BIRZEIT UNIVERSITY

Faculty of Engineering

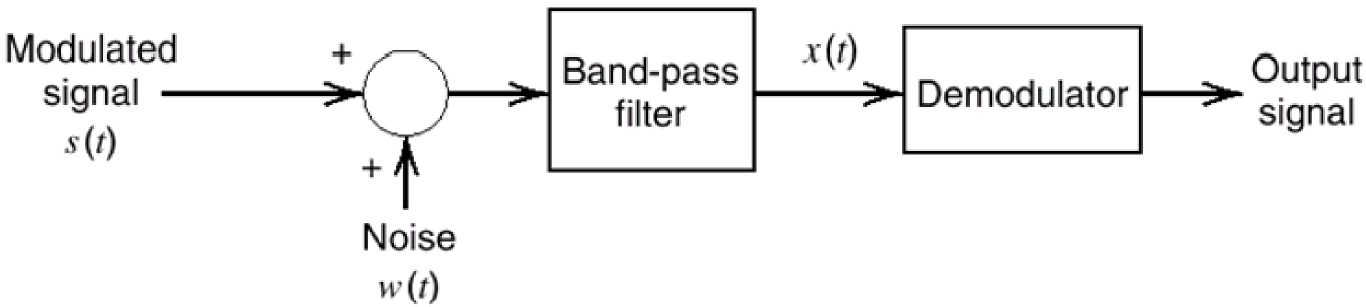
Electrical Engineering Department

Principles Of Communication Systems, ENEE3303

BIRZEIT UNIVERSITY
ECE Dept.

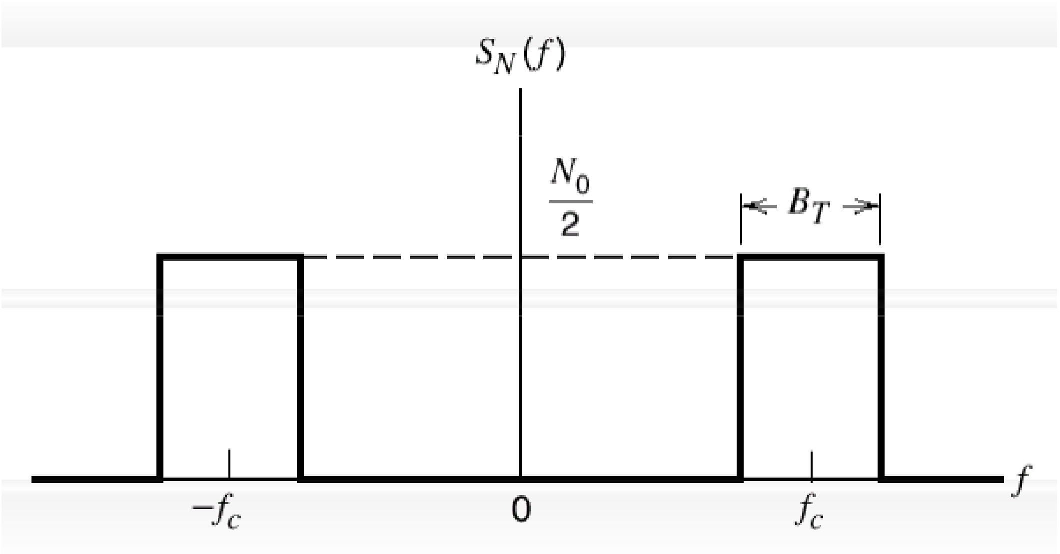
Introduction

- To undertake an analysis of noise in continuous-wave (CM) modulation systems, we need a receiver model.
- The customary practice is to model the receiver noise (channel noise) as *additive*, *white*, and *Gaussian*. These simplifying assumptions enable us to obtain a basic understanding of the way in which noise affects the performance of the receiver.



- $s(t)$ denotes the incoming modulated signal.
- $w(t)$ denotes front-end receiver noise. The power spectral density of the noise $w(t)$ is denoted by $N_0/2$, defined for both positive and negative frequencies. N_0 is the average noise power per unit bandwidth measured at the front end of the receiver.
- The bandwidth of this band-pass filter is just wide enough to pass the modulated signal without distortion.
- Assume the band-pass filter is ideal, having a bandwidth equal to the transmission bandwidth B_T of the modulated signal $s(t)$, and a mid- band frequency equal to the carrier frequency f_c, $f_c >> B_T$.

