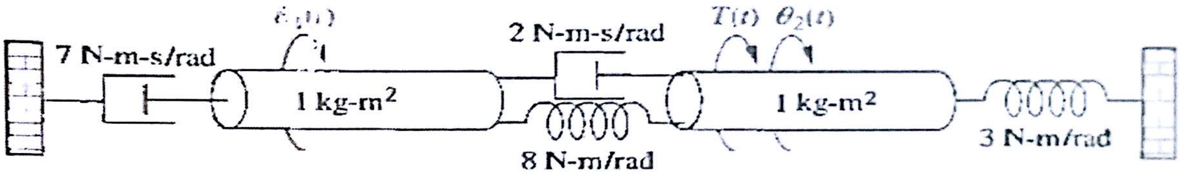
Birzeit University-Faculty Of Engineering and Technology
Electrical and Computer Engineering Department
EE3302-Control Systems

Inst.: Dr. J. Siam | Midterm Exam | 2nd semester 2016

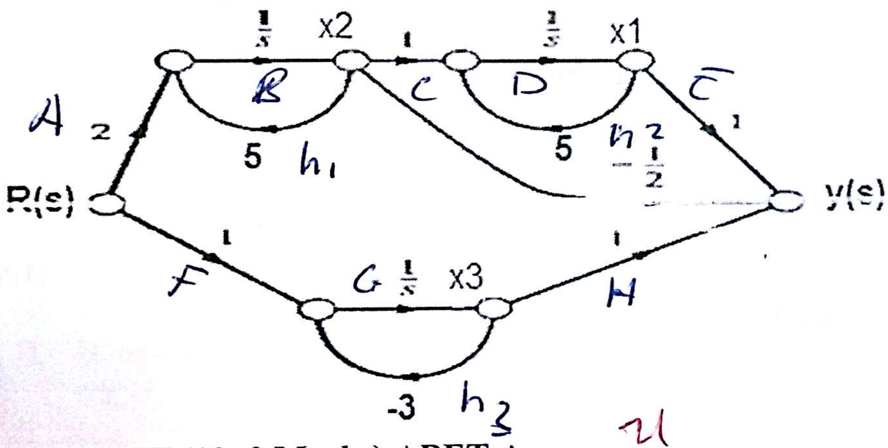
Question I (8+8 Marks) ABET A:

For the system in figure:

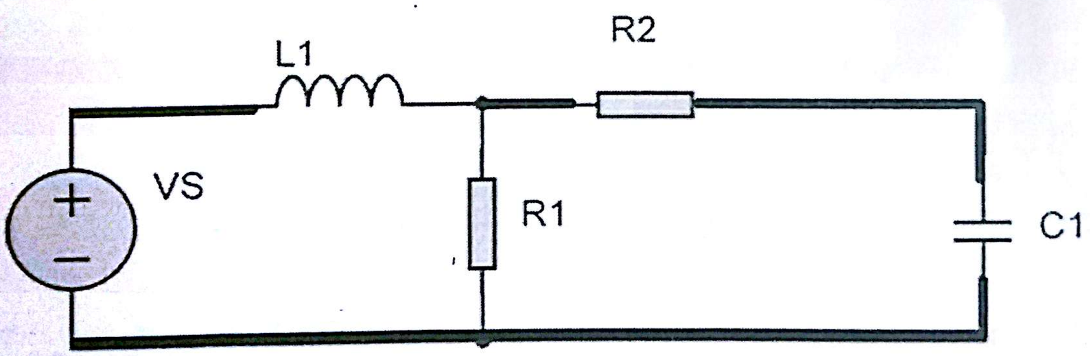


1. Design the analogous electric circuit
2. Write the equations of the system in matrix form

Question II (8Marks) ABET A:
Determine the equivalent transfer function using Mason's formula



Question III (12+9 Marks) ABET A:
1. Determine the state space representation of the circuit considering v_{L1} as output: VS =5 u(t), R1 =R2= 10kΩ, L1= 0.1 mH, and C1=1μF



2. Determine the phase variable state space representation in matrix form for the following system and draw the state flow diagram

$$G(s) = \frac{s^2 + 12}{s^4 + 2s^3 + s + 10}$$

Question IV (6+6+6+6 Marks) ABET C:

A rotational head of moment of inertia J = 4 Kg.m^2 is placed in a viscous medium with coefficient f_θ and connected by a torsion-spring of coefficient K_θ = 9N.m/rad to a fixed holder ,the system has to be designed to achieve a position under damped response when a unit step input force is applied ,

1. Determine the system dynamic equation and the possible range of f_θ to achieve the desired under-damped response.
2. Determine(if exists) the value of the parameter f_θ necessary to achieve a settling time smaller than 10 seconds at 2% error.
3. Determine the overshoot time and the value and the natural frequency of the system.
4. Determine the value of the step input torque that achieves the steady state position of 45 degrees.

