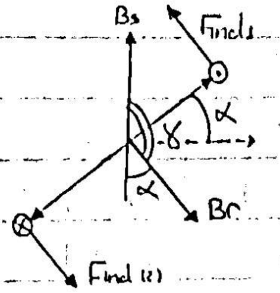
Assume that the field distribution of stator is given by:

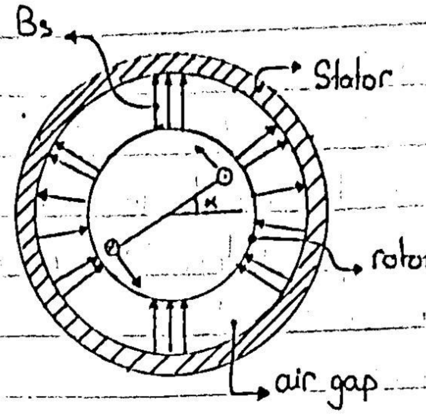
$B_{S_{stator}} = B_S \sin\alpha$

In general,

$F_{ind} = i\, l \times B$

$\quad = i\, l B \sin\alpha$





$F_{ind(1)} = i_1 l_1 B_s \sin\alpha$ C.C.W

$F_{ind(2)} = i_2 l_2 B_s \sin\alpha$ C.C.W

$\tau_{ind(1)} = r i l B_s \sin\alpha$

$\tau_{ind(2)} = r i l B_s \sin\alpha$

The net induced torque is:

$\tau_{ind} = 2 r i l B_s \sin\alpha$

Note:

From the first figure:

γ is the angle between B_r & B_s

$\gamma + \alpha = 180$

$\sin\alpha = \sin\gamma$

So

$\tau_{ind} = k B_r \times B_s$

According to Ampere's law:

$\oint H \cdot dl = I_{enc}$

$H_r = C i$, where C is constant.

$\tau_{ind} = 2rl \dfrac{B_r}{\mu C} B_s \sin\alpha$

$= \dfrac{2rl}{\mu C} B_r B_s \sin(180 - \gamma)$

$= k B_r \times B_s$, k is not constant since the magnetic permability (μ) varies with the amount of magnetic saturation in the machine

The net magnetic field in the Machine:

$\vec{B}_{net} = \vec{B}_r + \vec{B}_s$

$\vec{B}_s = \vec{B}_{net} - \vec{B}_r$

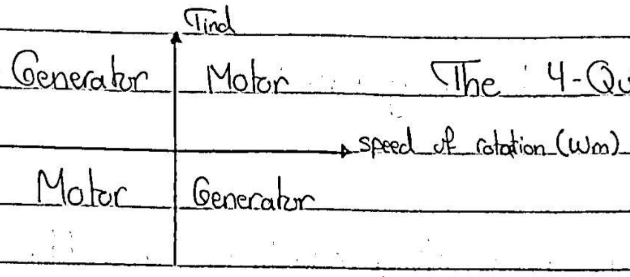
$\rightarrow \tau_{ind} = k \vec{B}_r \times \vec{B}_{net}$

$= k \vec{B}_r B_{net} \sin\delta$, where δ is the angle between $\vec{B}_r$ & B_{net}.

Note:

1. If the direction of induced torque is opposite to the direction of rotation, then the Machine acts like a generator.

2. If the direction of induced torque is the same as the direction of rotation, then the Machine acts like a Motor.



The 4-Quadrants of Operation

The efficiency of the Machine:

Power losses in AC Machines:

1. Electrical (current) copper losses

This is due to the winding resistance

a) $P_{SCL} = 3 I_s^2 R_s$ → R_s: resistance winding of the stator / phase; P_{SCL}: stator copper losses

b) $P_{RCL} = I_r^2 R_r$ → R_r: resistance winding of the rotor; P_{RCL}: rotor copper losses

Note: The power of the stator is a 3-ϕ power.

2. Core losses:

Eddy currents & Hystresis

core losses $\propto B_r^2 \, \omega_m^{3/2}$

3 Mechanical losses:

i) Friction: due to bearings.

ii) Windage: Friction between air and any of the moving parts of the machine.

Mechanical losses: $a_0 \omega_m + a_1 \omega_m^2 + a_2 \omega_m^3$

4 Stray losses:

They represent 1% of full-load. They are losses, which are not accounted for.

* Note: The mechanical and core losses of a machine are often lumped togather and called the no-load rotational loss of the machine.
- At no-load, all the input power must be used to overcome these losses. Therefore, measuring the input power to the stator of an AC machine acting as a motor at no-load will give an approximate value for these losses.

→ Efficiency calculations:

$\eta = \frac{P_{out}}{P_{in}} \times 100\%$

$= \frac{P_{in} - P_{loss}}{P_{in}} \times 100\%$, Since the difference between the input power and output power is the losses occure inside AC Machine.

→ For generator operation:

$P_{in} = \underbrace{T_{app}}_{\text{Torque applied}} \ \underbrace{\omega_m}_{\text{speed}}$ * This power is mechanical.

$P_{out} = 3 V_\phi I_\phi \underbrace{\cos\theta_\phi}_{\text{The power factor.}}$

$= \sqrt{3} V_L I_L \cos \underbrace{\theta_\phi}_{\text{The phases angle.}}$

- For Motor calculations:

$P_{in} = 3 V_\phi I_\phi \cos\theta_\phi$

$= \sqrt{3} V_L I_L \cos\theta_\phi$

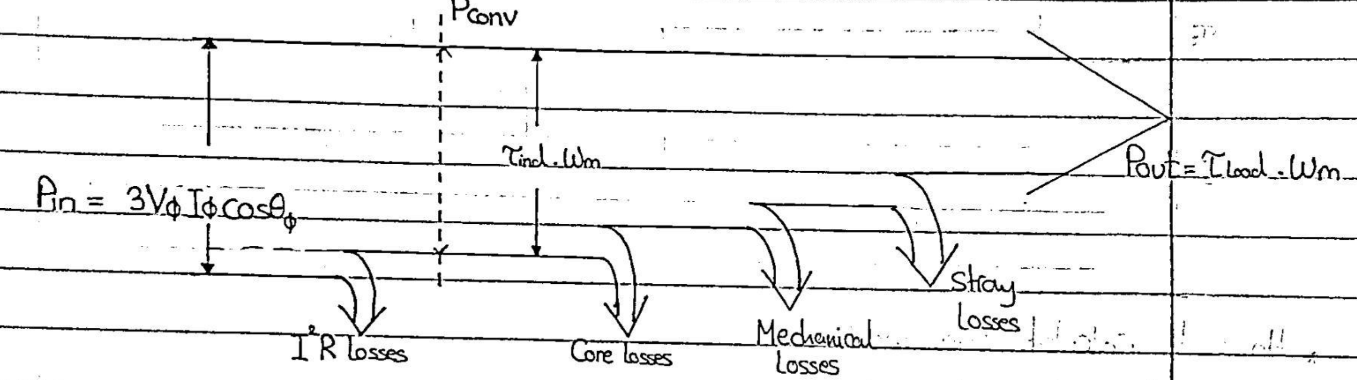
$P_{out} = \underline{\underline{\tau_L}} \cdot \omega_m$

τ_L → Torque load applied to the shaft.

- Power-Flow Diagram:

It's one of the most convenient techniques for accounting for power losses

[1] The Power-flow diagram of a 3-ϕ ac Motor:



P_{conv}: is the converted power $\begin{cases} \text{to mechanical if Motor} \\ \text{to electrical if Generator.} \end{cases}$

It's given by:

$P_{conv} = \underline{\underline{\tau_{ind}}} \cdot \omega_m$

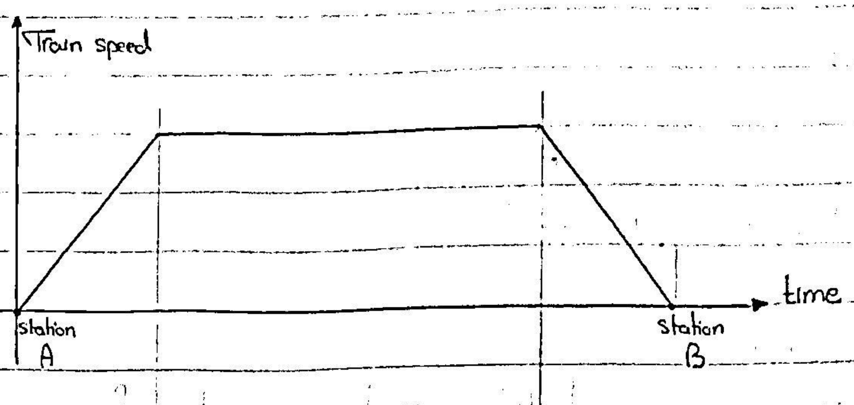
τ_{ind} → The induced torque.

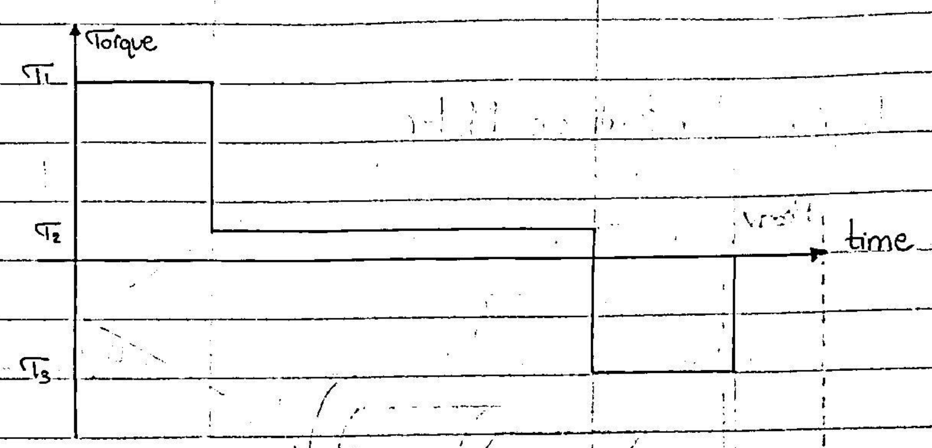
From Newton's second law:

$\Sigma\tau = J \dfrac{d\omega_m}{dt}$, ∴ $\Sigma\tau$: The net torque, J: inertia, $\dfrac{d\omega_m}{dt}$: acceleration.

$\tau_{ind} - \tau_L = J \dfrac{d\omega_m}{dt} - \tau_{friction}$

To understand the previous law, lets study the Train example:





- How to calculate the inertia:

$$\frac{1}{2} m V^2 = \frac{1}{2} J \omega_m^2$$

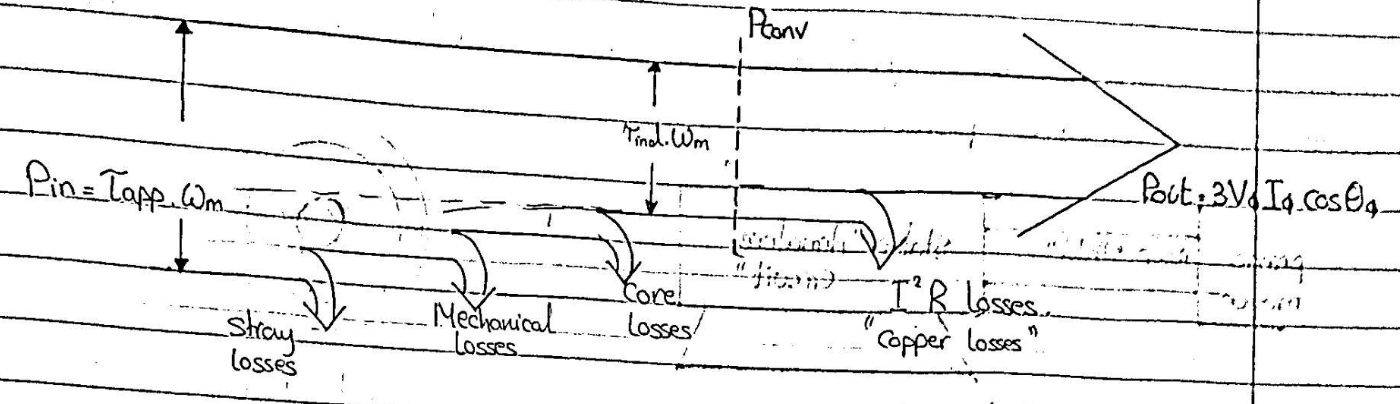
$$J = m\left(\frac{V}{\omega_m}\right)^2 \quad , \text{but } V = \omega . r$$

$$= m\left(\frac{\omega . r}{\omega_m}\right)^2 \quad , \text{if the shaft speed} = \text{wheel speed} \Rightarrow [\omega = \omega_m]$$

$$J = m\left(\frac{r}{(\omega_m/\omega)}\right)^2$$

$$= m\left(\frac{r}{\tau}\right)^2$$; where τ is the gear box ratio.

2 The Power-flow diagram of the 3-ϕ ac Generator:



▸ Voltage Regulation and Speed Regulation:

VR: It's a measure of generator ability to keep a constant voltage under load condition (as load varies)

$$VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%$$

∴ V_{NL}: The no-load terminal voltage of the generator
V_{FL}: The full-load terminal voltage of the generator.

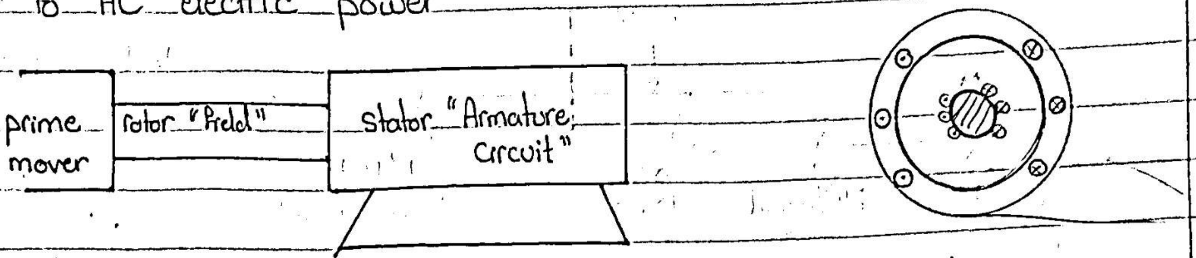
SR: It's the ability of Motor to keep constant speed as load varies.

$$SR = \frac{n_{NL} - n_{FL}}{n_{FL}} \times 100\%$$

i) positive SR means, The motor's speed drops with increasing load.
ii) negative SR means, The motor's speed rises with increasing load.
iii) if $SR = 0$, Then $n_{NL} = n_{FL}$

Chapter 4: Synchronous generators.