BIRZEIT UNIVERSITY
ECE Dept.



Idealized characteristic of band-pass filtered noise.

- The filtered noise $n(t)$ may be treated as a narrow band noise represented in the canonical form:

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

where $n_I(t)$ is the in-phase noise component and $n_Q(t)$ is the quadrature noise component, both measured with respect to the carrier wave $A_c\cos(2\pi f_c t)$.

BIRZEIT UNIVERSITY
ECE Dept.

- The filtered signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t)$$

- The average noise power at the demodulator input is equal to the total area under the curve of the power spectral density $S_N(f)$:

$$P_{\text{avg-noise}} = 2 \times B_T \times \frac{N_0}{2} = B_T N_0$$

- ◇ *Input signal-to-noise ratio* $(SNR)_I$ is defined as:

$$(SNR)_I = \frac{\text{average power of the modulated signal } s(t)}{\text{average power of the filtered noise } n(t)}$$

- ◇ *Output signal-to-noise ratio* $(SNR)_O$ is defined as:

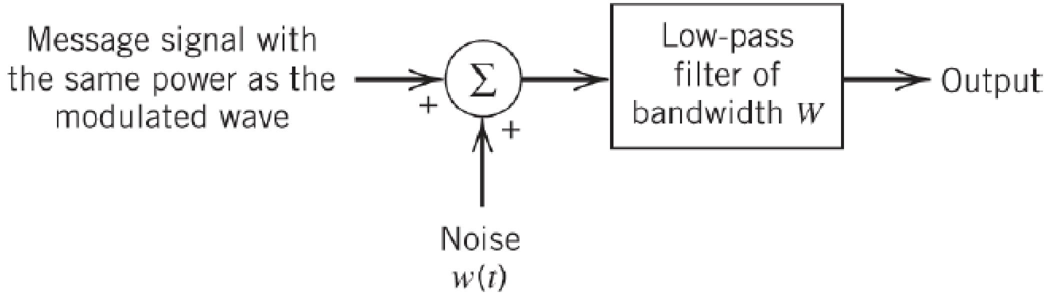
$$(SNR)_O = \left.\frac{\text{average power of the demodulated message signal}}{\text{average power of the noise}}\right|_{\text{measured at the receiver output}}$$

BIRZEIT UNIVERSITY
ECE Dept.

- *Channel signal-to-noise ratio*

$$(SNR)_C = \left.\frac{\text{average power of the modulated signal}}{\text{average power of noise in the message BW}}\right|_{\text{measured at the receiver input}}$$

- This ratio may be viewed as the signal-to-noise ratio that results from baseband (direct) transmission of the message signal $m(t)$ without modulation, as demonstrated in the following figure:



  - The message power at the low-pass filter input is adjusted to be the same as the average power of the modulated signal
  - The low-pass filter passes the message signal and rejects out-of-band noise.

BIRZEIT UNIVERSITY
ECE Dept.

- *Figure of merit*
  - For the purpose of comparing different continuous-wave (CW) modulation systems, we normalize the receiver performance by dividing the output signal-to-noise ratio by the channel signal-to-noise ratio.
  - The higher the value of the figure of merit, the better will the noise performance of the receiver be.
  - The figure of merit may equal one, be less than one, or be greater than one, depending on the type of modulation used.

