Solution of Assignment 8

Problem 1:

Uncompensated system: Search along the $\zeta = 0.5$ line and find the operating point is at -1.5356 ± j2.6598 with K = 73.09. Hence, $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \text{x}100$ = 16.3%; $T_s = \frac{4}{1.5356}$ = 2.6 seconds; $K_p = \frac{73.09}{30}$ =2.44. A higher-order pole is located at -10.9285.

Compensated: Add a pole at the origin and a zero at -0.1 to form a PI controller. Search along the ζ= 0.5 line and find the operating point is at -1.5072 ± j2.6106 with K = 72.23. Hence, the estimated performance specifications for the compensated system are: $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \text{x}100$ = 16.3%; $T_s = \frac{4}{1.5072}$ = 2.65 seconds; $K_p = \infty$. Higher-order poles are located at -0.0728 and -10.9125. The compensated system should be simulated to ensure effective pole/zero cancellation.

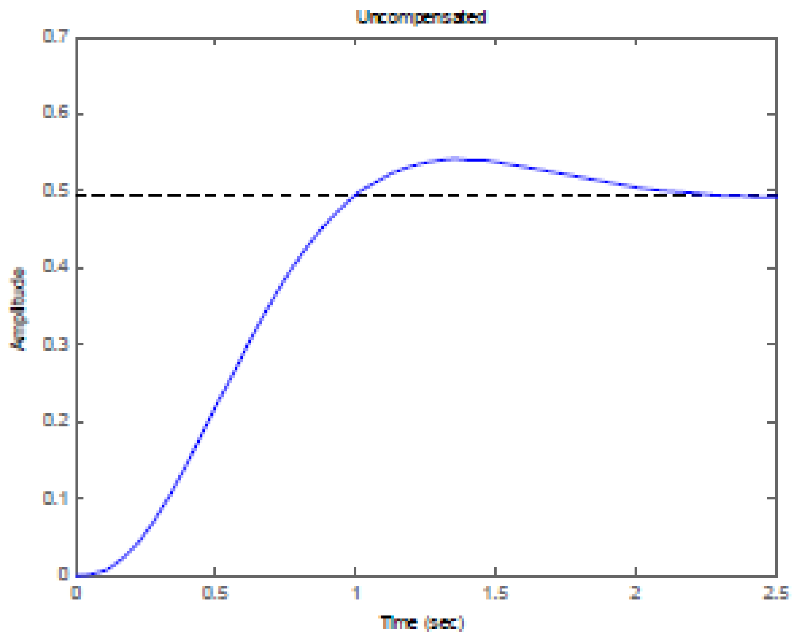
Problem 2:

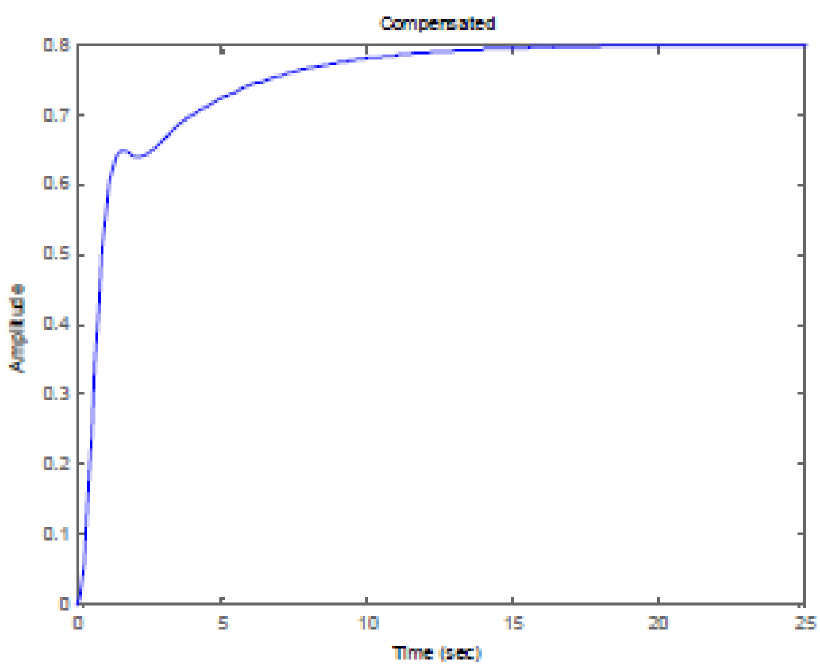
a. Searching along the 126.16° line (10% overshoot, $\zeta = 0.59$), find the operating point at

