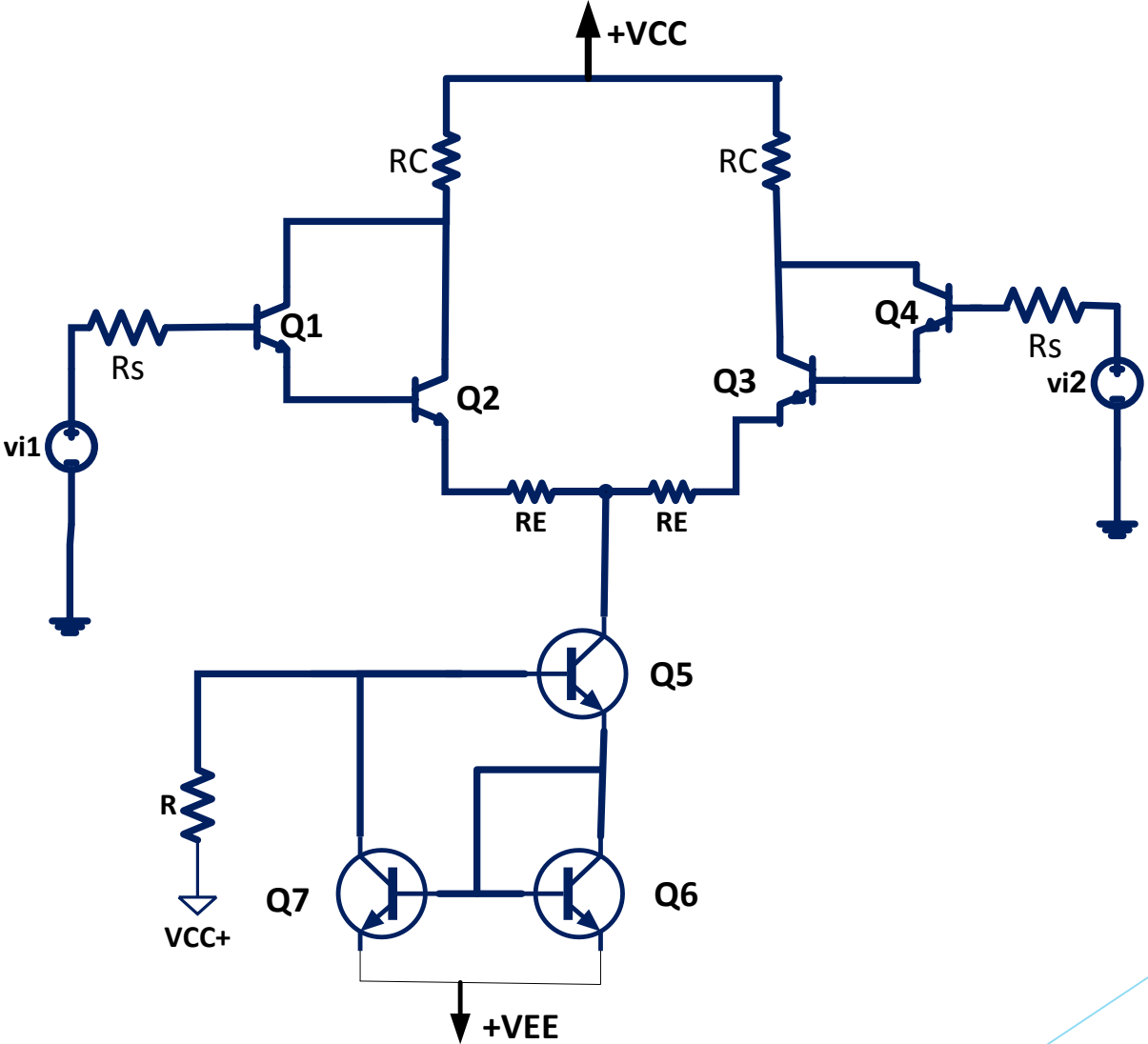
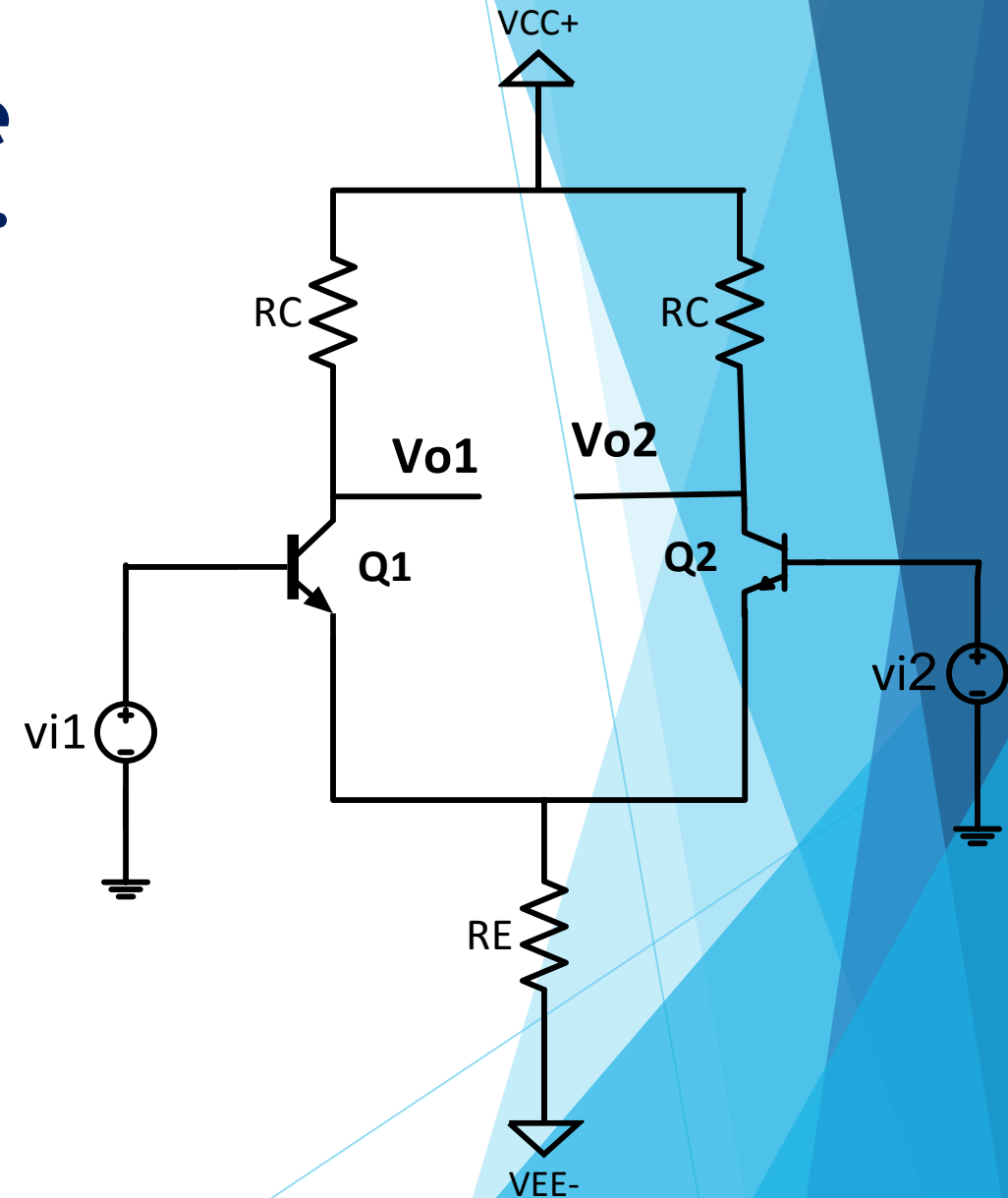


# Differential Amplifiers



# Differential Amplifiers

- ▶ Designed to amplify the difference between two input signal voltages.
- ▶ Found in many electronic circuits, including low and high frequency amplifiers.
- ▶ Almost always used as the input stage inside an IC operational amplifier to provide :
  - ▶ **Large input impedance** .
  - ▶ **Rejection of the noise** .



# Differential Amplifiers

## Simple Differential Amplifier

### DC Analysis:

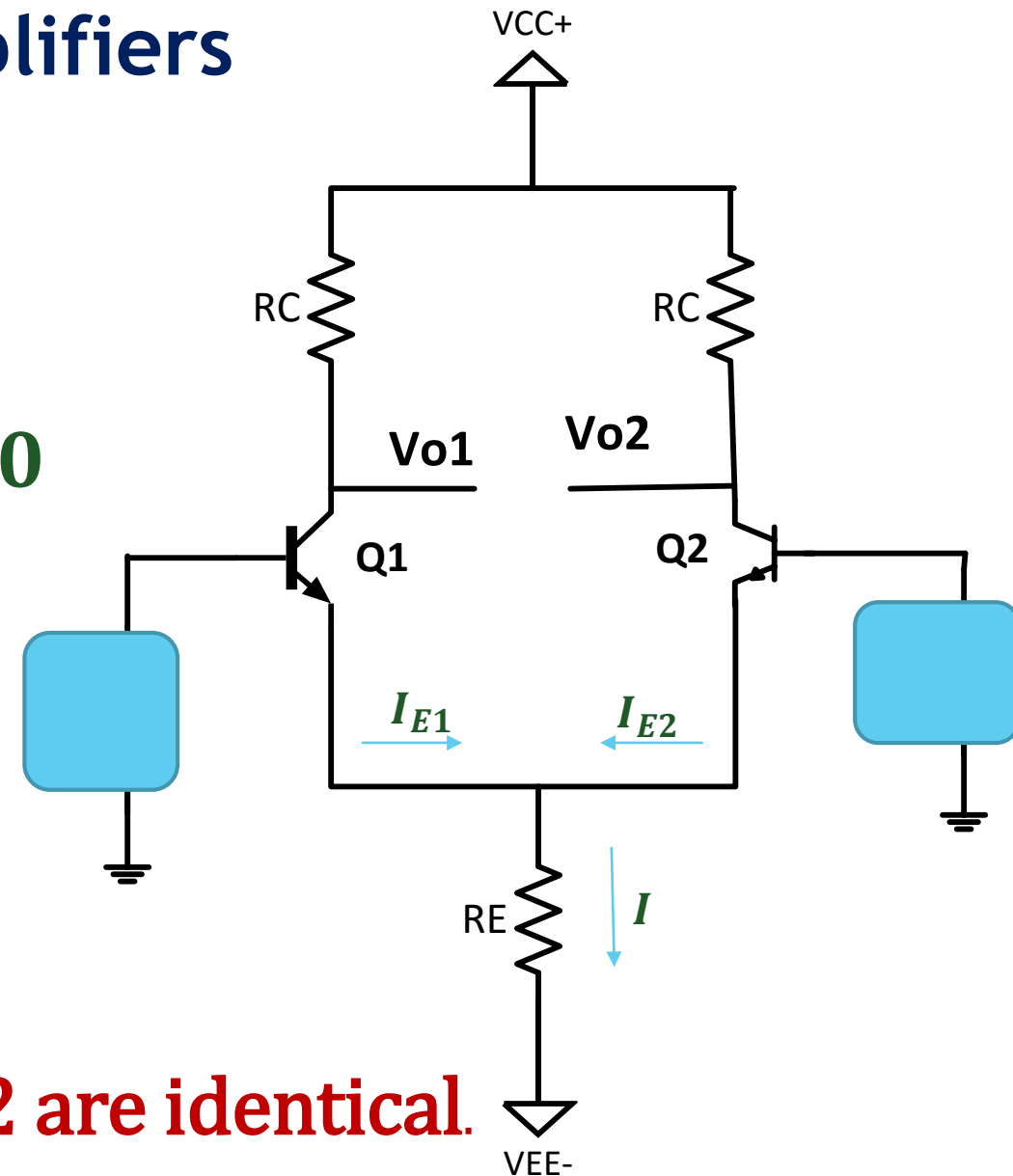
$$V_{B1} = V_{B2} = 0 \quad \text{since } v_{i1} = v_{i2} = 0$$

$$V_{E1} = V_{E2} = -0.7 \text{ V}$$

$$I = \frac{-0.7 + V_{EE}}{R_E}$$

$$I_{E1} = I_{E2} = \frac{1}{2} I$$

*symmetry*



**Q1 & Q2 are identical.**

$$\therefore \beta_1 = \beta_2$$

# Differential Amplifiers

## Simple Differential Amplifier

### Ac small Signal Analysis:

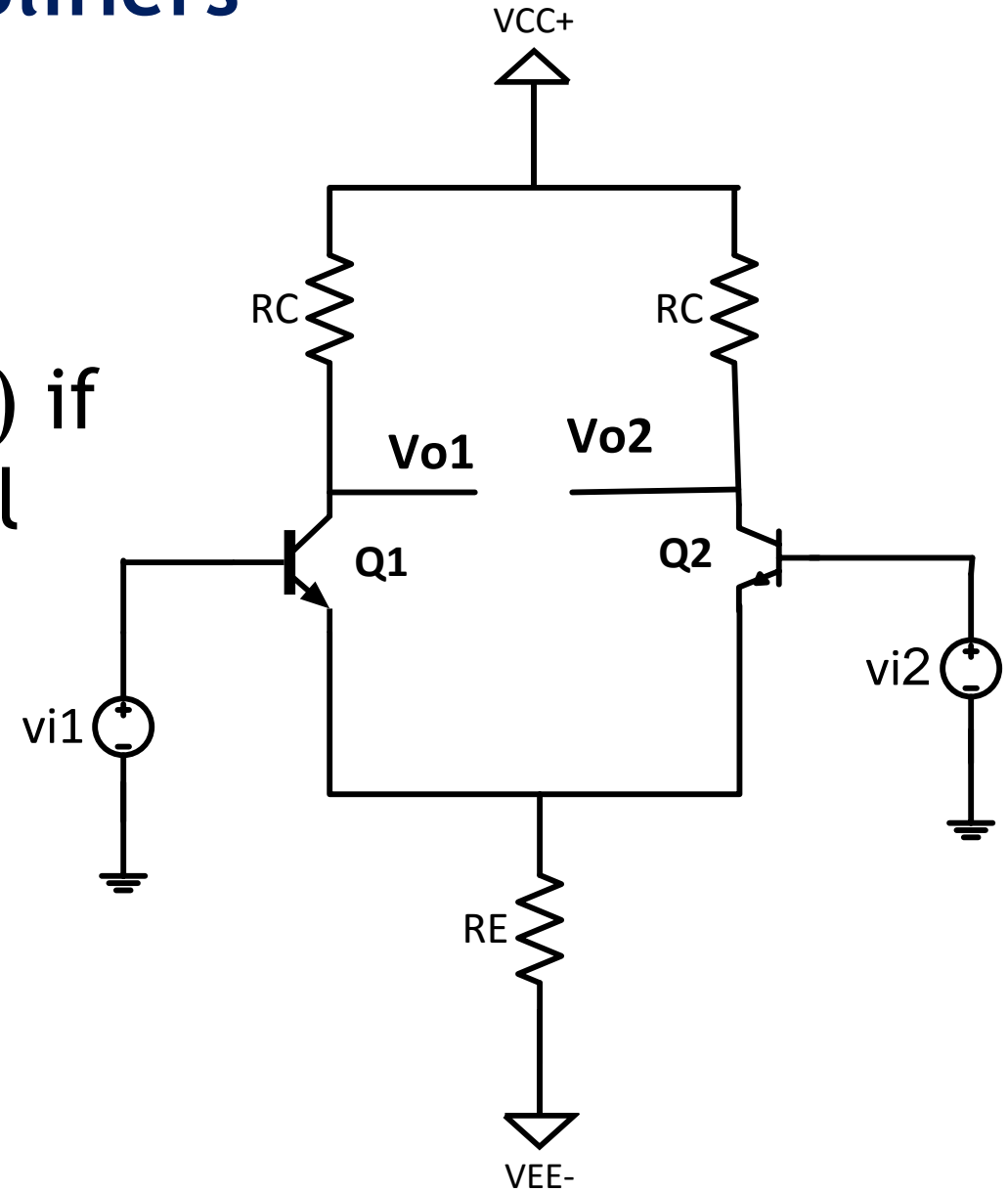
Calculate and sketch  $V_{o1}(t)$  and  $V_{o2}(t)$  if  $V_{i1}(t) = V_{i2}(t) = 100\text{mV}$  Peak sinusoidal

*assuming*

$$\beta_1 = \beta_2$$

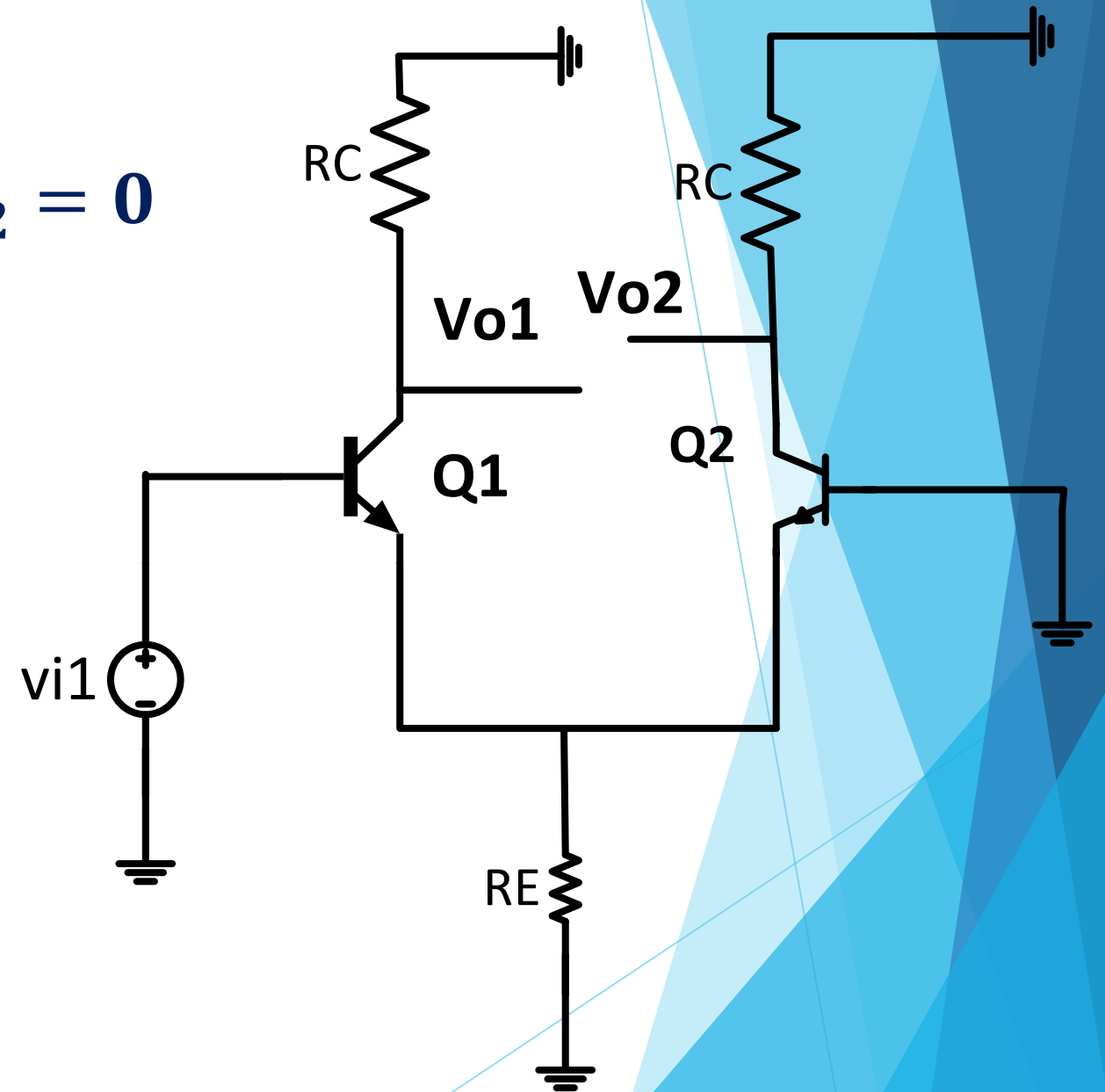
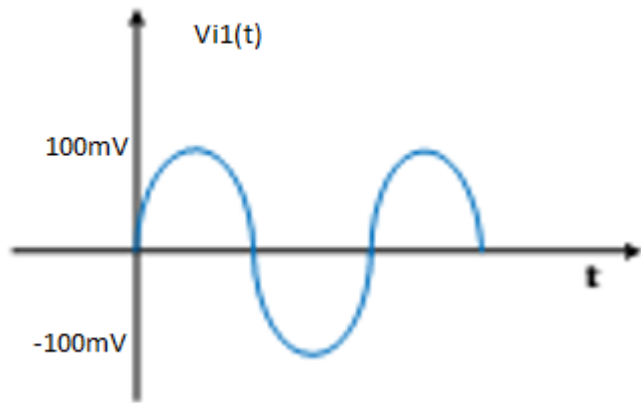
and

$$\frac{v_{c1}}{v_{be1}} = \frac{v_{c2}}{v_{be2}} = -100$$
$$\frac{v_{o1}}{v_{be1}} = \frac{v_{o2}}{v_{be2}} = -100$$



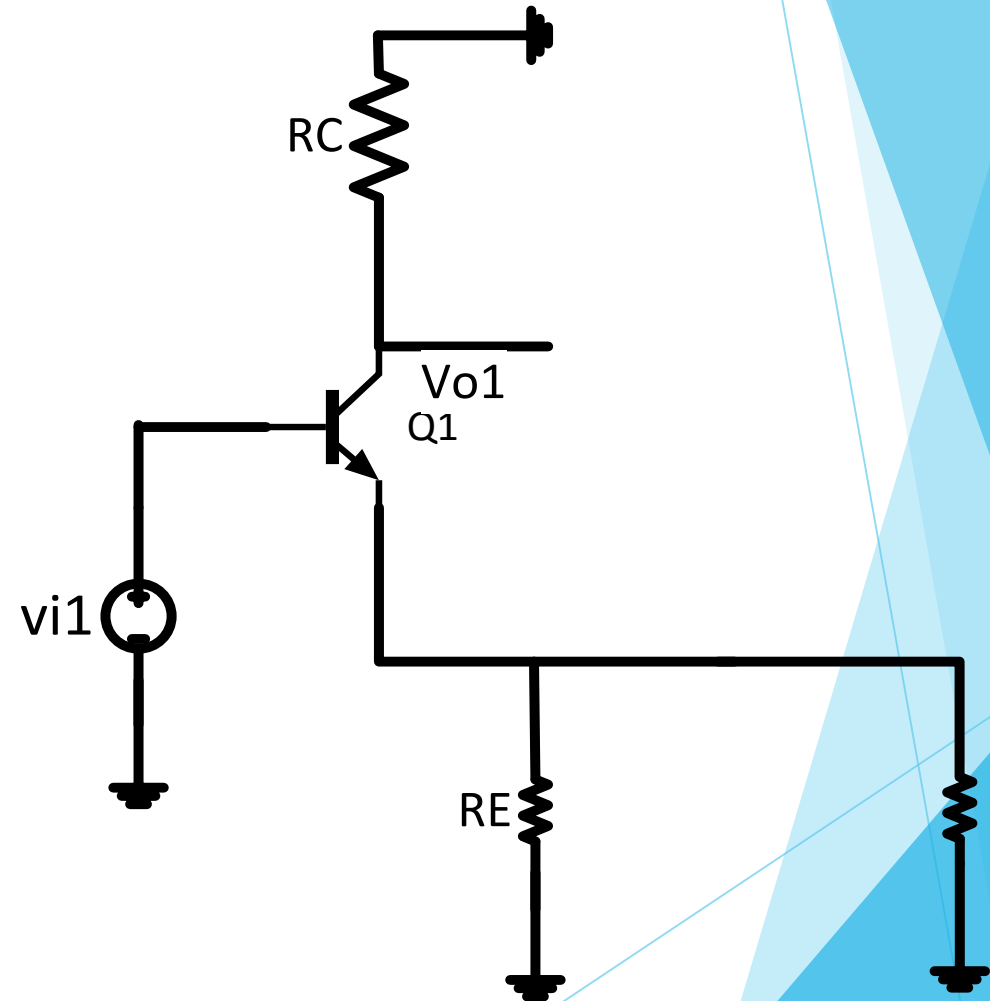
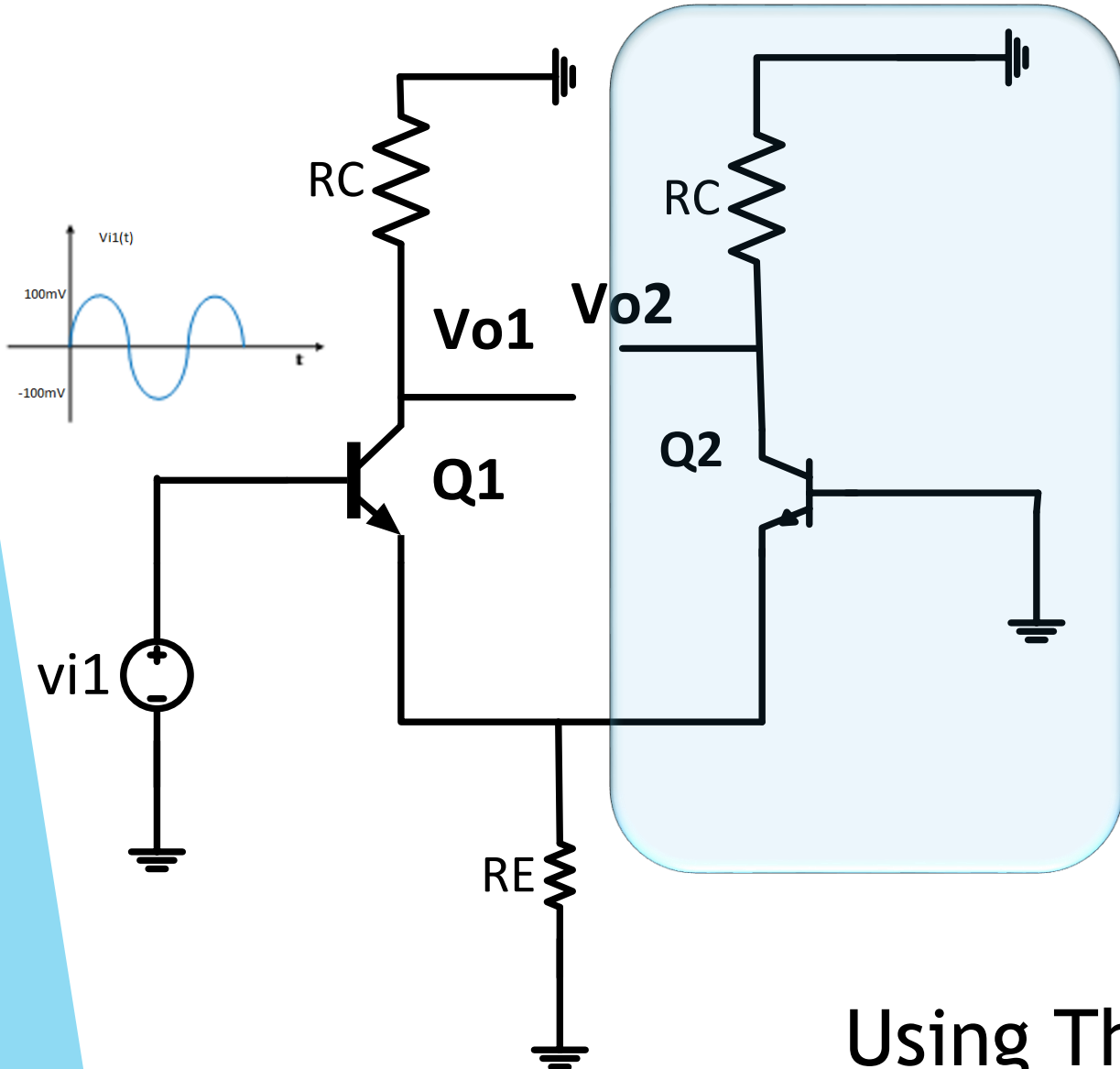
# Using superposition

1) *let  $v_{i1} = 100\text{ mV peak}$ ,  $v_{i2} = 0$*



# Simple Differential Amplifier

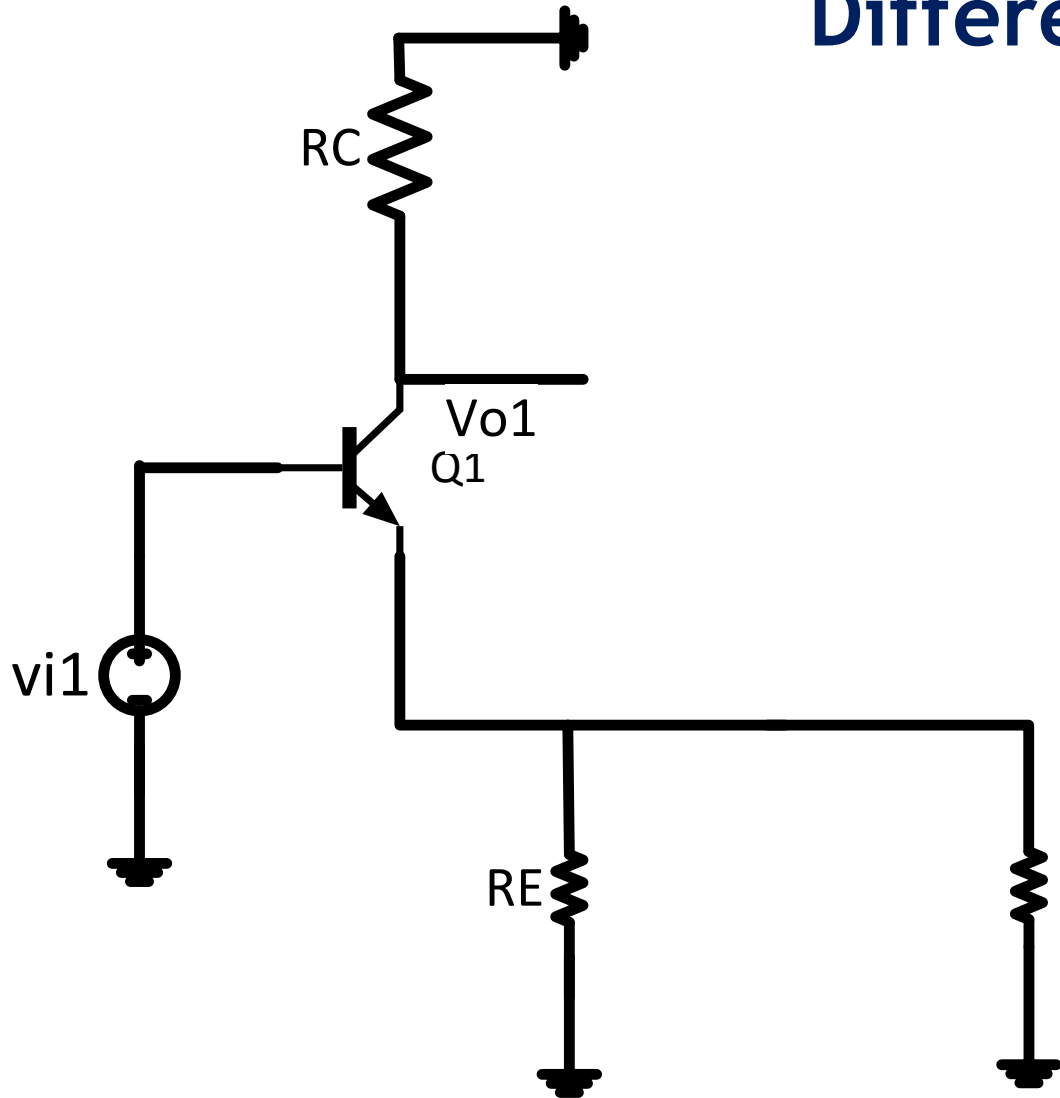
1) let  $v_{i1} = 100\text{ mV peak}$ ,  $v_{i2} = 0$



$$\frac{h_{ie2}}{h_{fe+1}} = h_{ib2}$$

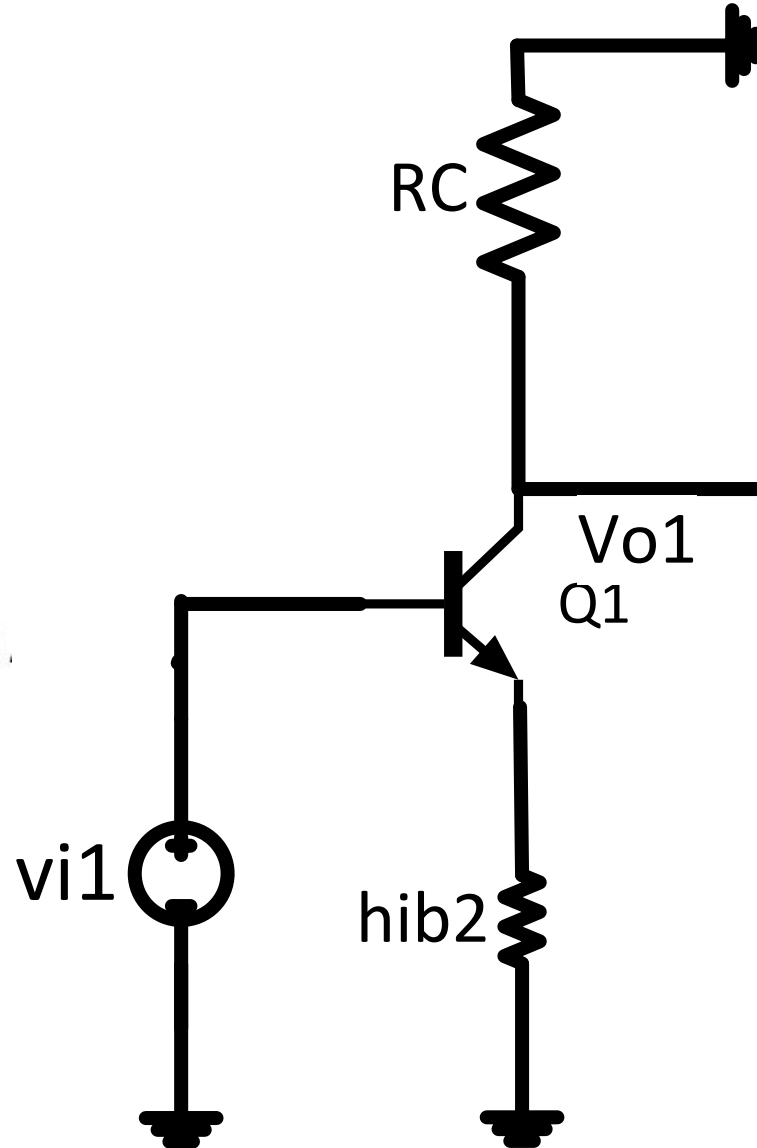
Using Thevenin theorem

# Differential Amplifiers



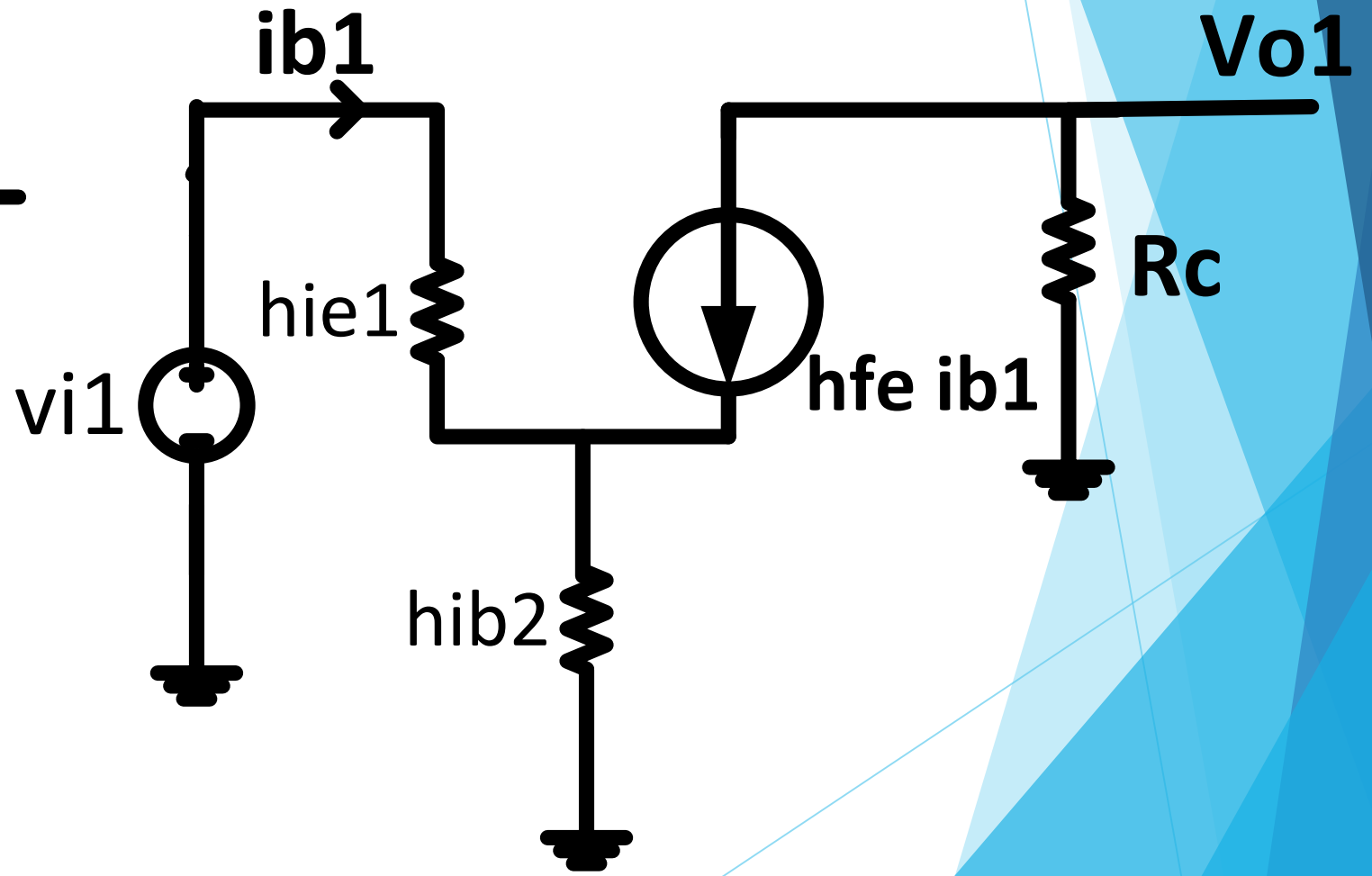
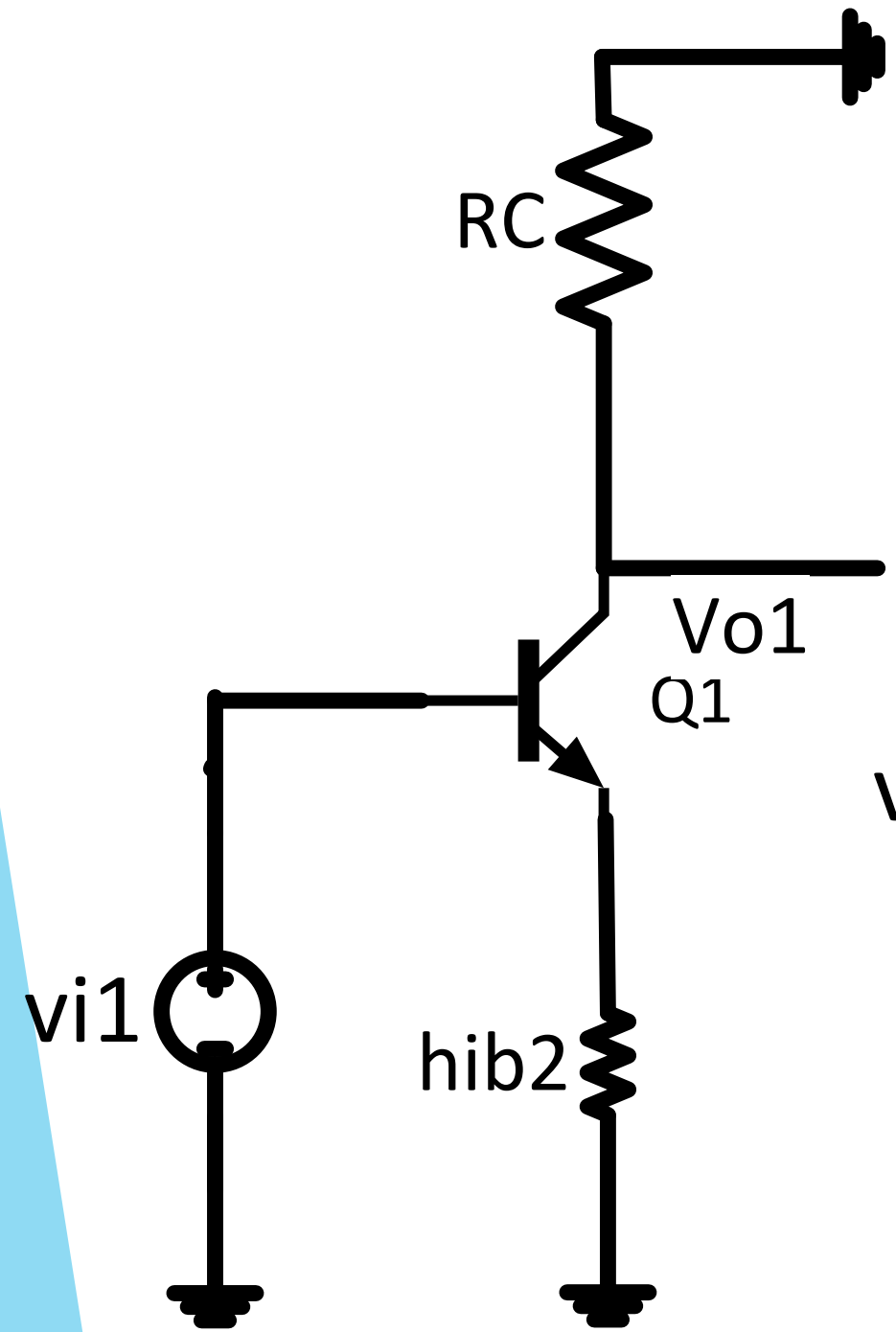
$$\frac{h_{ie2}}{h_{fe}+1} = h_{ib2}$$

*if  $R_E \gg h_{ib2}$   
 $\therefore R_E \parallel h_{ib2} \approx h_{ib2}$*



$$\frac{v_{o1}}{v_{be1}} = \frac{v_{o2}}{v_{be2}} = -100$$

Ac small signal equivalent circuit

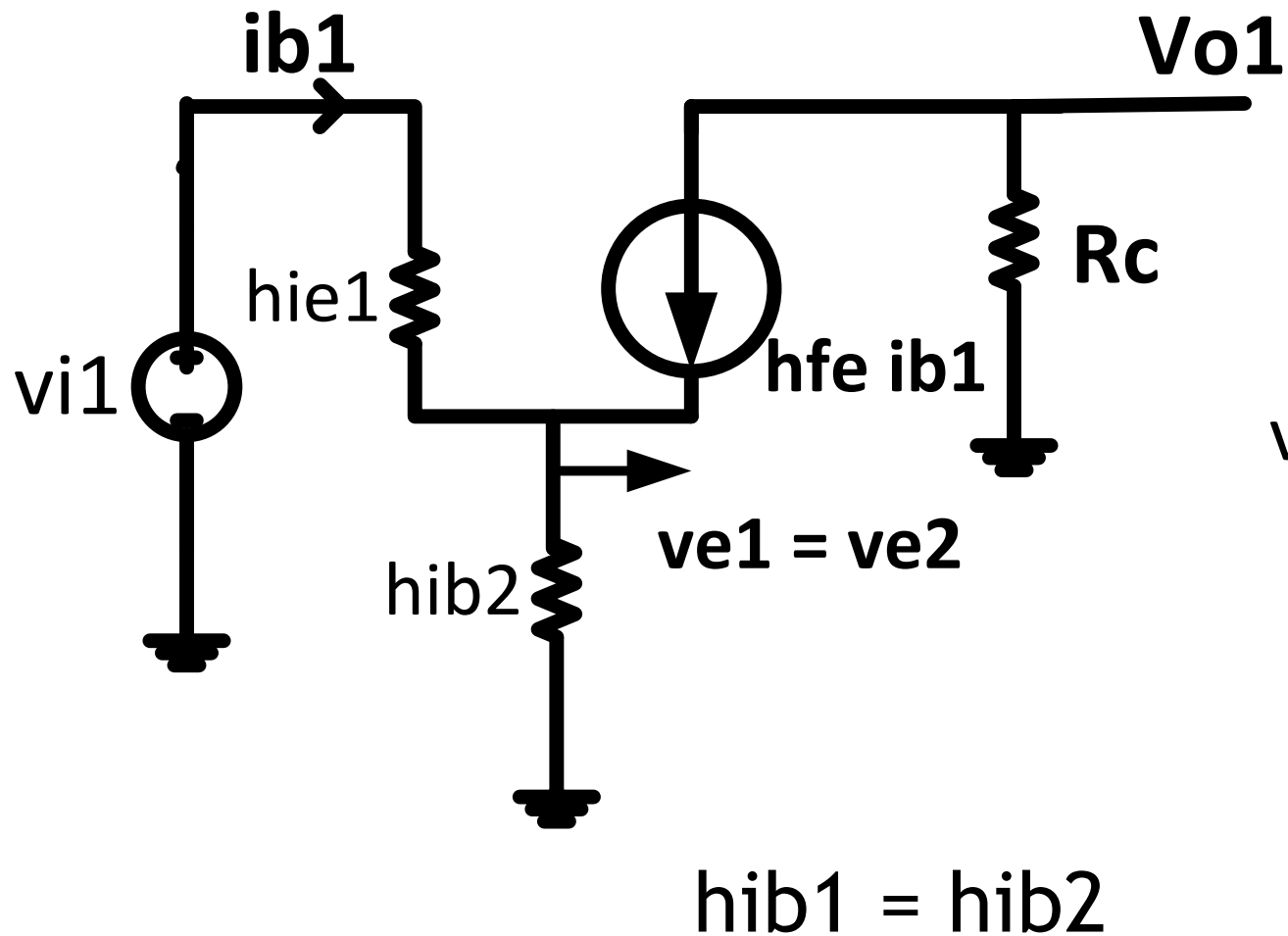




# Differential Amplifiers

## Simple Differential Amplifier

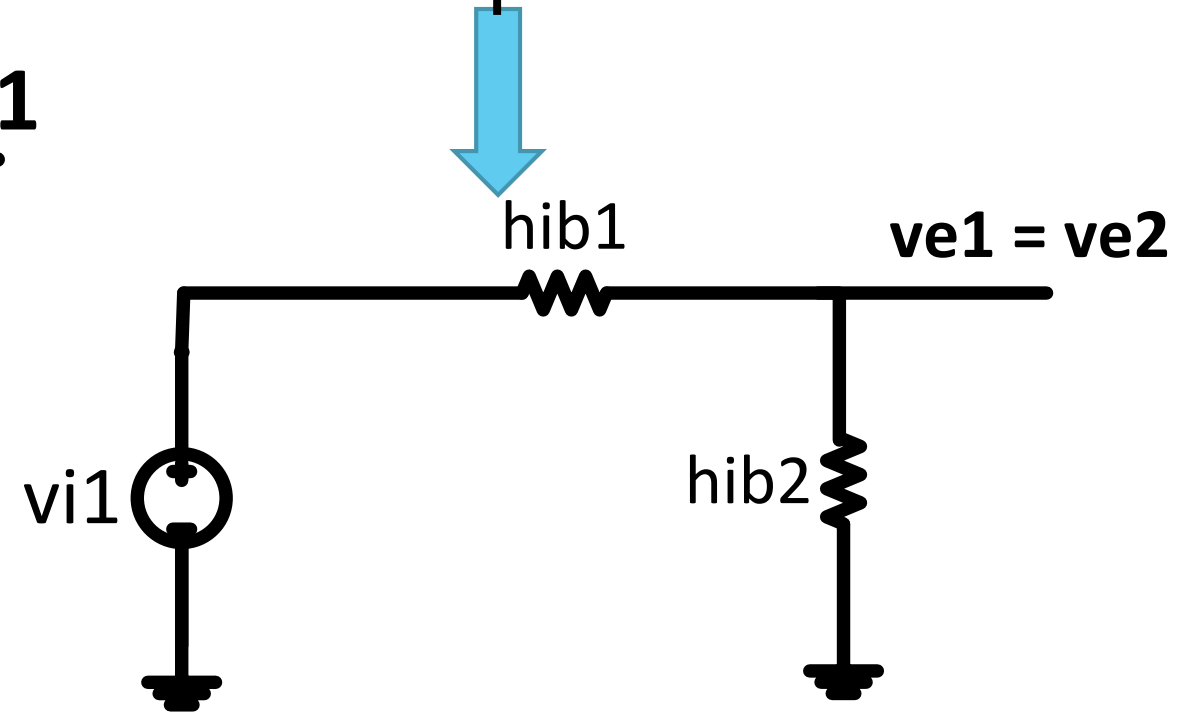
### AC Small Signal Analysis:



To find  $v_{e1} = v_{e2}$



Emitter equivalent circuit



$$v_{e1} = \frac{1}{2} v_{i1} = 50 \text{ mV peak}$$

# Differential Amplifiers

## AC Small Signal Analysis:

$$\frac{v_{o1}}{v_{be1}} = \frac{v_{o2}}{v_{be2}} = -100$$

$$v_{e1} = v_{e2} = \frac{1}{2} v_{i1} = 50 \text{ mV peak}$$

$$v_{be1} = v_{b1} - v_{e1}$$

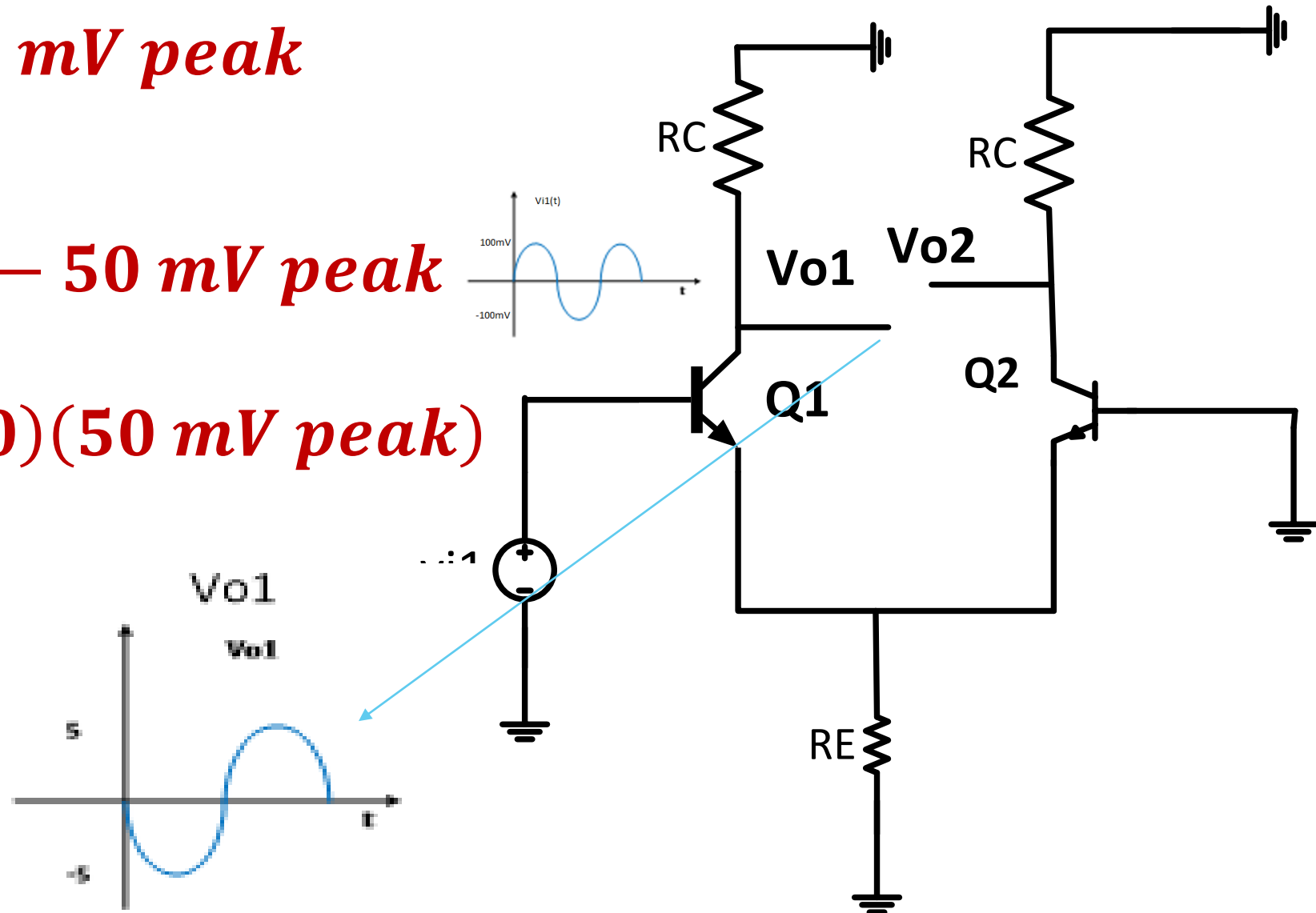
$$v_{be1} = 100 \text{ mV peak} - 50 \text{ mV peak}$$

$$v_{be1} = 50 \text{ mV peak}$$

$$\therefore v_{c1} = v_{o1} = (-100)(50 \text{ mV peak})$$

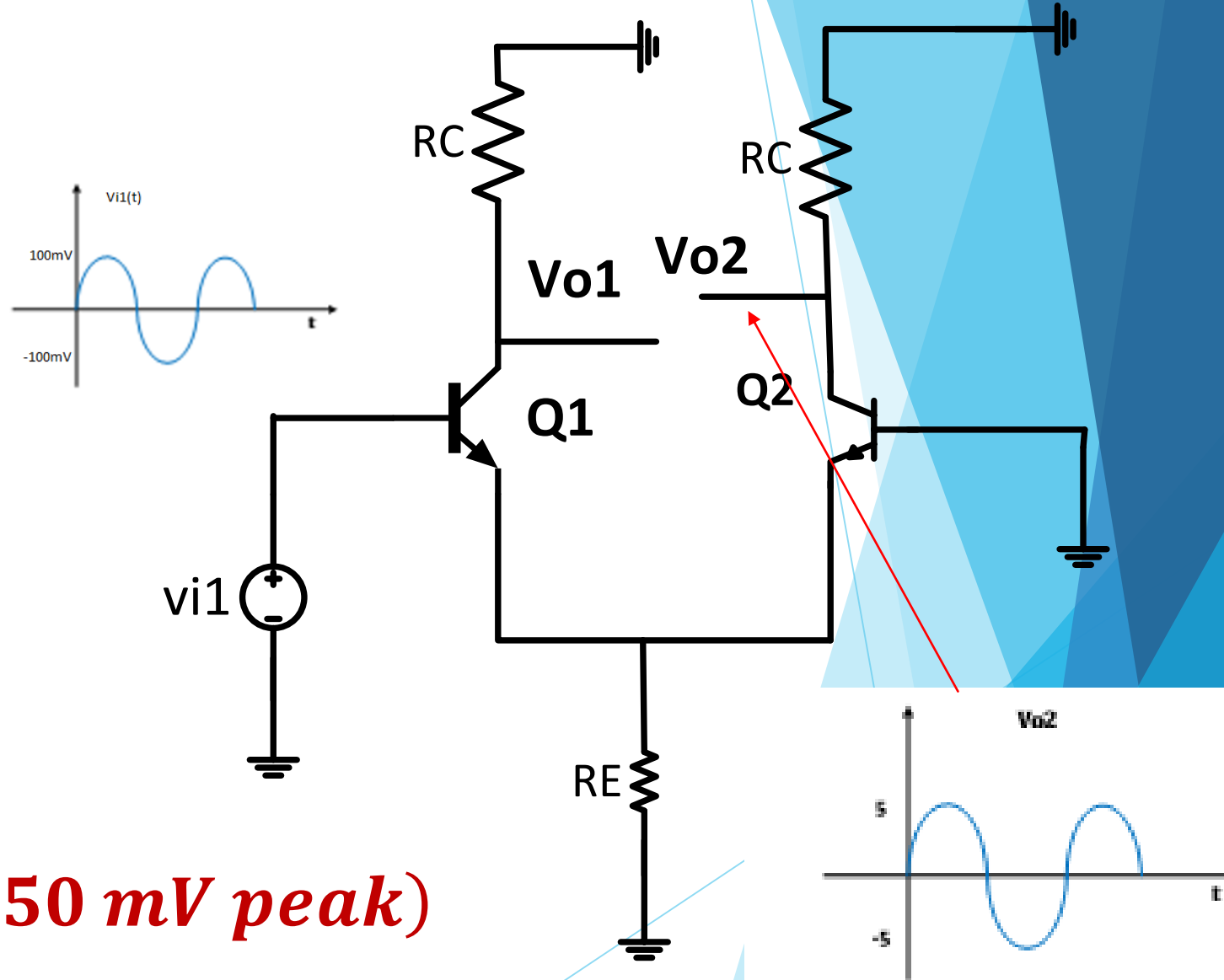
$$\therefore v_{c1} = -5 \text{ V peak}$$

$$v_{o1} = -5 \text{ V peak}$$



$$\frac{v_{o2}}{v_{be2}} = -100$$

- ▶ *to find  $v_{c2} = v_{o2}$  ,*
- ▶ *we need to find  $v_{be2}$*
- ▶  *$v_{be2} = v_{b2} - v_{e2}$*
- ▶  *$= v_{b2} - v_{e2}$*
- ▶  *$v_{be2} = 0 - 50 \text{ mV peak}$*
- ▶  *$v_{be2} = -50 \text{ mV peak}$*
- ▶  *$\therefore v_{o2} = v_{c2} = (-100)(-50 \text{ mV peak})$*
- ▶  *$v_{o2} = +5 \text{ V peak}$*



