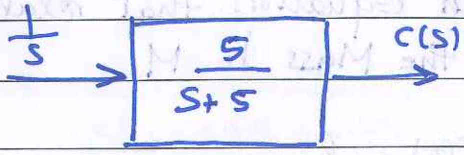
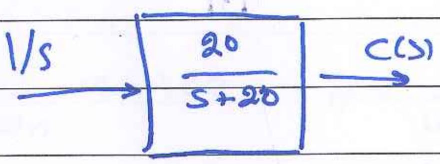
Eamples On Chapter Four

Ex: Find the output response, c(t), for each of the system shown below. Also, find the time constant, rise time



(a)



(b)

Ans :

ⓐ $C(s) = \frac{1}{s} \cdot \frac{5}{s+5}$

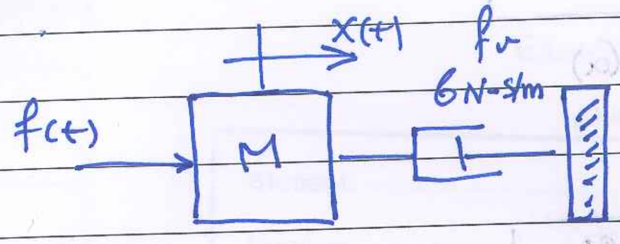
$c(t) = 1 - e^{-5t}$

$T = \frac{1}{5}$, $T_r = \frac{2.2}{a} = \frac{2.2}{5} = 0.44$, $T_s = \frac{4}{a} = \frac{4}{5} = 0.8$

ⓑ $C(s) = \frac{20}{(s)(s+20)} \Rightarrow c(t) = 1 - e^{-20t}$, $T = \frac{1}{20}$

$T_r = \frac{2.2}{a} = \frac{2.2}{20} = 0.11$, $T_s = \frac{4}{a} = \frac{4}{20} = 0.2$

Ex2: For the system shown below, ⓐ find an equation that relates settling time of the velocity of the mass to M; ⓑ find an equation that relates rise time of the velocity of the Mass to M



Ans: $F(s) = (Ms^2 + f_v s)\, X(s)$

$$T(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s}$$

$$= \frac{sX(s)}{F(s)} = \frac{1}{Ms + f_v} = \frac{1/M}{s + f_v/M} \cdot \frac{f_v/M}{f_v/M}$$

$$= \left(\frac{1}{M}\right) \cdot \frac{1}{(f_v/M)} \cdot \frac{f_v/M}{s + f_v/M}$$

$$= \frac{1}{f_v} \cdot \frac{f_v/M}{s + f_v/M}$$

$\boxed{a = 6/M}$; $T_s = \frac{4}{a} = \frac{4}{6/M} = \frac{4M}{6}$

$$T_r = \frac{2.2}{a} = \frac{2.2}{6/M} = \frac{2.2M}{6}$$

EX: For each of the second-order systems that follow, find $\zeta, \omega_n, T_p, T_r$, and %OS

ⓐ $T(S) = \dfrac{0.04}{S^2+0.02S+0.04}$

$\omega_n^2 = 0.04$, $2\zeta\omega_n = 0.02 \Rightarrow \zeta = 0.05, \omega_n = 0.2$

$T_s = \dfrac{4}{\zeta\omega_n} = 400$, $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 15.73$

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 85.45\%$, $\omega_n T_r =$

$\omega_n T_r = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1$, $\Rightarrow T_r = 5.26\,s$

EX:- For each pair of second-order system specifications that follow, find the location of the second-order pair of poles

ⓐ $\%OS = 12\%$, $T_s = 0.6$ sec

ⓑ $\%OS = 10\%$, $T_p = 5$ sec.

ⓒ $T_s = 7$ sec, $T_p = 3$ sec

Ans:- ⓐ $\zeta = \dfrac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.56$

$\omega_n = \dfrac{4}{\zeta T_s} = 11.92 \Rightarrow$ poles $= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$= -6.67 \pm j9.88$

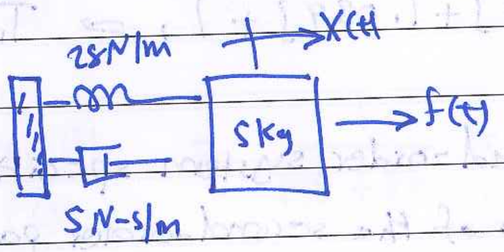
(b) $\zeta = \dfrac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.591$, $\omega_n = \dfrac{\pi}{T_p\sqrt{1-\zeta^2}} = 0.779$

