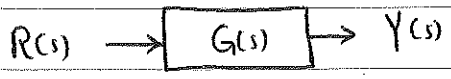
Question # 1



$$G(s) = \frac{Y(s)}{R(s)} \Rightarrow \quad y(t) = 1 + \sin t + 2e^{-2t} \qquad t \geq 0$$

$$Y(s) = \frac{1}{s} + \frac{1}{s^2+1} + \frac{2}{s+2}$$

$$= \frac{3s^3 + 3s^2 + 5s + 2}{s^4 + 2s^3 + s^2 + 2s} = \frac{3s^3 + 3s^2 + 5s + 2}{s(s^2+1)(s+1)}$$

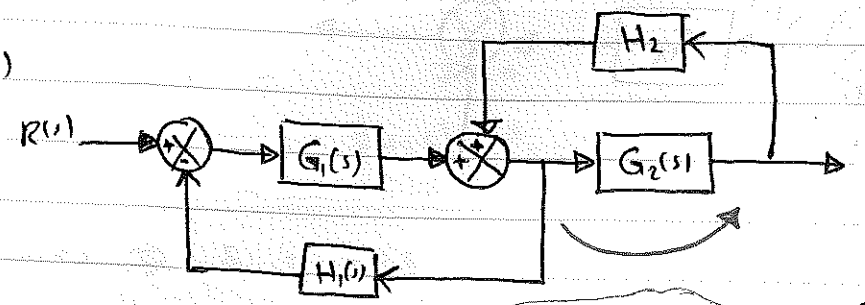
$$R(s) = \frac{1}{s^2}$$

$$\therefore \quad G(s) = \frac{Y(s)}{R(s)} = Y(s) * \frac{s^2}{1}$$

$$\boxed{G(s) = \frac{3s^4 + 3s^3 + 5s^2 + 2s}{s^3 + 2s^2 + s + 2}}$$

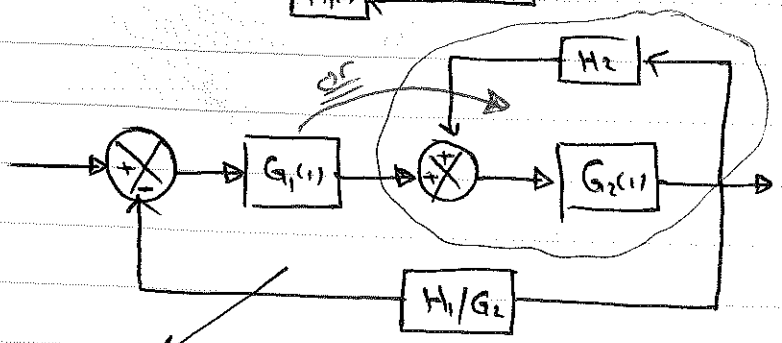
Question #2 :-

(a)



positive feedback.

$$\leftarrow \quad \frac{G_2}{1 - G_2 H_2}$$



negative feedback

$$T.F = \frac{\frac{G_1 G_2}{1 - G_2 H_2}}{1 + \left(\frac{G_1 G_2}{1 - G_2 H_2}\right)\left(\frac{H_1}{G_2}\right)} = \frac{G_1 G_2}{1 - G_2 H_2} * \frac{(1 - G_2 H_2)}{1 - G_2 H_2 + G_1 H_1}$$

$$= \frac{G_1 G_2}{1 - \underbrace{G_2 H_2}_{\text{fixed}} + G_1 H_1} \leftarrow \frac{1}{1}$$

$$\longrightarrow G_1 = \frac{1}{G_2}$$

$$\longrightarrow H_1 = \frac{G_2 H_2}{G_1} = H_2 G_2^2$$

Question #1 :-

$t_p = 0.182$

$t_r = 0.116$

$t_s = 0.375$

$M_p = 0.233$

$t_p = \frac{\pi}{w_d}$, $t_s = \frac{3}{\zeta w_n}$ or $\frac{4}{\zeta w_n}$

$t_r = \frac{\pi - \beta}{w_d}$

$\beta = \cos^{-1}\zeta$

$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$

$T(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2} = \frac{361}{s^2 + 15.855 + 361}$

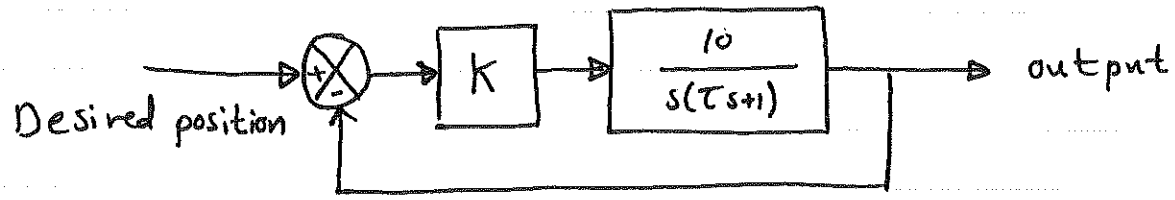
$w_d = 17.25$

$\zeta = 0.417$

$w_n = 18.98$

Question #4

a)



b) $$\frac{C(s)}{R(s)} = \frac{10k}{\tau s^2 + s + 10k} = \frac{10k/\tau}{s^2 + \frac{1}{\tau}s + \frac{10k}{\tau}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \frac{10k}{\tau}$$

$$2\zeta\omega_n = \frac{1}{\tau} \quad \text{for } \tau = 0.001 \Rightarrow \omega_n = \frac{1000}{2\zeta} = \frac{500}{\zeta}$$

but $$\omega_n^2 = \frac{10k}{\tau} = 10{,}000k$$

$$\omega_n^2 = 10{,}000k$$

$$\frac{(500)^2}{\zeta^2} = 10{,}000k \Rightarrow \frac{250000}{\zeta^2} = 10{,}000k$$

$$k = \frac{25}{\zeta^2}$$

for under-damped $0 < \zeta < 1$

$$\boxed{25 < k < \infty}$$