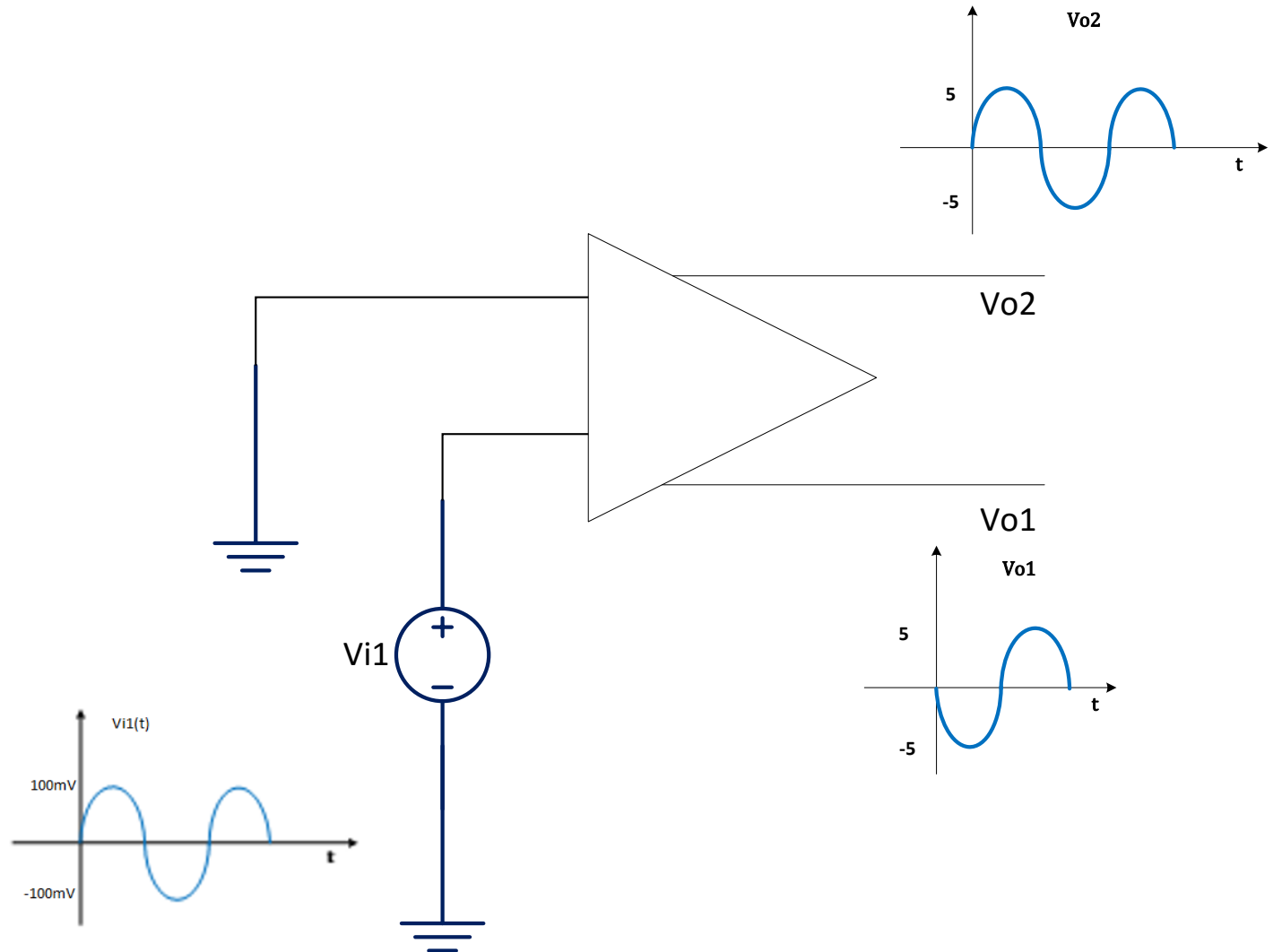
# Differential Amplifiers

## Simple Differential Amplifier

### AC Small Signal Analysis:

$$v_{o1} = -5 V \text{ peak}$$

$$v_{o2} = +5 V \text{ peak}$$



# Differential Amplifiers

## AC Small Signal Analysis:

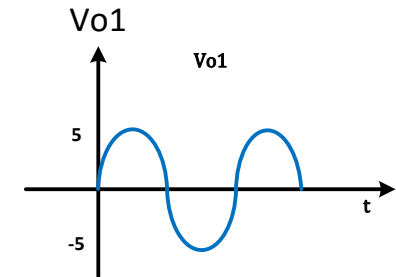
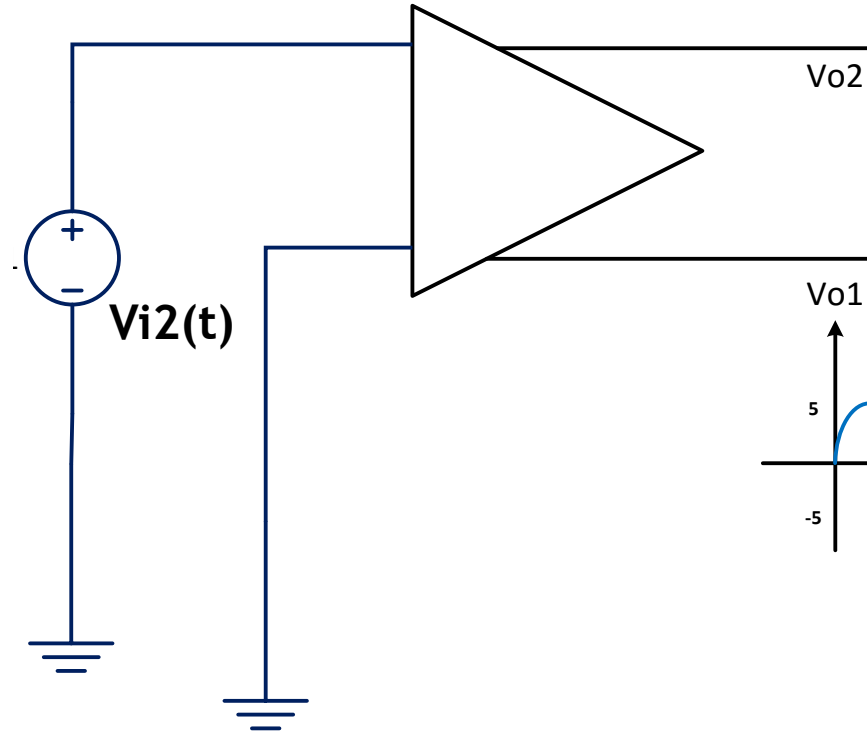
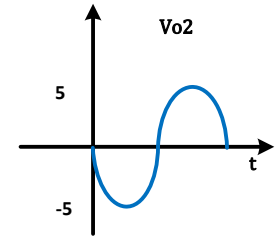
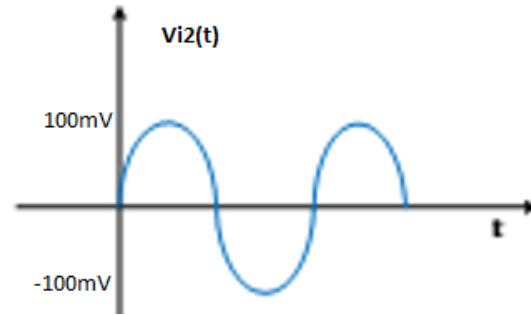
Using same steps for

$$v_{i2} = 100 \text{ mV peak}$$

$$v_{i1} = 0$$

$$v_{o1} = +5 \text{ V peak}$$

$$v_{o2} = -5 \text{ V peak}$$



# Differential Amplifiers

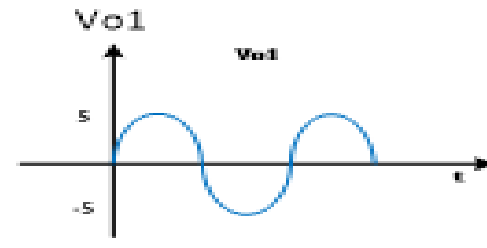
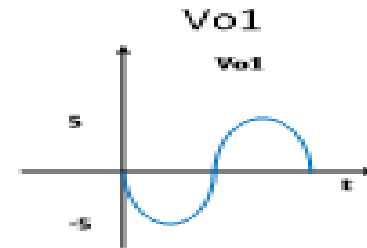
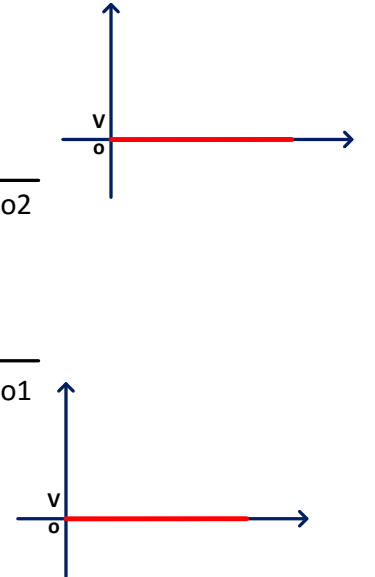
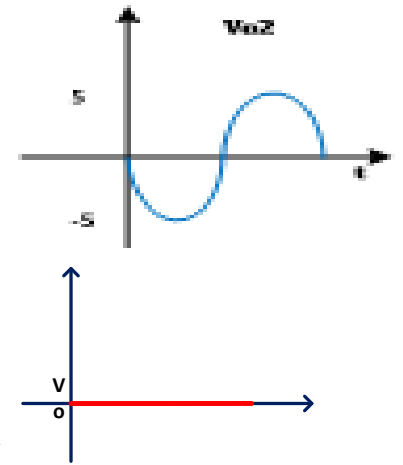
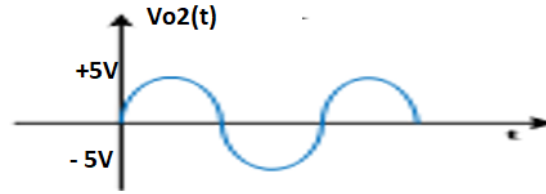
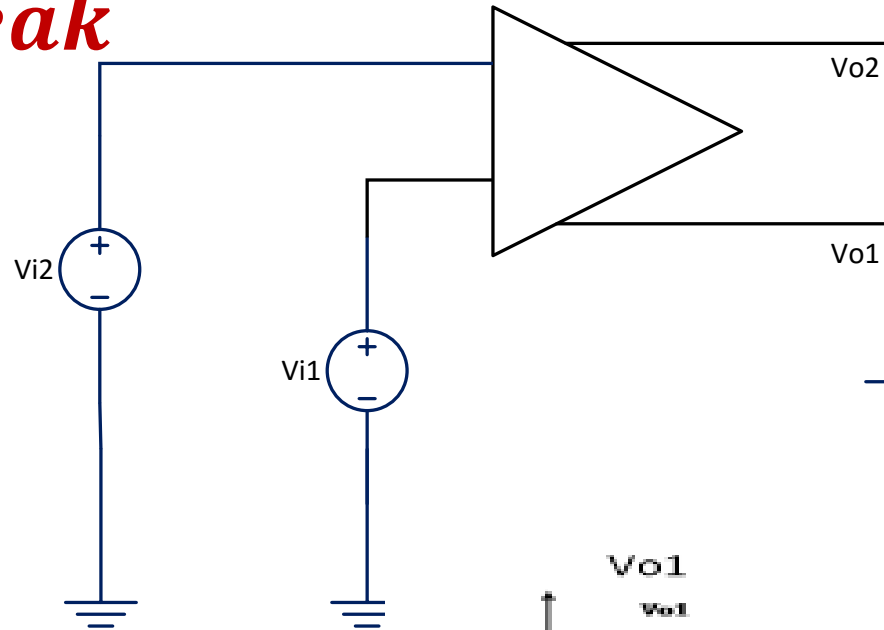
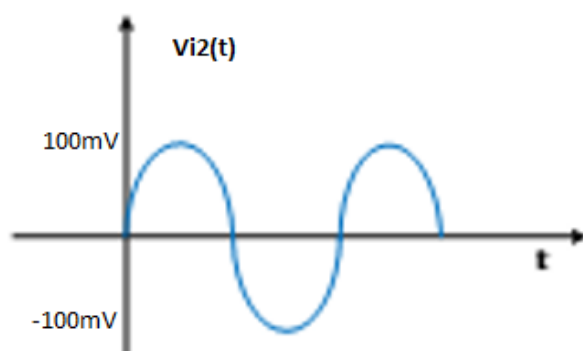
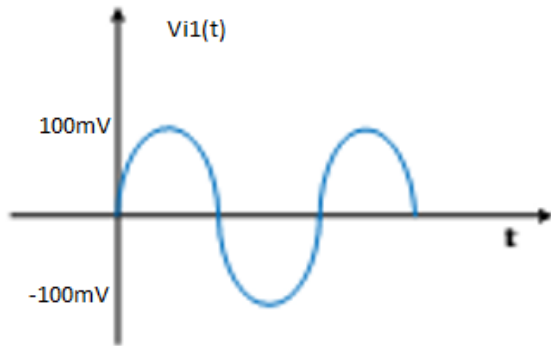
## Simple Differential Amplifier

### AC Small Signal Analysis:

*now if  $v_{i1} = v_{i2} = 100 \text{ mV peak}$*

$$v_{o1} = 0$$

$$v_{o2} = 0$$



# Differential Amplifiers

## Simple Differential Amplifier

### AC Small Signal Analysis:

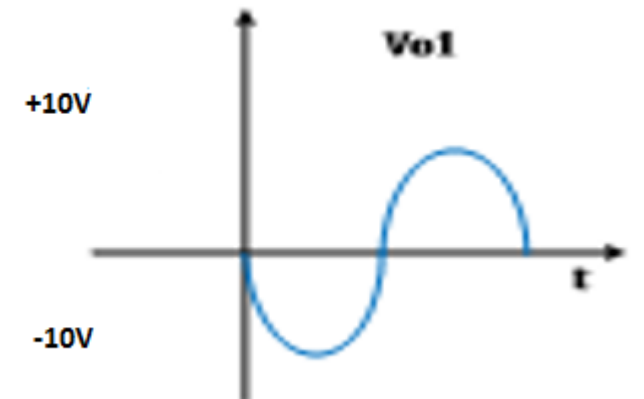
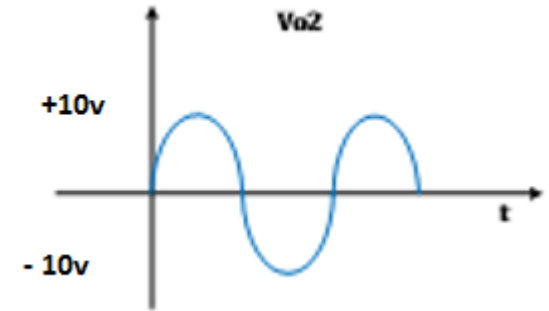
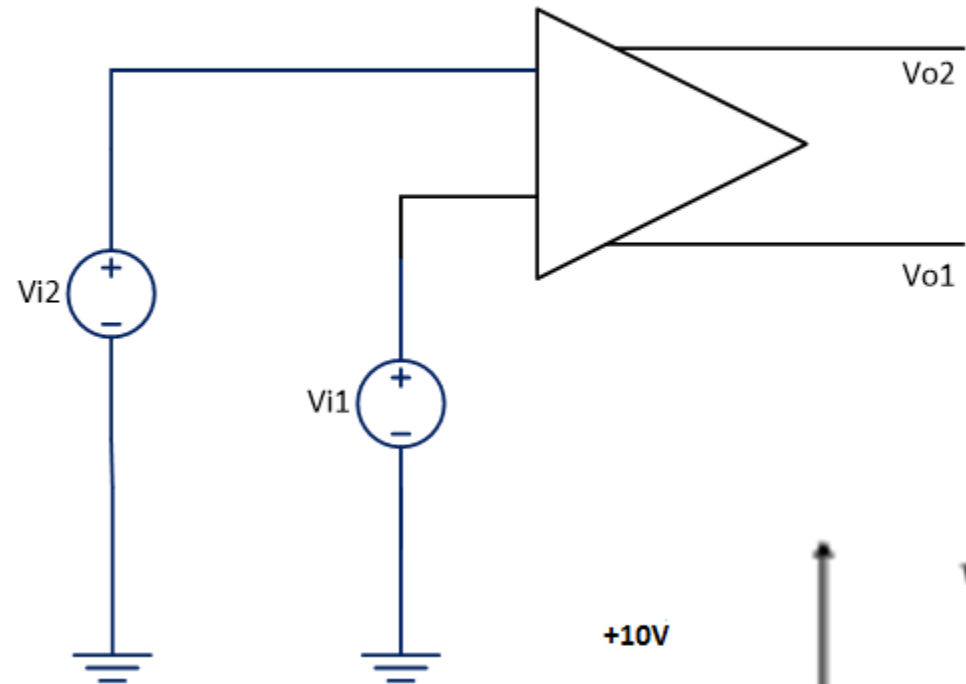
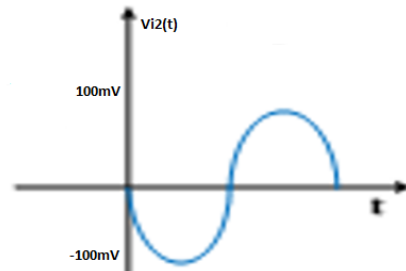
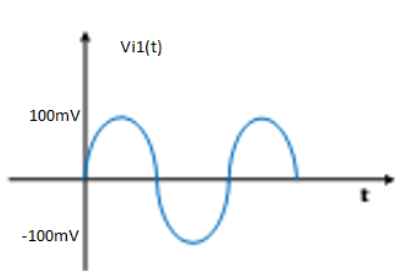
Using same steps for

$$v_{i1} = 100 \text{ mV peak}$$

$$v_{i2} = -100 \text{ mV peak}$$

$$v_{o2} = +10 \text{ V peak}$$

$$v_{o1} = -10 \text{ V peak}$$



## Differential Amplifier Circuit:

### Common Mode & Differential mode Signal

Since the differential amplifier is most often used to amplify the difference between two input signals.

$$\text{let } v_d = v_{i2} - v_{i1}$$

$v_d \equiv$  *Differential mode input signal*

$$\text{let } v_c = \frac{v_{i2} + v_{i1}}{2}$$

$v_c \equiv$  *Common mode input signal*

$$\therefore v_{i2} = v_c + \frac{v_d}{2}$$

$$v_{i1} = v_c - \frac{v_d}{2}$$



# Differential Amplifiers

## Differential Amplifier Circuit:

### Common Mode & Differential mode Signal

Input voltage can be represented in terms of a common mode and differential mode input signals.

**In the usual application of the differential amplifier , the differential mode input is desired and to be amplified , while the common mode input is to be rejected.**

# Differential Amplifier Circuit:

DC Analysis:  $\rightarrow v_{i1} = v_{i2} = 0$

$$R_S I_{B1} + V_{BE1} + R_E I - 12 = 0$$

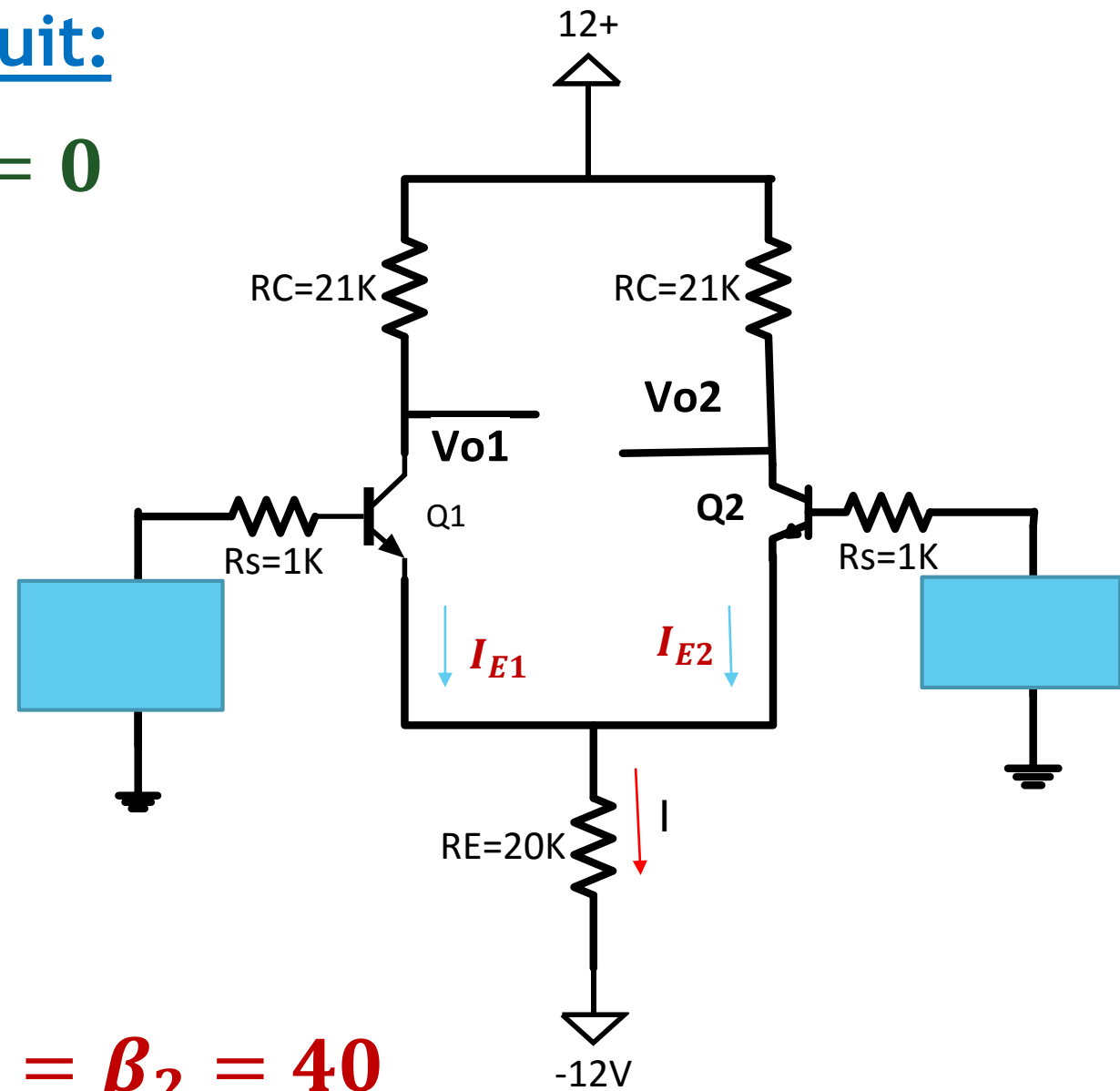
$$I = I_{E1} + I_{E2}$$

$$I_{E1} = I_{E2} \text{ [symmetry]}$$

$$I_{E1} = I_{E2} = \frac{12 - 0.7}{\frac{1k}{41} + (2)(20k)}$$
$$= 0.2825 \text{ mA}$$

$$\beta_1 = \beta_2 = 40$$

*Q<sub>1</sub> and Q<sub>2</sub> identical*



# Differential Amplifiers

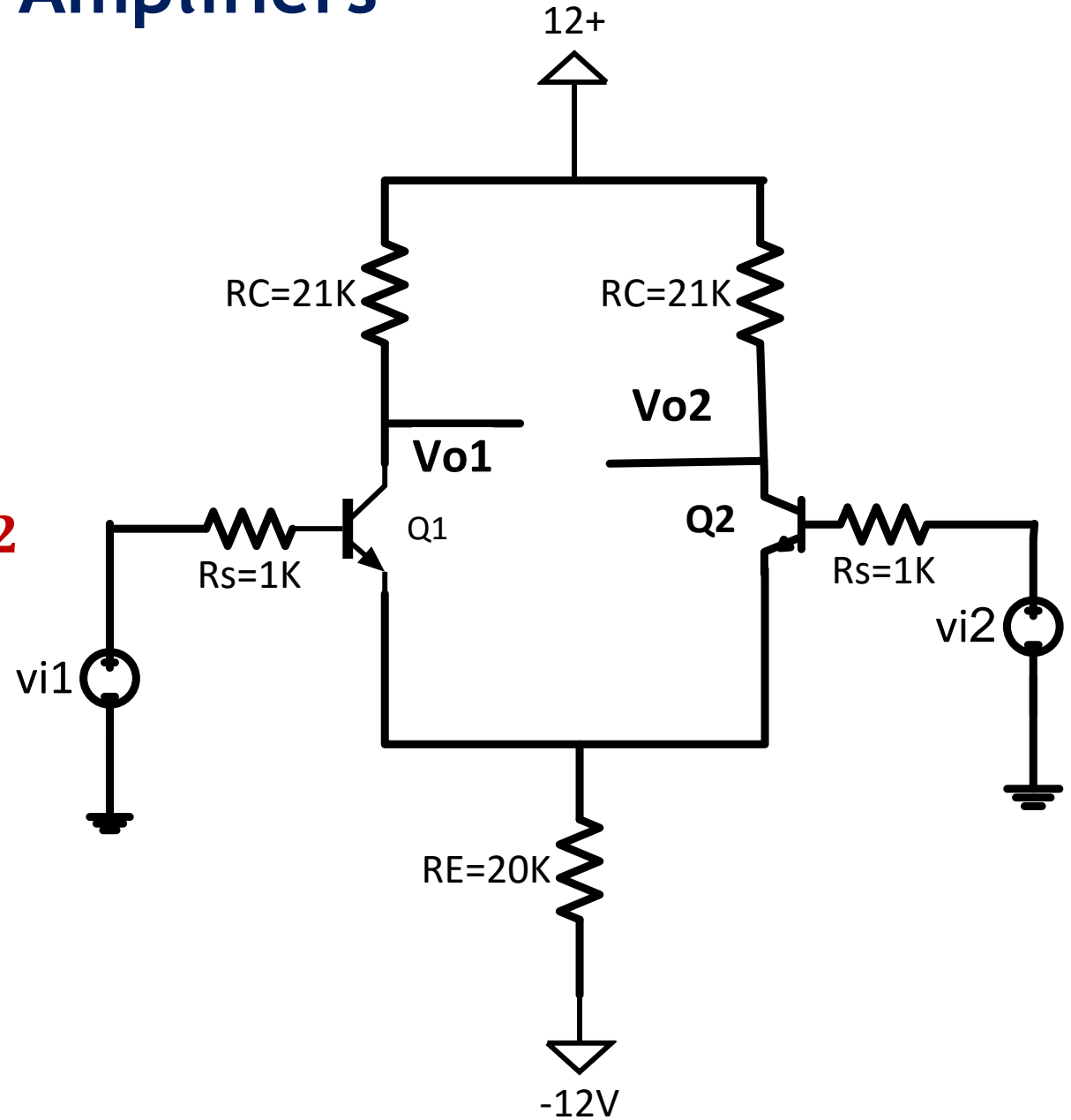
## Differential Amplifier Circuit:

### AC Analysis:

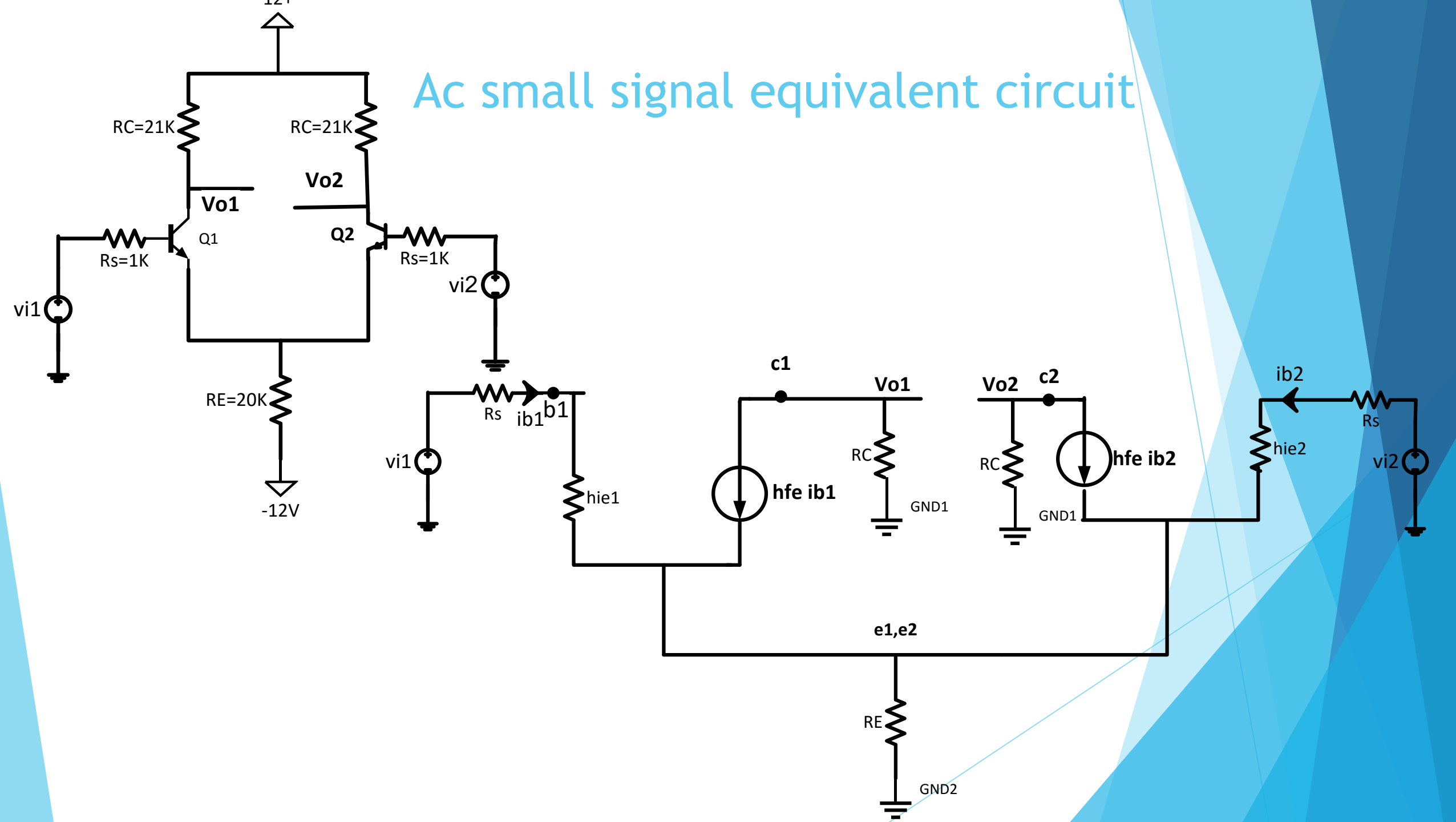
*since  $I_{E1} = I_{E2}$  and  $\beta_1 = \beta_2$*

$$h_{ie1} = h_{ie2} = h_{ie}$$

$$h_{ib1} = h_{ib2} = h_{ib}$$



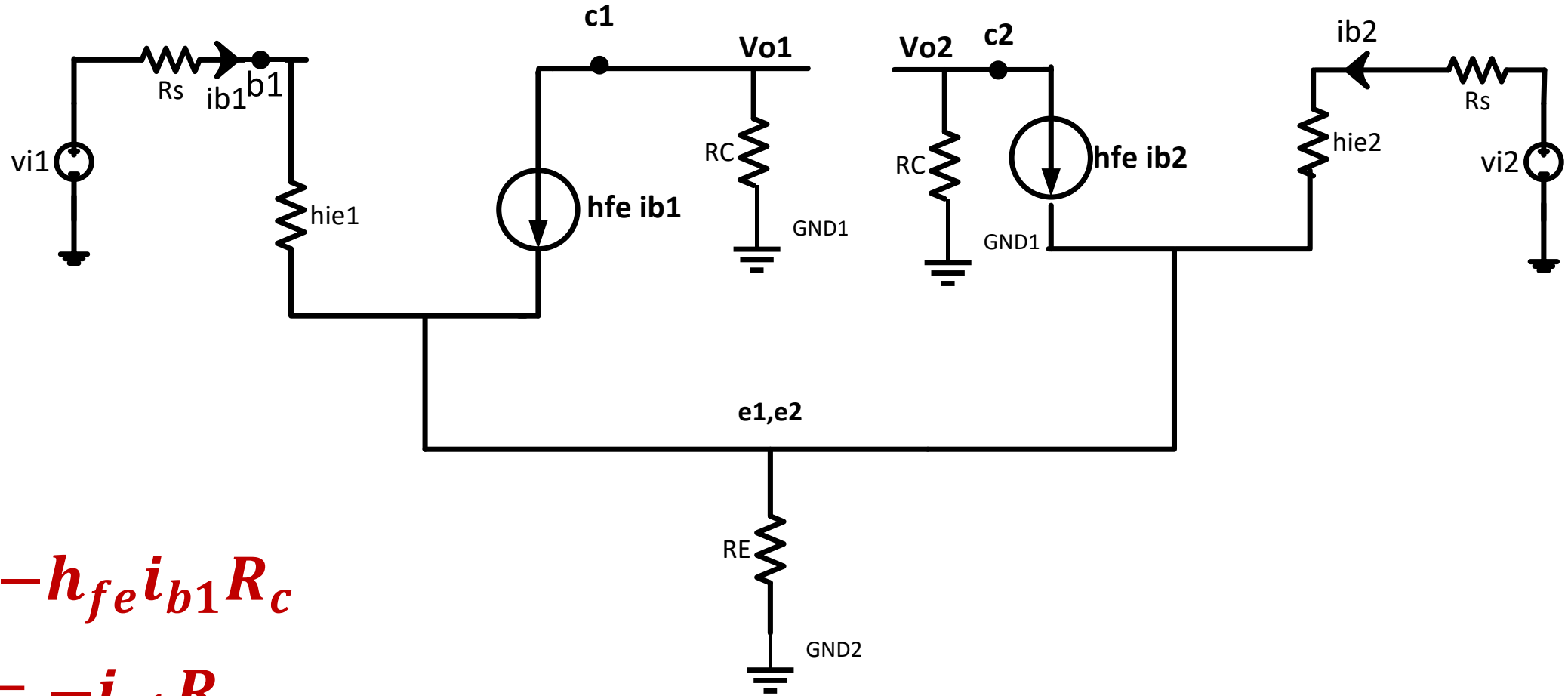
# Ac small signal equivalent circuit



# Differential Amplifiers

## Differential Amplifier Circuit:

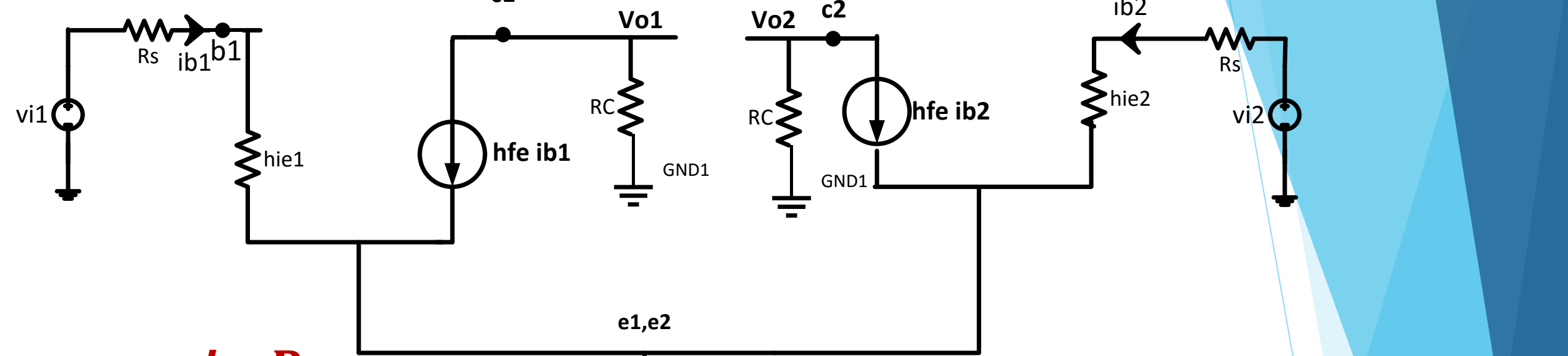
### AC Analysis:



$$v_{o1} = -h_{fe} i_{b1} R_c$$

$$v_{o1} = -i_{c1} R_c$$

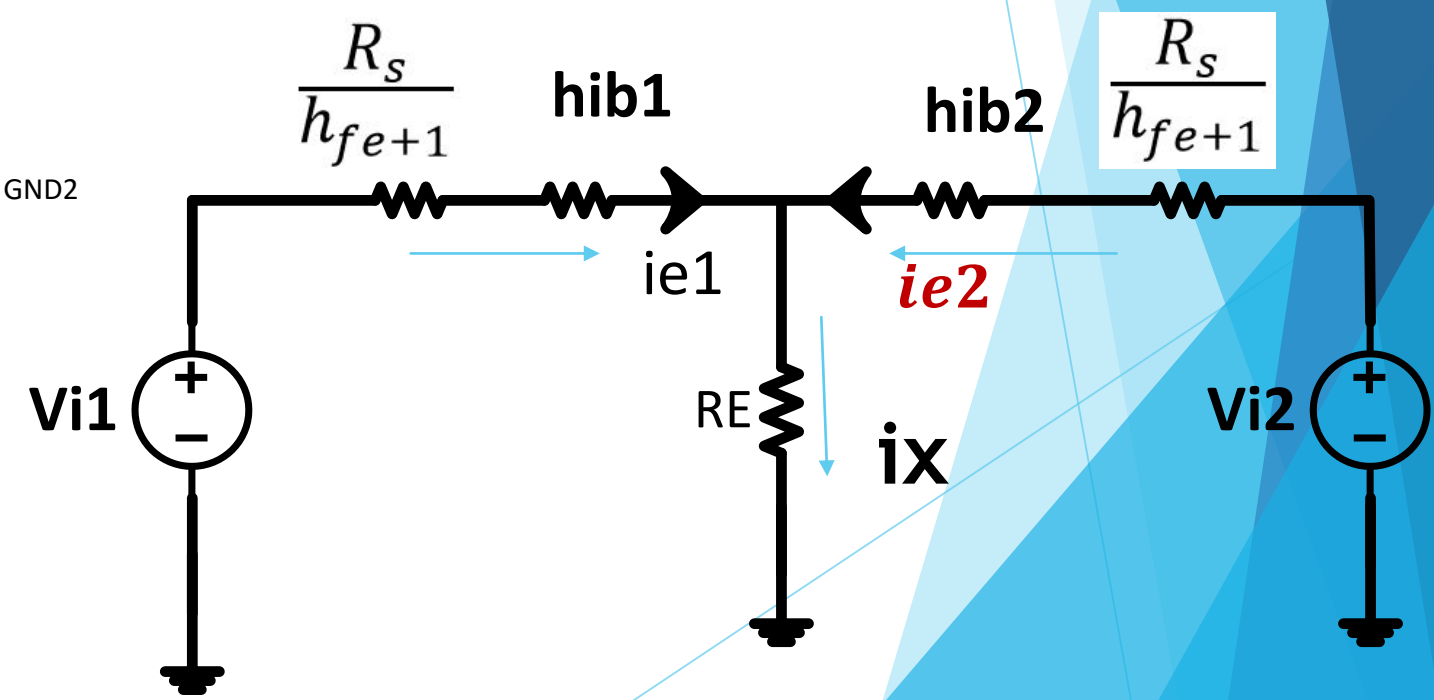
$$v_{o1} \approx -i_{e1} R_c$$



$v_{o1} \approx -i_{e1}R_c$

To find  $i_{e1}$

Emitter equivalent circuit



# Differential Amplifiers

Differential Amplifier Circuit:

## Small Signal Analysis

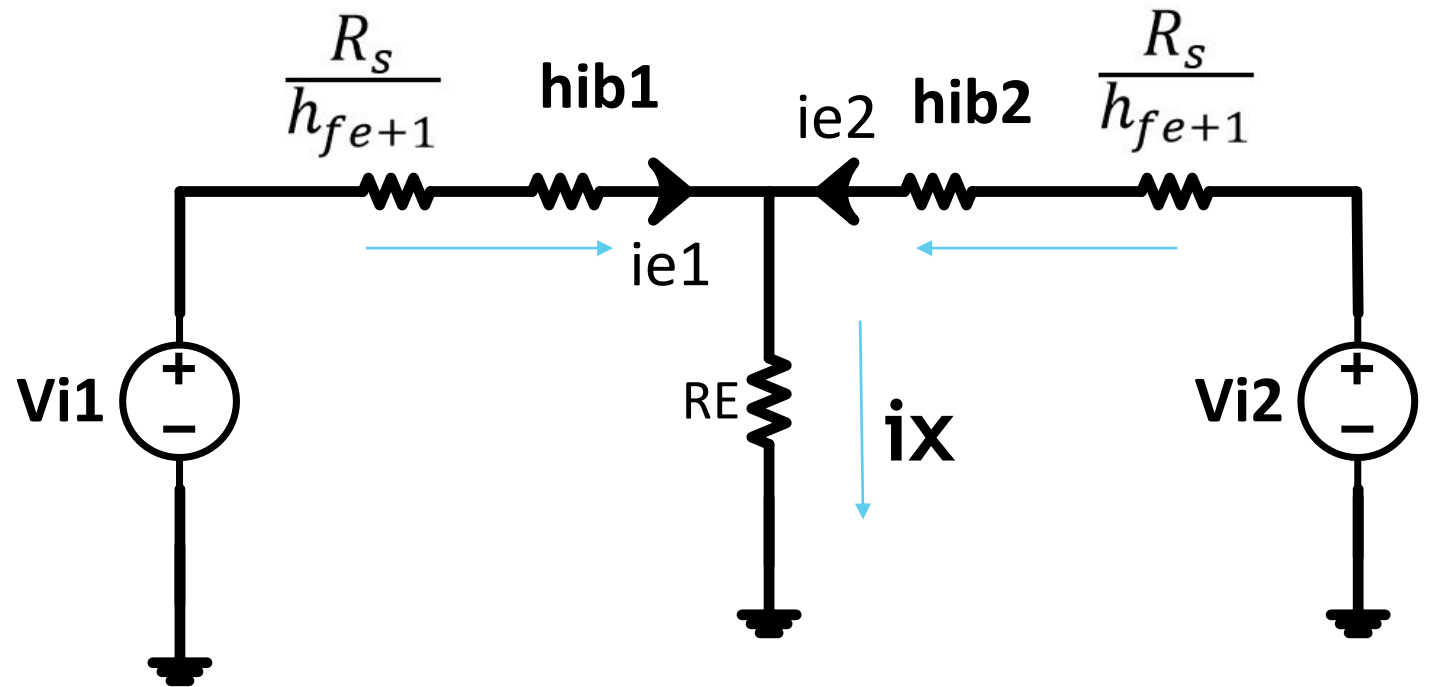
To find  $ie1$  →

Using emitter equivalent ckt

$$v_{i1} = v_c - \frac{v_d}{2}$$

$$v_{i2} = v_c + \frac{v_d}{2}$$

$$v_{o1} \cong -ie1 \cdot R_c$$



# Differential Amplifiers

$$v_{o1} \cong -ie1.Rc$$

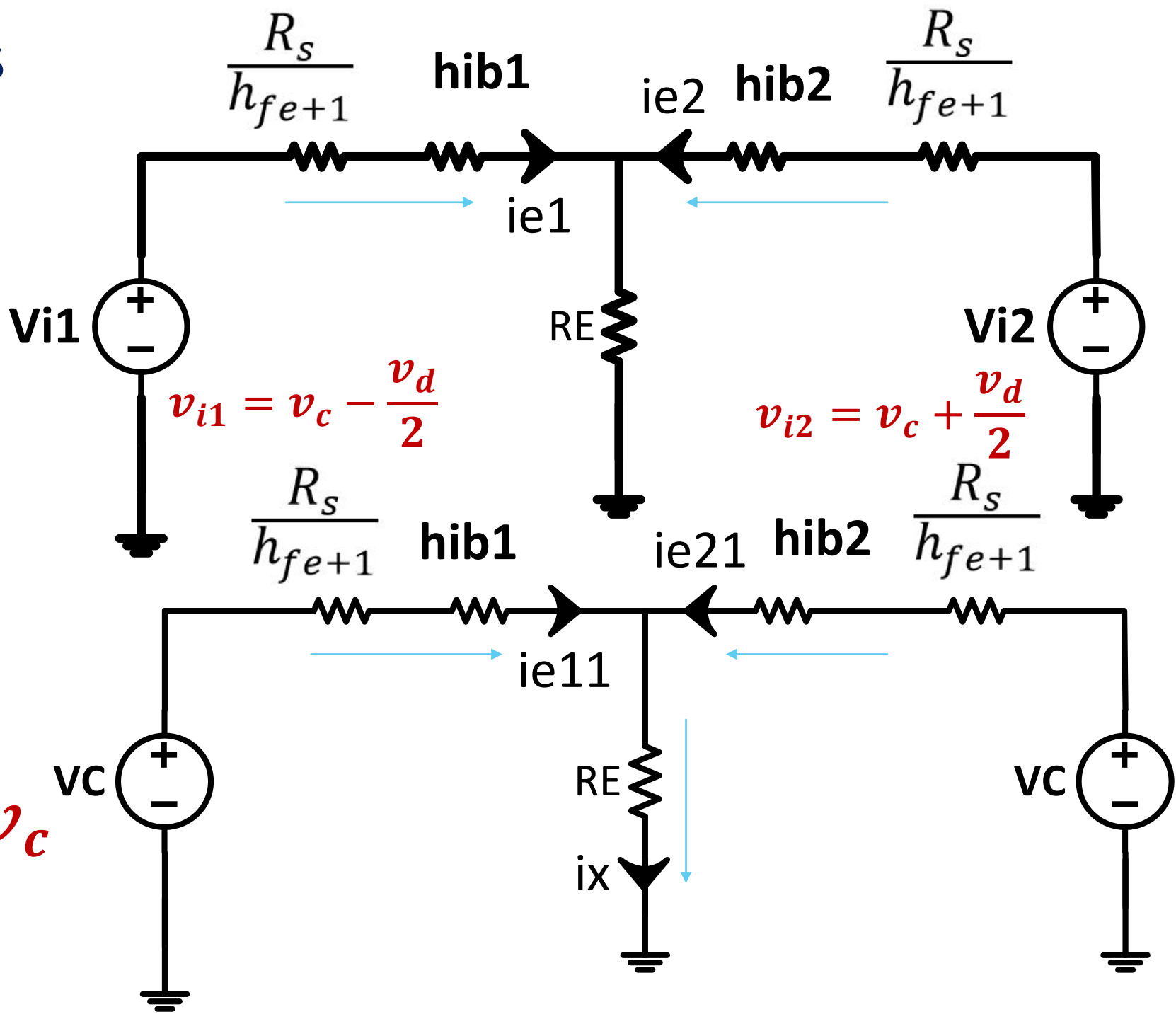
Using Superposition

$$ie1 = ie11 + ie12$$

To find  $ie11$

$$\text{let } v_d = 0$$

$$v_{i1} = v_c ; v_{i2} = v_c$$





# Differential Amplifiers

## Small Signal Analysis

Using emitter equivalent ckt

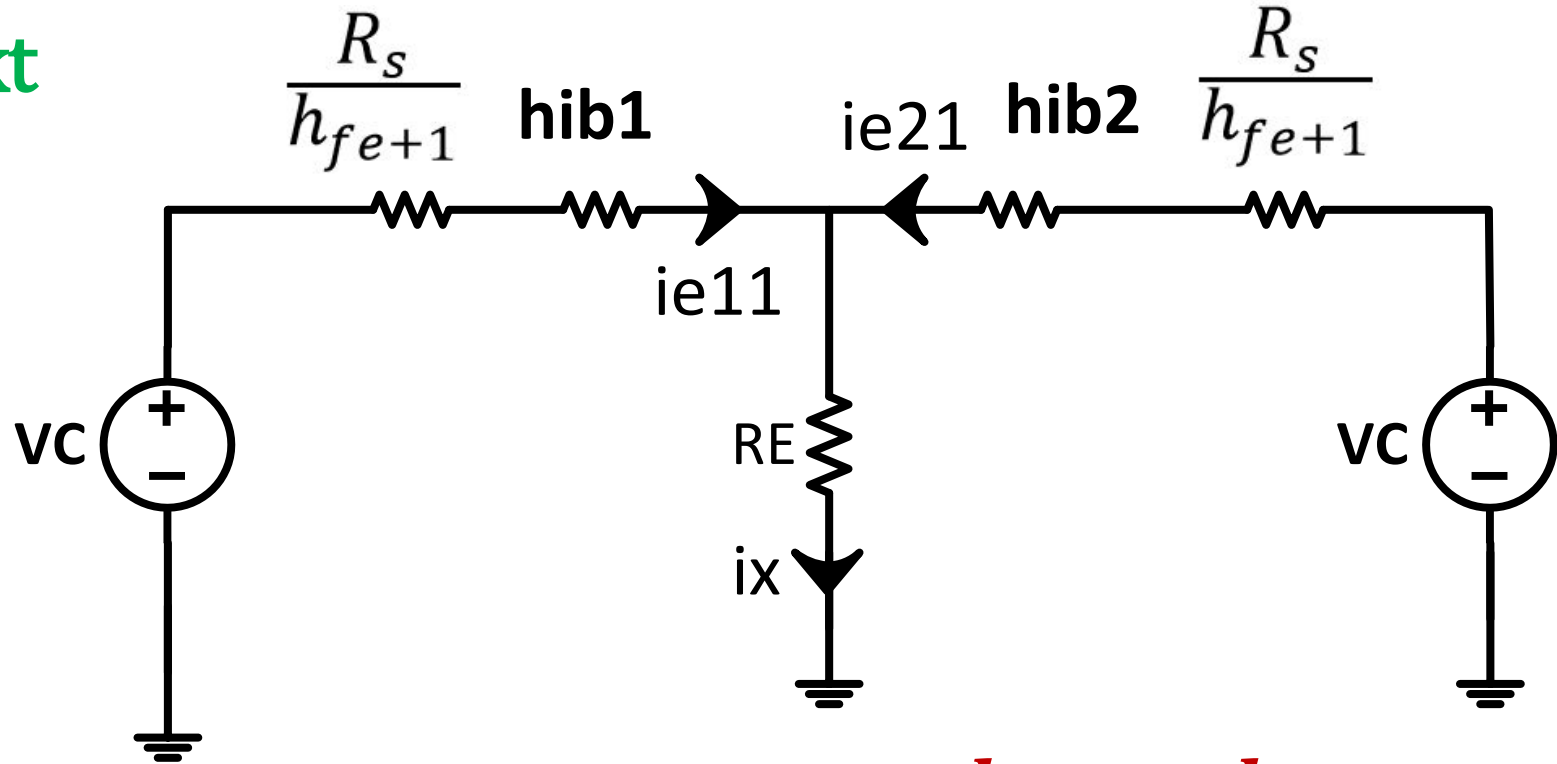
$$v_{i1} = v_c ; v_{i2} = v_c$$

By Symmetry

$$i_{e11} = i_{e21} ; i_x = 2i_{e11}$$

$$i_{e11} = \frac{v_c}{h_{ib} + \frac{R_s}{h_{fe+1}} + 2R_E}$$

$$i_{e21} = \frac{v_c}{h_{ib} + \frac{R_s}{h_{fe+1}} + 2R_E}$$



$$h_{ib1} = h_{ib2}$$

# Differential Amplifiers

2) let  $v_c = 0$

$$v_1 = -\frac{v_d}{2}; \quad v_2 = \frac{v_d}{2}$$

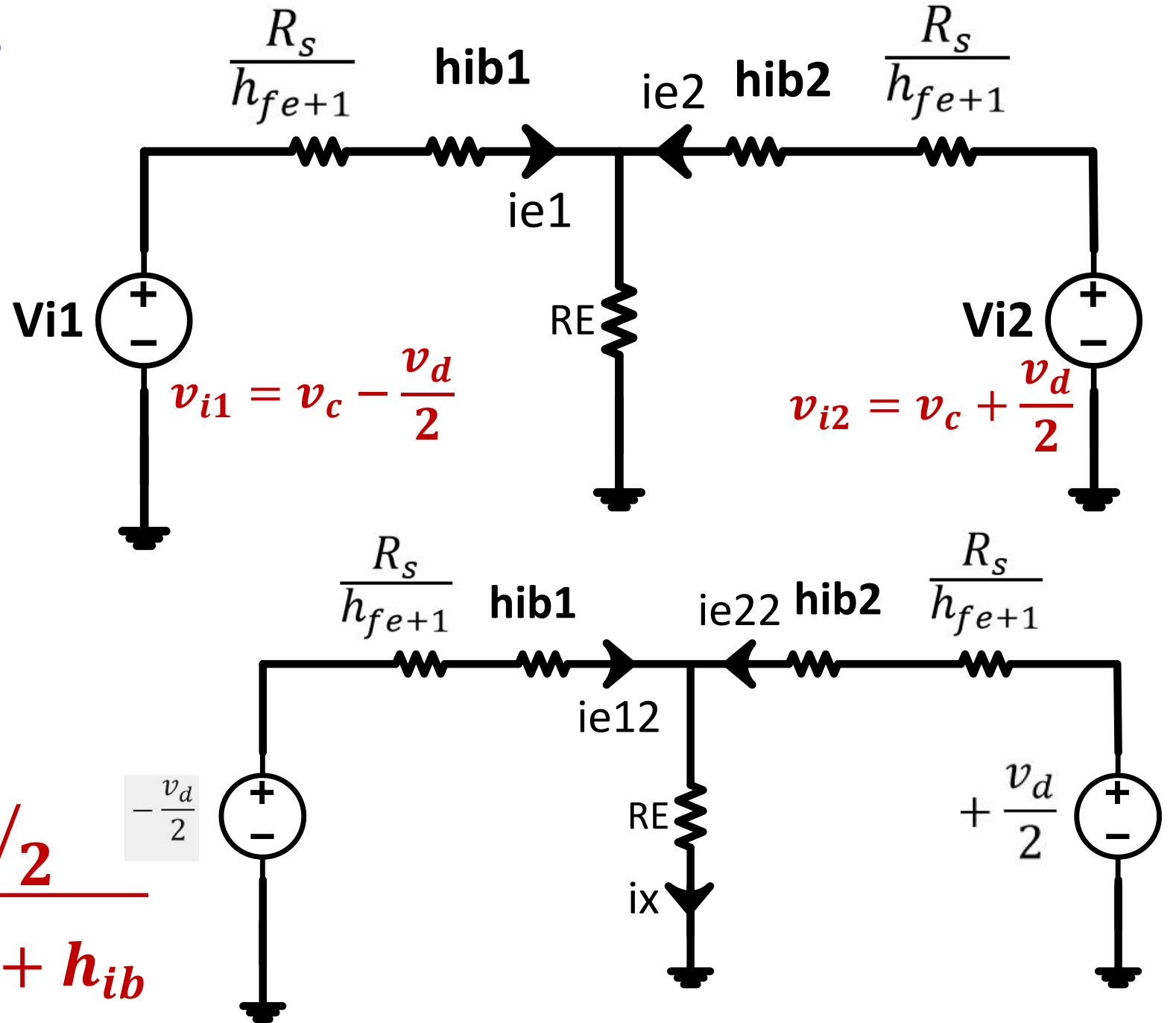
(Using symmetry)

$$i_{e12} = -i_{e22}$$

$$i_x = 0$$

$$i_{e12} = \frac{-v_d/2}{\frac{R_s}{h_{fe+1}} + h_{ib}}$$

$$i_{e22} = \frac{v_d/2}{\frac{R_s}{h_{fe+1}} + h_{ib}}$$



# Differential Amplifiers

## Differential Amplifier Circuit:

### Using Superposition

$$i_{e1} = \frac{v_c}{h_{ib} + \frac{R_s}{h_{fe+1}} + 2R_E} - \frac{v_d/2}{\frac{R_s}{h_{fe+1}} + h_{ib}}$$

$$i_{e2} = \frac{v_c}{h_{ib} + \frac{R_s}{h_{fe+1}} + 2R_E} + \frac{v_d/2}{\frac{R_s}{h_{fe+1}} + h_{ib}}$$

$$v_{o1} = + \frac{R_c v_d}{2 \left( h_{ib} + \frac{R_s}{h_{fe+1}} \right)} - \frac{R_c v_c}{h_{ib} + \frac{R_s}{h_{fe+1}} + 2R_E}$$

$$v_{o2} = - \frac{R_c v_d}{2 \left( h_{ib} + \frac{R_s}{h_{fe+1}} \right)} - \frac{R_c v_c}{h_{ib} + \frac{R_s}{h_{fe+1}} + 2R_E}$$

$$v_{o1} - v_{o2} = \frac{R_c}{h_{ib} + \frac{R_s}{h_{fe+1}}} v_d$$

# Differential Amplifiers

$$v_{o1} = + \frac{R_c v_d}{2 \left( h_{ib} + \frac{R_s}{h_{fe+1}} \right)} - \frac{R_c v_c}{h_{ib} + \frac{R_s}{h_{fe+1}} + 2R_E}$$