-1.8731 + j2.5633 with K = 41.1905. Hence, $K_p = \dfrac{41.1905}{2*3*7} = 0.9807$

b. A 4.0787 x improvement will yield $K_p = 4$. Use a lag compensator, $G_c(s) = \dfrac{s+0.40787}{s+0.1}$.

c.





Problem 3:

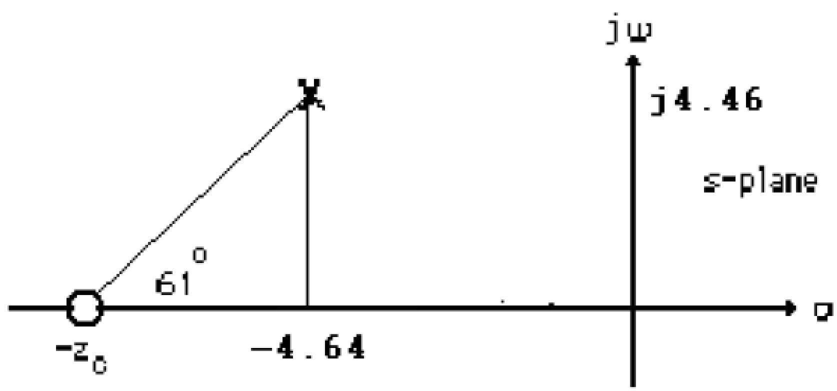
Uncompensated: Searching along the 135° line ($\zeta = 0.707$), find the operating point at

-2.32 + j2.32 with K = 4.6045. Hence, $K_p = \frac{4.6045}{30} = 0.153$; $T_s = \frac{4}{2.32} = 1.724$ seconds; $T_p = \frac{\pi}{2.32} = 1.354$ seconds; $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 4.33\%$;

$\omega_n = \sqrt{2.32^2 + 2.32^2} = 3.28$ rad/s; higher-order pole at -5.366.

Compensated: To reduce the settling time by a factor of 2, the closed-loop poles should be – 4.64 ± j4.64. The summation of angles to this point is 119° . Hence, the contribution of the compensating zero should be 180° -119° =61° . Using the geometry shown below,

$\frac{4.64}{z_c - 4.64} = \tan(61°)$. Or, $z_c = 7.21$.



After adding the compensator zero, the gain at -4.64+j4.64 is K = 4.77. Hence,

$K_p = \frac{4.77x6x7.21}{2x3x5} = 6.88$. $T_s = \frac{4}{4.64} = 0.86$ second; $T_p = \frac{\pi}{4.64} = 0.677$ second;

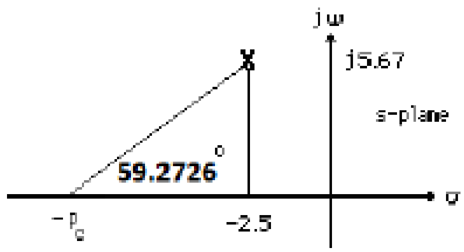
$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 4.33\%$; $\omega_n = \sqrt{4.64^2 + 4.64^2} = 6.56$ rad/s; higher-order pole at -5.49. The problem with the design is that there is steady-state error, and no effective pole/zero cancellation. The design should be simulated to be sure the transient requirements are met.

