Introduction.

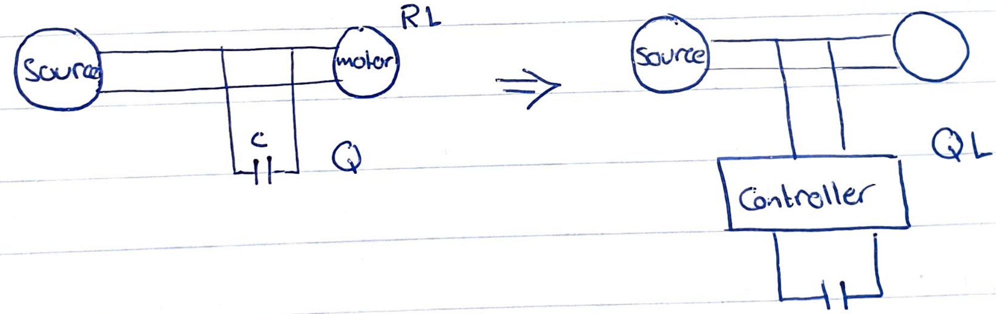
Definition of electronics; It is application of Solid - State Power devices and circuits for the Control and Conversion of electric energy from one to another form suitable for the load.

- Applications:

- Motor or Machine Control.
- Heat Control.
- Power supply.
- High voltage DC transmision system (HVDC)
- light Control
- Charging & battery Cells.
- Electric vehicks.
- Flexible AC transmision (FACT)

$S = \sqrt{P^2 + Q^2}$

$S = \sqrt{P^2}$



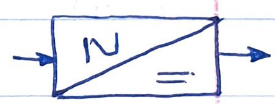
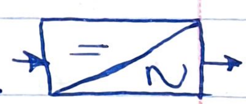
- Solid - State Devices.

1. Power Diodes.
2. Silicon Control Rectifier (SCR).
3. Metal - Oxide Field efect transistor (MOSFET)
4. Insulated Gate Bipolar transistor (IGBT).
5. Bipolar Junction Transistor (BJT).

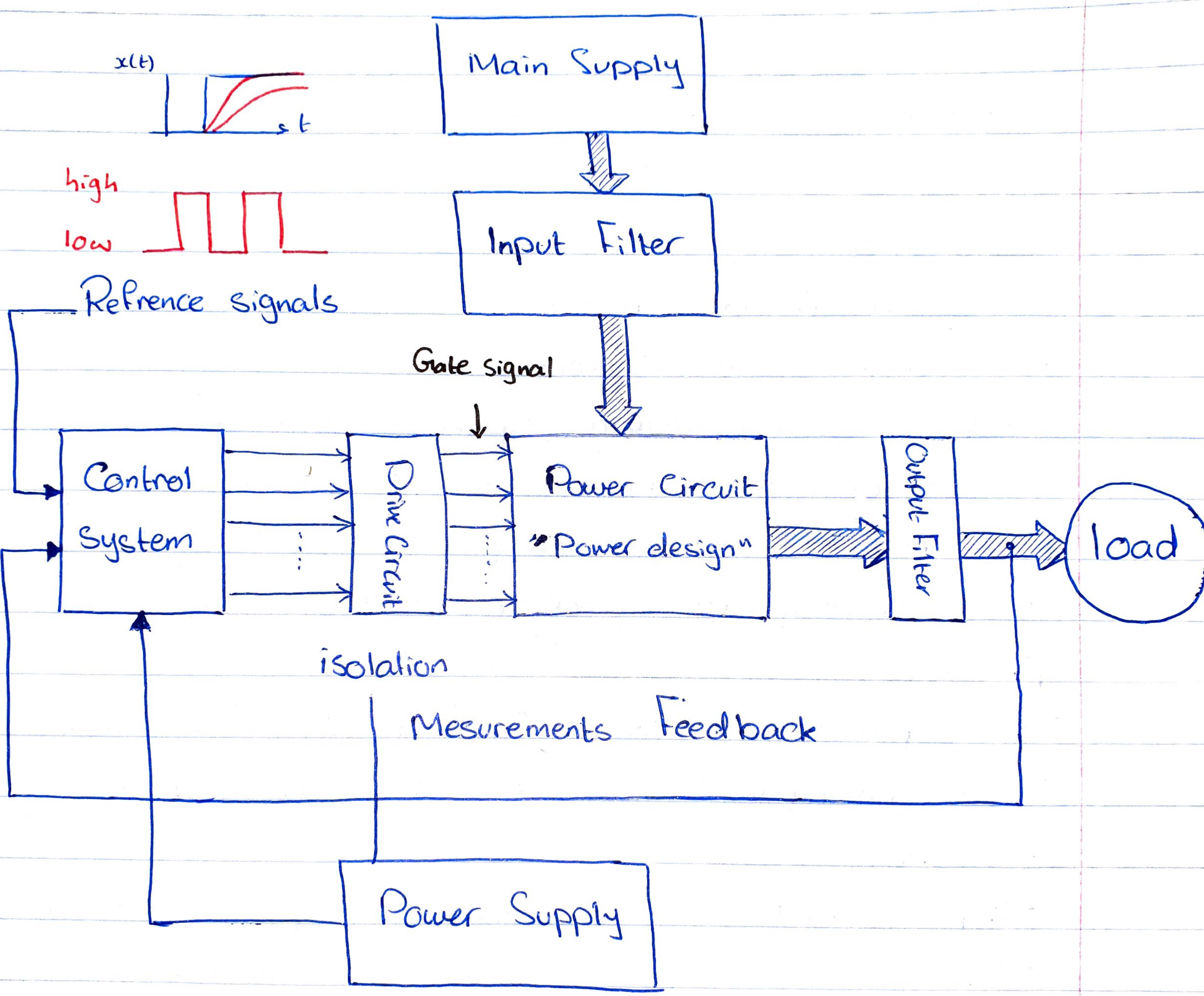
- Types of Power electronic Circuits.

1. Diode rectifiers AC $\Rightarrow$ DC uncontral rectifier.
2. AC _ DC Converters (Controlled rectifier).
3. DC _ DC (DC choppers) stepup or stepdown (Boost Converters or Buck Converters).
4. DC - AC Converters (Inverters); Variable voltage variable frequency.
5. AC _ AC Converters (AC voltage Controller).

- Conversion types and symbols.

	Conversion From / to	Conversion Name	Conversion function	Conversion Symbol
1	AC / DC	Rectifier "Controller or uncontroller"	AC to Unipolar DC Current.	
2	DC / DC	Chopper	Constant DC to Constant or variable DC.	
3	DC / AC	Inverter	Fixed DC to AC of desired voltage & frequency.	
4	AC / AC	AC Voltage regulater or Controller	AC of desinved voltage and/or frequency from line AC Supply.	

- A typical Block Diagram of Power Conversion system.



- "Main Components"

1. Power Circuit: It is the power electronic Converter which Consists of Solite-State devices and some passive elements.
(AC/DC, DC/DC, DC/AC, AC/AC).

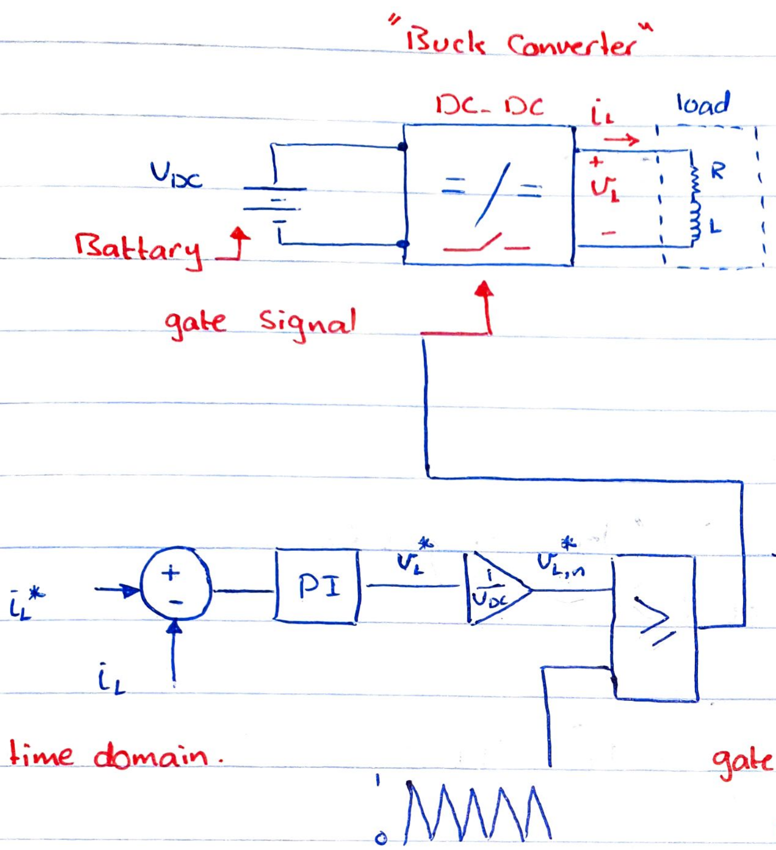
2. Control System: It matches the power Converter with the load to meet the load requirments.
- Inputs of the Control System:
  - Reference current, voltage, speed or torque.
  - Measure of Current, voltage, speed or torque.
  - Gain of the PI Controller proportinal Integral Controller.
  - Measured tempreture, presure, torque for monotoring purposes.
- The Output of Controller is the find gating signal, which are issued directly to the base/gate of Power or devices.
- Example:
  - Micro Controller
  - Micro Processor
  - Digital signal processor DSP
  - Field programmeble Gate Arry FPGA

3. loads:

loads

- Static load
  Residantal lighting & Heaters
- Dynamic load.
  Motor & Machin in general.

- Current Controller in DC_DC Converts with static RL_load.



i_L: Load current "Measured current"

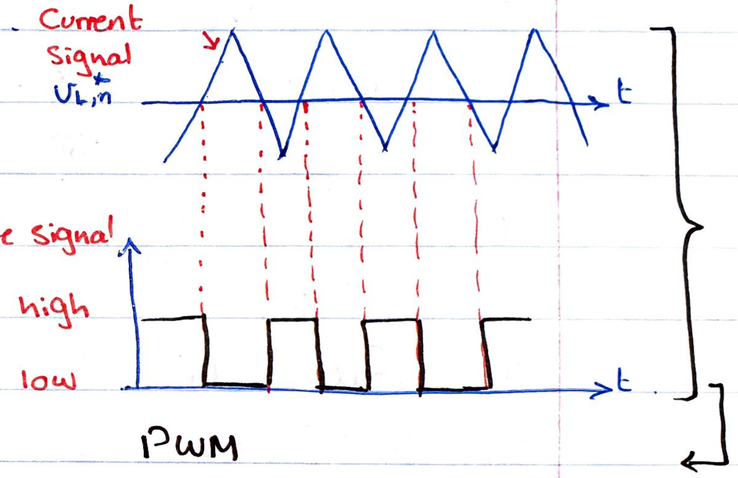
U_L: Load voltage.

i_L^*: Reference load current.

PI: Proportional_Integral Controller.

$U_{L,n}^*$: Normalized value of U_L^*

$$U_{L,n}^* = \frac{U_L^*}{U_{DC}}$$



PWM

Pulse_width_Modulation.

- PI Controller:

$U_L^* = k_p e + k_i \int e\,dt.$

k_p: Prepertional gain of the PI.

k_i: integral gain of the PI $e(t)$ is the error signal.

- Design of PI current controller:

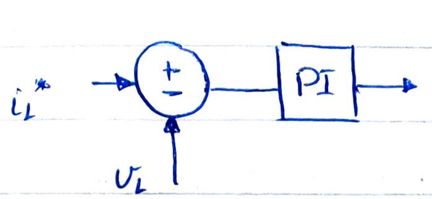
KVL in the power circuit.

$U_L = R i_L + L \frac{di_L}{dt}$; 1^{st} order differential equation.

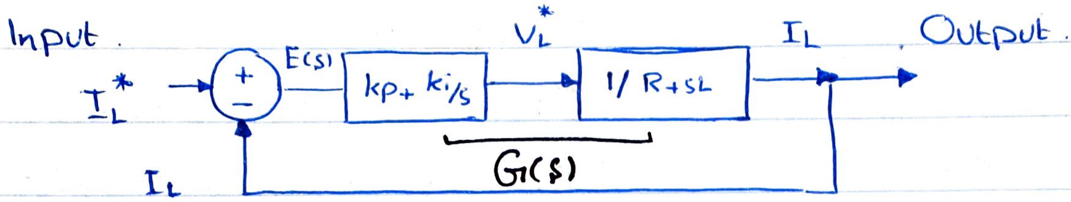
t_domain, Linear & time inveriant.

$V_L(s) = R I_L(s) + L s I_L(s)$

$V_L(s) = (R + sL) I_L(s)$ S_domain.



t_domain.



S-domain.

- Transfer function:

$$T(s) = \frac{\text{Output}}{\text{Input}} = \frac{I_L(s)}{I_L^*(s)}.$$

$$I_L = \frac{1}{R+sL} V_L = \left(\frac{1}{R+sL}\right)\left(kp + \frac{ki}{s}\right) E(s).$$

$$I_L(s) = \underbrace{\left(\frac{1}{R+sL}\right)\left(kp + \frac{ki}{s}\right)}_{G(s)}\left(I_L^* - I_L\right).$$

$$I_L = G(s)\,(I_L^* - I_L)$$

$$I_L = G\,I_L^* - G\,I_L \Rightarrow (1+G)\,I_L = G\,I_L^*$$

$$\frac{I_L}{I_L^*} = \frac{G}{1+G}$$

$$\frac{I_L}{I_L^*} = \frac{(1/R+sL)(kps+ki/s)}{1+(1/R+sL)((kps+ki)/s)}$$

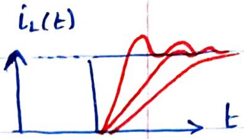
$$\frac{I_L}{I_L^*} = \frac{kps+ki}{s(R+sL)+kps+ki}$$

$$\frac{I_L}{I_L^*} = \frac{kps+ki}{s^2L+s(R+kp)+ki}$$

$$= \left(\frac{kp}{L}\right)\frac{s+(ki/kp)}{s^2+\left(\frac{kp+R}{L}\right)s+\left(\frac{ki}{L}\right)}$$

- The steady-state value of i_L.

$$i_{L,ss} = \lim_{t \to \infty} i_L(t) = \lim_{s \to 0} s\, I_L(s).$$



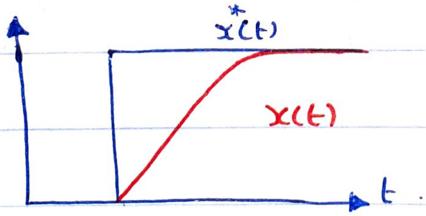
$$i_{L,ss} = \lim_{s \to 0} \frac{k_p}{L} \cdot \frac{s + (k_i/k_p)}{s^2 + \left(\frac{k_p + R}{L}\right)s + \frac{k_i}{L}} \cdot \underbrace{I_L^*(s) \cdot s}_{I} \qquad I_L(s) = \frac{I}{s}.$$

$$i_{L,ss} = \frac{k_p}{L} \cdot \frac{(k_i/k_p)}{(k_i/L)} \cdot I = I$$

- The general transfer function of 2^{nd} order closed-loop control system is given by:

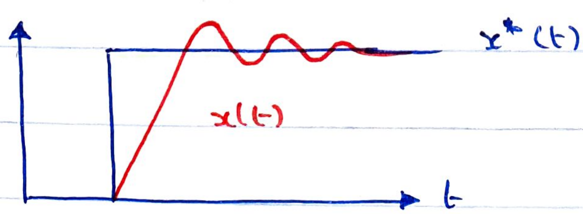
$$T(s) = k \frac{s + a}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{——— (2).}$$

- ζ is called the damping ratio $\zeta > 1 \Rightarrow$ Over damped response.
the actual value will reach its steady-state value without oscillation.



