

CONTROL THEORY ASSIGNMENT – LAPLACE TRANSFORMS

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1 Homework

Task 1 Find the Laplace Transforms of the following functions:

(a) $f(t) = (t^2 - 3)^2$

(e) $f(t) = e^{-3t} \cos \pi t$

(b) $f(t) = \sin^2(4t)$

(f) $f(t) = (t + 1)^3$

(c) $f(t) = \sin(3t - \frac{1}{2})$

(g) $f(t) = e^{-t} \sin 2t$

(d) $f(t) = -3t^4 e^{-0.5t}$

(h) $f(t) = u(t - 2\pi) \sin t$

Task 2 Find the Inverse Laplace Transforms of the following functions

(a) $F(s) = \frac{\pi}{(s+\pi)^2}$

(e) $F(s) = \frac{5(s+2)}{s^2(s+1)(s+3)}$

(b) $F(s) = \frac{s-6}{(s-1)^2+4}$

(f) $F(s) = \frac{s^2+2s+3}{(s+1)^3}$

(c) $F(s) = \frac{2s+16}{(s-4)(s+4)}$

(g) $F(s) = \frac{s^2+2s+4}{s^2}$

(d) $F(s) = \frac{(4s-2)}{s^2-6s+18}$

(h) $F(s) = \frac{2s^2+4s+5}{s(s+1)}$

Task 3 Solve the following differential equations using Laplace Transforms:

(a) $9\ddot{y} - 6\dot{y} + y = 0$ $y(0) = 3$ $\dot{y}(0) = 1$

(b) $\ddot{y} + 10\dot{y} + 24y = 144t^2$ $y(0) = \frac{19}{12}$ $\dot{y}(0) = -5$

(c) $\ddot{y} + 2\dot{y} + 2y = e^{-t} + 5\delta(t-2)$ $y(0) = 0$ $\dot{y}(0) = 0$

(d) $\ddot{y} + 4y = 0$ $y(0) = 5$ $\dot{y}(0) = 0$

(e) $\ddot{y} + \omega_n^2 y = t$ $y(0) = 0$ $\dot{y}(0) = 0$

(f) $\ddot{y} + y = \sin 3t$ $y(0) = 0$ $\dot{y}(0) = 0$

(g) $2\ddot{y} + 2\dot{y} + y = 1$ $y(0) = 0$ $\dot{y}(0) = 2$

2 Answers

Task 1

$$(a) F(s) = \frac{24}{s^5} - \frac{12}{s^3} + \frac{9}{s}$$

$$(b) F(s) = \frac{32}{s(s^2+64)}$$

$$(c) F(s) = \frac{3 \cos \frac{1}{2} - s \sin \frac{1}{2}}{s^2+9}$$

$$(d) F(s) = -\frac{72}{(s+0.5)^5}$$

$$(e) F(s) = \frac{s+3}{(s+3)^2+\pi^2}$$

$$(f) F(s) = \frac{s^3+3s^2+6s+6}{s^4}$$

$$(g) F(s) = \frac{2}{(s+1)^2+4}$$

$$(h) F(s) = e^{-2\pi s} \frac{1}{s^2+1}$$

Task 2

$$(a) f(t) = \pi t e^{-\pi t}$$

$$(b) f(t) = e^t(\cos 2t - \frac{5}{2} \sin 2t)$$

$$(c) f(t) = 3e^{4t} - e^{-4t}$$

$$(d) f(t) = e^{3t}(4 \cos 3t + \frac{10}{3} \sin 3t)$$

$$(e) f(t) = \frac{10}{3}t - \frac{25}{9} + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t}$$

$$(f) f(t) = e^{-t}(t^2 + u(t))$$

$$(g) f(t) = \delta(t) + 2u(t) + 4t$$

$$(h) f(t) = 2\delta(t) + 5u(t) - 3e^{-t}u(t)$$

Task 3 Solve the following differential equations using Laplace Transforms:

$$(a) y(t) = 3e^{\frac{t}{3}}$$

$$(b) y(t) = 6t^2 - 5t + \frac{19}{12}$$

$$(c) y(t) = e^{-t} \sin t + e^{-t} - e^{-t} \cos t + 5e^{-(t-2)} \sin(t-2)u(t-2)$$

$$(d) y(t) = 5 \cos 2t$$

$$(e) y(t) = \frac{1}{\omega_n^2}(t + \frac{1}{\omega_n} \sin \omega_n t)$$

$$(f) y(t) = \frac{3}{8} \sin t - \frac{1}{8} \sin 3t$$

$$(g) y(t) = u(t) - e^{-\frac{t}{2}}(\cos \frac{t}{2} - \sin \frac{t}{2})$$