Problem 4:

a. $\zeta\omega_n = \frac{4}{T_s} = 2.5$; $\zeta = \frac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.404$. Thus, $\omega_n = 6.188$ rad/s and the operating point is $-2.5 \pm j5.67$.

b. Summation of angles including the compensating zero is -120.7274^o. Therefore, the compensator pole must contribute $120.7274^o - 180^o = -59.2726^o$.

c. Using the geometry shown below, $\frac{5.67}{P_c - 2.5} = \tan 59.2726^\circ$. Thus, $P_c = 5.87$.

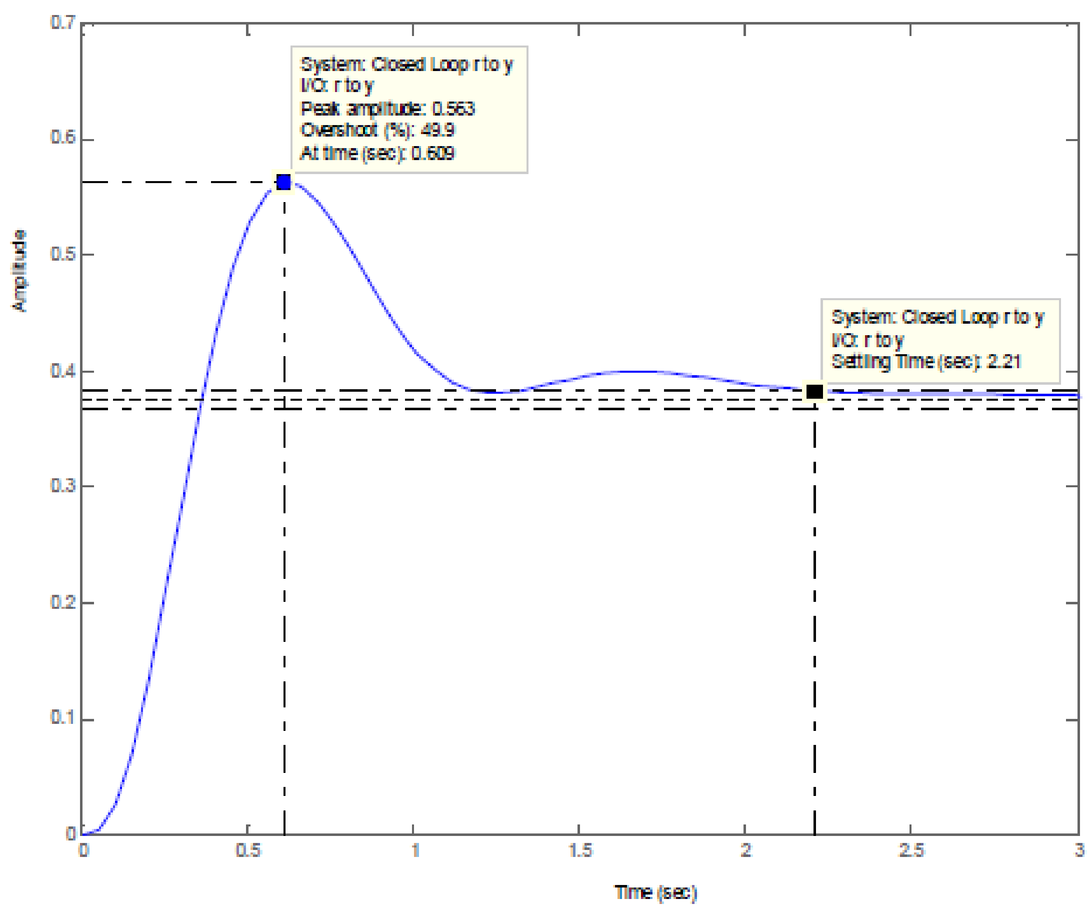


d. Adding the compensator pole and using -2.5 + j5.67 as the test point, K = 225.7929.

e. Searching the real axis segments for K = 225.7929, we find higher-order poles at -11.5886, and --1.3624.

f. Pole at -11.5886 is 4.64 times further from the imaginary axis than the dominant poles. Pole at --1.3624 may not cancel the zero at -1. Questionable second-order approximation. System should be simulated.