"slaw response"

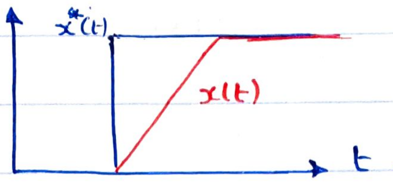
- $\zeta < 1 \Rightarrow$ Under damped response.
The actual value with oscillate about its final value.



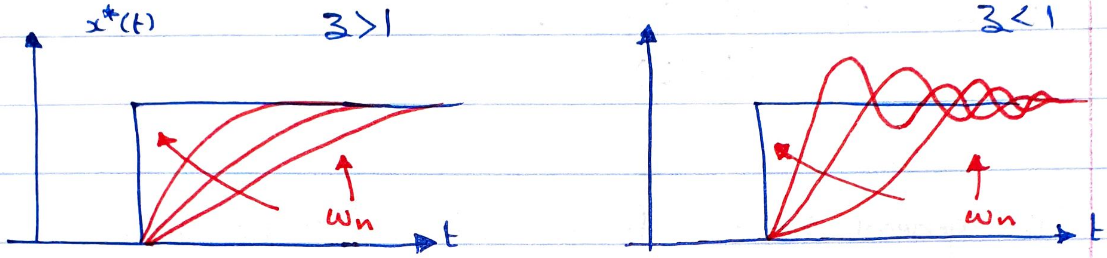
" Fast response"

- $\zeta = 1 \Rightarrow$ Critically damped.

The actual value is on the edge of oscillation.



- ω_n is called the natural frequency of the Controller.



By equating the equations (1) & (2), we get.

$k_i = \omega_n^2 L.$

$k_p = 2\zeta\omega_n L - R.$

- AC - DC Uncontrolled Converters "Rectifier".

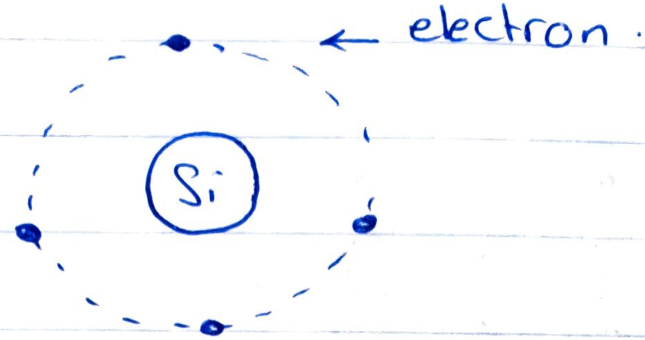
- Power diods. وطن قراءة

The diode acts as a switch to perform various function such as switching in rectifiers, free wheeling in switching regulator, energy transfer between Components, and energy feedback from load to source.

- Basic Semiconductor physics.

* The most Commonly used semiconductors are silicon and germanium.
The silicon material is cheaper and allows the diode to opreate of higher temprature, therefor, the germanium diodes are rarely used.

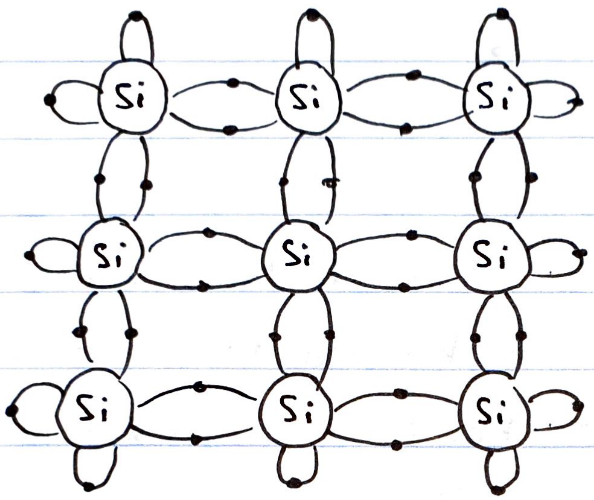
* The Silicon (Si) has 4 electrons in its Outer orbit and lies in the group IV of the periodic table.



* The array or Crystal of Semiconductor material is composed of Si atoms. Each atom is bonded to the nearest 4 atoms by تساهمية Covalent bonds. Each bond is composed of shared electrons between the two adjacent atoms.

- Semiconductor material

It is formed by using elements in group IV of periodic table such as silicon (Si) and germanium (Ge).

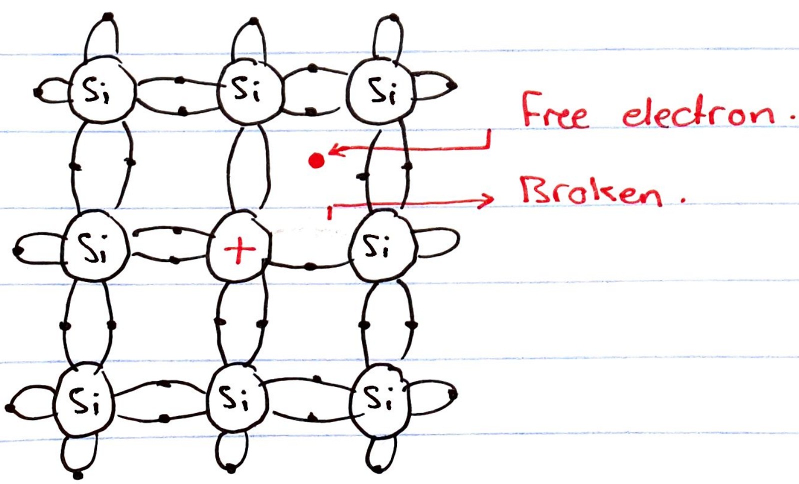


Array of Si atoms.

* At tempreture beyond absolute zero, some of covalent bonds will be broken due to thermal ionization, which creates positive changes + electrons.

$$A + kT \longrightarrow A^+ + e^-$$

A; neutral atom, kT; Thermal energy.



- Doped semiconductor. شوائب

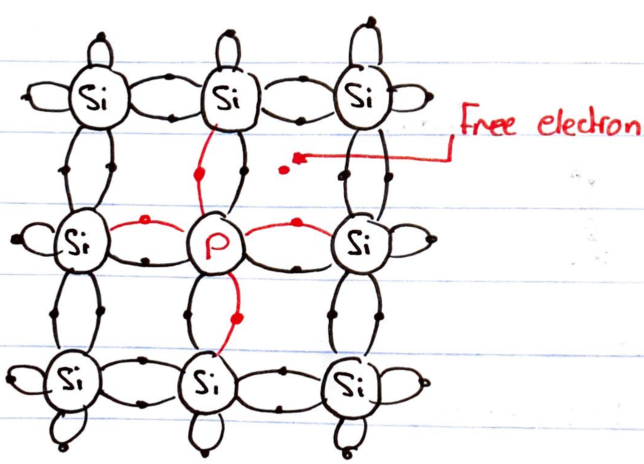
عملية تنشيط Doping process: It is the process of adding impurties to the semiconductor material to form the n-type & p-type material.

Impurties.

- Elements in group III of periodic table.
- They have 3 electrons in their outer shell.
- Example; Boron.

- Elements in group V of periodic table.
- They have 5 electrons in their outer shell.
- Example; phosphorous.

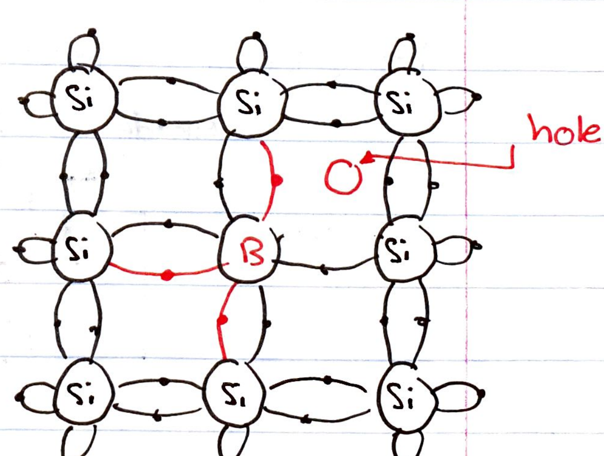
- n-type: The semiconductor (Si) is doped with some elements in group V (ph) → Free electrons auilable.
- p-type: The semiconductor (Si) is doped withe elements in group III (B) → Free holes.



n-type material

Majority charge carriers → electrons.

Minority carriers → holes.



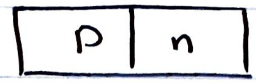
p-type material.

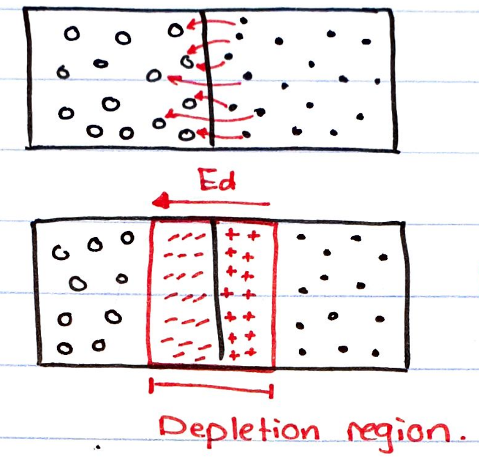
Majority charge carriers → holes.

Minority charge carriers → electrons.

- Pn- junction.

It is formed by connecting the p-type material with the n-type material.

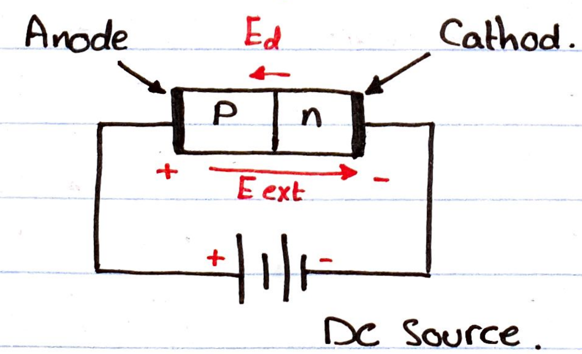




Ed : Depletion electric field.

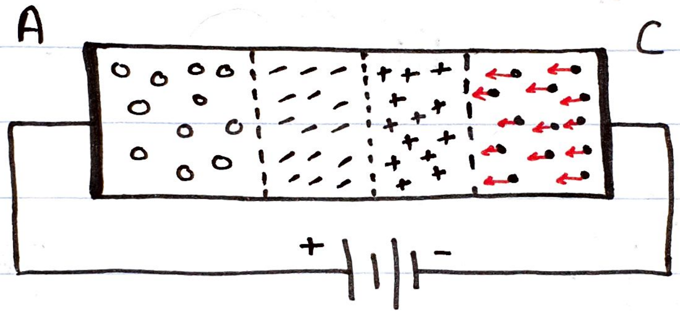
- The electrons near the junction will diffuse (تنتشر) over it to fill the holes (لملئ الثقوب) in the p-type material. As a result, positive and negative space charges are formed in a region called the depletion region.

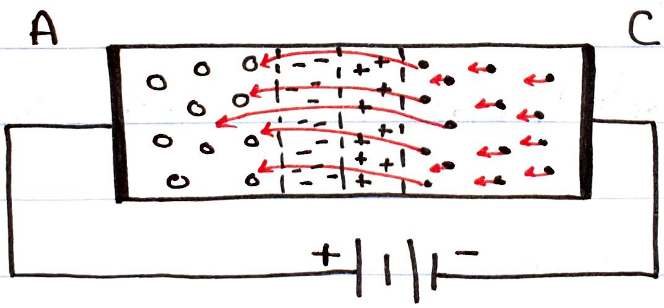
- Pn junction "Conducting".



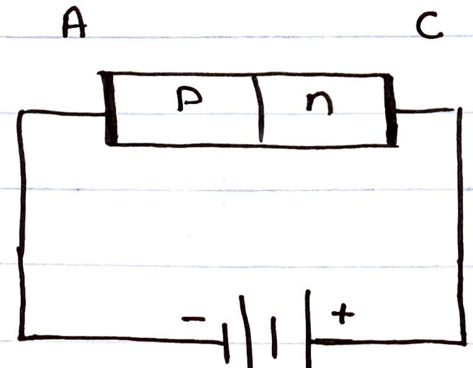
- The negative terminal of the DC source is connected to the n-type (Cathod) and the positive terminal is connected to the p-type (Anode).

- Since $E_{ext} >> E_d$, the electrons will move to the left and the depletion region will become smaller.
- Any of electrons close to the depletion region will jump over it and other electrons will keep moving to the left.
- New electrons will be generated from the Cathod by the negative terminal of the DC source. These electrons will replace that have been pushed (تدفع) to the left, resulting in a steady flow of electrons.
- At the same time, the holes will diffuse over the depletion region, resulting in a steady flow of positive charge carriers.
- By definition of Current, we have current flowing.