*Ad*  $\equiv$  *Differential mode gain*

$$Ad = \frac{v_o}{v_d} \Big|_{v_c=0}$$

let  $v_o = v_{o1}$

$$Ad = \frac{v_{o1}}{v_d} \Big|_{v_c=0}$$

$$Ad = \frac{R_c}{2 \left( h_{ib} + \frac{R_s}{h_{fe+1}} \right)}$$

# Differential Amplifiers

$$v_{o1} = + \frac{R_c v_d}{2 \left( h_{ib} + \frac{R_s}{h_{fe+1}} \right)} - \frac{R_c v_c}{h_{ib} + \frac{R_s}{h_{fe+1}} + 2R_E}$$

*Ac*  $\equiv$  *Common mode gain*

$$Ac = \frac{v_o}{v_c} \Big|_{v_d=0}$$

*let*  $v_o = v_{o1}$

$$Ac = \frac{v_{o1}}{v_c} \Big|_{v_d=0}$$

$$Ac = \frac{-R_c}{2R_E + h_{ib} + \frac{R_s}{h_{fe+1}}}$$

# Differential Amplifiers

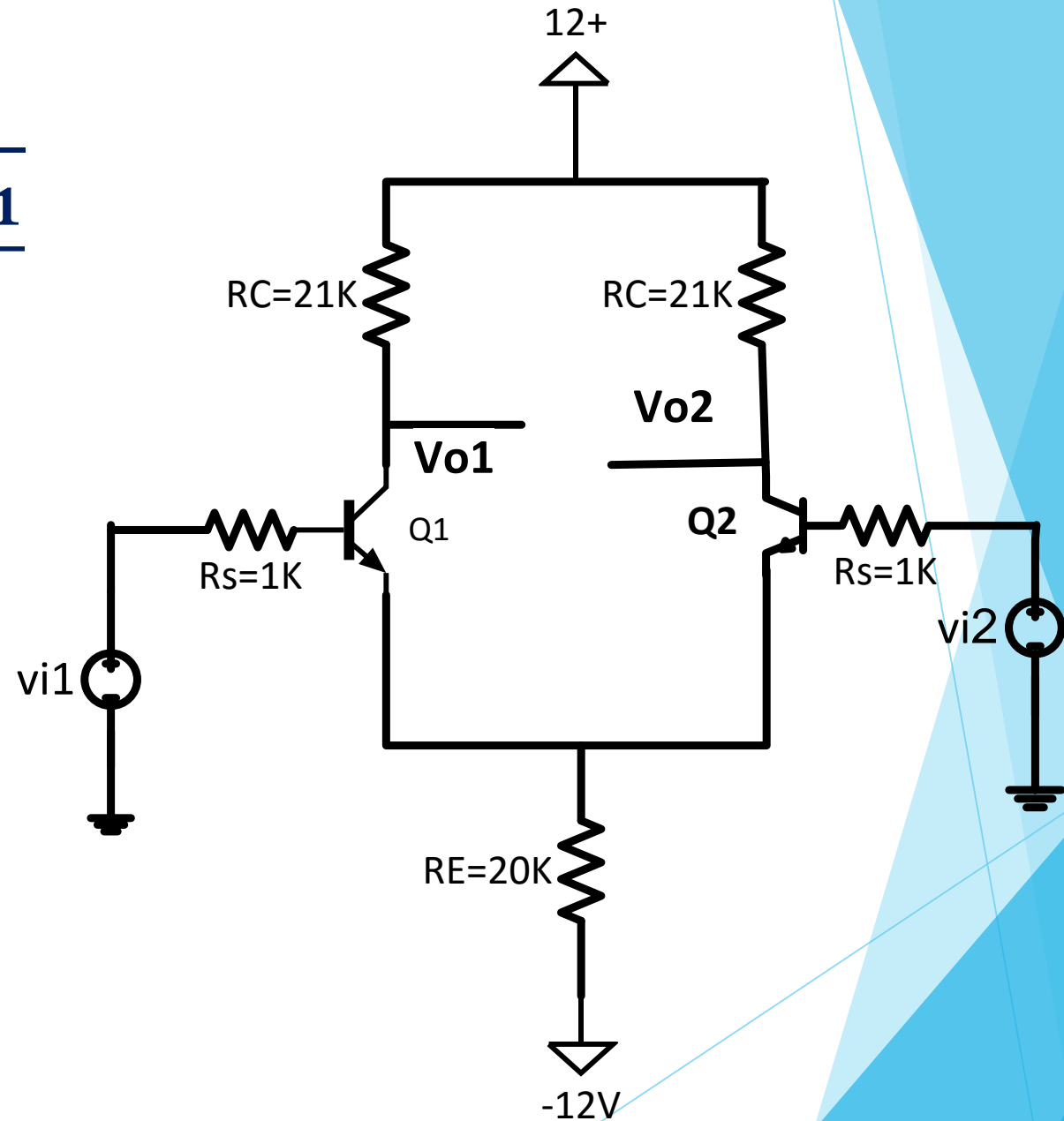
***CMRR***  $\equiv$  ***Common Mode Rejection Ratio***

$$CMRR = \left| \frac{A_d}{A_c} \right|$$

$$CMRR = \frac{2R_E + h_{ib} + \frac{R_s}{h_{fe+1}}}{2 \left( h_{ib} + \frac{R_s}{h_{fe+1}} \right)}$$

***To increase CMRR we need to increase  $R_E$***

$$CMRR = \frac{2R_E + h_{ib} + \frac{R_s}{h_{fe+1}}}{2 \left( h_{ib} + \frac{R_s}{h_{fe+1}} \right)}$$



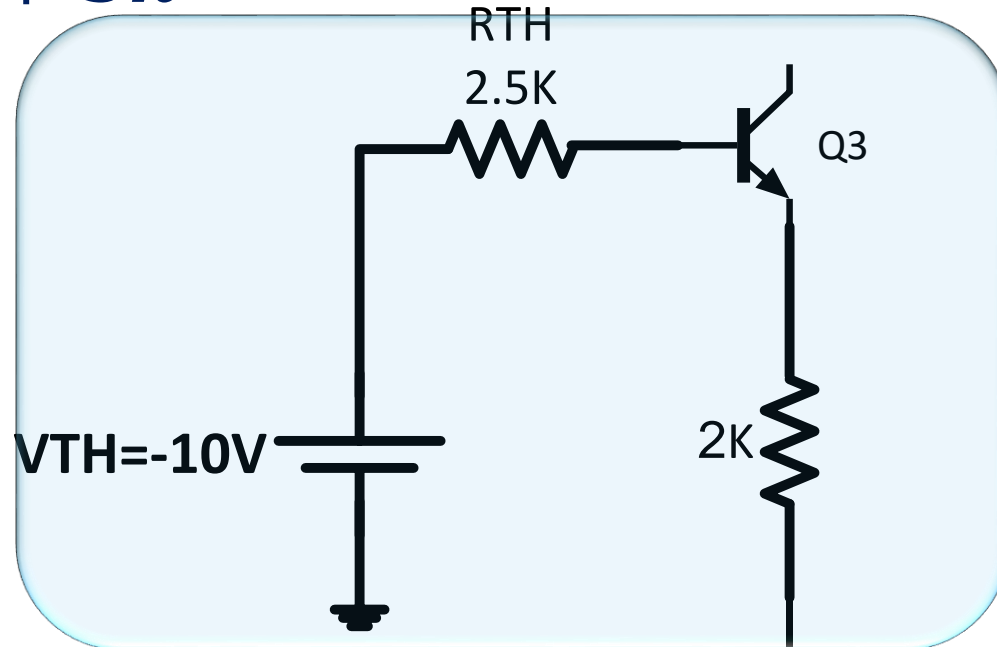
$$I_{E1} = I_{E2} = \frac{12 - 0.7}{\frac{R_s}{41} + 2R_E}$$



# Differential Amplifier with constant current source:

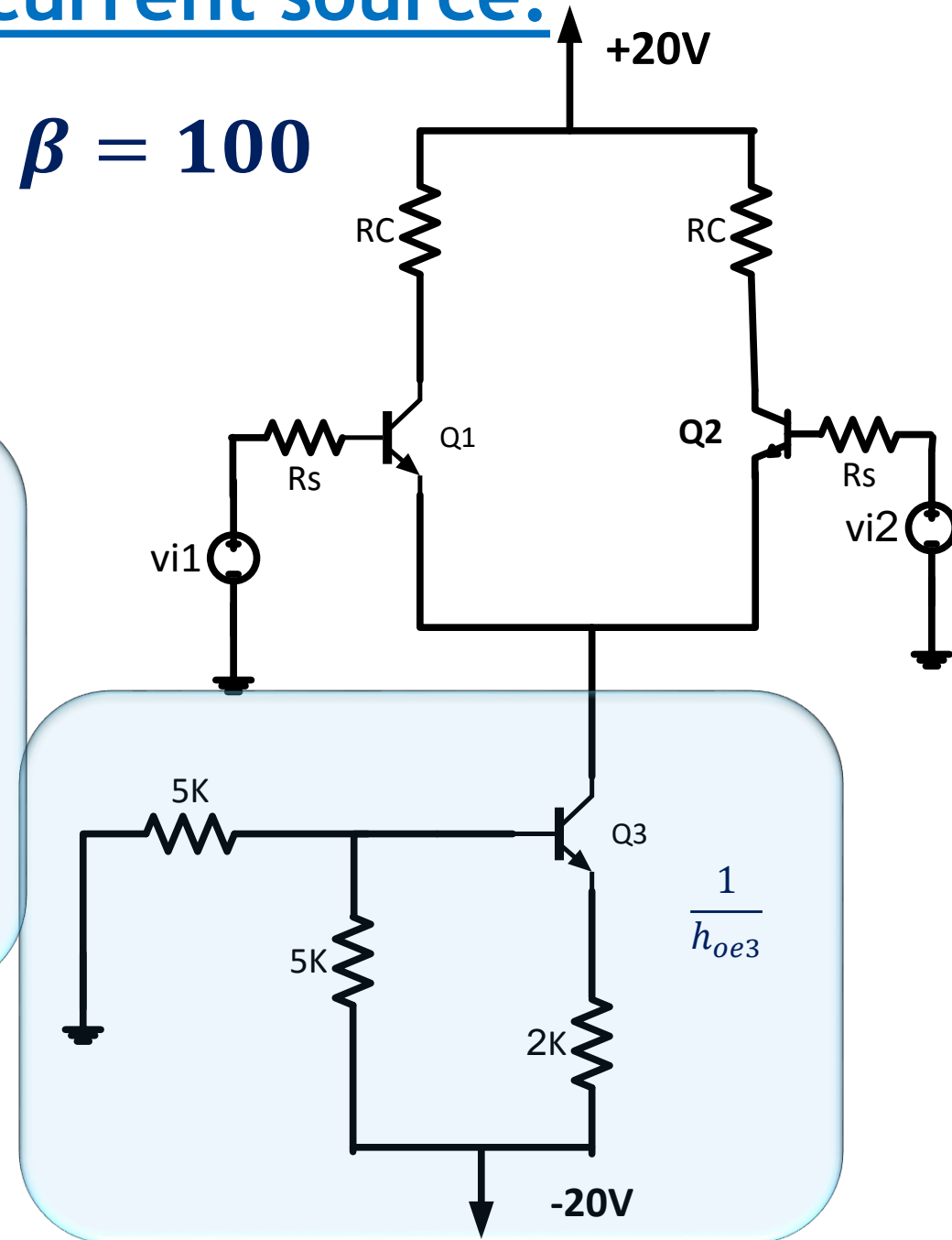
$$R_{TH} = 5k \parallel 5k = 2.5k$$

$$V_{TH} = \frac{5k}{5k + 5k} (-20) = -10V$$



$$I_{E3} = \frac{10 - 0.7}{\frac{2.5k}{101} + 2k} = 4.65 mA$$

$$\beta = 100$$

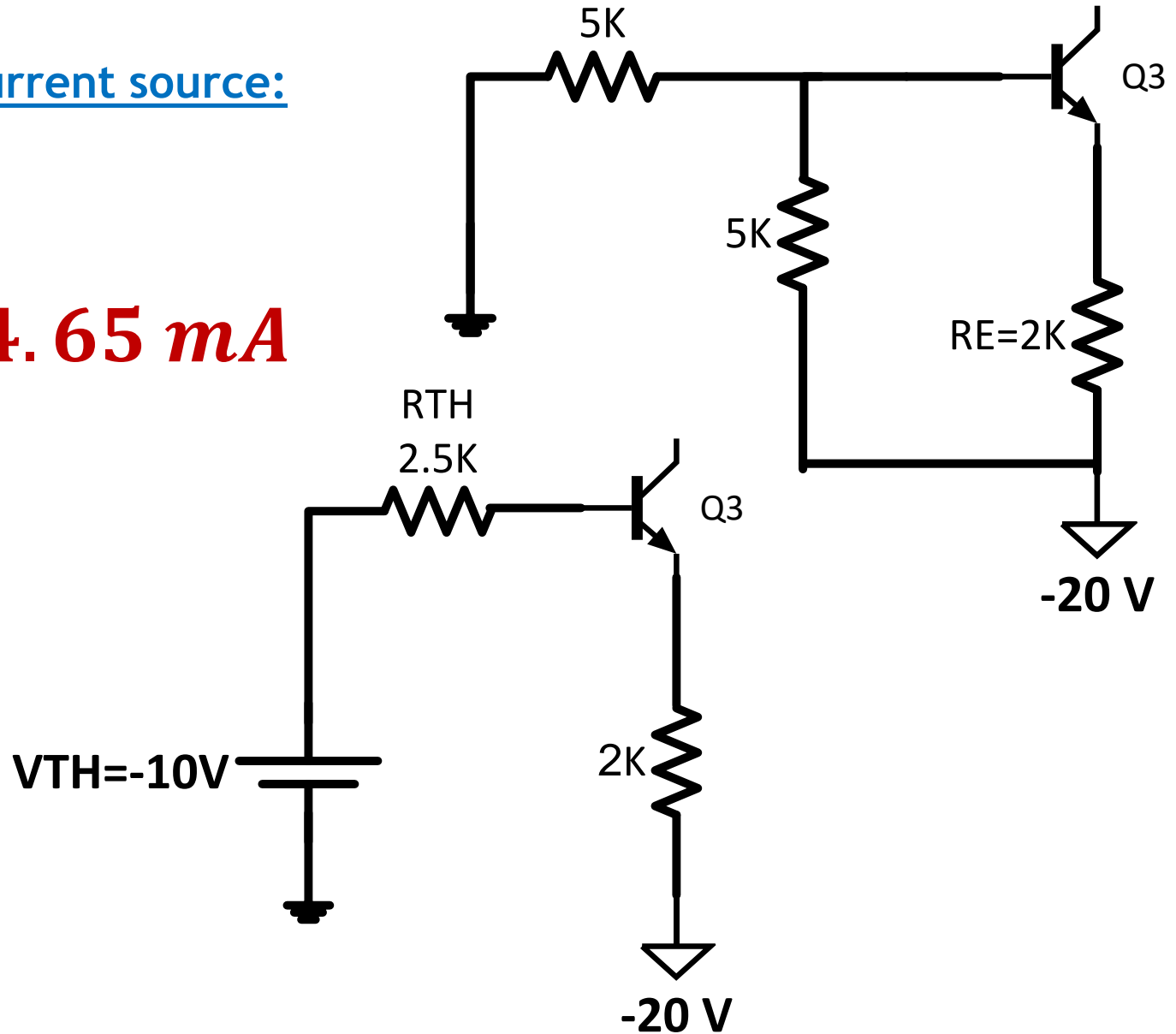


# Differential Amplifiers

Differential Amplifier with constant current source:

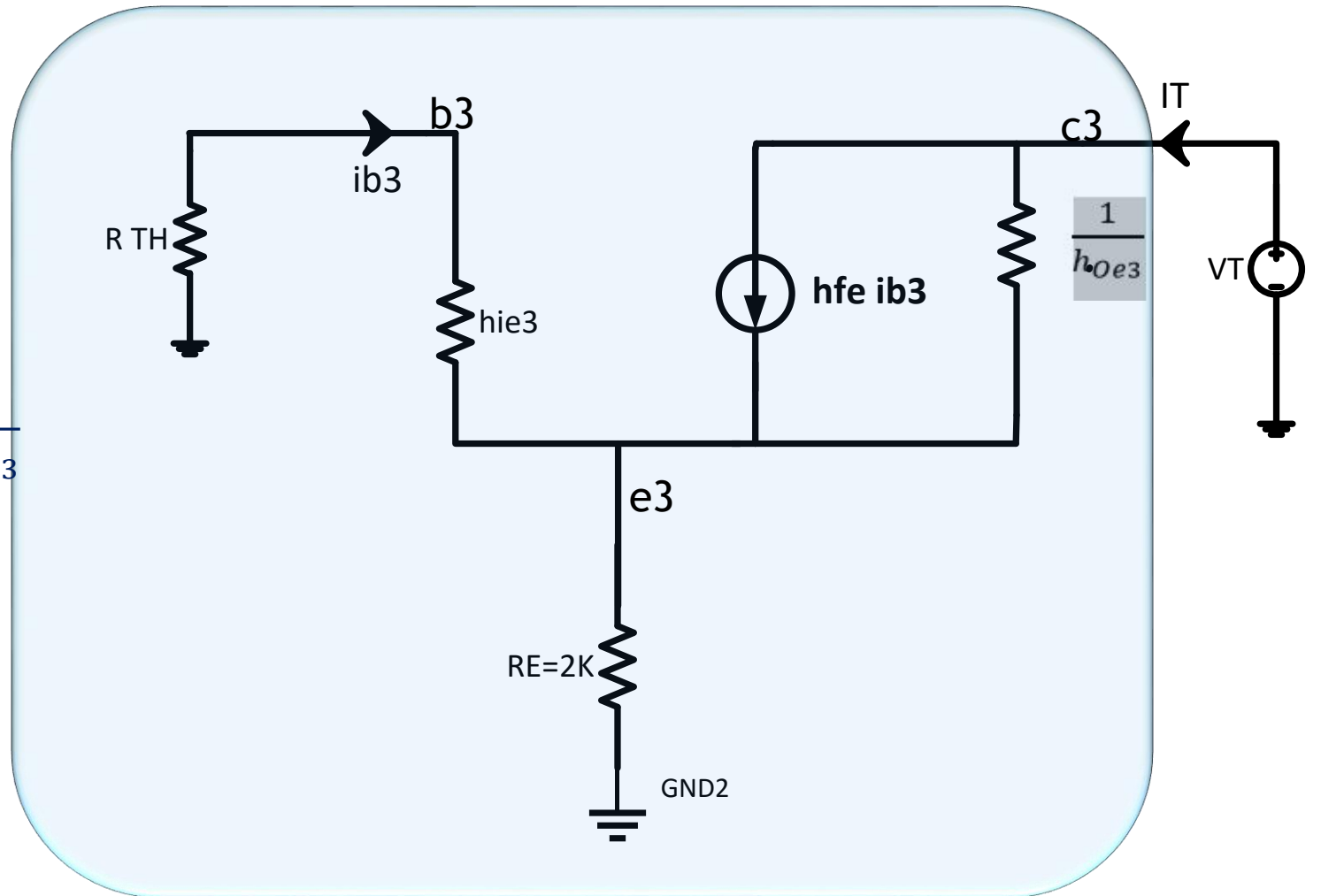
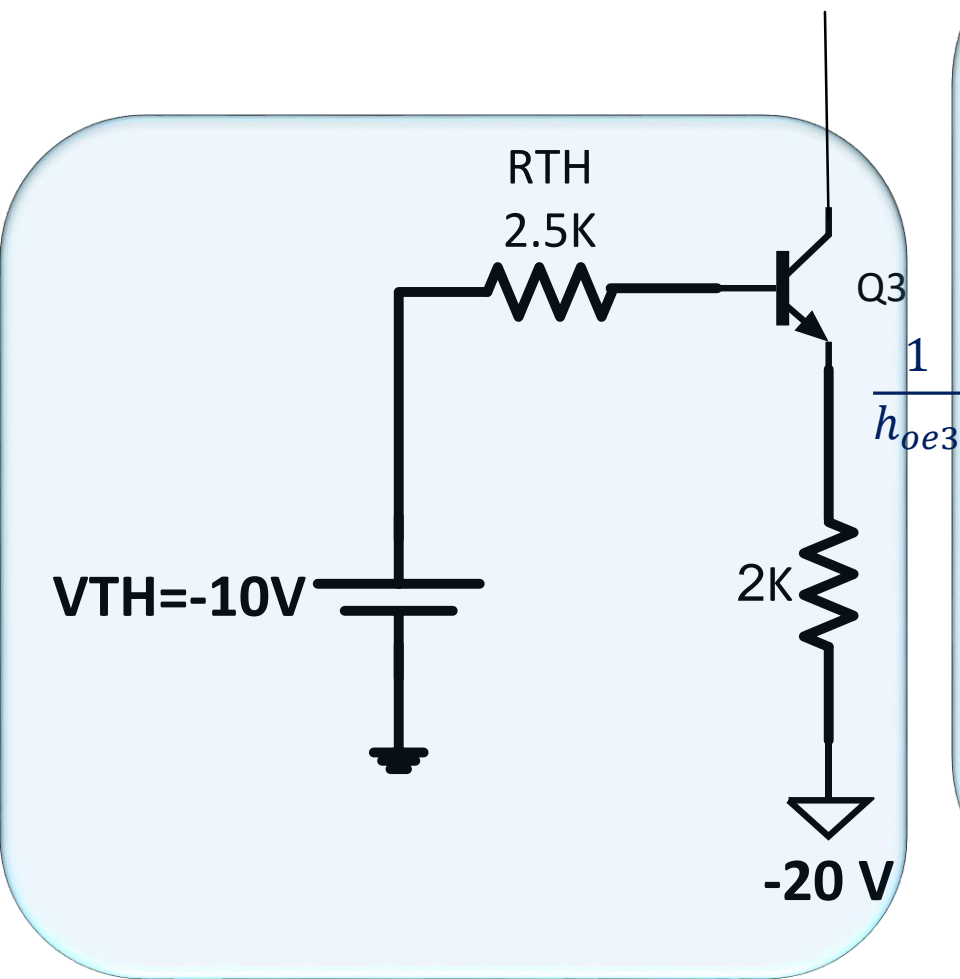
$$I_{E3} = \frac{10 - 0.7}{\frac{2.5k}{101} + 2k} = 4.65 \text{ mA}$$

$$h_{ie3} = \beta \frac{V_T}{I_{CQ3}} = 0.559 \text{ K}$$



# Differential Amplifiers

Ac small signal equivalent circuit for the constant current source:

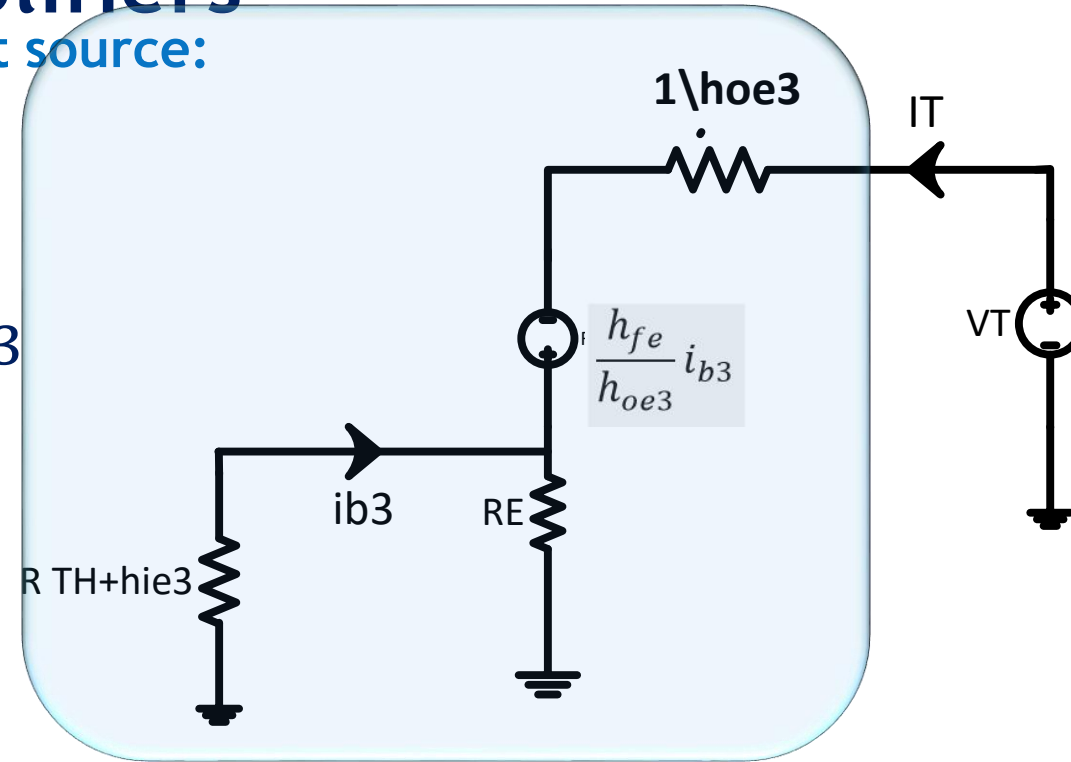


# Differential Amplifiers

Ac small signal equivalent circuit for the constant current source:

$$V_T = \frac{1}{h_{oe3}} I_T - \frac{h_{fe}}{h_{oe3}} i_{b3} - (h_{ie3} + R_{TH}) i_{b3}$$

$$i_{b3} = - \frac{R_E}{R_E + h_{ie3} + R_{TH}} I_T$$



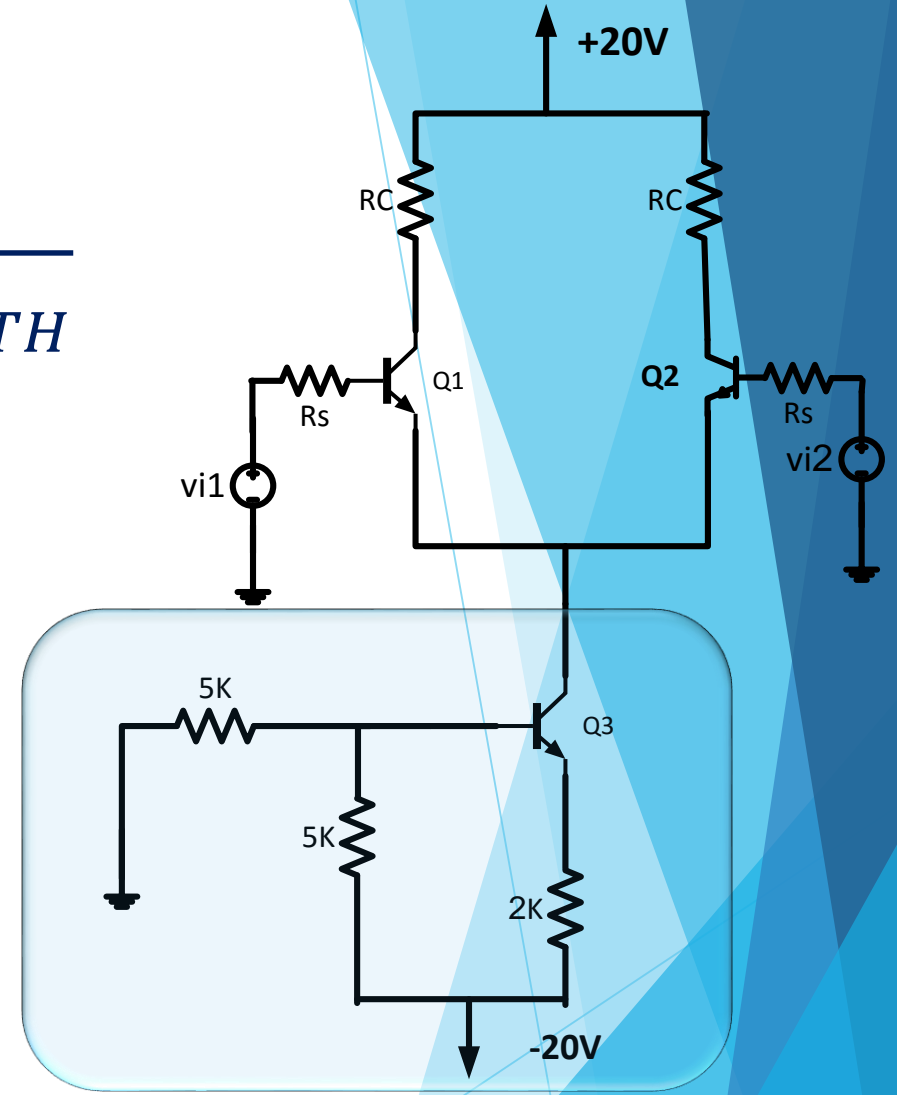
$$\begin{aligned} \therefore R_o &= \frac{V_T}{I_T} \\ &= \frac{1}{h_{oe3}} + \frac{h_{fe}}{h_{oe3}} \frac{R_E}{R_E + h_{ie3} + R_{TH}} + \frac{(h_{ie3} + R_{TH})}{R_E + h_{ie3} + R_{TH}} R_E \end{aligned}$$

$$R_o \approx \frac{1}{h_{oe3}} + \frac{h_{fe}}{h_{oe3}} \frac{R_E}{R_E + h_{ie3} + R_{TH}}$$

$$R_o \approx \frac{1}{h_{oe3}} + \frac{h_{fe}}{h_{oe3}} \frac{R_E}{R_E + h_{ie3} + R_{TH}}$$

- ▶ let  $\frac{1}{h_{oe}} = 80K$ ,  $h_{ie} = 0.559k$
- ▶  $h_{fe} = 100$ ,  $R_E = 2k$ ,  $R_{TH} = 2.5k$
- ▶  $R_o = 3.25 M\Omega$

$$CMRR = \frac{2R_E + h_{ib} + \frac{R_s}{h_{fe+1}}}{2 \left( h_{ib} + \frac{R_s}{h_{fe+1}} \right)}$$



# Differential Amplifiers

## Bipolar transistor current sources:

Q1 and Q2 are in the active region

### 1. Current mirror : Simple

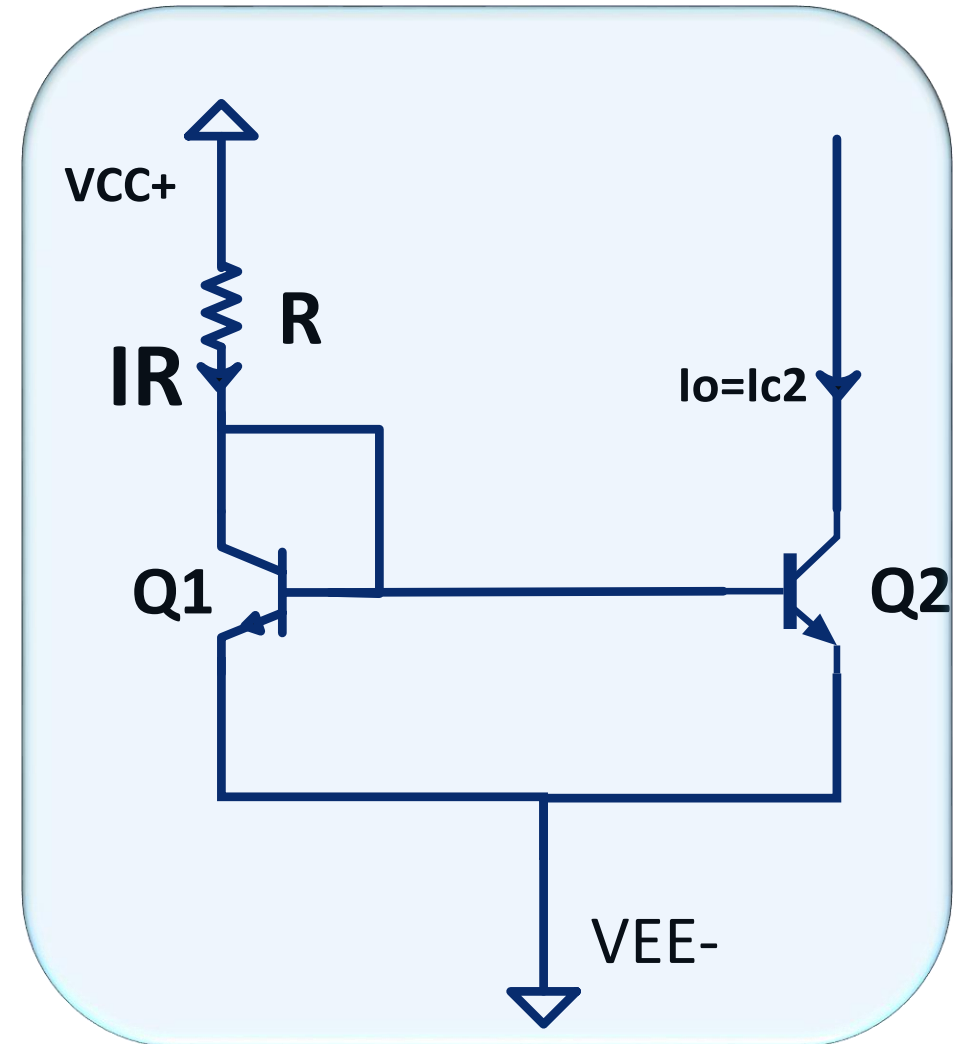
$$I_C = I_S e^{\frac{V_{BE}}{V_T}}$$

*If Q<sub>1</sub> is matched to Q<sub>2</sub>*

$$\beta_1 = \beta_2 ; I_{S1} = I_{S2} ; V_{T1} = V_{T2}$$

And since  $V_{BE1} = V_{BE2}$

$$\therefore I_{C1} = I_{C2} = I_o$$

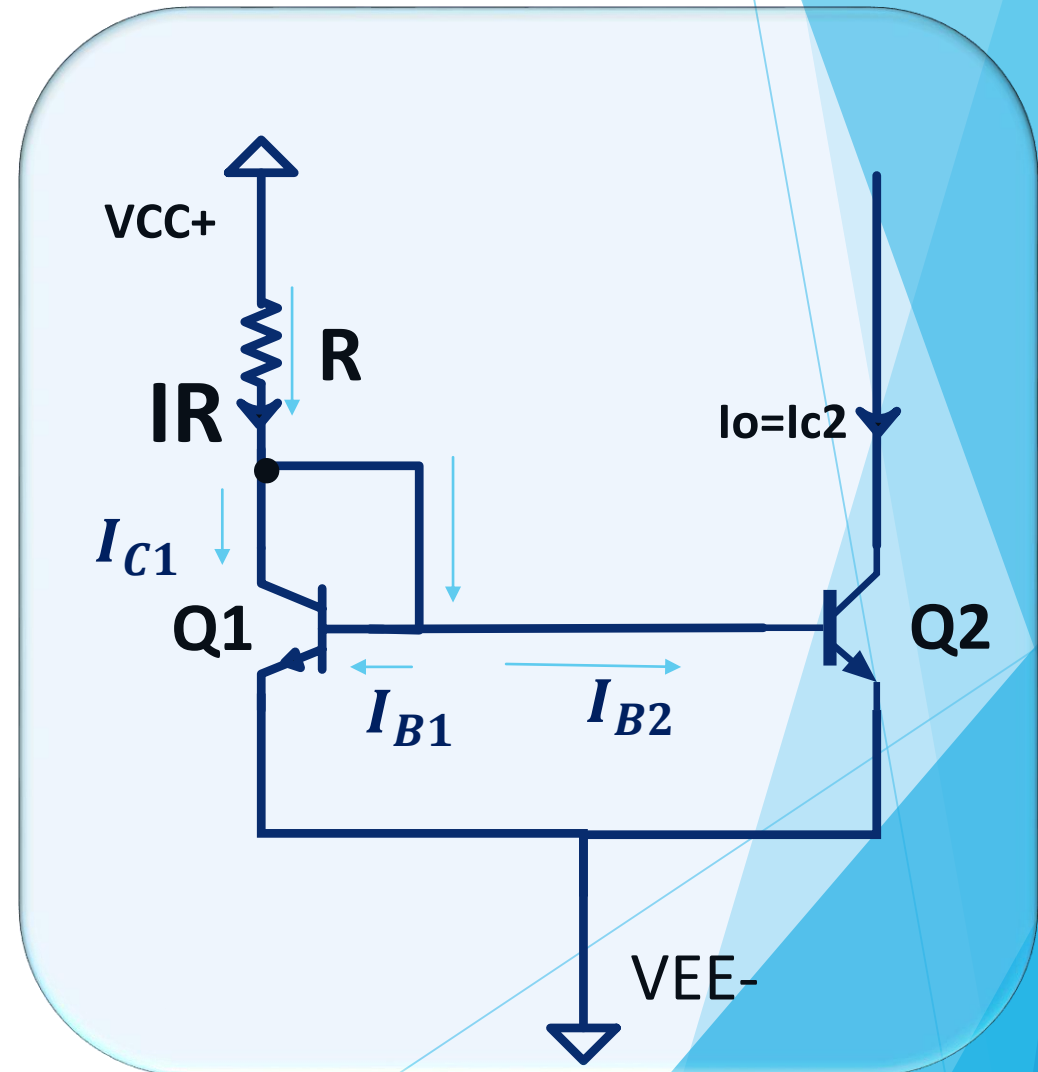


To find  $I_o$  in terms of  $I_R$

▶ **KCL**  $I_R = I_{C1} + I_{B1} + I_{B2}$

▶  $I_{C1} = I_{C2}$  ;  $I_{B1} = I_{B2}$

▶  $\therefore I_o = I_{C2} = \frac{I_R}{1 + \frac{2}{\beta}}$



# Differential Amplifiers

Bipolar transistor current sources:

## 1. Current mirror : Simple

1) if  $\beta = \infty$

$$I_o = I_R$$

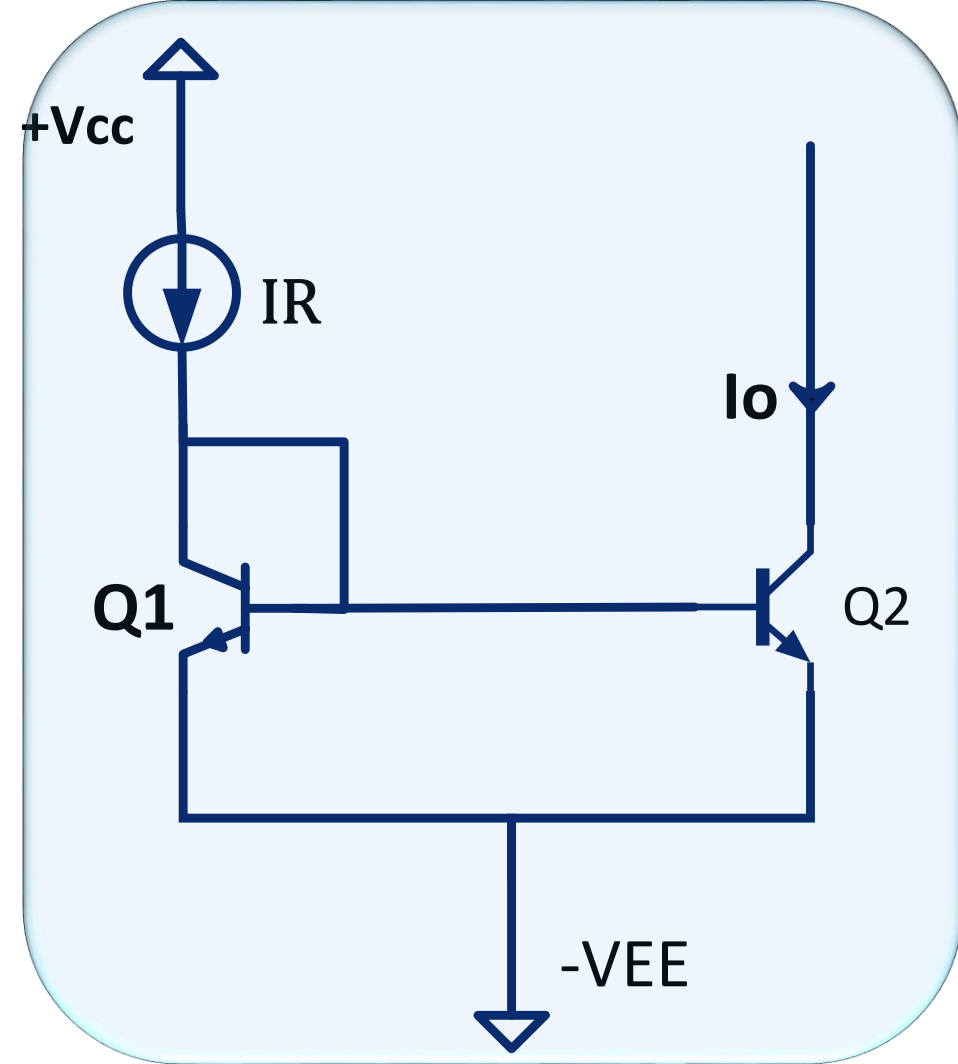
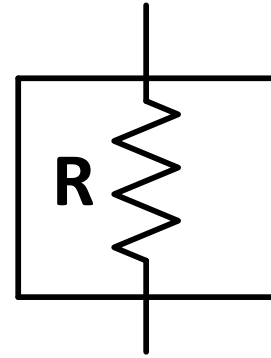
2) if  $\beta = 100$

$$I_o = I_R \quad ; \quad 2\% \text{ error}$$

To find  $I_R$

$$\text{KVL: } V_{CC} = R I_R + V_{BE1} - V_{EE}$$

$$I_R = \frac{V_{CC} + V_{EE} - V_{BE}}{R}$$



$$I_o = I_{C2} = \frac{I_R}{1 + \frac{2}{\beta}}$$



## Bipolar transistor current sources:

$$I_C = I_S e^{\frac{V_{BE}}{V_T}}$$

### 1. Current mirror : Simple

*If the area of the EB junction of Q2 is m times that of Q1*

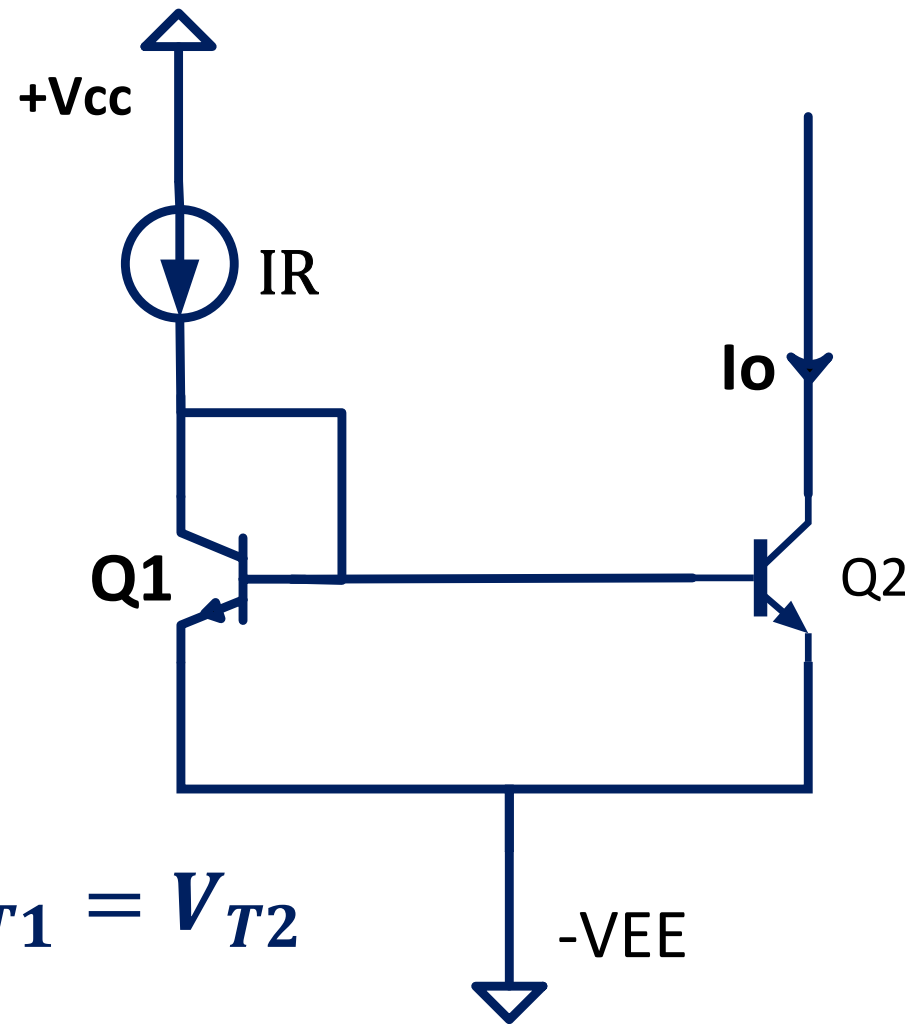
$$\therefore I_{S2} = m I_{S1}$$

And since  $V_{BE1} = V_{BE2}$  and  $\beta_1 = \beta_2$  ;  $V_{T1} = V_{T2}$

$$\therefore I_{C2} = m I_{C1}$$

**KCL:**  $I_R = I_{C1} + I_{B1} + I_{B2}$

$$I_R = \frac{I_{C2}}{m} + \frac{I_{C2}}{\beta m} + I_{B2}$$



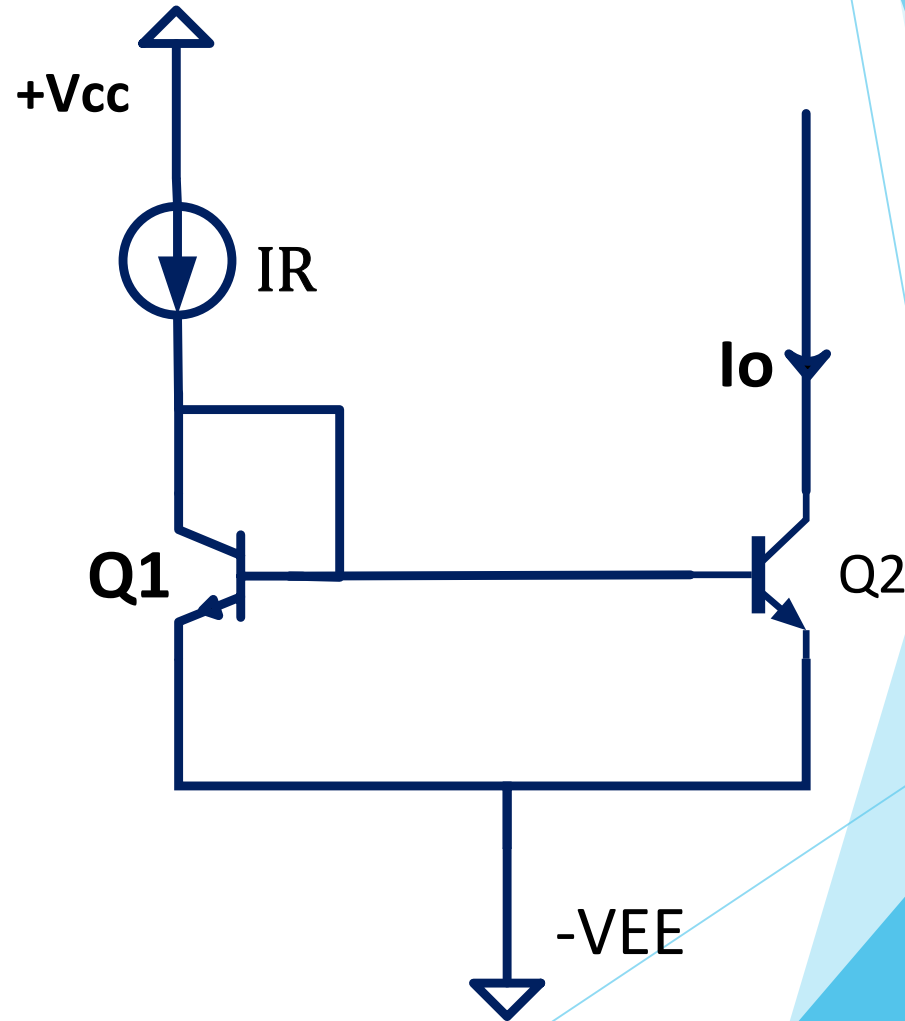
$$I_R = \frac{I_{C2}}{m} + \frac{I_{C2}}{\beta m} + I_{B2}$$

$$I_{C2} = m I_{C1}$$

$$\therefore I_{C2} = I_o = I_R \frac{m}{1 + \frac{m+1}{\beta}}$$

if  $\beta = \infty$

$$I_o = I_{C2} = m I_R$$

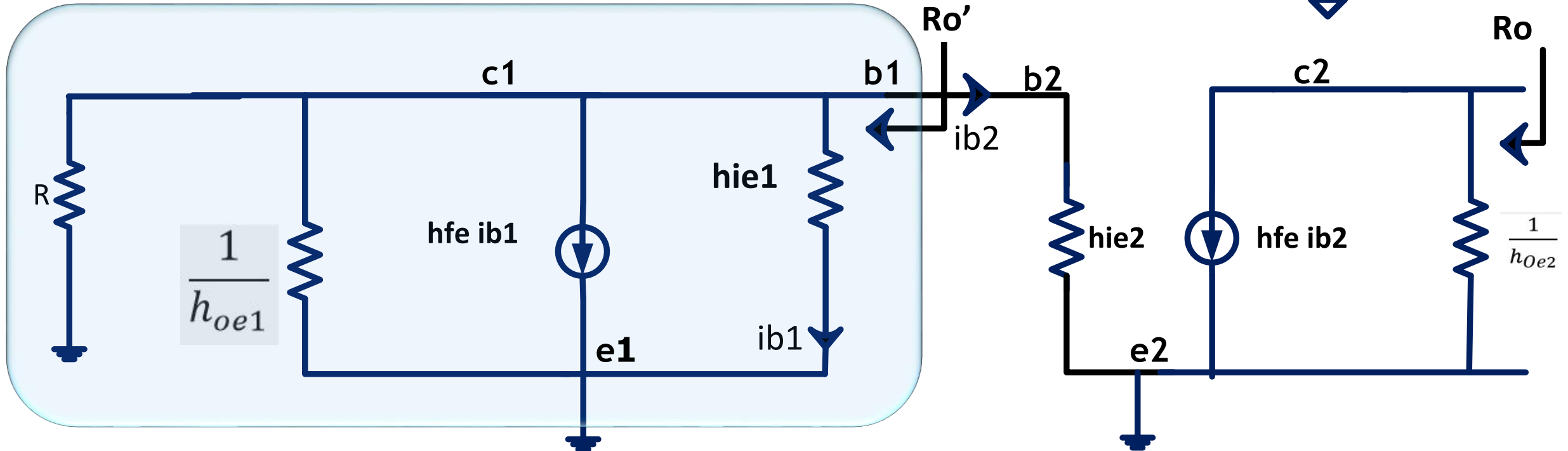
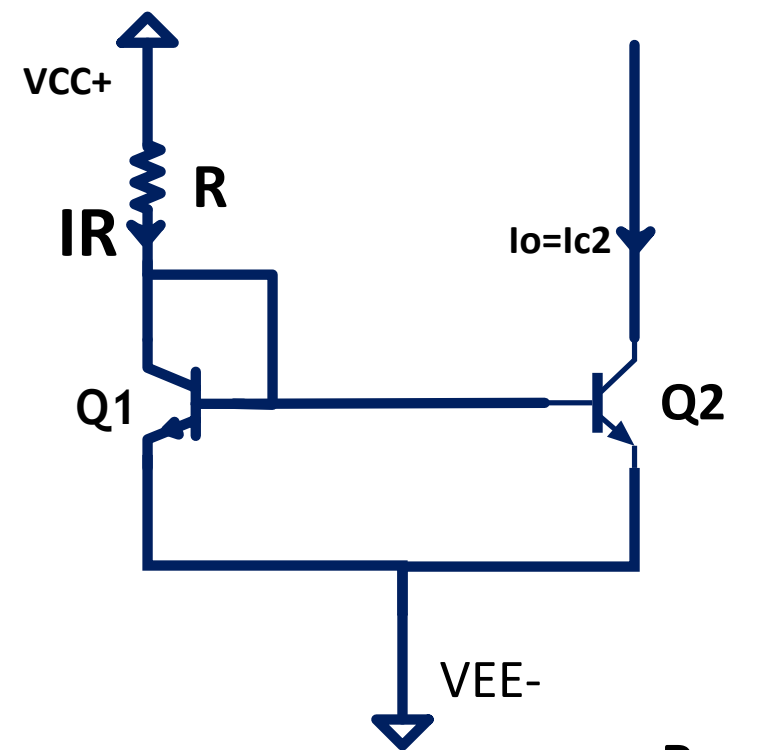


# Bipolar transistor current sources:

## Output - Impedance:

To find  $R_o = \frac{V_T}{I_T}$

Ac small signal equivalent circuit

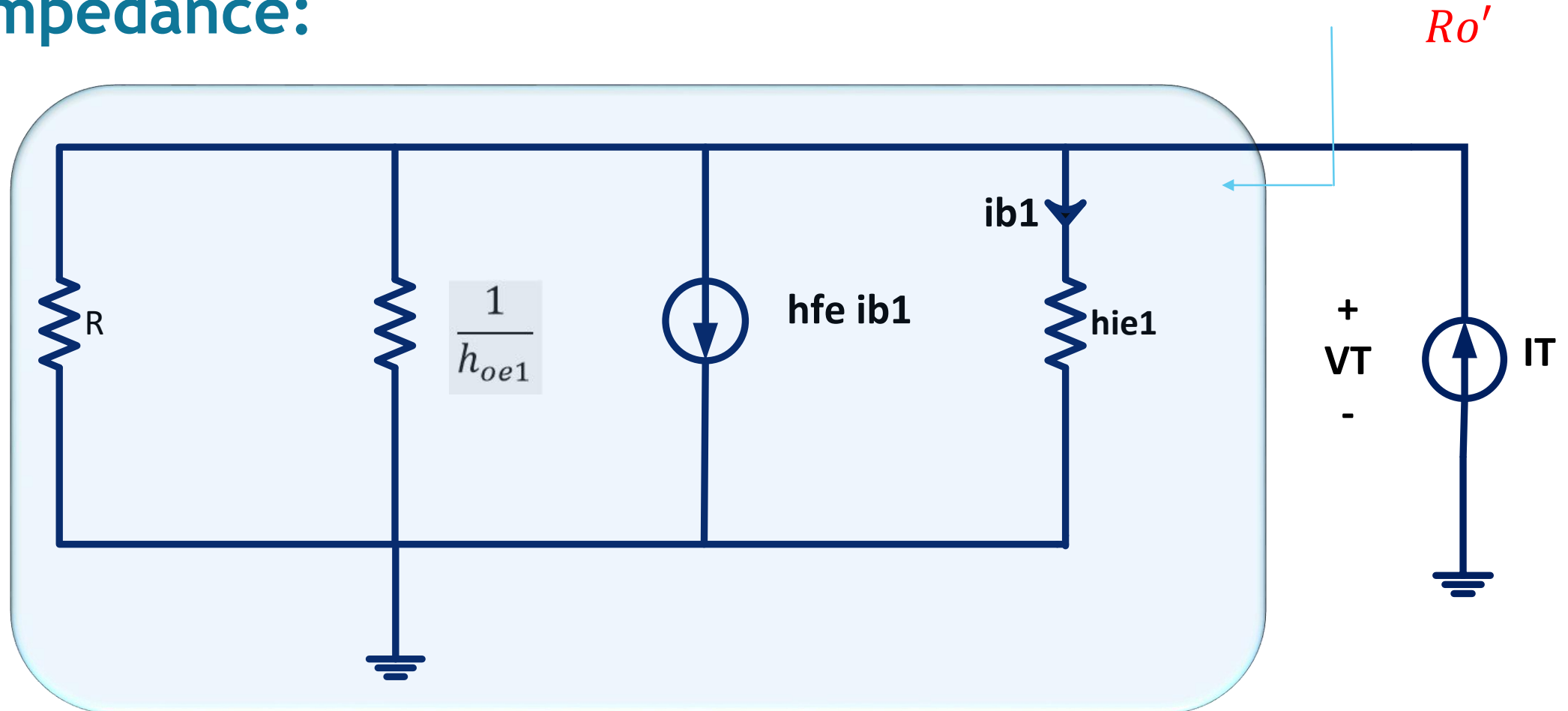


# Differential Amplifiers

## Bipolar transistor current sources:

### Output - Impedance:

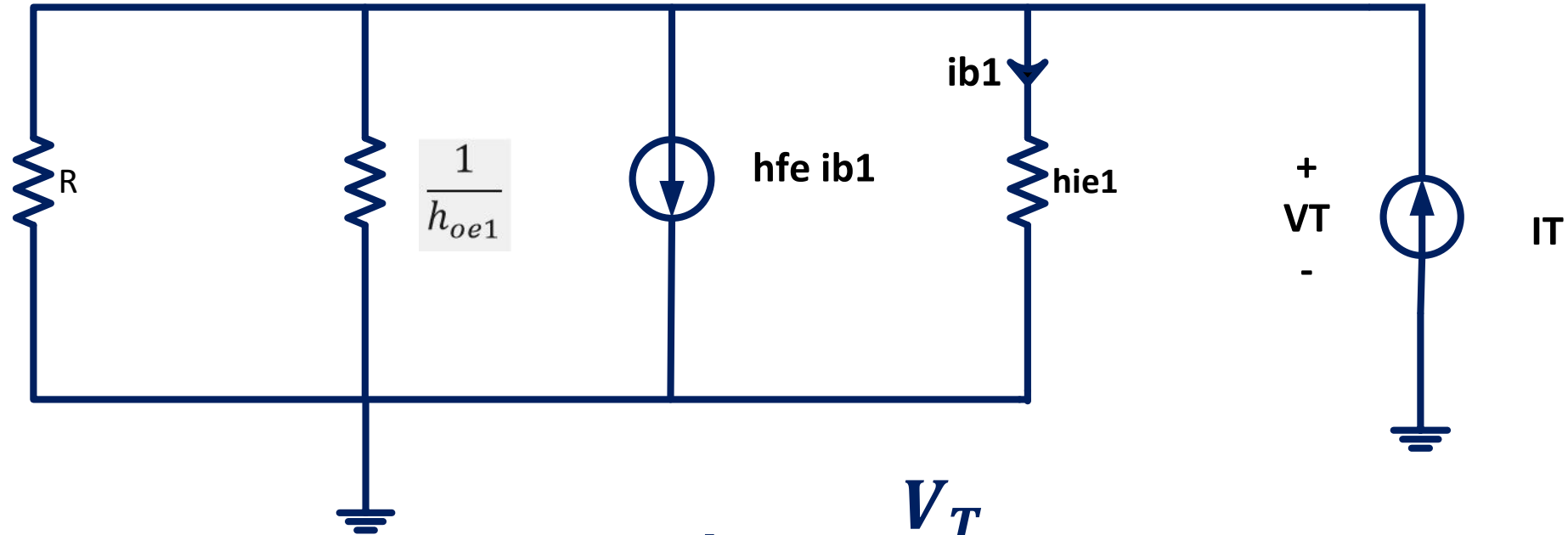
*To find  $R_{o'}$*



# Bipolar transistor current sources:

To find  $R_{o'} = \frac{V_T}{I_T}$

Output - Impedance:



$$I_T = \frac{V_T}{R \parallel \frac{1}{h_{oe1}} \parallel h_{ie1}} + h_{fe} i_{b1}$$

$$i_{b1} = \frac{V_T}{h_{ie1}}$$

$$g_{m1} = \frac{h_{fe1}}{h_{ie1}}$$

$$I_T = \frac{h_{fe}}{h_{ie1}} V_T + \frac{V_T}{R \parallel h_{ie1} \parallel \frac{1}{h_{oe1}}}$$

$$R_{o'} = R \parallel h_{ie1} \parallel \frac{1}{h_{oe1}} \parallel \frac{1}{g_{m1}}$$