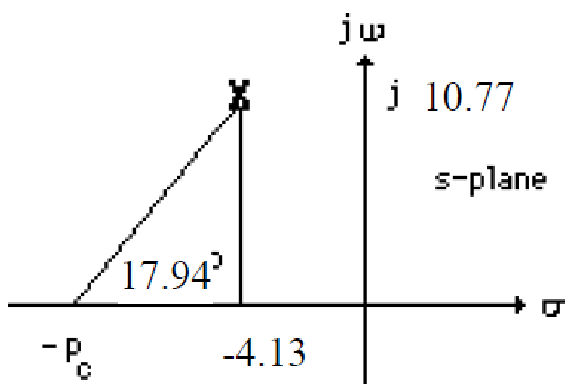
g.



A simulation of the system shows a percent overshoot of 49.9% and a settling time of 2.21 seconds. Thus, the specifications were not met because pole-zero cancellation was not achieved. A redesign is required.

Problem 5:

a. Searching along the 110.97° line ($\%OS = 30\%$; $\zeta = 0.358$), find the operating point at -2.065 + j5.388 with $K = 366.8$. Searching along the real axis for $K = 366.8$, we find a higher-order pole at –16.87. Thus, $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{2.065} = 1.937$ seconds. For the settling time to decrease by a factor of 2, Re = $-\zeta\omega_n$ = -2.065 x 2 = - 4.13. The imaginary part is $-4.13\tan 110.97^0 = 10.77$. Hence, the compensated dominant poles are $-4.13 \pm j10.77$. The compensator zero is at -7. Using the uncompensated system's poles along with the compensator zero, the summation of angles to the design point, $-4.13 \pm j10.77$ is –162.06°. Thus, the contribution of the compensator pole must be –162.06° - 180° = -17.94°. Using the following geometry, $\frac{10.77}{p_c - 4.13}$ = tan 17.94°, or $p_c = 37.4$.



Adding the compensator pole and using $-4.13 \pm j10.77$ as the test point, $K = 5443$.

b. Searching the real axis segments for $K = 5443$ yields higher-order poles at approximately –8.12 and –42.02. The pole at –42.02 can be neglected since it is more than five times further from the imaginary axis than the dominant pair. The pole at –8.12 may not be canceling the zero at -7. Hence, simulate to be sure the requirements are met.

c.

Program:

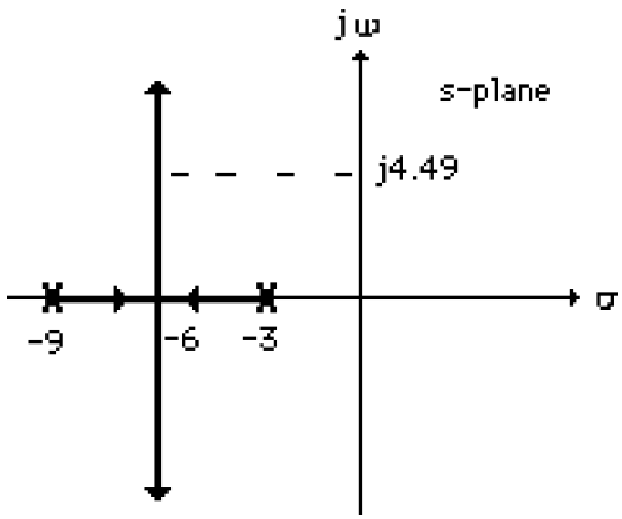
```
'Uncompensated System G1(s)'
numg1=1;
deng1=poly([-15 (-3+2*j) (-3-2*j)]);
G1=tf(numg1,deng1)
G1zpk=zpk(G1)
K1=366.8
'T1(s)'
T1=feedback(K1*G1,1);
T1zpk=zpk(T1)
'Compensator Gc(s)'
numc=[1 7];
denc=[1 37.4];
Gc=tf(numc,denc)
'Compensated System G2(s) = G1(s)Gc(s)'
K2=5443
G2=G1*Gc;
G2zpk=zpk(G2)
'T2(s)'
T2=feedback(K2*G2,1);
T2zpk=zpk(T2)
step(T1,T2)
title(['Uncompensated and Lead Compensated Systems'])
```