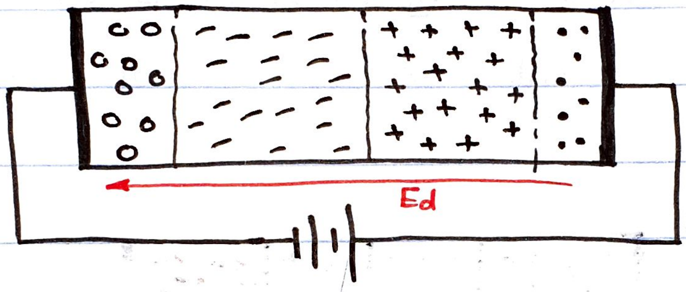


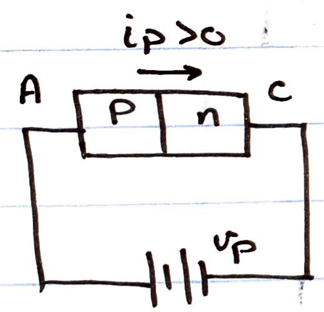


- Pn junction "Non_Conducting"

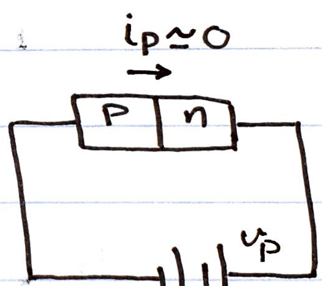


- The positive terminal is connected to the cathod and the negative terminal is connected to the anode.
- The electrons are تنجذب attracted by the anode and the holes are attracted by the cathode.
- The depletion region is explected. (expands).
- It becomes very difficult for the electrons to move from n-type material to the anode.
- The current is blocked.





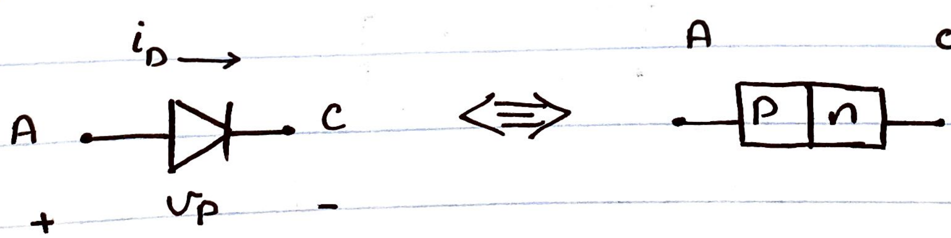
Conducting; Forward biased



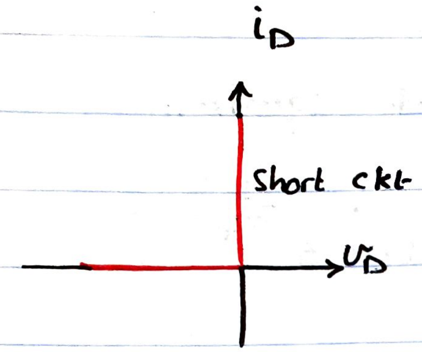
Non-Conducting; Reversed biased

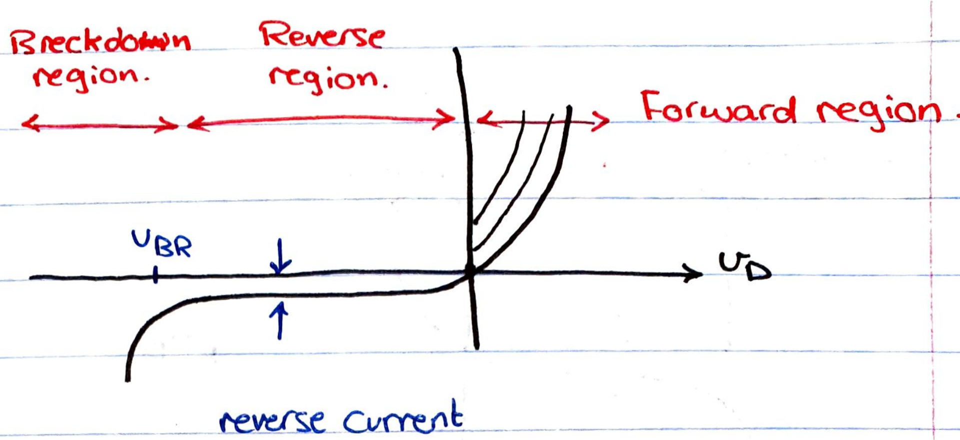
- Diode characteristic.

- The diode is a two terminal pn-junction device.
- Symbol of diode



i-v Characteristic of diode.





- Diode equation.

$i_D = I_s \left[e^{(v_D/(nV_T))} - 1 \right]$ ← empirical equation.

i_D: Diode current.

V_D: Diode voltage.

I_s: Reverse leakage current ; $(10^{-6} - 10^{-15})$ A.

n: Empirical Constant or idealety factor.

$n = 1 - 2$.

V_T: Terminal voltage.

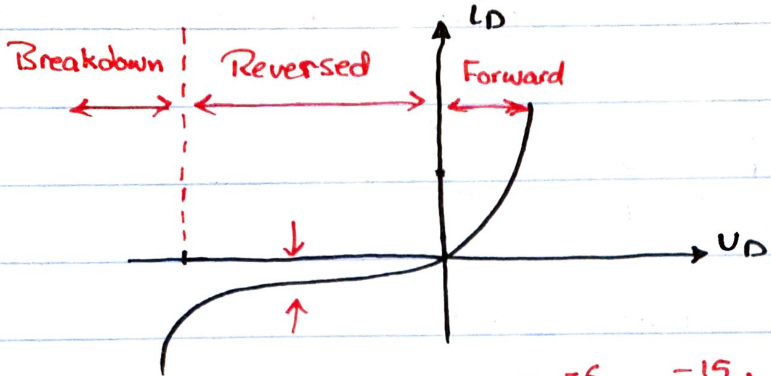
$V_T = \frac{kT}{q}$, $k = 1.38 \times 10^{-23}$ J/k.

$q = 1.6 \times 10^{-19}$

- Junction tempreture of 25°C.

$V_T = \frac{1.38 \times 10^{-23} \times (25 + 273)}{1.6 \times 10^{-19}} = 25.7$ mV.

- Region of Diode charachteristic:



leakage current $(10^{-6} - 10^{-15})$ A.

Diode equation:

$$i_D = I_s\left[e^{\left(\frac{V_D}{nV_T}\right)} - 1\right]$$

1- Forward - biased region:

- In this region, the diode is conducting as long as $V_D > V_{TD}$. Where V_{TD} is called threshold or turn-on voltage of diode, which is around 0.7 V.
- when the diode is conducting, a small forward voltage will يبرز appear across its terminals. This voltage depends on the عملية التصنيع manufacturing process and the junction temprature.
- The diod current in this region can be approximetely given by: $i_D \approx I_s\, e^{(V_D/(nV_T))}$

2- Reversed baisd region:

In this region the diode is consider to be non-conducting.

- To turn-off the diode,
  - $V_{BR} < V_D < 0$; V_{BR}: Breakdown voltage.
  - $V_D >> V_T$.
- The diode current in this region is given by:

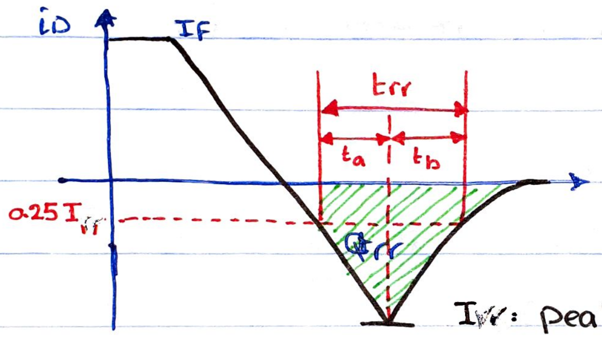
$$i_D = I_s\left[\underbrace{e^{V_D/(nV_T)}}_{0} - 1\right] \simeq -I_s$$

3- Breakdown region.   التيار بنزل بشكل سريع ⇐ فولت أعلى من VBR

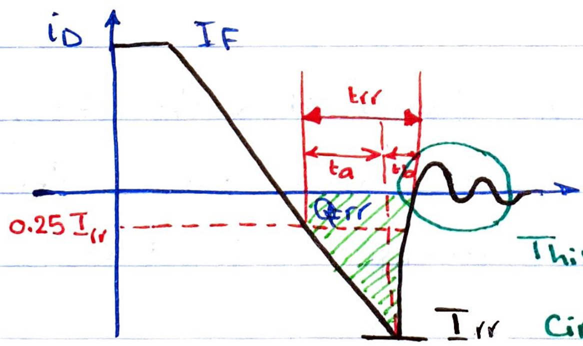
- The diode goes to this region when $v_D < -V_{BR}$, where V_{BR} is the breakdown voltage (typically 1kV).
- Since V_{BR} is usually high voltage, it is necessary to limit the reverse current to limit the power dissipation within a permissible value.   حتى ما يمشي تيار عالي ويحرقها
- The reverse current decrease rapidly.

- Reverse Recovery Charactristic.

Soft recovery.



Irr: peak value of revers current



Abrupt recovery.

time.

This overshoot is due to the inductive circuit "stray inductance".

Irr

الفترة ما بين مرور التيار ع diode ذروة ما بين بالصفر لحد ما يوصل 0.25 Irr

- Reverse recovery time (trr): It is the time between the instant when the diode current passes through zero and the moment when the peak reverse current (Irr) has decayed to 25% of its value.   هذا الزمن حتى أقدر ألم الالكترونات ع Positive ions من جديد للتأكيد وأرجع السيستم

» This time is required to recombine the minority charge carriers with their opposite charge.   Thermal isonization بسبب

$t_{rr} = t_a + t_b$.

ta: due to the charge stored in the depletion region.

tb: due to the charge stored in the bulk semi conductor material

In realuty, $t_a \gg t_b$.

» Soft Factor (SF):

$$SF = \frac{t_a}{t_b}.$$

SF↓ ⇒ Over voltage ↑

- The rate of change of reverse current is given by:

$$\frac{di}{dt} = \frac{I_{rr}}{t_a} \approx \frac{I_{rr}}{t_{rr}}.$$

- The peak reverse current is given by:

$$I_{rr} = t_{rr} \frac{di}{dt} \quad \text{......} (1)$$

- Reverse recovery charge (Q_{rr}): It is the charge that flows in the opposite direction in the diode due to the charge over from forward mode to reverse mode. It can be calculated as: when I make cheak over forward to revers.

$$Q_{rr} \approx \frac{1}{2} t_{rr} I_{rr} \quad \text{......} (2).$$

$(1) \to (2) \Rightarrow$ $Q_{rr} = \frac{1}{2} t_{rr} \left(t_{rr} \frac{di}{dt}\right).$

$$Q_{rr} = \frac{1}{2} t_{rr}^2 \frac{di}{dt}.$$

$$\Rightarrow t_{rr} = \sqrt{\frac{2Q_{rr}}{(di/dt)}}$$

- Note:

Q_{rr}, t_{rr}, I_{rr} & SF ; are usually given in the datasheet of the diode.

- Example: The recovery reverse time of diode is $3\,\mu sec$, the rate of fall of the diode current is $30\,A/\mu sec$.

1. Determine the peak reverse current.
2. Determine the Storage Current.

1. $t_{rr} = 3\,\mu sec = 3 \times 10^{-6}$.

$$\frac{di}{dt} = 30\,A/\mu sec = 30 \times 10^{6}.$$

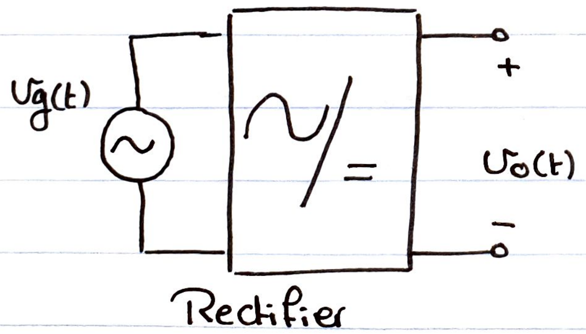
$$\frac{di}{dt} = \frac{I_{rr}}{t_{rr}} \Rightarrow I_{rr} = \frac{di}{dt} \cdot t_{rr} = 90\,A.$$

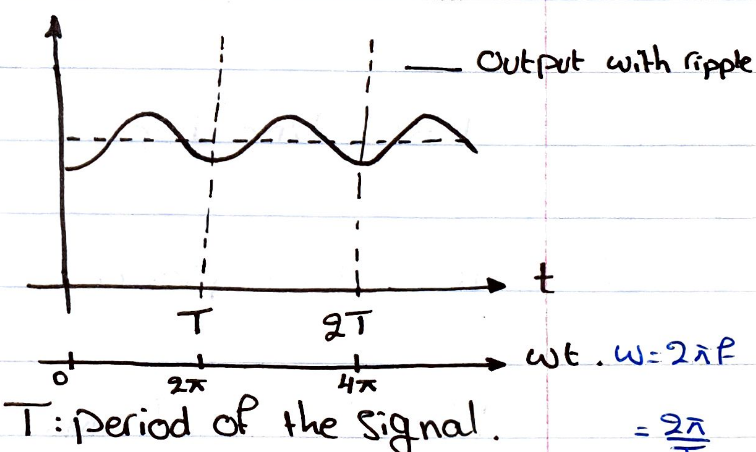
2. $Q_{rr} = \frac{1}{2} t_{rr} \cdot I_{rr}$

$$= \frac{1}{2}(3 \times 10^{-6})(90) = 135\,\mu C.$$

- Diode Rectifier:

A rectifier is a Circuit that Converts the AC signal into a unidirection signal.





T: period of the signal.

- Performance Parameters:

1. Average value of Output (load) voltage.

$$\text{average} \Rightarrow V_{DC} = \frac{1}{T}\int_0^T v_o(t)\,dt = \frac{1}{2\pi}\int_0^{2\pi} v_o(t)\,d(\omega t)$$

2- Average value of Output current.

$$I_{DC} = \frac{1}{T}\int_0^T i_o(t)\,dt = \frac{1}{2\pi}\int_0^{2\pi} i_o(t)\,d(\omega t)$$

3- RMS value of output voltage.

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v_o^2(t)\,dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_o^2(t)\,d(\omega t)}$$

4- RMS value of load current.

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i_o^2(t)\,dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_o^2(t)\,d(\omega t)}$$

5- DC output power:

$$P_{DC} = V_{DC}\, I_{DC}.$$

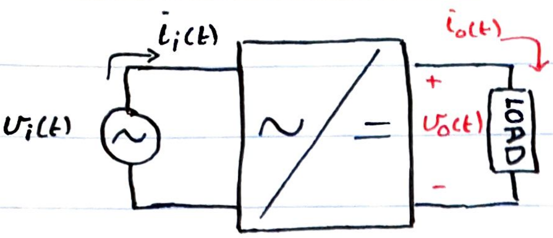
6- AC output power.

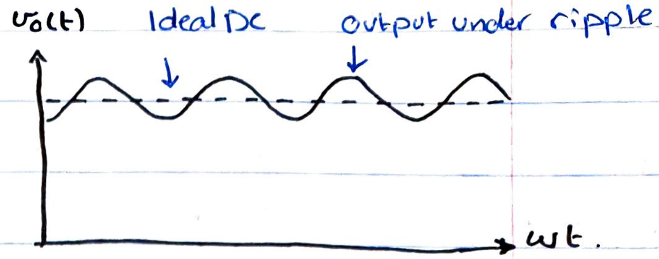
$$P_{AC} = V_{RMS} \cdot I_{RMS}$$

7- Efficiency or rectification ratio.

$$\eta = \frac{P_{DC}}{P_{AC}}$$; This is not a power efficiency. This is a conversion efficiency that measures the quality of output waveform.