# Differential Amplifiers

Bipolar transistor current sources:

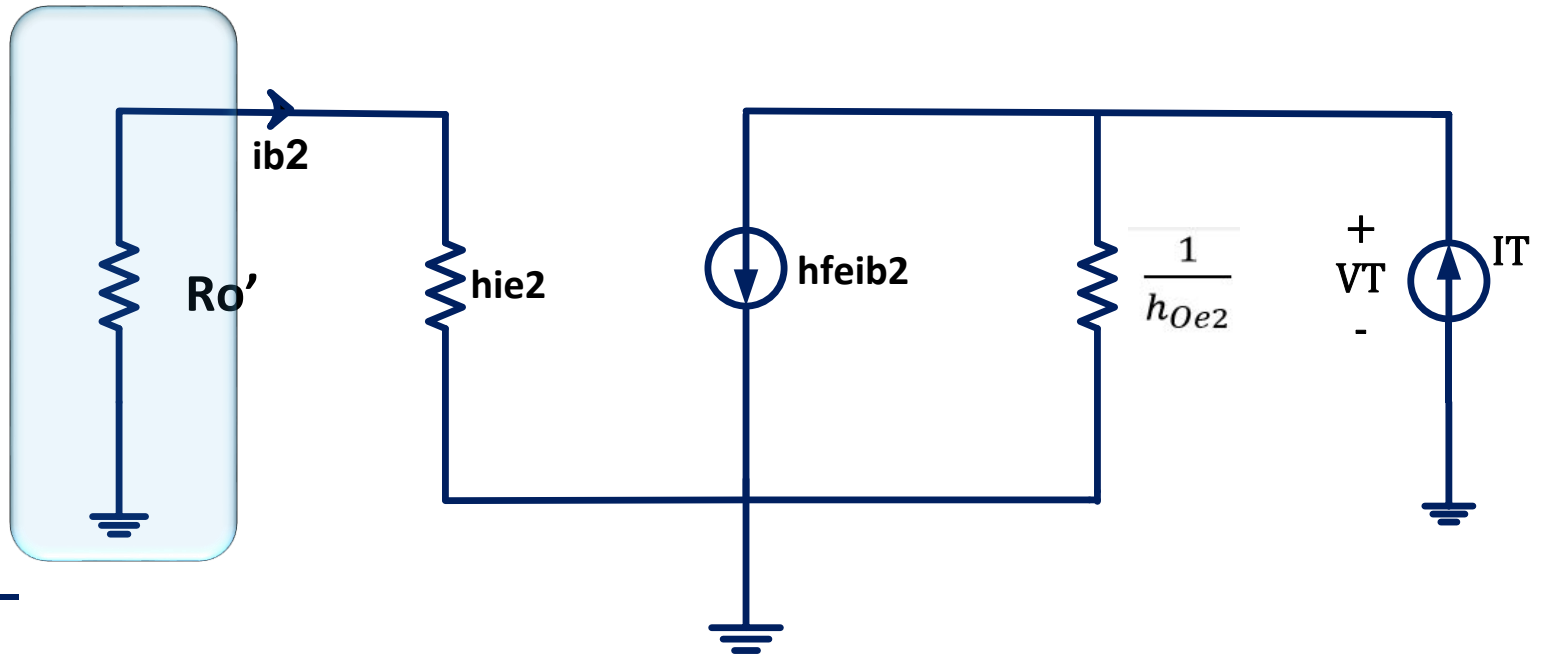
Output - Impedance:

To find  $R_o = \frac{V_T}{I_T}$

$$I_T = h_{fe} i_{b2} + \frac{V_T}{\frac{1}{h_{oe2}}}$$

$$i_{b2} = 0$$

$$\therefore \frac{V_T}{I_T} = \frac{1}{h_{oe2}}$$



# Bipolar transistor current sources:

## 2. Bipolar mirror with base-current compensation:



$Q_1, Q_2$  are matched

$$\beta_1 = \beta_2 ; I_{S1} = I_{S2} ; V_{T1} = V_{T2}$$

And since  $V_{BE1} = V_{BE2}$

$$\therefore I_{C1} = I_{C2} = I_o$$

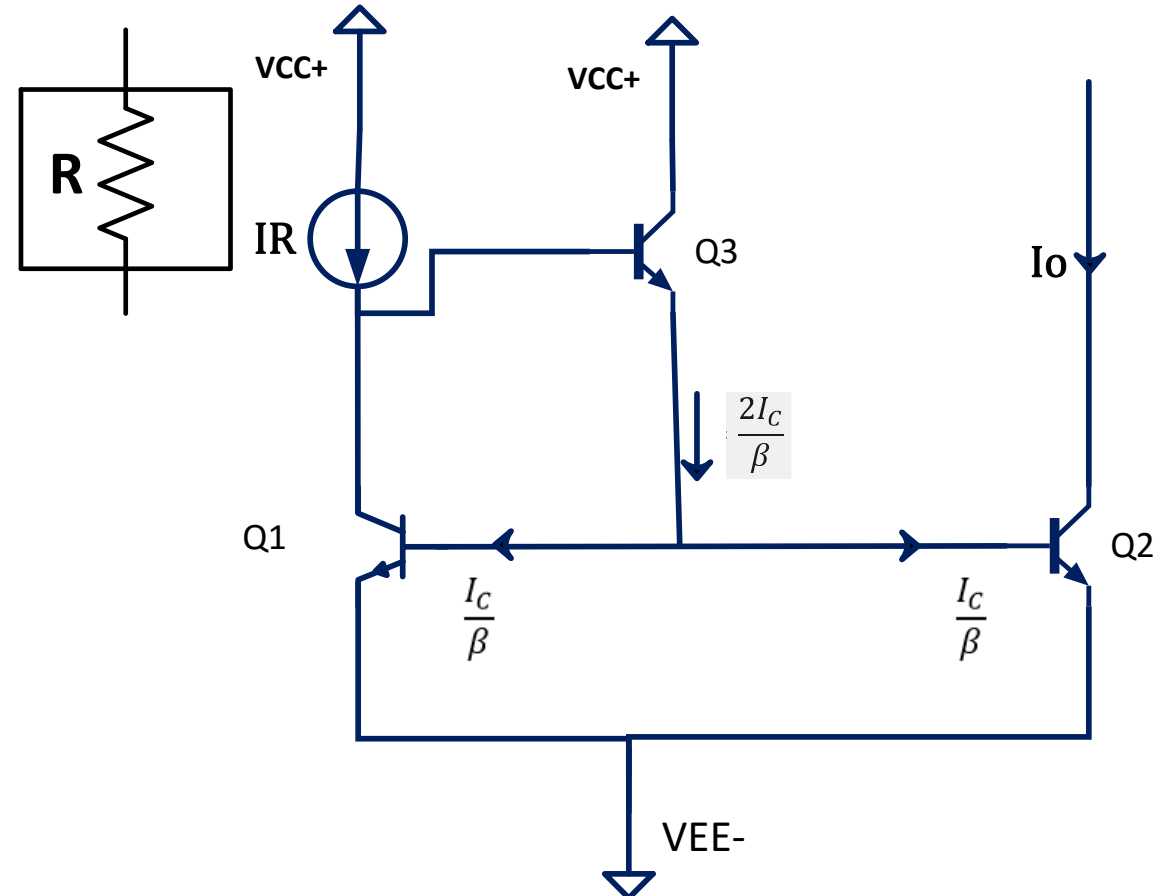
$$I_{B1} = I_{B2}$$

$$\therefore I_R = I_{C1} + I_{B3}$$

$$I_{B3} = \frac{I_{E3}}{\beta + 1}$$

$$I_{E3} = \frac{2I_C}{\beta}$$

Reduce the  $\beta$  dependence

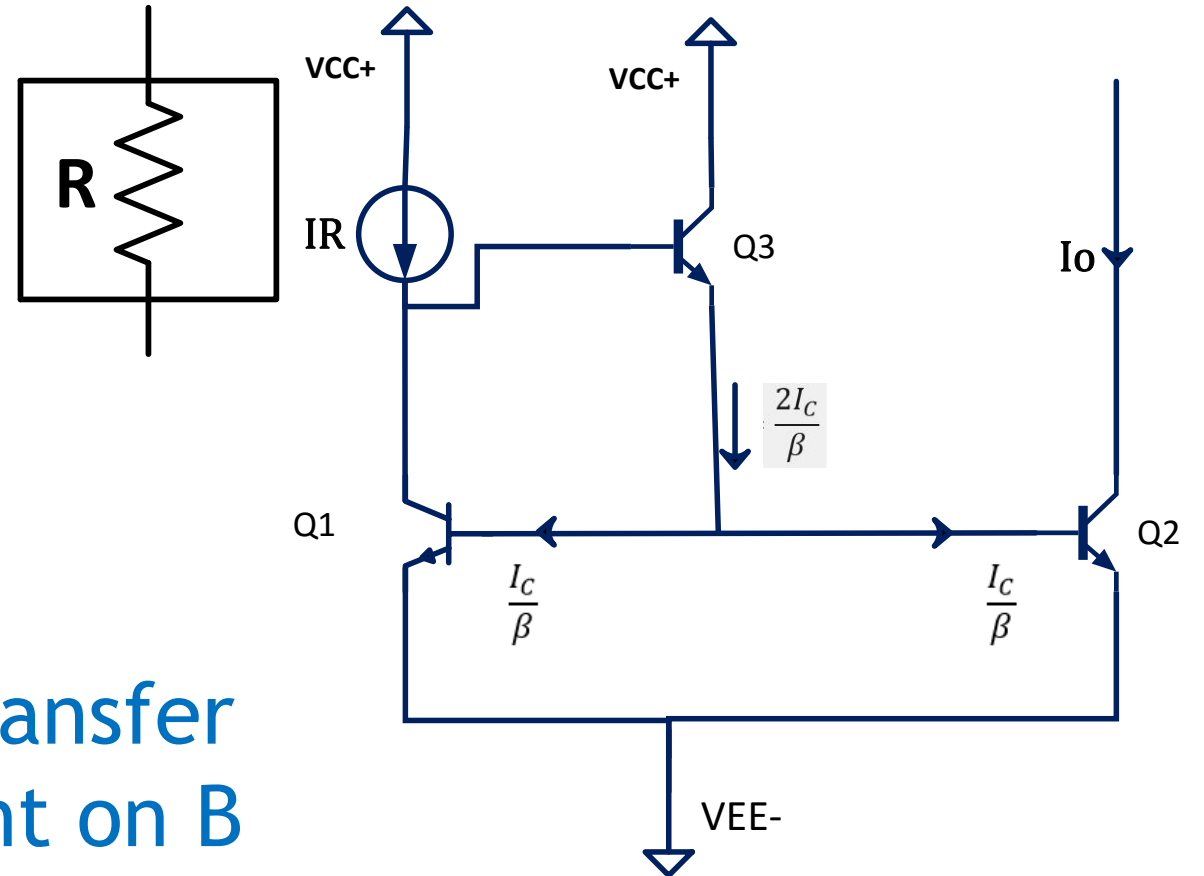


# Bipolar transistor current sources:

## 2. Bipolar mirror with base-current compensation:

$$\therefore I_R = I_C + \frac{2I_C}{\beta(\beta + 1)}$$

$$\therefore I_O = I_R \cdot \frac{1}{1 + \frac{2}{\beta^2 + \beta}}$$



Current source with a current transfer ratio that is much less dependent on  $\beta$  than that of the simple current mirror.



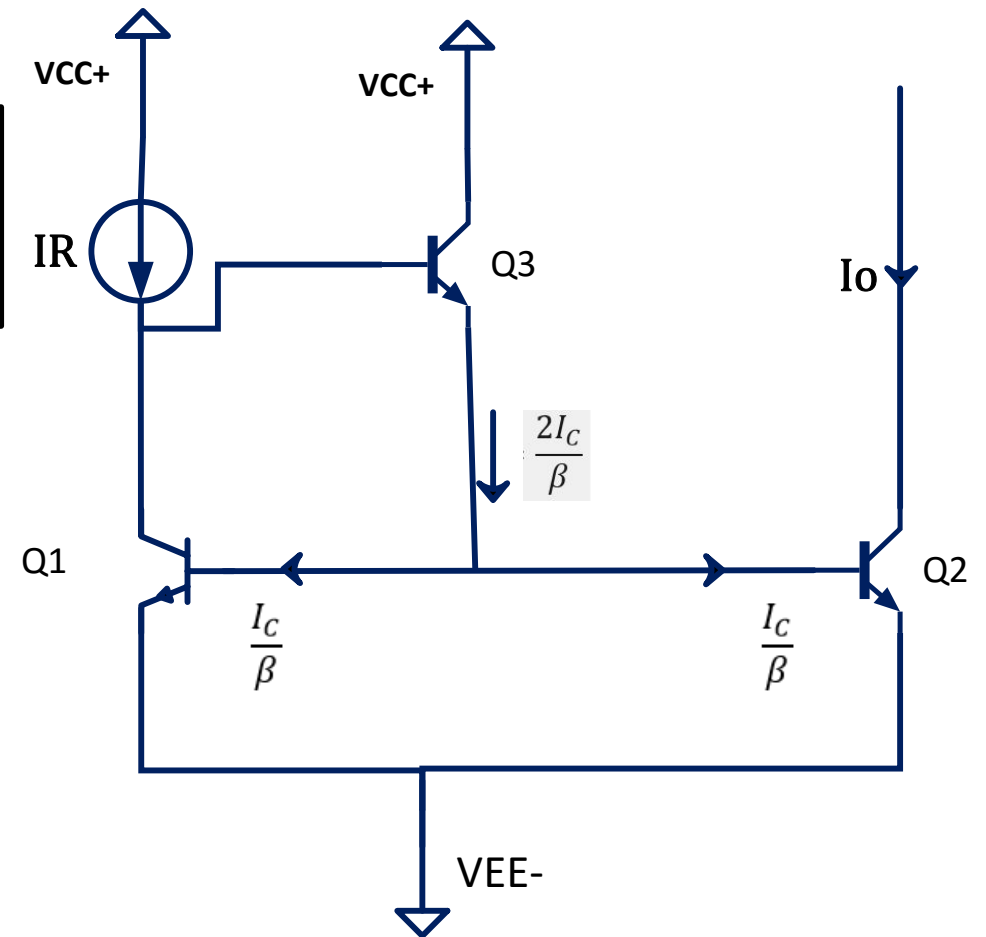
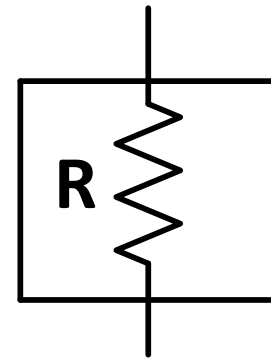
# Bipolar transistor current sources:

## 2. Bipolar mirror with base-current compensation:

Show that :

$$R_o = \frac{1}{h_{oe2}}$$

$$I_R = \frac{V_{CC} + V_{EE} - V_{BE3} - V_{BE1}}{R}$$



# Bipolar transistor current sources:

## 3. The Wilson Current:

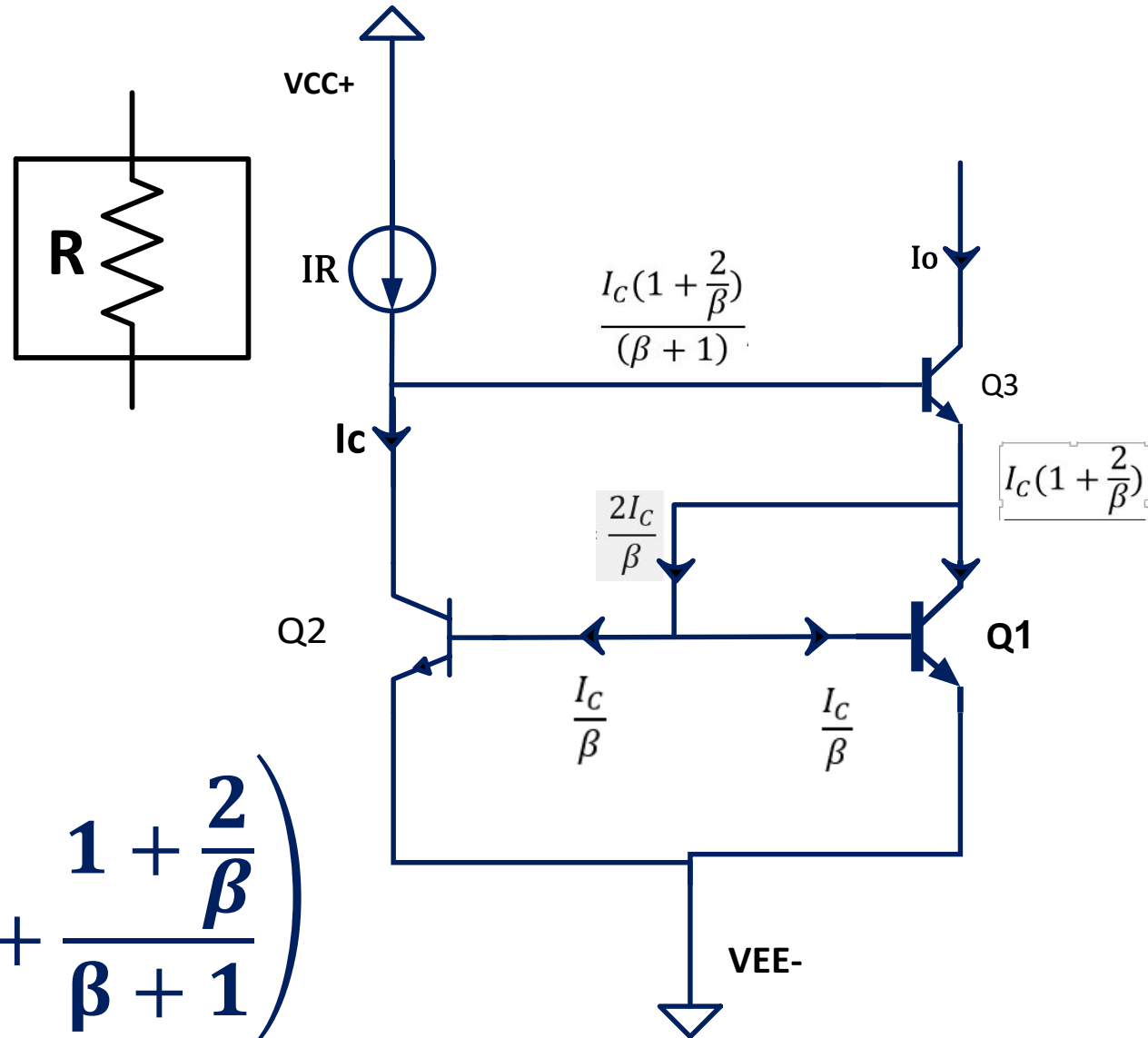
1) Reduce the B dependence

2) Increase  $R_o = \frac{h_{fe}}{2} \frac{1}{h_{oe3}}$

$Q_1, Q_2$  are matched

$$I_{C1} = I_{C2} = I_C$$

$$I_R = I_C + \frac{I_C \left(1 + \frac{2}{\beta}\right)}{\beta + 1} = I_C \left(1 + \frac{1 + \frac{2}{\beta}}{\beta + 1}\right)$$



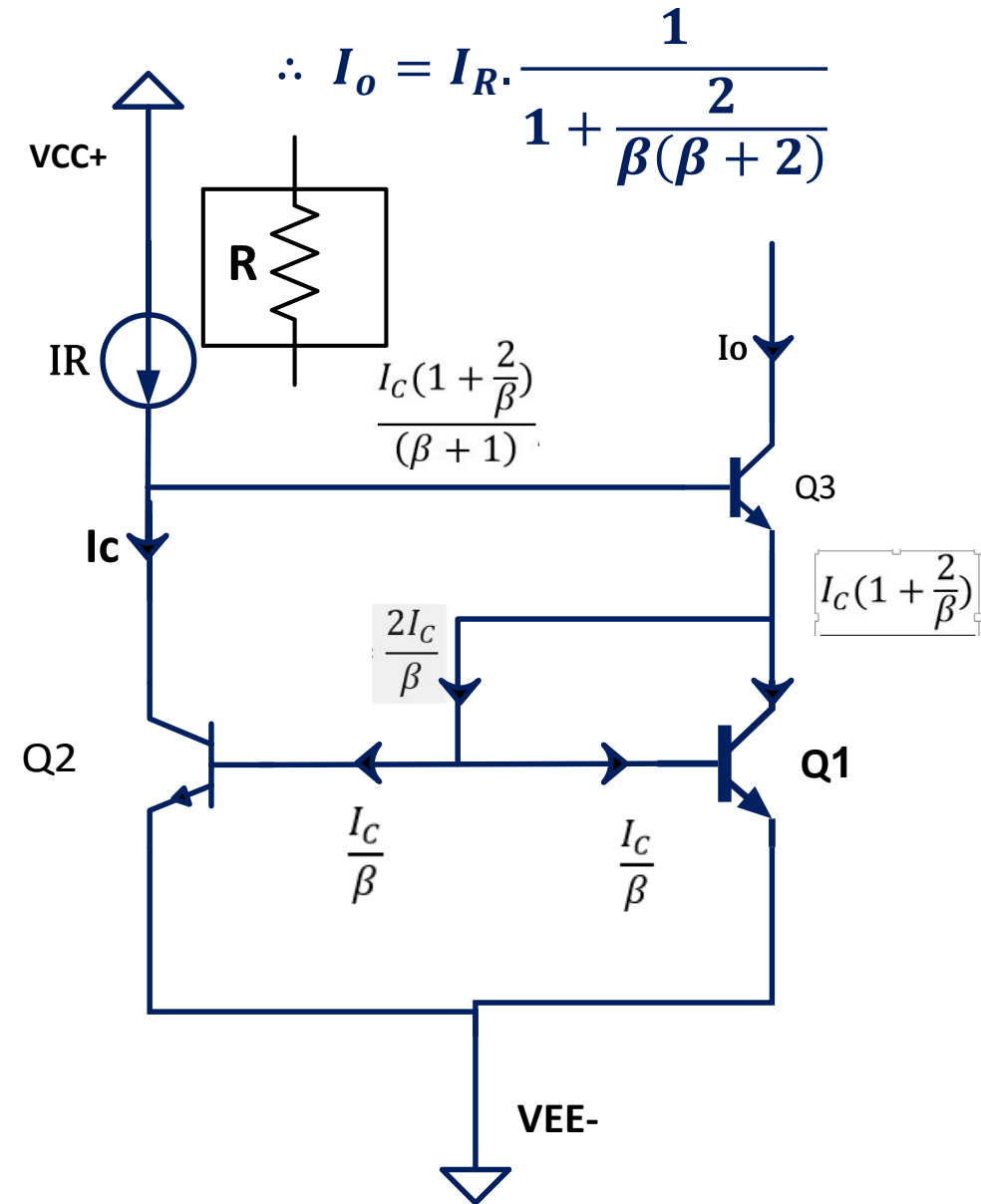
# Bipolar transistor current sources:

## 3. The Wilson Current:

$$I_o = I_{C3} = \frac{\beta}{\beta + 1} I_{E3} = \frac{\beta \left(1 + \frac{2}{\beta}\right)}{\beta + 1} I_C$$

$$\frac{I_o}{I_R} = \frac{I_C \left(1 + \frac{2}{\beta}\right) \left(\frac{\beta}{\beta + 1}\right)}{I_C \left(1 + \frac{1 + \frac{2}{\beta}}{\beta + 1}\right)}$$

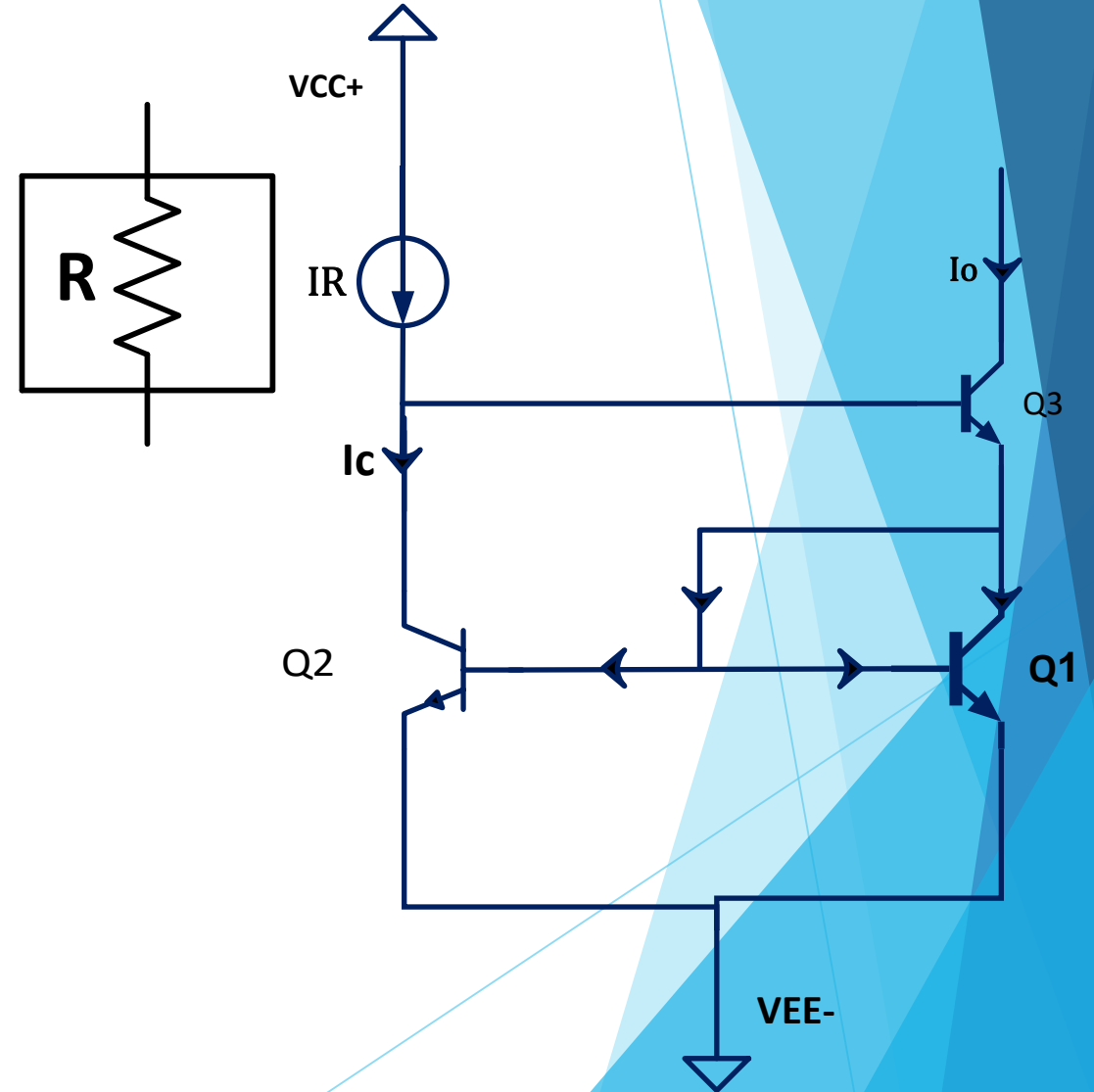
$$I_o = I_R \frac{1}{1 + \frac{2}{\beta(\beta + 2)}}$$



# Differential Amplifiers

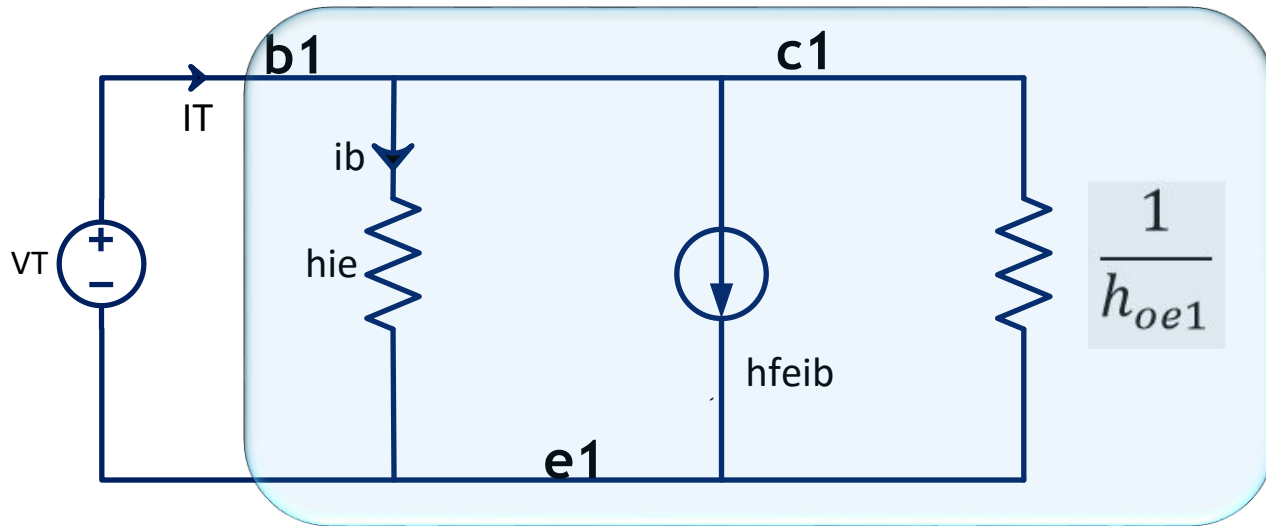
## The Wilson current mirror

To prove that  $R_o \approx \frac{h_{fe}}{2} \frac{1}{h_{oe}}$



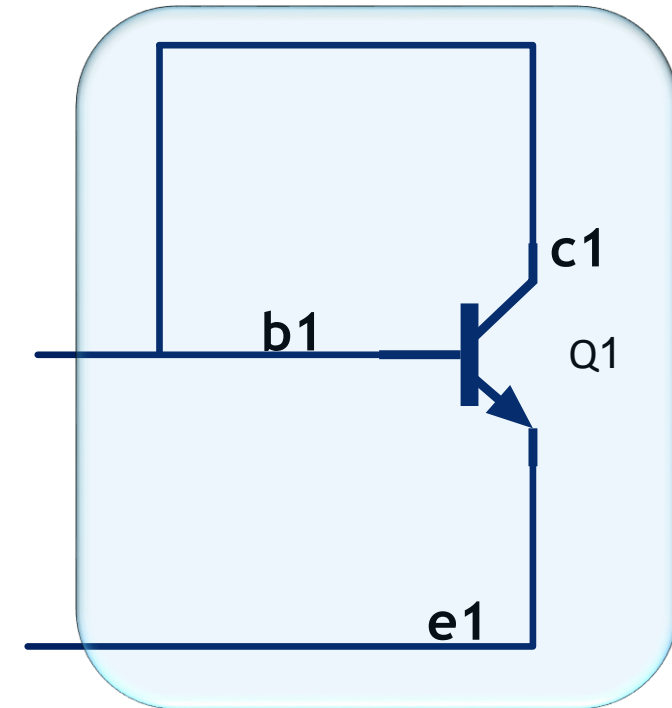
# Bipolar transistor current sources:

The Equivalent circuit for a diode connected transistor



$$I_T = h_{fe1} i_{b1} + \frac{V_T}{h_{ie1} \parallel \frac{1}{h_{oe1}}}$$

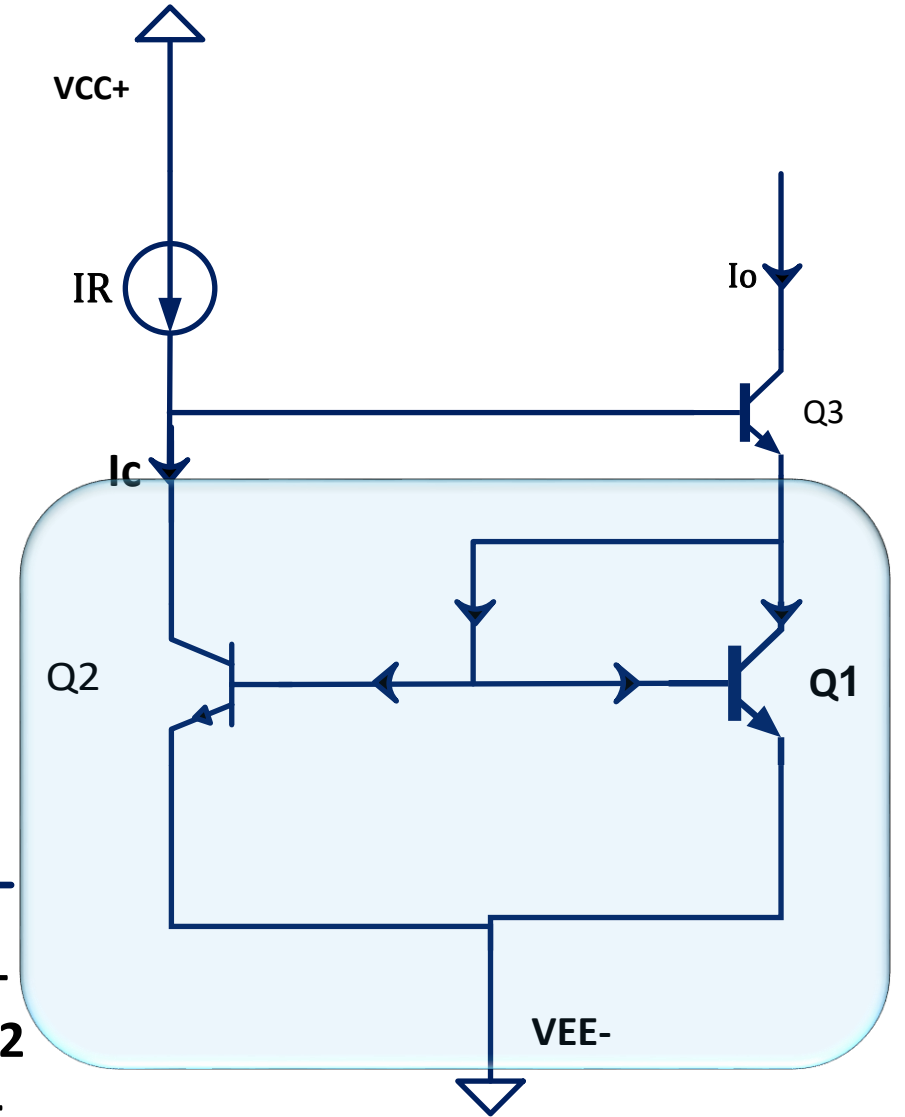
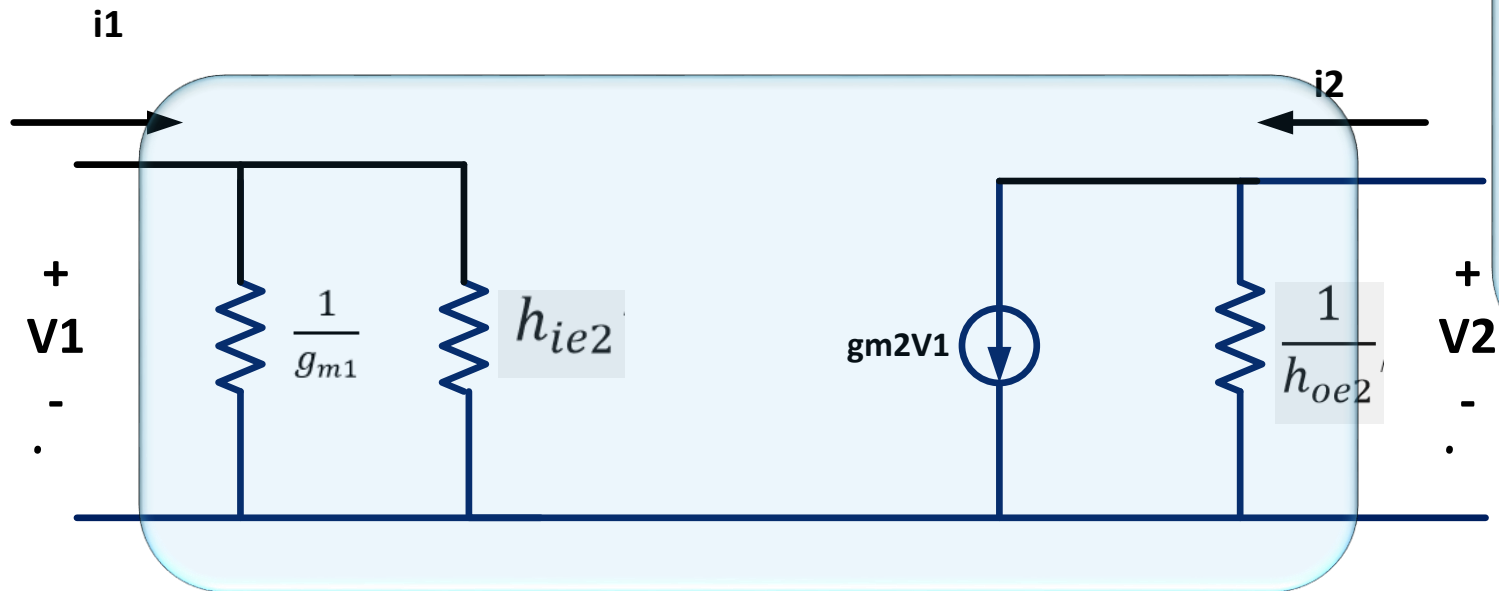
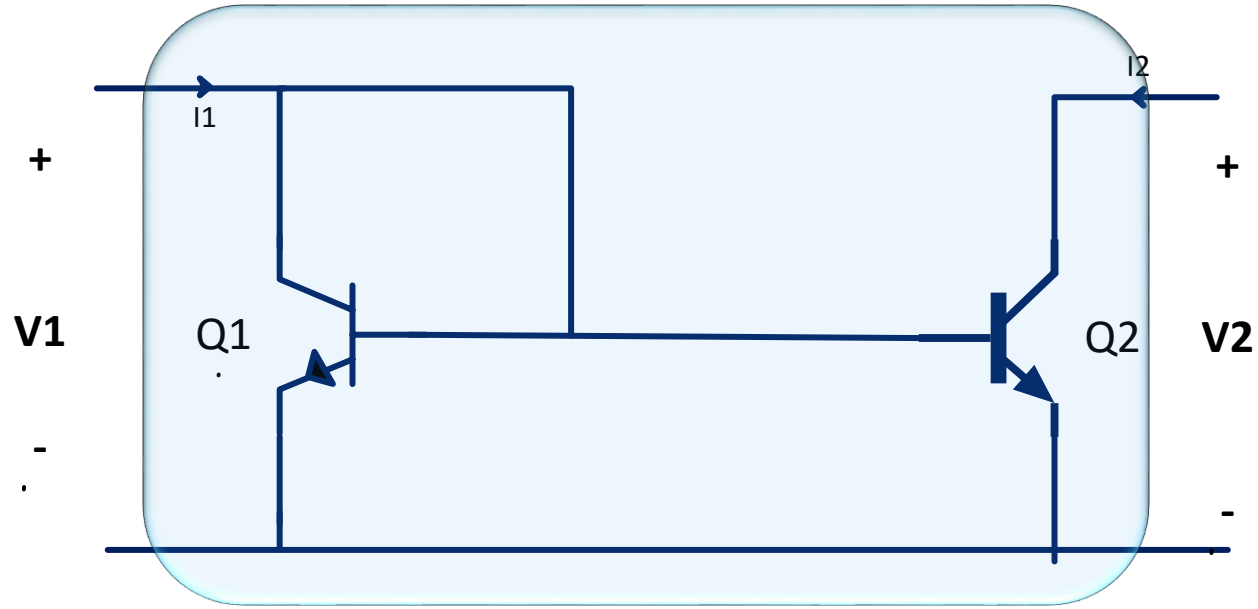
$$i_{b1} = \frac{V_T}{h_{ie1}}$$



$$\therefore \frac{V_T}{I_T} = \frac{1}{g_{m1}} \parallel h_{ie1} \parallel \frac{1}{h_{oe1}}$$

$$R_{TH} \approx \frac{1}{g_{m1}} \approx h_{ib1}$$

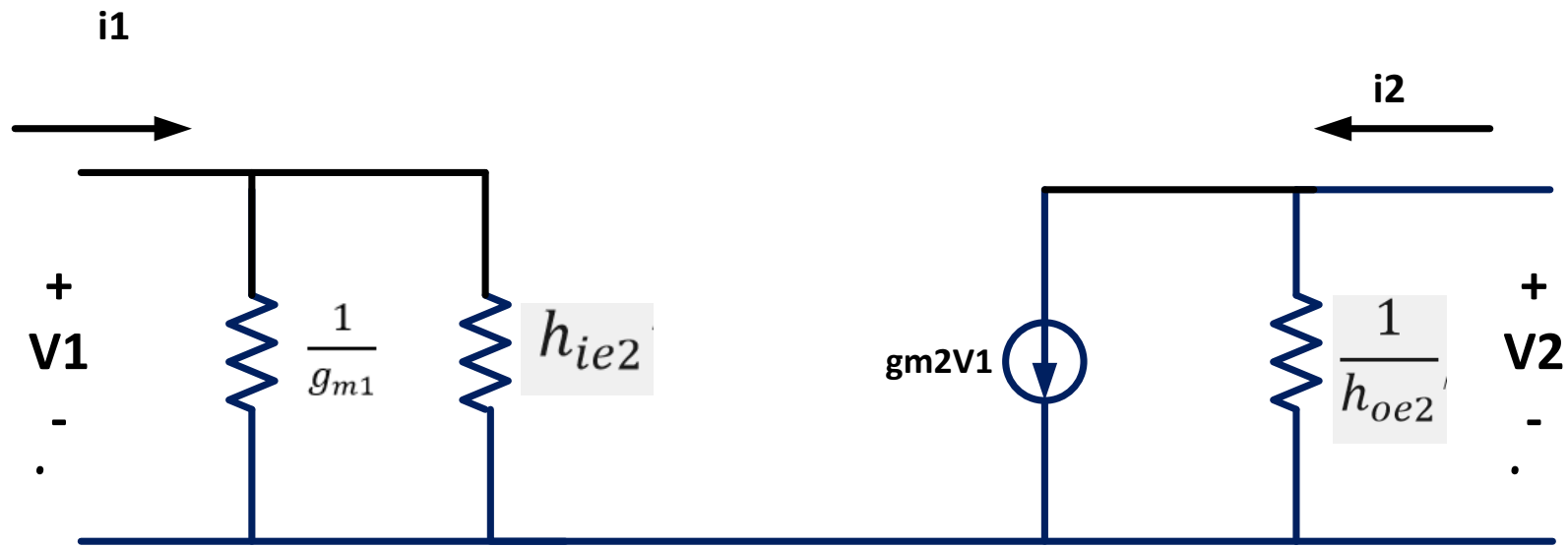
# Two Port Model for the current mirror



Two Port Model for the current mirror

$$V_1 = h_{11} i_1 + h_{12} V_2$$

$$i_2 = h_{21} i_1 + h_{22} V_2$$



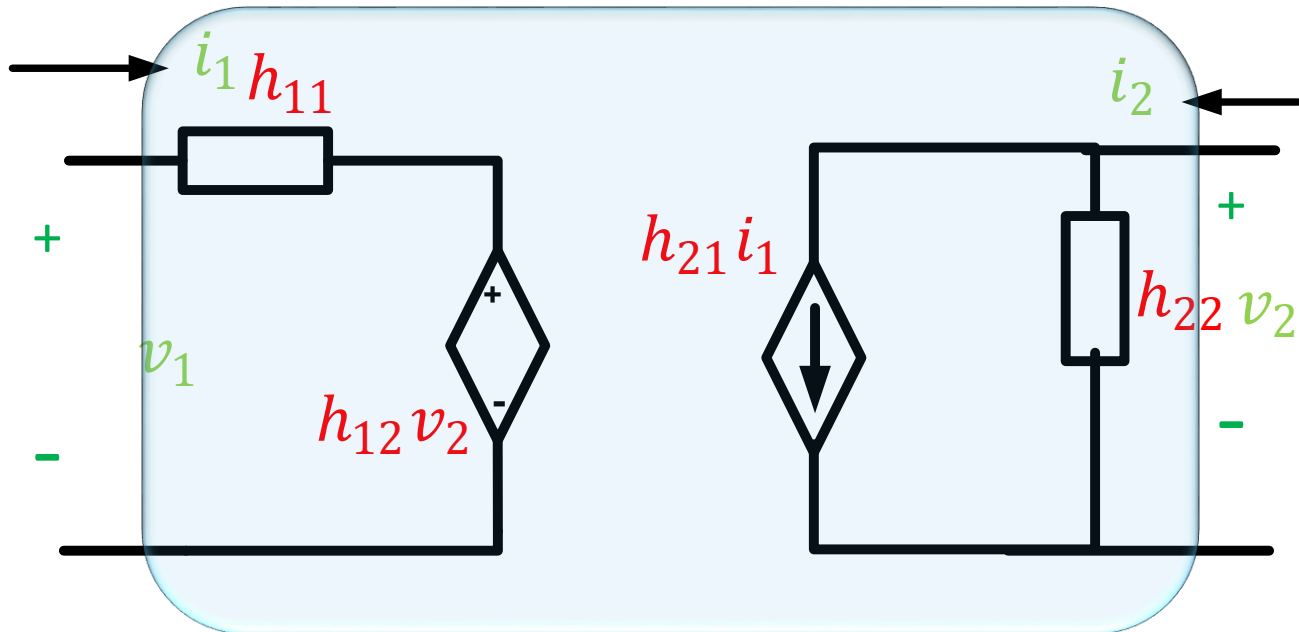
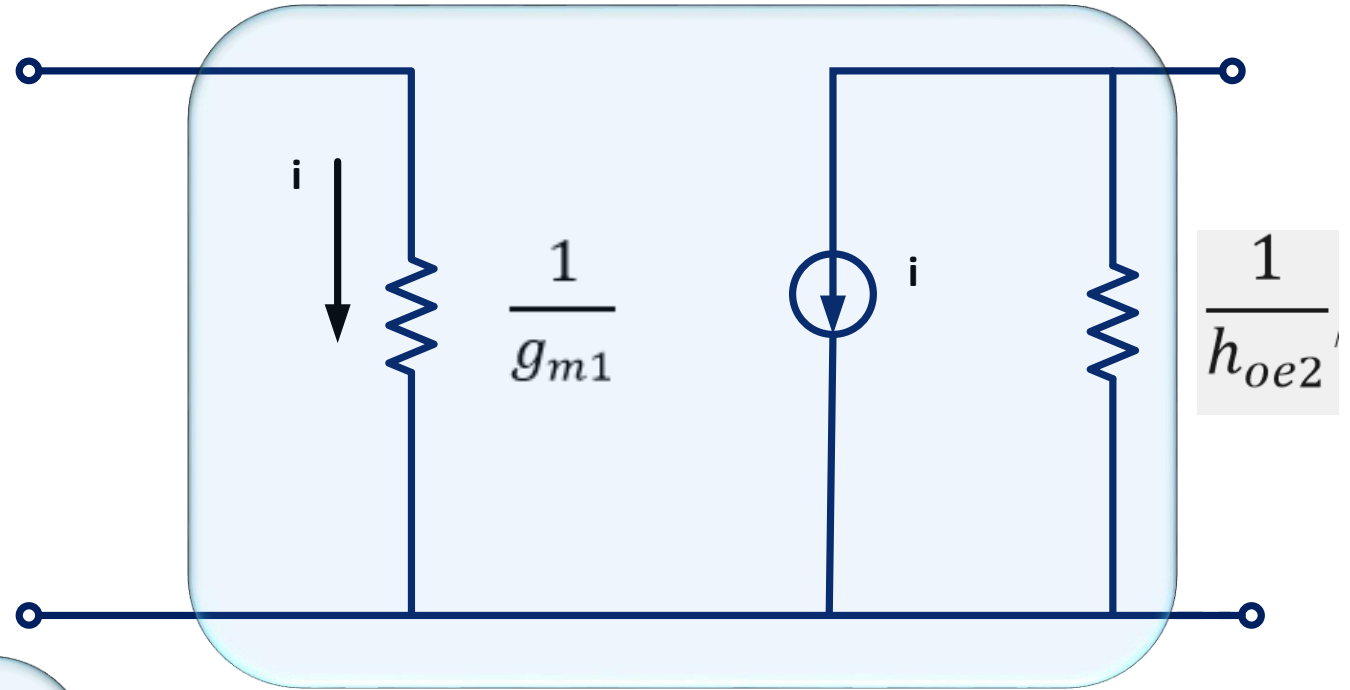
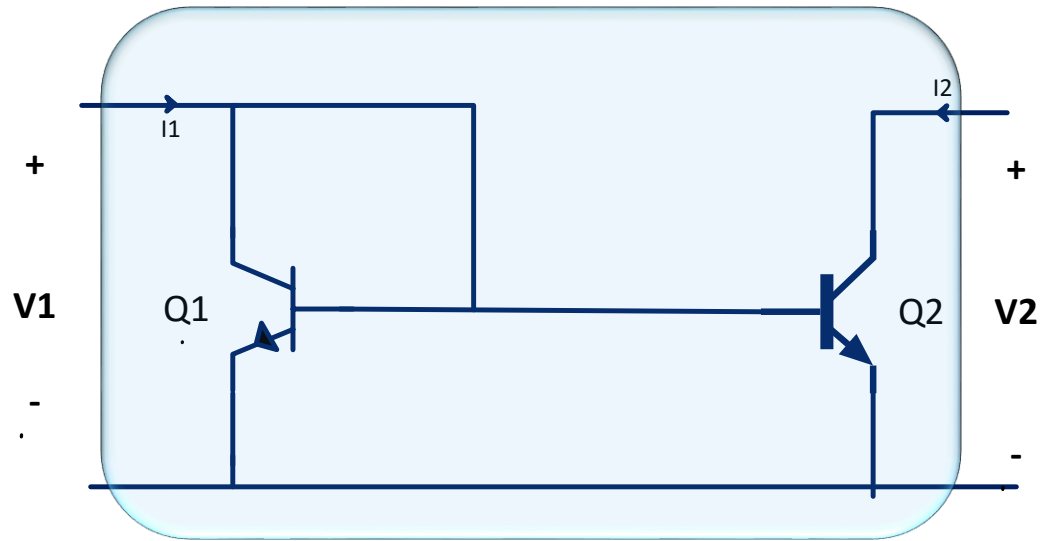
$$h_{11} = \frac{v_1}{i_1} \Big|_{v_2=0} = \frac{1}{g_{m1}} \parallel h_{ie2} \approx \frac{1}{g_{m1}}$$

$$h_{12} = \frac{v_1}{v_2} \Big|_{i_1=0} = 0$$

$$h_{22} = \frac{i_2}{v_2} \Big|_{i_1=0} = h_{oe2}$$

$$h_{21} = \frac{i_2}{i_1} \Big|_{v_2=0} = \frac{g_{m2} h_{ie2}}{1 + g_{m1} h_{ie1}} \approx \frac{g_{m2}}{g_{m1}} = 1$$

# Two Port Model for the current mirror



$$h_{11} = \frac{1}{g_{m1}}$$

$$h_{12} = 0$$

$$h_{21} = 1$$

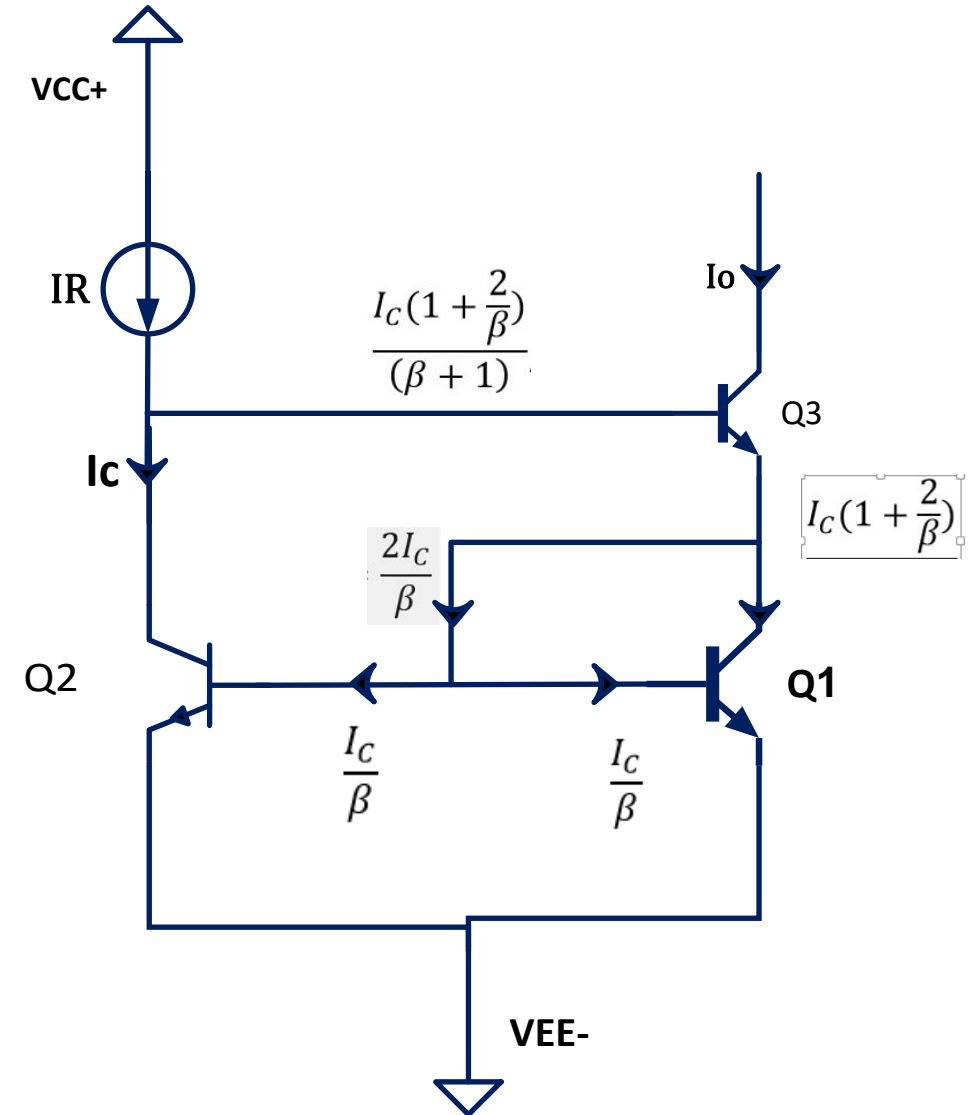
$$h_{22} = h_{oe2}$$



# Differential Amplifiers

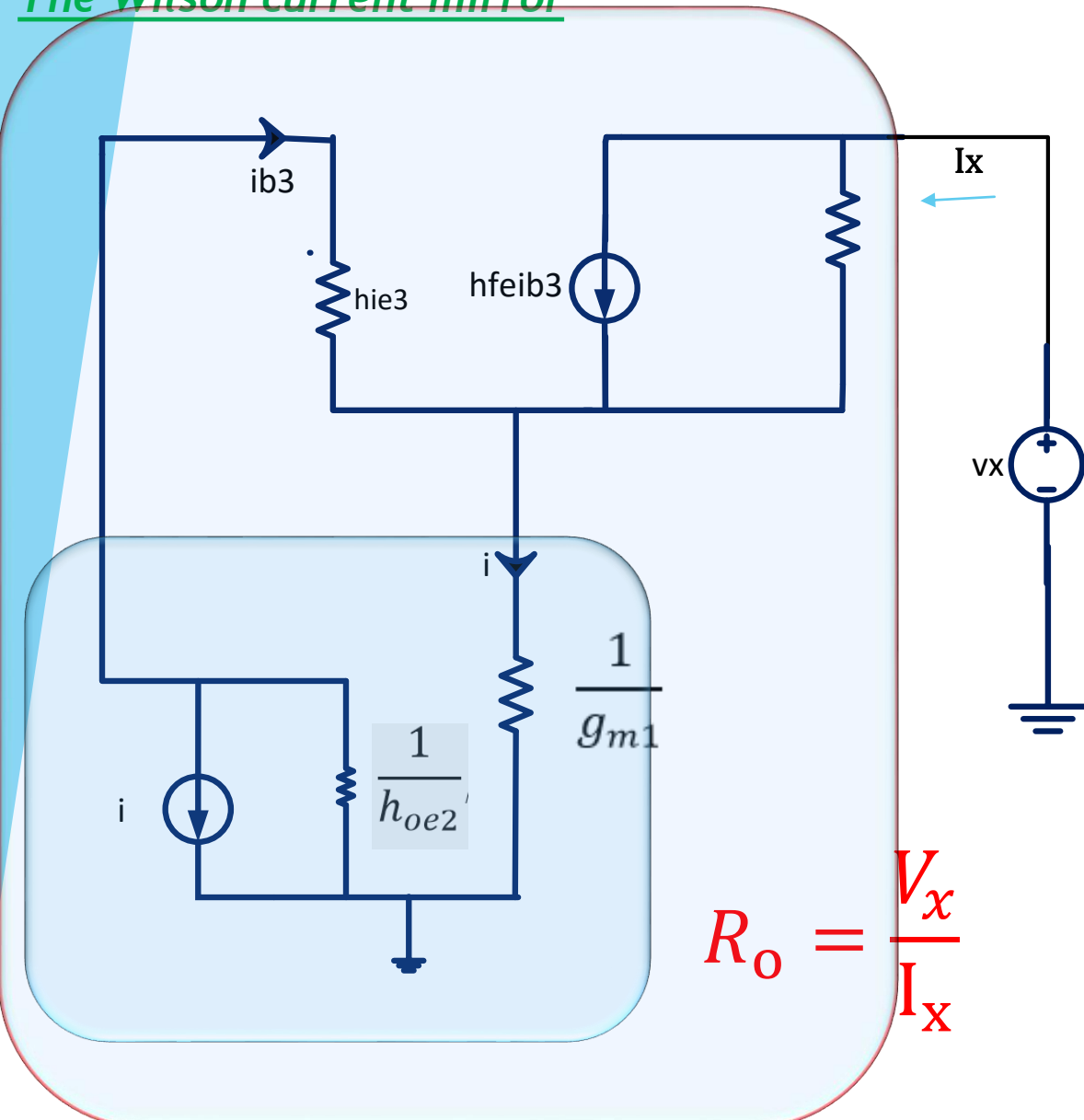
## The Wilson Current:

