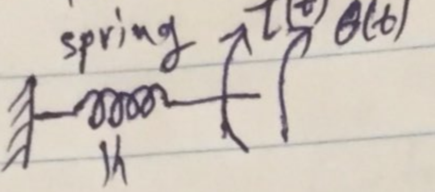
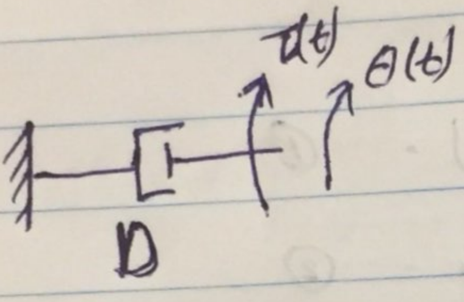
2.6 Rotational Mechenical system transfer function.

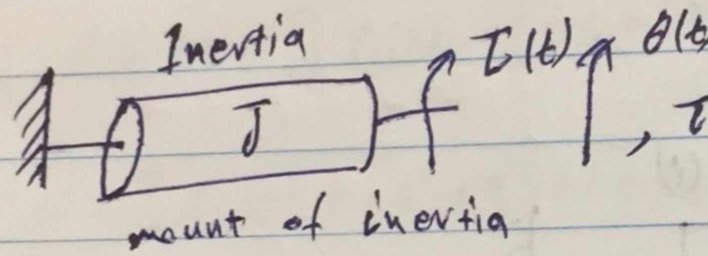
Impeda[nce]



