BIRZEIT UNIVERSITY

Faculty of Engineering and Technology
Department of Mechanical and Mechatronics Engineering
Final Examination – Fall 2016

ENME 438: Control Theory
Date of Examination: 1/2/2017
Instructor: Eng. Sima Rishmawi

Student ID: __________________
Time duration: 2 hours 30 minutes
Total Marks: 100

This exam contains 9 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter your Student ID number on the top of this page, and at the bottom of every page, in case the pages become separated.

You may *not* use your books, notes, or any other reference on this exam, except for a three-sided A4 cheat sheet. You can use your own calculator only. Borrowing calculators is not allowed.

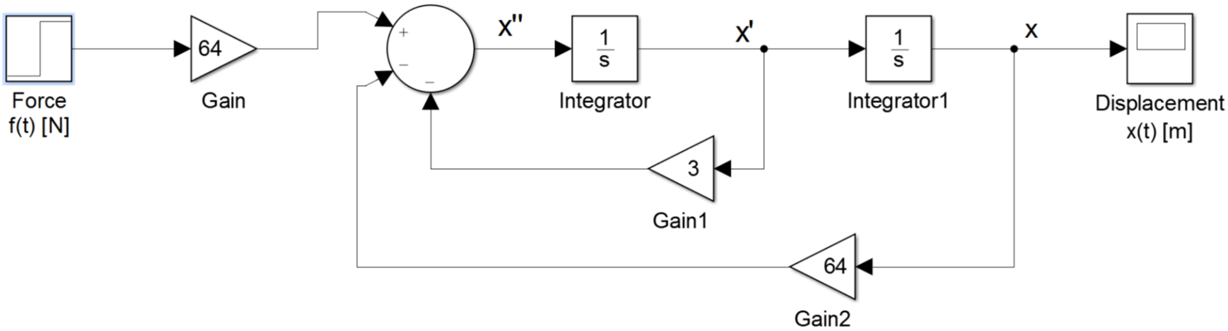
The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	15	
4	15	
5	40	
Total:	110	

1) The figure below shows the block diagram of a spring-mass-damper system, created on Simulink. Use the block diagram to find the following:



(a) The Transfer Function $T(s) = \frac{X(s)}{F(s)}$

(b) The natural frequency and the damping ratio. Is the system under-damped, over-damped, or critically damped? Explain.

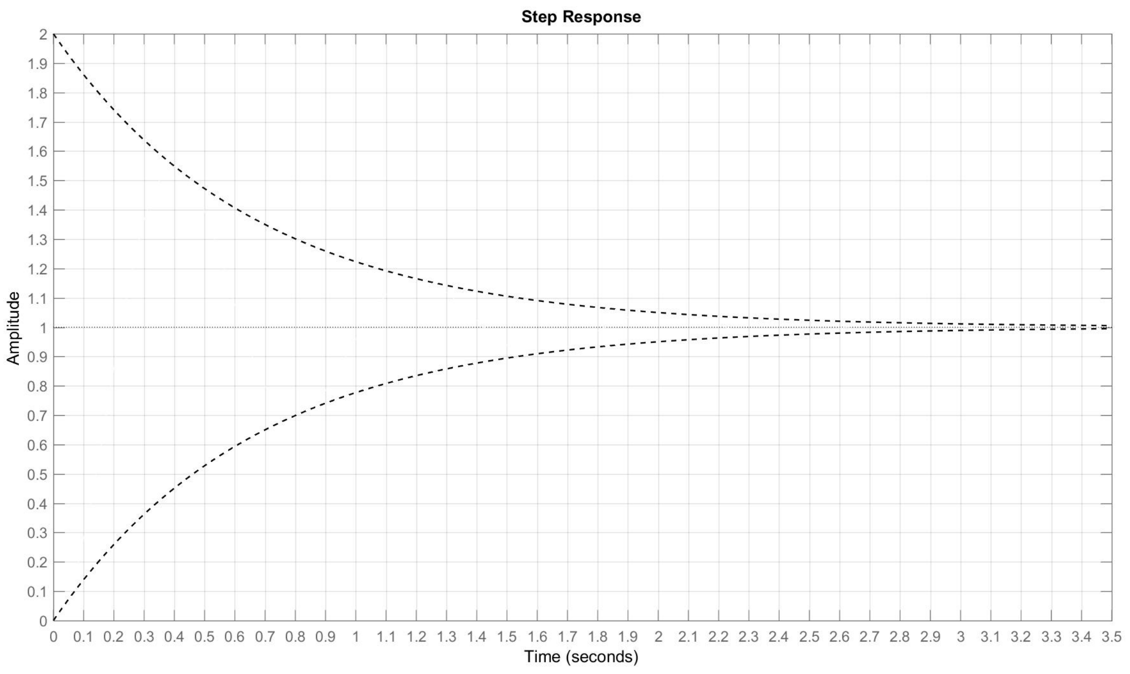
(c) Peak Time

(d) Rise Time

(e) Percentage Overshoot

(f) Settling Time

(g) Using the parameters calculated, sketch the response $x(t)$ in the designated area as expected to appear on the oscilloscope.