*To proof*  $R_o \cong \frac{h_{fe}}{2} \cdot \frac{1}{h_{oe3}}$

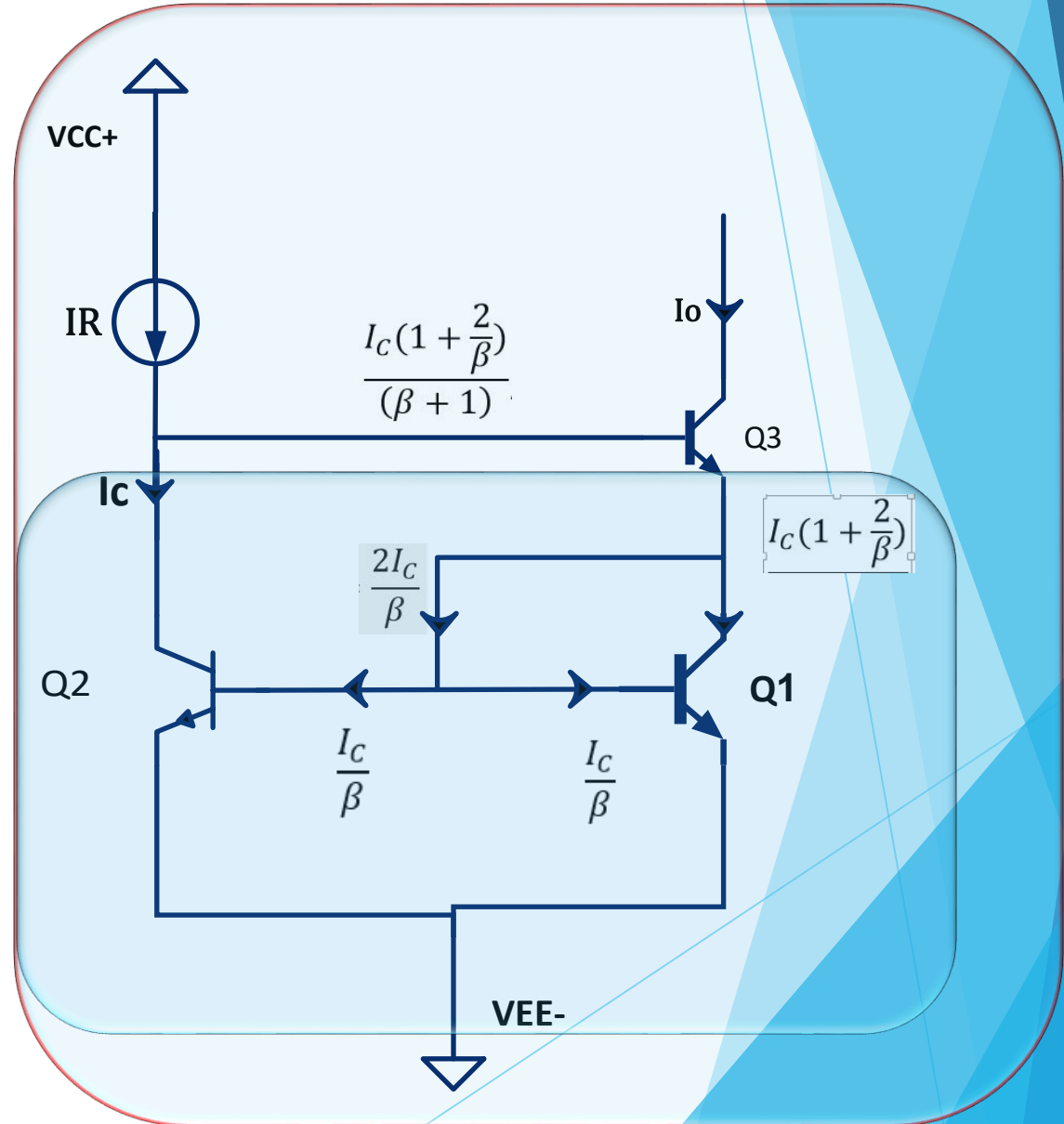


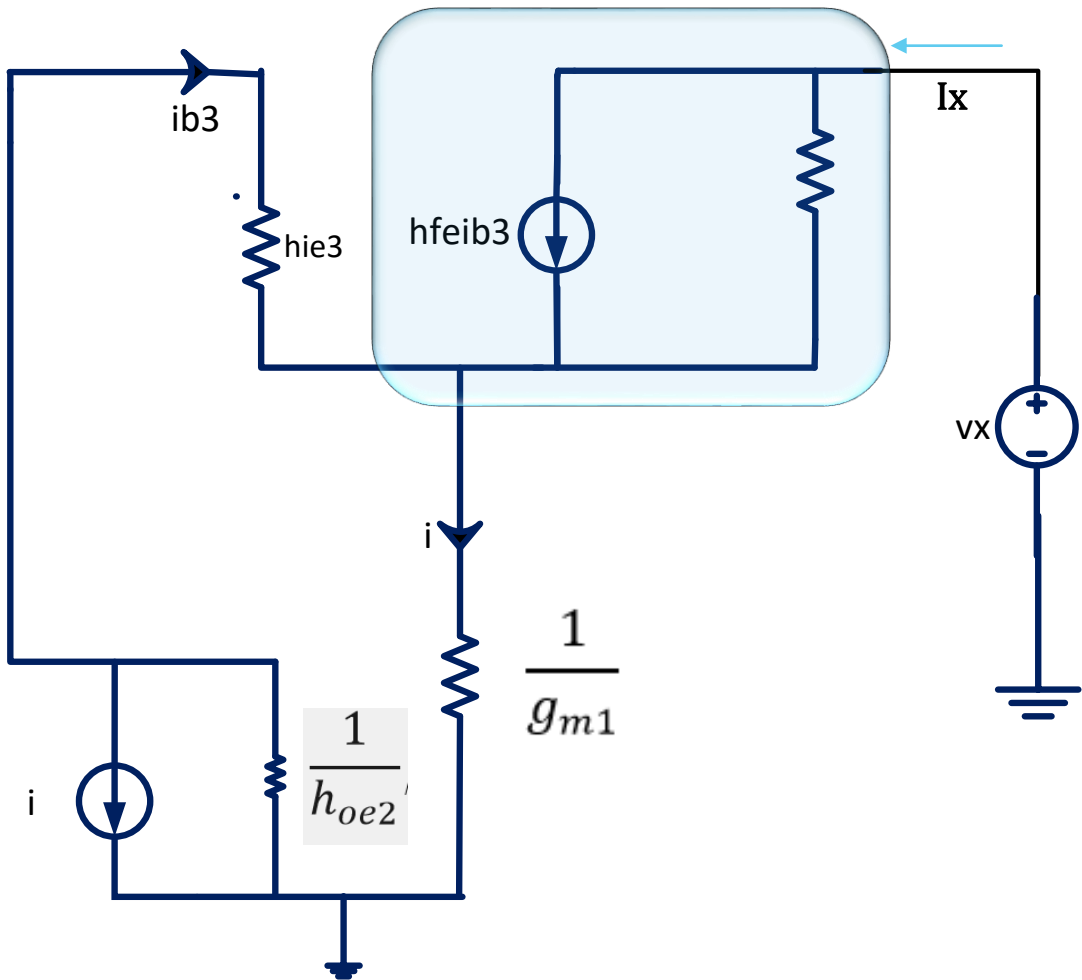
# Differential Amplifiers

## The Wilson current mirror

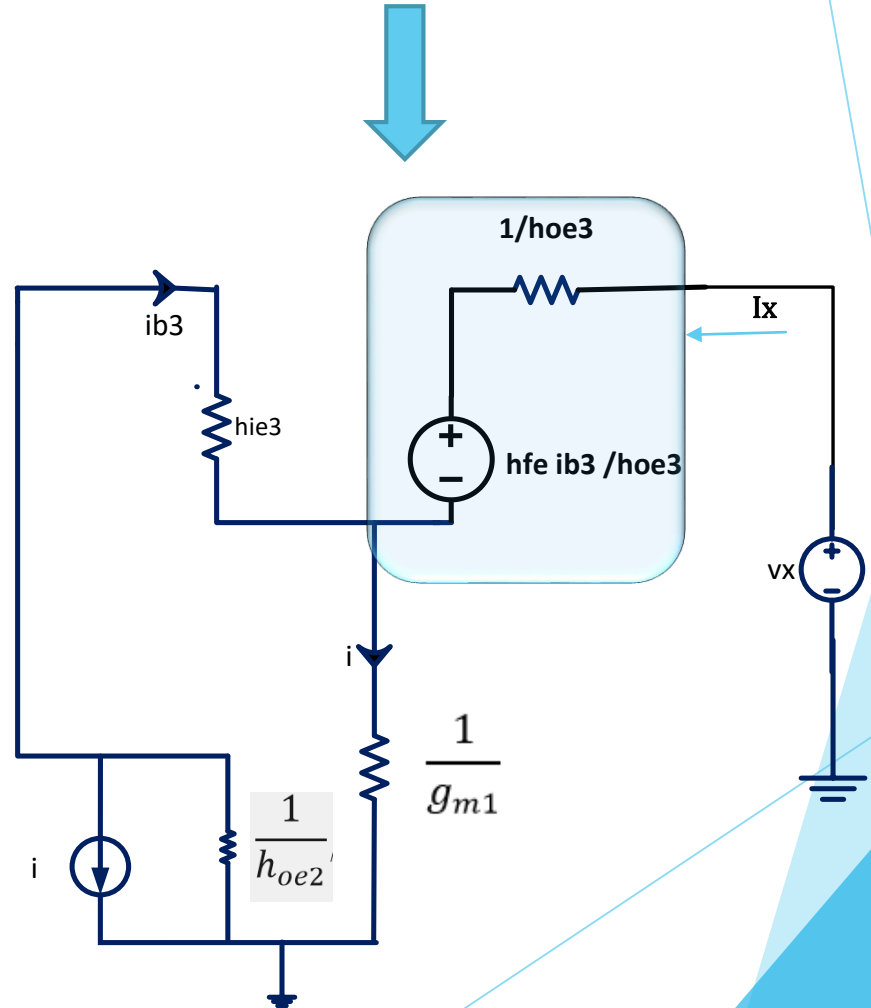


$$R_o = \frac{V_x}{I_x}$$





## Source transformation



# The Wilson current mirror

$$V_X + \frac{h_{fe}}{h_{oe3}} ib_3 = \left( \frac{1}{h_{oe3}} + \frac{1}{g_{m1}} \right) I_X + \frac{1}{g_{m1}} ib_3$$

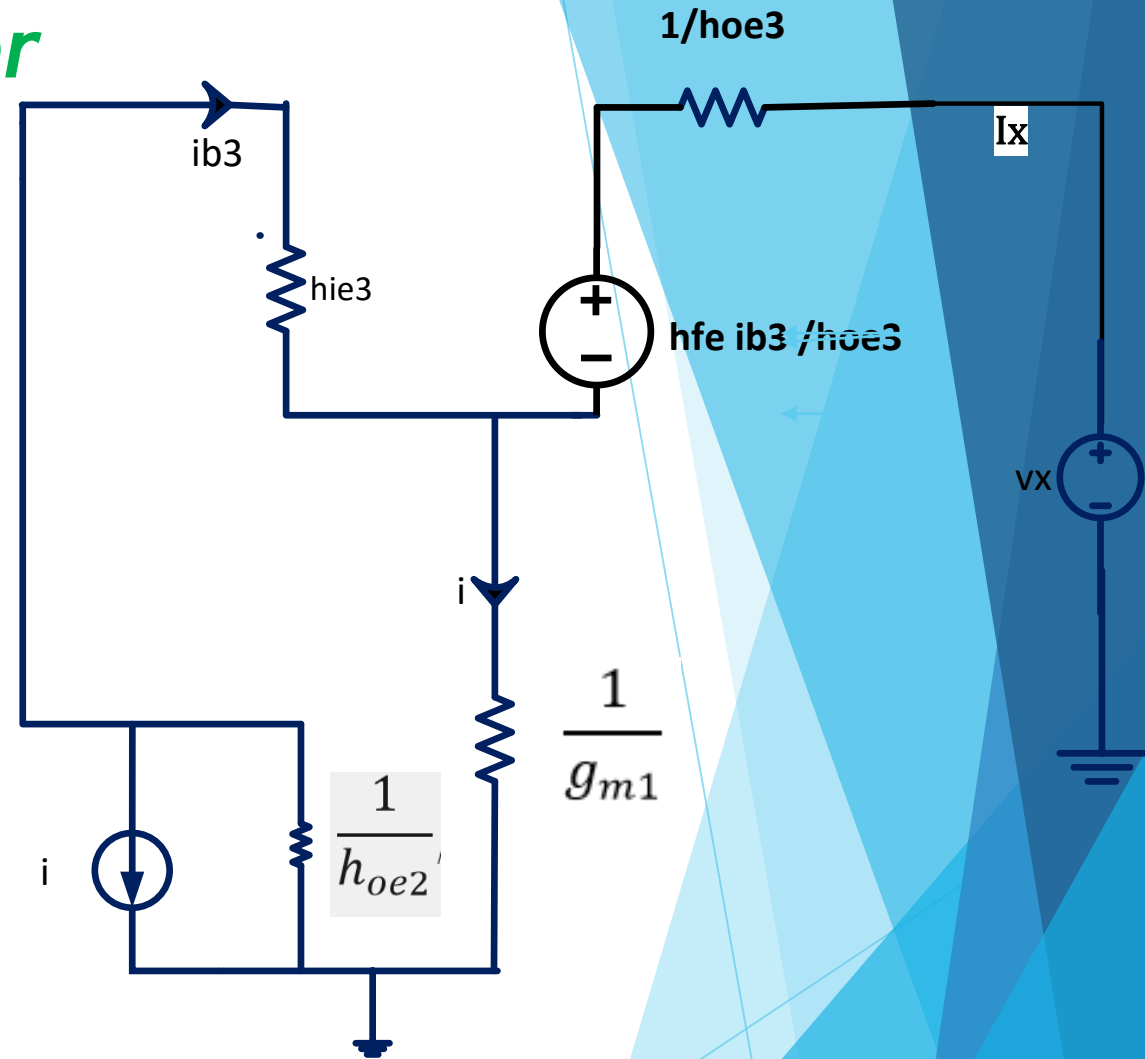
$$V_X = \left( \frac{1}{h_{oe3}} + \frac{1}{g_{m1}} \right) I_X + \left( \frac{1}{g_{m1}} - \frac{h_{fe}}{h_{oe3}} \right) ib_3$$

since  $\frac{1}{h_{oe3}} \gg \frac{1}{g_{m1}}$  and  $\frac{h_{fe}}{h_{oe3}} \gg \frac{1}{g_{m1}}$

$$V_X = \frac{1}{h_{oe3}} I_X - \frac{h_{fe}}{h_{oe3}} ib_3 \rightarrow (1)$$

$$-\frac{i}{h_{oe2}} = \frac{1}{g_{m1}} I_X + \left( h_{ie3} + \frac{1}{g_{m1}} + \frac{1}{h_{oe2}} \right) ib_3 \rightarrow (2)$$

$$i = ib_3 + I_X \rightarrow (3)$$

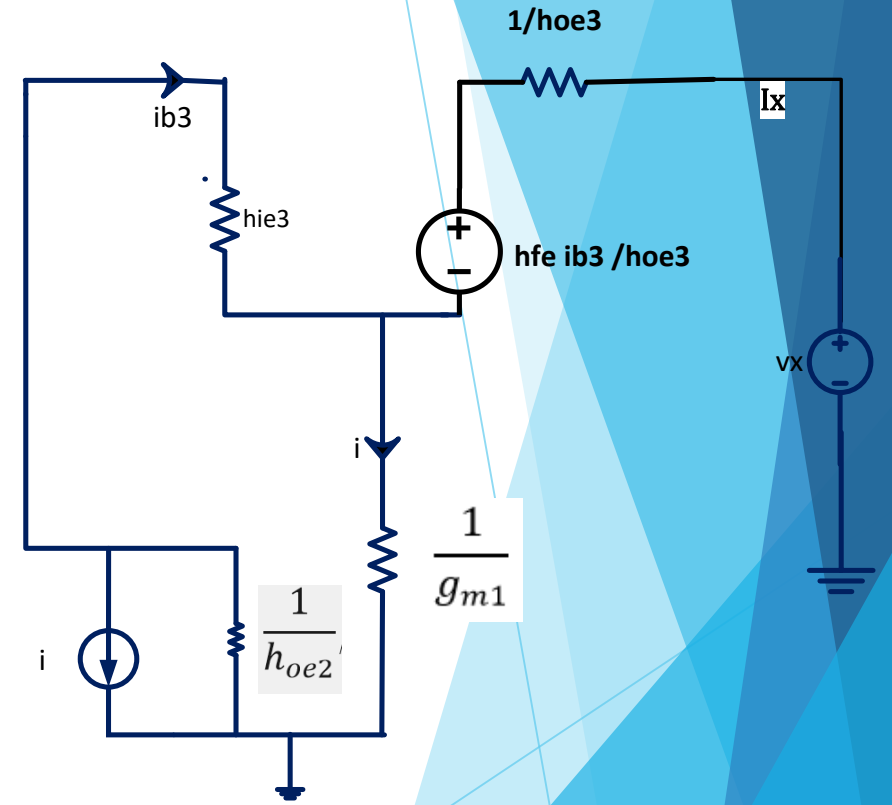


sub (3) into (2) ▶

$$0 = \left( \frac{1}{h_{oe2}} + \frac{1}{g_{m1}} \right) I_X + \left( \frac{1}{g_{m1}} + h_{ie3} + \frac{2}{h_{oe2}} \right) ib_3 \quad \blacktriangleright$$

since  $\frac{1}{h_{oe2}} \gg \frac{1}{g_{m1}}$  and  $\frac{2}{h_{oe2}} \gg \frac{1}{g_{m1}} + h_{ie3}$  ▶

$$0 = \frac{1}{h_{oe2}} I_X + \frac{2}{h_{oe2}} ib_3 \rightarrow (4) \quad \blacktriangleright$$



# The Wilson current mirror

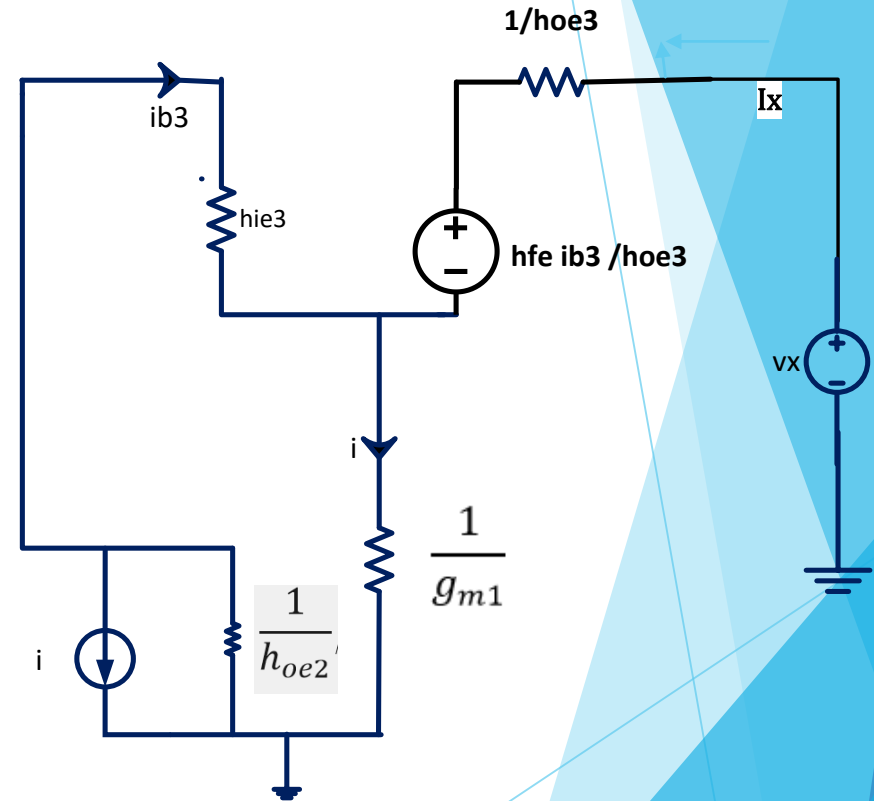
$$V_X = \frac{1}{h_{oe3}} I_X - \frac{h_{fe}}{h_{oe3}} ib_3 \rightarrow (1)$$

using (4) we get  $ib_3 = -\frac{I_X}{2}$

$$\therefore V_X = \frac{1}{h_{oe3}} I_X + \frac{h_{fe}}{2} \frac{1}{h_{oe3}} I_X$$

$$\therefore \frac{V_X}{I_X} = \left( \frac{h_{fe}}{2} + 1 \right) \frac{1}{h_{oe3}}$$

$$\therefore R_o = \frac{V_X}{I_X} \approx \frac{h_{fe}}{2} \frac{1}{h_{oe3}}$$



## 4. Widlar Current Source:

- To Produce Very small  $I_o$
- To increase  $R_o$

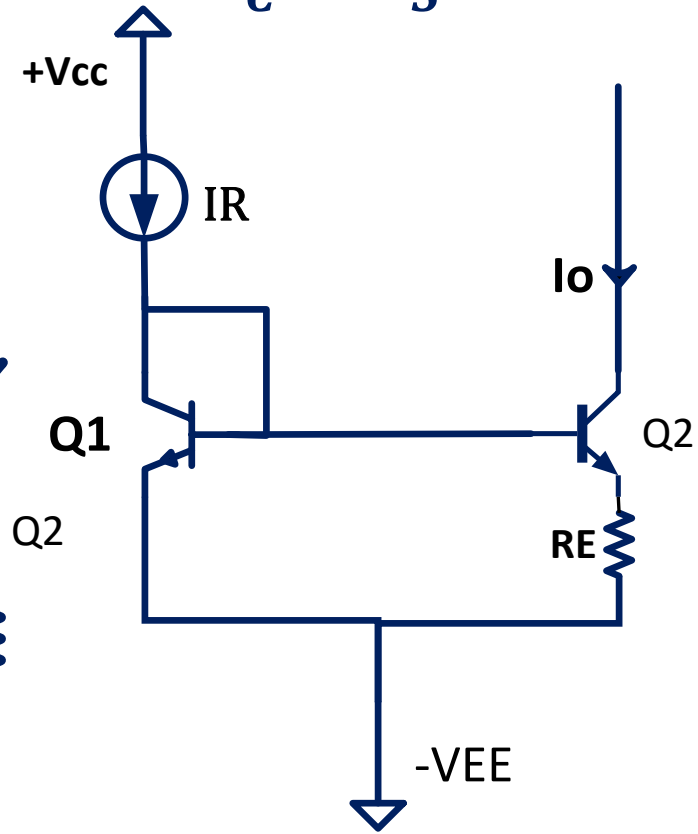
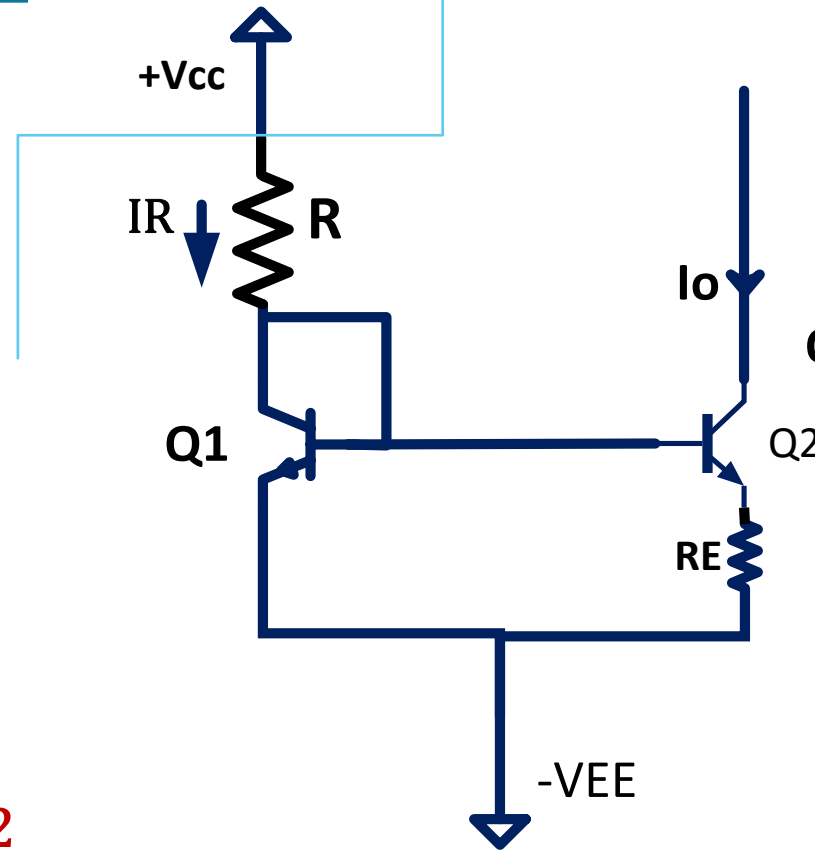
$$KVL : V_{BE1} = V_{BE2} + R_E I_{E2}$$

$$V_T \ln \frac{I_{C1}}{I_{S1}} = V_T \ln \frac{I_{C2}}{I_{S2}} + R_E I_{E2}$$

$$V_T \ln \frac{I_{C1}}{I_{S1}} - V_T \ln \frac{I_{C2}}{I_{S2}} = R_E I_{E2}$$

if  $Q_1$  and  $Q_2$  are matched

$$I_{C1} > I_{C2} = I_o \quad I_C = I_S e^{\frac{V_{BE}}{V_T}}$$



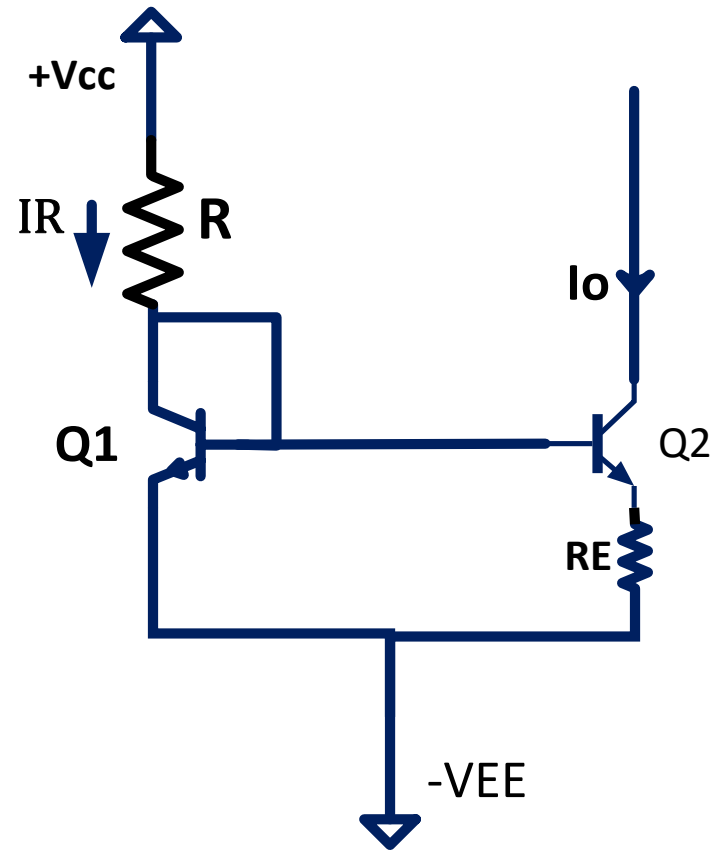
$$V_T \ln \frac{I_{C1}}{I_{C2}} = R_E I_{E2}$$

$$V_T \ln \frac{I_{C1}}{I_{C2}} = R_E I_{E2}$$

*if*  $\beta = \infty$

$$V_T \ln \frac{I_R}{I_O} = R_E I_O$$

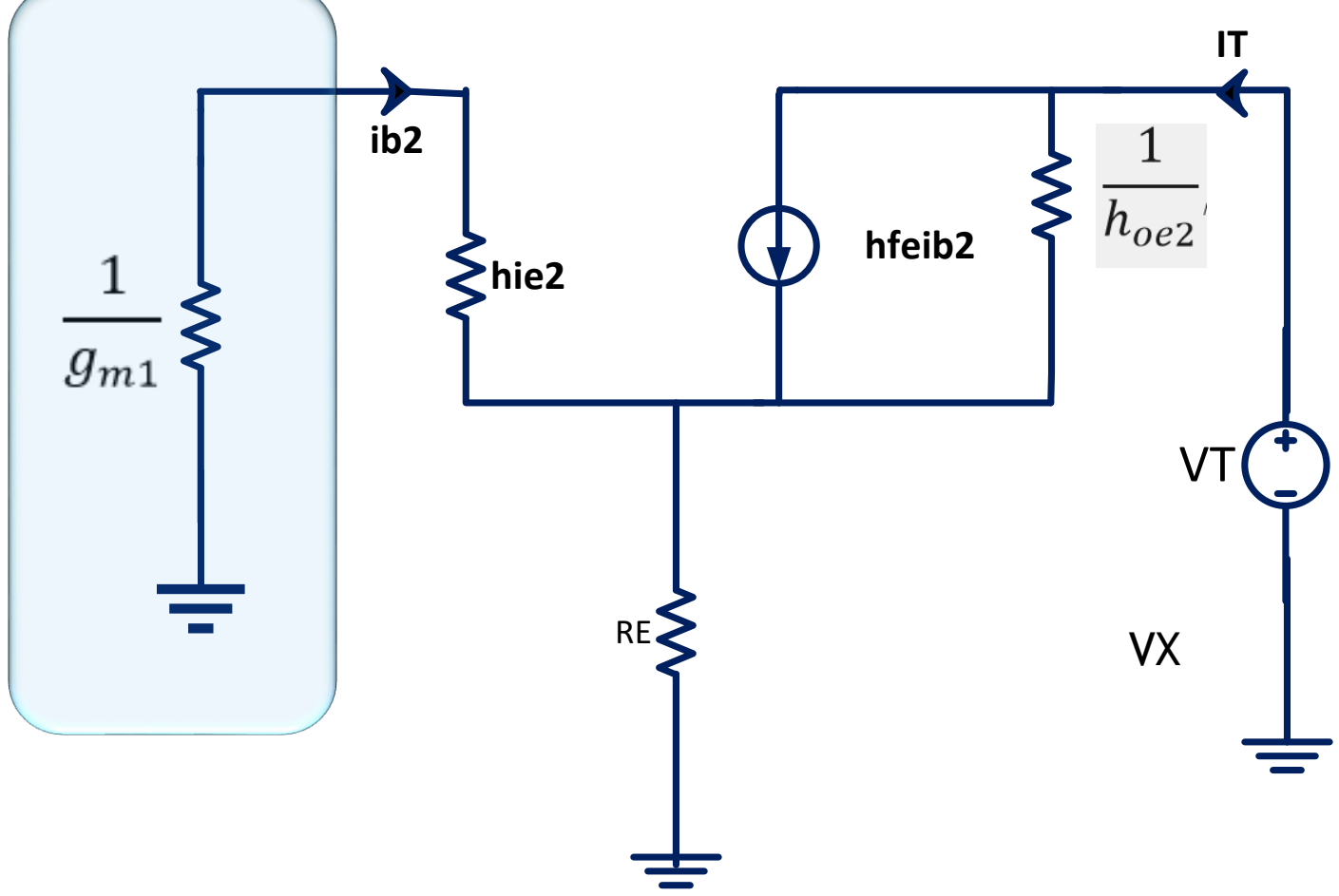
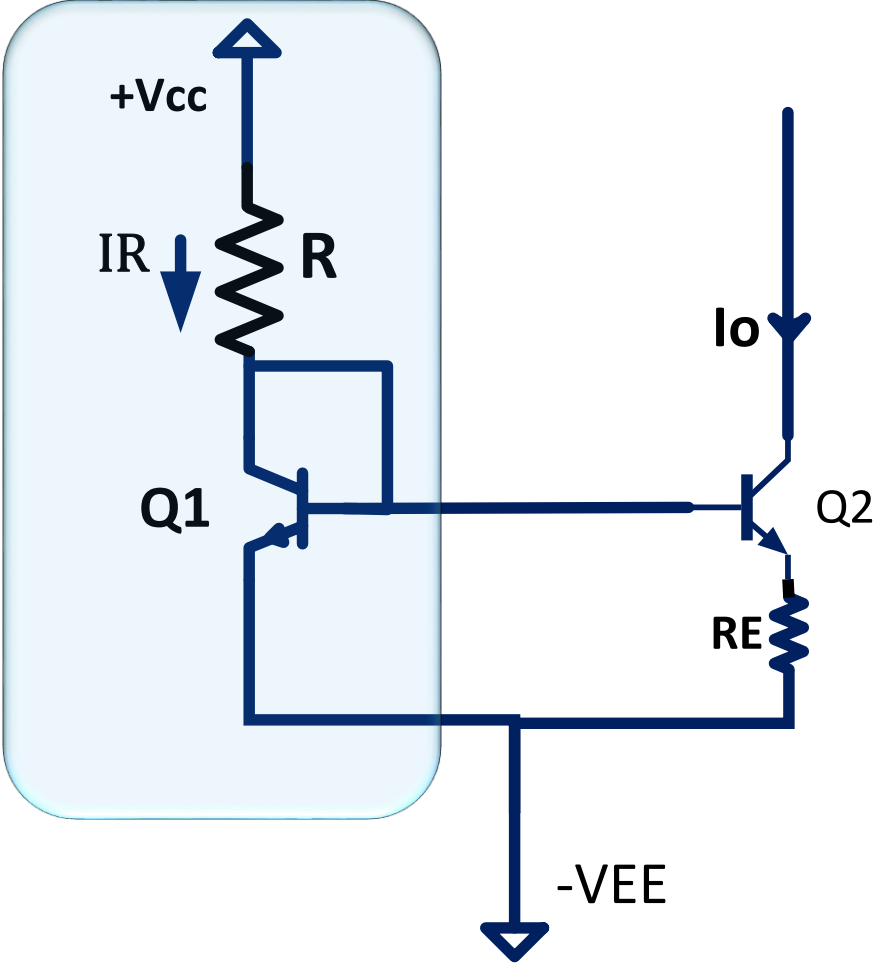
$$I_R = \frac{V_{CC} + V_{EE} - V_{BE1}}{R}$$





# 4. Widlar Current Source:

- To Find  $R_o$ :



$$R_o \approx \frac{1}{h_{oe2}} + \frac{h_{fe}}{h_{oe2}} \frac{R_E}{R_E + h_{ie2} + \frac{1}{g_{m1}}}$$

Design a simple current mirror to generate  $I_o = 10\mu\text{A}$   
given that  $V_{BE} = 0.7\text{ V}$  @  $1\text{mA}$

Assume  $\beta = \infty$ ,  $V_{CC} = 10\text{V}$  and  $V_{EE} = 0\text{V}$

We need to find the value of  $V_{BE}$  @  $10\mu\text{A}$

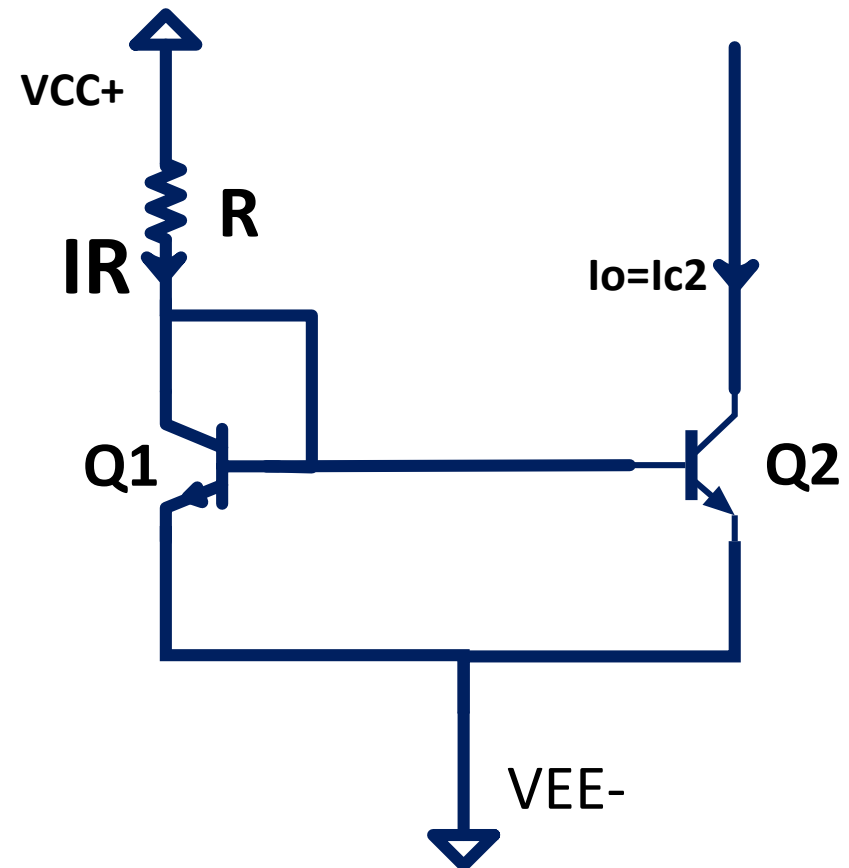
$$V_{BE} = V_T \ln \frac{I_C}{I_S}$$

$$V_{BE2} - V_{BE1} = V_T \ln \frac{I_{C2}}{I_{C1}}$$

$$V_{BE2} = V_{BE1} + V_T \ln \frac{I_{C2}}{I_{C1}}$$

$$= 0.7 + V_T \ln \frac{10\mu\text{A}}{1\text{mA}}$$

$$= 0.58\text{ V}$$



# Differential Amplifiers

Design a simple current mirror to generate  $I_o = 10\mu A$   
given that  $V_{BE} = 0.7 V @ 1mA$

Assume  $\beta = \infty$ ,  $V_{CC} = 10V$  and  $V_{EE} = 0V$

$$V_{BE} = 0.7 V @ I_C = 1mA$$

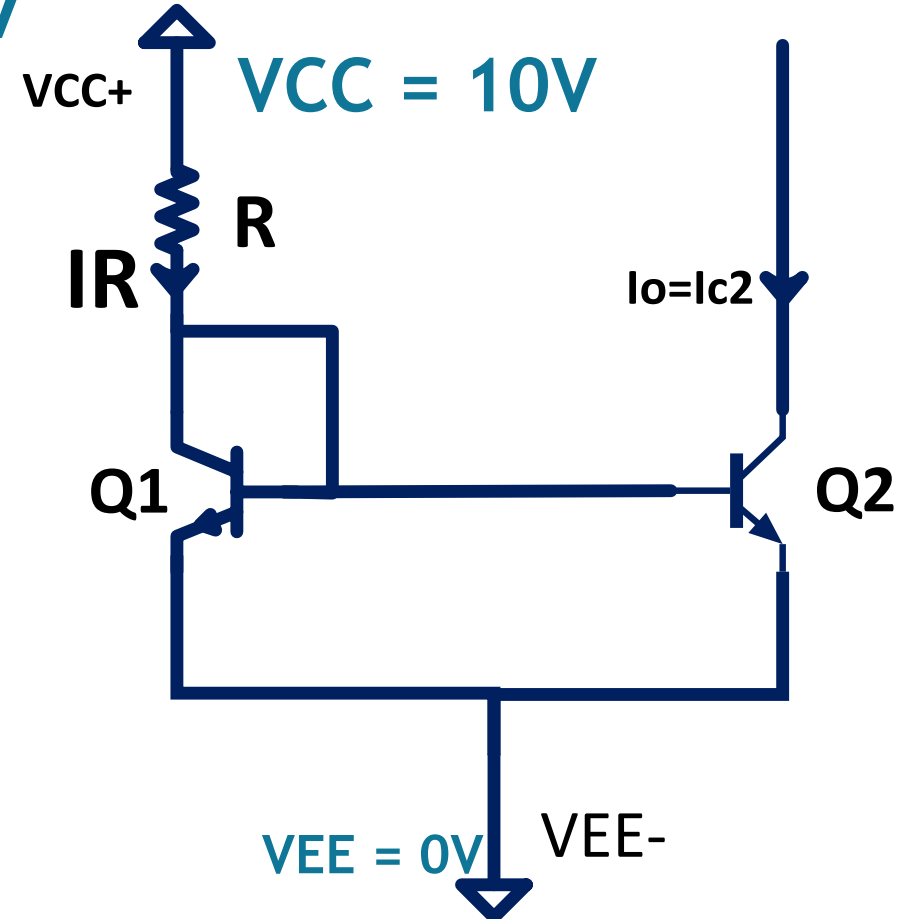
$$V_{BE} = 0.58 V @ I_C = 10\mu A$$

$$KVL : V_{CC} = R I_R + V_{BE}$$

$$\text{since } \beta = \infty$$

$$I_R = I_o = 10\mu A$$

$$\therefore R = 942K$$



Too Large , Not Practical

Design a Wedlar current source to generate  $I_o = 10\mu A$

given that  $V_{BE} = 0.7 V$  @  $I_c = 1mA$

Assume  $B = \infty$ ,  $V_{CC} = 10V$  and  $V_{EE} = 0V$

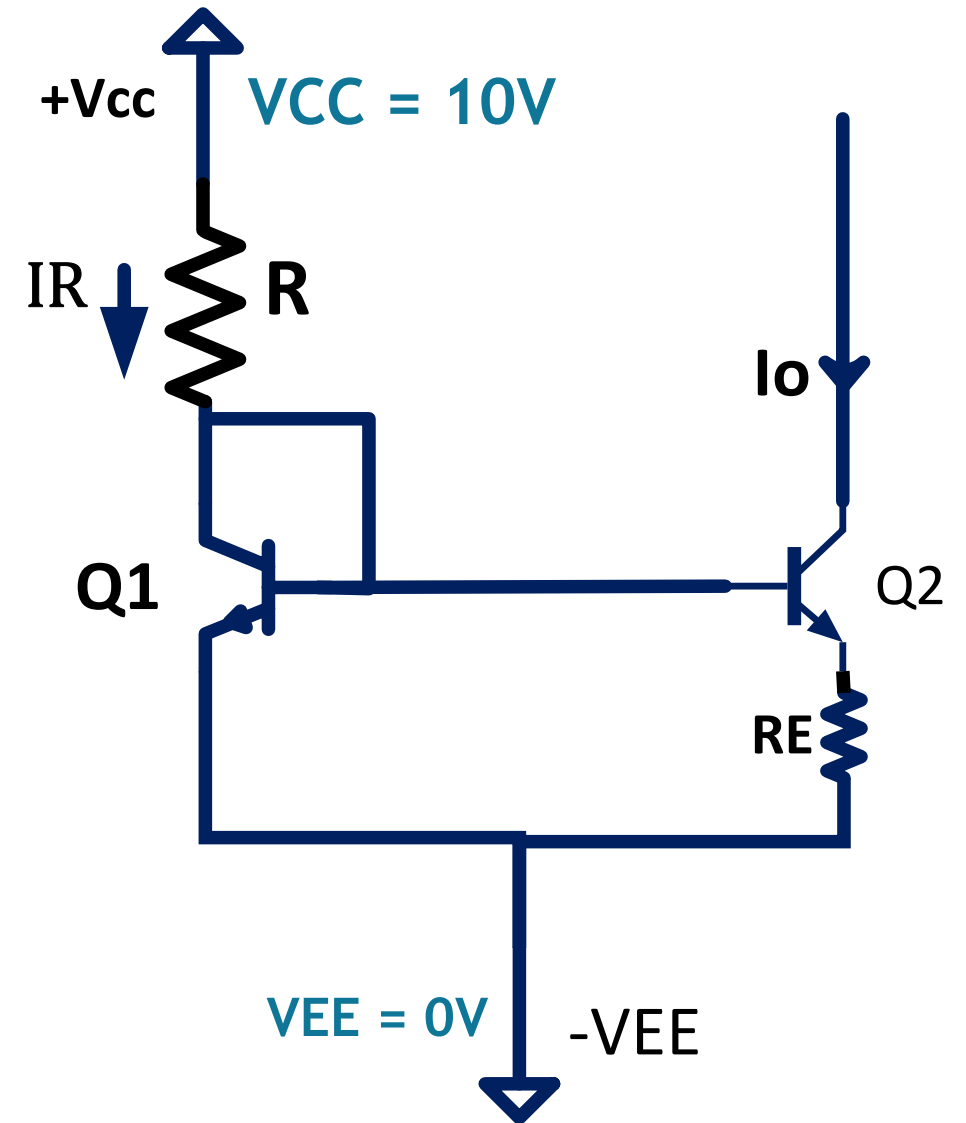
Assume that  $I_R = 1mA$

$$\therefore V_{BE1} = 0.7 V$$

$$R = \frac{V_{CC} - V_{BE1}}{I_R} = 9.3k$$

$$I_o R_E = V_T \ln \frac{I_R}{I_o}$$

$$\therefore R_E = 11.5k$$

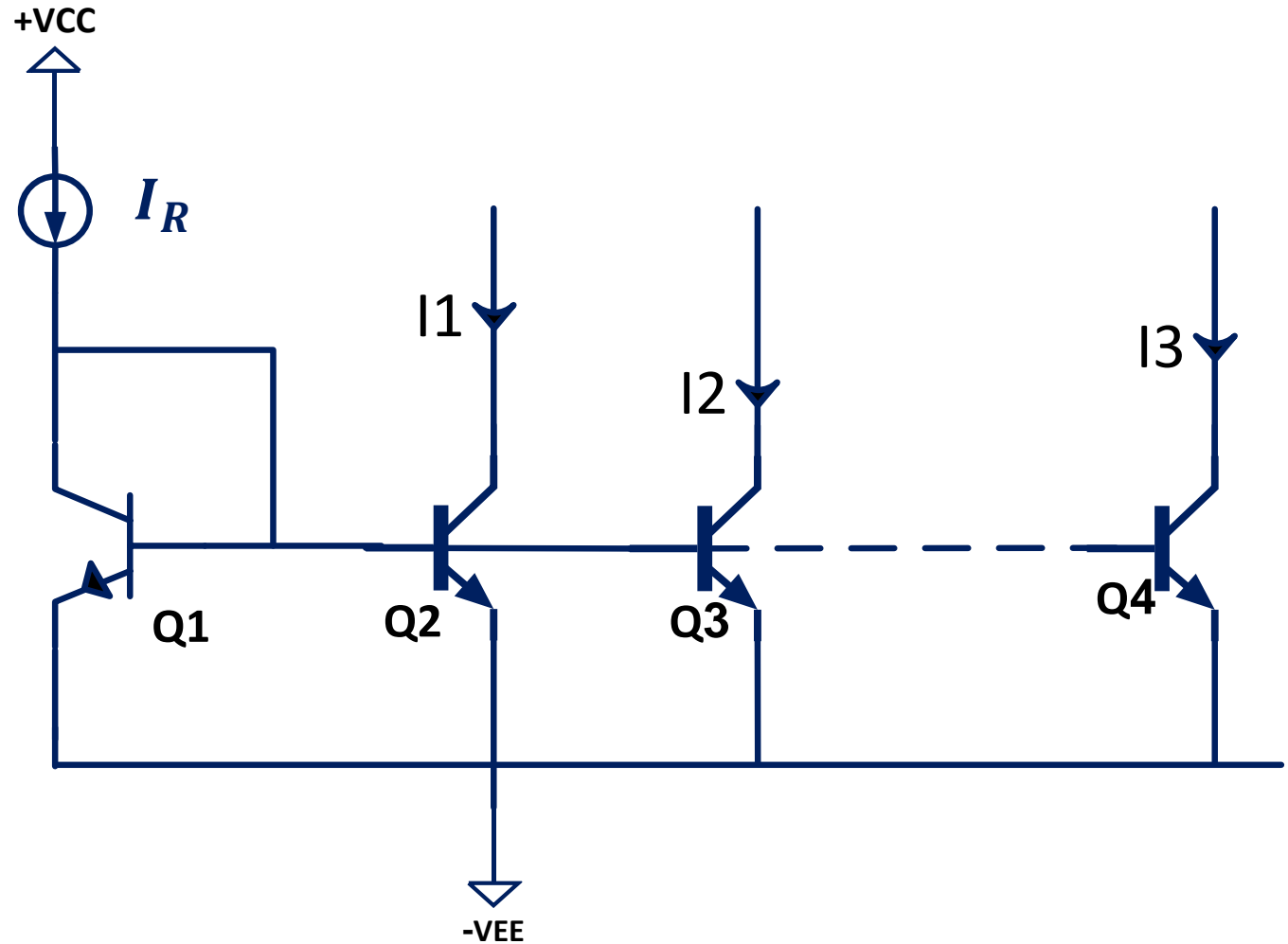


*smaller resistor ; smaller chip area*

# Multitransistor Current Mirror

If  $\beta$  is finite and all the transistors are matched:

$$I_1 = I_2 = I_3 = \frac{I_R}{1 + \frac{(N + 1)}{\beta}}$$



# Generalized Current Mirror

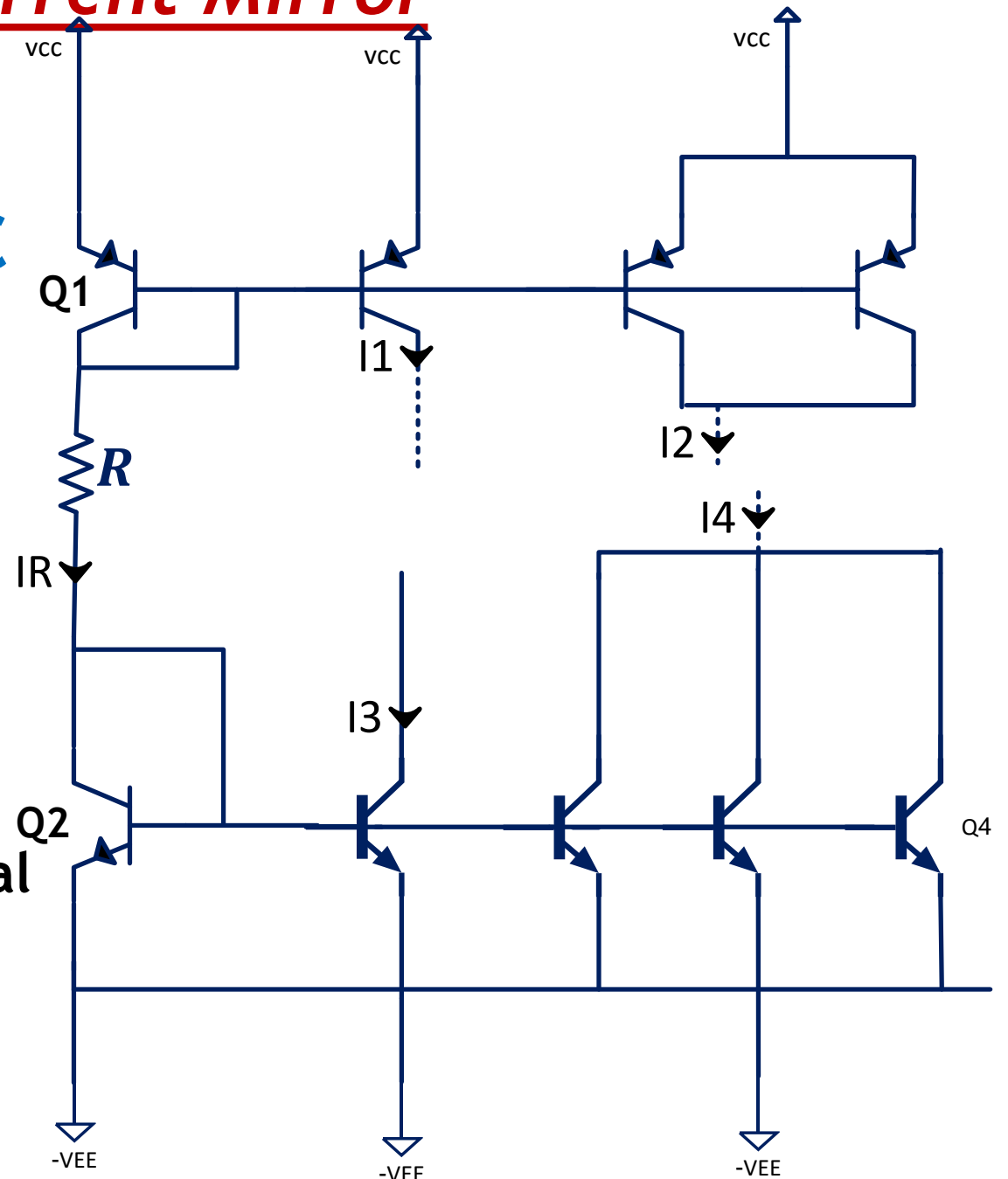
To Generate bias currents for different amplifier stages in an IC

$$I_R = \frac{V_{CC} + V_{EE} - V_{EB1} - V_{BE2}}{R}$$

if  $\beta \rightarrow \infty$

Since  $|V_{BE}|$  For all the transistors are equal

$$I_1 = I_R ; I_2 = 2I_R ; I_4 = 3I_R$$



# Mosfet Current Sources:

## The Basic mosfet Current Source

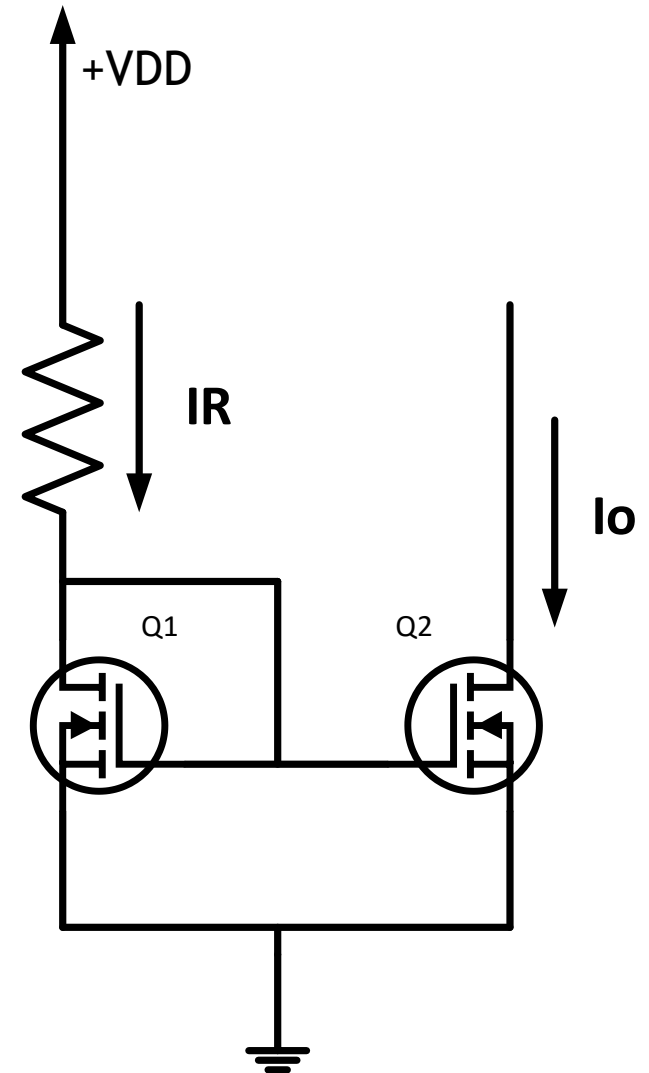
Since  $V_{G1} = V_{D1}$  and  $V_{GS} = V_{DS}$   
 $|V_{DS}| > |V_{GS} - V_T|$