Question V (4+12 Marks, ABET C) :

An electric plant has the following transfer function:

$$G(s) = \frac{1}{s^4 + s^3 + 10s^2 + 3s - 1}$$

A unity negative feedback system with a direct path proportional controller gain $k \in R$ has to be designed to stabilize electric system

1. Plot the feedback control system and determine its transfer function
2. Determine the range of the proportional controller gain in which the system is stable,

Question VI (10+5 Marks, ABET C) :

Consider the system in **Question V** and :

1. Design using the gain k the system to be type 0, then compute the position, velocity and acceleration errors.
2. Determine the value of K that minimizes the value of the finite steady state error

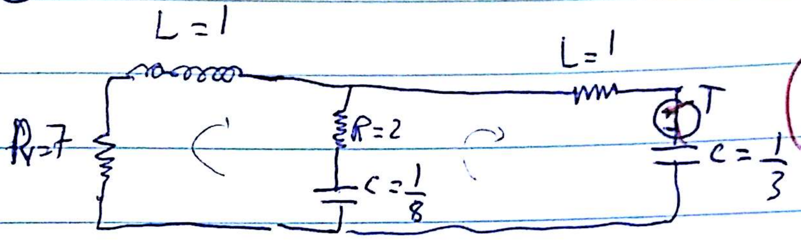
Good Luck

Q1) $\left(Ms + f_v + \frac{k_s}{s}\right) s\, X(s) = f_{ex}$

$\left(Ls + R + \frac{1}{Cs}\right) I = V_i$

$\Rightarrow$ $M = L$, $f_v = R$, $k_s = \frac{1}{C}$

①



-1

②

$(s^2 + 9s + 8)\theta_1 - (2s+8)\theta_2 = 0$

$(s^2 + 2s + 11)\theta_2 - (2s+8)\theta_1 = T$

$$\Rightarrow \begin{bmatrix} s^2+9s+8 & -2s-8 \\ -2s-8 & s^2+2s+11 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ T \end{bmatrix}$$

Q2) forward path

3 pathes

$p1 = ABCDE = \frac{2}{s^2}$

$p2 = ABM = \frac{-1}{s}$

$p3 = FGH = 1/s$

loop gain

$L1 = BH1 = \frac{5}{s}$

$L2 = DH_2 = \frac{5}{s}$

$L3 = GH_3 = \frac{-3}{s}$

non tuching (2,2)

$L_{12} = BH1DH_2 = \frac{25}{s^2}$

$L_{13} = BGH_1H_3 = -15/s^2$

$L_{23} = DGH_2H_3 = \frac{25}{s^2}$

non tuching (3,3)

$L_{123} = BDGH_1H_2H_3 = \frac{-75}{s^3}$

$$\boxed{H(s) = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}}$$

$$\Delta = 1 - (BH_1 + DH_2 + GH_3) + (BDH_1H_2 + BGH_1H_3 + DGH_2H_3) - (BDGH_1H_2H_3)$$

Complete

$$\Delta_1 = 1 - (GH_3)$$

Complete

$$\Delta_2 = 1 - (DH_2 + GH_3) + (DH_2GH_3)$$

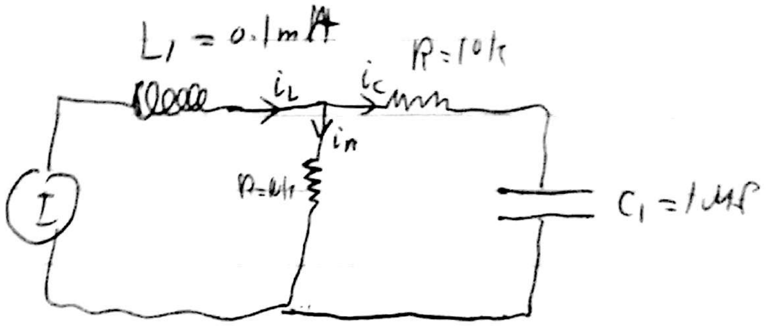
$$\Delta_3 = 1 - (BH_1 + DH_2) + (BDH_1H_2)$$

$$\boxed{H(s) = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}}$$

Q3 $y = V_{L1}$

let $x_1 = i_L$

$x_2 = V_C$



(1)

$V_{R_1} - V_c = I_c R_2$

$I_R R_1 - X_2 = C\dot{X}_2 R_2$; $\boxed{i_R = i_L - i_c = X_1 - C\dot{X}_2}$

$(X_1 - C\dot{X}_2)R - X_2 = C\dot{X}_2 R$

$10k X_1 - 0.01\dot{X}_2 = 0.01\dot{X}_2$

$\Rightarrow \boxed{\dot{X}_2 = 0.5M X_1}$

(2)