- Rectifier Circuit.





- Performance parameters.

1. Average value of output voltage.

$$V_{DC} = \frac{1}{2\pi}\int_0^{2\pi} v_o(t)\, d(\omega t).$$

2. Average value of output current.

$$I_{DC} = \frac{1}{2\pi}\int_0^{2\pi} i_o(t)\, d(\omega t).$$

3. RMS value of $v_o(t)$.

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_o^2(t)\, d(\omega t)}$$

4. RMS value of $i_o(t)$.

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_o^2(t)\, d(\omega t)}$$

5. DC output power.

$$P_{DC} = V_{DC}\, I_{DC}.$$

6. AC output Power.

$$P_{AC} = V_{RMS}\, I_{RMS}$$

7. Rectification

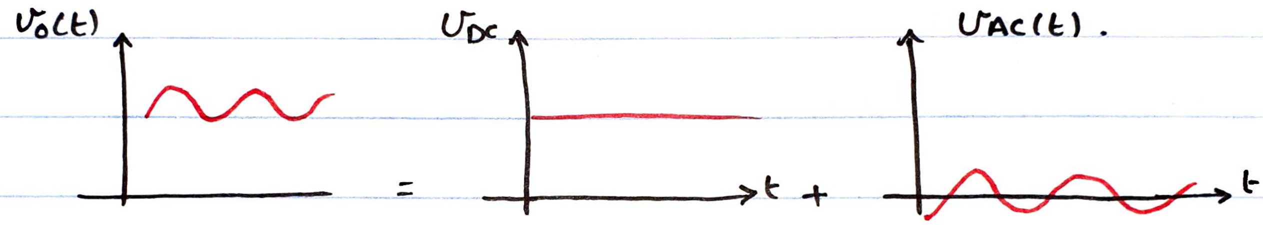
$$\eta = \frac{P_{DC}}{P_{AC}}.$$

8. Effective RMS value of the AC component of $v_o(t)$.

$v_o(t) = V_{DC} + V_{AC}(t)$.

RMS value of $V_{AC}(t)$. $\rightarrow$ $V_{DC} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_{AC}(t)\, d(\omega t)}$

$= \sqrt{V_{RMS}^2 - V_{AC}^2}$



- Recall the RMS value of $V_o(t)$.

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_o^2(t)\, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_{AC}(t) + V_{DC})^2\, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_{AC}^2(t)\, d(\omega t) + \frac{1}{2\pi}\int_0^{2\pi} 2 V_{DC} V_{AC}(t) d(\omega t) + \frac{1}{2\pi}\int_0^{2\pi} V_{DC}^2\, d}$$

$$V_{RMS} = \sqrt{V_{AC}^2 + V_{DC}^2}$$

9. Ripple Factor (RF) : It is measure of ripple content in the Output waveform.

$$RF = \frac{V_{AC}}{V_{DC}}$$, For ideal rectifier → $RF = 0\%$.

10. Form Factor (FF) : It is a measure of the shape of output waveform.

$$FF = \frac{V_{RMS}}{V_{DC}} = \frac{\sqrt{V_{AC}^2 + V_{DC}^2}}{V_{DC}} = \sqrt{\left(\frac{V_{AC}}{V_{DC}}\right)^2 + 1}$$

$FF = \sqrt{(RF)^2 + 1}$, For ideal rectifier $FF = 100\%$.

11. Transformation utilization factor (TUF).

$$TUF = \frac{P_{DC}}{V_s I_s}$$

where, V_s is the rms value of input voltage.
I_s is the rms value of input current.

12. Total harmonic distortion (THD) or harmonic factor (HF) of input current.

$$THD = \sqrt{\frac{I_s^2 - I_1^2}{I_1^2}}$$, Ideally $THD = 0\%$.

$$I_s = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_s^2(t)\, d(\omega t)}$$

I_1 : It is RMS value of the fundemental component of $i_s(t)$.

13. Crest Factor (CF).

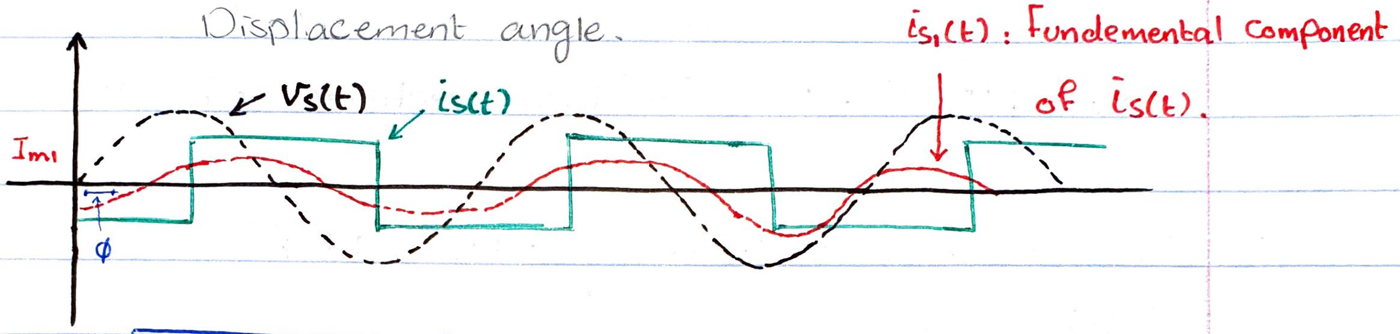
$$CF = \frac{I_{s\,Peak}}{I_s}$$, It is used to determine the rated peak current of power device.

121- Input Power Factor.

$$PF = \frac{I_1}{I_s} \cos \phi$$

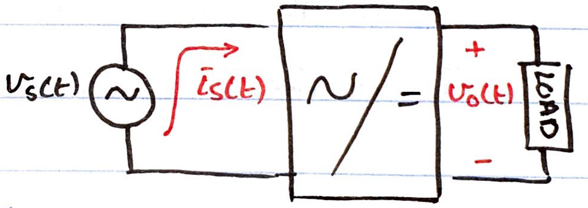
$\frac{I_1}{I_s}$, distortion factor, $\cos\phi$: displacement factor.

ϕ : Phase difference between the fundamental components of input voltage and current.



$$I_s = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_i^2(s)\, dt}$$

$$I_1 = \frac{I_{m1}}{\sqrt{2}}.$$



Distortion Factor = I_1 / I_s.

Displacement Factor = $\cos\phi$

- Fourier Series.

It is used to represent any Periodic signal by a discrete. sum of sinusoidal functions with average value.

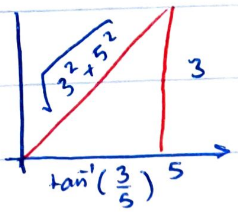
$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t).$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\, dt = \frac{1}{2\pi} \int_0^{2\pi} x(t)\, d(\omega t).$$

$$a_n = \frac{2}{T} \int_0^T x(t) \cos(n\omega t) dt = \frac{1}{\pi} \int_0^{2\pi} x(t) \cos(n\omega t) d\omega t$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin(n\omega t)\,dt = \frac{1}{\pi}\int_0^{2\pi} x(t)\sin(n\omega t)\,d(\omega t).$$

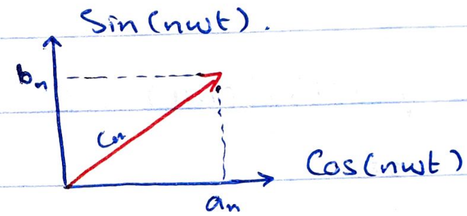
$5\cos(\omega t) + 3\sin(10t)$.



$$x(t) = a_0 + \sum_{n=1}^{\infty} C_n\cos(n\omega t + \phi_n).$$

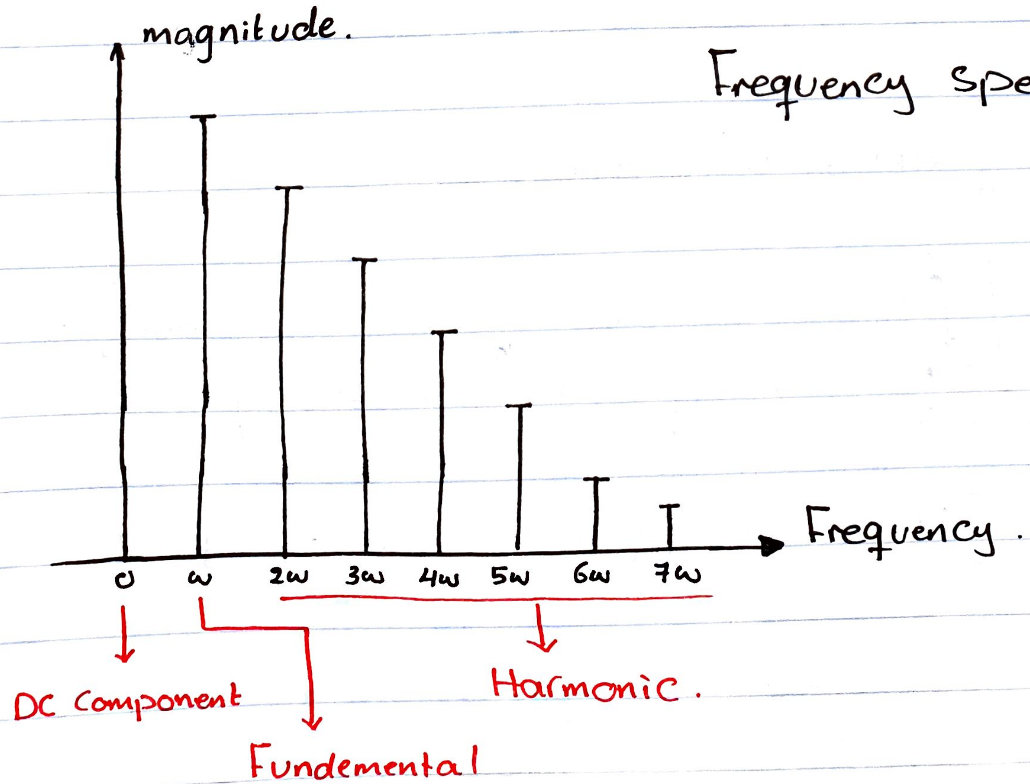
$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right).$$

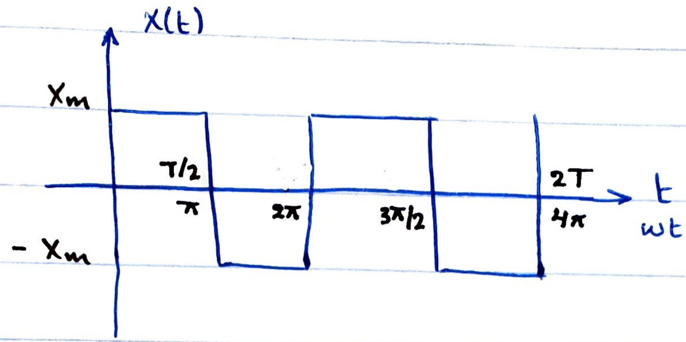


$$x(t) = \underbrace{a_0}_{\text{DC Component}} + \underbrace{C_1\cos(\omega t + \phi_1)}_{\text{Fundemantal or main Component.}} + \underbrace{C_2\cos(2\omega t + \phi_2) + C_3\cos(2\omega t + \phi_3) + \dots}_{\text{Harmonics.}}$$

Frequency spectrum.



Example: Fourier Series for square wave.



Solution:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} x(t)\, d(\omega t) = 0 \quad \text{, NO DC Component.}$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} x(t) \cos(n\omega t)\, d(\omega t).$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} X_n \cos(n\omega t)\, d\omega t - \int_{\pi}^{2\pi} X_m \cos(n\omega t)\, d\omega t \right]$$

$$a_n = \frac{X_m}{\pi} \left[\frac{1}{n} \sin(n\omega t) \Big|_0^{\pi} - \frac{1}{n} \sin(n\omega t) \Big|_0^{2\pi} \right]$$

$a_n = 0$. symmetric about x-axis (odd Function).

$$b_n = \frac{1}{\pi} \int_0^{2\pi} x(t) \sin(n\omega t)\, d\omega t$$

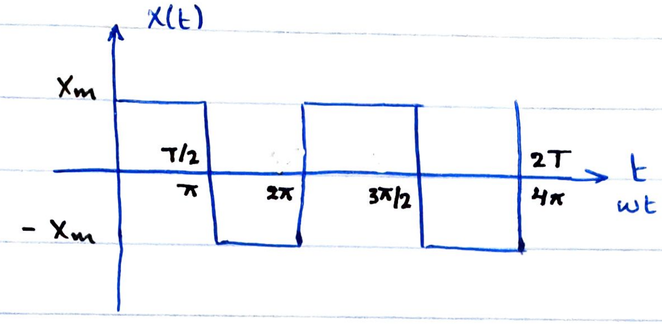
$$b_n = \frac{1}{\pi} \left[\int_0^{\pi} X_m \sin(n\omega t)\, d\omega t - \int_{\pi}^{2\pi} X_m \sin(n\omega t)\, d\omega t \right].$$

$$= \frac{X_m}{\pi} \left[\frac{1}{n} \cos(n\omega t) \Big|_{\pi}^{0} + \frac{1}{n} \cos(n\omega t) \Big|_{\pi}^{2\pi} \right].$$

$$= \frac{X_m}{n\pi} \left[(1 - \cos(n\pi)) + (\cos(2n\pi) - \cos(n\pi) \right]$$

$$= \frac{2X_m}{m\pi} (1 - \cos(m\pi)) = \begin{cases} 0 & \text{, n is even} \\ \frac{4X_m}{n\pi} & \text{, m is odd.} \end{cases}$$

- Example: Fourier Series for square wave.



Solution:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t).$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x(t)\, d(\omega t) = 0$$, NO DC component.

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x(t)\cos(n\omega t)\, d(\omega t).$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} X_n \cos(n\omega t)\, d\omega t - \int_{\pi}^{2\pi} X_m \cos(n\omega t)\, d\omega t\right]$$

$$a_n = \frac{X_m}{\pi}\left[\frac{1}{n}\sin(n\omega t)\Big|_0^{\pi} - \frac{1}{n}\sin(n\omega t)\Big|_0^{2\pi}\right]$$

$a_n = 0$. symmetric about x-axis (odd Function).