$T(t) = K\,\theta(t)$, K

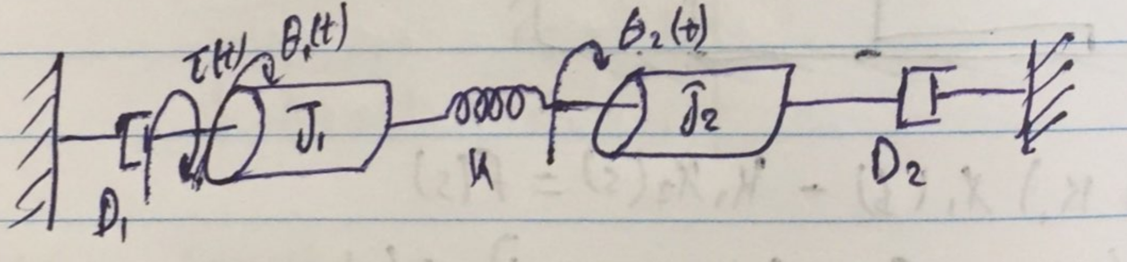


$T(t) = D\frac{d}{dt}\theta(t)$, Ds



$T(t) = J\frac{d^2}{dt^2}\theta(t)$, Js^2

EX1

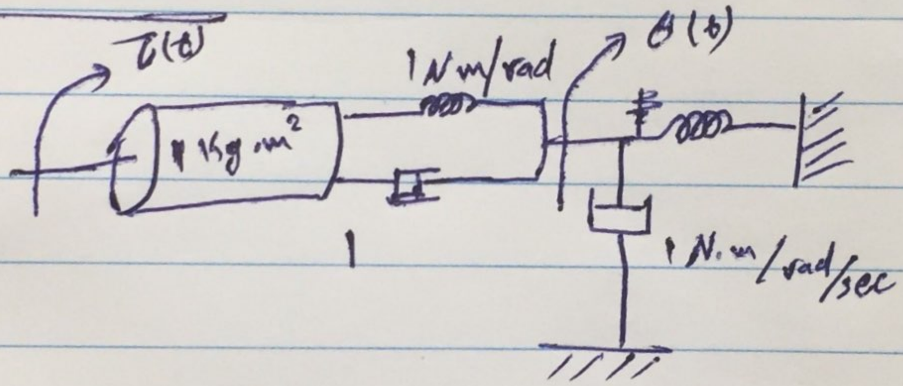


TF $\frac{\theta_2(s)}{T(s)}$

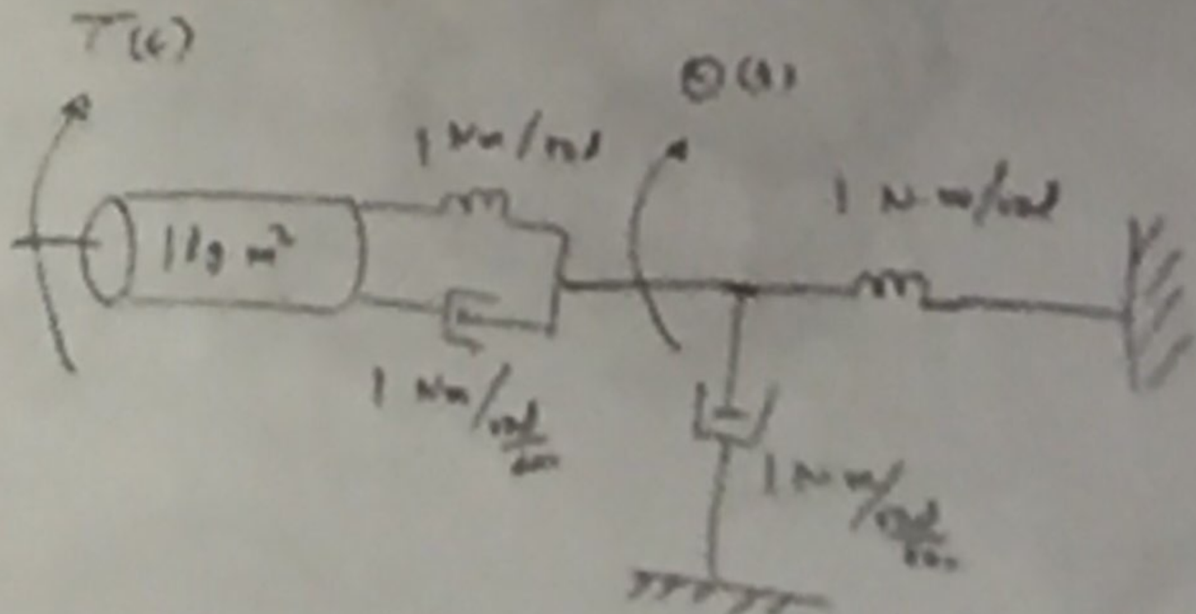
$$[J_1 s^2 + D_1 s + K]\,\theta_1(s) - K\theta_2(s) = T(s)$$