Question 1 continues on the next page...

20 marks

Solution:

(a)

$$G_1(s) = \frac{\frac{1}{s}}{1+\frac{3}{s}} \times \frac{1}{s} = \frac{1}{s^2+3s}$$

$$T(s) = \frac{\frac{1}{s^2+3s}}{1+\frac{64}{s^2+3s}} \times 64 = \frac{64}{s^2+3s+64}$$

(b)

$$\omega_n = \sqrt{64} = 8\ rad/s\ ,\ \zeta = \frac{3}{2\omega_n} = 0.1875$$

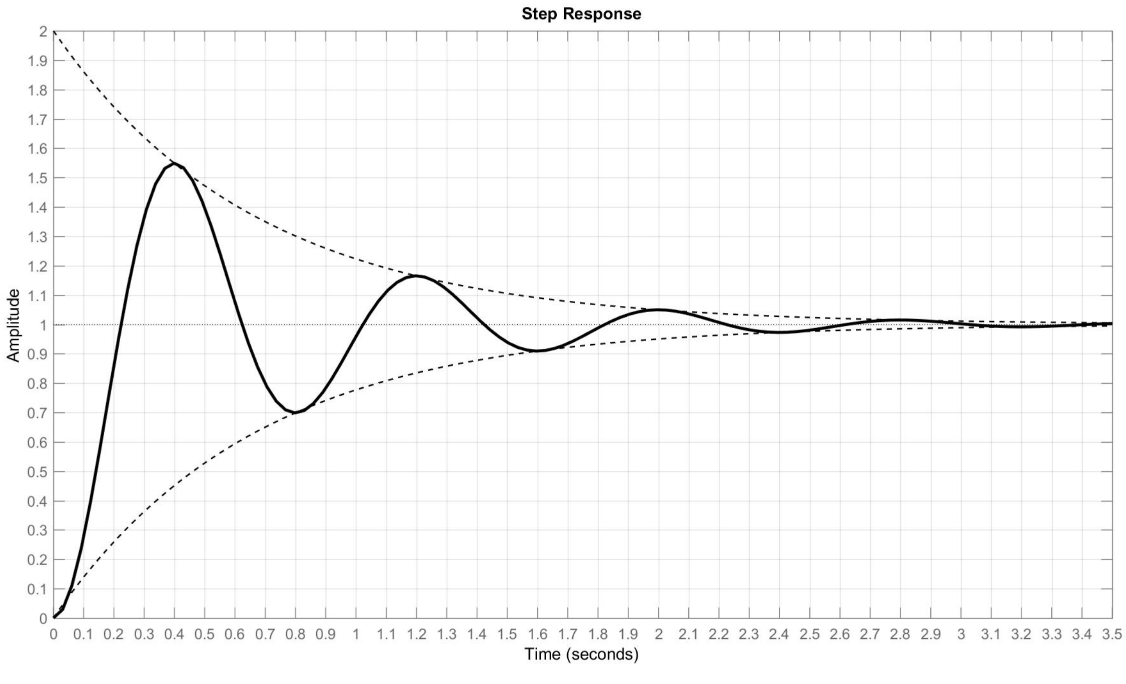
System is under-damped, $\zeta < 1$

(c) $T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.4\ s$

(d) $T_r = \frac{1.203}{wn} = 0.15\ s$

(e) $\%OS = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% = 54.9\%$

(f) $T_s = \frac{4}{\zeta\omega_n} = 2.7\ s$



2) The open-loop function of a unity feedback system is:

$$G(s) = \frac{K}{s(0.5s+1)(2s+1)}$$

(a) Determine the range of values for K over which the closed-loop system will be stable.

(b) If we want to increase the stability range to 3 times the range you obtained using a unity feedback, by adding a feedback transfer function $H(s) = s + a$, what would be the minimum value of a? ($a \neq 0$)

20 marks

Solution:

(a)

$$T(s) = \frac{K}{s^3 + 2.5s^2 + s + K}$$

Set up the Routh-Hurwitz Table:

$$\begin{array}{c|cc} s^3 & 1 & 1 \\ s^2 & 2.5 & K \\ s^1 & \frac{2.5-K}{2.5} & 0 \\ s^0 & K & 0 \end{array}$$

For stability: $K > 0$ and $K < 2.5$, thus $\boxed{0 < K < 2.5}$

(b)

$$T(s) = \frac{K}{s^3 + 2.5s^2 + (K+1)s + Ka}$$

Choose $K = 7.5$ to increase the stability region to three times the previous one.