Synchrouns generators are synchronous machines used to convert mechanical power to AC electric power



Side view — Front view

Prime mover: It's the initial source of mechanical energy. It's any machine that converts (feul / steam) into a mechanical energy.

- The synchronous machine has two main windings:
  1. The "rotor" or "field" windings, where the machine flux is induced (produced).
  2. The "stator" or "armature" windings, where the 3-ϕ voltages are induced (produced).

- Operating principle:

  A DC current is supplied to the rotor coil to produce a rotor Magnetic field.

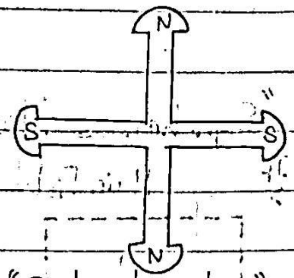
  The rotor rotats by means of external prime mover to produce a rotating magnetic field

  The rotating magnetic field will induce 3-ϕ voltages across the stator or armature windings

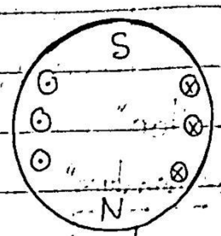
- Types of rotor in synchronous generator:

1. Salient polar rotor:
   The poles are sticking out from the rotor surface.

2. Cylinderical rotor:
   The poles are constructed flush with the rotor surface



"Salient rotor"
$P = 4$

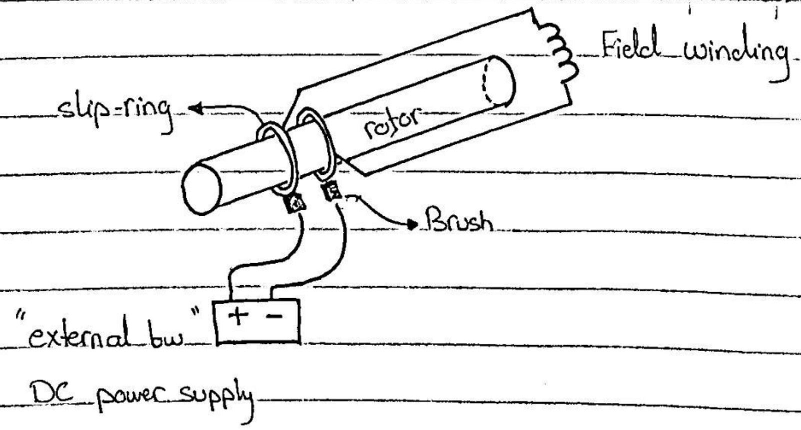


"cylinderical rotor"
$P = 2$

Notes:
When $P > 4$, salient pole machine is selected.
when $P = 2, 4$, cylinderical pole machine is used.

- Methods to feed the rotor coil

1. Slip-rings and brushes:
   slip-rings: Are Metal rings encircling the shaft, but insulated from it.
   Brush: It's a block of carbon compound that conducts electricity freely, but has very low friction.



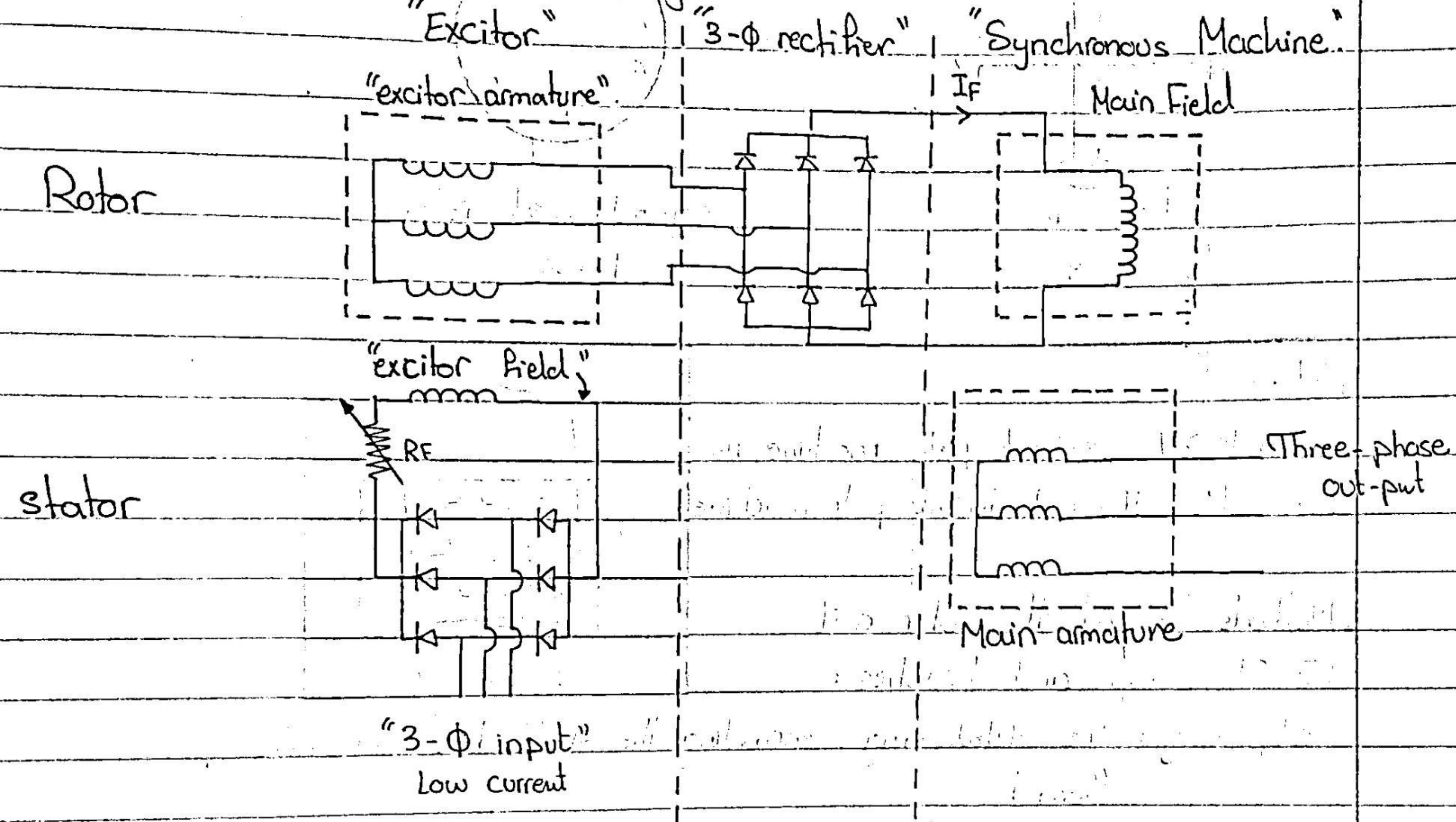
→ Sliprings and brushes problems:

1) The brushes need regular maintenances

2) The voltage drop across brushes may cause a power losses.

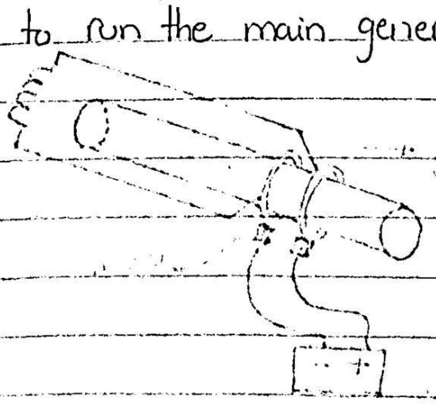
[2] Brushless excitor:

It's a small ac generator with it's field circuit mounted on the stator of main generator, and it's armature windings mounted on the rotor of the main generator.



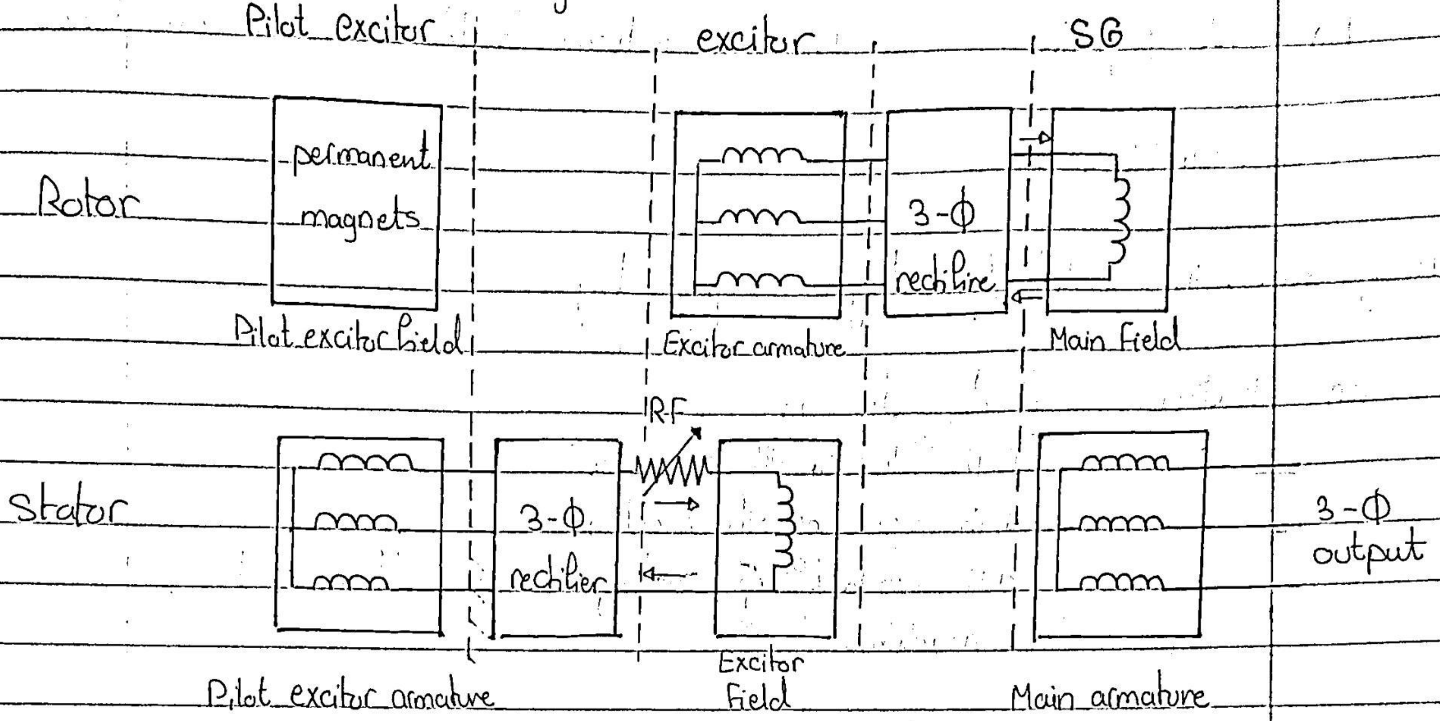
The drawback or disadvantage of the Brushless excitor:

→ it needs a 3-Φ input power to run the main generator



③ Pilot excitor:

It's a small ac generator with permenant magnet mounted on the rotor, and its armature circuit mounted on the stator of the main generator



▸ Mechanical speed of synchronous Generator:

Synchronous mechanical speed of the rotor is synchronoized with the electric frequency.

$$n_m = n_s = \frac{120}{P} f_e$$

: n_m: is the speed of the rotor [Mechanical speed].

n_s: is the synchronous speed.

P: The number of poles

f_e: Electrical frequency in Hertz.

Notes:

① $n = 60 f \rightarrow n_m = \frac{2}{P} n_e$

② $$\omega_m = \frac{2}{P} \omega_e$$

→ Internal voltage of synchronous generator:
The RMS value of the Line-Neutral [L-N] induced voltage in Synchronous generator is given by:

$$E_A = \frac{N_c \, \phi \, \omega_e}{\sqrt{2}} \quad \because E_A: \text{The induced voltage on the armature.}$$

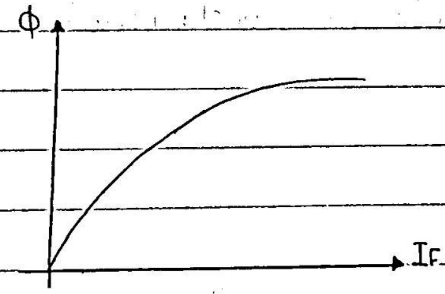
$$= \sqrt{2} \, \pi \, N_c \, \phi \, f_e$$

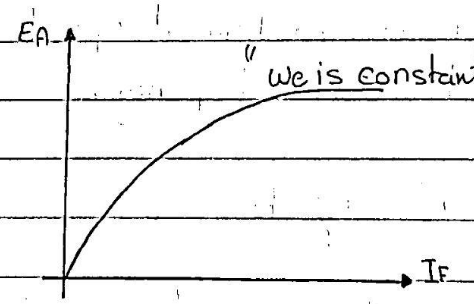
In general, the induced voltage is given by:

$E_A = K \phi \omega_e$, it depends on:

1. The flux of the machine.
2. The speed of the machine.
3. A constant representing the construction of the machine.

Note:
The flux is linearly propotional to the field current (I_F).

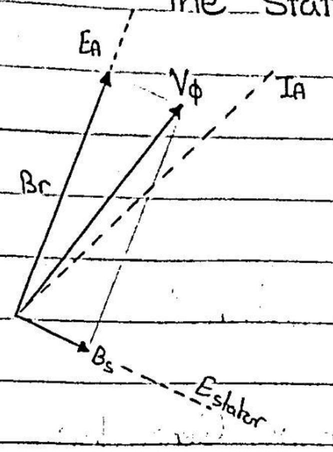




→ The equivalent circuit of synchronous machine:
The terminal voltage (V_ϕ) of synchronous machine [generator] is not equal to the induced voltage (E_A), because of the following:

1. Armature reaction.
2. Self inductance of armature circuit [L_A].
3. Self resistance of armature circuit [R_A].

[1] Armature reaction: is a distortion of air gap magnetic flux due to the current flowing in the stator winding.



$\vec{V_\phi} = \vec{E_A} + \vec{E_{stator}}$

$\vec{E_{stator}} = -jX\vec{I_A}$

$\vec{V_\phi} = \vec{E_A} - jX\vec{I_A}$

↳ reactance of the armature reaction.