$$\text{Figure of merit} = \frac{(SNR)_O}{(SNR)_C}$$

Noise in DSB-SC Receiver

- The model of a DSB-SC receiver using a coherent detector

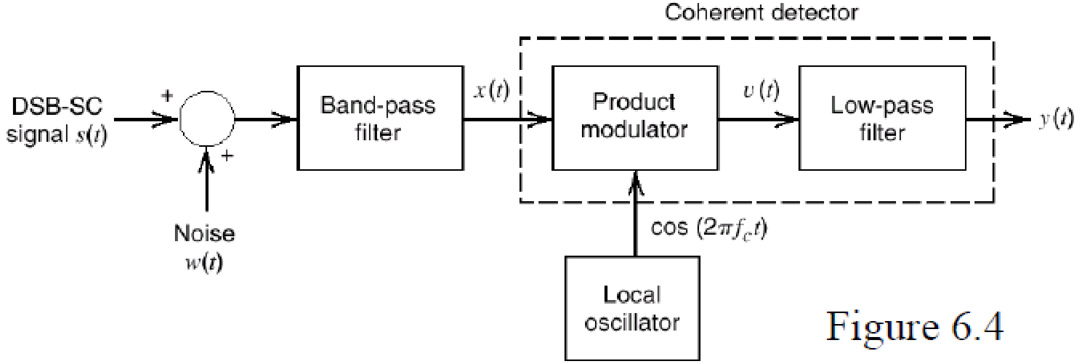


Figure 6.4

- The amplitude of the locally generated sinusoidal wave is assumed to be unity.
- For the demodulation scheme to operate satisfactorily, it is necessary that the local oscillator be synchronized both in phase and in frequency with the oscillator generating the carrier wave in the transmitter. We assume that this synchronization has been achieved.

Noise in DSB-SC Receiver

BIRZEIT UNIVERSITY
ECE Dept.

- The DSB-SC component of the modulated signal $s(t)$ is expressed as

$$s(t) = CA_c \cos(2\pi f_c t)\, m(t)$$

  where C is the system dependent scaling factor. The purpose of which is to ensure that the signal component $s(t)$ is measured in the same units as the additive noise component $n(t)$.
- $m(t)$ is the sample function of a stationary process of zero mean, whose power spectral density $S_M(f)$ is limited to a maximum frequency W, i.e. W is the message bandwidth.
- The average power P of the message signal is the total area under the curve of power spectral density

$$P = \int_{-W}^{W} S_M(f)\, df$$

- Mixing of a Random Process with a Sinusoidal Process

$$Y(f) = X(t)\cos(2\pi f_c t + \Theta)$$

$$R_Y(\tau) = \frac{1}{2} R_X(\tau)\cos(2\pi f_c \tau)$$

$$S_Y(f) = \frac{1}{4}\left[S_X(f - f_c) + S_X(f + f_c)\right]$$

- Therefore, the average power of the DSB-SC modulated signal component $s(t)$ is given by: $\dfrac{C^2 A_c^2 P}{2}$
- The average power of the noise in the message BW is WN_0
- The channel signal-to-noise ratio of the DSB-SC modulation system is:

$$(\mathrm{SNR})_{C,\mathrm{DSB}} = \frac{C^2 A_c^2 P}{2WN_0}$$

BIRZEIT UNIVERSITY
ECE Dept.

- Next, we wish to determine the output signal-to-noise ratio.
- Using the narrowband representation of the filtered noise $n(t)$, the total signal at the coherent detector input may be expressed as:

$$x(t) = s(t) + n(t)$$
$$= CA_c \cos(2\pi f_c t) m(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

- The output of the product-modulator component of the coherent detector is:

$$\upsilon(t) = x(t)\cos(2\pi f_c t)$$
$$= \frac{1}{2}CA_c m(t) + \frac{1}{2}n_I(t)$$
$$+\frac{1}{2}\left[CA_c m(t) + n_I(t)\right]\cos(4\pi f_c t) - \frac{1}{2}A_c n_Q(t)\sin(4\pi f_c t)$$

cos(α+β) + cos(α−β) = 2cos(α)cos (β)
sin(α+β) + sin(α−β) = 2sin(α)cos(β)

low-pass filter, BW=W

$$y(t) = \frac{1}{2}CA_c m(t) + \frac{1}{2}n_I(t)$$

Noise in DSB-SC Receiver

BIRZEIT UNIVERSITY
ECE Dept.

- The message signal $m(t)$ and in-phase noise component $n_I(t)$ of the filtered noise $n(t)$ appear additively at the receiver output.
- The quadrature component $n_Q(t)$ of the noise $n(t)$ is completely rejected by the coherent detector.
- We note that these two results are independent of the input signal-to-noise ratio.
- Thus, coherent detection distinguishes itself from other demodulation techniques in the important property: <u>the output message component is unmutilated and the noise component always appears additively with the message, irrespective of the input signal-to-noise ratio</u>.

Noise in DSB-SC Receiver

- The receiver output signal : $y(t)=\frac{1}{2}CA_c m(t)+\frac{1}{2}n_I(t)$
- The average power of message component may be expressed as

$$P_{avg} = \frac{C^2 A_c^2 P}{4}$$

- The average power of the noise at the receiver output is

$$\left(\frac{1}{2}\right)^2 2WN_0 = \frac{1}{2}WN_0$$

BIRZEIT UNIVERSITY
ECE Dept.