Set up the Routh-Hurwitz Table:

$$\begin{array}{c|cc} s^3 & 1 & 8.5 \\ s^2 & 2.5 & 7.5a \\ s^1 & \frac{21.25-7.5a}{2.5} & 0 \\ s^0 & 7.5 & 0 \end{array}$$

$\boxed{a = 2.83}$

3) Use the Ziegler-Nichols Method to design a PID controller for the system that has the following open-loop transfer function:

$$G(s) = \frac{K}{s^3 + 2s^2 + 2s + 1}$$

(a) Write the expression for the controller transfer function

(b) What are the poles and/or zeros that were added to the system?

(c) If the response of the controlled system still had a significant steady-state error, which parameter should be fine-tuned? Why?

15 marks

Solution:

$$G(s) = \frac{K}{s^3 + 2s^2 + 2s + 1}$$

$$T(s) = \frac{K}{s^3 + 2s^2 + 2s + (1 + K)}$$

Set up the Routh-Hurwitz Table:

s^3	1	2
s^2	2	$(1+K)$
s^1	$\frac{4-(1+K)}{2}$	0
s^0	$1+K$	0

For stability:

$$K > -1 \rightarrow K > 0$$

$$4 - K - 1 > 0 \rightarrow K < 3$$

$$\boxed{K_{cr} = 3}$$

Using the value of K_{cr} in the second row to find P_{cr}:

$$2s^2 + 4 = 0, s = j\omega_{cr}$$

$$2(j\omega_{cr})^2 + 4 = 0 \rightarrow \omega_{cr} = \sqrt{2}$$

$$P_{cr} = \frac{2\pi}{\omega_{cr}} = 4.44\ s$$

$$K_p = 0.6K_{cr} = 0.6 \times 3 = 1.8$$

$$T_i = 0.5P_{cr} = 0.5 \times 4.44 = 2.22\ s$$

$$T_d = 0.125P_{cr} = 0.125 \times 4.44 = 0.555\ s$$

 Question 3 continues on the next page...

(a)

$$G_c(s) = K_p\Big(1 + \frac{1}{T_i s} + T_d s\Big) = 1.8\Big(1 + \frac{1}{2.22s} + 0.555s\Big)$$

$$\boxed{G_c(s) = \frac{4.435s^2 + 7.992s + 1.8}{2.22s} = \frac{0.999s^2 + 1.8s + 0.811}{s}}$$

(b) Zeros are at: -0.99 and -0.82 Pole is at: 0

(c) To improve the steady-state error we need to tune the T_i parameter, because the PI controller is responsible for improving the steay-state error.

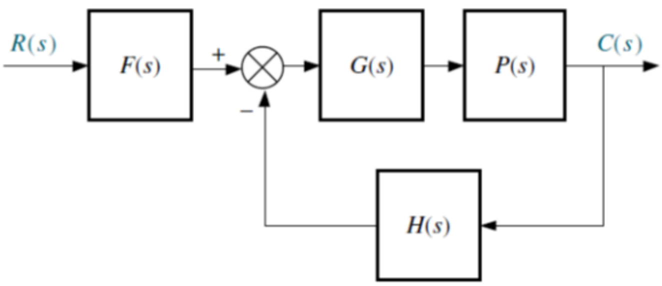
4) The transfer function from elevator deflection to altitude change in a Tower Trainer Unmanned Vehicle is:

$$P(s) = \frac{h(s)}{\delta(s)} = \frac{-34.16s^3 - 144.4s^2 + 7047s + 557.2}{s^5 + 13.18s^4 + 95.93s^3 + 14.61s^2 + 31.94s}$$

An Autopilot is built around the aircraft as shown in the figure with $F(s) = H(s) = 1$, and

$$G(s) = \frac{0.00842(s + 7.895)(s^2 + 0.108s + 0.3393)}{(s + 0.07895)(s^2 + 4s + 8)}$$

The steady-state error for a ramp input in this system is $e_{ss} = 25$. Find the slope of the ramp input.