[2] + [3] : The inductance & resistance of the armature windings:

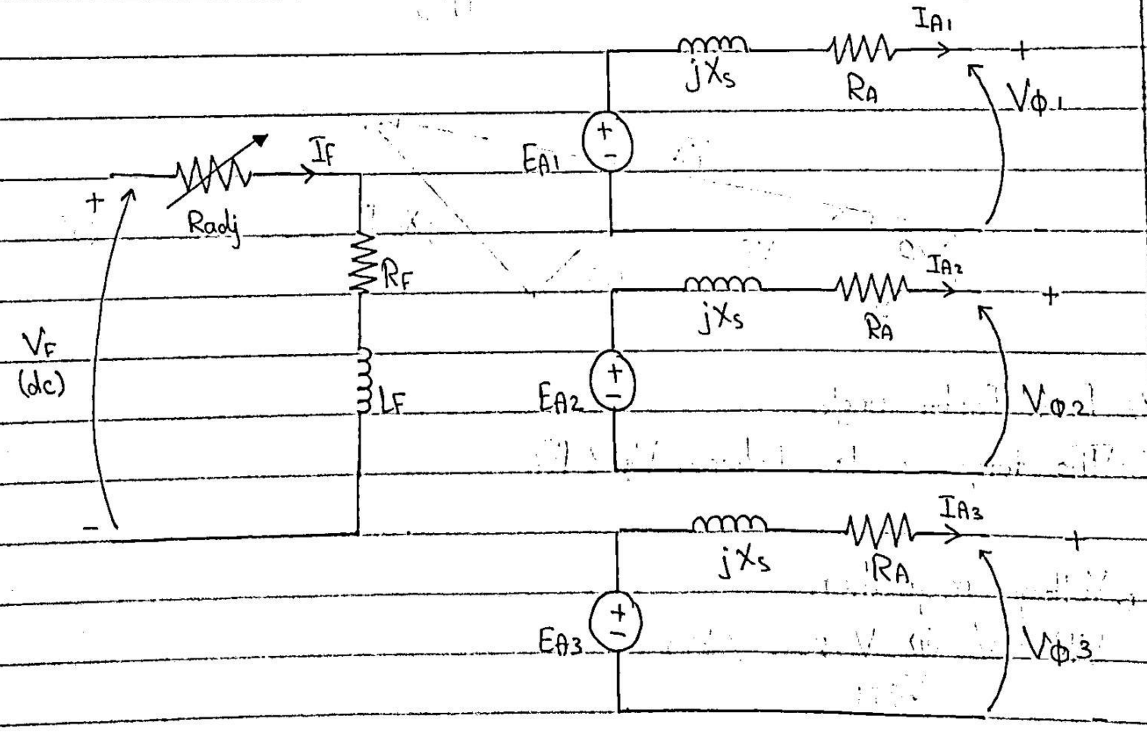
$\vec{V_\phi} = \vec{E_A} - jX\vec{I_A} - jX_A\vec{I_A} - R_A\vec{I_A}$

X_A: The self reactance of the stator armature winding.

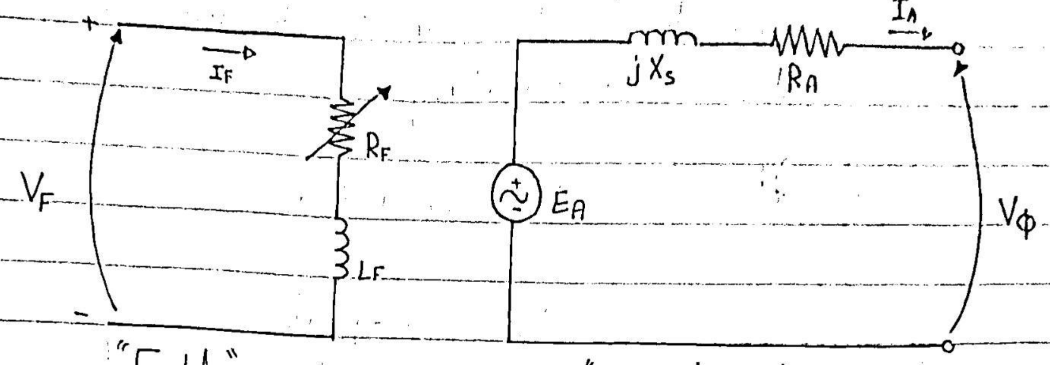
Then we can conclude that the equivalent circuit is:

$\vec{V_\phi} = \vec{E_A} - j(X_A + X)\vec{I_A} - R_A\vec{I_A}$

$\therefore X_A + X = X_s$, synchronous reaction [synchronous reactance].



- The per-phase equivalent circuit:



"Field" "Armature"

→ Phase Diagram of 3ϕ synchronous Generator:

By applying KVL in the per-phase armature circuit, we get:

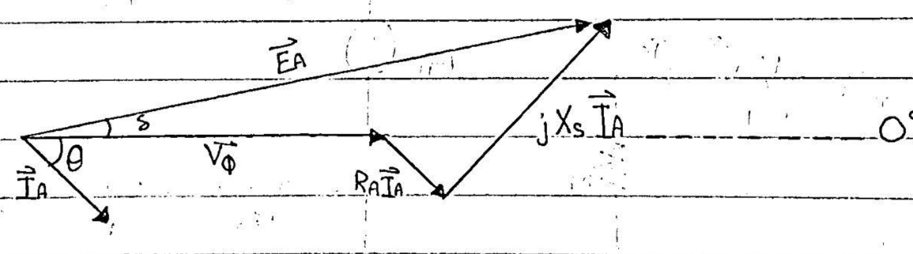
$$\vec{E_A} = R_A \vec{I_A} + jX_s \vec{I_A} + \vec{V_\phi}$$

Assume that the angle of $\vec{V_\phi}$ is zero, (refrence angle):

$$\vec{V_\phi} = |V_\phi| \angle 0°$$

[1] Lagging PF:

"The generator is connected to a lagging loads [inductive load]".



θ: Power Factor angle.

δ: The torque angle, between V_ϕ & E_A

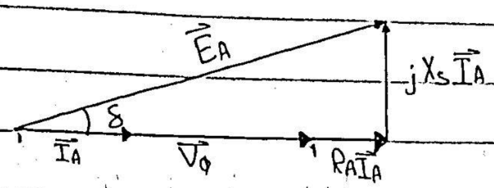
→ Voltage regulation

$$VR = \frac{V_{\phi(NL)} - V_{\phi FL}}{V_{\phi FL}} \times 100\%$$

$$= \frac{E_A - V_\phi}{V_\phi} \times 100\% \quad > 0$$

[2] Unity PF:

"The generator is connected to resistive load"

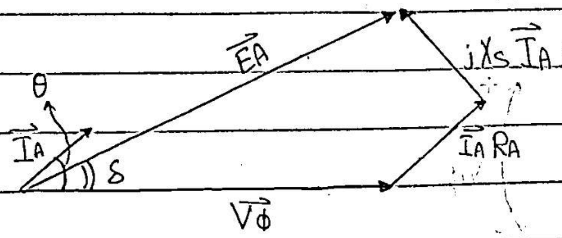


VR is also positive.

[3] Leading PF:

"The generator is connected to a leading load [capacitive load]"

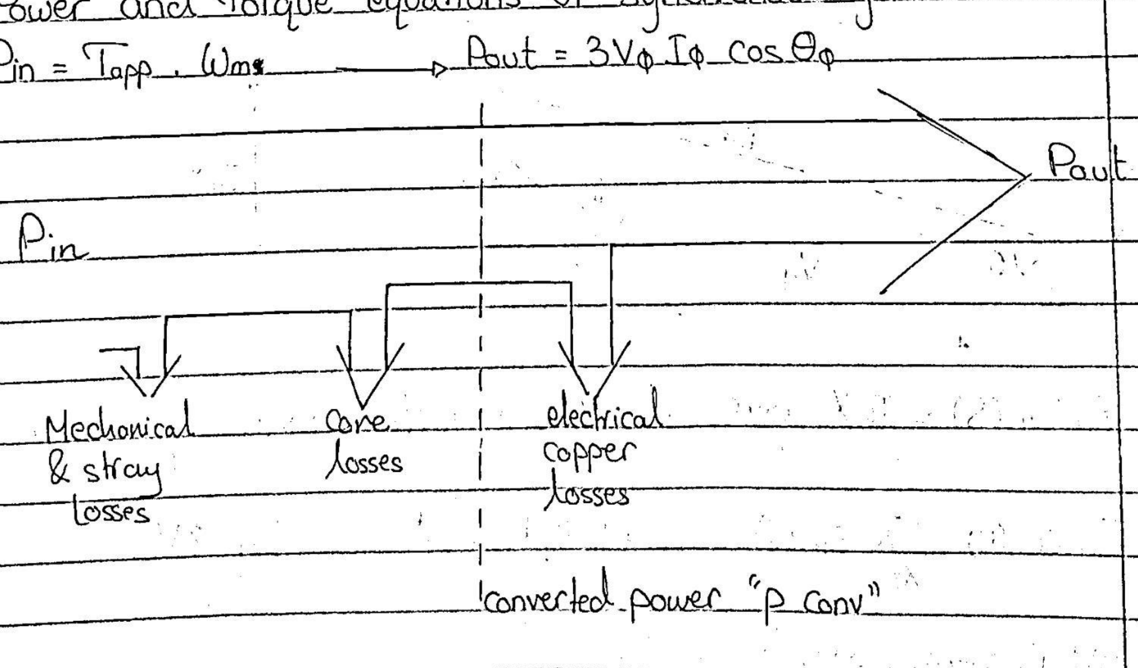
Note: This type is not used in real life application.



$E_A < V_\phi \rightarrow VR = \frac{E_A - V_\phi}{V_\phi} \times 100\% < 0$

→ Power and Torque equations of synchronous generator:

$P_{in} = T_{app} \cdot \omega_m \longrightarrow P_{out} = 3V_\phi I_\phi \cos\theta_\phi$

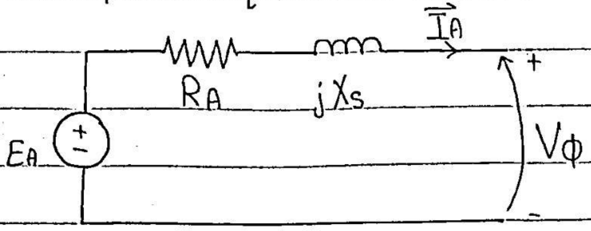


$P_{conv} = \tau_{ind} \cdot \omega_{ms} = 3 E_A I_A \cos(\gamma)$

① from $\vec{B_c}$ & $\vec{B_s}$ ② γ: The angle between $\vec{E_A}$ & $\vec{I_A}$.

i. electro-magnet torque

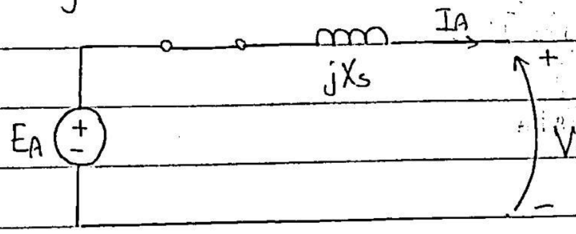
developed torque



$P_{out} = 3 V_\phi I_\phi \cos \phi_\phi$

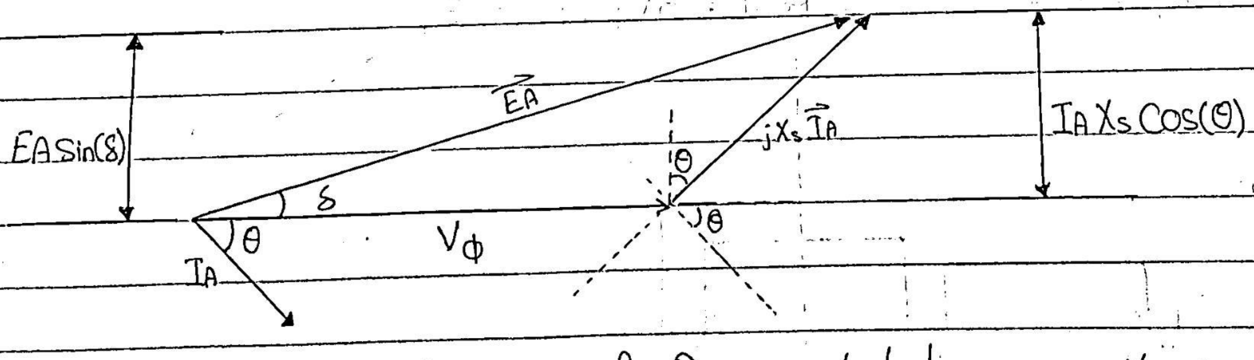
$Q_{out} = 3 V_\phi I_\phi \sin \theta_\phi$

Assume that the load is connected to an inductive load. Since R_A is smaller than X_s ($X_{self\,A} + X_A$), then we can neglect it.



$\vec{E_A} = \vec{V_\phi} + jX_s \vec{I_A}$

Phasor Diagram:



$E_A \sin(\delta) = I_A X_s \cos(\theta)$ if R is neglected, since $\frac{X_s}{R} \gg 10$

$I_A \cos(\theta) = \frac{E_A}{X_s} \sin(\delta)$. Multiply both sides by $3V_\phi$.

$3 V_\phi I_A \cos(\theta) = 3 V_\phi \frac{E_A}{X_s} \sin(\delta)$

$p_{out} = p_{conv}$, since R is neglected.

$P_{out} = P_{conv} = \tau_{ind}\,\omega_{ms}$

The induced torque in synchronous machine is:

$\tau_{ind} = \frac{3E_A}{X_s} \cdot \frac{V_\phi}{\omega_{ms}} \cdot \sin(\delta)$.

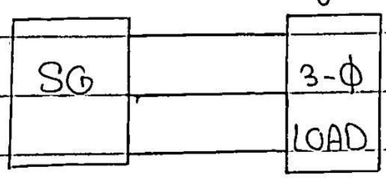
Since Pout is a sin function:

$P_{out} = \frac{3V_\phi E_A}{X_s} \sin(\delta)$, it has a max value:

$P_{out\,max} = \frac{3V_\phi E_A}{X_s}$, $\therefore \delta_{max} = 90°$ [steady-state stability limit]

→ Effect of load changes on SG operation:

1) Assume that the generator is connected to 3-ϕ load.



2) Assume that the machine flux is constant:

ϕ is constant mean that the field current (I_F) is constant.