$$-K\theta_1(s) + [J_2 s^2 + D_2 s + K]\,\theta_2(s) = 0$$

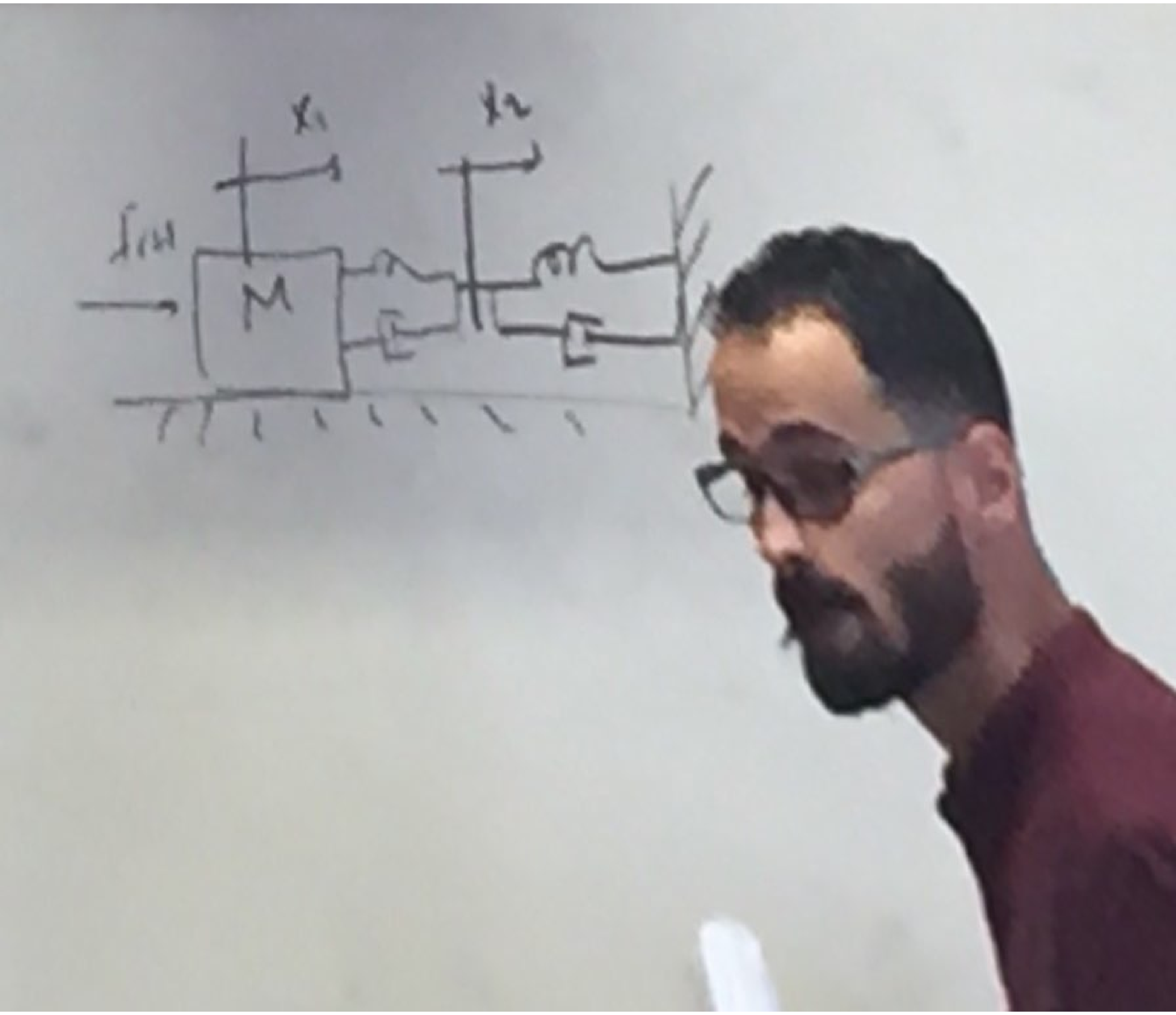
EX|



EX

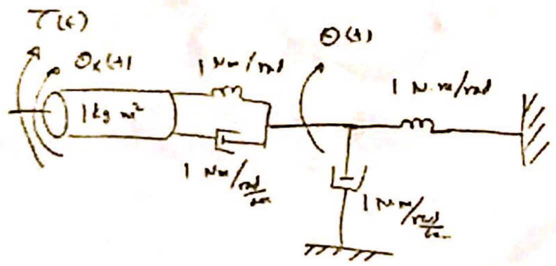


Find the TF $\frac{\Theta(s)}{T(s)}$



x₁
x₂
f(t)
M

EX



Find the TF $\dfrac{\theta(s)}{T(s)}$

$$[s^2+s+1]\,\theta_x(s) - [s+1]\,\theta(s) = T(s) \quad — (1)$$

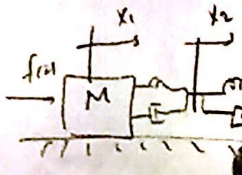
$$-[s+1]\,\theta_x(s) + 2(s+1)\,\theta(s) = 0 \quad — (2)$$

$$\theta(s) = \frac{\begin{vmatrix} s^2+s+1 & T(s) \\ -(s+1) & 0 \end{vmatrix}}{\begin{vmatrix} s^2+s+1 & -(s+1) \\ -(s+1) & 2(s+1) \end{vmatrix}}$$

$$\frac{\theta(s)}{T(s)} = \frac{s+1}{2(s+1)(s^2+s+1) - (s+1)^2}$$

$$= \frac{1}{2s^2+2s+2-s-1}$$

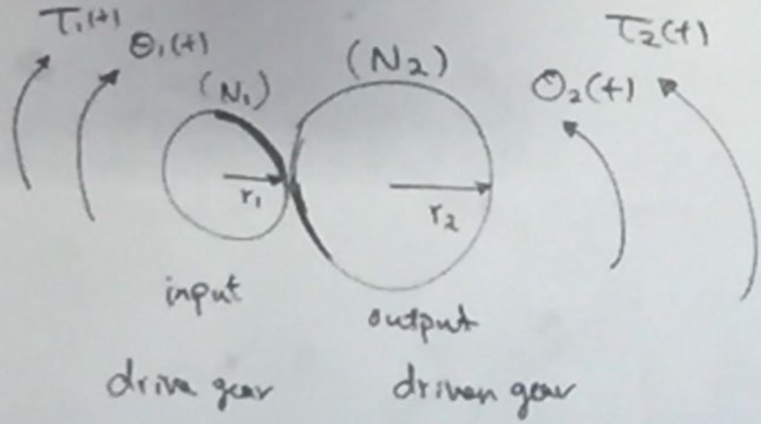
$$= \frac{1}{2s^2+s+1}$$



$+1) - (s+1)^2$

$2 - s - 1$

2.7 | Transfer functions of mechanical systems with gears.