$\therefore Q1$  is operated in the pinch off region

$$I_{DS1} = \frac{1}{2} \bar{K}_{n1} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_T)^2$$

$$I_o = I_{DS2} = \frac{1}{2} \bar{K}_{n2} \left(\frac{W}{L}\right)_2 (V_{GS2} - V_T)^2$$

$$I_R = I_{DS1} + I_{G1} + I_{G2} = I_{DS1}$$

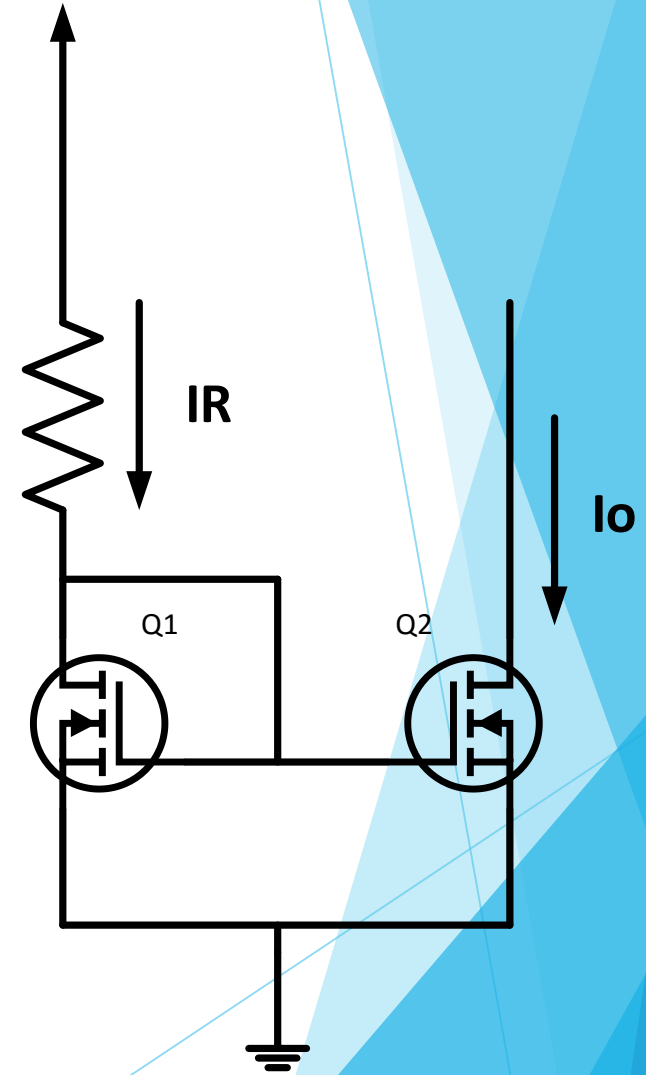


$$I_R = I_{DS1} = \frac{1}{2} \bar{K}_{n1} \left( \frac{W}{L} \right)_1 (V_{GS1} - V_T)^2$$

$$I_o = I_{DS2} = \frac{1}{2} \bar{K}_{n2} \left( \frac{W}{L} \right)_2 (V_{GS2} - V_T)^2$$

*Since  $V_{GS1} = V_{GS2}$   
and  $V_{T1} = V_{T1}$ , and  $\bar{K}_{n1} = \bar{K}_{n2}$*

$$\therefore \frac{I_o}{I_R} = \frac{\left( \frac{W}{L} \right)_2}{\left( \frac{W}{L} \right)_1} \equiv \text{current gain}$$





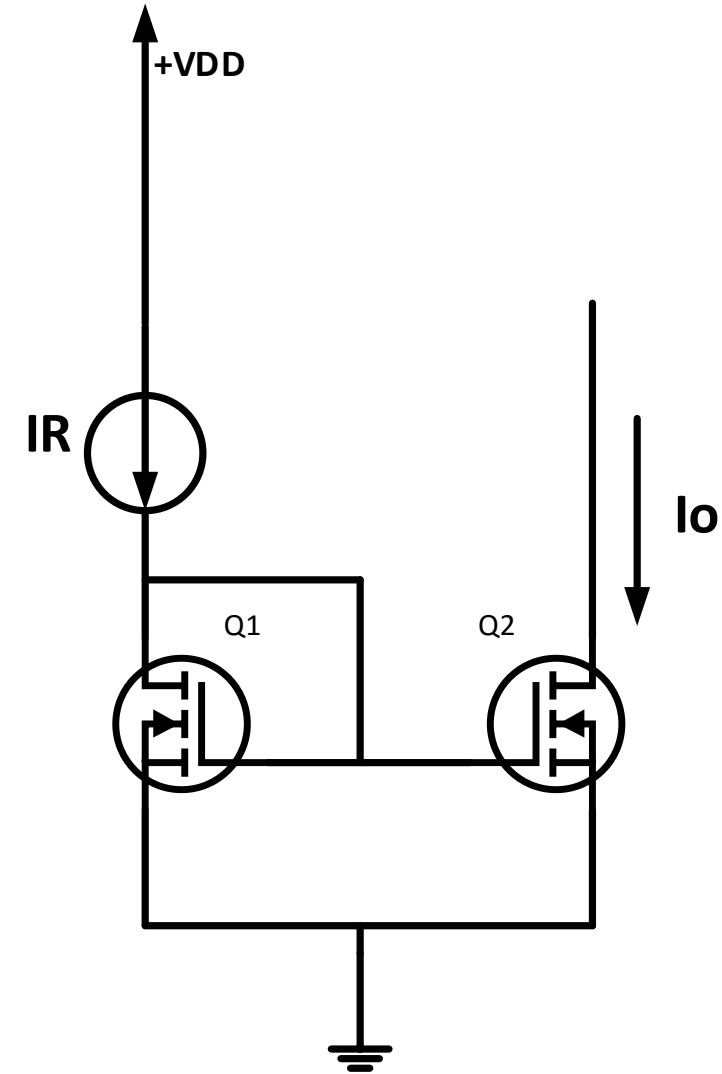
# Mosfet Current Sources:

## The Basic mosfet Current Source

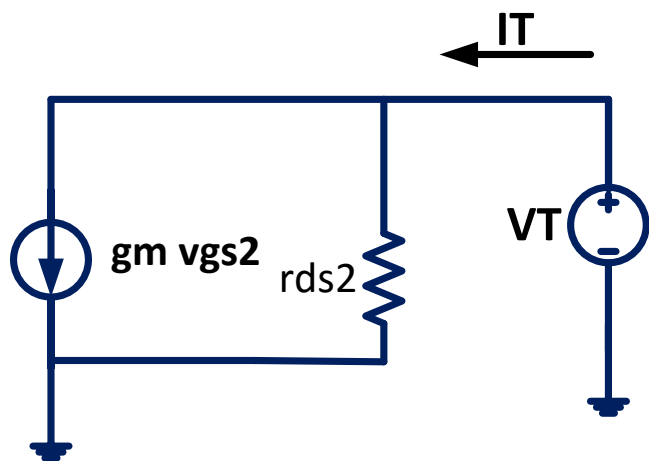
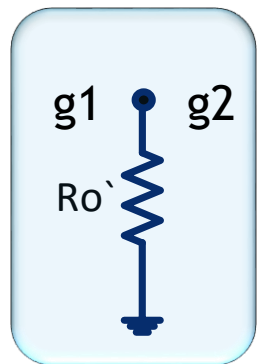
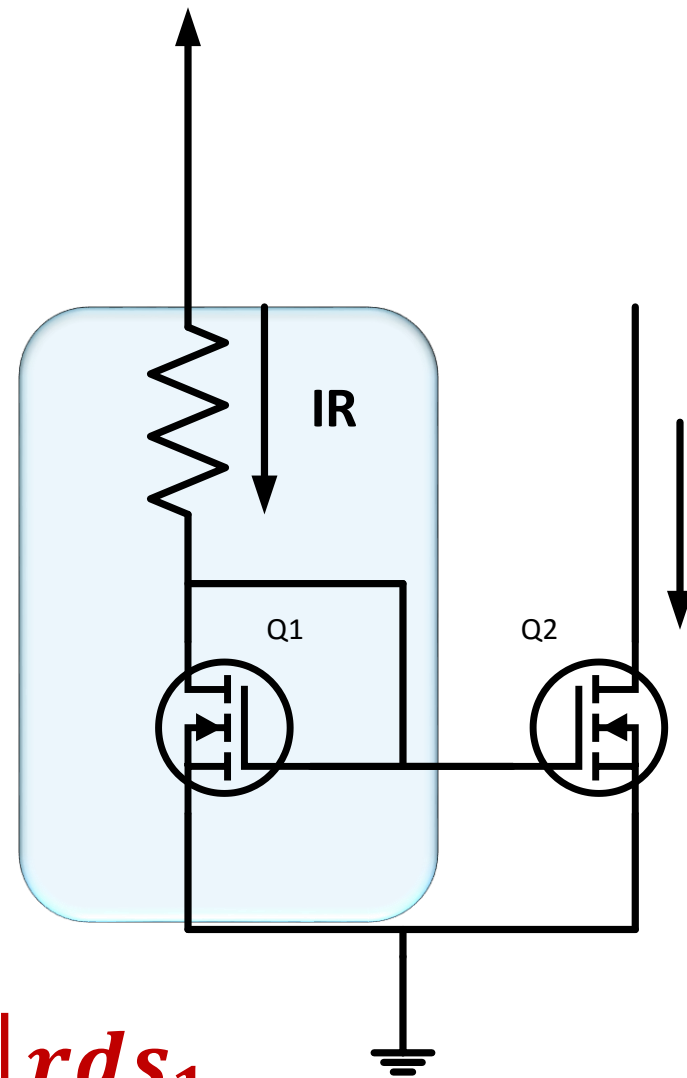
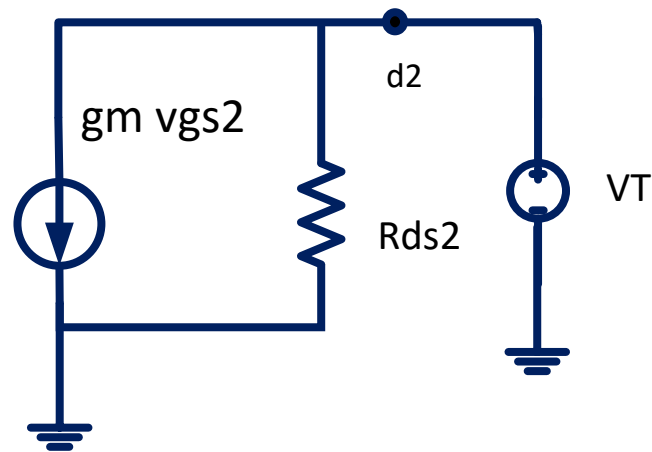
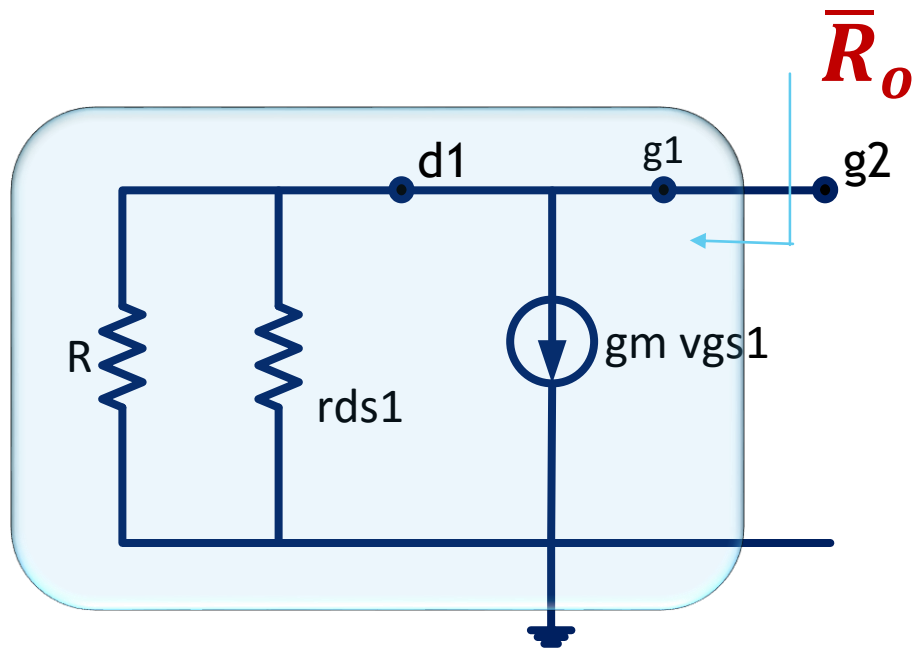
*If we have matched Transistors*

$$\frac{I_o}{I_R} = 1$$

**$I_o = I_R$  current mirror**



# Mosfet Current Sources:



$$\bar{R}_o = \frac{1}{g_{m1}} \parallel R \parallel r_{ds1}$$

$$R_o = \frac{V_T}{I_T} = r_{ds2}$$

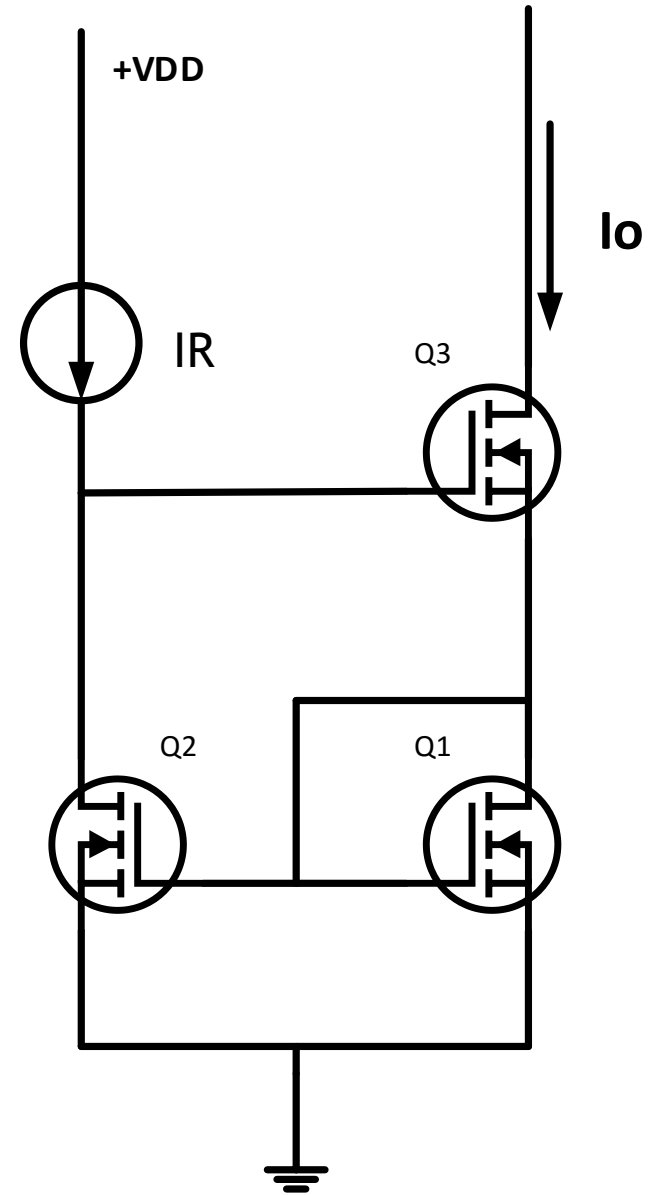
# Mosfet Current Sources:

The Wilson mosfet current mirror

$Q_1, Q_2$  are matched

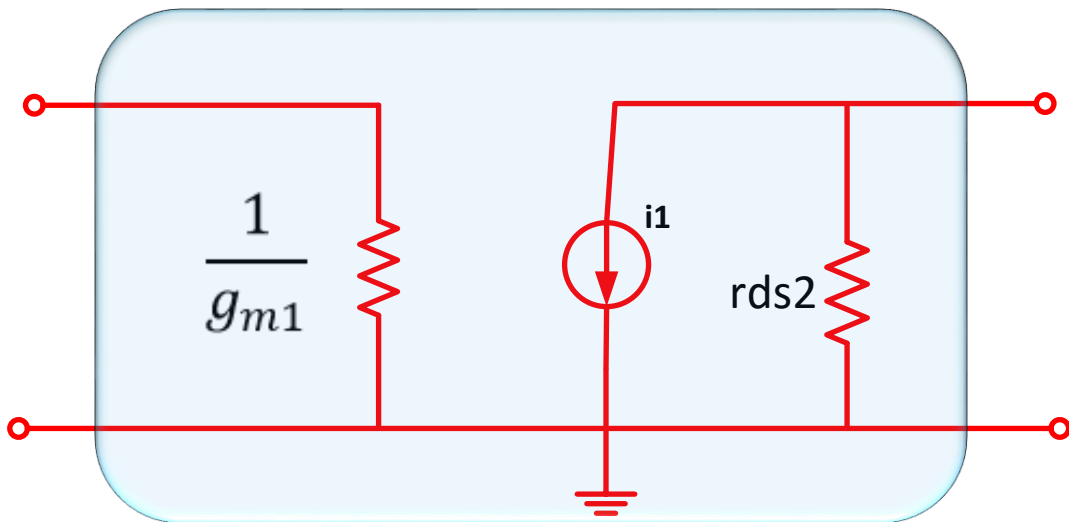
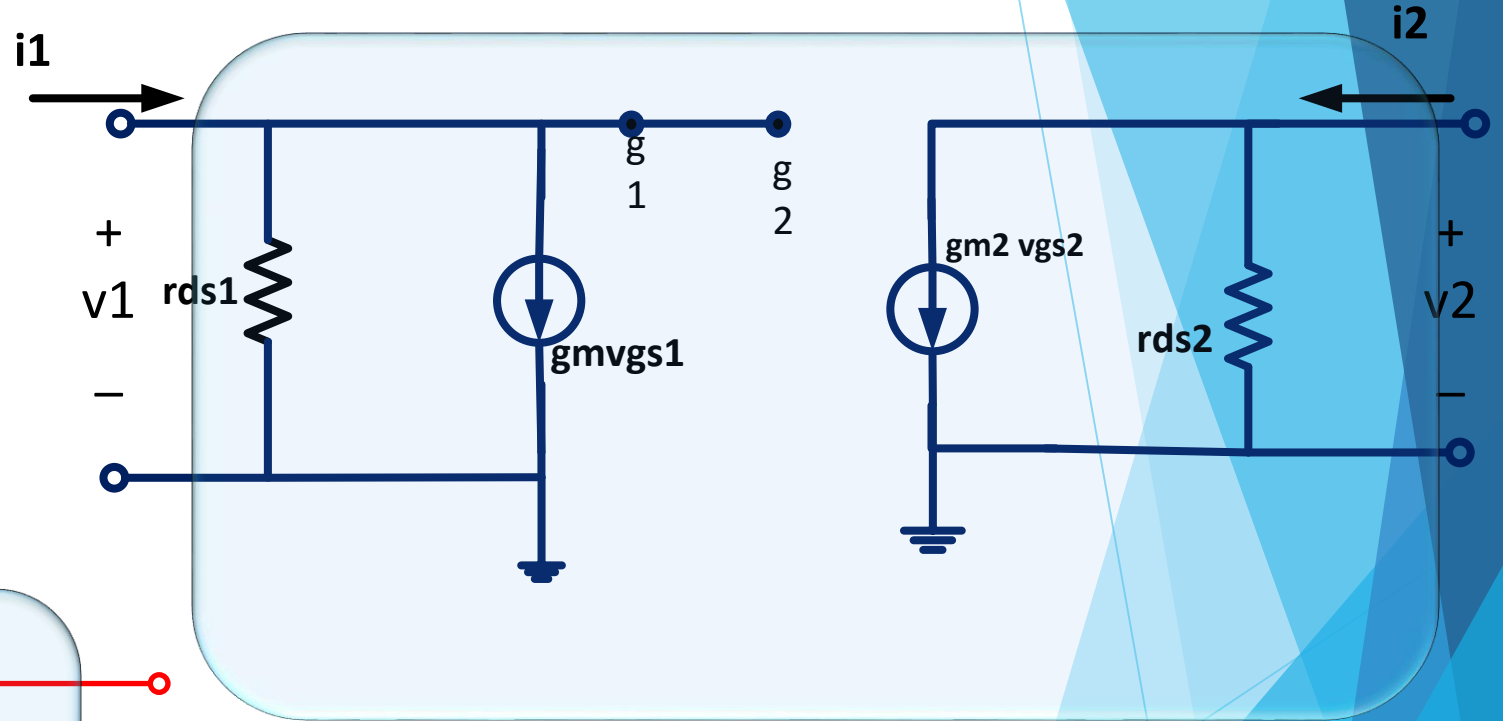
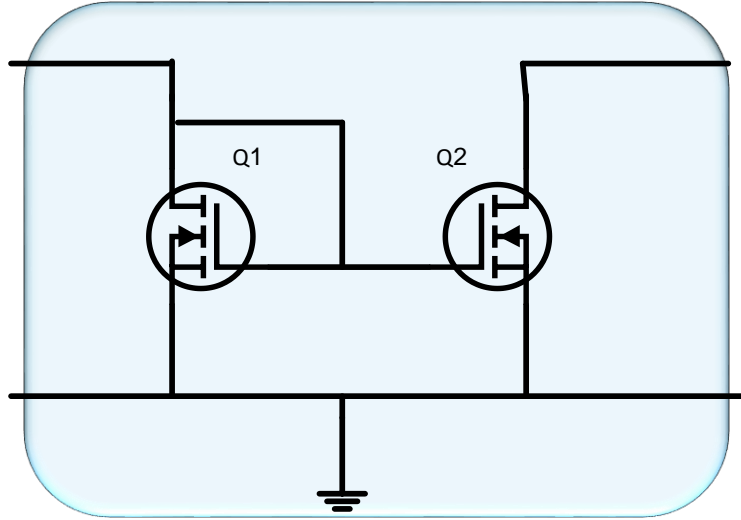
$$\therefore I_o = I_R$$

$$R_o \cong gm_3 r_{ds3} r_{ds2}$$



# Mosfet Current Sources:

## Two Port Model for Mosfet Current Mirror



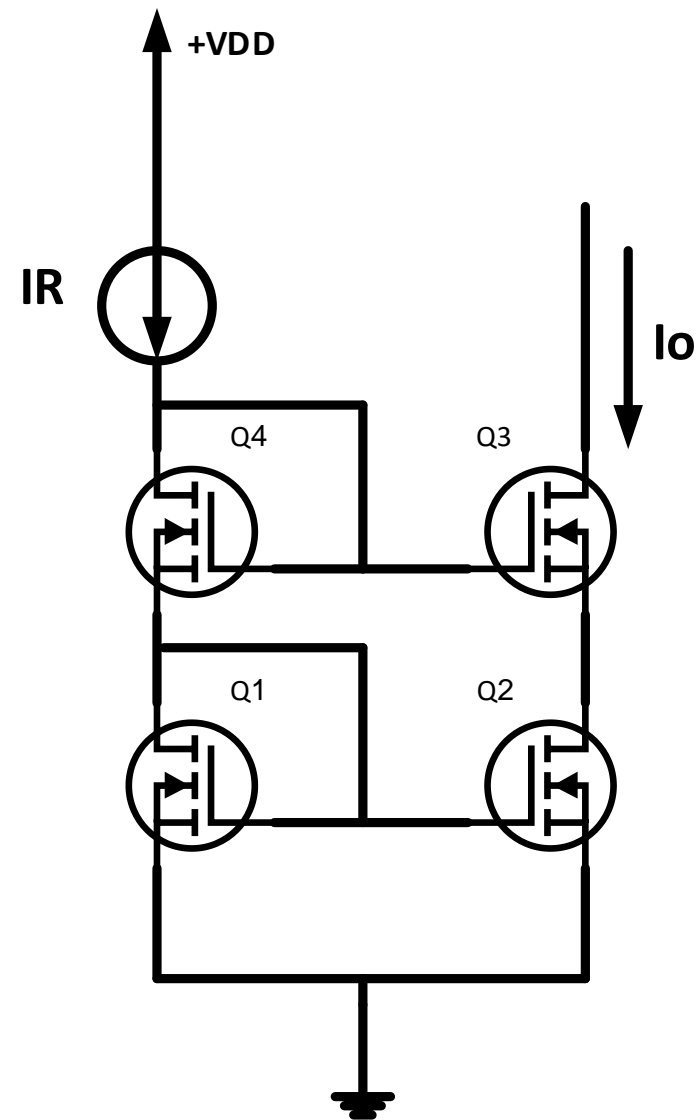
# Mosfet Current Sources:

## Cascode mosfet current mirror

$$I_{DS1} = \frac{1}{2} \bar{K}_n (V_{GS1} - V_T)^2 \left(\frac{W}{L}\right)_1 = I_R$$

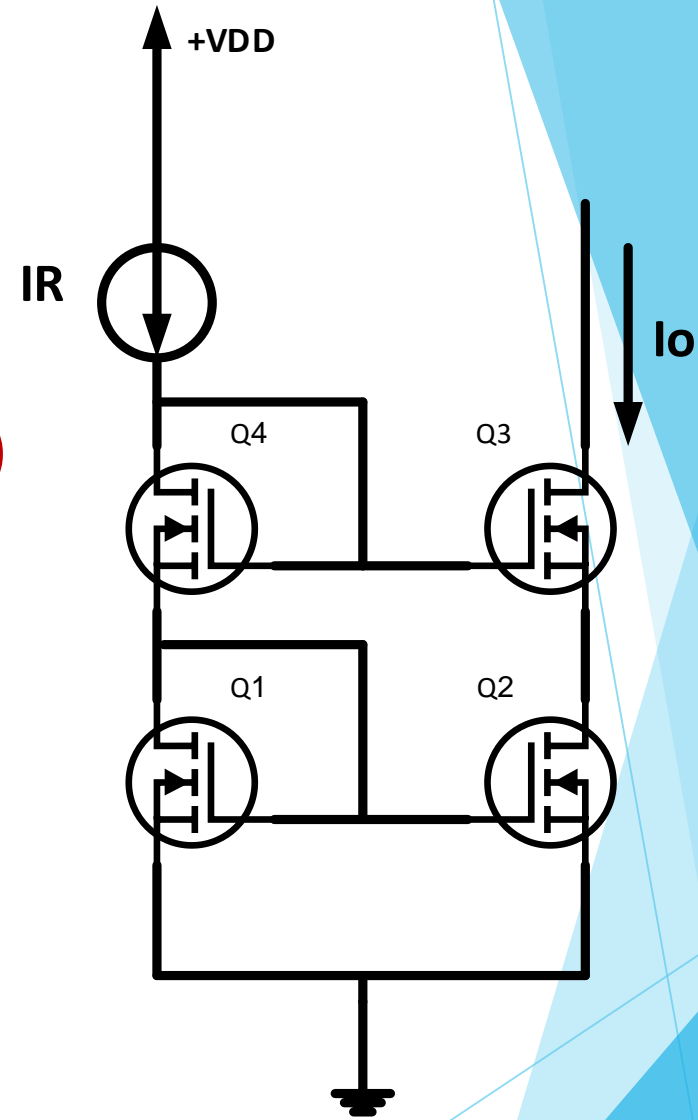
$$I_{DS2} = \frac{1}{2} \bar{K}_n (V_{GS2} - V_T)^2 \left(\frac{W}{L}\right)_2 = I_o$$

$$\frac{I_o}{I_R} = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}$$



►  $R_o = rds_3 + rds_2(1 + gm rds_3)$

$R_o \approx rds_2 rds_3 gm$



# Circuits With Active Load

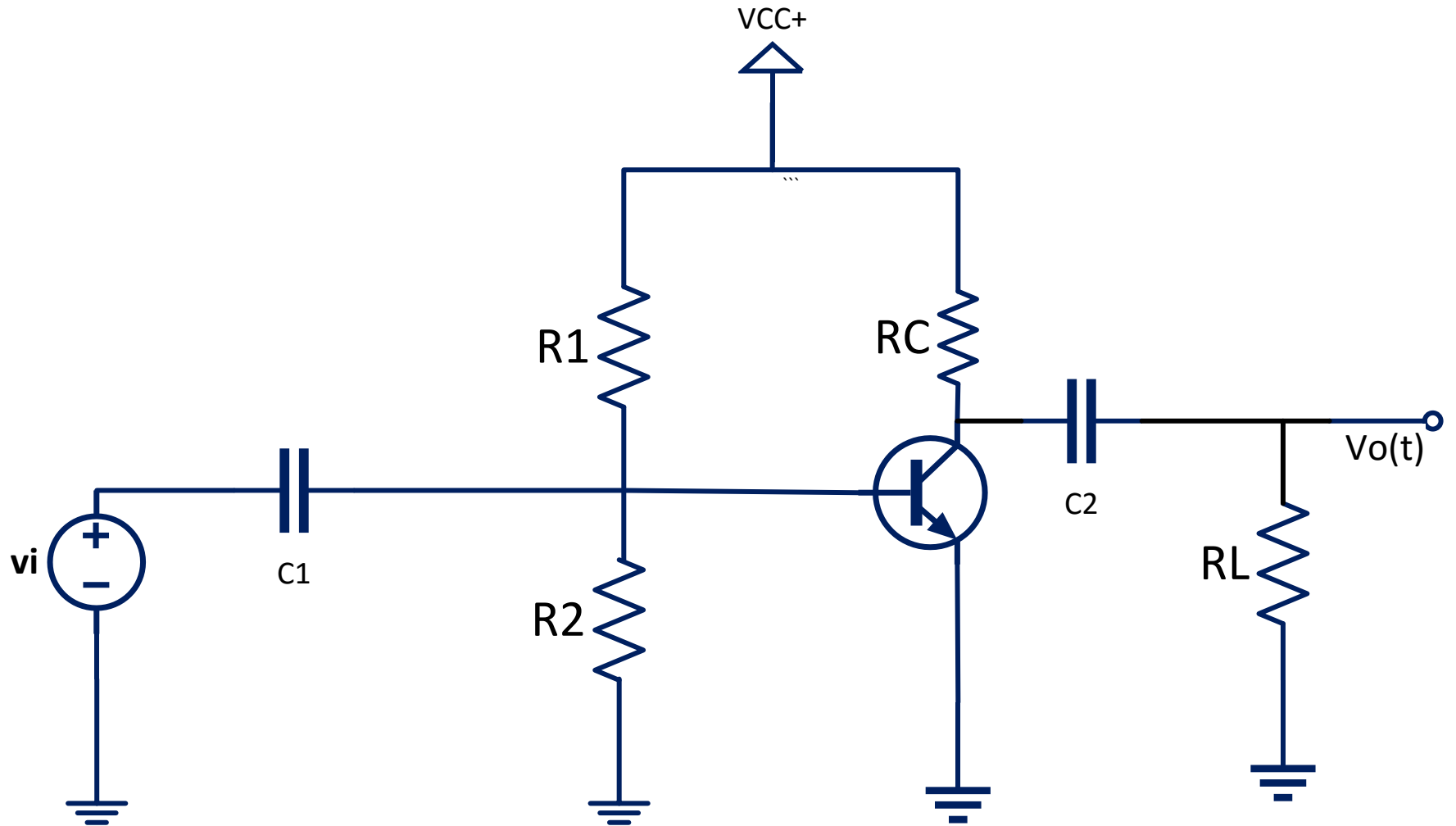
Find  $A_v = \frac{V_o}{V_i}$

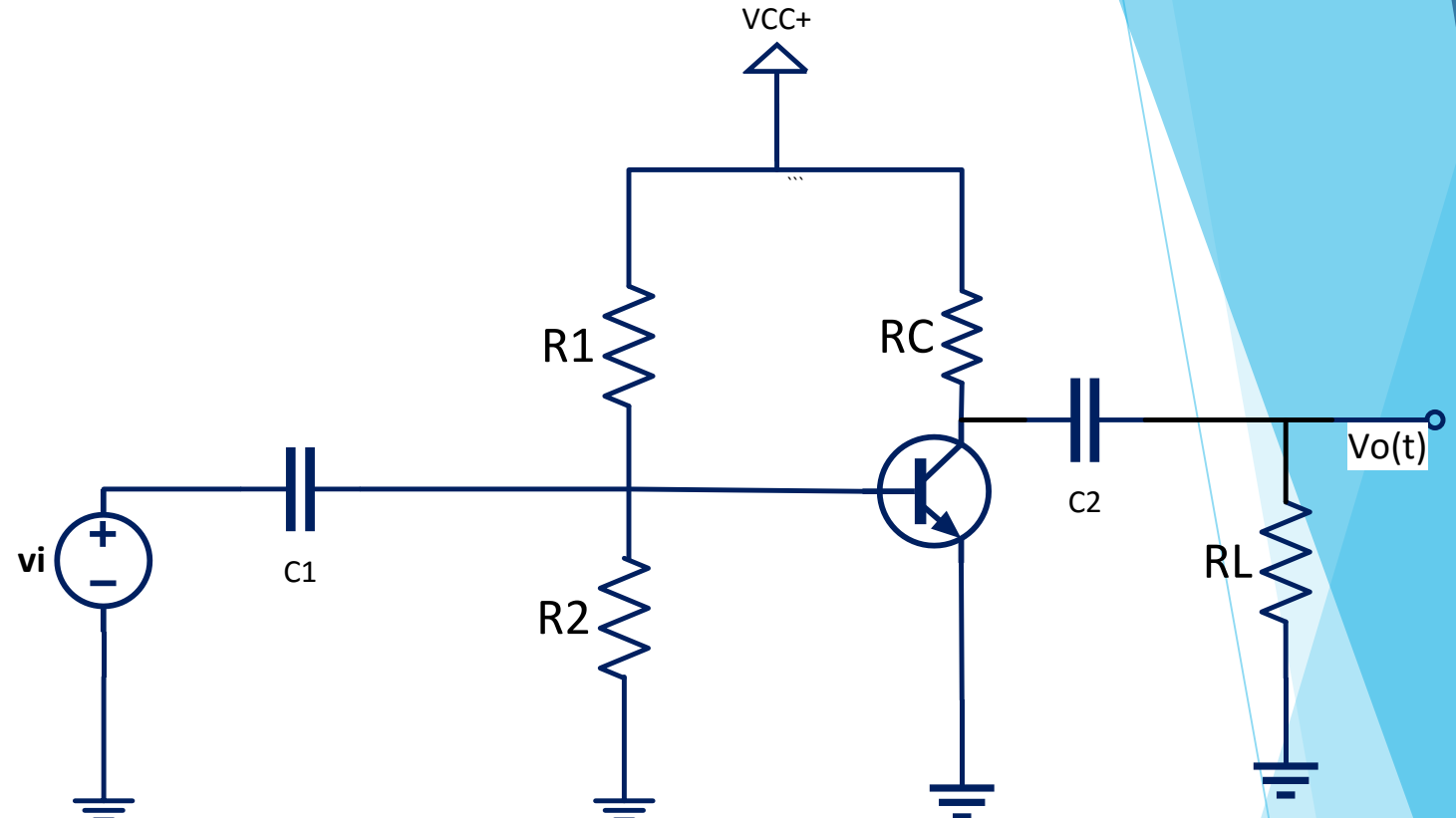
*Let  $R_C = 5k$ ,*

*$I_{CQ} = 1mA$ ,*

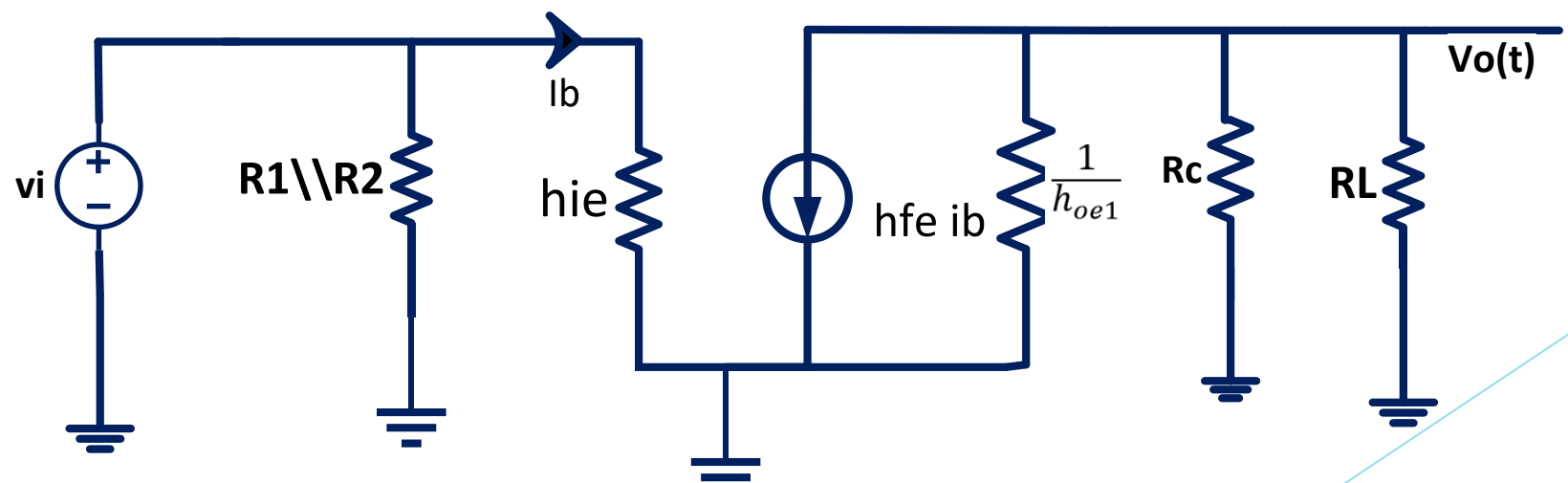
*and  $\frac{1}{h_{oe}} = 120k$*

*$R_L = 10k$*



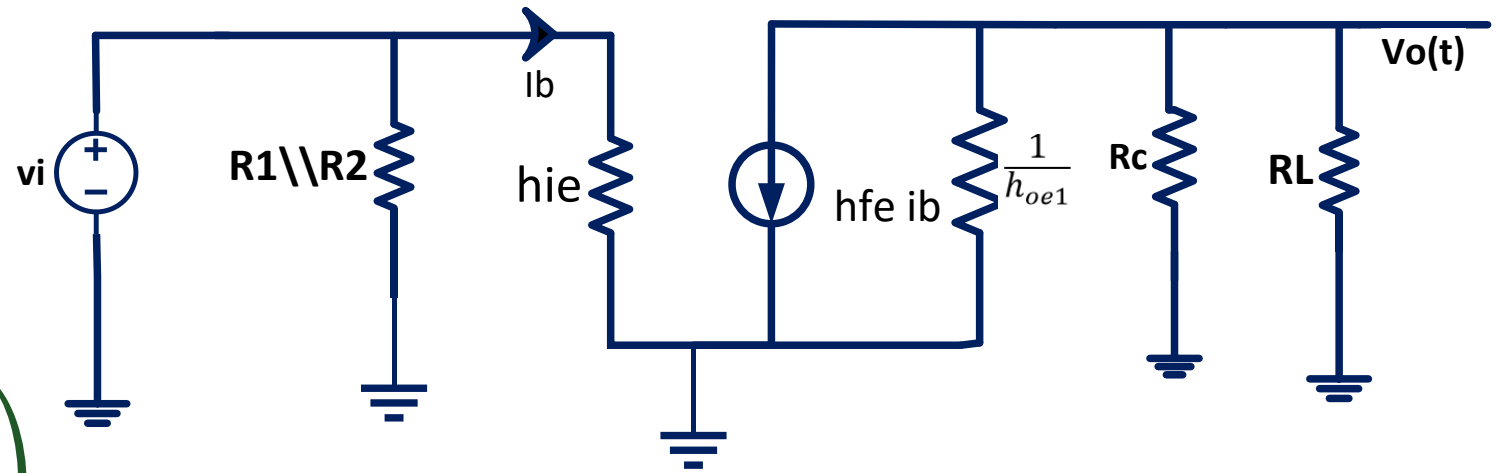


## Ac Small signal equivalent Circuit





# Circuits With Active Load



$$V_o = -h_{fe} i_b \left( R_C \parallel R_L \parallel \frac{1}{h_{oe}} \right)$$

$$i_b = \frac{v_i}{h_{ie}}$$

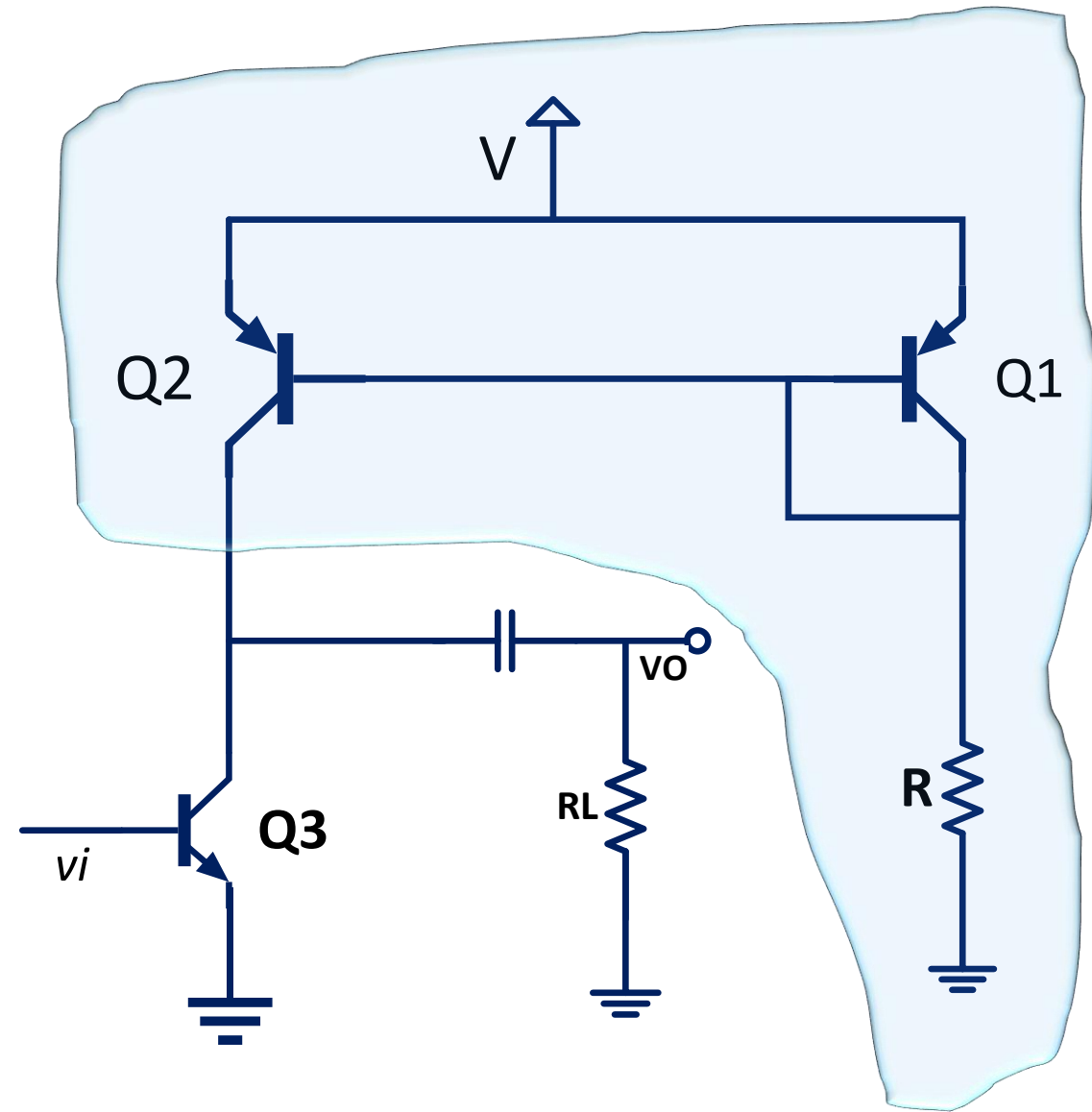
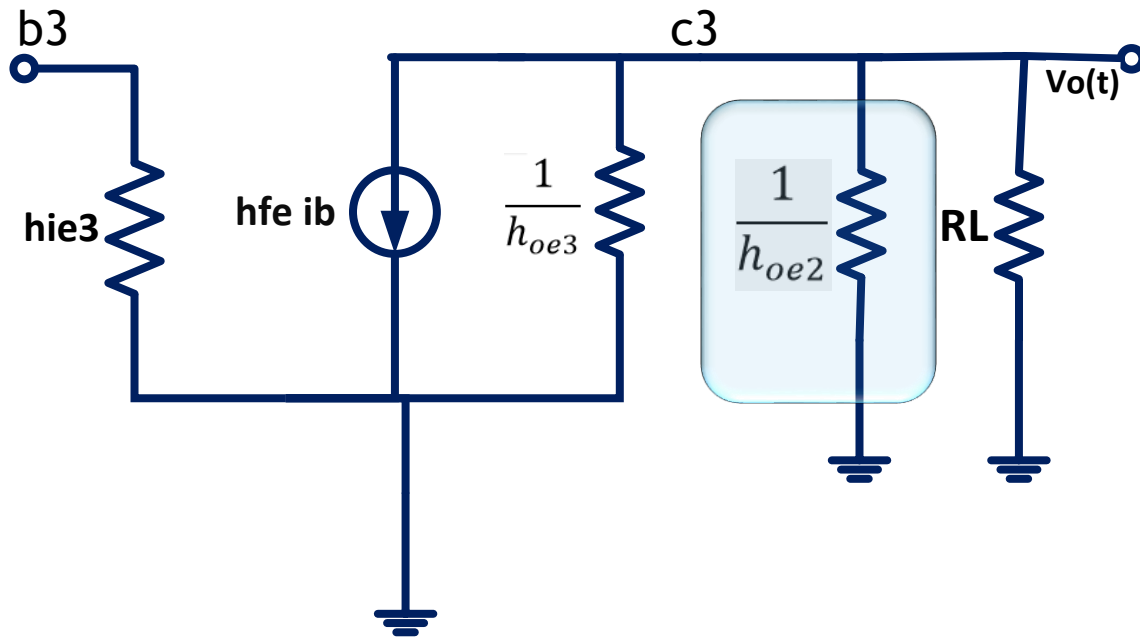
*To increase  $|A_v|$ ,  $R_C \uparrow$ ,  $V_{CE} \downarrow$   
 → Transistor enters saturation*

$$\therefore A_v = -\frac{h_{fe}}{h_{ie}} \left( R_C \parallel R_L \parallel \frac{1}{h_{oe}} \right)$$

$$A_v = -gm \left( R_C \parallel R_L \parallel \frac{1}{h_{oe}} \right) = -125 \quad V_{CE} = V_{CC} - R_C I_{CQ}$$

# Amplifier With an Active Load

## Ac Small signal equivalent Circuit



## Amplifier With an Active Load

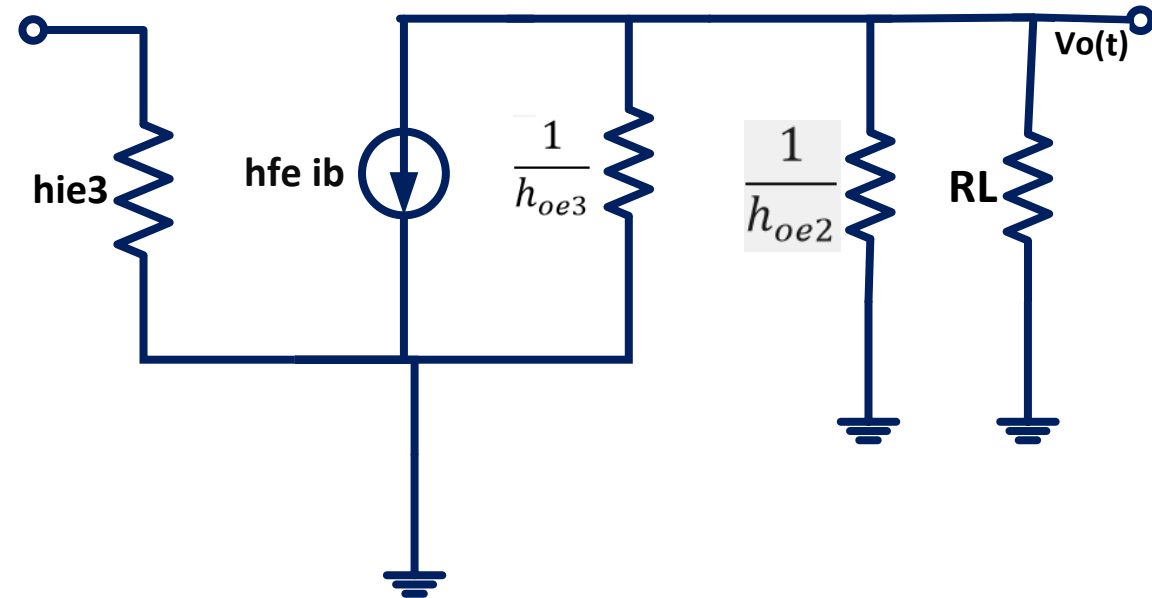
$$A_v = -g_m \left( \frac{1}{h_{oe3}} \parallel \frac{1}{h_{oe3}} \parallel R_L \right)$$

$$\text{let } \frac{1}{h_{oe3}} = 120k$$

$$\frac{1}{h_{oe2}} = 80k$$

$$, I_{CQ} = 1mA$$

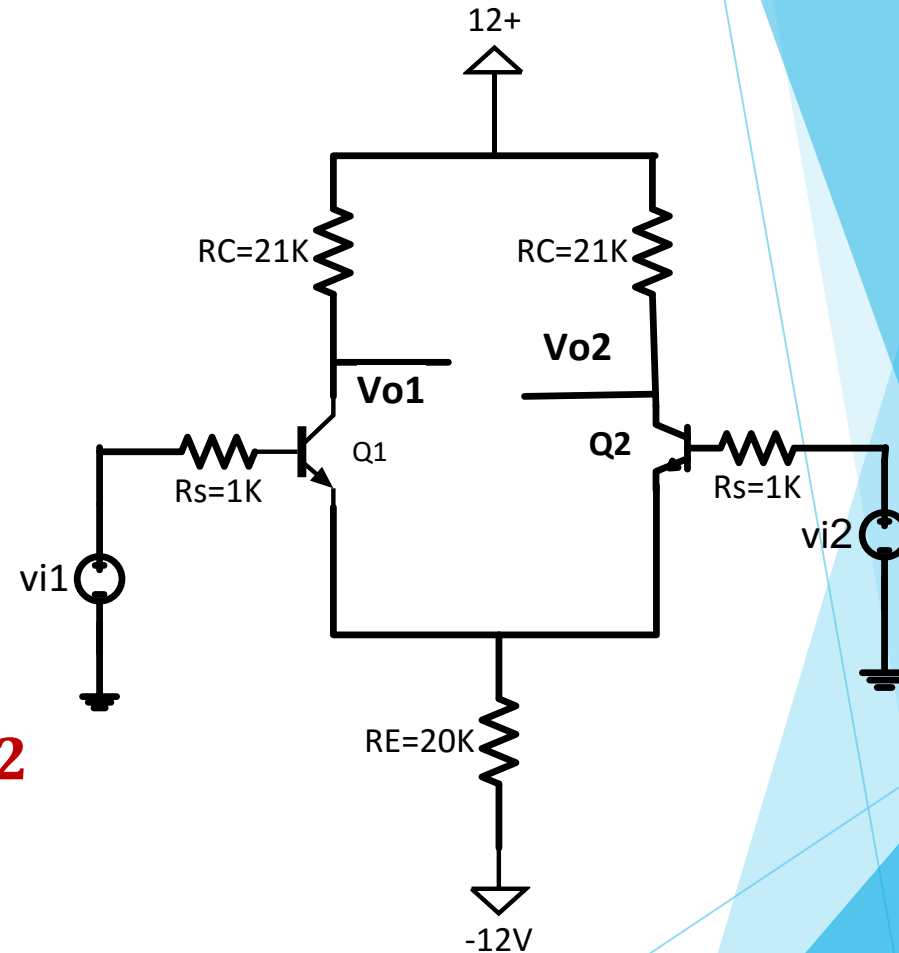
$$\therefore g_m = \frac{h_{fe3}}{h_{ie3}} = \frac{I_{CQ}}{V_T} = 38.46 \Omega$$



$$V_o = \begin{cases} -1846 v_i ; R_L = \infty \\ -1247 v_i ; R_L = 100k \\ -318 v_i ; R_L = 10k \end{cases}$$