3) Assume that the prime-mover is rotating at constant speed

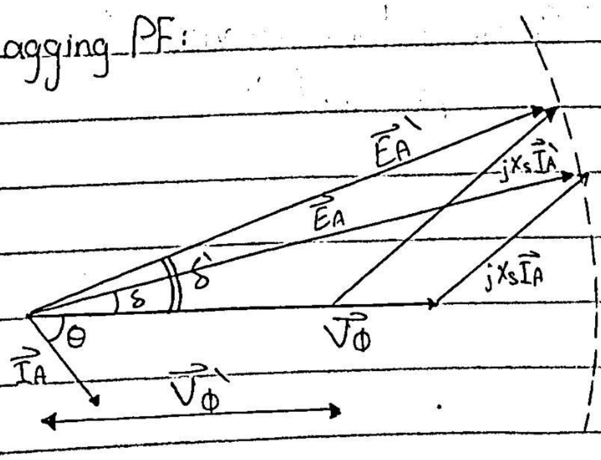
ω_{ms} is constant, so the induced voltage (E_A):

$E_A = K\phi\omega_{ms}$ is constant.

4) Assume that the load increased at the same PF:

Note: load increased means that a higher (I_A) is consumed.

[i] Lagging PF:

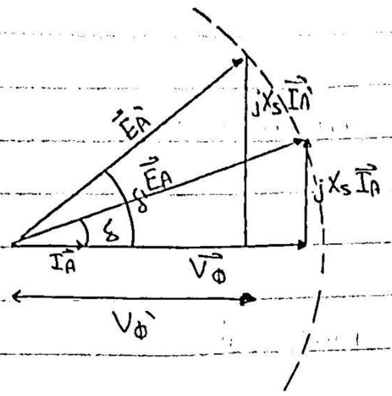


Notes:

① The terminal voltage decreases as the load increases.

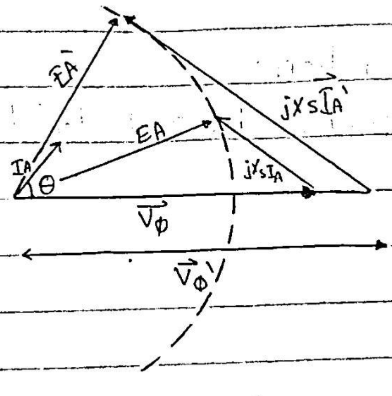
② $|I_A'| > |I_A|$

(ii) Unity P.F:



Note:
① The terminal voltage decreases as the load increases.

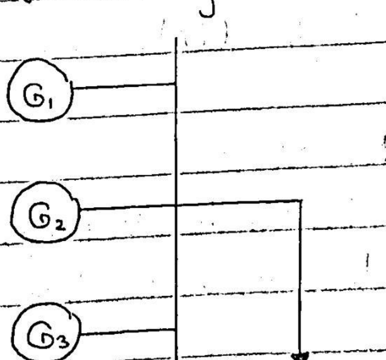
(iii) Leading P.F:



Note:
① The terminal voltage increases as the load increases.

→ The Automatic Voltage regulator (AVR)
It's a closed controller to keep the terminal voltage of Synchronous generator constant.

→ Parallel operation of synchronous generator:
Other than emergency generators, rarely there is a case where a single (isolated) generator supplies a load in power systems.

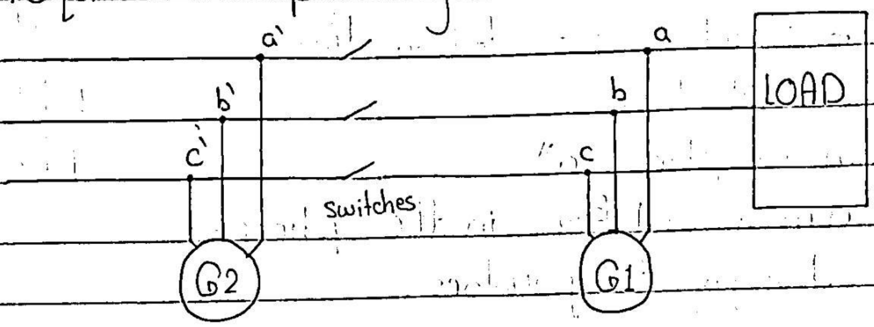


"3-generators connected in parallel".

Major Advantages:

1. Supplies a larger load in the system
2. high reliability of power system.
3. One or more generators can be removed for shutdown a preventive maintenance.
4. If only one generator is used and it's not running at full load then it will be relatively inefficient process.

Conditions required for paralleling:

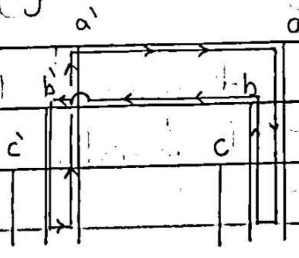


G1: The running generator.

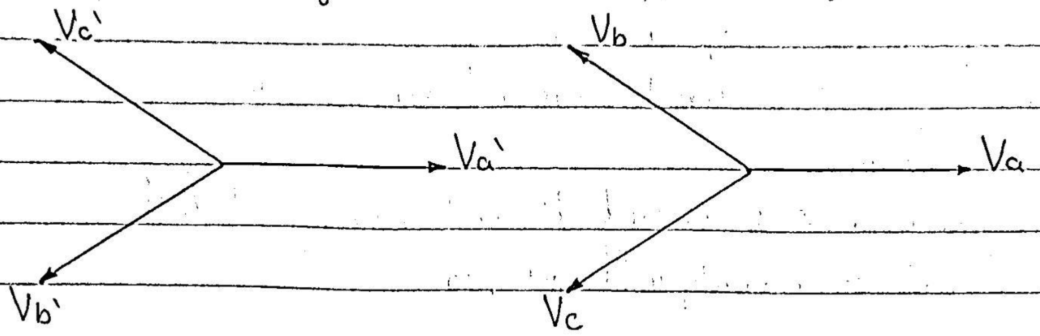
G2: The oncoming generator.

1. The generators must have the same RMS Line voltages: To avoid huge circulating current that may flow in the phases, so damaging the generator.

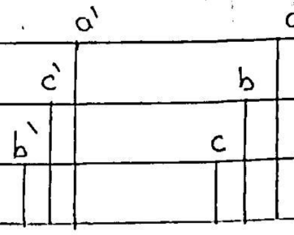
"circulating current" path



2 The phase sequence of the generators must be the same:
Assume that the generators has an opposite sequence:



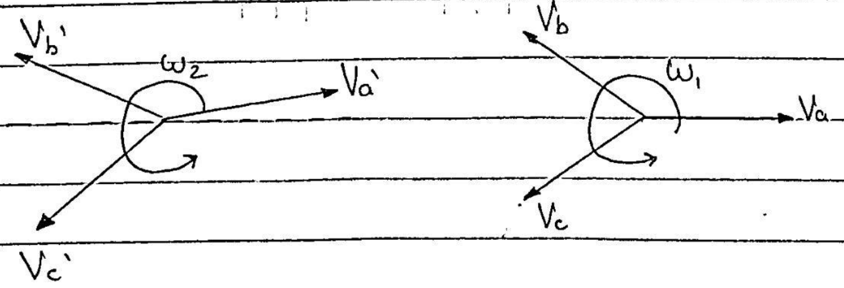
If the generators where connected in this mannar, there would be:
1) no problem in phase "a"
2) a huge current will flow in the phases "b" & "c" damaging the generators.



To correct the phase sequence, simply swap any two of the three phases on the generator.

3 The phase angle of the "a", "b", "c" phases must be equal.
4 The frequency of the on-coming generator must be slightly higher than the frequency of the running generator.

Explanation for 3 & 4:
Assume that the frequency of generator 1 is $\omega_1 = 2\pi f_1$
Assume that the frequency of generator 2 is $\omega_2 = 2\pi f_2$



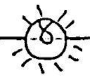
"In phase" "Out of phase"

To study the importance of slightly different between the frequencies:

lets study the two cases:

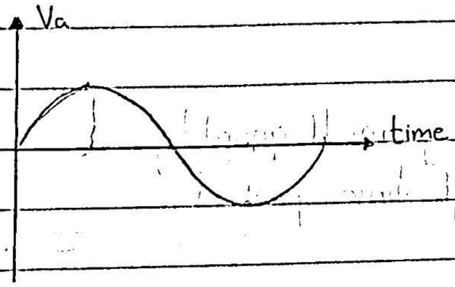
1 Case ~~#~~1:

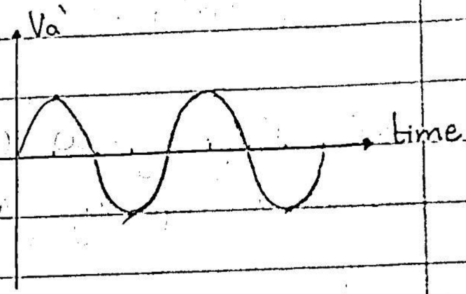
$\omega_2 \neq \omega_1$ and the difference $(\omega_2 - \omega_1)$ is high.

V_a'  V_a → light bulb [on-off quickly].

Assume $V_a = A\sin(\omega t)$

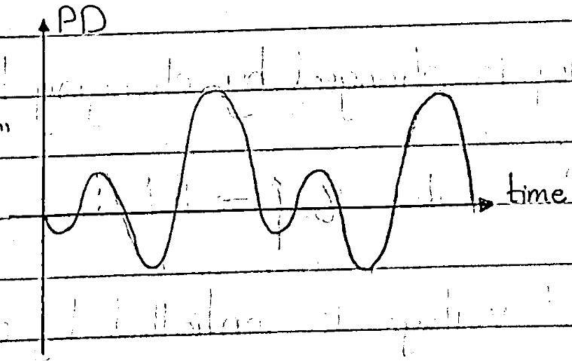
$V_a' = A\sin(2\omega t)$.





$\therefore PD = V_a - V_a'$

"The Potential difference"



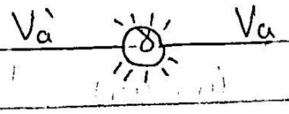
At $t=0$: $V_a = V_a'$ [They are in phase].

At the peak, They are 180° out of phase.

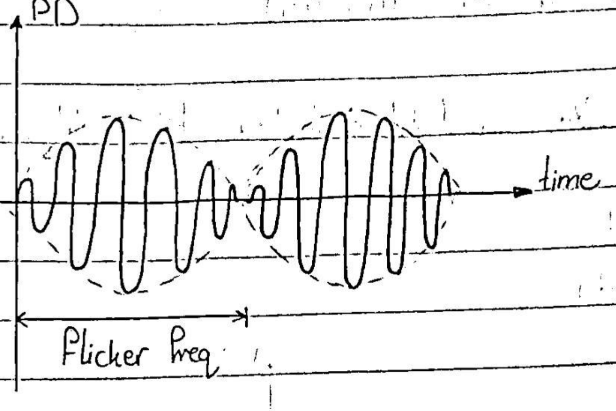
Notation: Flicker frequency: $2\omega - \omega = \boxed{\omega}$.

Case #2:

$\omega_2 \neq \omega_1$ and the difference $(\omega_2 - \omega_1)$ is very small.



$\therefore PD = V_a - V_a`$



Flicker Frequency $= \omega_2 - \omega_1$ = a very small quantity.

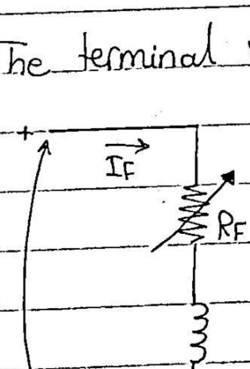
a small frequency gives a high time period.

* Note:

The frequency is changed by changing the speed of the prime mover.

$n_{s,m} = \frac{120}{P} f_e$, when $n_m \uparrow \rightarrow f_e \uparrow$

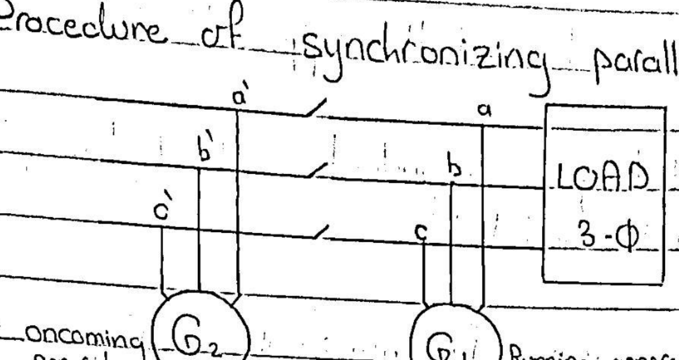
→ The terminal voltage is controlled by changing R_F:



$R_F \downarrow \rightarrow I_F \uparrow \rightarrow \phi \uparrow \rightarrow (E_A = k\phi\omega_m) \uparrow \rightarrow V_\phi \uparrow$

$R_F \uparrow \rightarrow I_F \downarrow \rightarrow \phi \downarrow \rightarrow (E_A = k\phi\omega_m) \downarrow \rightarrow V_\phi \downarrow$

→ Procedure of synchronizing parallel generator:



[1] Adjust the speed of G_2 to match the speed of G_1

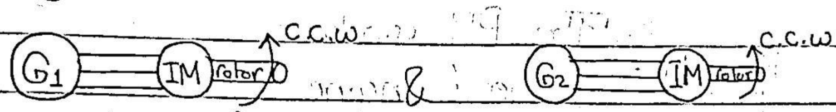
[2] Using Voltmeters adjust V_T (terminal voltage) of G_2 to be equal to V_T of G_1.

Note: "V_T is controlled via I_F".

[3] Phase sequence: We can adjust it by two methods:

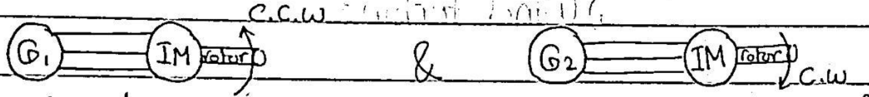
i) Using small 3-phase induction motor

IF:



Then they have the same phase sequence.

IF



Then they have opposite phase sequence, So two of the three phases of G_2 must be reversed.

ii) Three light bulb method:

IF G_1 & G_2 have the same phase sequence then the three lamps will glow togather and turn off togather.

IF G_1 & G_2 have opposite phase sequence then the three lamps will glow in succession.

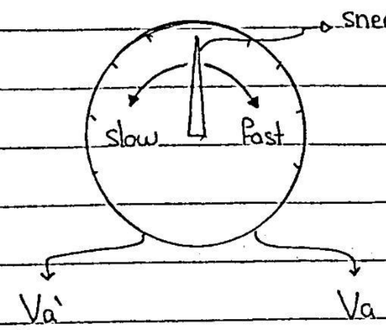
[4] Set the frequency of G_2 to be slightly higher than the frequency of G_1.