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x(t)\sin(n\omega t)\, d\omega t$$

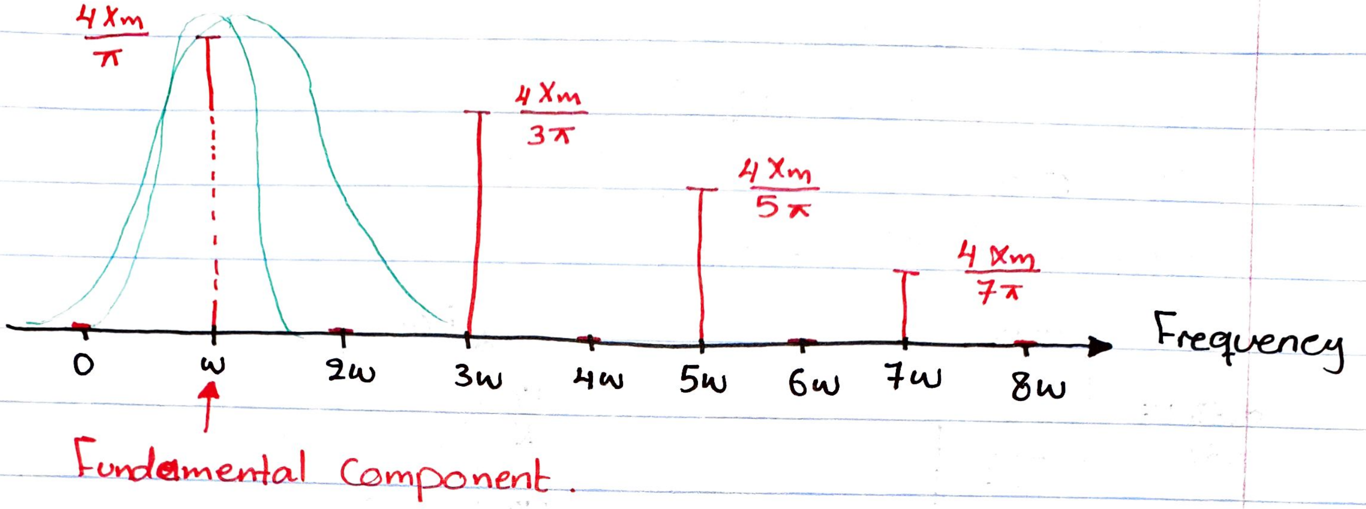
$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} X_m \sin(n\omega t)\, d\omega t - \int_{\pi}^{2\pi} X_m \sin(n\omega t)\, d\omega t\right].$$

$$= \frac{X_m}{\pi}\left[\frac{1}{n}\cos(n\omega t)\Big|_{\pi}^{0} + \frac{1}{n}\cos(n\omega t)\Big|_{\pi}^{2\pi}\right].$$

$$= \frac{X_m}{n\pi}\left[(1 - \cos(n\pi)) + (\cos(2n\pi) - \cos(n\pi)\right]$$

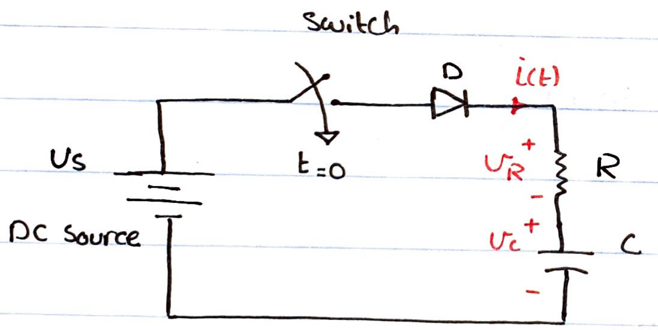
$$= \frac{2X_m}{m\pi}(1 - \cos(m\pi)) = \begin{cases} 0 & , n \text{ is even} \\ \frac{4X_m}{n\pi} & , m \text{ is odd.} \end{cases}$$

- $$x(t) = \sum_{m=1}^{\infty} \frac{4X_m}{n\pi} \sin(m\omega t).$$



- Diode Circuits and Rectifier.

1. Diode Circuit with RC load.



Assume that the switch is closed at $t=0$.

KVL in the loop:

$V_s = R\,i(t) + V_c(t).$

$i(t) = C \frac{dV_c}{dt}$

$V_s = RC \frac{dV_c(t)}{dt} + V_c(t).$

$X(t) = X_f + (X_i - X_f)\, e^{-(t-t_0)/\tau} \quad , t \geq 0$

The solution is given by:

$X_i = X(t_0) = \text{initial value}$

$V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)]\, e^{-t/\tau}$

$X_f = X(\infty) = \text{Final value}$

τ: time Constant.

$\tau = RC$.

- Assume that the capacitor is initially fully discharged $\Rightarrow V_C(0) = 0$.

when $t \to \infty \Rightarrow$ C is open circuit.

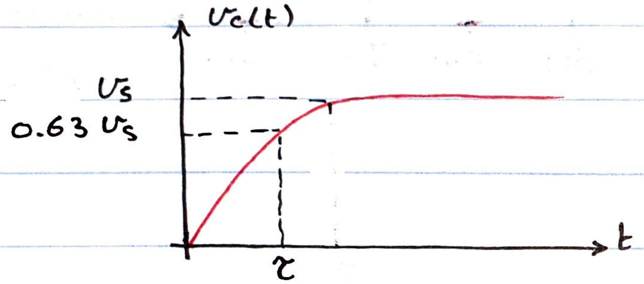
$$V_C(\infty) = V_s$$

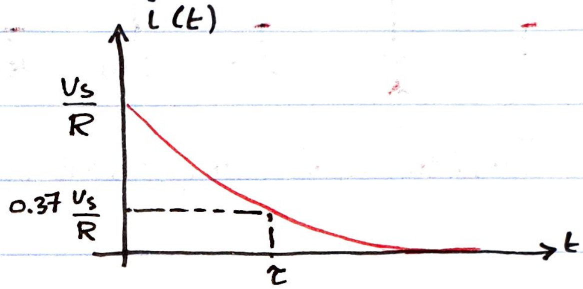
$$V_C(t) = V_s + (0 - V_s)\, e^{-t/\tau}$$

$$V_C(t) = V_s\,(1 - e^{-t/\tau}) \;, \quad t \geq 0 .$$

$$i(t) = C\,\frac{dV_C(t)}{dt} = C\,V_s\left(\frac{1}{\tau}\, e^{-t/\tau}\right)$$

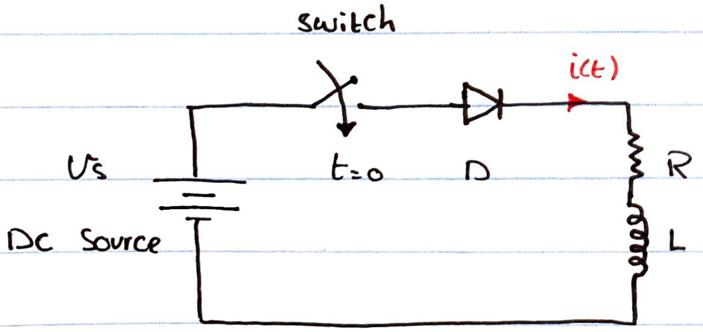
$$i(t) = \frac{V_s}{R}\, e^{-t/\tau} \;, \quad t \geq 0 .$$





- The energy stored in the capacitor under steady state condition $(t \to \infty)$; $E_C = \frac{1}{2} C V_s^2$.

2. Diode circuit with RL load.



Assume that the switch is closed at $t=0$ By applying KVL in the loop:

$$V_s = R\, i(t) + L\,\frac{di(t)}{dt}$$

The solution is given by:

$$i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-t/\tau} .$$

τ: time constant.

$$\tau = L/R$$

- Assume that the inductor is initially fully discharged $\Rightarrow i(0) = 0$.

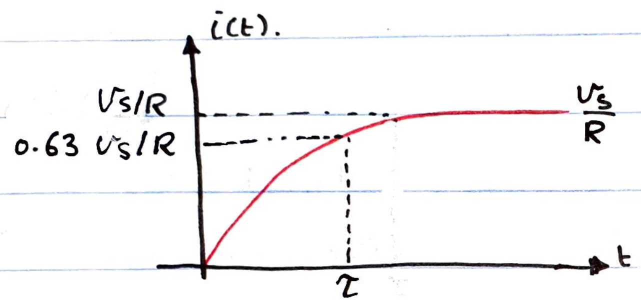
At $t \to \infty \Rightarrow L$ is short circuit.

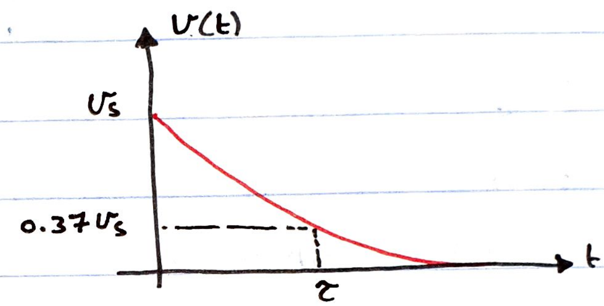
$i(\infty) = \frac{V_s}{R}$; $i(\infty)$: steady state conductor current.

$i(t) = \frac{V_s}{R} + (0 - \frac{V_s}{R}) e^{-t/\tau}$

$i(t) = \frac{V_s}{R}(1 - e^{-t/\tau})$; $t \geq 0$.

$V_L(t) = L\frac{di(t)}{dt} = L \frac{V_s}{R} \frac{1}{\tau} e^{-t/\tau} = V_s e^{-t/\tau}$, $t \geq 0$.





- The energy stored in the inductor under steady state condition is:

$E_L = \frac{1}{2} L I^2$; $I = \frac{V_s}{R}$.

- Freewheeling diode with RL load.

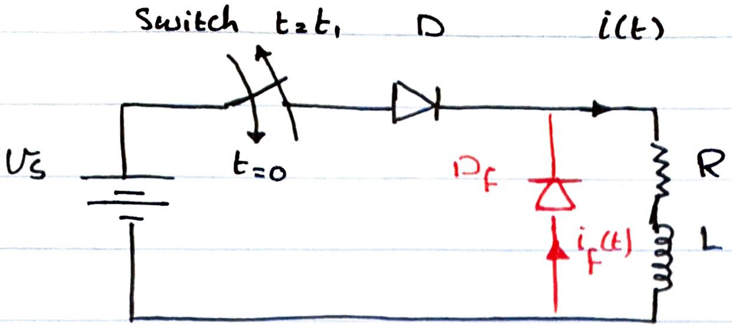
- The steady state current in the inductor is given by:

$i_{ss} = I = \frac{V_s}{R}$; i_{ss}: steady state current.

An attempt (محاولة) to open the switch will result in transfering (نقل) the energy stored in the inductor ($\frac{1}{2} L I^2$) into a high voltage across the diode and switch. The energy will be dissipated (تتبدد) in the form of spark (شرارة) that may damage the circuit component.

- To overcome this problem, a freewheeling diode is connected across the RL load to provide an alternative path for the current when the switch is opened.

توفير طريق بديل للتيار عندما نفتح المفتاح.



D_f : Freeweeling diode.

Modes of operation

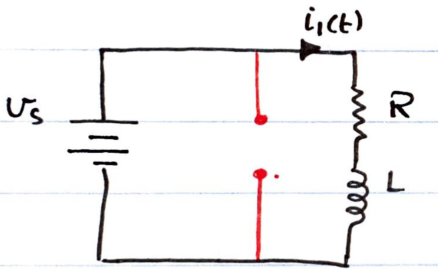
Mode I

$0 \leq t \leq t_1$

The switch is ON "closed".

D is Conducting.

Df is reversed - biased.



$$i_1(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right)$$

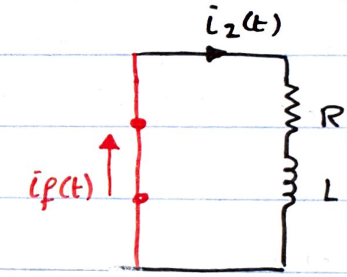
Step response.

Mode II

$t_1 \leq t \leq t_2$.

The switch is OFF "Opened"

Df is Conducting.

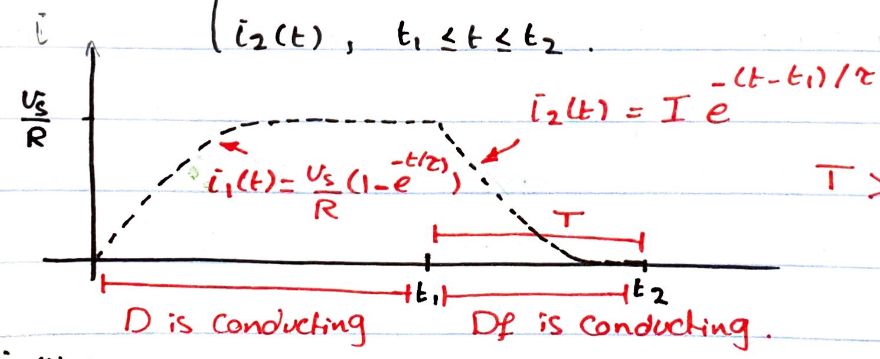


$$i_2(t) = I \, e^{-(t-t_1)/\tau}$$

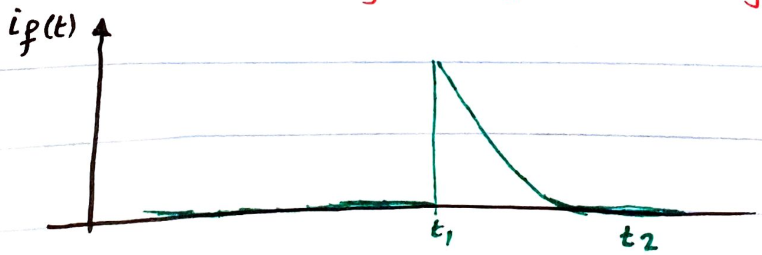
where I is the initial current of $i_2(t) \Rightarrow i_2(t_1) = i_1(t_1)$.