$\theta_1 r_1 = \theta_2 r_2$ → The distance traveled along each gear circumference is the same

$$\boxed{\frac{\theta_1}{\theta_2} = \frac{r_2}{r_1} = \frac{N_2}{N_1}}$$

input energy = output energy

$T_1 \theta_1 = T_2 \theta_2$

$$\frac{\theta_1}{\theta_2} = \frac{T_2}{T_1} = \frac{N_2}{N_1}$$

$\theta_1 \rightarrow \boxed{\frac{N_1}{N_2}} \rightarrow \theta_2$

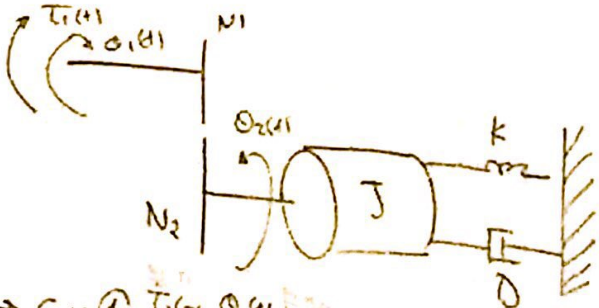
$\frac{T_1}{T_2} = \frac{N_1}{N_2}$

$T_2 = T_1 \frac{N_2}{N_1}$

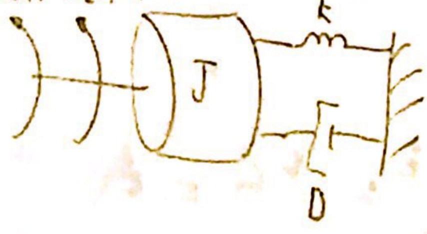
$T_1 \rightarrow \boxed{\frac{N_2}{N_1}} \rightarrow T_2$

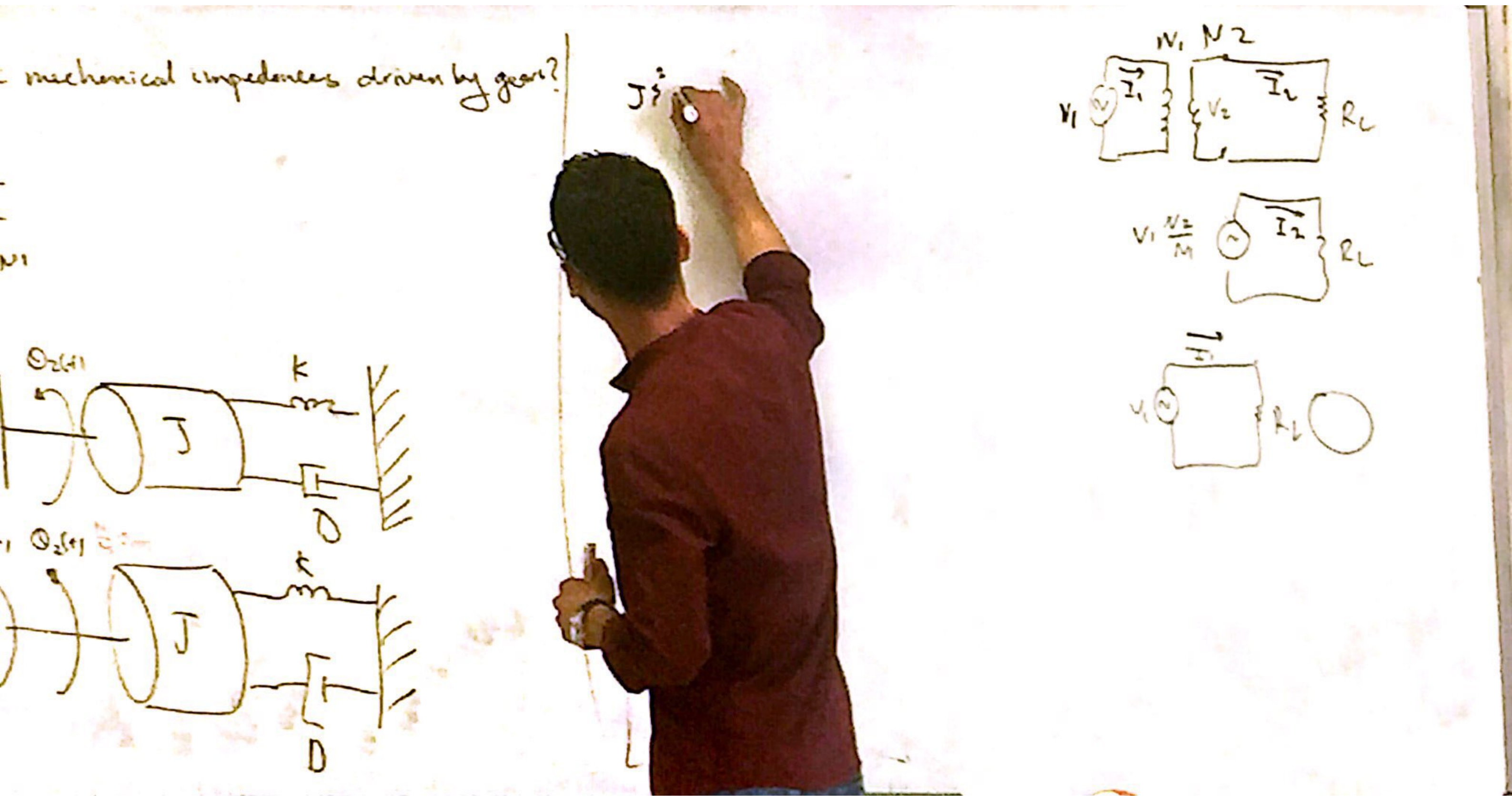
→ what about mechanical impedances driven by gears?