When the bulbs (lamps) are turned off V_a is in phase with $V_{a'}$ and paralleling can be done.

"See case 2 in the previous page".

This method is not very accurate, a better approach is to use a synchroscope.

∴ Synchroscope: is a device that gives an induction about the phase shift between Va and Va'.

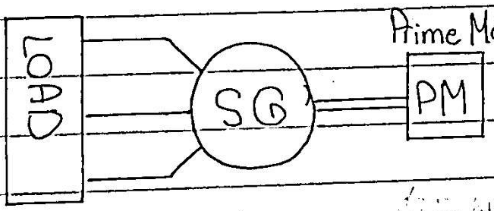


Fast: The oncoming frequency ≫ The running frequency
The needle will rotate C.W.

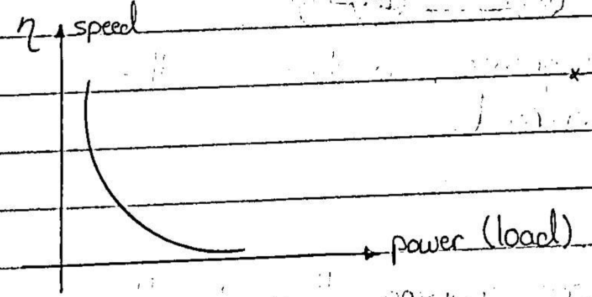
slow: The oncoming frequency ≪ The running frequency
The needle will rotate C.C.W. ⚠

Then:

→ Frequency - Power and Voltage - Reactive Power characteristics:



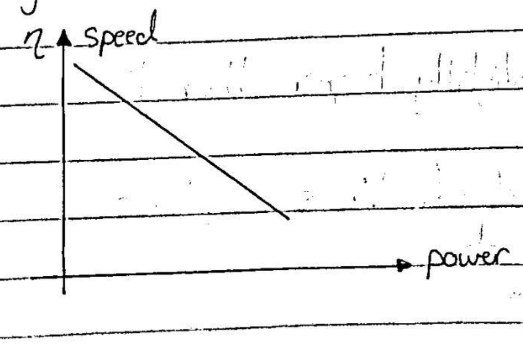
The PM can be:
1) Diesel engine.
2) Water turbine.
3) Wind turbine.



* The speed droop is non-linear.

To study this curve it's important to find some way to make it linear:

The governor mechanism makes the droop characteristics linear.

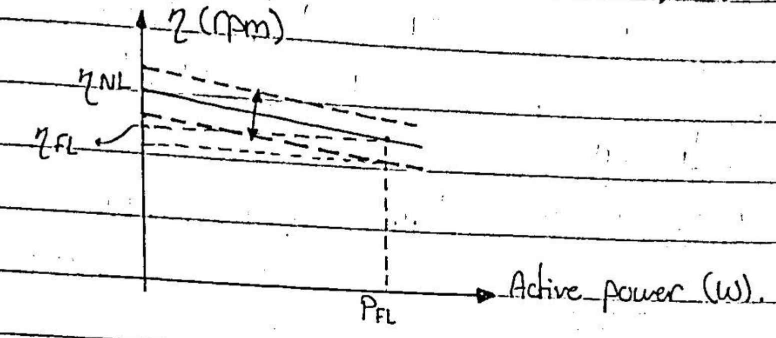


* a missing note of the "Synchroscope".

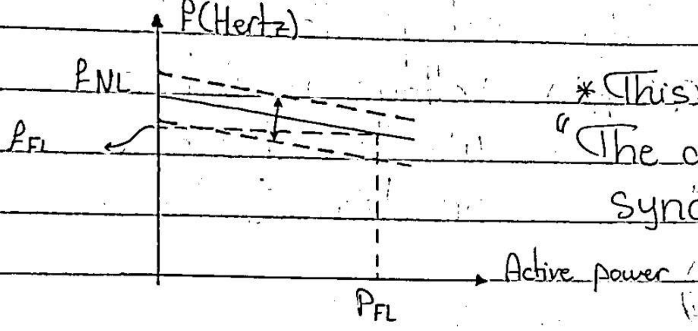
When the sneedle is in vertical Position as shown in the figure, Va is in phase with Va', you can close the switch.

The speed droop (SD) = $\frac{\eta_{NL} - \eta_{FL}}{\eta_{FL}} \times 100\%$

it's usually fall between (2 - 4)%.



Since $\eta = \frac{120 f}{P}$, then:



* This curve is known as: "The droop characteristic of synchronous generator."

Note: While adjusting the set points of governor mechanism you can increase or decrease η_{NL} "see the 1st figure".

To study the Power from the figures above, we need to study the slope;

$$\text{Slope} = \frac{\Delta Y}{\Delta X} \rightarrow \frac{f_{FL} - f_{NL}}{P_{FL}} \quad [\text{Hertz} / \text{KW}]$$

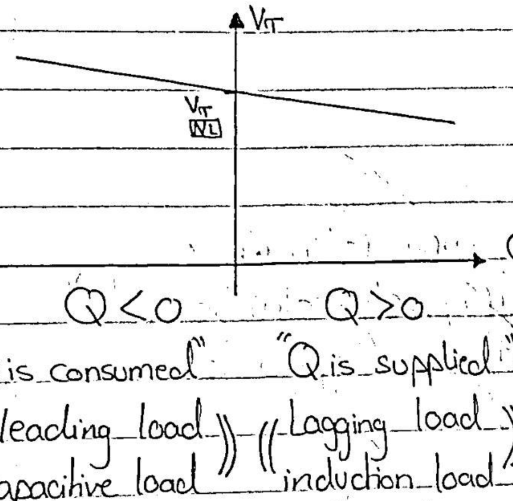
$$P_{FL} = \frac{1}{\text{slope}} (f_{FL} - f_{NL})$$

$$= \frac{1}{\text{slope}'} (f_{NL} - f_{FL})$$

In general, $P = S_P (f_{NL} - f_{FL})$

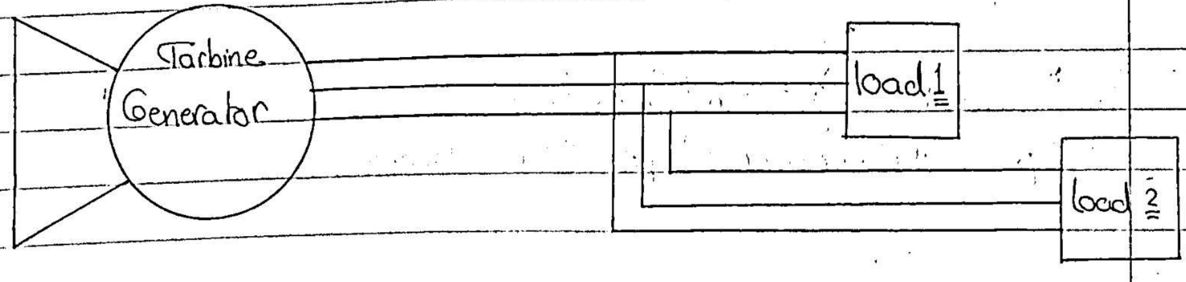
∴ S_P is the power slope [KW / Hertz]

By increasing the lagging loads,
1) The load will consume more Q.
2) The generator will supply more Q.
3) The terminal voltage (V_T) will decrease; The relation between the voltage and the reactive power is non-linear, but they found a way to solve this:
By "The AC Voltage regulation", this device will make the relation linear



* Note:
while adjusting the set points of the voltage regulation we can increase or decrease $V_{T(N-L)}$

Example: The figure shows a generator supplying a load. A second load is to be connected in parallel with the first one. The generator has a no load frequency of 61 Hz and a slope s_P = 1MW/Hz. load 1 consumes a real power of 1000 kW = 1MW at PF 0.8 lagging, while the second load consumes a real power of 800kW at 0.707 lagging PF.
a) Before the switch is closed, what is the operating frequency of the system?
b) After load 2 is connected, what is the operating frequency of the system?
c) After load 2 is connected, what action could an operator take to restore the system frequency to 60 Hz.



Solution:

[1] lets summarize the given info:

$S_p = 1MW/Hz$ $P_{load1} = 1000kW$

$f_{NL} = 61\ Hz$ $P_{load2} = 800kW$

[2] we know the relation:

$$P = s_p(f_{NL} - f_{FL}) \rightarrow f_{FL} = f_{NL} - \frac{P}{s_p}$$

a) $f_{FL} = f_{system}$

$$= 61 - \frac{1000k}{1000k} \Rightarrow \boxed{60\ Hz}$$ ∴ P here is only the load 1 power.

b) $$f_{FL} = 61 - \frac{1800k}{1000k} \Rightarrow \boxed{59.2\ Hz}$$ ∴ P here = $P_{load\,1} + P_{load\,2}$.

c) $$60 = f_{NL} - \frac{1800k}{1000k} \rightarrow f_{NL} = 61.8$$, lets study whats happened here:

when load2 is connected the frequency falls to 59.2Hz, we need to restore the system to its proper operating frequency, we know that f_{NL} at the starting was 61, so we need to use the governor to rise the f_{NL} 0.8 Hz so we will restore the system frequency to 60 Hz.

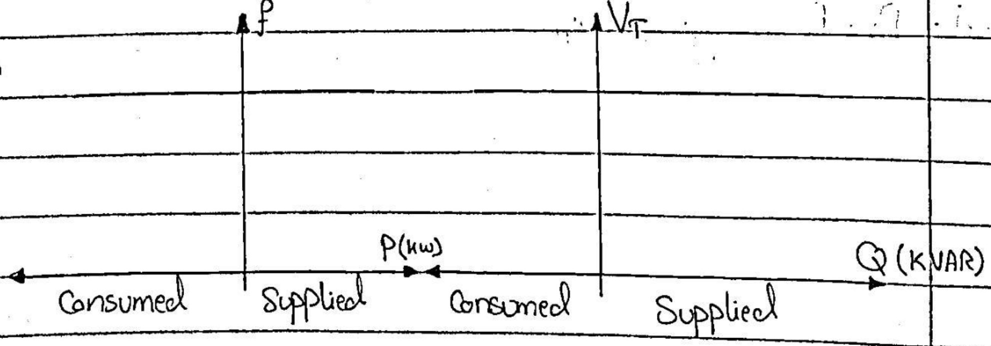
→ Operation of generators in Parallel with large power systems.

This idea is idealized in the concept of an infinite bus;

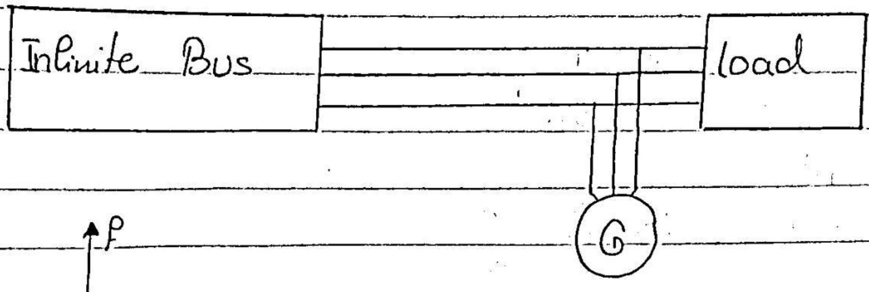
Infinite Bus: Is a large power system that its voltage and frequency do not vary regardless of how much real and reactive power is drawn from or supplied to it.

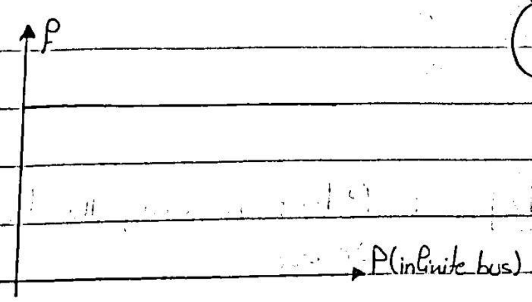
[1] "The frequency is" constant

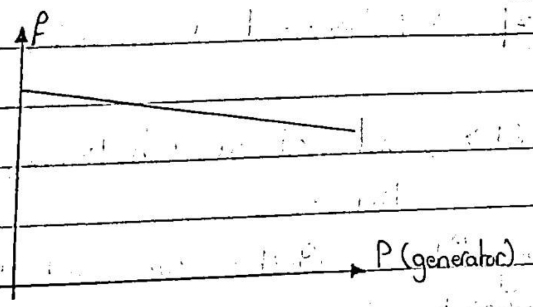
[2] "The voltage is" constant



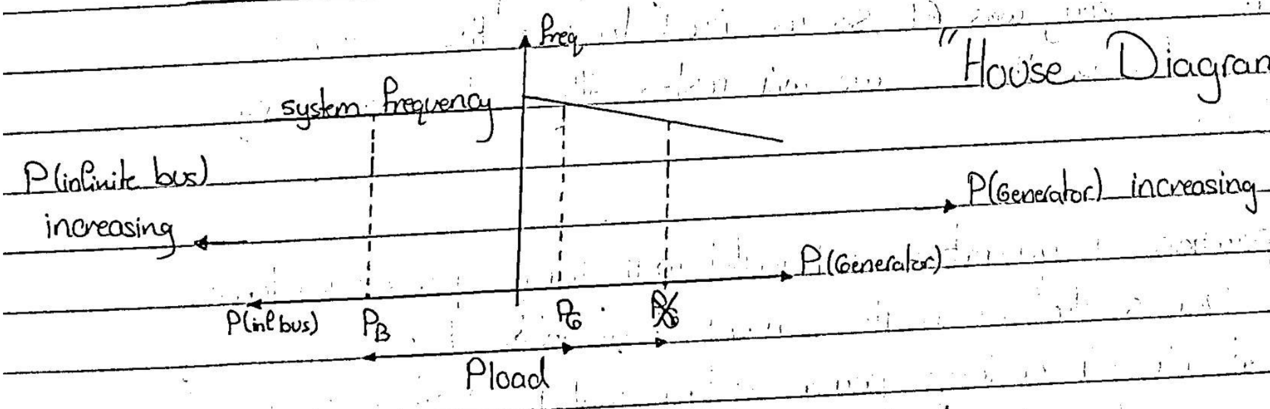
→ Parallel operation with generators:





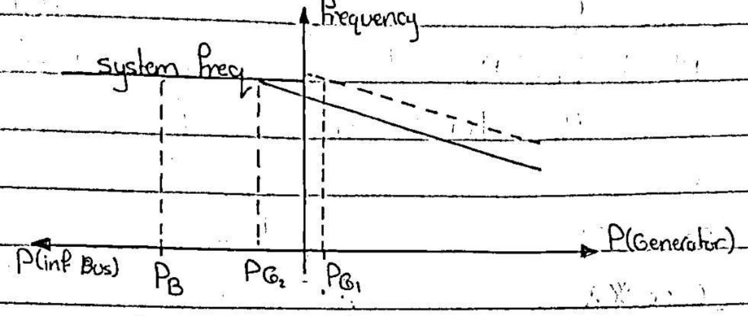


"House Diagram":



∴ P_B: Power supplied by the infinite bus to the load.
P_G: Power supplied by the oncoming generator to the load.
$P_{load} = P_B + P_G$

At Synchronizing:



If the frequency of the oncoming generator is slightly higher than the system frequency, then:

① P_G is small and positive.