$-V_s + V_L + V_R = 0$

$-V_s + L\dot{X}_1 + (X_1 - CsX_2) = 0$

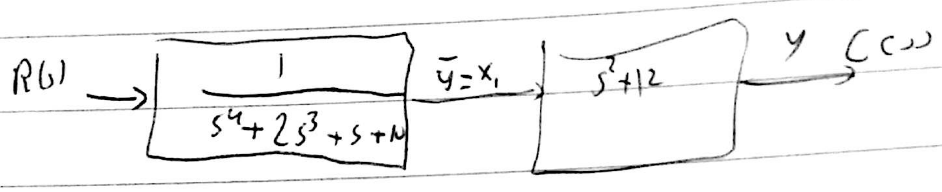
$\dot{X}_1 = \frac{V_s}{L} - \frac{X_1}{L} + \frac{CsX_2}{L}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -10^4 & 0.015 \\ 0.5M & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 10^4 \\ 0 \end{bmatrix} V_s$$

$$y = V_L = L \dot{i}_L = L \dot{x}_1 = L s x_1 = 0.1m\, s\, x_1$$

$$y = \begin{bmatrix} 0.1m & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

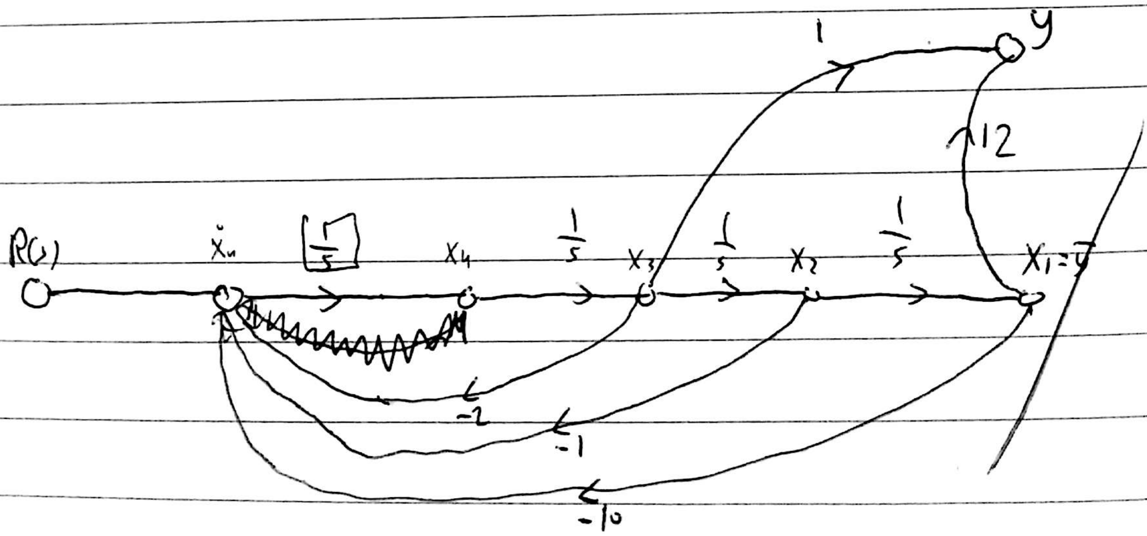
Q3 b)



$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ x_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -10 & -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$\bar{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$y = \bar{y}\left[s^2+12\right] = x_3 + 12x_1$$



Q4)

$$H(s) = \frac{1}{Js^2 + f_o s + K_o} = \frac{1}{4s^2 + f_o s + 9}$$

$$= \frac{1}{4} \frac{1}{s^2 + \frac{f_o}{4} + \frac{9}{4}} = \frac{1}{9} \frac{\frac{9}{4}}{s^2 + \frac{f_o s}{4} + \frac{9}{4}}$$

to make it under damp $\Delta < 0$

$$\Delta = \left(\frac{f_o}{4}\right)^2 - 4(1)\frac{9}{4} < 0$$

$$\Rightarrow \frac{f_o^2}{16} - 9 < 0 \Rightarrow f_o^2 < 144 \Rightarrow |f_o| < 12$$

$$\Rightarrow \boxed{-12 < f_o < 12}$$

2) $t_{settling} = 10 \le \frac{4}{\alpha} \Rightarrow \boxed{\alpha = 0.4}$

$$2\alpha = \frac{f_o}{4} \Rightarrow \alpha = \frac{f_o}{8} \Rightarrow 0.4 = \frac{f_o}{8} \Rightarrow \boxed{f_o \geq 3.2}$$

