# Control Theory Assignment – Stability

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**1** Using the Routh-Hurwitz table, tell how many poles of the following closed-loop function are in the right-half plane, in the left-half plane, or on the $j\omega$-axis:

$$T(s) = \frac{s+8}{s^5 - s^4 + 4s^3 - 4s^2 + 3s - 2}$$

**2** The closed-loop transfer function of a system is:

$$T(s) = \frac{s^3 + 2s^2 + 7s + 21}{s^5 - 2s^4 + 3s^3 - 6s^2 + 2s - 4}$$

Determine how many closed-loop poles lie in the right-half plane, in the left-half plane and on the $j\omega$-axis.

**3** How many poles of the following open-loop system lie in the left-half plane, in the right-half plane or on the $j\omega$-axis?

$$T(s) = \frac{-6}{s^6 + s^5 - 6s^4 + s^2 + s - 6}$$

Use Matlab to find the pole locations and verify your answer.

**4** Consider the following open-loop system:

$$G(s) = \frac{1}{4s^2(s^2+1)}$$

Using the Routh-Hurwitz criterion, find the region of the $s$-plane where the poles of the closed-loop system are located.

**5** In the following open-loop function:

$$G(s) = \frac{K(s+2)}{s(s-1)(s+3)}$$

Find the range of $K$ for closed-loop stability.

**6** Consider the following open-loop system:

$$G(s) = \frac{8}{s(s^6 - 2s^5 - s^4 + 2s^3 + 4s^2 - 8s - 4)}$$

Using the Routh-Hurwitz criterion, find the region of the $s$-plane where the poles of the closed-loop system are located.

**7** Consider the following Routh-Hurwitz table. Notice that the $s^5$ row was originally all zeros. Tell how many roots of the original polynomial were in the right-half plane, in the left-half plane, and on the $j\omega$-axis.

| | | | | |
|---|---|---|---|---|
| $s^7$ | 1 | 2 | -1 | -2 |
| $s^6$ | 1 | 2 | -1 | -2 |
| $s^5$ | 3 | 4 | -1 | 0 |
| $s^4$ | 1 | -1 | -3 | 0 |
| $s^3$ | 7 | 8 | 0 | 0 |
| $s^2$ | -15 | -21 | 0 | 0 |
| $s^1$ | -9 | 0 | 0 | 0 |
| $s^0$ | -21 | 0 | 0 | 0 |

**8** For the following open-loop systems, find the range of $K$ that ensures closed-loop stability.

a.

$$G(s) = \frac{K(s+6)}{s(s+1)(s+4)}$$

b.

$$G(s) = \frac{K(s+1)}{s^4(s+2)}$$

c.

$$G(s) = \frac{K(s-2)(s+4)(s+5)}{s^2+12}$$

**9** Using the Routh-Hurwitz Criterion, find the range of $K$ that will yield oscillations for the unity feedback system with:

$$G(s) = \frac{K(s+2)}{(s^2+1)(s+4)(s-1)}$$

**10** Given a unity feedback system with an open-loop function of:

$$G(s) = \frac{K(s+4)}{s(s+1.2)(s+2)}$$

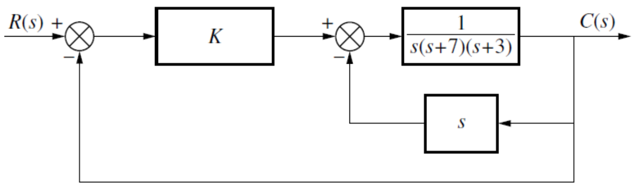
find the following:

a. The range of $K$ that keeps the system stable.

b. The value of $K$ that makes the system oscillate.

c. The frequency of oscillation when $K$ is set to the value that makes the system oscillate.

**11** For the system shown in the figure, find the value of gain $K$, that will make the system oscillate. Also, find the frequency of oscillation.




**12** For a unity feedback system with an open-loop function of:

$$G(s) = \frac{K(s+2)}{(s^2+1)(s+4)(s-1)} \tag{1}$$

find the range of $K$ for which there will be only two, closed-loop, right-half-plane poles.

**13** The closed-loop transfer function of a system is:

$$T(s) = \frac{s^2 + K_1 s + K_2}{s^4 + K_1 s^3 + K_2 s^2 + 5s + 1}$$

Determine the range of $K_1$ in order for the system to be stable. What is the relationship between $K_1$ and $K_2$ for stability?