for example

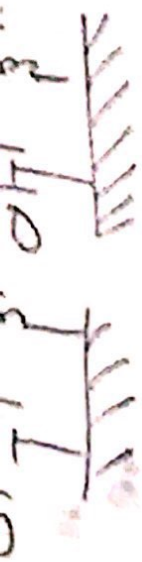


⇒ Case ① $T_2(t)$ $\theta_2(t)$




mechanical impedances driven by gear?
J
K
D
N1 N2
V1
I1
V2
I2
RL

...es driven by gears?



$$(Js^2 + Ds + k)\Theta_2(s) = T_2(s)$$

$$\therefore G(s) = \frac{\Theta_2(s)}{T_2(s)}$$

Case (2)

$$G(s) = \frac{\Theta_1(s)}{T_1(s)}$$

$$\therefore (Js^2 + Ds + k)\Theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

Case (3)

$$G(s) = \frac{\Theta_1(s)}{T_1(s)}$$

$$\left[(Js^2 + Ds + k)\frac{N_1}{N_2}\Theta_1(s) = T_1(s)\frac{N_2}{N_1}\right] \times \frac{N_1}{N_2}$$

... are driven by gears?

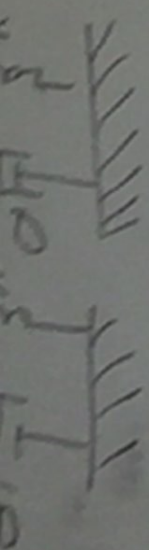
$$(Js^2 + Ds + k)\,\theta_2(s) = T_2(s)$$