- The output signal-noise ratio for DSB-SC

$$(\mathrm{SNR})_{O,\mathrm{DSB\text{-}SC}} = \frac{C^2 A_c^2 P/4}{WN_0/2} = \frac{C^2 A_c^2 P}{2WN_0}$$

- We obtain the figure of merit

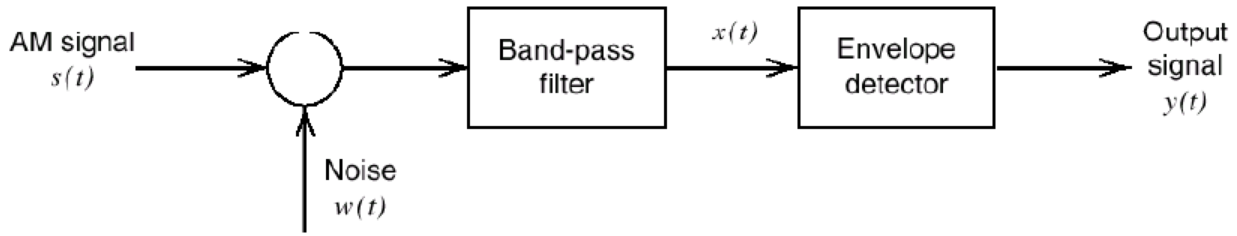
$$\left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\mathrm{DSB\text{-}SC}} = 1$$

- A full AM signal is given by

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

where $A_c\cos(2\pi f_c t)$ is the carrier wave, $m(t)$ is the message signal and bandwidth is W, k_a is a constant that determines the percentage modulation.

- We would like to perform noise analysis for an AM system using an envelope detector.



Noise in AM Receiver

- We perform the noise analysis of the AM receiver by first determining the channel signal-to-noise ratio, and then the output signal-to-noise ratio.
- We can easily obtain average power of the AM signal

$$s(t) = A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t)$$

$$P_s = \frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 k_a^2 P$$

- The average power of noise in the message bandwdith is WN_0 (the same as the DSB-SC system)
- The channel signal-to-noise ratio for AM is therefore:

$$(\text{SNR})_{C,\text{AM}} = \frac{A_c^2 \left(1 + k_a^2 P\right)}{2WN_0}$$

BIRZEIT UNIVERSITY
ECE Dept.

- The filtered signal $x(t)$ applied to the envelope detector in the receiver is given by:

$$x(t) = s(t) + n(t)$$
$$= \left[A_c + A_c k_a m(t) + n_1(t)\right]\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$
$$y(t) = \text{envelope of } x(t)$$
$$= \left\{\left[A_c + A_c k_a m(t) + n_I(t)\right]^2 + n_Q^2(t)\right\}^{1/2}$$

⇩ Assume average carrier power >> average noise power

$$y(t) \simeq A_c + A_c k_a m(t) + n_I(t)$$

- The dc term or constant term A_c may be removed simply by means of a blocking capacitor.
- If we ignore the dc term A_c, we find that the remainder has a form similar to the output of a DSB-SC receiver using coherent detection.

Noise in AM Receiver

- The output signal-to-noise ratio of an AM using an envelope detector is approximately

$$(\mathrm{SNR})_{O,\mathrm{AM}} \simeq \frac{A_c^2 k_a^2 P}{2WN_0}$$

- This Eq. is valid only if the following two conditions are satisfied
  - The average noise power is small compared to the average carrier power at the envelope detector input.
  - The amplitude sensitive k_a is adjusted for a percentage modulation less than or equal to 100 percent. ($|k_a m(t)| <= 1$)

BIRZEIT UNIVERSITY
ECE Dept.

- Comparison of figure of merit (AM, DSB-SC, SSB)

$$\left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\substack{\text{AM using}\\ \text{Envelope Detector}}} \simeq \frac{k_a^2 P}{1+k_a^2 P} \leq 1 \iff \left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\text{DSB-SC}} = \left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\text{SSB}} = 1$$

- The figure of merit of a DSB-SC receiver or that of an SSB receiver using coherent detection is always unity.
- The corresponding figure of merit of an AM receiver using envelope detection is always less than unity.
- In other words, the noise performance of an AM receiver is always inferior to that a DSB-SC receiver. This is due to the wastage of transmitter power, which results from transmitting the carrier as a component of AM wave.