Natural Response.

$$i(t) = \begin{cases} i_1(t), & 0 \leq t \leq t_1 \\ i_2(t), & t_1 \leq t \leq t_2 \end{cases}$$

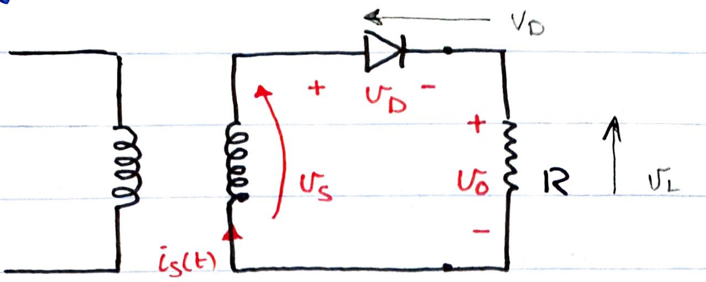


$$T >> \tau = \frac{L}{R}$$

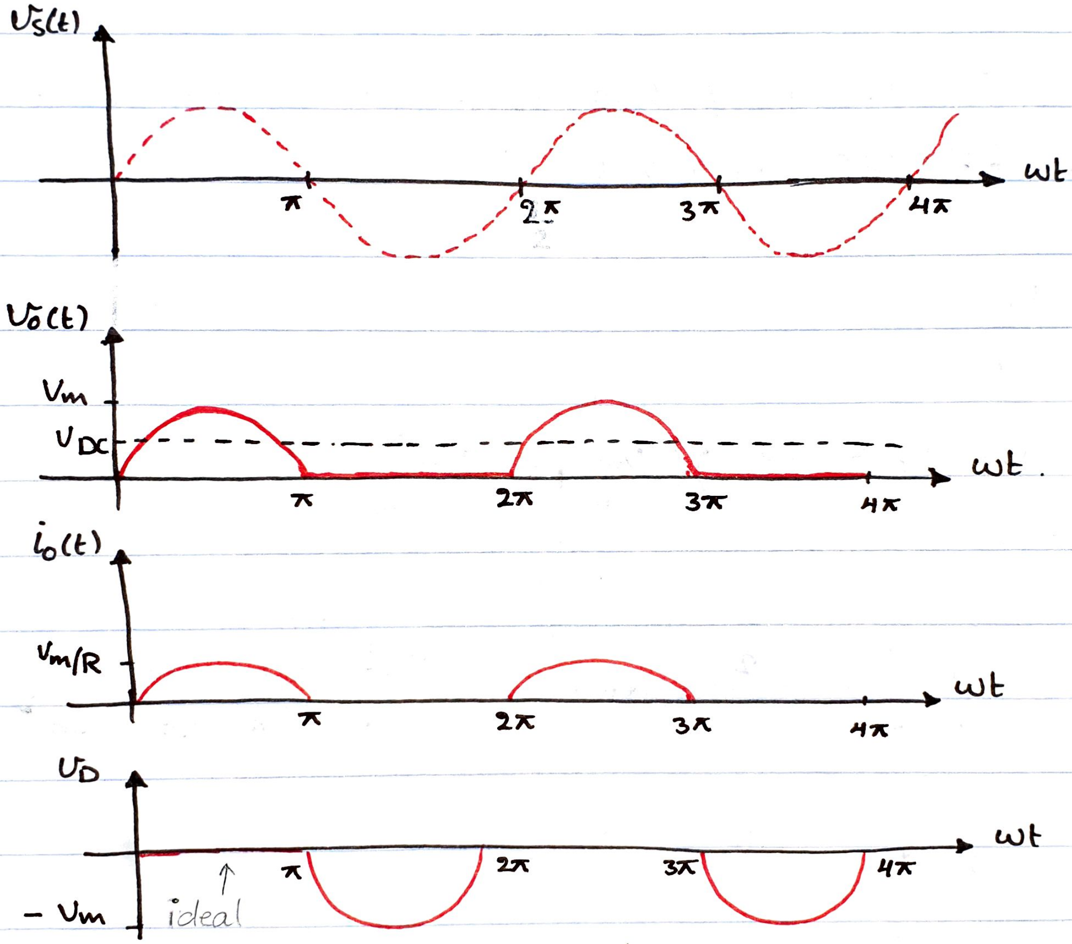


- Rectifiers:

1. Single-phase (1-ϕ) Half-wave Rectifiers.



$v_s = V_m \sin(\omega t)$.



- The average value of output voltage.

$$V_{DC} = \frac{1}{2\pi}\int_0^{2\pi} v_o(t)\, d(\omega t).$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)\, d(\omega t).$$

$$= \frac{V_m}{2\pi} \cos(\omega t) \Big|_{\pi}^{0} = \frac{V_m}{2\pi}\left[\cos(0) - \cos(\pi)\right].$$

$$V_{DC} = \frac{V_m}{2\pi}\left(1-(-1)\right) = \frac{V_m}{\pi}$$

- The average Output Current is:

$$I_{DC} = \frac{V_{DC}}{R} = \frac{V_m}{\pi R}.$$

- Peak inverse voltage (PIV):

It is the maximum reverse voltage across the diode when the diode is in blocking state.

$PIV = V_m$.

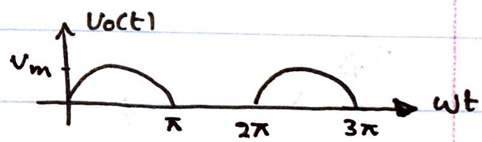
- Example: For the 1-ϕ half-wave rectifier with resistive load, Find η, FF, RF, TUF & CF?

1- $$\eta = \frac{P_{DC}}{P_{AC}}.$$

$$P_{DC} = V_{DC} I_{DC} = \left(\frac{V_m}{\pi}\right)\left(\frac{V_m}{\pi R}\right) = \frac{V_m^2}{\pi^2 R}$$

$$P_{AC} = V_{RMS} I_{RMS}.$$

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2(\omega t)\, d\omega t}.$$



$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} \frac{V_m^2}{2}(1-\cos(2\omega t))\, d\omega t}.$$

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} \frac{V_m^2}{2}\, d\omega t} = \frac{V_m}{2}.$$

$$I_{RMS} = \frac{V_{RMS}}{R} = \frac{V_m}{2R}.$$

$$P_{AC} = \left(\frac{V_m}{2}\right)\left(\frac{V_m}{2R}\right) = \frac{V_m^2}{4R}.$$

$$\eta = \frac{V_m^2/\pi^2 R}{V_m^2/4R} = \frac{4}{\pi^2} \simeq 40.5\,\%.$$

2- $FF = \frac{V_{RMS}}{V_{DC}} = \frac{V_m/2}{V_m/\pi} = \frac{\pi}{2} = 157\%$.

3- $RF = \sqrt{FF^2 - 1}$

$= \sqrt{(1.57)^2 - 1} \simeq 121\% = 1.21$.

4- $TUF = \frac{P_{DC}}{V_s I_s} = \frac{V_{DC} I_{DC}}{V_s I_s}$

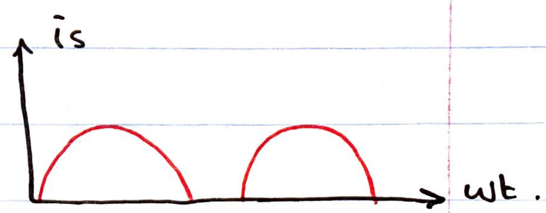
$P_{DC} = \frac{V_m^2}{\pi^2 R}$, $V_{DC} = \frac{V_m}{\pi}$, $I_{DC} = \frac{V_m}{\pi R}$.

$V_s = \frac{V_m}{\sqrt{2}}$, $I_s = \frac{V_m}{2R}$

$TUF = \frac{V_m^2}{\pi^2 R} \cdot \frac{2\sqrt{2} R}{V_m^2} = \frac{2\sqrt{2}}{\pi^2} \simeq 28\%$.

$\frac{1}{TUF} \simeq 3.5$.

- The transformer has to be around 3.5 times larger than it is needed to deliver (يوصل) the same power when it is supplied by a pure AC sinusoidal source.
- The transformer carriers a DC current saturation problem in the transformer core.

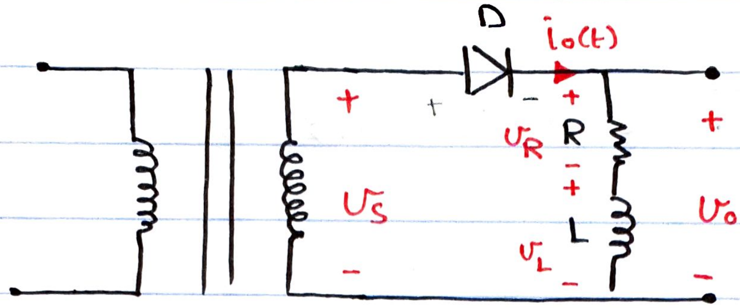


$i_s(peak) = \frac{V_m}{R}$, $I_s = \frac{V_m}{2R}$

$CF = \frac{i_s(Peak)}{I_s} = \frac{V_m/R}{V_m/2R} = 2$.

CF (Creast Factor).

- 1-ϕ Half-wave rectifier with RL load.



$v_S(t) = V_m \sin(\omega t)$.

- Due to the inductive load, the condition period of the diode will extend beyond π until the current becomes zero at $\omega t = \pi + \theta$
- The average value of $v_L(t)$ is zero, since i_o is periodic.

$$\bar{v}_L = \frac{1}{T}\int_0^T v_L(t) = 0$$

- Proof:

$$v_L = L\frac{di_o}{dt}$$

$$v_L\,dt = L\,di_o.$$

$$\frac{1}{T} v_L\,dt = \frac{L}{T}\,di_o.$$

$$\frac{1}{T}\int_0^T v_L\,dt = \frac{L}{T}\int_{i_o(0)}^{i_o(T)} di_o.$$

$$\bar{v}_L = \frac{L}{T}\left[\cancel{i_o(T) - i_o(0)}\right]$$

0

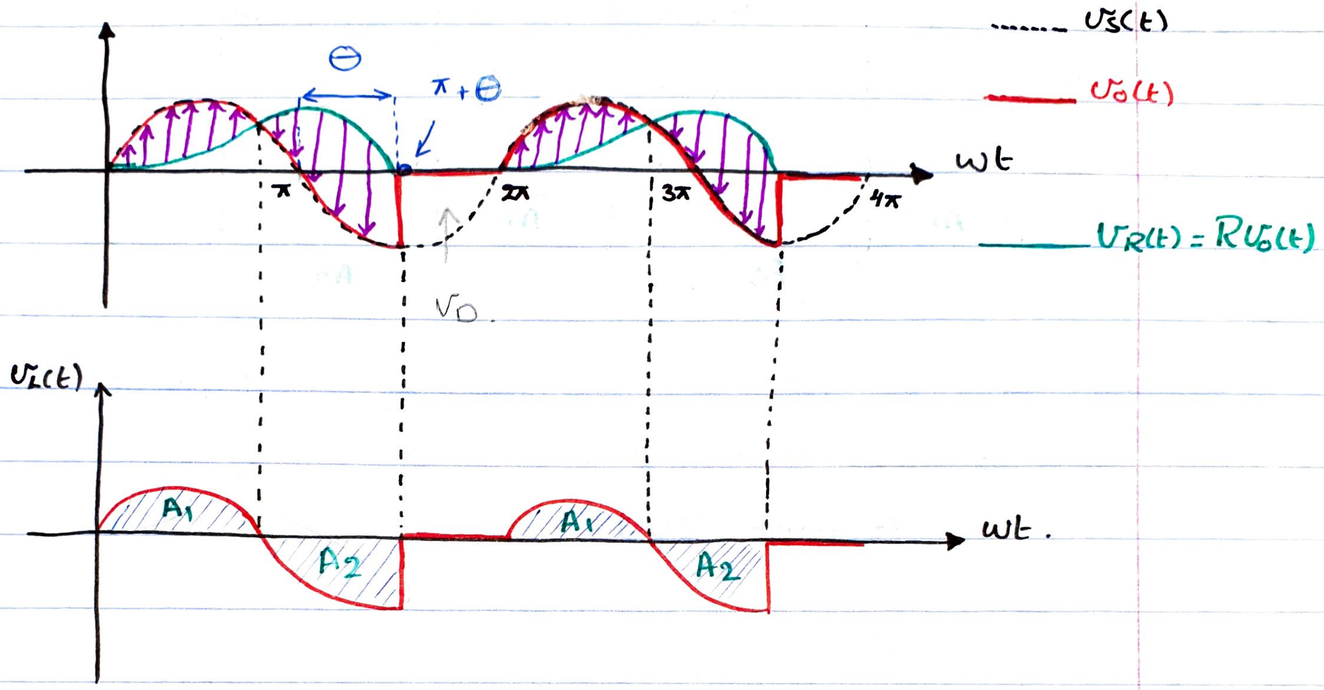
$$\bar{v}_L = 0$$

- Apply KVL:

$$-v_S + Ri_o + L\frac{di_o}{dt} = 0 \Rightarrow V_m \sin(\omega t) = Ri_o(t) + L\frac{di_o}{dt}$$

$$i_o(t) = \frac{V_m}{Z}\sin\beta\, e^{-t/\tau} + \frac{V_m}{Z}\sin(\omega t - \beta), \quad 0 \le \omega \le \pi + \theta$$

- $Z = \sqrt{R^2 + (\omega L)^2}$
- $\tau = L/R$
- $\beta = \tan^{-1}\left(\frac{\omega L}{R}\right)$



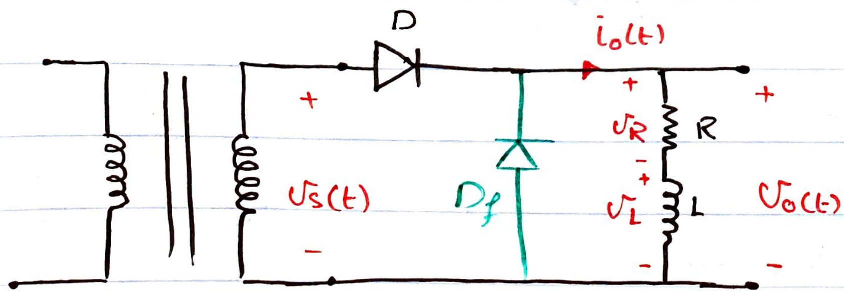
- The average output voltage is

$$V_{DC} = \frac{1}{2\pi}\int_0^{2\pi} v_o(t)\, d(\omega t) = \frac{1}{2\pi}\int_0^{\pi+\theta} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi} \cos(\omega t)\Big|_{\pi+\theta}^{0}$$

$$V_{DC} = \frac{V_m}{2\pi}\left(1 - \cos(\pi+\theta)\right) = \frac{V_m}{2\pi}(1+\cos\theta) < \frac{V_m}{\pi}$$

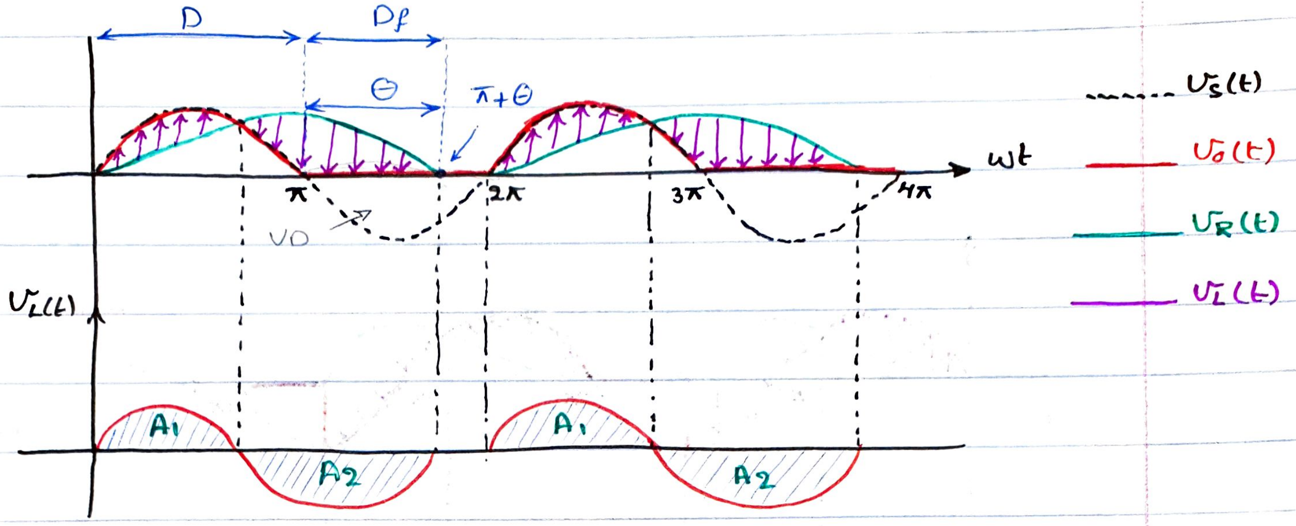
$\cos(a+b) = \cos A \cos B - \sin A \sin B$

- Addition of freewheeling diode.