$\Rightarrow$ poles $= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -0.4605 \pm j\,0.6283$

(c) $\zeta\omega_n = \dfrac{4}{T_s} = 0.571$, $\omega_n\sqrt{1-\zeta^2} = \dfrac{\pi}{T_p} = 1.047$

poles $= -0.571 \pm j\,1.047$

Ex: For the system shown below



(a) Find the transfer function $G(s) = X(s)/F(s)$

(b) Find ζ, ω_n, $\%OS$, T_s, T_p and T_r

Ans: $F(s) = (K + f_v s + M s^2) X(s)$

$G(s) = \dfrac{X(s)}{F(s)} = \dfrac{1}{Ms^2 + f_v s + K} = \dfrac{1/5}{s^2 + \frac{f_v}{M}s + K/M} \cdot \dfrac{K/M}{K/M}$

$= \dfrac{M}{K} \cdot \dfrac{K/M}{s^2 + \frac{f_v}{M}s + K/M}$

$\Downarrow$

$T_s = \dfrac{4}{\zeta\omega_n}$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

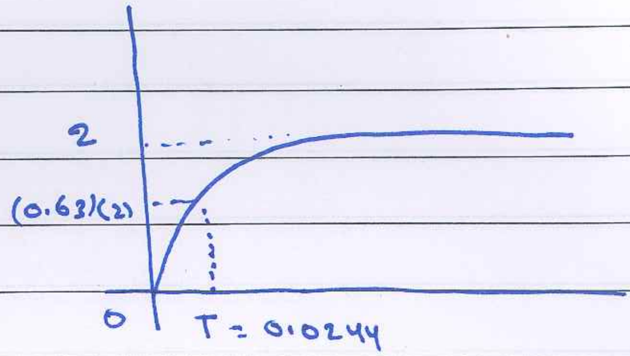
$2\zeta\omega_n = \dfrac{f_v}{M}$, $\omega_n^2 = \dfrac{K}{M}$

$\omega_n T_r = 1.768\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1 \Rightarrow T_r = 0.514$ sec

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$

Ex:- For each of the unit step response shown below, find the transfer function of the system.

ⓐ

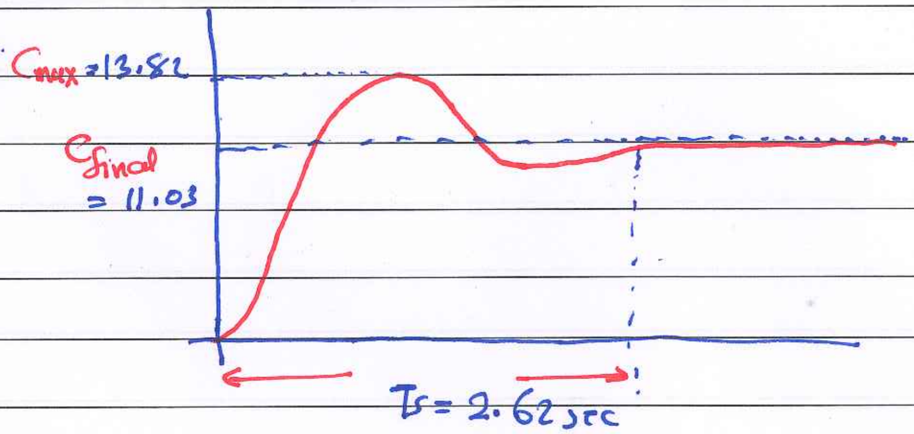


$$G(s) = \frac{K}{s+a} \;,\; a = \frac{1}{T} = 40.984 \;,\; \frac{K}{a} = 2 \Rightarrow K = 81.967$$

$\Rightarrow$

$$T(s) = \frac{81.967}{s+40.984}$$

ⓑ



$$\%OS = \frac{C_{max} - C_{final}}{C_{final}} \times 100\%$$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.4$$

$$T_s = \frac{4}{\zeta \omega_n} \Rightarrow \omega_n = 3.82 \Rightarrow$$

$$G(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2} \;;\; \frac{K}{\omega_n^2} = 11.03$$

$$\Rightarrow K = 160.95$$

$$\Rightarrow G(s) = \frac{160.95}{s^2 + 3.056s + 14.59}$$