Problem 6:

a. Since %OS = 1.5%, $\zeta = \dfrac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = 0.8$. Since $T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{2}{3}$ second,

ω_n = 7.49 rad/s. Hence, the location of the closed-loop poles must be -6±j4.49. The summation of angles from open-loop poles to -6±j4.49 is -226.3°. Therefore, the design point is not on the root locus.

b. A compensator whose angular contribution is 226.3°-180° = 46.3° is required. Assume a compensator zero at -5 canceling the pole. Thus, the breakaway from the real axis will be at the required -6 if the compensator pole is at -9 as shown below.

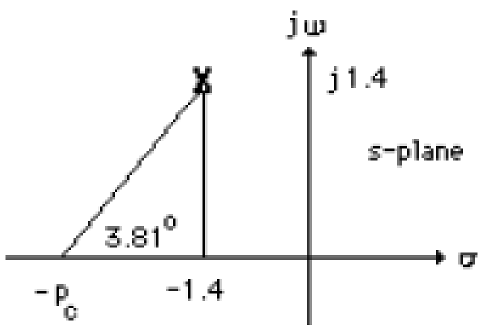


Adding the compensator pole and zero to the system poles, the gain at the design point is found to be 29.16. Summarizing the results: $G_c(s) = \frac{s+5}{s+9}$ with K = 29.16.

Problem 7:

a. For the settling time to be 2.86 seconds with 4.32% overshoot, the real part of the compensated dominant poles must be $\frac{4}{T_s} = \frac{4}{2.86} = 1.4$. Hence the compensated dominant poles are $-1.4 \pm j1.4$.

Assume the compensator zero to be at -1 canceling the system pole at -1. The summation of angles to the design point at $-1.4 \pm j1.4$ is $-176.19°$. Thus the contribution of the compensator pole is $176.19° - 180° = 3.81°$. Using the geometry below, $\frac{1.4}{p_c - 1.4} = \tan 3.81°$, or $p_c = 22.42$.



Adding the compensator pole and using $-1.4 \pm j1.4$ as the test point, $K = 88.68$.

b. Uncompensated: Search the 135° line (4.32% overshoot) and find the uncompensated dominant pole at $-0.419 + j0.419$ with $K = 1.11$. Thus $K_v = \frac{1.11}{3} = 0.37$. Hence, $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.419} = 9.55$ seconds and %OS = 4.32%. Compensated: $K_v = \frac{88.68}{22.42 \times 3} = 1.32$ (Note: steady-state error improvement is greater than 2). $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.4} = 2.86$ seconds and %OS = 4.32%.

c. Uncompensated: $K = 1.11$; Compensated: $K = 88.68$.

d. Uncompensated: Searching the real axis segments for $K = 1.11$ yields a higher-order pole at -3.16 which is more than five times the real part of the uncompensated dominant poles. Thus the second-order approximation for the uncompensated system is valid.

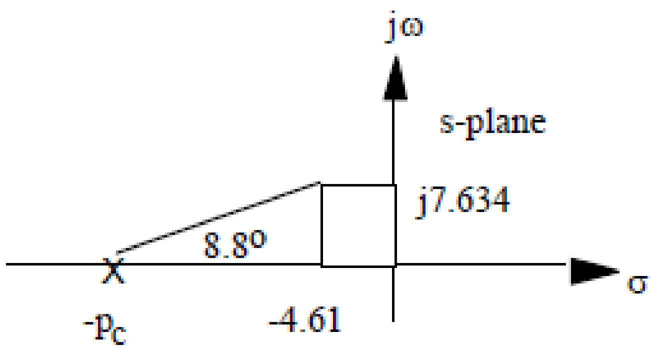
Compensated: Searching the real axis segments for $K = 88.68$ yields a higher-order pole at -22.62 which is more than five times the real part of the compensated dominant poles' real part. Thus the second order approximation is valid.