② The oncoming machine acts like a generator.

If the frequency of the oncoming generator is slightly lower than the system frequency, then:

① P_G is small and negative.

② The oncoming machine acts like a Motor.

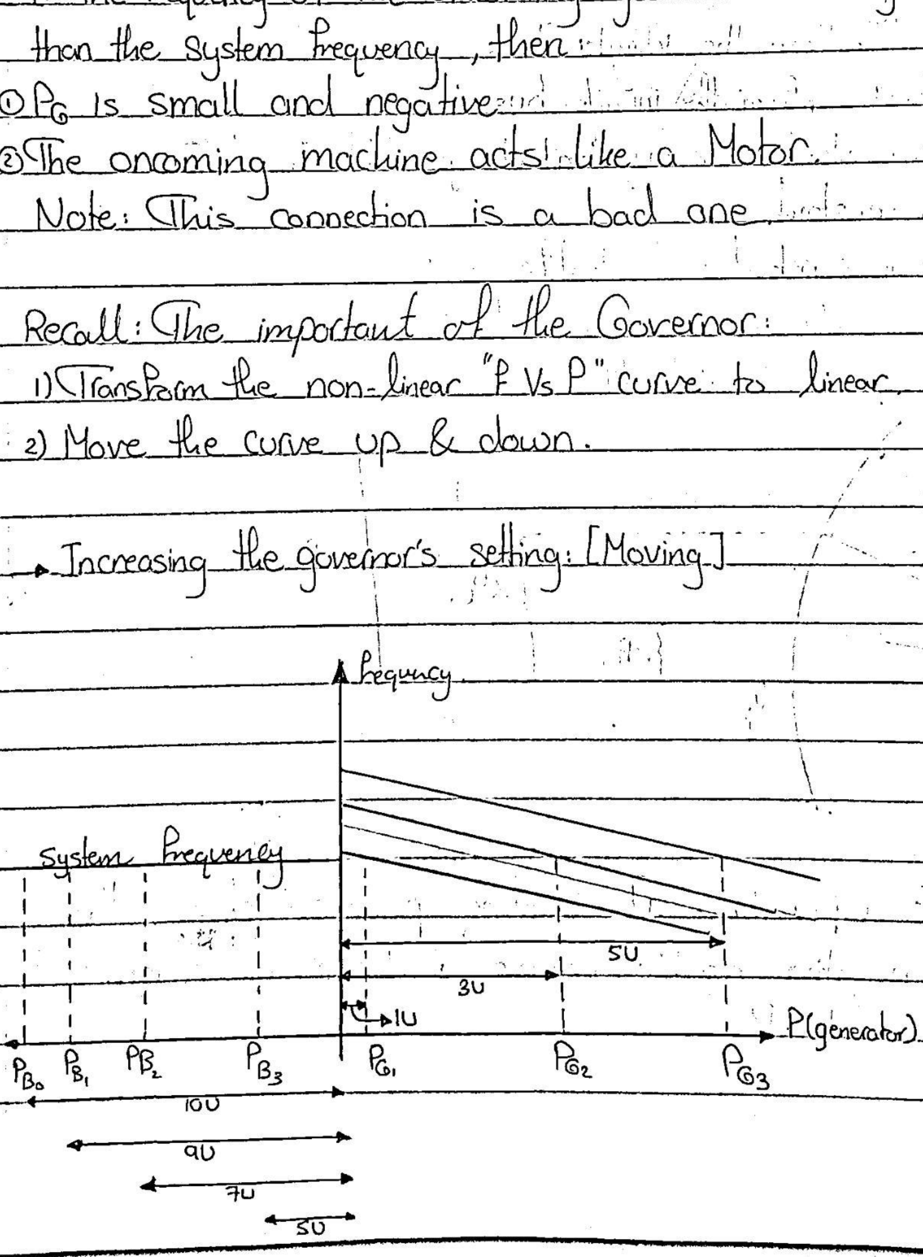
Note: This connection is a bad one.

Recall: The important of the Governor:

1) Transform the non-linear "f Vs P" curve to linear.

2) Move the curve up & down.

→ Increasing the governor's setting: [Moving]



→ see the ※ as example.

I let the load need 10U of power, U: unit (KW, MW, GW) ...

Case ※0:

$P_{load} = P_{B0} + P_{G0}$

$= 10U + 0$

Case ※1

$P_{load} = P_{B1} + P_{G1}$

$= 9U + 1U$

Case ※2:

$P_{load} = P_{B2} + P_{B2}$

$= 7U + 3U$

Case ※3

$P_{load} = P_{B3} + P_{G3}$

$= 5U + 5U$

If the generator settings ↑ → Then P_G↑ & P_B↓

→ The Phasor Diagram:

$P(generator) = 3V_\phi \frac{E_A}{X_s} \sin\delta$

→ V_ϕ is constant → from the infinite bus.

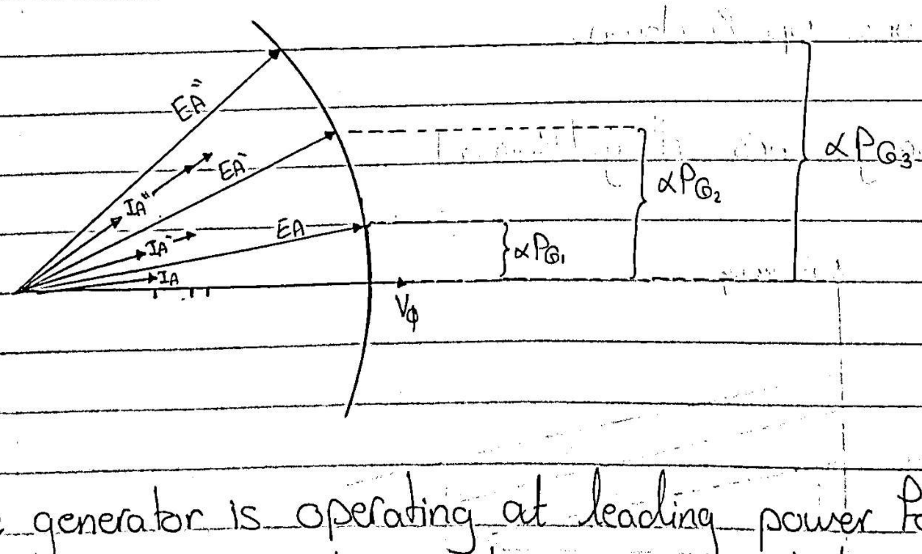
$X_s = \omega L_s$ is constant → from the infinite bus

$E_A = K\phi\omega$, ϕ is constant since we don't change field current (I_F)

ω is constant

→ E_A is constant.

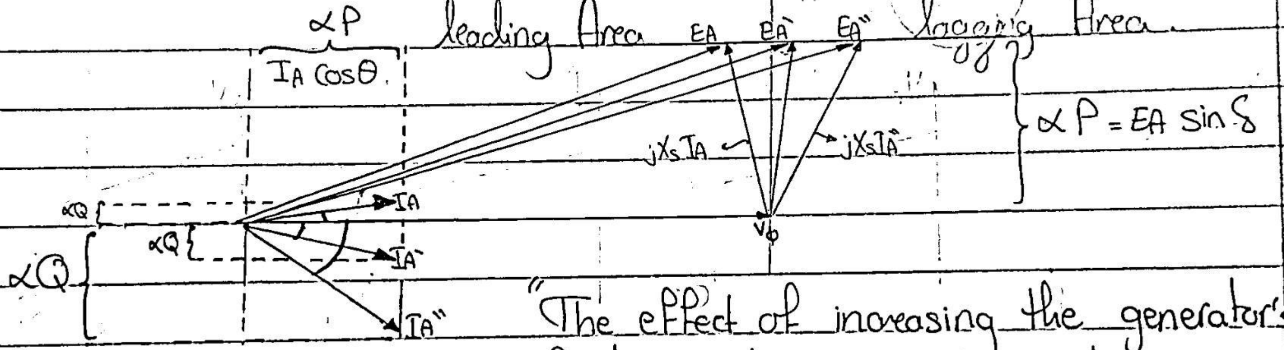
$P_G \propto E_A \sin\delta$



The generator is operating at leading power factor, the generator supplies negative reactive power or it absorbs reactive power from the infinite Bus.

Now a new question appears "How Can the Generator be adjusted to supply reactive power".
By Changing the excitation (field current):
The Q_G is supplied by controlling the field current.
Adjustment of generator reactive power.
P_G is constant.
$P_G = 3V_\phi \frac{E_A}{X_s} \sin\delta = 3V_\phi I_A \cos\theta$. , $\because V_\phi$ is constant [infinite bus].

Since $P_G \propto E_A \sin\delta$, $P_G \propto I_A \cos\theta$, and it's constant.
$\rightarrow E_A \sin\delta$ = Constant.
$I_A \cos\theta$ = constant.



"The effect of increasing the generator's field current on the phasor diagram of the Machine".

The generators consume Q from the infinite bus "under-excitation" process, leading Area.

If $I_F \downarrow$: then $Q_{absorbed} \uparrow$
$\theta \uparrow$, PF $[\cos\theta] \downarrow$

$\rightarrow$ The generators supply Q [reactive power] "Over-excitation". lagging Area,
If $I_F \uparrow$: $Q_{supplied} \uparrow$.

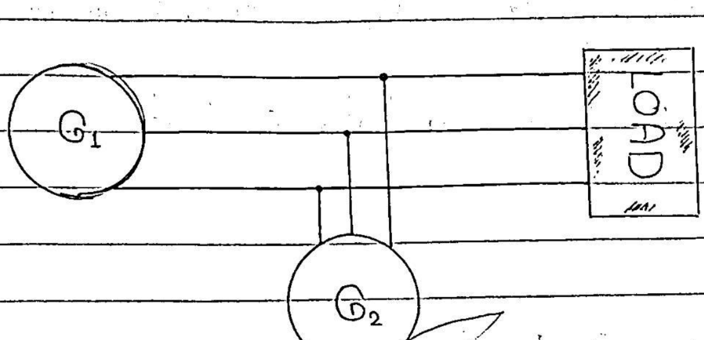
Conclusion:

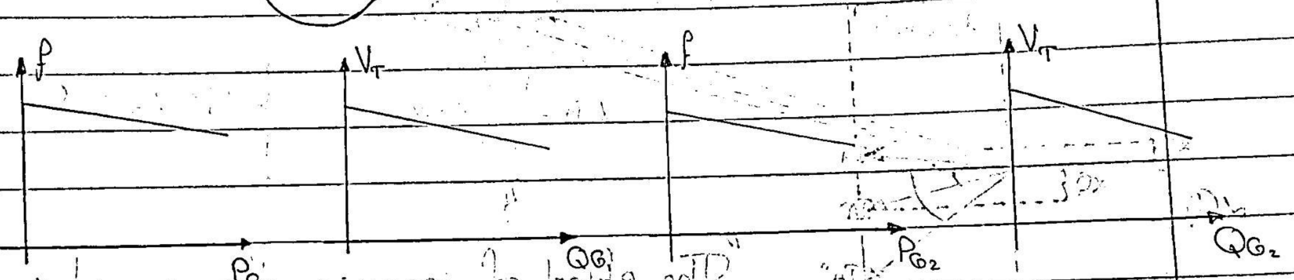
① V_T & f_{sys} are controlled via the infinite bus.

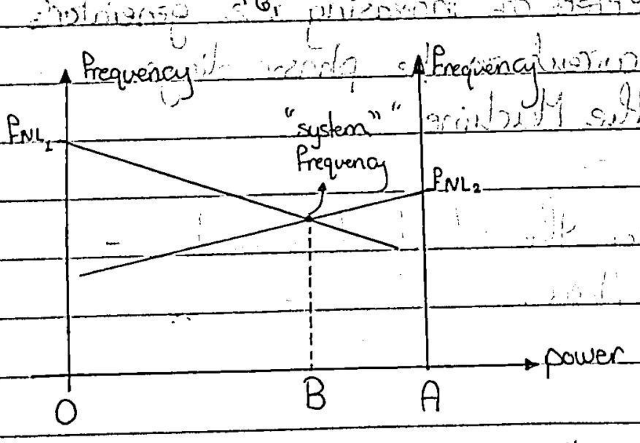
② P_G is controlled via the governor setting.

③ Q_G is controlled via the excitation.

→ Parallel operation of generators with other generators of the same size:







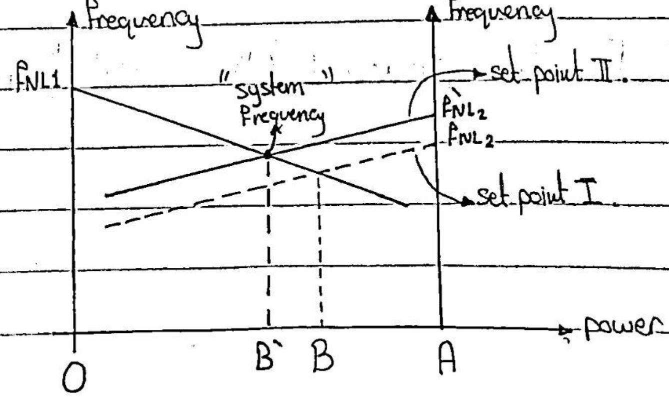
$\overline{OA} = P_{load}$

$\overline{OB} = P_{G1}$

$\overline{BA} = P_{G2}$

$P_{load} = P_{G1} + P_{G2}$

Changing the governor setting.