$$\therefore G(s) = \frac{\theta_2(s)}{T_2(s)}$$

Case ②

$$G(s) = \frac{\theta_2(s)}{T_1(s)}$$

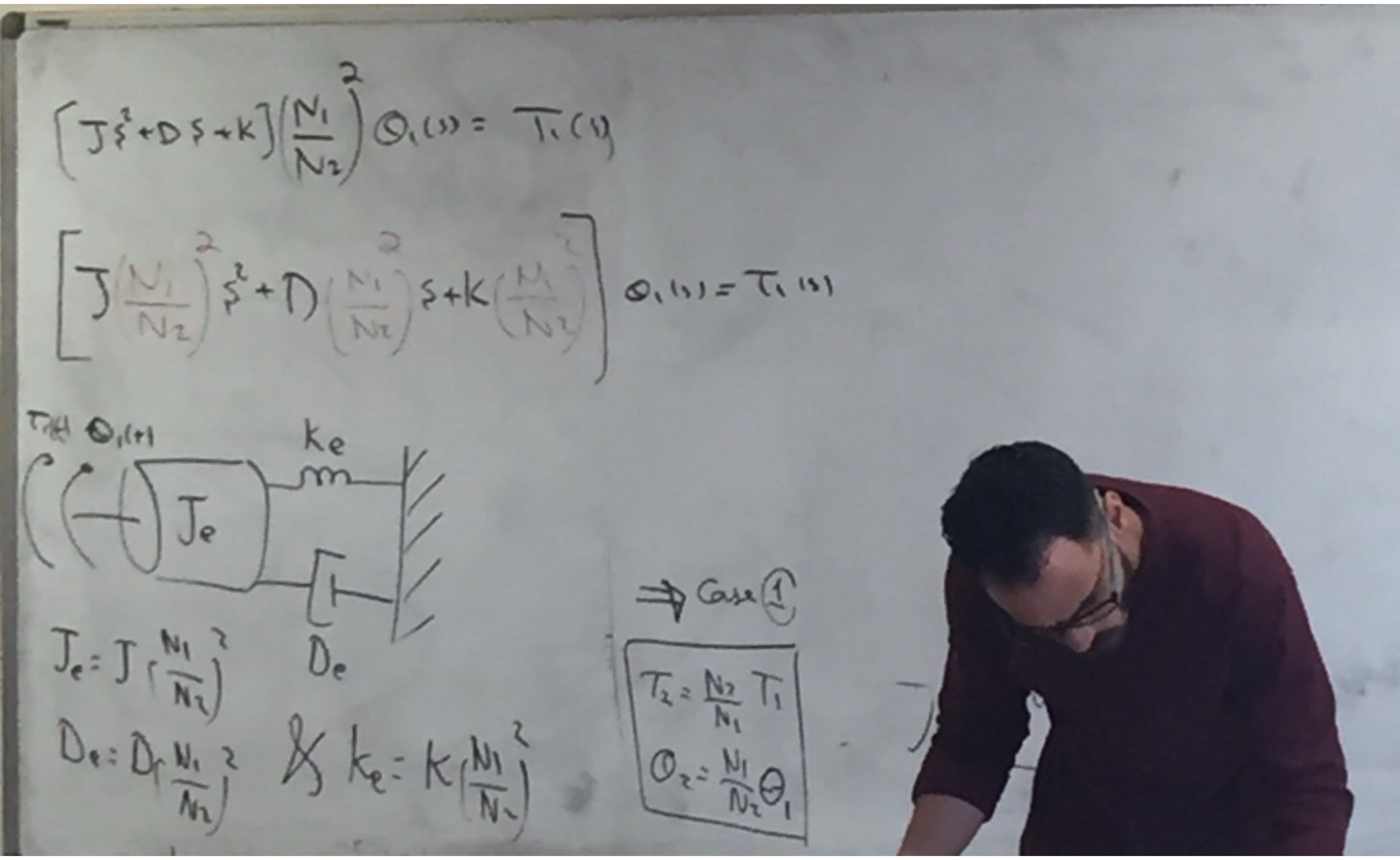
$$\therefore (Js^2 + Ds + k)\,\theta_2(s) = T_1(s)\,\frac{N_2}{N_1}$$



Case ③

$$G(s) = \frac{\theta_1(s)}{T_1(s)}$$

$$\left[(Js^2 + Ds + k)\,\frac{N_1}{N_2}\,\theta_1(s) = T_1(s)\,\frac{N_2}{N_1}\right] \times \frac{N_1}{N_2}$$



$\left[J s^2 + D s + K\right]\left(\frac{N_1}{N_2}\right)^2 \Theta_1(s) = T_1(s)$
$\left[J\left(\frac{N_1}{N_2}\right)^2 s^2 + D\left(\frac{N_1}{N_2}\right)^2 s + K\left(\frac{N_1}{N_2}\right)^2\right] \Theta_1(s) = T_1(s)$
$T_1(t)$ $\Theta_1(t)$
K_e
J_e
D_e
$J_e = J\left(\frac{N_1}{N_2}\right)^2$
$D_e = D\left(\frac{N_1}{N_2}\right)^2$ & $K_e = K\left(\frac{N_1}{N_2}\right)^2$
⇒ Case 1
$T_2 = \frac{N_2}{N_1} T_1$
$\Theta_2 = \frac{N_1}{N_2} \Theta_1$

$\theta_1(s) = T_1(s)$

$\Rightarrow$ Case ①

$$T_2 = \frac{N_2}{N_1} T_1$$

$$\theta_2 = \frac{N_1}{N_2} \theta_1$$

∴ Rotational mechanical impedances can be reflected through gears trains by multiplying the mechanical impedances by the ratio

$$\left(\frac{N_{destination}}{N_{source}} \right)^2$$