Problem 8:

a. Searching the 30% overshoot line ($\zeta = 0.358$; 110.97°) for 180° yields -1.464 + j3.818 with a gain, K = 218.6.

b. $T_p = \frac{\pi}{\omega_d} = \frac{\pi}{3.818} = 0.823$ second. $K_v = \frac{218.6}{(5)(11)} = 3.975$.

c. Lead design: From the requirements, the percent overshoot is 15% and the peak time is 0.4115 second. Thus, $\zeta = \frac{-\ln(\%/100)}{\sqrt{\pi^2+\ln^2(\%/100)}} = 0.517$; $\omega_d = \frac{\pi}{T_p} = 7.634 = \omega_n\sqrt{1-\zeta^2}$. Hence, $\omega_n = 8.919$. The design point is located at $-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} = -4.61 + j7.634$. Assume a lead compensator zero at -5. Summing the angles of the uncompensated system's poles as well as the compensator zero at -5 yields −171.2°. Therefore, the compensator pole must contribute (171.2° - 180°) = -8.8°. Using the geometry below,



$\frac{7.634}{p_c - 4.61} = \tan(8.8°)$. Hence, $p_c = 53.92$. The compensated open-loop transfer function is $\frac{K}{s(s+11)(s+53.92)}$. Evaluating the gain for this function at the point, -4.61 + j7.634 yields K = 4430.

Lag design: The uncompensated $K_v = \frac{218.6}{(5)(11)} = 3.975$. The required K_v is 30*3.975 = 119.25.

The lead compensated $K_v = \frac{4430}{(11)(53.92)} = 7.469$. Thus, we need an improvement over the lead compensated system of 119.25/7.469 = 15.97. Thus, use a lag compensator

$G_{lag}(s) = \frac{s+0.01597}{s+0.001}$. The final open-loop function is $\frac{4430(s+0.01597)}{s(s+11)(s+53.92)(s+0.001)}$.

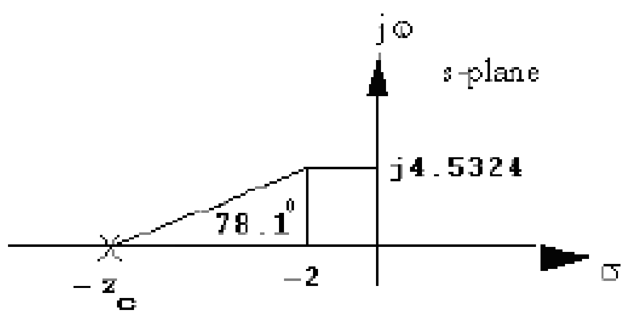
Problem 9:

a. The desired operating point is found from the desired specifications. $\zeta\omega_n = \frac{4}{T_s} = \frac{4}{2} = 2$ and

$\omega_n = \frac{2}{\zeta} = \frac{2}{0.4037} = 4.954$. Thus, $\text{Im} = \omega_n\sqrt{1-\zeta^2} = 4.954\sqrt{1-0.4037^2} = 4.5324$. Hence

the design point is –2 +j4.5324. Now, add a pole at the origin to increase system type and drive error to zero for step inputs.

Now design a PD controller. The angular contribution to the design point of the system poles and pole at the origin is 101.9^0. Thus, the compensator zero must contribute $180^0 - 101.9^0 = 78.1^0$. Using the geometry below,



$\frac{4.5324}{z_c - 2} = \tan(78.1^0)$. Hence, $z_c = 2.955$. The compensated open-loop transfer function with PD

compensation is $\frac{K(s+2.955)}{s(s+4)(s+6)(s+10)}$. Adding the compensator zero to the system and

evaluating the gain for this at the point $-2 + j4.5324$ yields K = 294.51 with a higher-order pole at -2.66 and -13.34.