15 marks

Solution:

$$K_v = \lim_{s\to 0} sP(s)G(s)$$
$$= \lim_{s\to 0} s\frac{-34.16s^3 - 144.4s^2 + 7047s + 557.2}{s^5 + 13.18s^4 + 95.93s^3 + 14.61s^2 + 31.94s}\frac{0.00842(s + 7.895)(s^2 + 0.108s + 0.3393)}{(s + 0.07895)(s^2 + 4s + 8)}$$
$$= 0.623$$

For a unit step input

$$e_{ss}(\infty) = \frac{1}{K_v} = 1.605$$

The input slope is

$$\boxed{slope = \frac{25}{1.605} = 15.6}$$

5) A permanent magnet linear electromagnetic actuator has voltage as an input and force as an output. It has the transfer function shown below:

$$G_a(s) = \frac{30}{s + \frac{R}{L}}$$

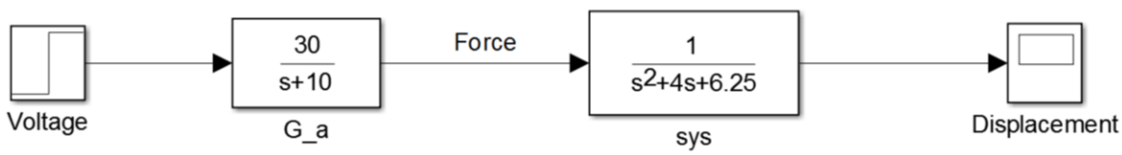
where R is the resistance of the coil of the electromagnet, and has a value of 100 $m\Omega$; L is the inductance of the coil and has a value of 10 mH.
The linear electromagnetic actuator will exert a force on a mass in a spring-mass-damper system, where the mass m is 1 kg, the spring has a spring constant k_s of 6.25 N/m, and the damper has a value b of 4 Ns/m.

(a) Draw a block diagram of the open loop system that has Voltage as its input and the mass Displacement as its output.

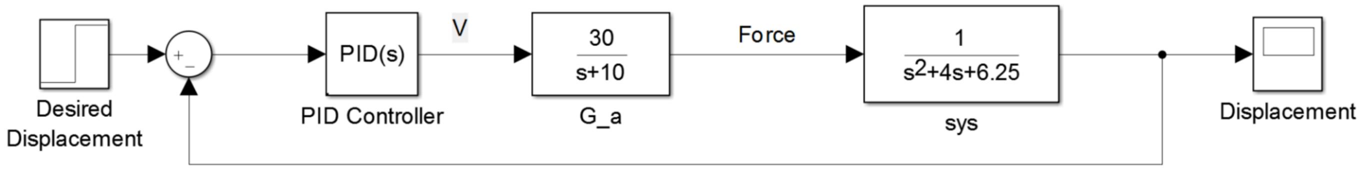
$$G_a = \frac{30}{s+10}$$

$$G_s = \frac{1}{s^2 + 4s + 6.25}$$

$$G = \frac{30}{(s+10)(s^2+4s+6.25)}$$



(b) Draw a block diagram of the closed loop system after adding the appropriate controller. This system has the desired value of the displacement as its input, and the actual value of the displacement as its output.



(c) Draw the root locus of the system. Show all your calculations and mark all the critical or important points on the diagram. Place the origin of your axes at point $(3, 9)$ cm starting from the lower right corner. Use a scale of 1 rad/s = 20 mm. Use a landscape format.
Open loop poles are:

$$p_1 = -10, \; p_2 = -2 + j1.5, \; p_3 = -2 - j1.5$$

Question 5 continues on the next page...

$$\sigma_a = \frac{-10 - 2 + j1.5 - 2 - j1.5}{3} = 143 = 4.7$$

$$\theta_a = \frac{(2k+1)\pi}{3} = 60^o,\ 180^o,\ 300^o$$

$$\phi_d = 180 - (\theta_1 + \theta_2) = 180 - 101 = 79^o$$

$$T(s) = \frac{30K}{(s+10)(s^2+4s+6.25)+30K} = \frac{30K}{s^3+14s^2+46.25s+62.5+30K}$$

Set up the Routh-Hurwitz Table:

s^3	1	46.25
s^2	14	$(62.5+30K)$
s^1	$\frac{647.5-(62.5+30K)}{14}$	0
s^0	$(62.5+30K)$	0

$$647.5 - (62.5+30K) = 0 \rightarrow K = 19.5$$

$$14s^2 + 647.5 = 0 \rightarrow s = +- j6.8$$

(d) With the help of the root locus you have just drawn, design a controller that achieves an overshoot of % while decreasing the settling time to half its original value. Your controller should also limit the steady-state error to within %. Your design is complete if you provide the transfer function of the controller showing zeros and/or poles and the gain K.

40 marks