$v_s(t) = V_m \sin(\omega t)$.

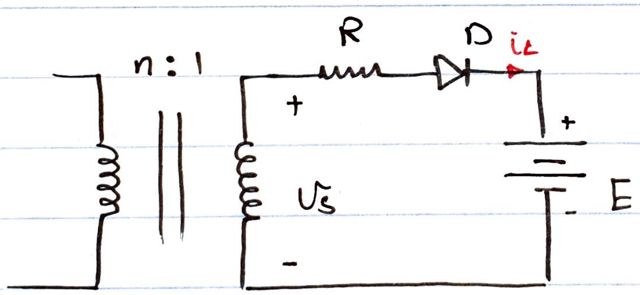
$V_s(t) = V_m \sin\left[\int \omega(t)\,dt + \theta_o\right]$ المعادلة الأصلية للابريز



- The diode prevents the negative voltage to appears across the load $\Rightarrow$ V_{DC} increases and becomes similer to V_{DC} in Case of purely resistive load.

$(0 \to \pi+\theta) \to$ without freewhealing diode, $(0 \to \pi) \to$ with freewhealing diode.

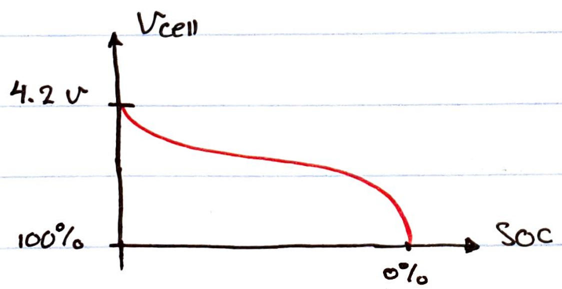
- 1-Ø Half-wave rectifiers (Battery Charger).



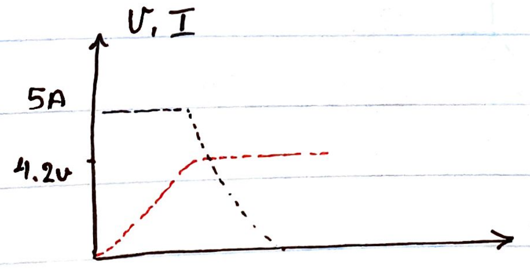
i_c: charging current.

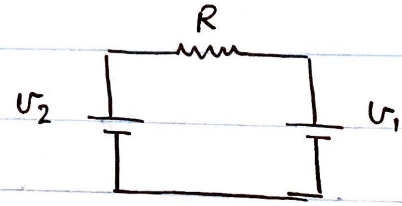
The diode conducts when $V_s(t) > E$.

R: It is used to limit the Charging resistor.

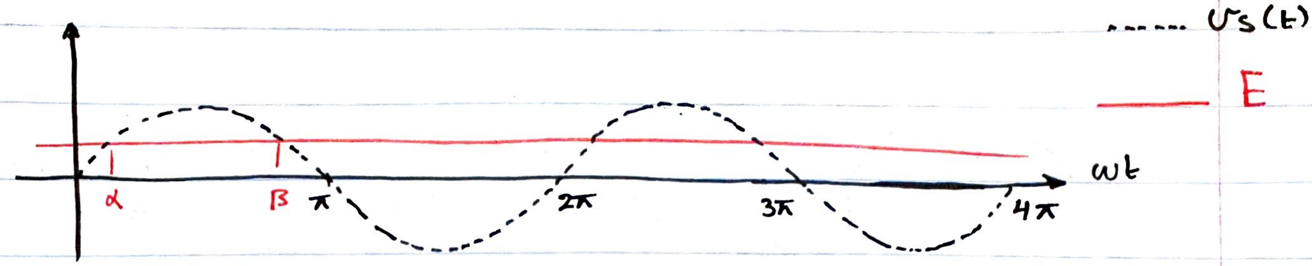


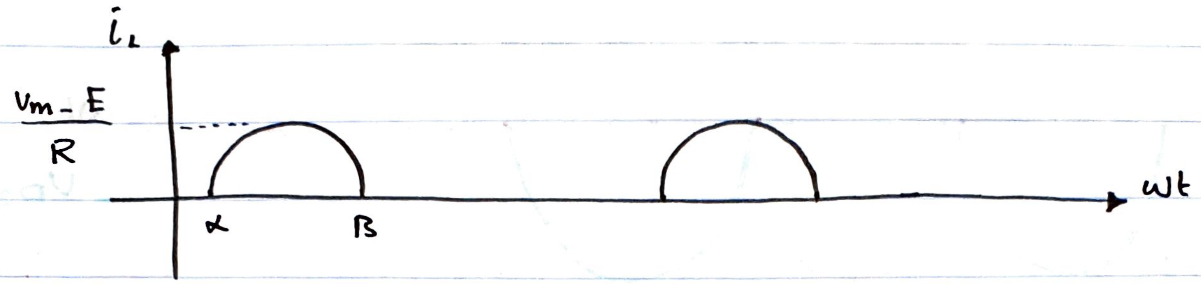
 Li ion.





$I\downarrow = \frac{V_2 - V_1\uparrow}{R}$





- α & β are found from:

$v_s(t) = E \Rightarrow V_m \sin(\alpha) = E$

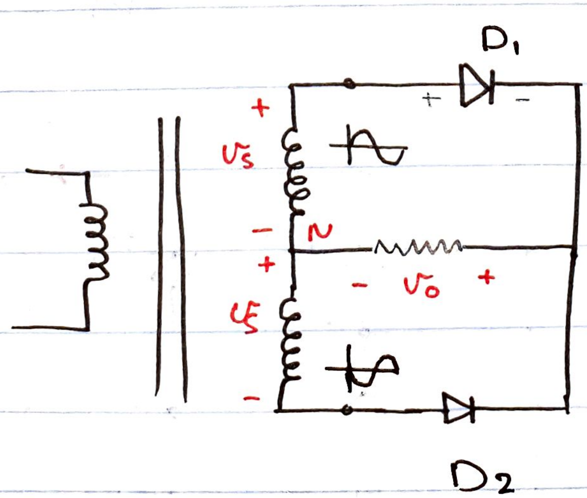
$\omega t = \alpha$, $\alpha = \sin^{-1}\left(\frac{E}{V_m}\right)$, $\beta = \pi - \alpha$.

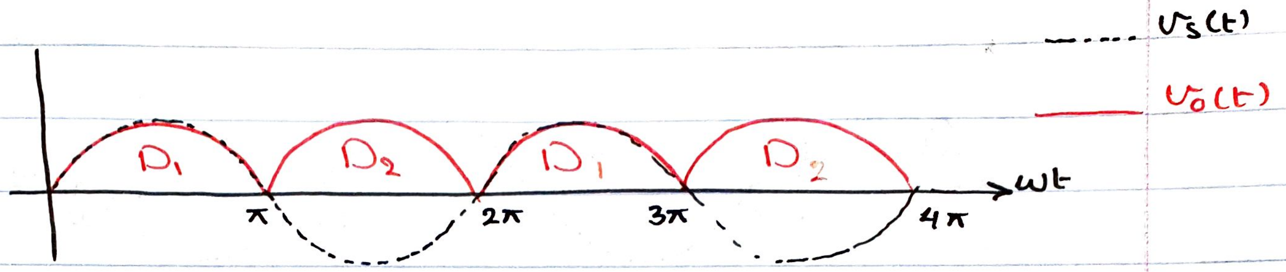
- Charging current is given by:

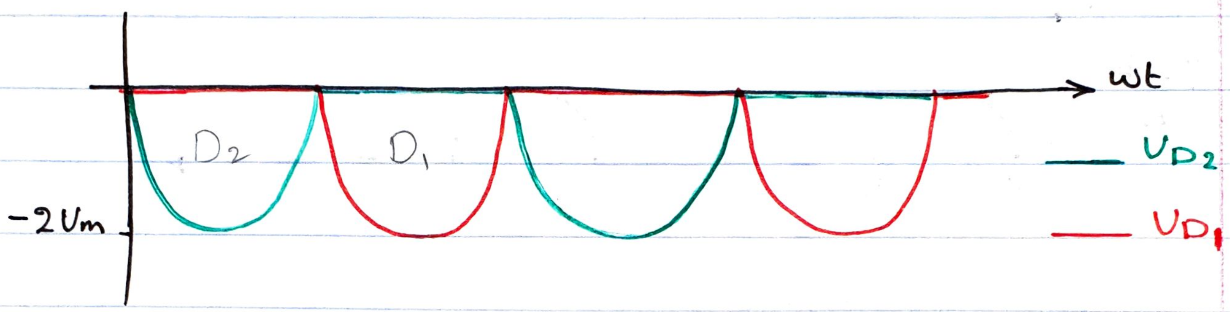
$$i_L = \frac{v_s(t) - E}{R} = \frac{V_m \sin\omega t - E}{R}$$

2 - Full-wave Rectifiers.

2-1 Full-wave rectifier with center-tapped transformer.







$$V_{DC} = \frac{2V_m}{\pi}$$

- Rectifiers.

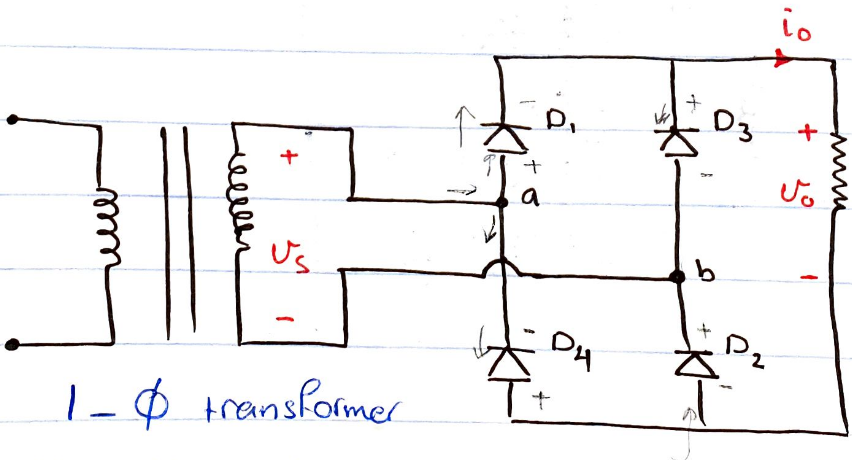
1. $1-\phi$ Half-wave.
2. $1-\phi$ Full-wave.

   2.1) $1-\phi$ Full wave with Center-Tupped Transformer.

   2.2) $1-\phi$ Full-wave bridge rectifier.

2.2) $1-\phi$ Full-wave bridge rectifier.

It is Commonly used for industrial application.



لما يكون load ⇐ R

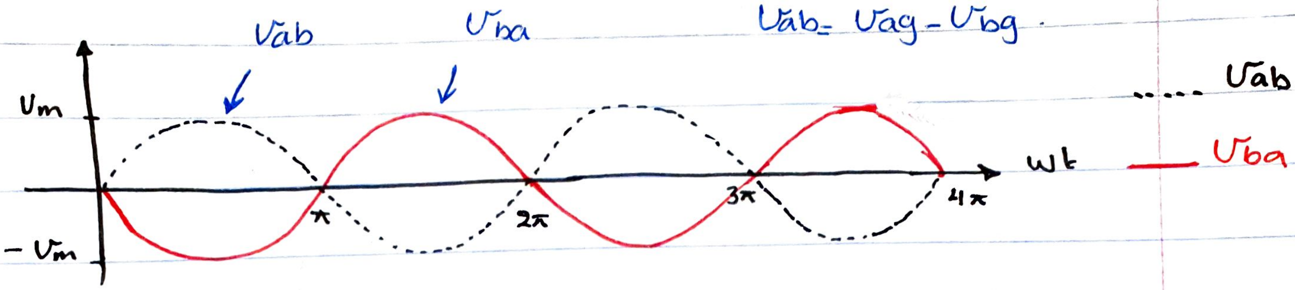
R ← التيار والفولتج

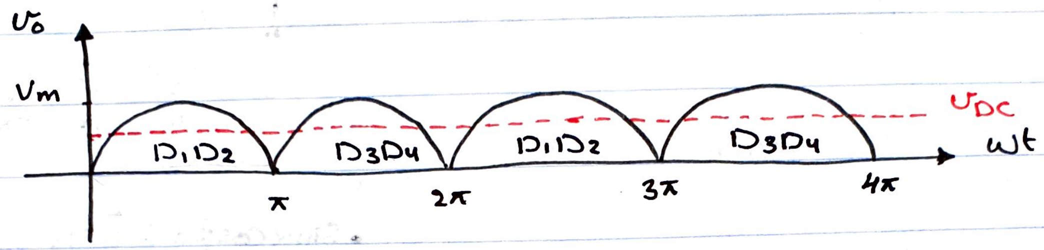
in phase

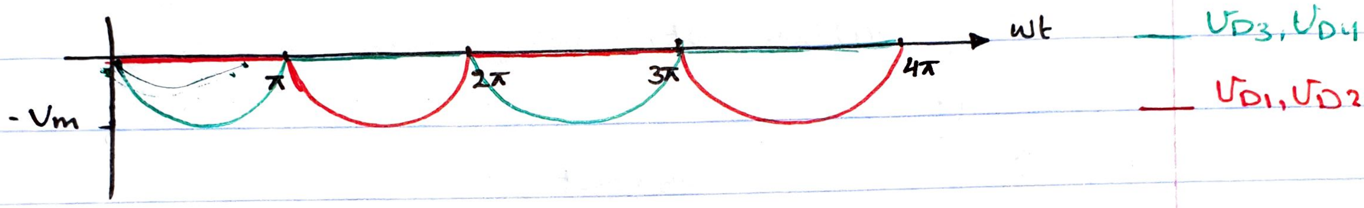
يعني الثيتا دايمًا صفر

$1-\phi$ transformer

$V_s = V_m \sin(\omega t)$.