BIRZEIT UNIVERSITY
ECE Dept.

- Example
  - Consider a sinusoidal wave of frequency f_m and amplitude A_m as the modulating wave, as shown by

$$m(t) = A_m \cos(2\pi f_m t)$$

  - The corresponding AM wave is

$$s(t) = A_c\,[1+ \mu\cos(2\pi f_m t)]\,\cos(2\pi f_c t)$$

$$\text{modulation factor : } \mu = k_a A_m$$

  - The average power of the modulation wave $m(t)$ is (assuming a load resistor of 1ohm)

$$P = \frac{1}{2} A_m^2$$

- We obtain the figure of merit

$$\left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\mathrm{AM}} = \frac{\frac{1}{2}k_a^2 A_m^2}{1+\frac{1}{2}k_a^2 A_m^2} = \frac{\mu^2}{2+\mu^2}$$

- When $\mu = 1$ (100% modulation using envelope detection), we get a figure of merit = 1/3.
- This means that, other factors being equal, an AM system (using envelope detection) must transmit three times as much average power as a suppressed-carrier system (using coherent detection) in order to achieve the same quality of noise performance.

Noise in FM Receiver

- In FM system, increasing the carrier power has a noise-quieting effect.

  $$\text{Average power of output noise} = \frac{N_0}{A_c^2}\int_{-W}^{W} f^2 df = \frac{2N_0W^3}{3A_c^2} \propto \frac{2}{A_c^2}$$

- We obtain $(\text{SNR})_{O,\text{FM}} = \dfrac{k_f^2 P}{2N_0W^3/3A_c^2} = \dfrac{3A_c^2k_f^2P}{2N_0W^3}$

- The average power in the modulated signal $s(t)$ is $A^2{}_c/2$, and the average noise power in the message bandwidth is WN_o. The channel signal to noise ratio $(\text{SNR})_{C,FM}$ is

  $$(\text{SNR})_{C,\text{FM}} = \frac{A_c^2}{2WN_0}$$

- Figure of merit for frequency modulation:

  $$\left.\frac{(\text{SNR})_O}{(\text{SNR})_C}\right|_{\text{FM}} = \frac{3k_f^2P}{W^2}$$

- Example
  - A sinusoidal wave of frequency f_m as the modulating signal, and assume a peak frequency deviation Δf. The FM signal is define by

$$s(t) = A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t)\right]$$

  - Therefore, we may write

$$2\pi k_f \int_0^t m(\tau)\,d\tau = \frac{\Delta f}{f_m}(\sin 2\pi f_m t)$$

  - Differentiating both sides with respect to time and solving for $m(t)$

$$m(t) = \frac{\Delta f}{k_f}\cos(2\pi f_m t)$$

Noise in FM Receiver

- The average power of message signal $m(t)$

$$P = \frac{(\Delta f)^2}{2k_f^2}$$

- We get output signal-to-noise ratio

$$(\mathrm{SNR})_{O,\mathrm{FM}} = \frac{3A_c^2(\Delta f)^2}{4N_0W^3} = \frac{3A_c^2\beta^2}{4N_0W}$$

- Where $\beta = \Delta f / W$ is the modulation index and we get the figure of merit

$$\left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\mathrm{FM}} = \frac{3}{2}\left(\frac{\Delta f}{W}\right)^2 = \frac{3}{2}\beta^2$$

- It is important to note that the modulation index $\beta = \Delta f / W$ is determined by the bandwidth W of the postdetection low-pass filter and is related to the sinusoidal message frequency f_m.

- For an AM system operating with a sinusoidal modulating signal and 100 percent modulation, we have

$$\left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\mathrm{AM}} = \frac{1}{3}$$

- It is of particular interest to compare the noise performance of AM and FM systems.

$$\left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\mathrm{FM}} = \frac{3}{2}\left(\frac{\Delta f}{W}\right)^2 = \frac{3}{2}\beta^2 \xleftrightarrow{\text{compare}} \left.\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C}\right|_{\mathrm{AM}} = \frac{1}{3}$$

when $\frac{3}{2}\beta^2 > \frac{1}{3}$ ⇨ FM has better performance

⇨ $\beta > \frac{\sqrt{2}}{3} = 0.471$

- Define $\beta = 0.5$ as the transition between narrowband FM and wide-band FM.