Solution of Homework 5

Problem 1:

s^5	1	4	3
s^4	-1	-4	-2
s^3	ε	1	0
s^2	$\frac{1-4\varepsilon}{\varepsilon}$	-2	0
s^1	$\frac{2\varepsilon^2+1-4\varepsilon}{1-4\varepsilon}$	0	0
s^0	-2	0	0

3 rhp, 2 lhp

Problem 2:

s^5	1	3	2
s^4	-1	-3	-2
s^3	-2	-3	ROZ
s^2	-3	-4	
s^1	-1/3		
s^0	-4		

Even (4): 4 jω; Rest(1): 1 rhp; Total (5): 1 rhp; 4 jω

Problem 3:

s^6	1	-6	1	-6
s^5	1	0	1	
s^4	-6	0	-6	
s^3	-24	0	0	ROZ
s^2	ε	-6		
s^1	-144/ε	0		
s^0	-6			

Even (4): 2 rhp; 2 lhp; Rest (2): 1 rhp; 1 lhp; Total: 3 rhp; 3 lhp

Problem 4:

$$T(s) = \frac{1}{4s^4 + 4s^2 + 1}$$

s4	4	4	1	
s3	16	8	0	ROZ
s2	2	1	0	
s1	4	0	0	ROZ
s0	1	0	0	

Even (4): 4 jω

Problem 5:

The characteristic equation is:

$$1 + K\frac{(s+2)}{s(s-1)(s+3)} = 0 \text{ or}$$

$$s(s-1)(s+3) + K(s+2) = 0 \text{ or}$$

$$s^3 - 2s^2 + (K-3)s + 2K = 0$$

The Routh array is:

s^3	1	$K-3$
s^2	2	$2K$
s	-3	
1	$2K$	

The first column will always have a sign change regardless of the value of K. There is no value of K that will stabilize this system.

Problem 6:

$$T(s)=\frac{8}{s^7-2s^6-s^5+2s^4+4s^3-8s^2-4s+8}$$

s^7	1	-1	4	-4	
s^6	-2	2	-8	8	
s^5	-12	8	-16	0	ROZ
s^4	0.6667	-5.333	8	0	
s^3	-88	128	0	0	
s^2	-4.364	8	0	0	
s^1	-33.33	0	0	0	
s^0	8	0	0	0	

Even (6): 3 rhp, 3 lhp; Rest (1): 1 rhp; Total: 4 rhp, 3 lhp

Problem7 :

Even (6): 1 rhp, 1 lhp, 4 jω; Rest (1): 1 lhp; Total: 1 rhp, 2 lhp, 4 jω

Problem 8:

a.

$$T(s)=\frac{K(s+6)}{s^3+5s^2+(K+4)s+6K}$$

s^3	1	4 + K
s^2	5	6K
s^1	20 -K	0
s^0	6K	0

Stable for $0 < K < 20$

b.

$$T(s) = \frac{K(s+1)}{s^5 + 2s^4 + Ks + K}$$

. Always unstable since s^3 and s^2 terms are missing.

c.

$$T(s) = \frac{Ks^3 + 7Ks^2 + 2Ks - 40K}{Ks^3 + (7K+1)s^2 + 2Ks + (12 - 40K)}$$

s^3	K	$2K$
s^2	$7K+1$	$12-40K$
s^1	$\frac{54K^2-10K}{7K+1}$	0
s^0	12-40K	

For stability, $\frac{10}{54} < K < \frac{12}{40}$

Problem 9:

$$T(s) = \frac{K(s+2)}{s^4 + 3s^3 - 3s^2 + (K+3)s + (2K-4)}$$

s^4	1	- 3	2K - 4
s^3	3	K+ 3	0
s^2	$\frac{-(K+12)}{3}$	2K - 4	0
s^1	$\frac{K(K+33)}{K+12}$	0	0
s^0	2K - 4	0	0

Conditions state that $K < -12$, $K > 2$, and $K > -33$. These conditions cannot be met simultaneously.

System is not stable for any value of K.

Problem 10:

$$T(s) = \frac{5K(s+4)}{5s^3 + 16s^2 + (12+5K)s + 20K}$$

Making a Routh table,

s^3	5	12+5K
s^2	16	20K
s^1	192 - 20K	0
s^0	20K	0

a. For stability, $0 < K < 9.6$.

b. Oscillation for $K = 9.6$.

c. From previous row with K=9. 6, $16s^2 + 192 = 0$. Thus $s = \pm j\sqrt{12}$, or $\omega = \sqrt{12}$ rad/s.

Problem 11:

$$T(s) = \frac{K}{s^4 + 8s^3 + 17s^2 + 10s + K}$$

s^4	1	17	K
s^3	8	10	0
s^2	$\frac{126}{8}$	K	0
s^1	$-\frac{32}{63}K + 10$	0	0
s^0	K	0	0

a. For stability $0 < K < 19.69$.

b. Row of zeros when K = 19.69. Therefore, $\frac{126}{8}s^2 + 19.69$. Thus, $s = \pm j\sqrt{1.25}$, or

$\omega = 1.118$ rad/s.

Problem 12:

$$T(s) = \frac{K(s+2)}{s^4 + 3s^3 - 3s^2 + (K+3)s + (2K-4)}$$

s^4	1	-3	2K-4
s^3	3	K+3	0
s^2	$-\frac{K+12}{3}$	2K-4	0
s^1	$\frac{K(K+33)}{K+12}$	0	0
s^0	2K-4	0	0

For $K < -33$: 1 sign change; For $-33 < K < -12$: 1 sign change; For $-12 < K < 0$: 1 sign change; For $0 < K < 2$: 3 sign changes; For $K > 2$: 2 sign changes. Therefore, $K > 2$ yields two right-half-plane poles.

Problem 13:

	1	K_2	1
s^3	K_1	5	0
s^2	$\frac{K_1K_2 - 5}{K_1}$	1	0
s^1	$\frac{K_1^2 - 5K_1K_2 + 25}{5 - K_1K_2}$	0	0
s^0	1	0	0

For stability, $K_1K_2 > 5$; $K_1^2 + 25 < 5K_1K_2$; and $K_1 > 0$. Thus $0 < K_1^2 < 5K_1K_2 - 25$,

or $0 < K_1 < \sqrt{5K_1K_2 - 25}$.