PI design: Use $G_{PI}(s) = \frac{(s+0.01)}{s}$. Hence, the equivalent open-loop transfer function is

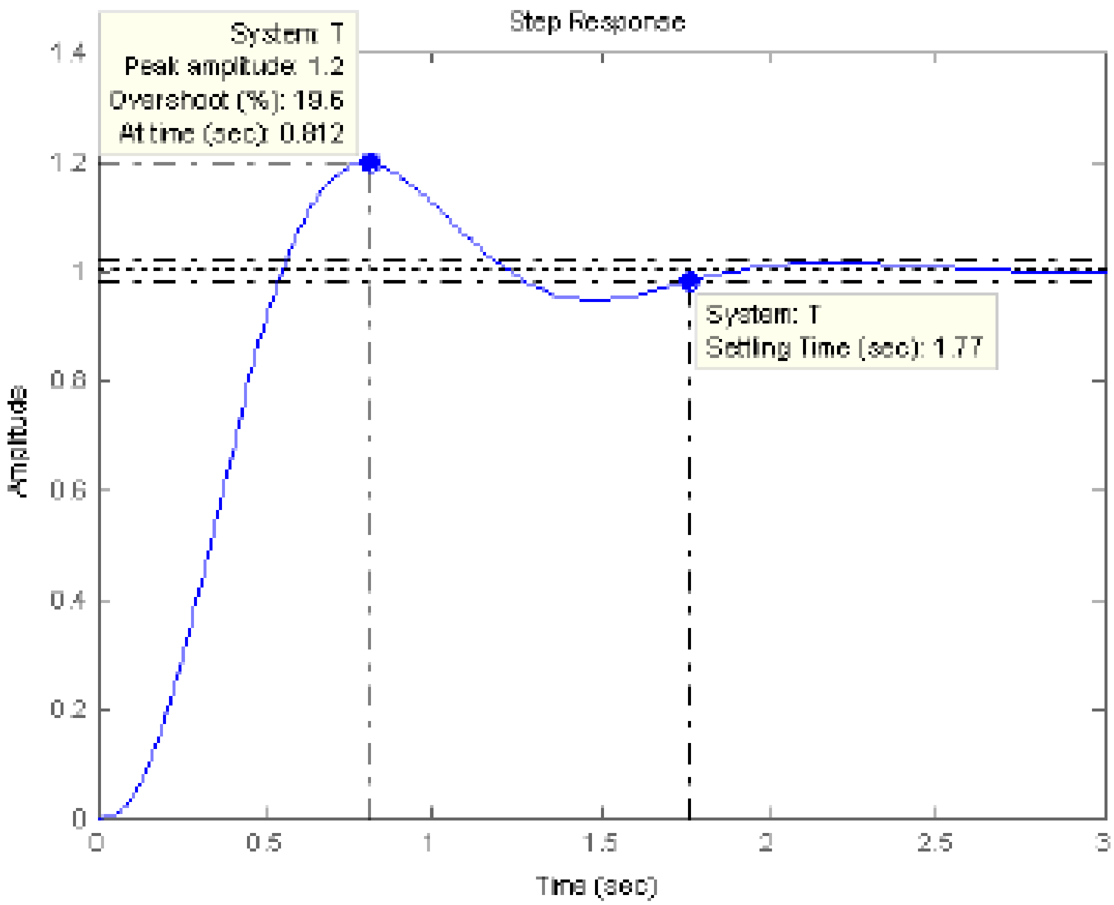
$$G_e(s) = \frac{K(s+2.955)(s+0.01)}{s^2(s+4)(s+6)(s+10)}$$ with K = 294.75.

b.

Program (Step Response):

```
numg=[-2.995 -0.01];
deng=[0  0 -4 -6 -10];
K=294.75;
G=zpk(numg,deng,K)
T=feedback(G,1);
step(T)
```

```
294.75 (s+2.995) (s+0.01)
-------------------------
 s^2 (s+4) (s+6) (s+10)
```



Program (Ramp Response):

```
numg=[-2.995 -0.01];
deng=[0 0 -4 -6 -10];
K=294.75;
G=zpk(numg,deng,K)
T=feedback(G,1);
Ta=tf([1],[1 0]);
step(T*Ta)
```

Computer response:

```
Zero/pole/gain:
294.75 (s+2.995) (s+0.01)
-------------------------
 s^2 (s+4) (s+6) (s+10)
```