$\overline{OB`} = P_{G1}'$ @ set point II

$\overline{B`A} = P_{G2}'$ @ set point II

$\overline{OA} = P_{load}$

$\overline{OB} = P_{G1}$ @ set point I

$\overline{BA} = P_{G2}$ @ set point I

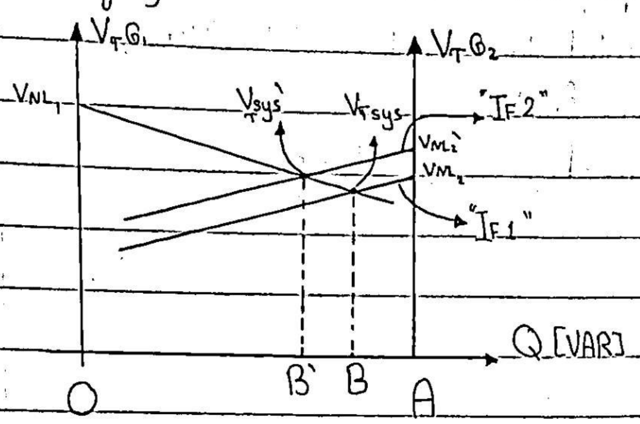
Conclusion :

Changing the governor setting "P_{G_2}" ↑

Then the system frequency ↑

P_{G_1} ↓ , P_{G_2} ↑ such that , $P_{G_1} + P_{G_2} = P_{load}$

Changing the excitation.



$\overline{OA} = Q_{load}$

$\overline{OB} = Q_{G_1}$ @ I_{F1}

$\overline{BA} = Q_{G_2}$ @ I_{F1}

$\overline{OB`} = Q_{G_1}$ @ I_{F2}

$\overline{B`A} = Q_{G_2}$ @ I_{F2}

Conclusion :

$I_{F_{G_2}} \uparrow \rightarrow V_{T_{G_2}} \uparrow \rightarrow Q_{G_1} \downarrow$ & $Q_{G_2} \uparrow$

Examples :

[1] Two generators supplying a load,

G_1 : $f_{NL} = 61.5$ Hz and $sp = 1$ MW/Hz

G_2 : $f_{NL} = 61.0$ Hz and $sp = 1$ MW/Hz .

The two generators are supplying a real load totaling 2.5 MW at 0.8 PF lagging.

a) At What frequency is the system operating?, How much power is supplied by each generator?

$P_1 = S_{P_1}(f_{NL1} - f_{sys})$

$P_2 = S_{P_2}(f_{NL2} - f_{sys})$

$P_{load} = P_1 + P_2$

$2.5M = \frac{1M}{Hz}(61.5 - f_{sys}) + \frac{1M}{Hz}(61 - f_{sys})$ solving

$\boxed{f_{sys} = 60 \text{ Hz}}$

Now $P_1 = \frac{1MW}{Hz}(61.5 - 60) = 1.5MW$.

$P_2 = P_{load} - P_1 = 1 MW$.

b) Suppose an additional 1 MW load were attached to this power system. What would the new system frequency be?, How much power would G_1 and G_2 supply now?

When the 1MW load is added → The total load will be 3.5 MW,

$P_{load} = S_{P_1}(f_{NL_1} - f_{sys}) + S_{P_2}(f_{NL_2} - f_{sys})$

$3.5M = 1M(61.5 - f_{sys}) + 1M(61.0 - f_{sys})$ → Solving ...

$f_{sys} = 59.5\ Hz$.

Now $P_1 = 1M(61.5 - 59.5) = 2 MW$

$P_2 = P_{load} - P_1 = 1.5 MW$.

c) With the system in part b; What will the system frequency and generators powers be if the governor set points on G_2 are increased by 0.5 Hz?

$P_{load} = S_{P_1}(f_{NL_1} - f_{sys}) + S_{P_2}(f_{NL_2} - f_{sys})$

$3.5M = 1M(61.5 - f_{sys}) + 1M(61.5 - f_{sys})$, solving ...

$f_{sys} = 59.75\ Hz$

Note they have the same frequency & slope so, $P_1 = P_2$

$P_1 = P_2 = S_P(f_{NL} - f_{sys})$

$= 1M(61.5 - 59.75)$

$= \boxed{1.75\ MW}$.

Notice: The f_{sys} rose, Power of G_2 rose, Power of G_1 fell.

[2] A 480-V, 50-Hz, Y connected, 6 poles SG is rated at 50K VA at 0.8 PF lagging, It has a synchronous reactance of 1Ω per-phase [~~Assume that this generator is connected to a steam turbine capable of supplying up to 45kW~~]

The friction and winding losses are 1.5 kW and the core losses are 1kW. It's rated current is 60A at 0.8 lagging

The field current has been adjusted so that $V_T = 480V$ at no-load.

a) What is the speed of rotation of this generator:

$\rightarrow W_m = \frac{2}{P} W_e$

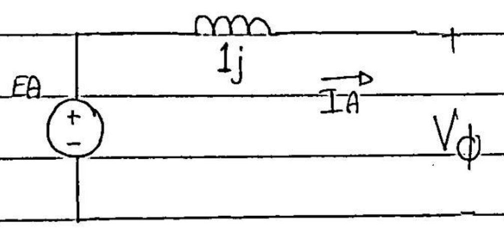
Note: $\eta = \frac{120}{P} f_e$

$= \frac{2}{6}(2\pi(50))$

$W_m = 1000 \frac{\pi}{30}$ rad/sec

b) What is the terminal voltage of the generator at full load:

$\triangleright$ per-phase equivalent circuit:



$\therefore I_A = 60 \angle -\cos^{-1}(0.8)$

$E_A = 480$ volt, at no-load.

$\vec{E_A} = jX_s\vec{I_A} + \vec{V_\phi} \Rightarrow \sqrt{\frac{2}{3}}(480)\angle\delta = j(60\angle -36.87) + V_\phi$

$\therefore \sqrt{\frac{2}{3}}$ is added $\Rightarrow \sqrt{2}$ to have the peak value

$1/\sqrt{3}$ to have a phase voltage not line voltage.

$\therefore$ Math note: $\vec{E_A} = jX_s\vec{I_A} + \vec{V_\phi}$ is two equations not one.

one is the magnitude & the other is the angle.

since we need $|V_\phi|$, we will use the magnitude equation:

$\therefore j(60\angle -36.87) = 36 + 48j$

$\sqrt{\frac{2}{3}}(480) = \sqrt{(V_\phi + 36)^2 + (48)^2}$

$V_\phi = 352.96$ volt "peak value" $\rightarrow V_\phi = 250$ volt rms.

V_T: The terminal voltage, The line voltage

$= \sqrt{3} V_\phi \Rightarrow 410$ volt

c) What is the efficiency under full-load condition?

$\eta = \frac{P_{out}}{P_{in}}$:

$P_{out} = \sqrt{3}\, V_T\, I_A\, PF$

$= \sqrt{3}\,(410)(60)\,0.8$

$= 34.1\ kW$

$P_{in} = P_{out} + P_{core\ losses} + P_{losses\_windage}$

$= 34.1 + 1 + 1.5$

$= 36.6\ kW$

$\eta = \frac{34.1}{36.6} = 93.2\%$

d) Calculate the applied Torque

$T_{app} = \frac{P_{in}}{\omega_s}$

$= \frac{36.6\ kW}{1000\left(\frac{\pi}{30}\right)}$

$= 350\ N.m.$

Chapter 5: Synchronous Motor

Principle of Operation.

i. The field current produces the ~~stator~~ rotor magnetic field $\vec{B_s}$.

ii. A set of 3ϕ voltage is applied to stator windings. which produces a set of 3ϕ current to flow in the stator windings.

iii. The current will produce a rotating stator magnetic field $\vec{B_s}$ which rotates at synchronous speed $\underline{\underline{\omega_s}}$

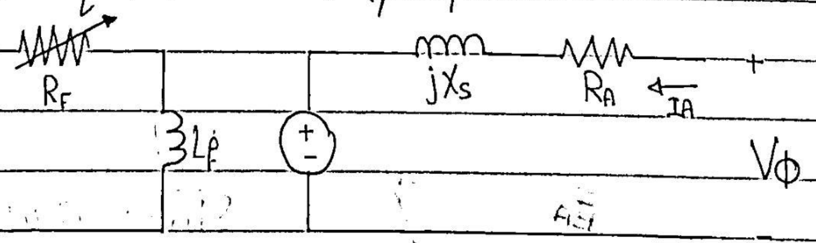
iv. The rotor is rotating at starting by some external means to obtain a magnetic locking between the rotor & stator poles, then produce a continous induced torque "τ_{ind}".

Recall:

$$\tau_{ind} = K\vec{B_r} \times \vec{B_s} = K\vec{B_r} \times \vec{B_{net}}$$

$$= K B_r B_{net} \sin\delta \quad \therefore \delta = \text{"Torque angle"}$$

- The Equivelant circuit (per-phase):



Field Armature

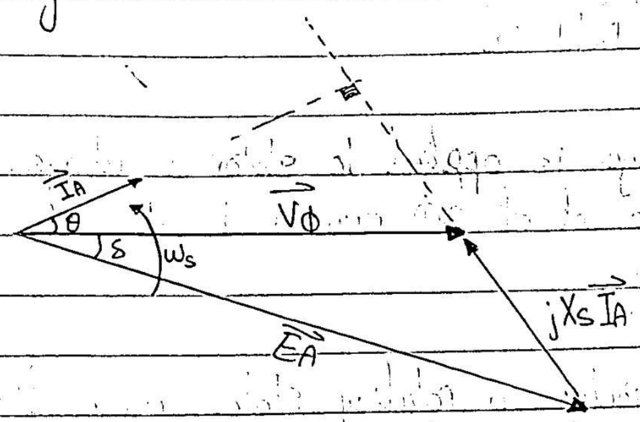
∴ R_A: Is the armature Resistance.
X_s: Synchronous Resistance.

→ KVL at the armature circuit:

$$\vec{V_\phi} = \vec{E_A} + R_A\vec{I_A} + jX_s\vec{I_A}$$

→ Phaser Diagram:

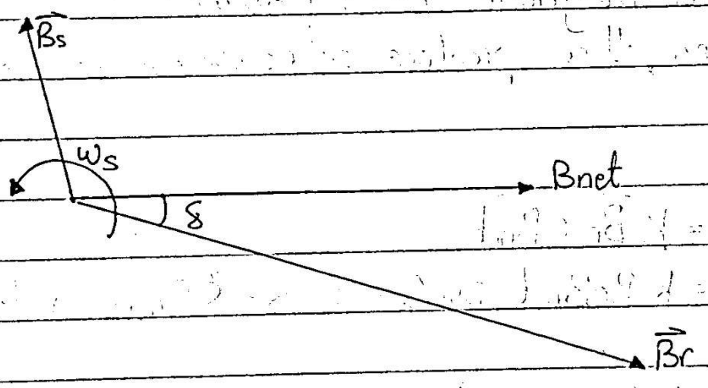
A) "Motor":



∴ $\vec{B_r}$ corresponds to $\vec{E_A}$

$\vec{B_{net}}$ 〃 〃 $\vec{V_\phi}$

$\vec{B_s}$ 〃 〃 $j X_s \vec{I_A}$

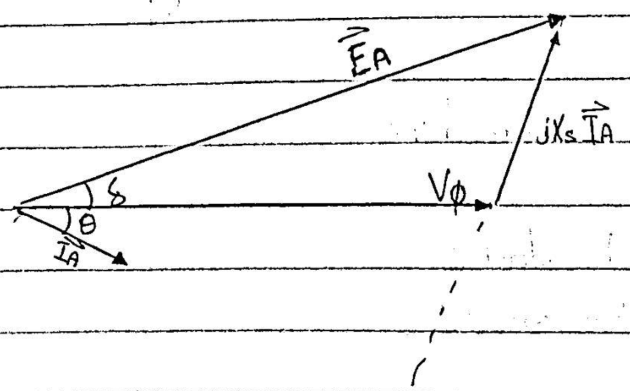


Note:

$\omega_s \Rightarrow$ C.C.W

$\tau_{ind} \Rightarrow$ C.C.W

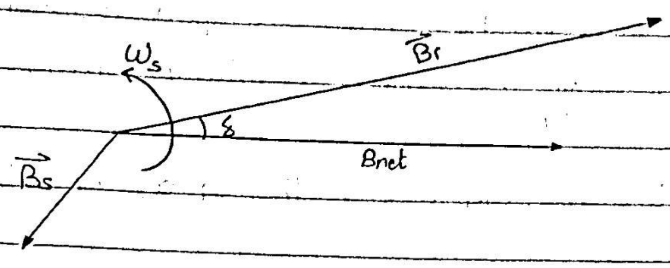
B) "Generator":



Note:

τ_{ind}: C.W

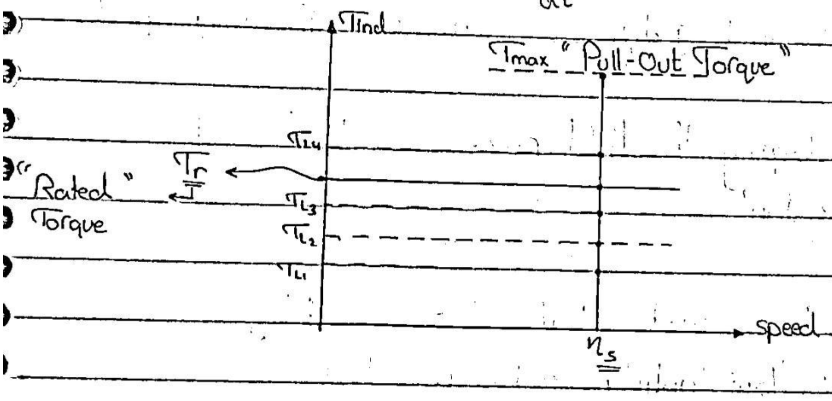
ω_s: C.C.W



ω_s : C.C.W.
τ_{ind} : C.W.

→ Torque speed characteristics:

$$\tau_{ind} - \tau_{load} = J \frac{d\omega}{dt}$$



*Note: Another Name for the rated torque is the nominal torque.

: n_s : Synchronous speed

The induced torque is always equal the load torque so,

$\tau_{ind} = \tau_{load} \rightarrow \tau_{ind} - \tau_{load} = 0$ " $\rightarrow 0 = J \frac{d\omega}{dt} \rightarrow \omega$ is constant $= n_s$

Def: Pull-Out torque: It is the maximum operating torque of "SM".

* $\tau_{max} = (2.5 - 3)\, \tau_r$

Torque equation:

$$\tau_{ind} = \frac{3 V_\phi E_A}{X_s \omega_s} \sin\delta$$

$$\tau_{max} = \tau_{pull\text{-}out} = \frac{3 V_\phi E_A}{X_s \omega_s}$$ since $\delta = 90°$ "max-torque angle".

If $\tau_{load} > \tau_{max}$, The motor will lose the synchronism.

"Slipping pole phenomenon"

Speed Regulation :

Since the SM always operates at the synchronous speed we can see that its speed regulation $= \frac{\eta_{NL} - \eta_{FL}}{\eta_{FL}} \times 100\%$ is always zero.

