Control Theory Assignment – Mathematical Modeling

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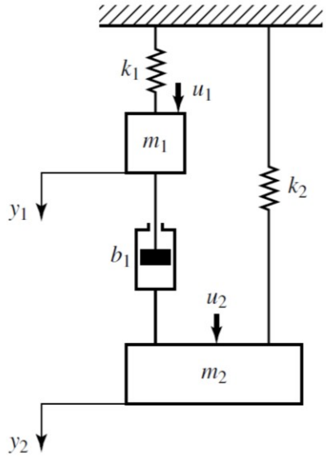
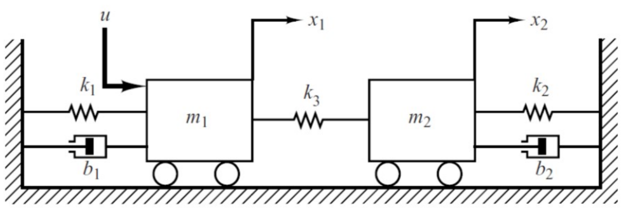
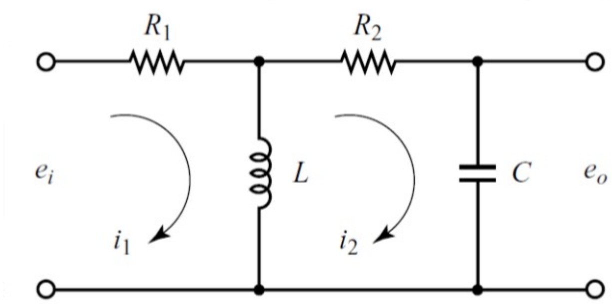
1 Find the differential equations representing the system shown in the Figure.



2 Obtain the Transfer Functions $X_1(s)/U(s)$ and $X_2(s)/U(s)$ of the mechanical system shown in the Figure.



3 Obtain the Transfer Function $E_o(s)/E_i(s)$ of the electrical circuit shown in the Figure.



4 A laser printer uses a laser beam to print copy rapidly for a computer. The laser is positioned by a control input $r(t)$, so that we have:

$$Y(s) = \frac{4(s+50)}{s^2+30s+200}R(s) \tag{1}$$

The input $r(t)$ represents the desired position of the laser beam.

(a) If $r(t)$ is a unit step input, find the output $y(t)$.

(b) What is the final value of $y(t)$?

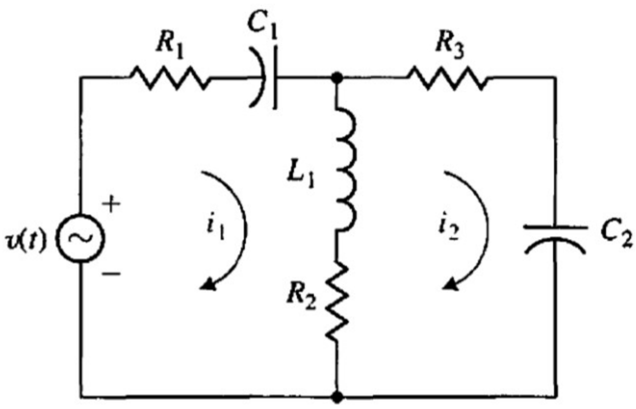
Solution: (a) $y(t) = 1 + 0.6e^{-20t} - 1.6e^{-10t}$, (b) $y_{ss} = 1$

5 The output y and the input x of a device are related by:

$$y = x + 1.4x^3 \tag{2}$$

Obtain a linearized model of the system for an equilibrium point $x_o = 1$.

6 Using the Laplace Transformation, obtain the current $I_2(s)$ from the circuit in the Figure below. Assume that all initial currents are zero, the initial voltage across C_1 is zero, $v(t)$ is zero and the initial voltage across C_2 is 10 v.



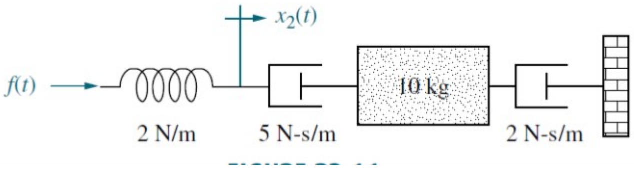
7 For each of the following Transfer Functions, write the corresponding differential equation:

1. $$\frac{X(s)}{F(s)} = \frac{7}{s^2+5s+10} \tag{3}$$

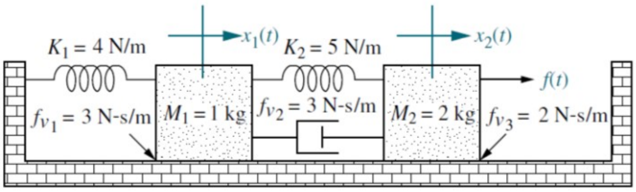
2. $$\frac{X(s)}{F(s)} = \frac{15}{(s+10)(s+11)} \tag{4}$$

3. $$\frac{X(s)}{F(s)} = \frac{s+3}{s^3+11s^2+12s+18} \tag{5}$$

8 Find the Transfer Function $G(s) = \frac{X_2(s)}{F(s)}$ for the translational mechanical system shown in the Figure. (Hint: place a zero mass at $x_2(t)$).



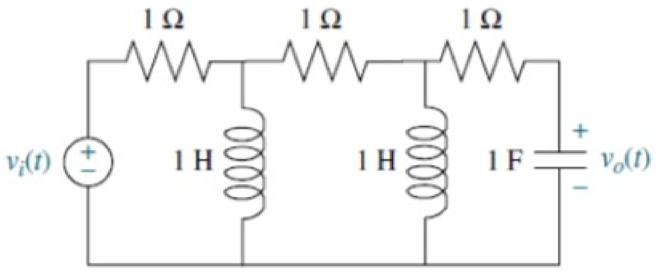
9 For the system in the Figure below, find the Transfer Function $G(s) = \frac{X_1(s)}{F(s)}$.



10 Linearize the following function for small changes of x about $x_o = 0$: $f(x) = e^{-x}$.

11 Obtain a state space representation of the system in Problem 1.

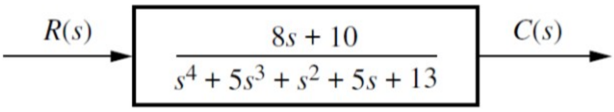
12 Represent the following network in state space knowing that $v_o(t)$ is the output.



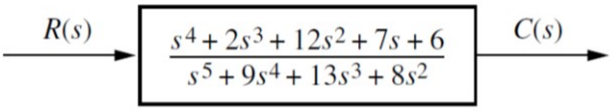
13 Obtain a state space representation of the system described by the following differential equation:

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 20u(t) \tag{6}$$

14 For each of the systems shown in the Figure, find the state equations and output equations.



(a)



(b)

15 For each of the following state space representations, find the corresponding Transfer Function:

$$(1)\ \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$(2)\ \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 3 & -5 & 2 \\ 1 & -8 & 7 \\ -3 & -6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$