# Differential Amplifiers

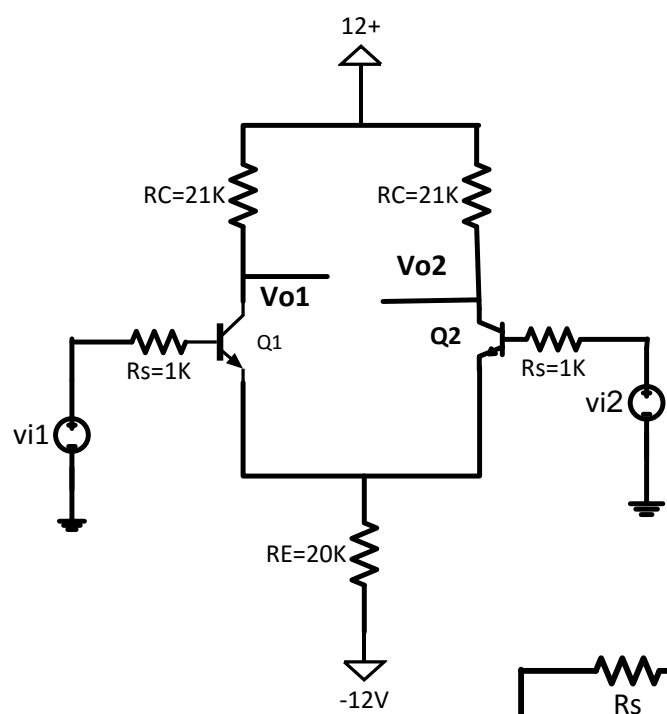
## AC Analysis: Input Impedance



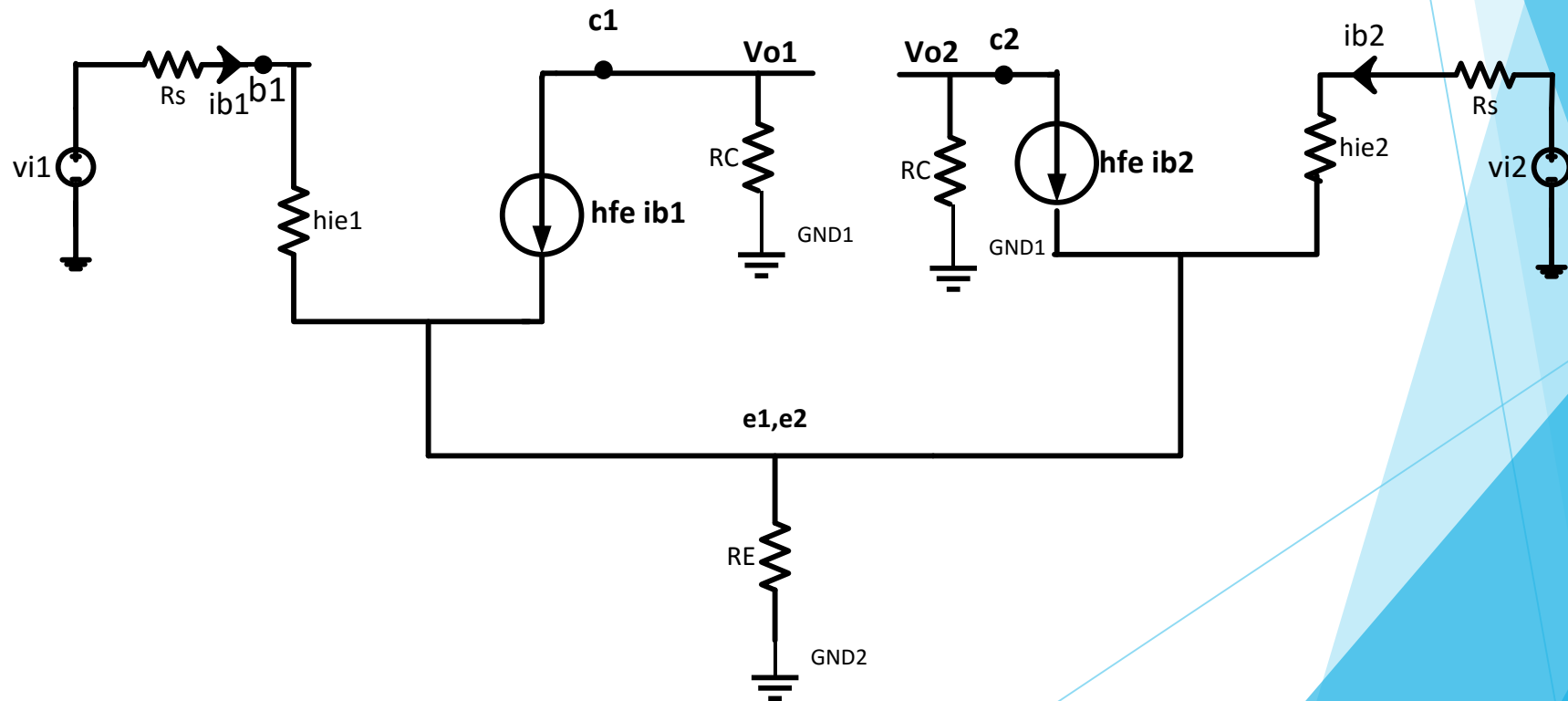
*since  $I_{E1} = I_{E2}$  and  $\beta_1 = \beta_2$*

$$h_{ie1} = h_{ie2} = h_{ie}$$

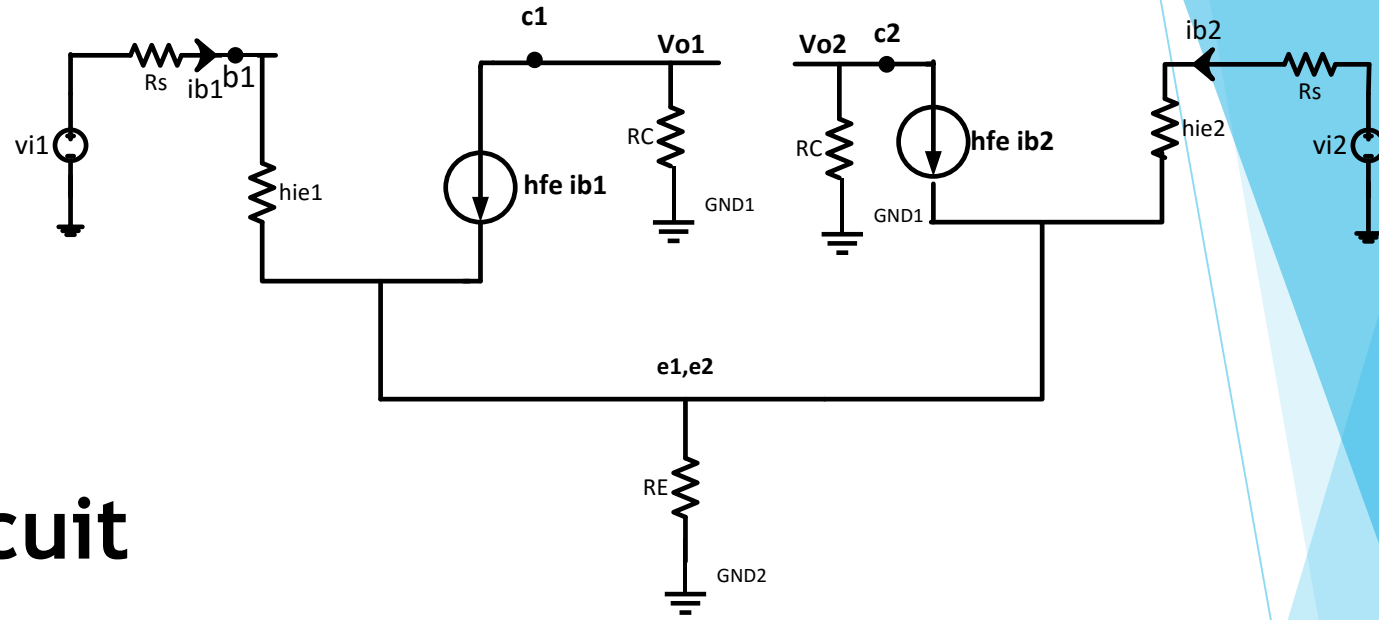
$$h_{ib1} = h_{ib2} = h_{ib}$$



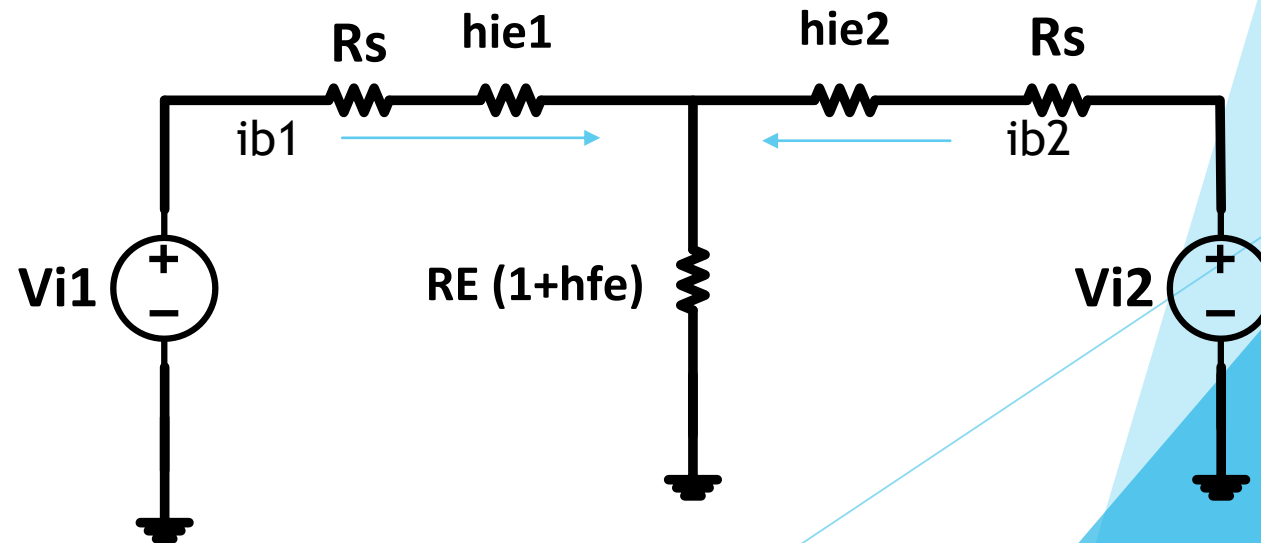
Ac small signal equivalent circuit



# To find the input impedance



## Base equivalent circuit



## ***Input Impedance:***

Base equivalent circuit

***Differential mode input impedance  $\equiv Z_{id}$***

$$Z_{id} = \left. \frac{v_d}{i_b} \right|_{v_c=0}$$

***Common mode input impedance  $\equiv Z_{ic}$***

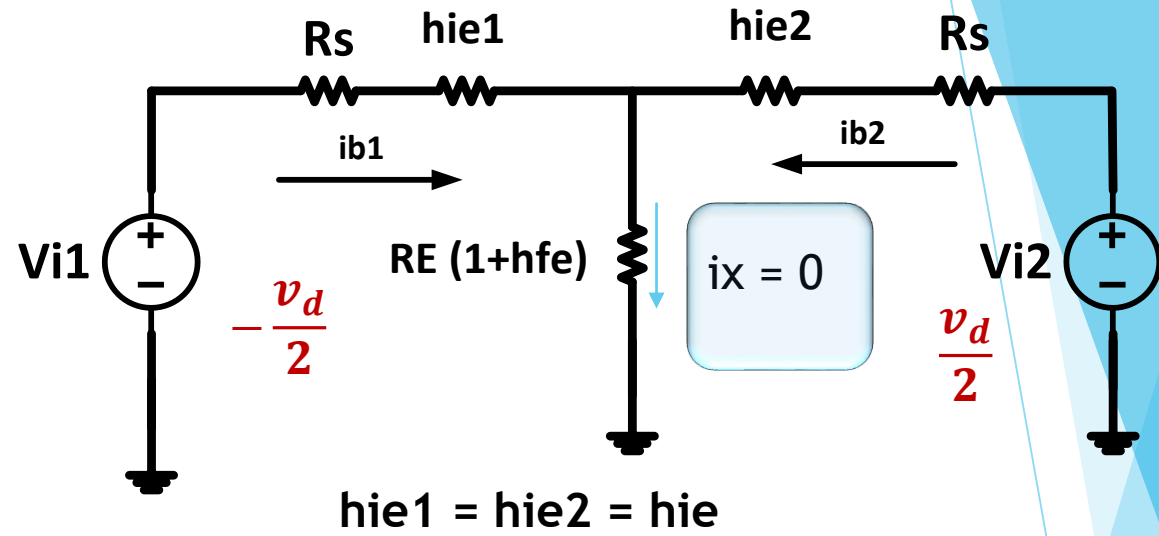
$$Z_{ic} = \left. \frac{v_c}{i_b} \right|_{v_d=0}$$

# Input impedance

$$Z_{id} = \left. \frac{v_d}{i_b} \right|_{v_c=0}$$

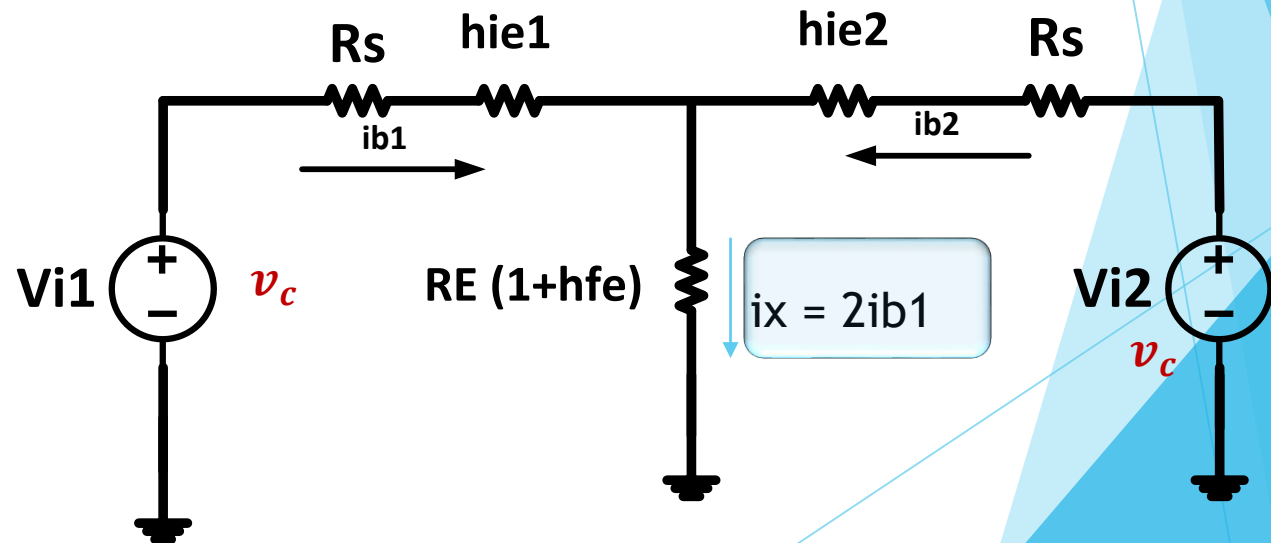
$$v_{i1} = -\frac{v_d}{2}; v_{i2} = \frac{v_d}{2}$$

$$Z_{id} = 2(R_S + h_{ie})$$



$$Z_{ic} = \left. \frac{v_c}{i_b} \right|_{v_d=0}$$

$$v_{i1} = v_c; v_{i2} = v_c$$



$$Z_{ic} = R_S + h_{ie} + 2(1 + h_{fe})RE$$

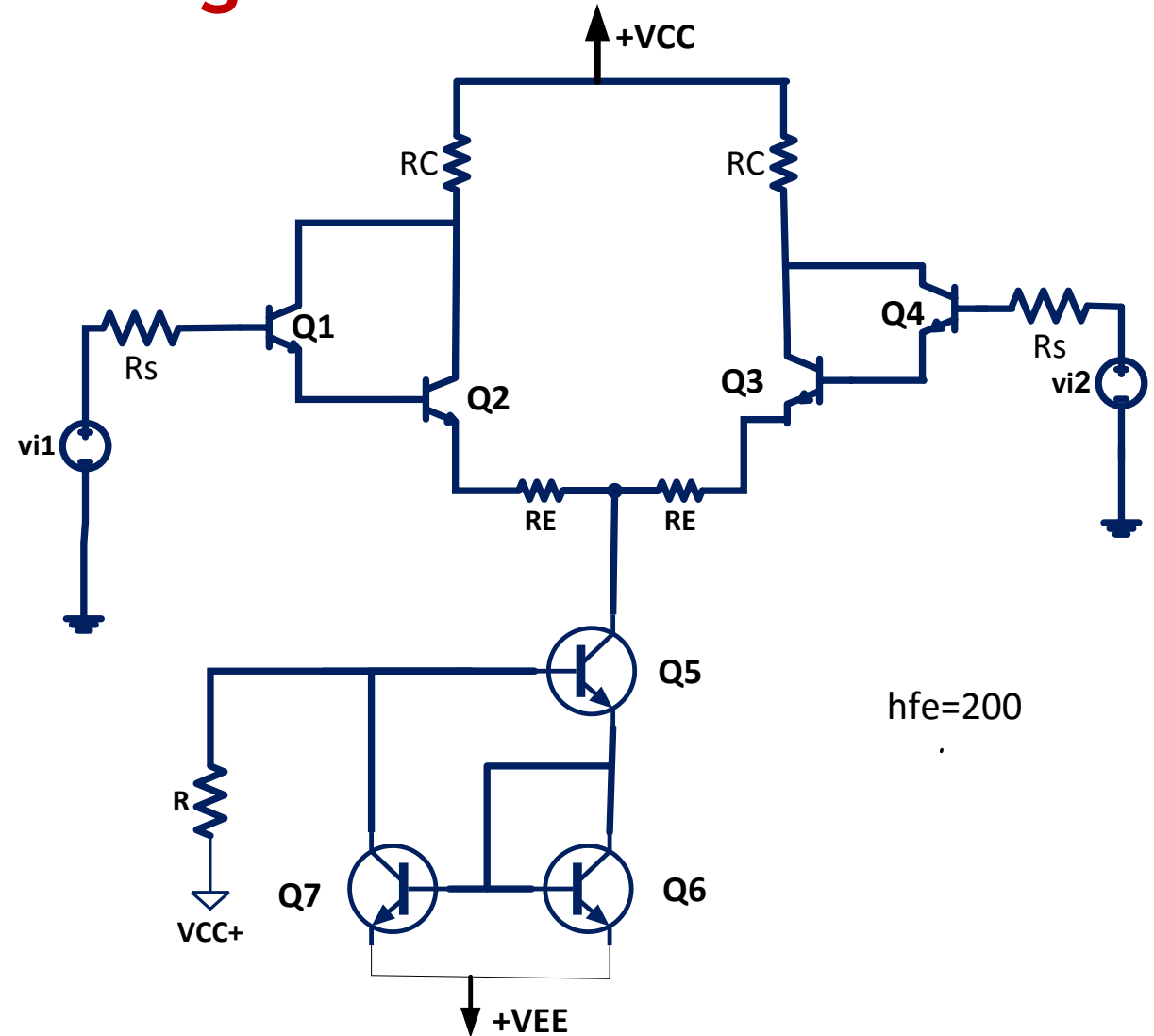
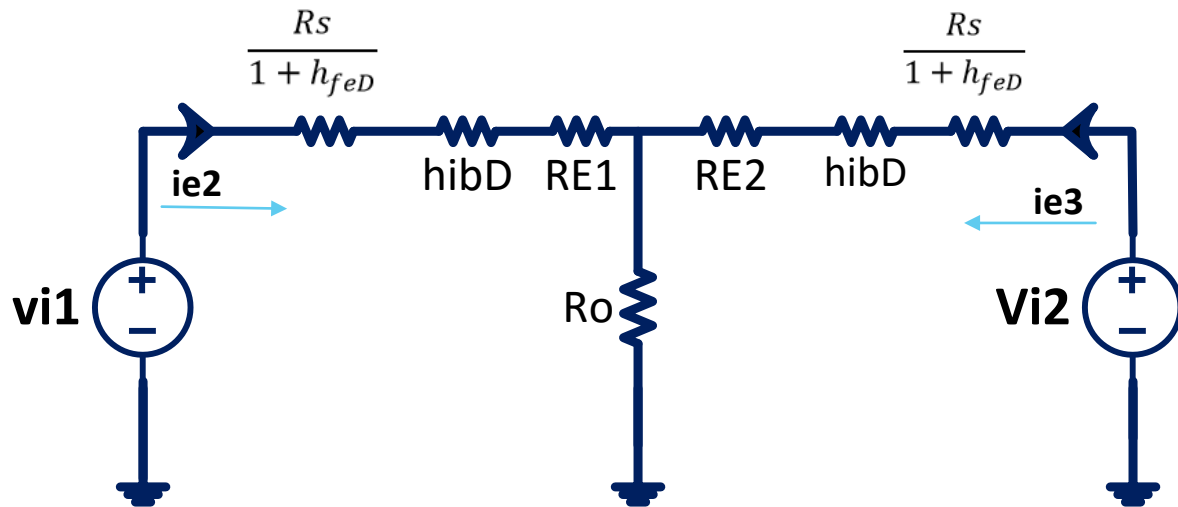


# Differential Amplifiers Using Darlington and FET

## Differential Amplifiers Using Darlington

To find  $A_c$  &  $A_d$ :

Emitter equivalent circuit

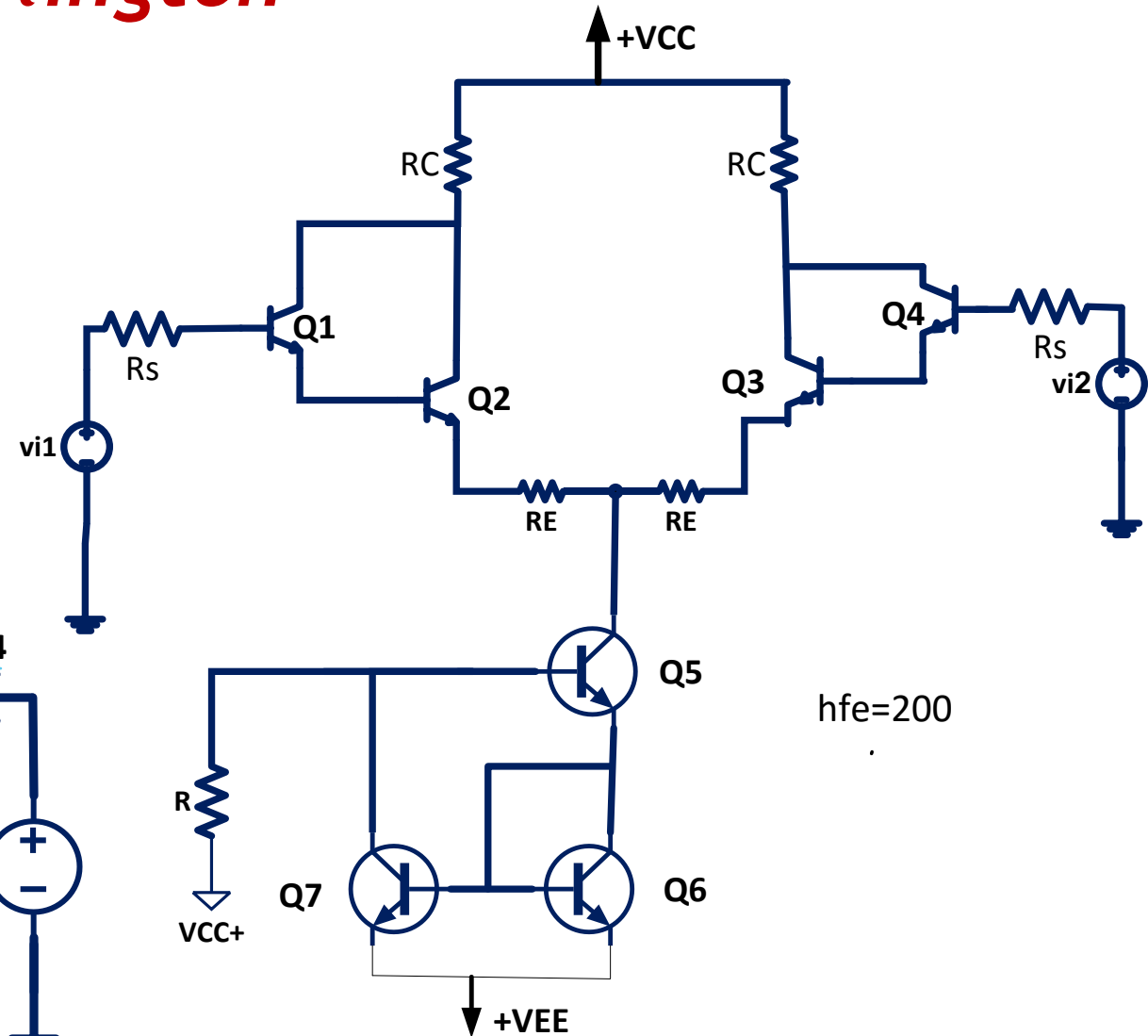
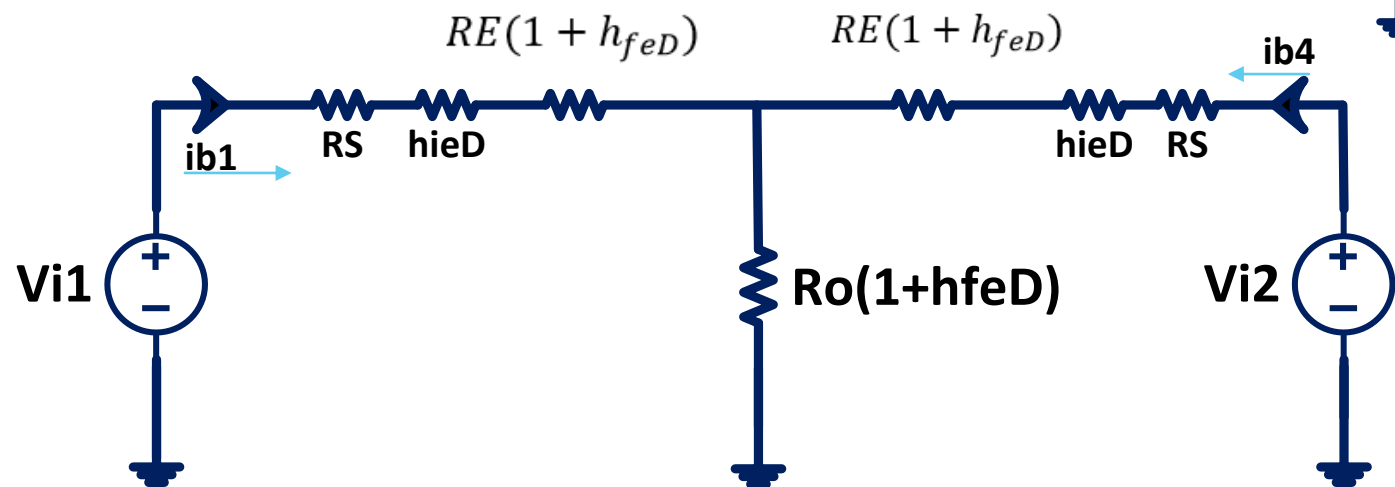


# Differential Amplifiers Using Darlington and FET

## Differential Amplifiers Using Darlington

To find  $Z_{id}$ :

Base equivalent circuit

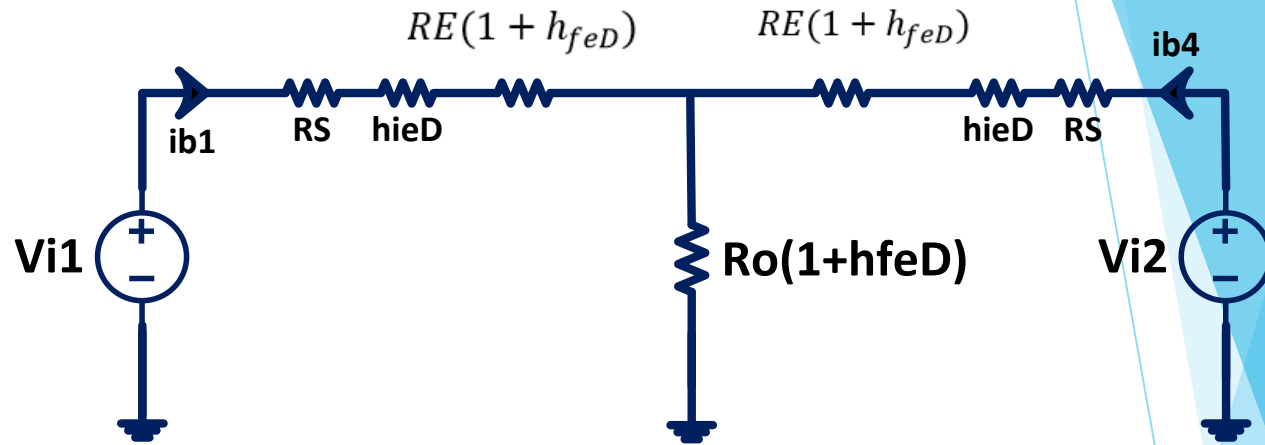


# Input Impedance:

Base equivalent circuit

$$Z_{id} = \left. \frac{v_d}{i_{b1}} \right|_{v_c=0}$$

$$v_{i1} = -\frac{v_d}{2}; v_{i2} = \frac{v_d}{2}$$



$$Z_{id} = 2 \left( R_S + h_{ieD} + R_E(1 + h_{feD}) \right)$$

$$Z_{ic} = \left. \frac{v_c}{i_b} \right|_{v_d=0}$$

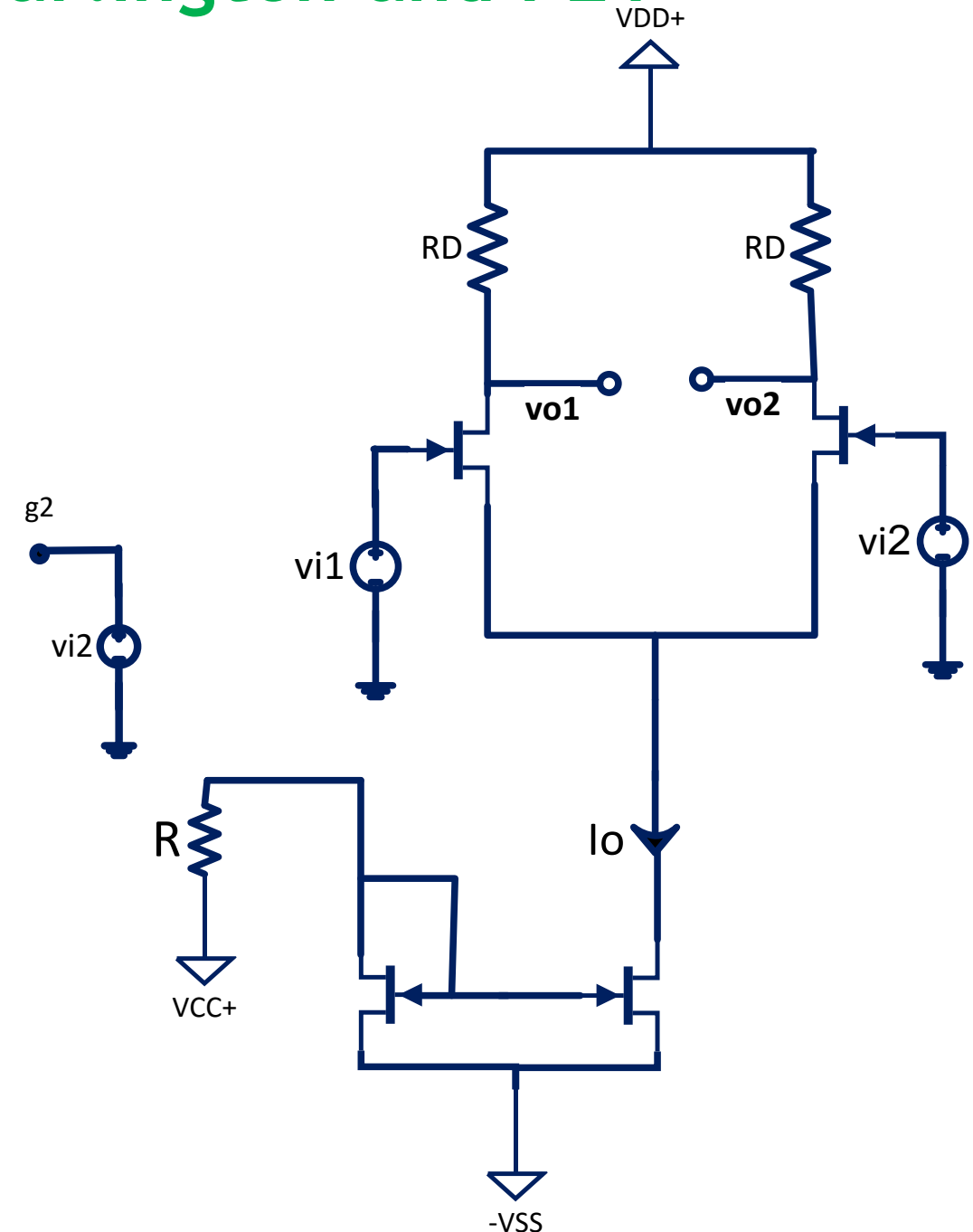
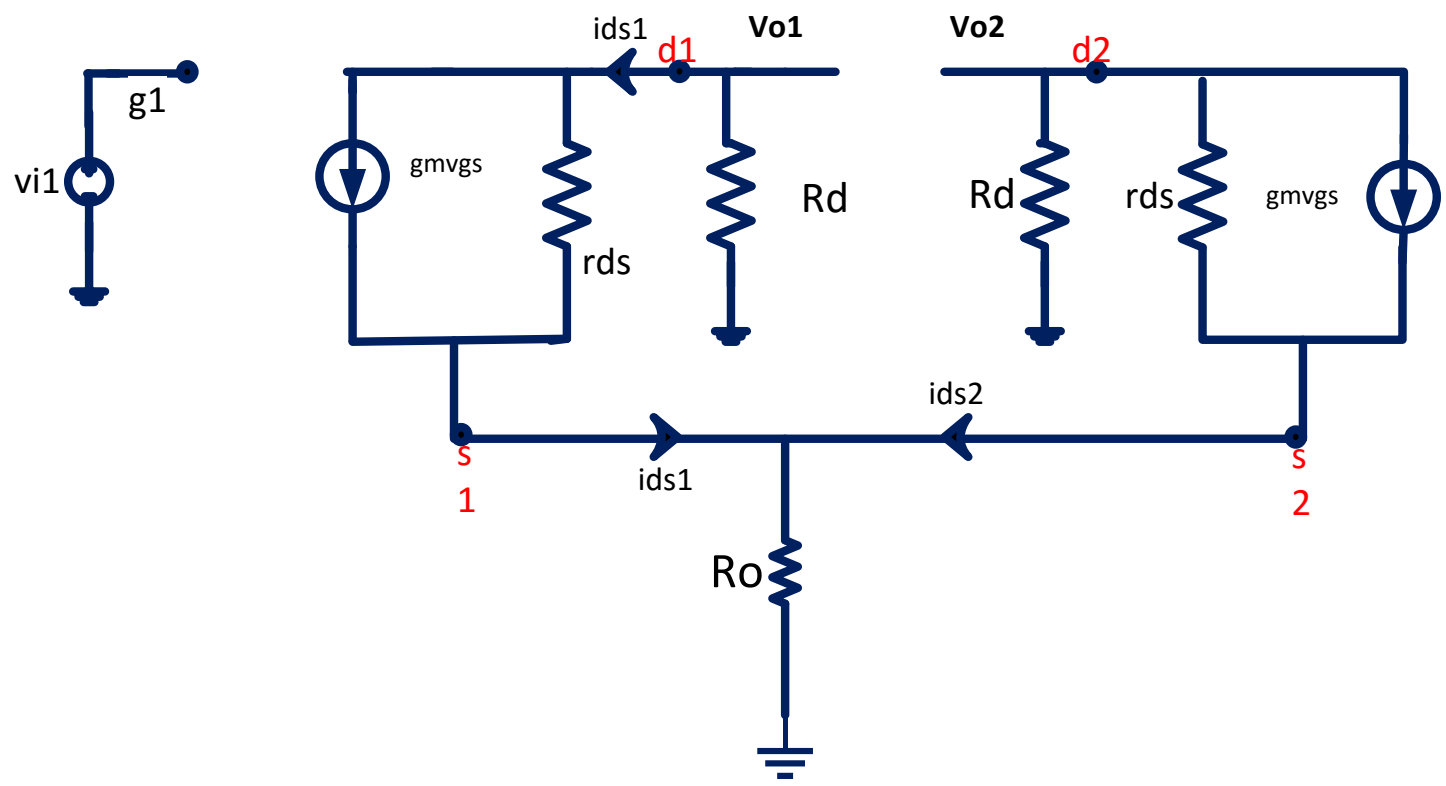
$$v_{i1} = v_c; v_{i2} = v_c$$

$$Z_{ic} = R_S + h_{ieD} + R_E(1 + h_{feD}) + 2R_o(1 + h_{feD})$$

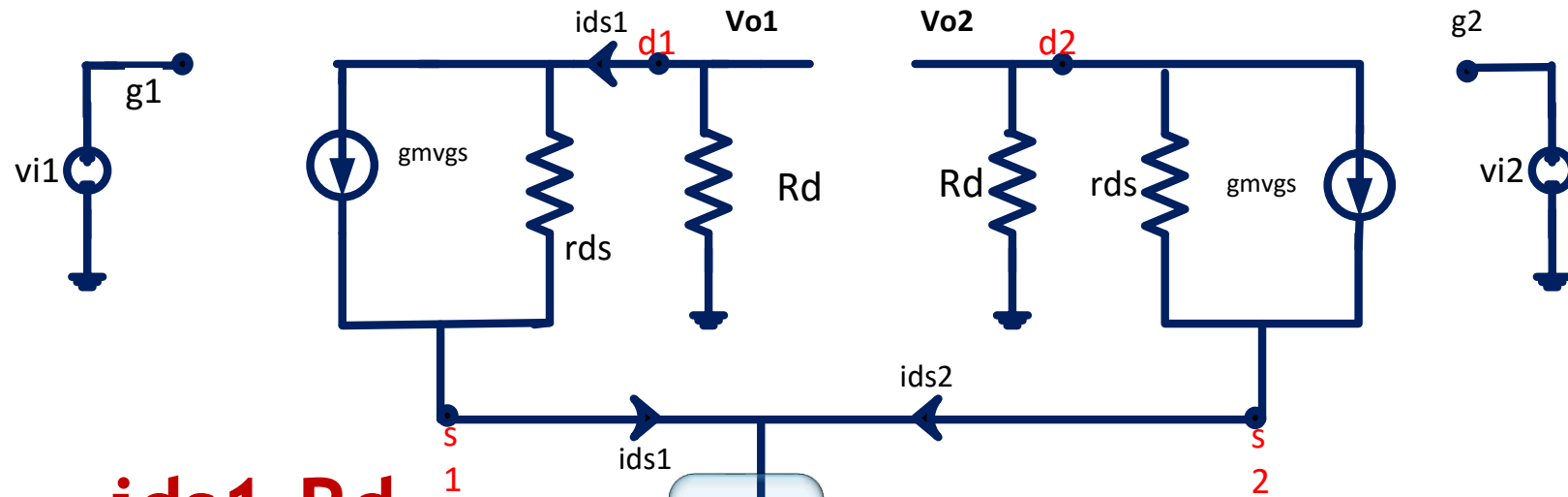
# Differential Amplifiers Using Darlington and FET

## Differential Amplifiers Using FET

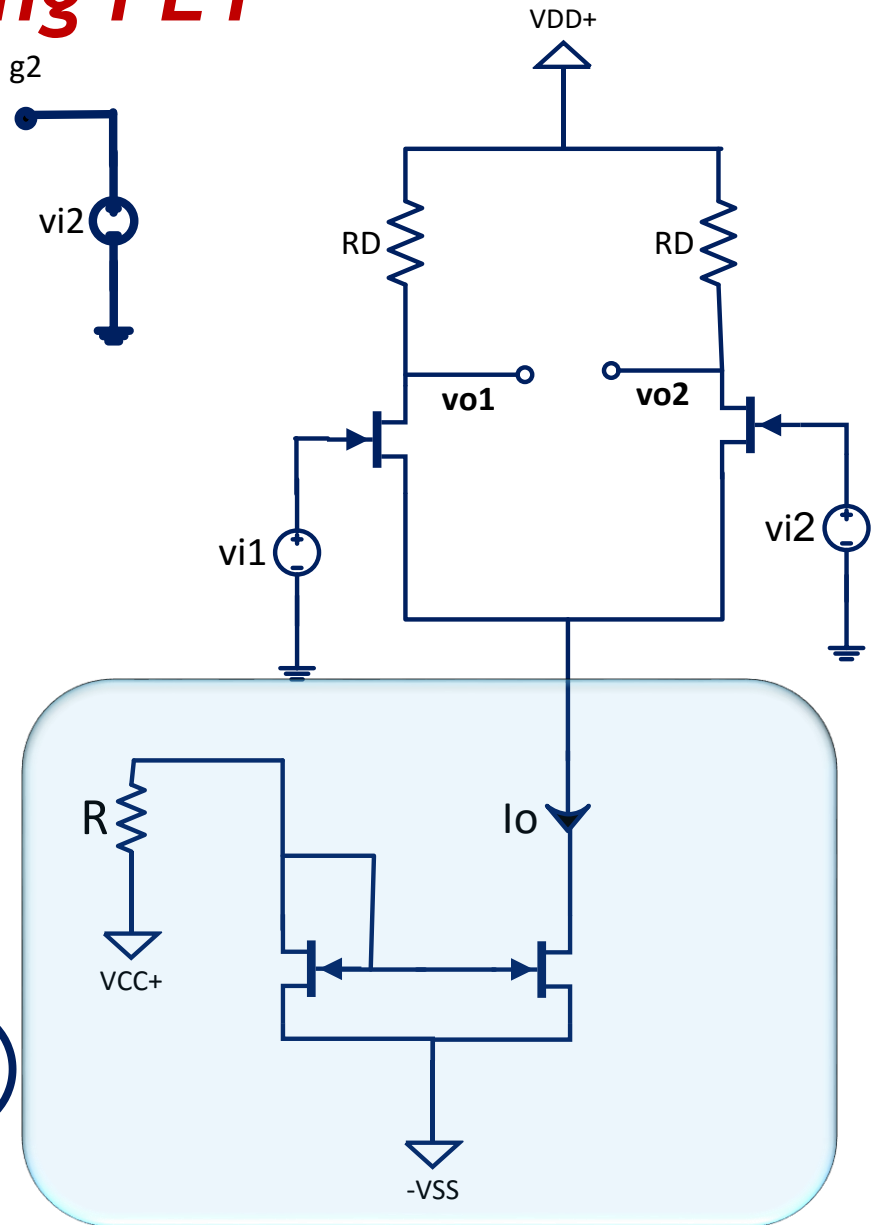
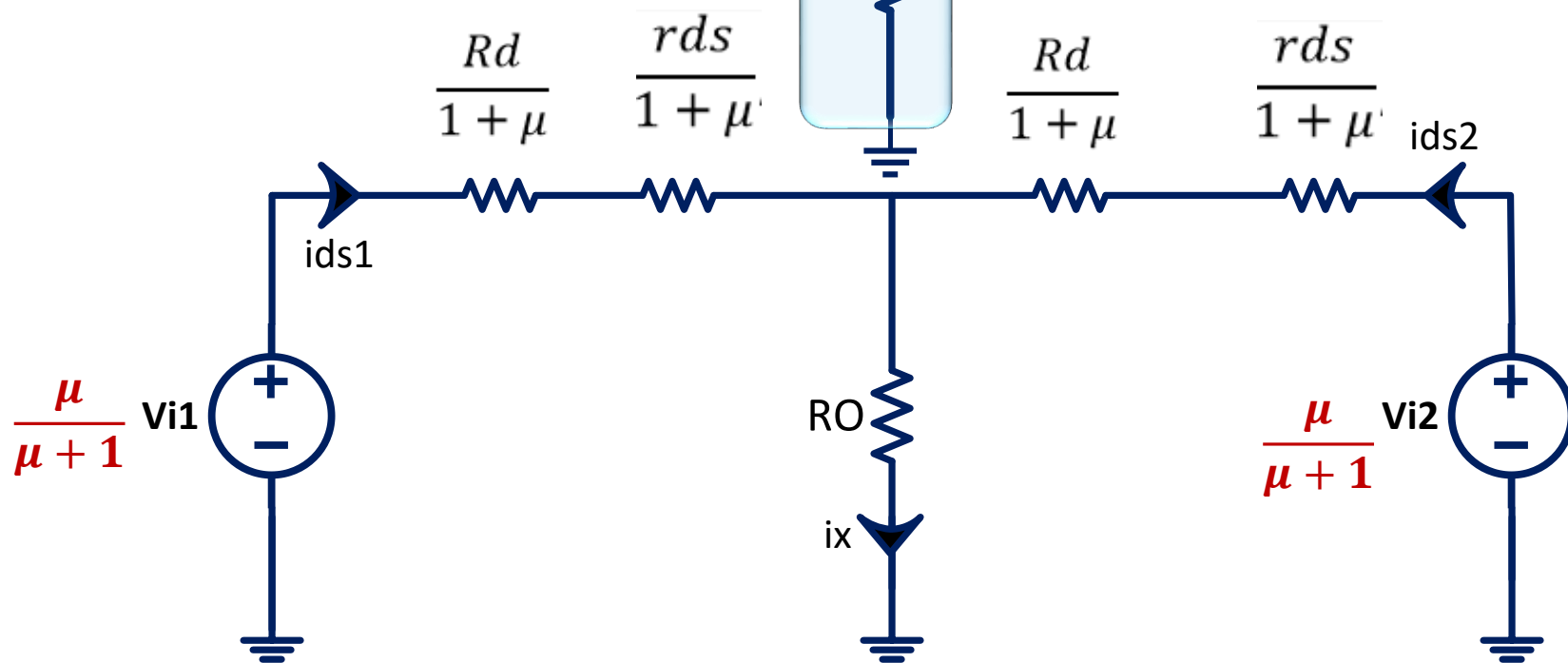
Ac Small signal equivalent Circuit:



# Differential Amplifiers Using FET



$$V_{o1} = -i_{ds1} \cdot R_d$$

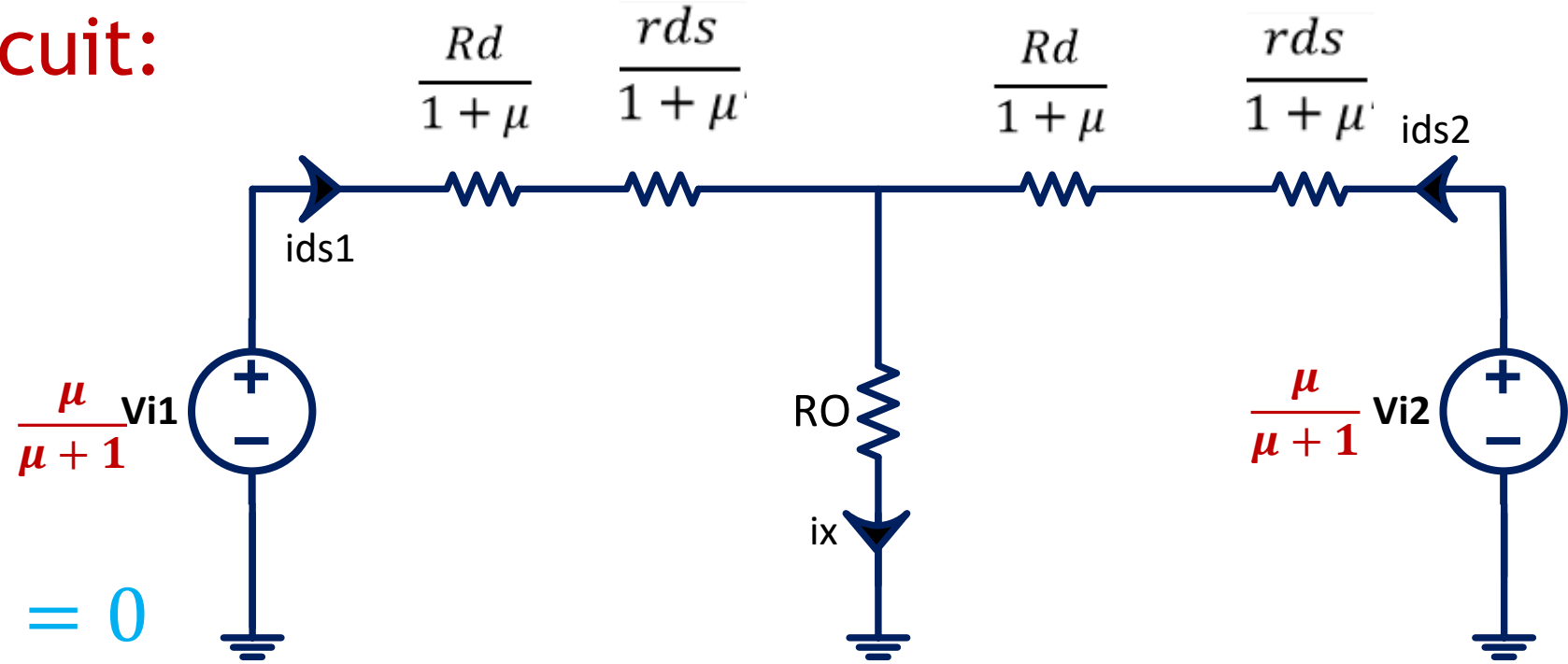


# Differential Amplifiers Using FET

Source equivalent Circuit:

$$v_{i1} = v_c - \frac{v_d}{2}$$

$$v_{i2} = v_c + \frac{v_d}{2}$$



1) To find  $A_d$ ; set  $v_c = 0$

$$v_{i1} = -\frac{v_d}{2}$$

$$v_{i2} = \frac{v_d}{2}$$

$$i_{ds1} = -i_{ds2}$$

$$i_{ds1} = \frac{\frac{\mu}{\mu+1} \left( -\frac{v_d}{2} \right)}{\frac{R_d + r_{ds}}{\mu+1}}$$

$$v_{o1} = \frac{R_d \frac{\mu}{\mu+1} v_d}{2 \left( \frac{R_d + r_{ds}}{\mu+1} \right)}$$

$i_x = 0$  symmetry

$$v_{o1} = \frac{R_d \frac{\mu}{\mu + 1} v_d}{2 \left( \frac{R_d + r_{ds}}{\mu + 1} \right)}$$

$$A_d = \left. \frac{v_{o1}}{v_d} \right|_{v_c=0}$$

$$A_d = \frac{R_d \frac{\mu}{\mu + 1}}{2 \left( \frac{R_d + r_{ds}}{\mu + 1} \right)}$$

# Differential Amplifiers Using FET

Ac Small signal equivalent Circuit:

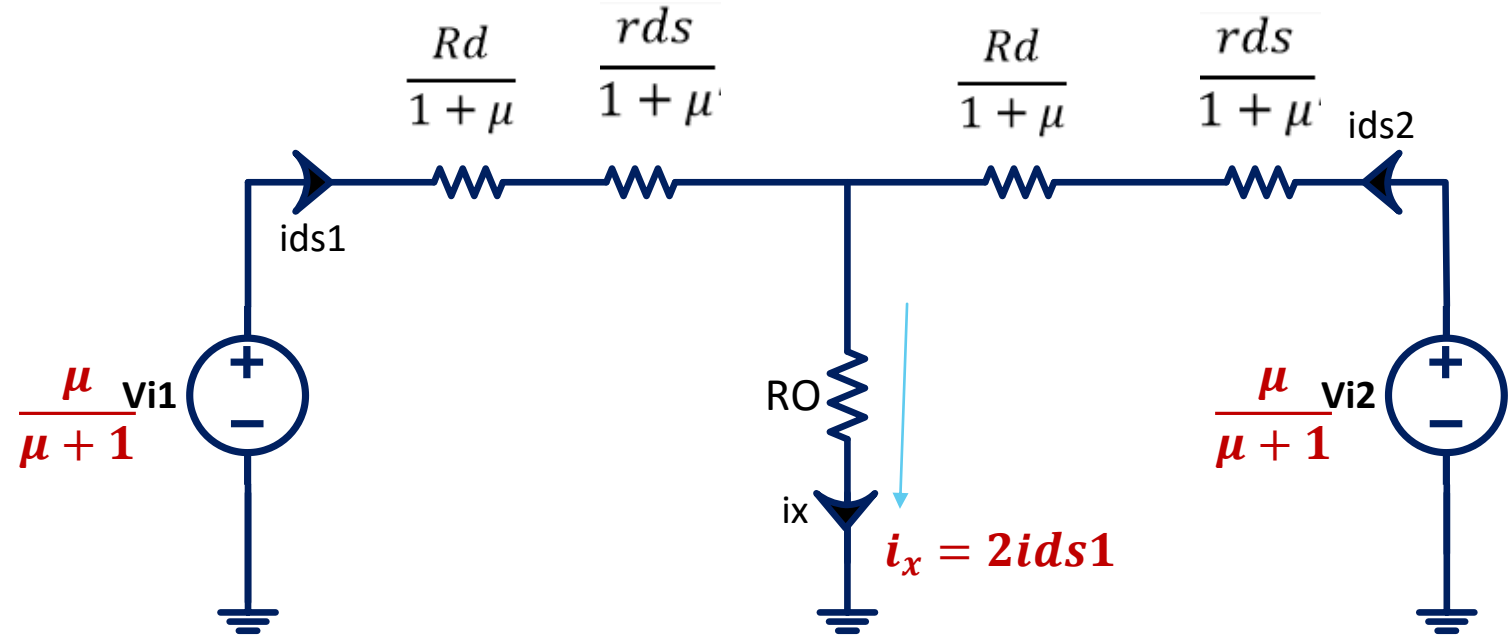
2) To find  $A_c = \frac{v_{o1}}{v_c} \Big|_{v_d=0}$

$$v_{i1} = v_c$$

$$v_{i2} = v_c$$

$$i_{ds1} = i_{ds2}$$

$$i_x = 2i_{ds1}$$



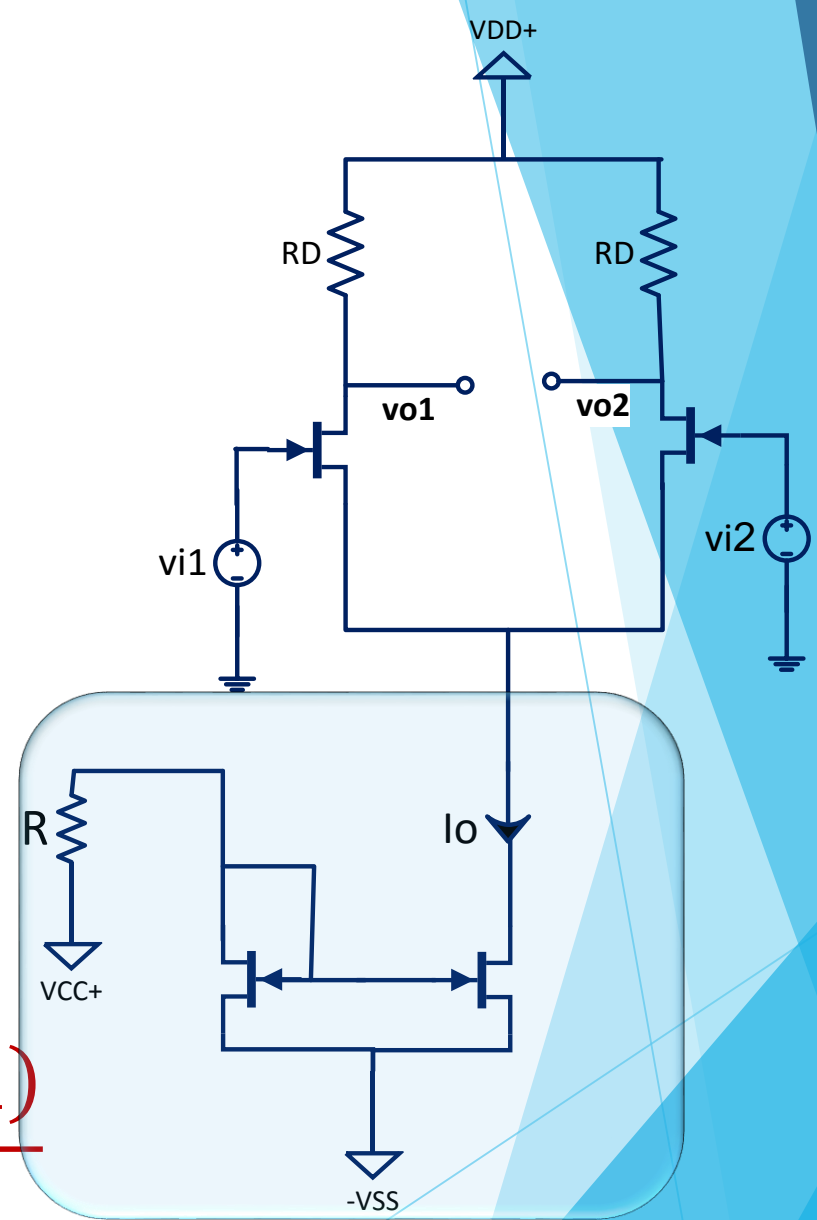
$$i_{ds1} = \frac{v_c \frac{\mu}{\mu + 1}}{\left( \frac{R_d + r_{ds}}{\mu + 1} \right) + 2R_o}$$



$$v_{o1} = \frac{-R_d \frac{\mu}{\mu + 1}}{\left(\frac{R_d + r_{ds}}{\mu + 1}\right) + 2R_o} v_c$$

$$\therefore A_c = \frac{R_d \frac{\mu}{\mu + 1}}{\left(\frac{R_d + r_{ds}}{\mu + 1}\right) + 2R_o}$$

$$CMRR = \left| \frac{A_d}{A_c} \right| = \frac{r_{ds} + R_d + 2R_o(\mu + 1)}{2(R_d + r_{ds})}$$



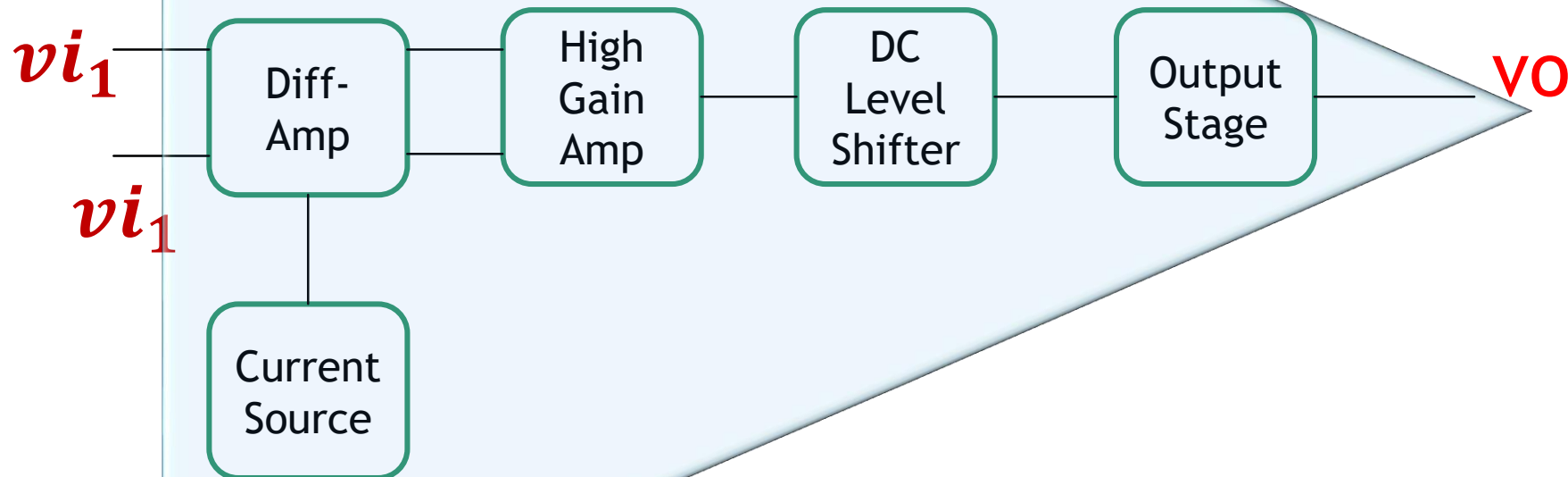
# *The Operational Amplifier*

Very high voltage gain ; 200,000

Very High input impedance ; 10M ohm

Very small output impedance ; 75ohm

Designed to do mathematical operations such as addition , subtraction ....