"$\eta_{NL} = \eta_{FL} = \eta_S$", from this we can consider the SM as an idealy performed Machine.

SR = 0 %

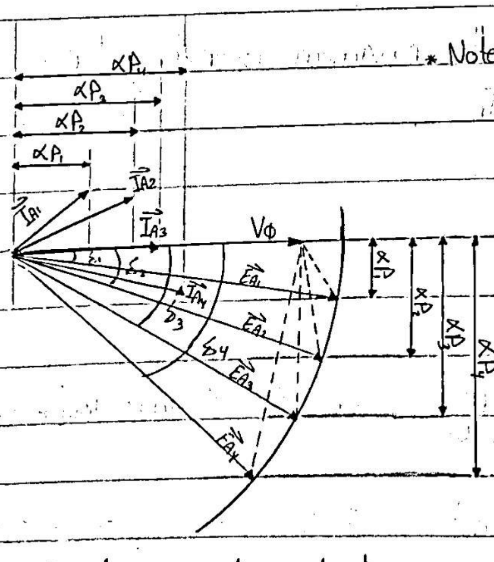
→ Effect of load changes:

$\tau_{ind} = \frac{3 V_\phi E_A}{X_s \omega_s} \sin\delta$ ① $\tau_{ind} - \tau_{load} = J \frac{d\omega_r}{dt}$ ② ; ω_r: The rotor speed.

Since we are changing the load but keeping the field current constant
The induced voltage $(E_A) = K\phi\omega_s$ is constant.

Assume that $\tau_{load} \uparrow$ → from Newton's law $\omega_r \downarrow$ → but $\delta \uparrow$ so $\sin\delta \uparrow$ →
→ $\tau_{ind} \uparrow$ from ① → $\omega_r \uparrow$ from Newton's law until it reaches ω_s but at larger δ.

Phasor Diagram: "Load increases ⇒ $\vec{I_A}$ increases"



* Note: leading : Q is supplied from the Motor.
unity : Q = 0
lagging : Q is absorbed by the Motor.

$E_{A1} = E_{A2} = E_{A3} = E_{A4}$

When the load ↑ → The P.F becomes less leading → unity → lagging → more lagging.

→ Effect of the Field current changes:

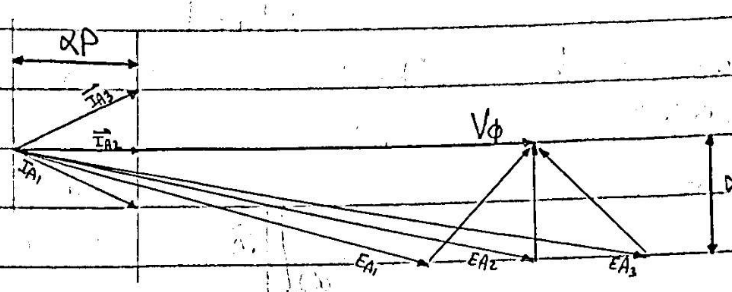
Assume P is constant.

$P = \frac{3 V_\phi E_A}{X_s} \sin\delta = 3 V_\phi I_A \cos\theta = \text{Constant}$.

$E_A \sin\delta = \text{constant}$, $I_A \cos\theta = \text{constant}$

$E_A = k\phi\omega$ ∴ $\phi \propto I_F$.

∴ $E_{A3} > E_{A2} > E_{A1}$



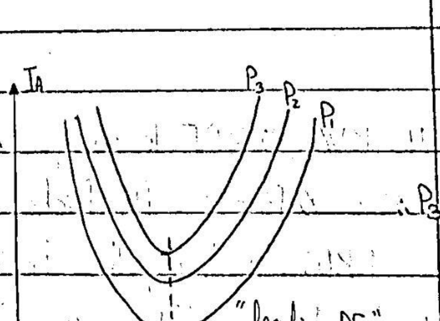
when $\vec{I_A}$ lags $\vec{V_\phi}$ then we are operating in the "Under-excited region"
when $\vec{I_A}$ leads V_ϕ then we are operating in the "Over-excited region"
Under-excited means E_A is less than the Unity E_A
Over-excited means E_A is more than the Unity E_A

When $I_F \uparrow$, The PF becomes less lagging → unity → leading → more leading.

Conclusion: 1) The real power is controlled by the load changes.
2) The reactive power is controlled by the field current.

→ V curves of SM:

1) The lagging PF region:
The field current is small "underexcited", Q is consumed, the motor is operating as an inductive load
$E_A \cos\delta < V_\phi$.



∴ $P_3 > P_2 > P_1$

2) The leading PF region:
The field current is large "Overexcited", Q is supplied. The Motor operates as a capacitive load. $E_A \cos\delta > V_\phi$.

→ Synchronous Condensor: "Synchronous Capacitor".

It is a synchronous motor without shaft operated at no load "$P = 0$" and if it is over-excited, [Q is supplied].

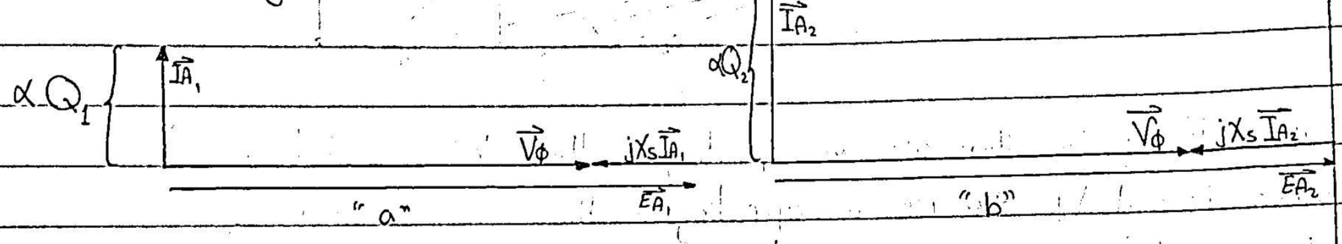
It was used historically as a power factor correction device.

$P = \frac{3V_\phi E_A}{X_s}\sin\delta = 3V_\phi I_A\cos\theta$, and [since "$P=0$" and ($E_A$ & I_A) have numerical values]

then the only way to have a "$P=0$":

"$\delta = 0$ & $\theta = 90°$"

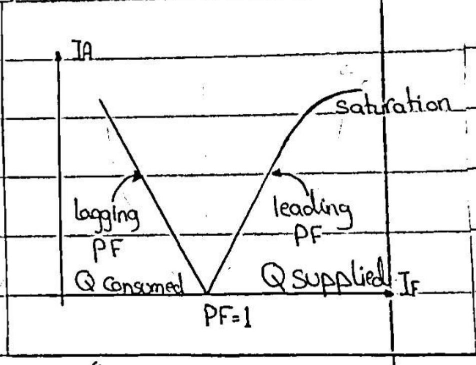
Phasor Diagram:



"a" & "b" represent how to control the reactive power.

Power Factor Correction can be adjusted by:

[1] Synchronous condensor.
[2] Static Capacitor.
[3] Power electronics, "Static VAR compensator".



"V curve for SC"

Example:

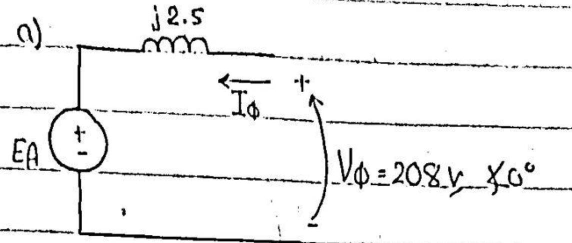
208 V, 45 KVA, 0.8 PF leading, Δ-connected, 60 Hz, synchronous machine Has $X_s = 2.5\,\Omega$ & $R_s = 0$. Its friction and windage losses are 1.5 KW and its core losses are 1 KW. Initially the shaft is supplied a 15 hp load and the motor PF is 0.8 leading. "1 hp = 746 W"

a) Find the value of $\vec{I_\phi}$, $\vec{I_L}$, $\vec{E_A}$?

b) Assume that the shaft load is now increased to 30 hp, find $\vec{I_\phi}$, $\vec{I_L}$ & $\vec{E_A}$? What is the new motor PF?

Solution:

a)



$V_\phi = V_L$ since its Δ connected.

KVL on the armature Circuit:

$\vec{V_\phi} = X_s \vec{I_A} + \vec{E_A}$

$208\angle 0° = (j2.5)\vec{I_A} + \vec{E_A}$, two unknowns and one equation:

given the output power = 15hp.

P_{out} [mechanical] = 15×746 = 11.19 KW.

to find $\vec{I_\phi} = \vec{I_A} \rightarrow P_{in} = P_{out} + P_{losses}$

$P_{in} = 13.69\,KW = 3V_\phi I_\phi \cos\theta$

$\Rightarrow I_\phi = \dfrac{13.69\,K}{3(208)(0.8)} \Rightarrow 27.4 \angle{+36.87°}\,A$ ①

$I_L = \sqrt{3}\,I_\phi = 47.5\,A$ ②

$\vec{E_A} = 208\angle 0° - j(2.5)(27.4)\angle 36.87°$

$= 255 \angle{-12.4°}$

b) KVL on the armature circuit:

$208\angle 0° = 2.5j\,\vec{I_A} + \vec{E_A}$,

since $\vec{E_A} = K\phi\omega$ and we don't change the ϕ so $|\vec{E_A}|$ is constant.

$\vec{E_A} = 255 \angle \delta'$ :: r

To find $\vec{I_A}$ lets go back to the KVL

$\vec{I_A} = \dfrac{208\angle 0° - \vec{E_A}}{(2.5)j}$

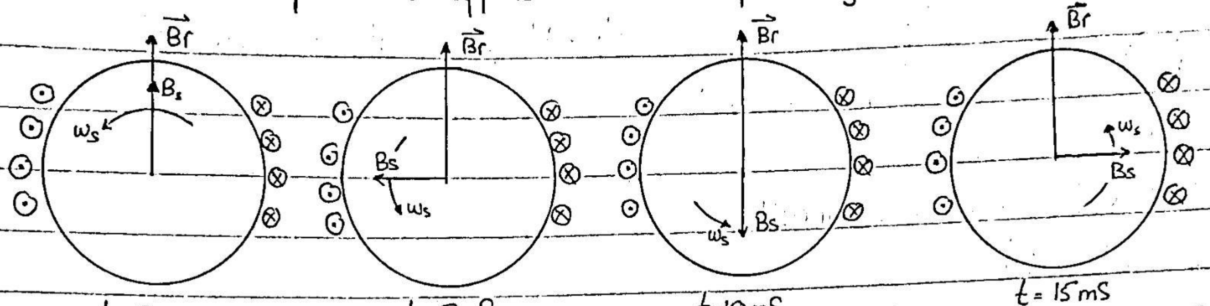
$I_A\angle\theta = \dfrac{208\angle 0° - 255\angle\delta'}{(2.5)j}$, the challenge now to find δ'

P_{out} = 30 hp = 22.38 KW	$\Rightarrow 24.88k = \dfrac{3V_\phi E_A}{X_s}\sin\delta$
$P_{in} = P_{out} + P_{losses}$	$\delta' = 23°$
$= (22.38k + 2.5k)$	$\vec{I_A} = \dfrac{208\angle 0° - 255\angle{-23}}{(2.5)j} = 41.27\angle 15°\,A$
$= 24.88K$	

PF = $\cos^{-1}(-15)$ = 0.966 leading.

→ Starting of Synchronous Motor

Assume that a 50 Hz power is applied to a two-pole synchronous machine "Motor".



t = 0 sec   t = 5mS   t = 10mS   t = 15mS

∴ ω_s = Synchronous Speed.

Note: 5ms, 10mS & 15ms are chosen since $f = 50$ Hz, so $T = \frac{1}{50} = 20$ mS

and we are dealing with 2-pole Motor "$\omega_m = \omega_s$"

$\tau_{ind} = k \vec{B_r} \times B_s$, So at:

$t = 0$ sec → $\tau_{ind} = 0$ * Here the problem has appeared,

$t = 5$ sec → τ_{ind} : C.C.W   The Motor will vibrate heavily causing overheating

$t = 10$ sec → $\tau_{ind} = 0$

$t = 15$ sec → τ_{ind} : C.W

Methods to start the synchronous Motor:

1 Using external prime mover:

Run the machine as a generator using the prime mover.

- increase the rotor speed up to ω_s.
- Disconnect the prime mover.

2. Using damper winding or amortisseur windings:

This method will start up the synchronous motor like 3-Φ induction Motor.

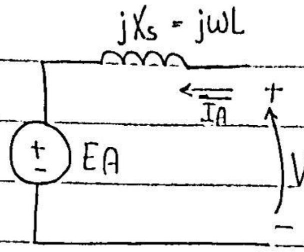
3. Reducing the electric frequency

$n_s = \frac{120 f_e}{P}$

When f_e is small the speed of Bs will be also small.

- The rotor can accelerate and lock in with Bs.
- When the motor is started, increase the electric frequency gradually up to its normal value.

⚠ But wait,

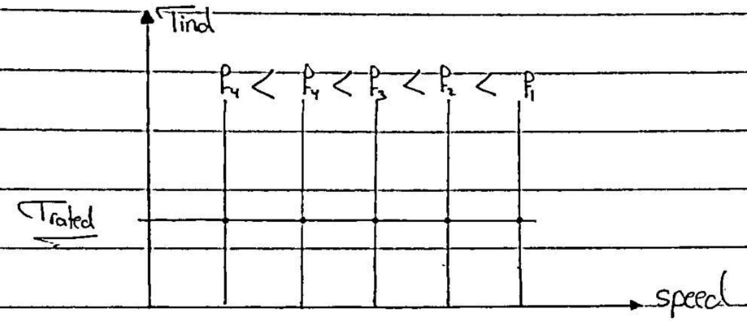


Since f_e is small the ω is small.
This will lead to "jX_s is small"
$\rightarrow V_\phi \simeq E_A$

Therefore, $V_\phi = K\phi\omega$; SO:

$\phi \propto \frac{V_\phi}{\omega}$, to avoid increasing the flux to the saturation, the ratio of $\frac{V_\phi}{\omega}$ should be appropriate.

Thus, the voltage must be decreased with f_e by the same factor to keep ϕ constant and not get saturated.



∴ $\frac{V_\phi}{f_e}$ is constant.

"VVVF Drive" Variable Voltage Variable Frequency

Block Diagram:

∴ f^R: refrence frequency
V^R: refrence Voltage