A

3) $W_n = \sqrt{\frac{9}{4}} = \frac{3}{2}$ rad/s

$$t_{os} = \frac{\pi}{W_d} = \frac{\pi}{W_n\sqrt{1-\xi^2}} \quad ; \quad \boxed{\xi = \frac{f_o}{12}}$$

$2\xi\omega_n = f_0/4$

$\xi = \frac{f_o}{8\omega_n} = \frac{f_0}{8\times\frac{3}{2}} = f_0/12$

$$\Rightarrow \boxed{t_{os} = \frac{\pi}{\frac{3}{2}\sqrt{1-\frac{f_o}{12}}} \quad 12 < f_o < 12}$$

-2

4) $45° = \frac{1}{9}$ input $\Rightarrow$ $\boxed{\text{input} = 405u(t)}$

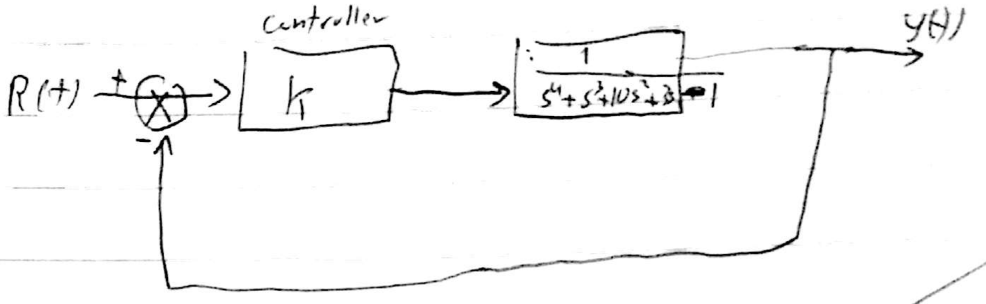
$9 \times \frac{1}{4}$

-3

$\frac{9}{4} \times 45 = \frac{B}{4}$

QV)

1)



2) $H(s) = \dfrac{K}{s^4+s^3+10s^2+3s+(-1+K)}$

to check stability we use Routh method

s^4	1	10	$K-1$
s^3	1	3	0
s^2	7	$K-1$	0
s^1	$22-K$	0	0
s^0	$K-1$	0	0

$\frac{21-(K-1)}{7} = \frac{22-K}{7}$

22

1

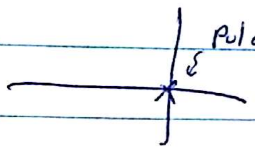
the system is Asymptotic stable in $K \in]1, 22[$

to check the stability at $k=1$

$\Rightarrow$

4	1	10	0
3	1	3	0
2	7	0	0
1	3	0	0
0	0	0	0 $\Leftarrow$

$\Rightarrow \quad s(s^3 + s^2 + 10s + 3)$



the system has one pole is origin $\Rightarrow$ simple stable

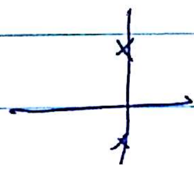
to check at 22

4	1	10	21
3	1	3	0
2	7	21	0
1	~~0~~ 14	~~0~~ 0	~~0~~ 0
0	21		

Auxelary polynomial $A(s) = 7s^2 + 21$

$\Rightarrow A'(s) = 14s$

$\Rightarrow$ the system is has two poles in the $j\omega$ axis (by the symetry of purly even polynomial)



$\Rightarrow$ the system is simple stable

$\Rightarrow$ the system $\begin{cases} \text{asymbtotic stable} & k \in]1, 22[\\ \text{simple stable} & k = 1, 22 \\ \text{unstable} & \text{o.w} \end{cases}$

Q6) the system to be in type 0,

1) it must be stable
2) $K \neq 1$

$$\Rightarrow G(s) = \frac{K}{s^4 + s^3 + 10s^2 + 3s + (k-1)} \quad ; \quad k \in]1, 22[$$

position error {input is $u(t)$}

$$\Rightarrow E(s) = \frac{R(s)}{1 + G(s)}$$

$$\Rightarrow e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{s \frac{1}{s}}{1 + \frac{k}{s^4 + s^3 + 10s^2 + 3s + (k-1)}}$$

$$= \boxed{\frac{1}{1 + \frac{k}{k-1}}} \quad ; \quad k \in]1, 22]$$

$$E(k) = \frac{k-1}{2k-1}$$

velocity error and acceleration error $= \infty$

since the refrencs dynamic is $>$ system dynamic

2) to mimize error

$E(K) = \frac{K-1}{2K-1}$; it clearly that the error decresing when k incresss

$\Rightarrow$ min error at $\boxed{K=22}$

$\rightarrow$ $\boxed{\text{error} = 0.488}$

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