# The Operational Amplifier

## DC Level Shifter

To make  $v_o = 0$ , when

$$v_{i1} = v_{i2} = 0$$

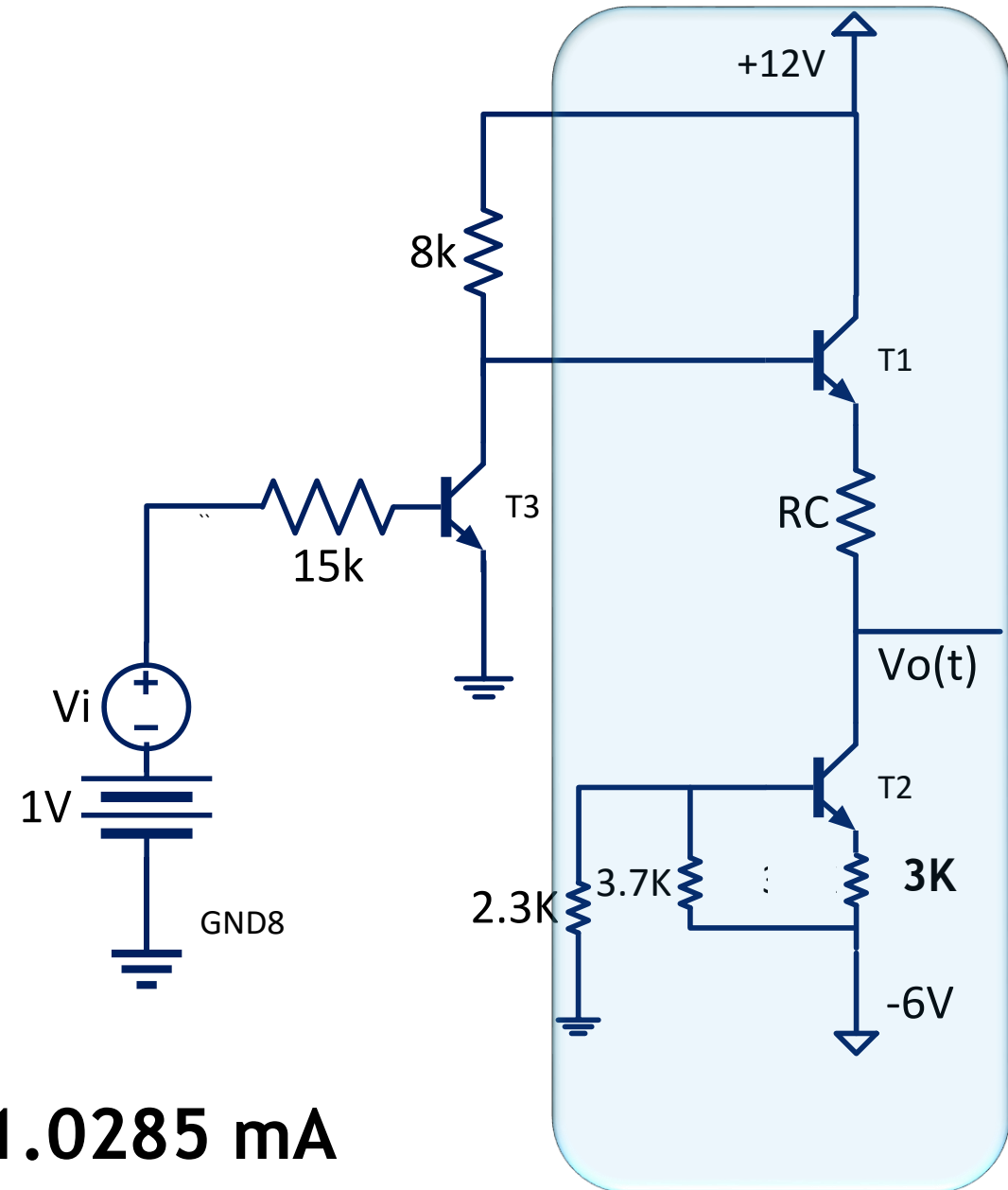
$$R_C = \text{?????}$$

$$R_{TH} = 2.3k \parallel 3.7k = 1.418k$$

$$V_{TH} = \frac{2.3k}{2.3k + 3.7k} (-6) = -2.2V$$

$$I_{E2} = \frac{V_{TH} - 0.7 + 6}{3k + \frac{R_{TH}}{\beta + 1}}$$

$$I_{E2} = 1.0285 \text{ mA}$$

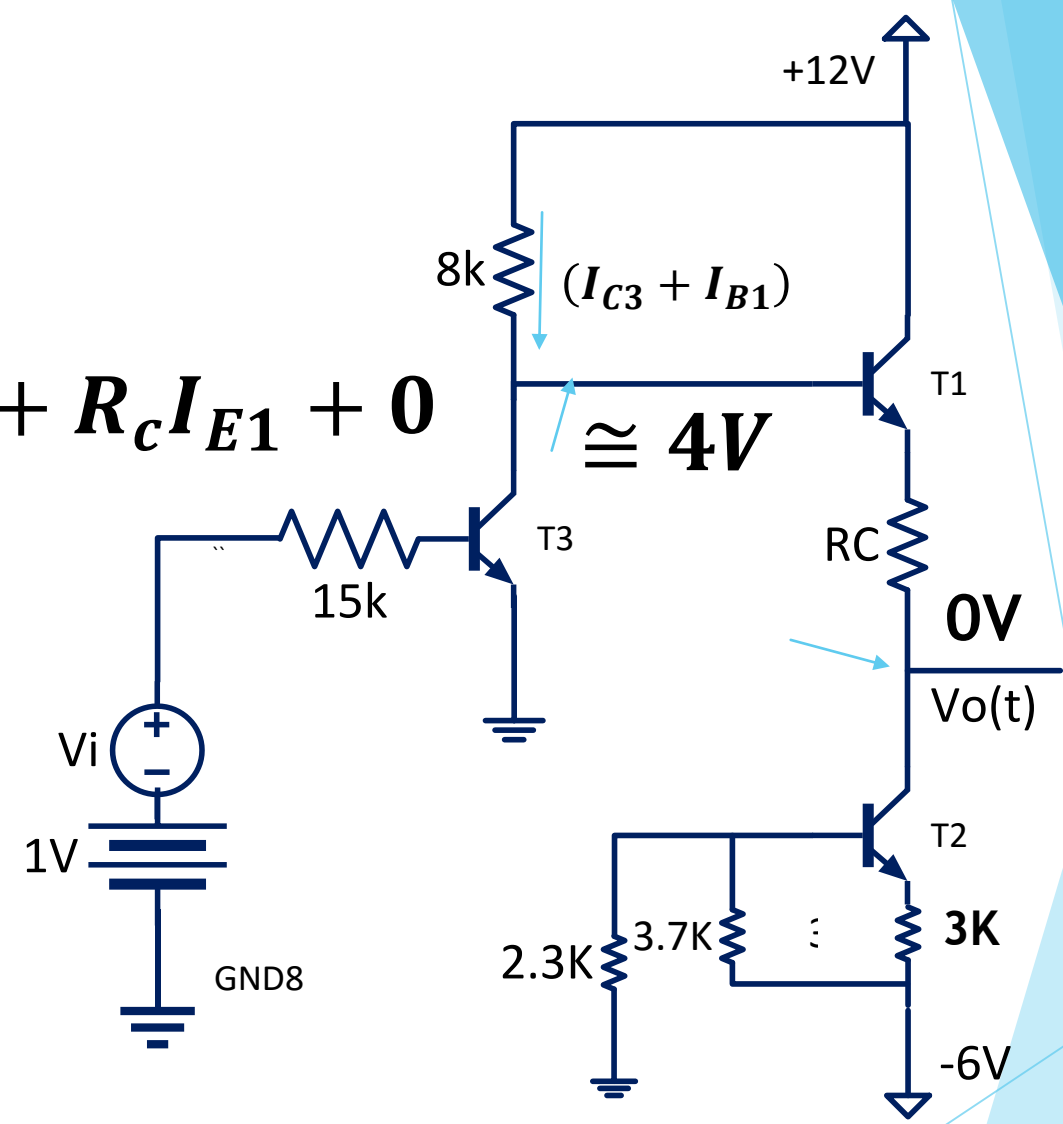


$$I_{B3} = \frac{1-0.7}{15k} = 0.02\text{mA} \blacktriangleright$$

$$I_{C3} = 1\text{mA}$$

$$12 = 8k(I_{C3} + I_{B1}) + 0.7 + R_c I_{E1} + 0$$

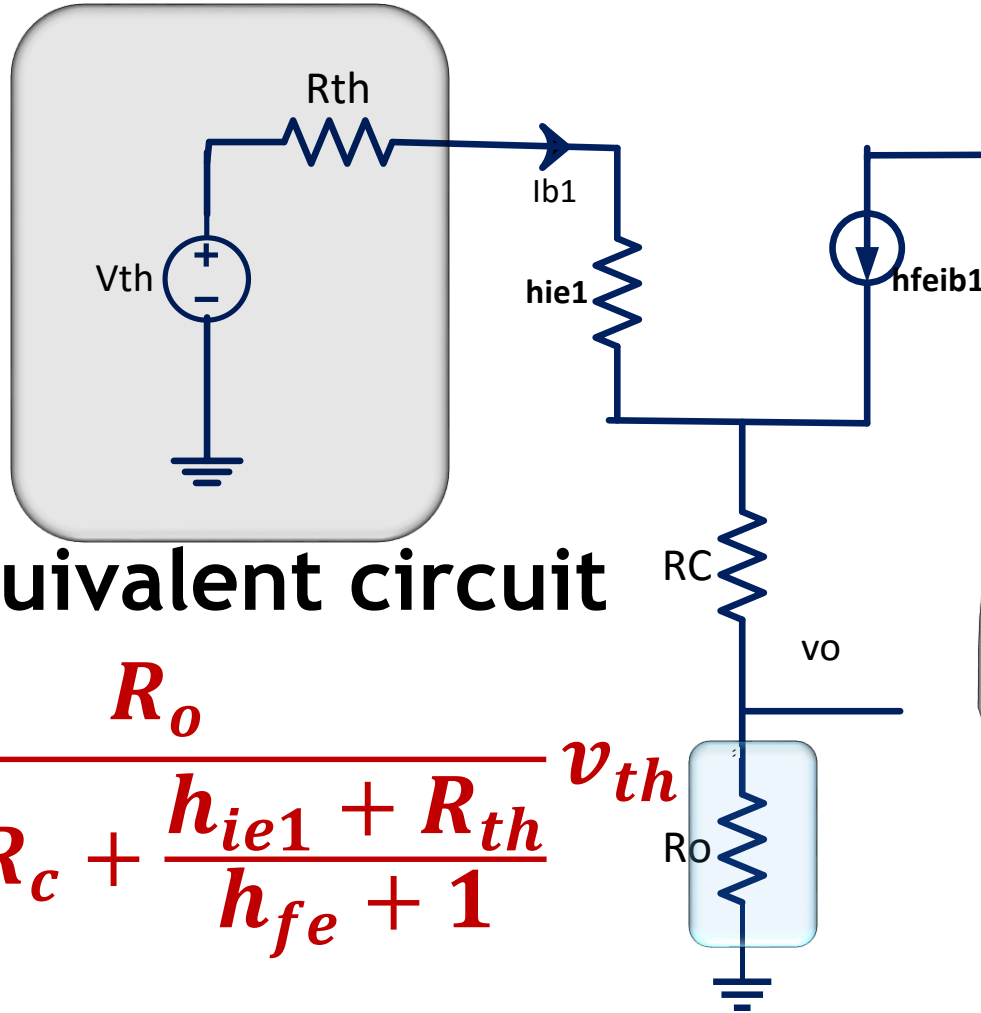
$$\therefore R_c \cong 3.3k$$



# THE FUNCTION OF THE CAPACITOR

# The Operational Amplifier

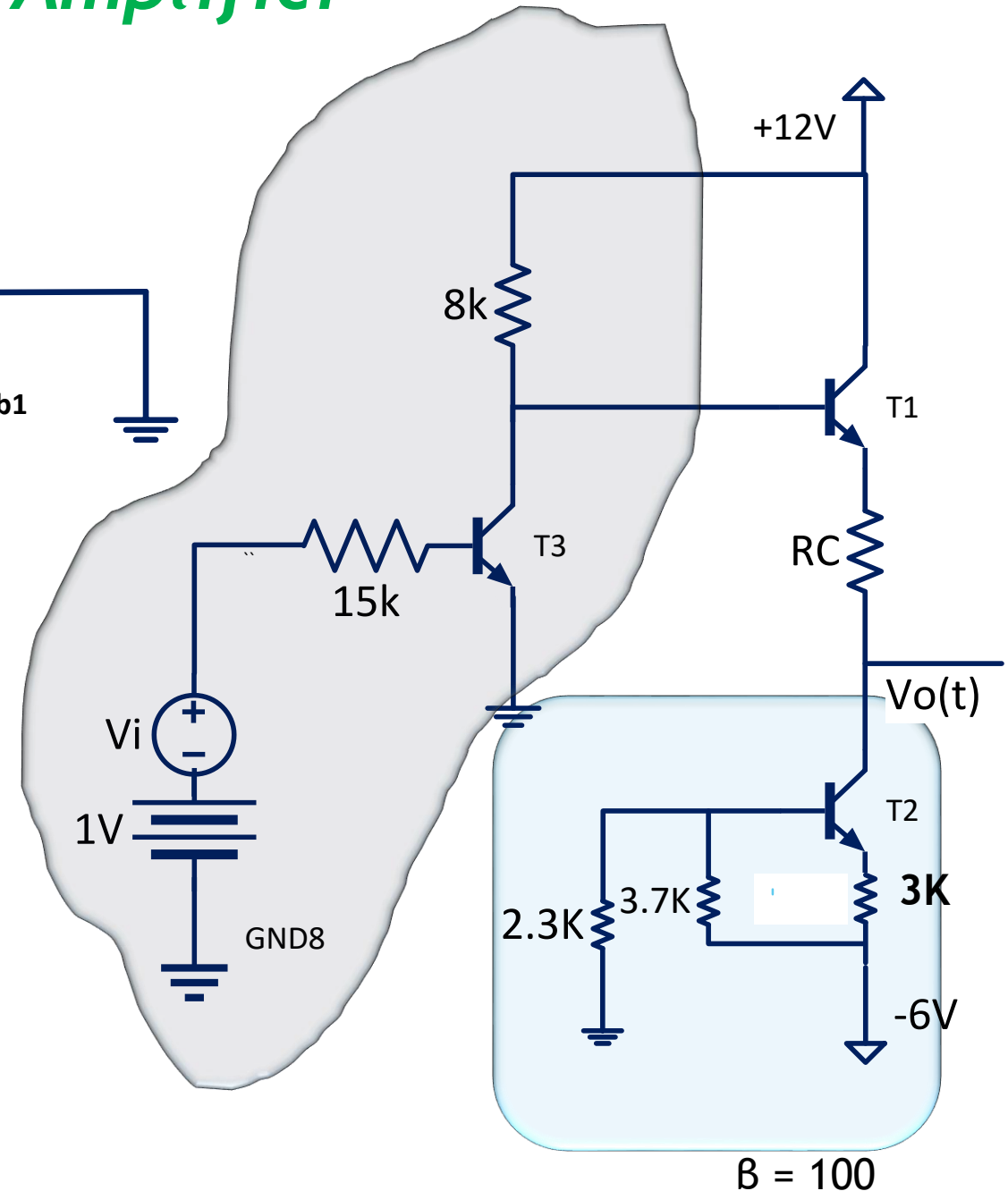
Ac Small signal equivalent Circuit:



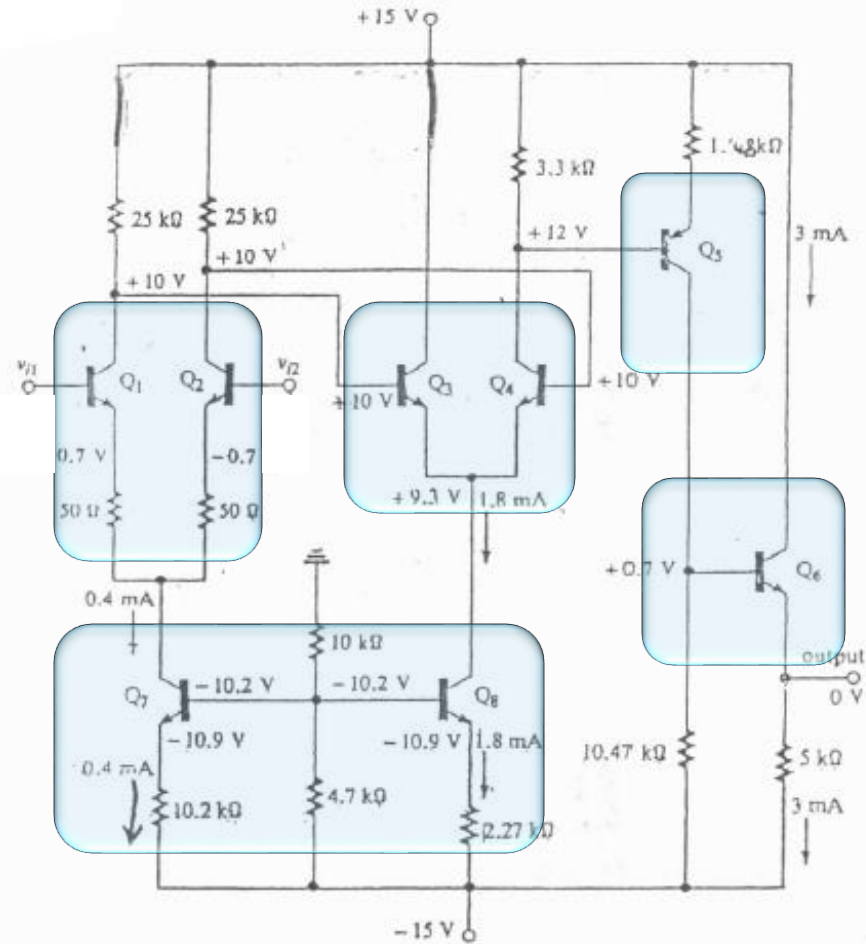
Emitter equivalent circuit

$$v_o = \frac{R_o}{R_o + R_c + \frac{h_{ie1} + R_{th}}{h_{fe} + 1}} v_{th}$$

$$v_o \approx v_{th}$$



# Complete Operational Amplifier



Q5 pnp DC Level Shifter

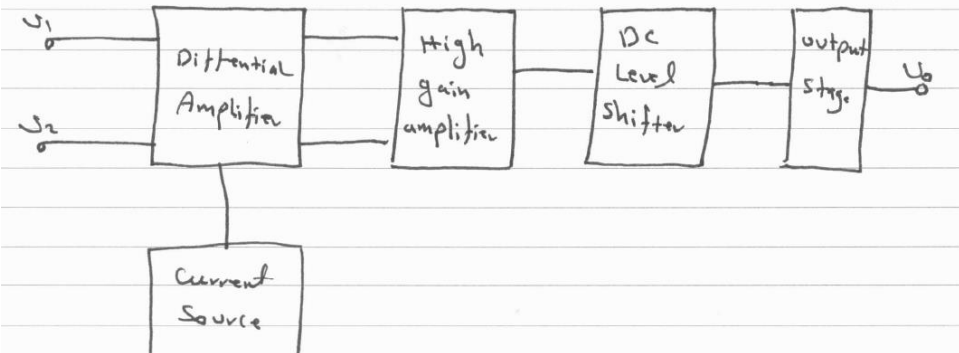
ALL  $\beta = 100$

Q6 Output Stage CC

Q1 and Q2 Differential Amplifier

Q3 and Q4 High Gain Amplifier

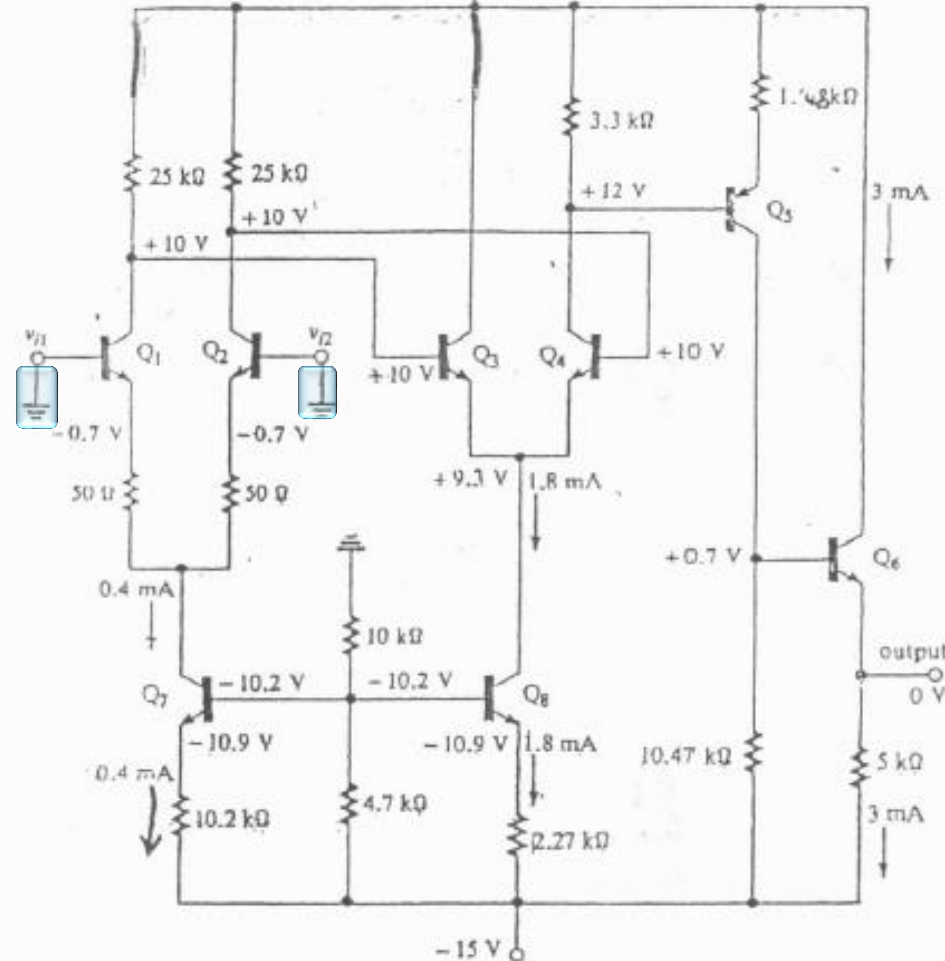
Q7 and Q8 Current Source



# Complete Operational Amplifier :

DC Analysis :

$$V_{i2} = V_{i1} = 0$$



All  $\beta = 100$

ALL  $\beta = 100$

$$V_{B7} = V_{B8} = (10K) \cdot (-15V) / (10K + 4.7K) = -10.2V$$

$$V_{E7} = V_{E8} = -10.2V - 0.7V = -10.9V$$

$$I_{E7} = (+15 - 10.9) / R_{E7} = 0.4mA$$

$$I_{E8} = (+15 - 10.9) / R_{E8} = 1.8mA$$

$$I_{CQ1} = I_{CQ2} = 0.5 I_{E7} = 0.2\text{mA}$$

$$I_{CQ3} = I_{CQ4} = 0.5 I_{E8} = 0.9\text{mA}$$

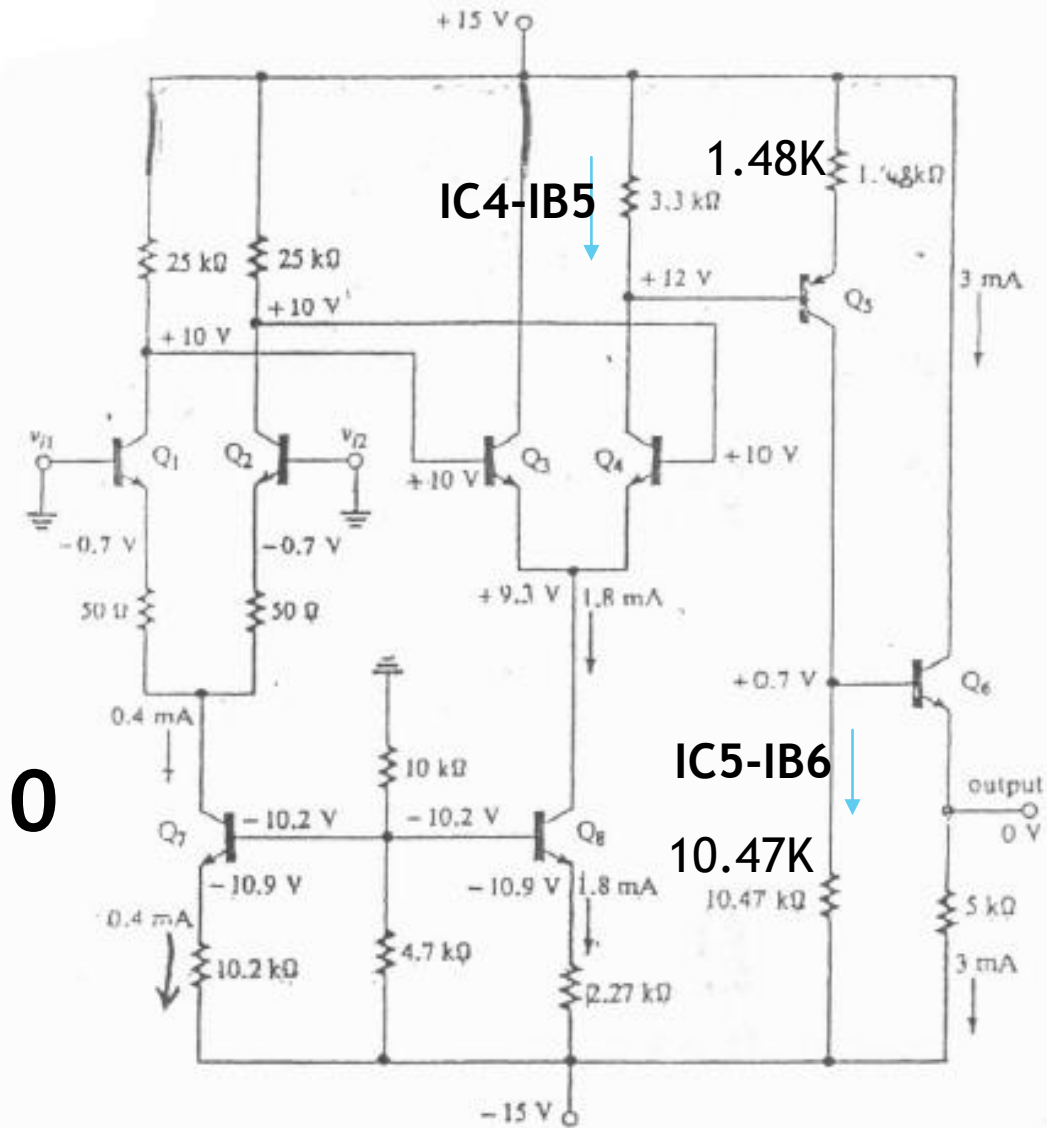
$$0 = 3.3\text{K} (I_{C4} - I_{B5}) + V_{BE5} - 1.48\text{K} I_{E5}$$

$$I_{E5} = 1.53\text{ mA}$$

$$V_{BE6} + 5\text{K} I_{E6} - 10.47\text{K} (I_{C5} - I_{B6}) = 0$$

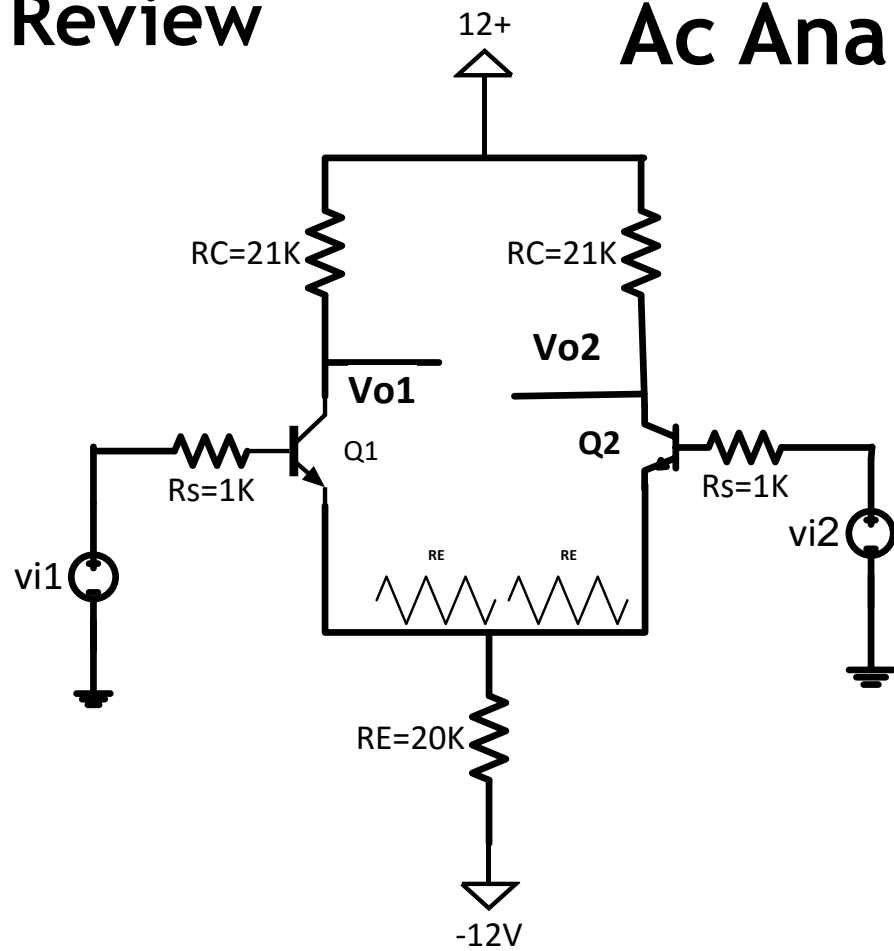
$$I_{E6} = 3\text{mA}$$

$$\text{Or } I_{E6} = \frac{0 - (-15)}{5\text{K}} = 3\text{mA}$$





# Review



# Ac Analysis : Differential Mode Analysis

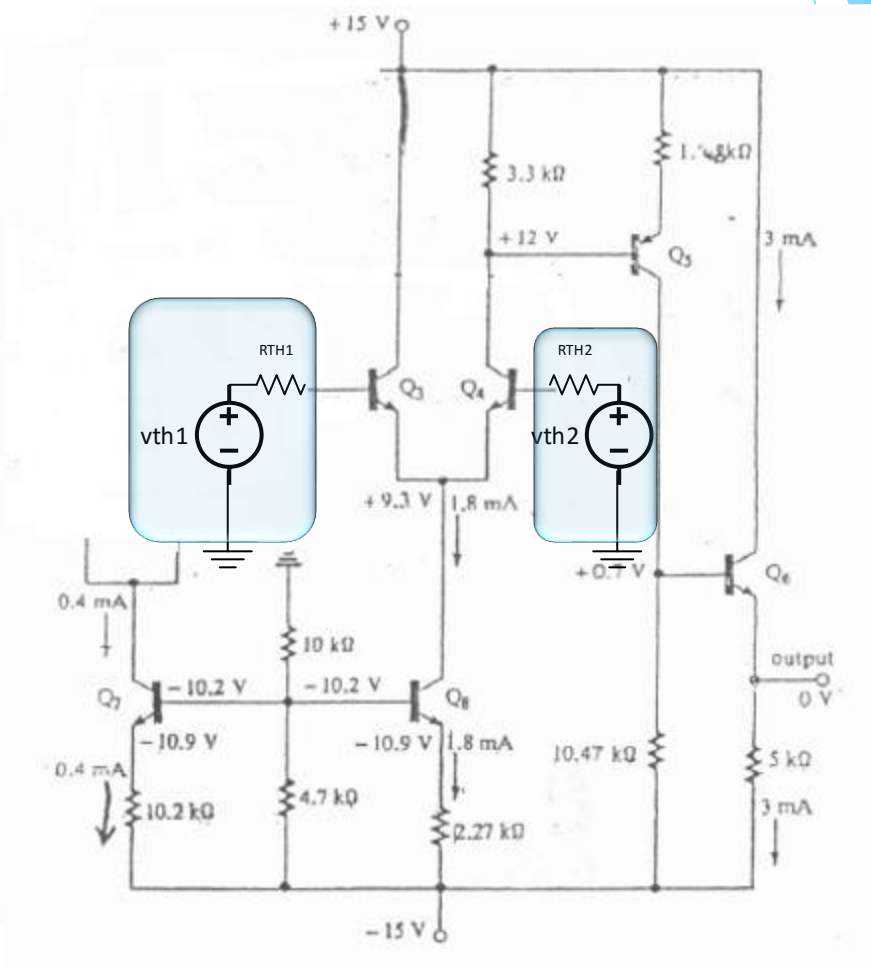
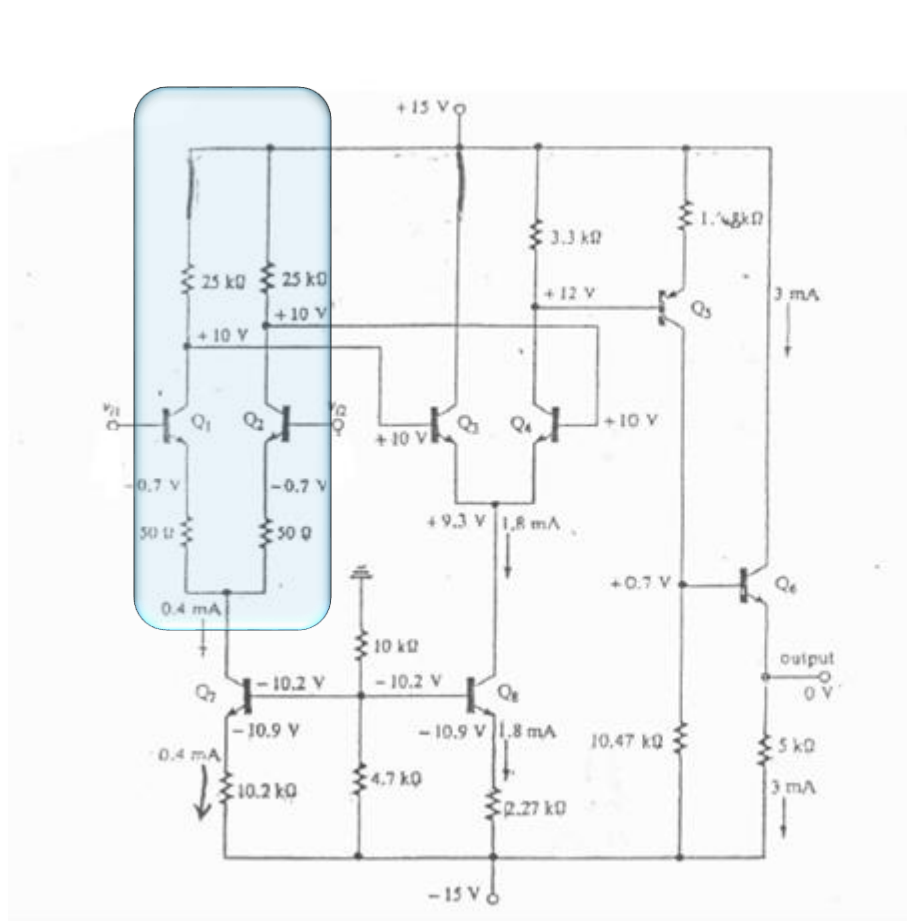
If we have  $RE1 = RE2$

$$v_{o1} = \frac{R_c v_d}{2 \left( h_{ib} + RE1 + \frac{R_s}{h_{fe+1}} \right)}$$

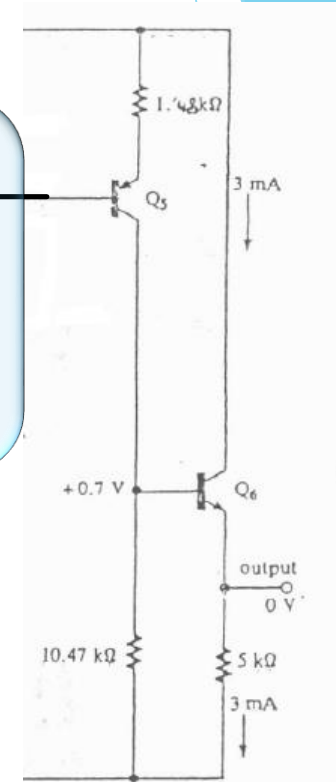
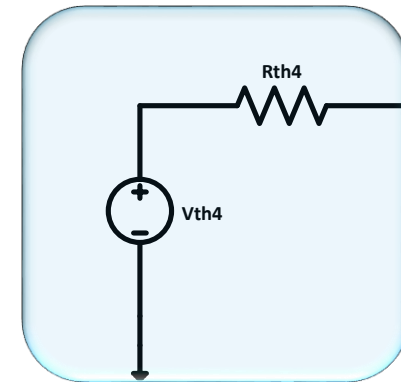
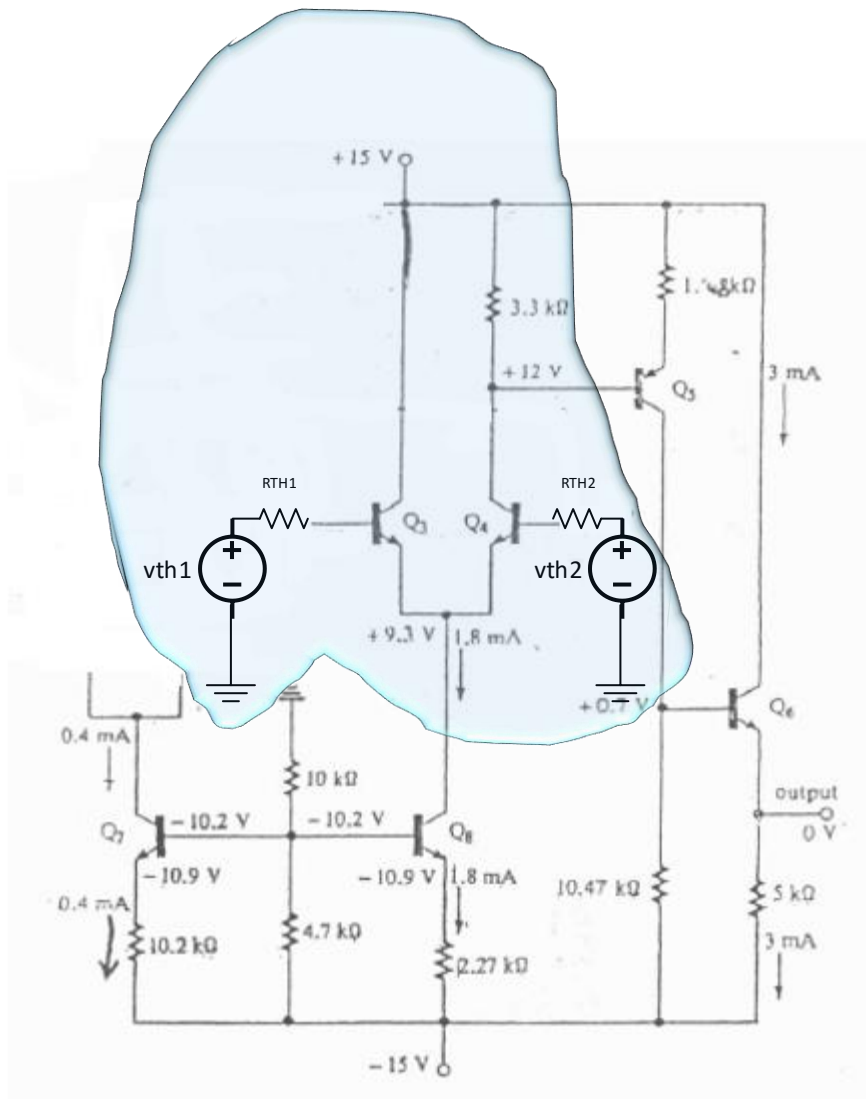
$$v_{o2} = - \frac{R_c v_d}{2 \left( h_{ib} + RE1 + \frac{R_s}{h_{fe+1}} \right)}$$

$$v_{o1} = \frac{R_c v_d}{2 \left( h_{ib} + \frac{R_s}{h_{fe+1}} \right)}$$

$$v_{o2} = - \frac{R_c v_d}{2 \left( h_{ib} + \frac{R_s}{h_{fe+1}} \right)}$$



All  $\beta = 100$



All  $\beta = 100$

# Ac Small signal Analysis

- hie1 = 13kΩ
- hie2 = 13kΩ
- hie3 = 2.89KΩ
- hie4 = 2.89KΩ
- hie5 = 1.7KΩ
- hie6 = 0.87KΩ

$$V_{th1} = \frac{\left(+\frac{vd1}{2}\right)R_{c1}}{R_{e1} + h_{ib1} + R_{s1}/(h_{fe} + 1)}$$

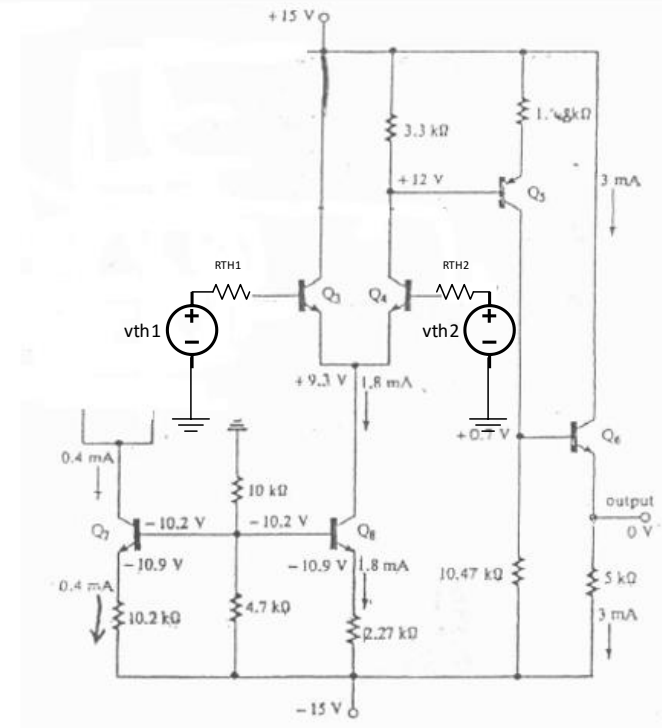
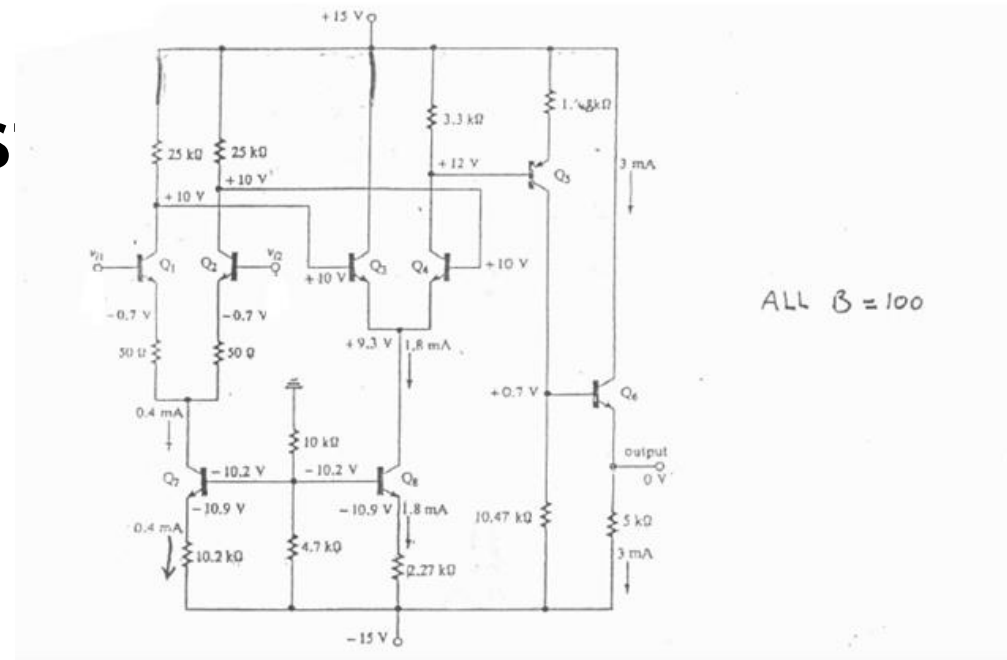
$$R_{s1} = 0 \quad v_{d1} = V_{i2} - V_{i1}$$

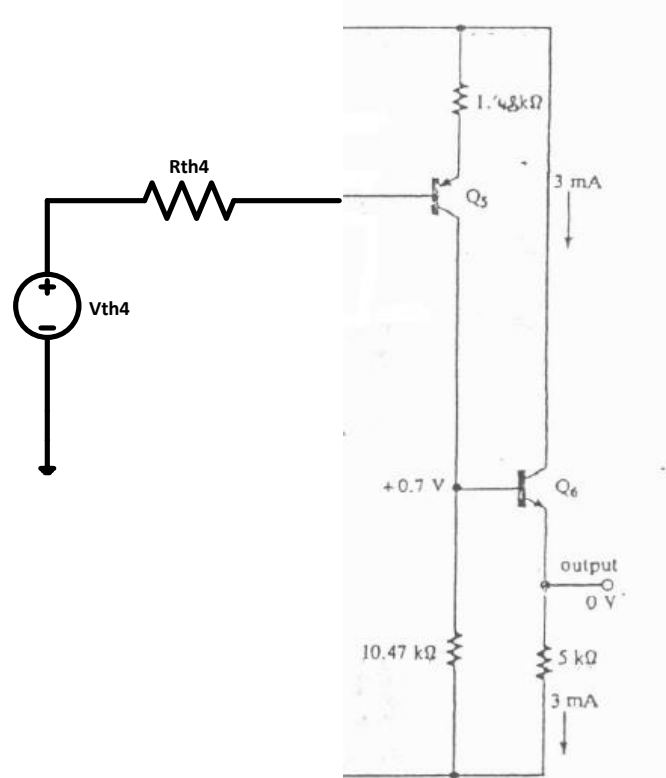
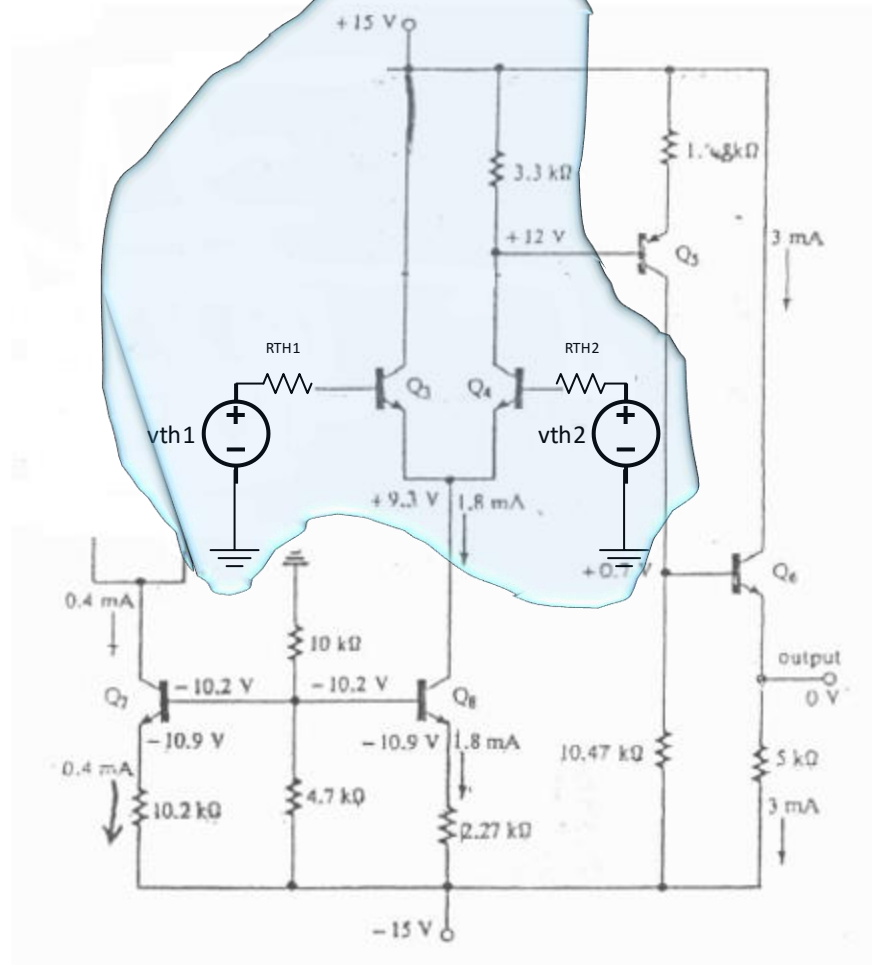
$$V_{th1} = 69.4 V_{d1} \quad R_{th1} = R_{c1} = 25K$$

$$V_{th2} = -V_{th1} = \frac{-\left(+\frac{vd1}{2}\right)R_{c1}}{R_{e1} + h_{ib1} + R_{s1}/(h_{fe} + 1)}$$

$$V_{th2} = -69.4 V_{d1}$$

$$R_{th2} = 25K$$





$$V_{th4} = \frac{-\left(+\frac{v_{d4}}{2}\right)RC4}{R_{e4} + h_{ib4} + R_{s4}/(h_{fe} + 1)} = 829 V_{d4}$$

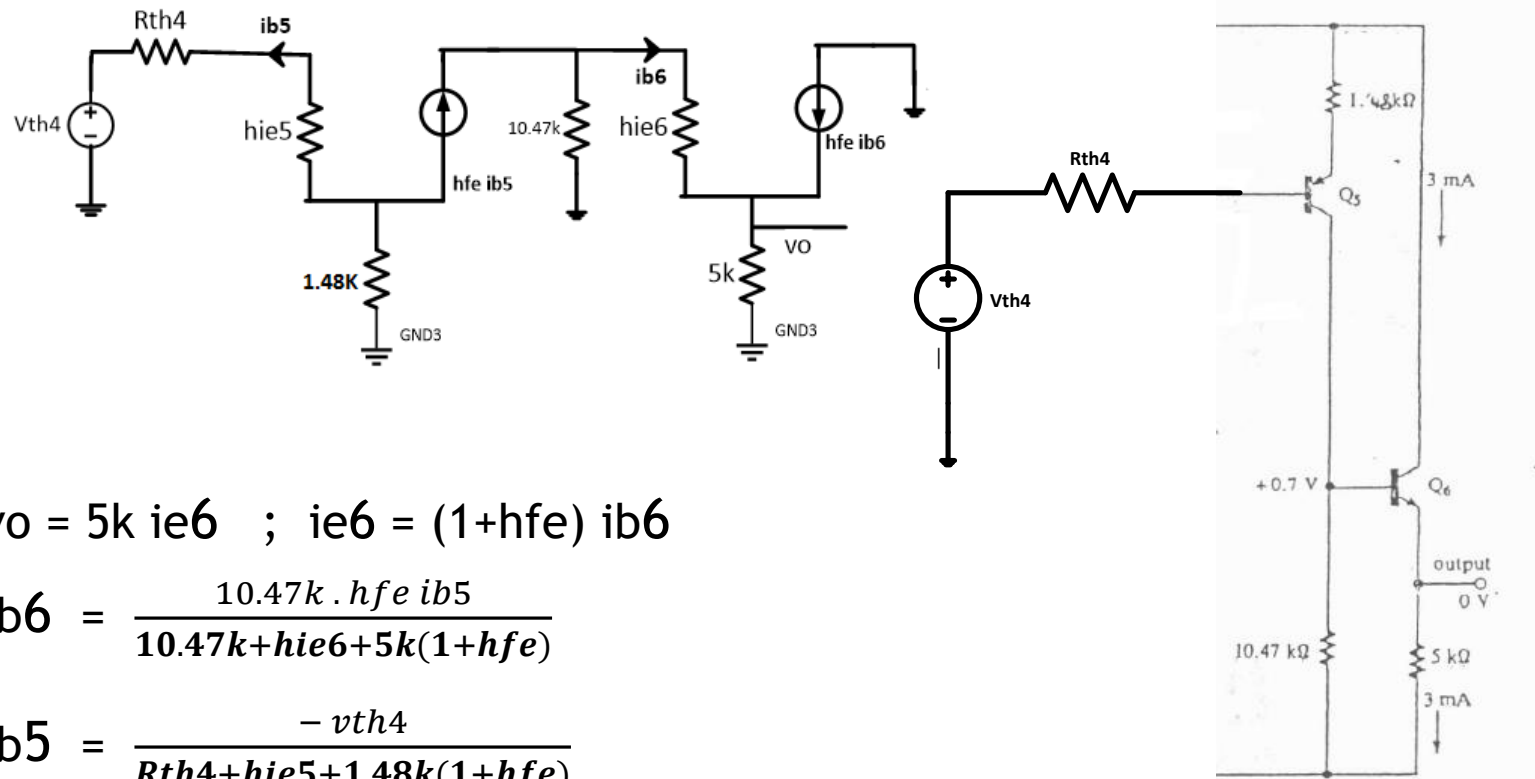
$$R_{e4} = 0$$

$$R_{s4} = 25K$$

$$V_{d4} = (V_{th2} - V_{th1})$$

$$R_{th4} = 3.3k$$

## Ac Small Signal Equivalent Circuit Of the DC Level Shifter and the Output Stages :

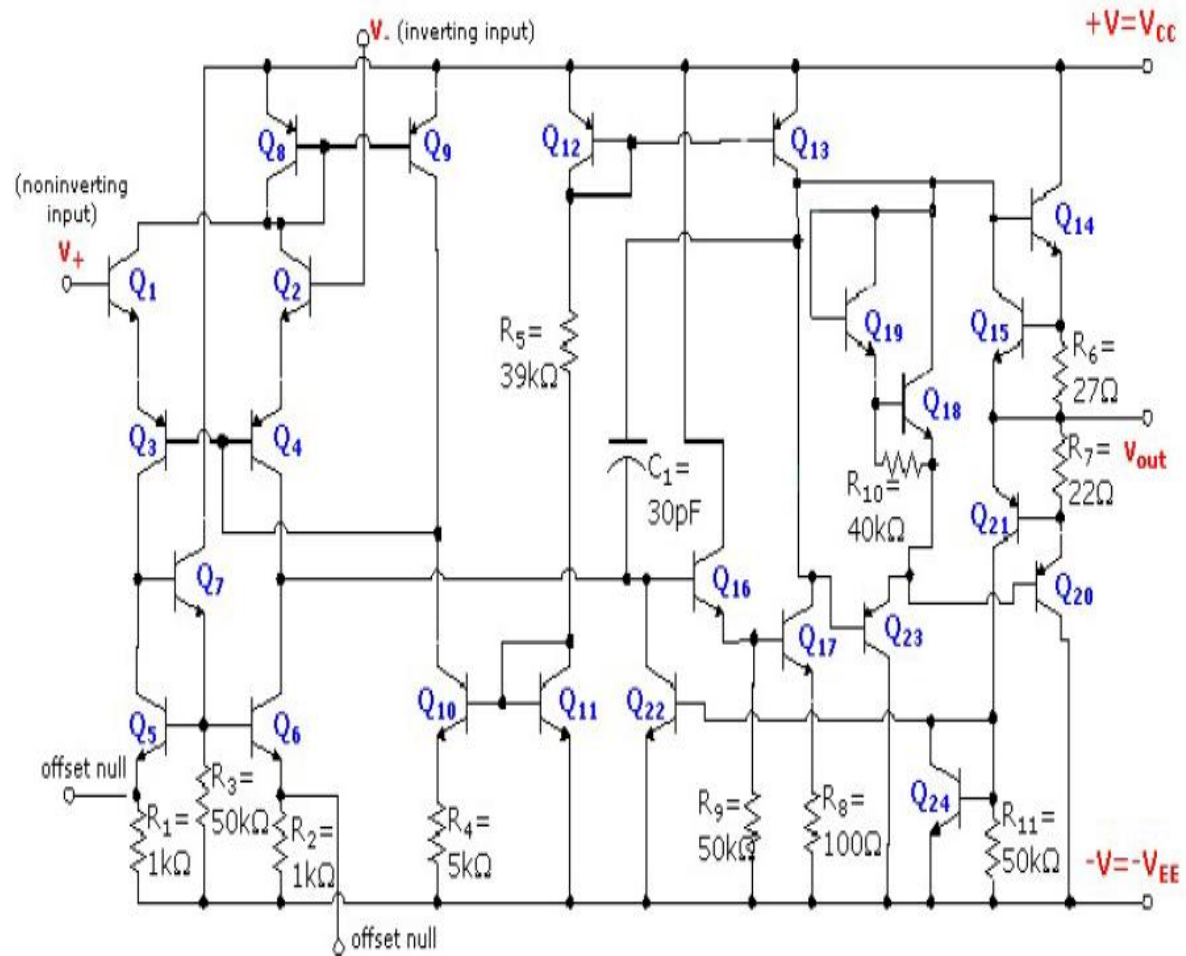


$$v_o = 5k i_{e6} \quad ; \quad i_{e6} = (1+h_{fe}) i_{b6}$$

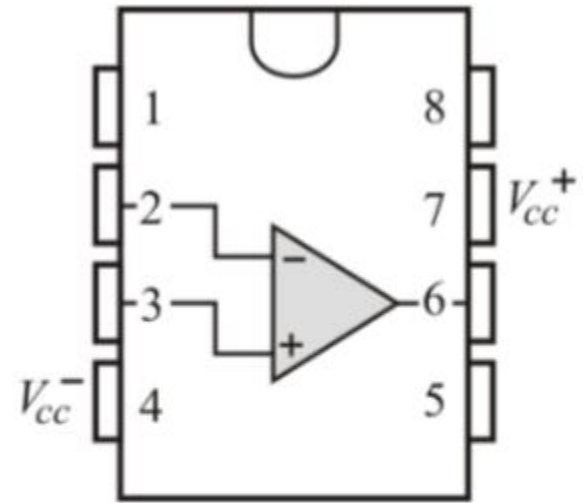
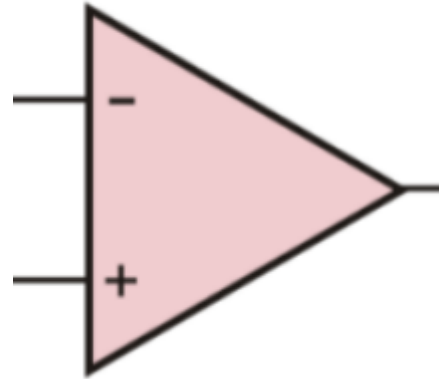
$$i_{b6} = \frac{10.47k \cdot h_{fe} i_{b5}}{10.47k + h_{ie6} + 5k(1+h_{fe})}$$

$$i_{b5} = \frac{-v_{th4}}{R_{th4} + h_{ie5} + 1.48k(1+h_{fe})}$$

$$V_o = - 5540 v_{d1} = + 5540 (v_{i1} - v_{i2})$$



# Operational Amplifier



**741 Op-Amp Pin out**