- The average output voltage:

$$V_{DC} = \frac{1}{2\pi} \int_0^{2\pi} V_o(t)\, d(wt).$$

$$V_{DC} = \frac{1}{\pi} \int_0^{\pi} V_m \sin(wt)\, d(wt).$$

$$V_{DC} = \frac{V_m}{\pi} \left[\cos(wt) \Big|_{\pi}^{0} \right] = \frac{V_m}{\pi} \left[1 - (-1) \right] = \frac{2V_m}{\pi}$$

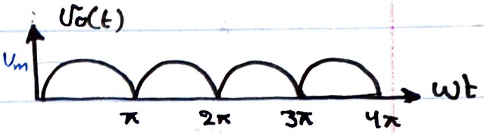
- Note: Two diodes are conducting at once and the forward voltage could be a problem for low voltage application (5 V power supply).
- Note: The transformer current does not carry DC current → No DC saturation problem in the transformer core.

2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 23
24, 27

- Example: Find the Fourier series for output voltage from the 1-ϕ Full-wave rectifier?

Solution:

$$V_o(t) = a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$



$$a_o = \frac{1}{2\pi} \int_0^{2\pi} V_o(t)\, d(\omega t) = \frac{2V_m}{\pi}$$

• $\sin\alpha \cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} V_o(t) \cos(n\omega t)\, d(\omega t).$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} V_m \sin(\omega t) \cos(n\omega t)\, d(\omega t).$$

$$a_n = \frac{V_m}{\pi} \int_0^{\pi} \sin((n+1)(\omega t))\, d(\omega t) - \int_0^{\pi} \sin((n-1)\omega t)\, d(\omega t).$$

$$a_n = \frac{V_m}{\pi}\left[\frac{1}{n+1} \cos(n+1)\omega t \Big|_{\pi}^{0} + \frac{1}{n-1} \cos(n-1)\omega t \Big|_{0}^{\pi} \right]$$

$$a_n = \frac{V_m}{\pi}\left[\frac{1}{n+1}(1 - \cos[(n+1)\pi]) + \frac{1}{n-1}(\cos[(n-1)\pi] - 1) \right].$$

When n is odd $\rightarrow a_n = 0$.

$$a_n = \frac{V_m}{\pi}\left[\frac{2}{n+1} - \frac{2}{n-1}\right] = \frac{2V_m}{\pi}\left(\frac{2n-2-2n-2}{n^2-1}\right)$$

$$a_n = -\frac{4V_m}{\pi(n^2-1)}$$ when n is even.

$$b_n = \frac{1}{\pi} \int_0^{2\pi} V_o(t) \sin(n\omega t)\, d(\omega t).$$

• $\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$

$$b_n = \frac{2}{\pi} \int_0^{\pi} V_m \sin(\omega t) \sin(n\omega t)\, d\omega t.$$

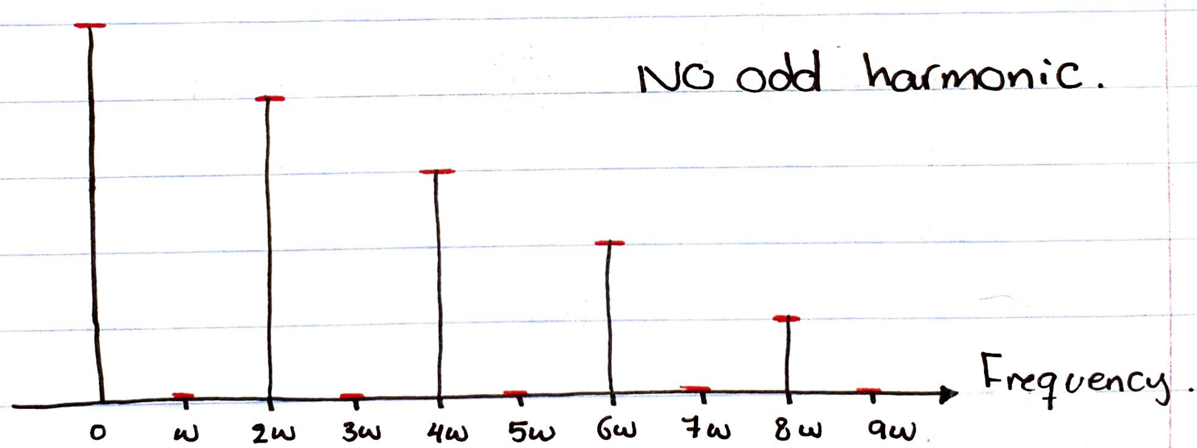
$$b_n = \frac{V_m}{\pi}\left[\int_0^{\pi} \cos(n+1)\omega t \, d(\omega t) - \int_0^{\pi} \cos(n-1)\omega t \, d(\omega t)\right]$$

$$b_n = \frac{V_m}{\pi}\left[\frac{1}{n+1}\sin(n+1)\omega t\Big|_0^{\pi} - \left(\frac{1}{n-1}\right)\sin(n-1)\omega t\Big|_0^{\pi}\right]$$

$$b_n = 0.$$

$$V_o(t) = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos(2\omega t)\,\cos(4\omega t) - \frac{4V_m}{15\pi}\cos(n\omega t) - \frac{4V_m}{35\pi}\cos(6\omega t) - \dots$$

$$V_o(t) = \frac{2V_m}{\pi} + \sum_{n=2,4,6,\dots}^{\infty} \frac{4V_m}{(n^2-1)\pi}\cos(n\omega t - \pi).$$



NO odd harmonic.

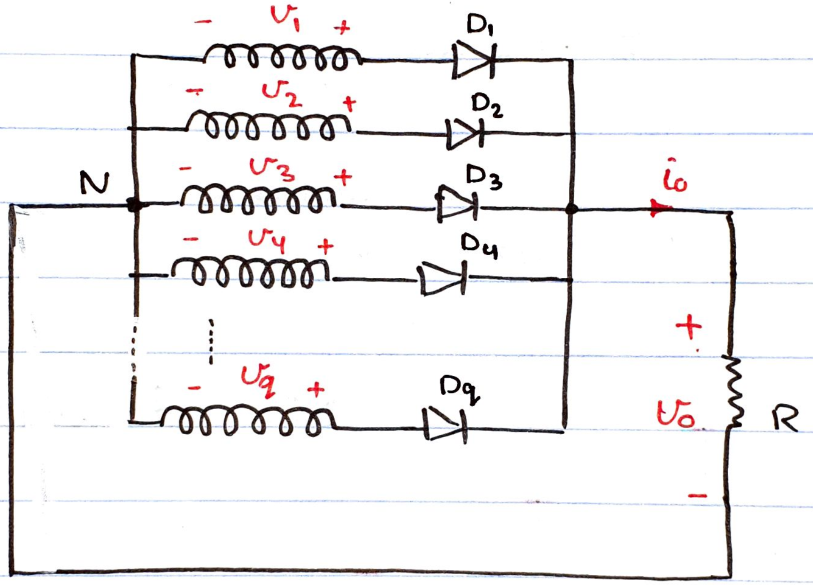
Dc Component

Fundamental Component.

3. Multi_Phase Star Rectifiers.

- The 1_ϕ Full-wave rectifier is usually used in application up to 5 kw.
- The Fundamental Frequency of 1_ϕ Full wave rectifier is 2w where w is the source frequency.

- A Filter is used to remove the harmonics. The size of the filter decreases with the increase in the frequency of harmonic.
- For higher powers, a multi-phase rectifiers are usually used to produce a fundemental compenent at frequency of $q\omega$ where q is the number of phases.
- The other harmonics at frequencies $nq\omega$, $n = 2, 3, \ldots$



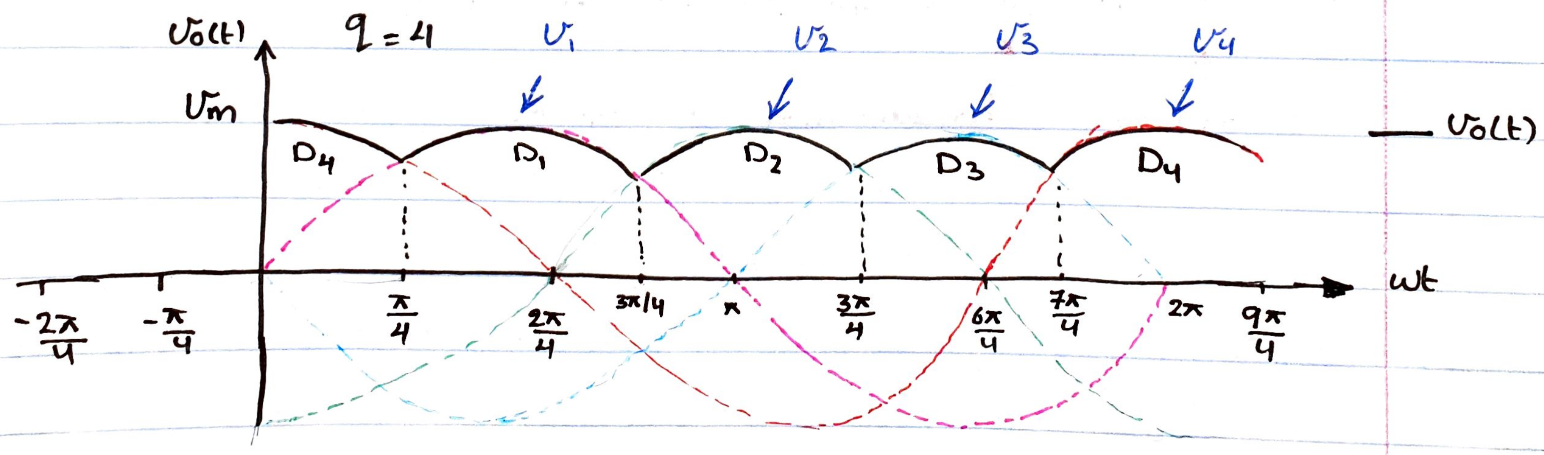
$v_1 = V_m \sin(\omega t)$

$v_2 = V_m \sin(\omega t - 90^\circ)$

$v_3 = V_m \sin(\omega t - 180^\circ)$

$v_4 = V_m \sin(\omega t - 270^\circ)$

$v_1, v_2, \ldots, v_q$ are shifted from each other by $\frac{2\pi}{q}$.



- Each diode conducts for $\frac{2\pi}{q}$

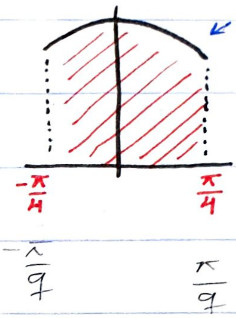
- The average output voltage:

$$\int_{-\pi/4}^{\pi/4} V_m \sin(\omega t + \tfrac{\pi}{2})\, d\omega t$$

$$V_{DC} = \frac{4}{2\pi} \int_{\pi/4}^{3\pi/4} V_m \sin(\omega t)\, d(\omega t) = \frac{4}{2\pi} \int_{-\pi/4}^{\pi/4} V_m \cos(\omega t)\, d(\omega t).$$

$$V_{DC} = \frac{4\times 2}{2\pi} \int_{0}^{\pi/4} V_m \cos(\omega t)\, d\omega t = \frac{2q}{2\pi}$$, $q = 4 \Rightarrow$ Pulses عدد

$$= \frac{2q}{2\pi} \int_{0}^{\pi/q} V_m \cos(\omega t)\, d\omega t \Leftarrow$$



$$= \frac{q}{\pi} \sin\left(\frac{\pi}{q}\right), \text{ valid } q > 2.$$

$$= \frac{q}{\pi} V_m \sin\left(\frac{\pi}{q}\right)$$

- The rms value of the output voltage:

$$V_{RMS} = \sqrt{\frac{2q}{2\pi} \int_{0}^{\pi/q} V_m^2 \cos^2(\omega t)\, d\omega t}$$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x).$

$$V_{RMS} = \sqrt{\frac{q}{2\pi} \int_{0}^{\pi/q} V_m^2 (1 + \cos 2\omega t)\, d(\omega t)}$$

$$= \sqrt{\frac{q}{2\pi}\left[\frac{\pi}{q} + \frac{1}{2}\sin\left(\frac{2\pi}{q}\right)\right]} \cdot V_m \quad V_{rms} = V_m \sqrt{\frac{q}{2\pi}\left(\frac{\pi}{q} - \frac{1}{2}\sin\frac{2\pi}{q}\right)}$$

- The rms value of the transformer secondary current.

$$I_s = \sqrt{\frac{1\times 2}{2\pi} \int_{0}^{\pi/q} I_m^2 \cos^2 \omega t\, d(\omega t)} \quad ; \quad I_m = \frac{V_m}{R}$$

Single phase

- Example: A 3-ϕ star rectifier has a purly resistive load with R ohms. Determine:

a- Efficiency. b- FF c- RF. d- TUF e- PIV ← أعلى Voltage على diode وهو off

f. Peak Current through a diode if the rectifier deliver. $I_{DC} = 30$ A at an output voltage of $V_{DC} = 140$ V.

$VI \Rightarrow$ RMS value

$\frac{1}{2} VI \Rightarrow$ Peak value

$\sqrt{3} VI \Rightarrow$ Line to Line

$3 VI$ Line to neutral

- Solution:

a- $$\eta = \frac{P_{DC}}{P_{AC}} = \frac{V_{DC} I_{DC}}{V_{RMS} I_{RMS}}$$

$3\text{-}\phi \rightarrow q$

$$V_{DC} = \frac{3 \times 2}{2\pi} \int_0^{\pi/3} V_m \cos(\omega t)\, d\omega t.$$

$$V_{DC} = \frac{3}{\pi} V_m \sin\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{2\pi} V_m = 0.827\, V_m.$$

$$I_{DC} = \frac{V_{DC}}{R} = 0.827 \frac{V_m}{R}.$$

$$V_{RMS} = \sqrt{\frac{3 \times 2}{2\pi} \int_0^{\pi/3} V_m^2 \cos^2(\omega t)\, d\omega t}$$

$$V_{RMS} = \sqrt{\frac{3}{2\pi} V_m^2 \int_0^{\pi/3} (1 + \cos(\omega t)\, d(\omega t)}$$

$$V_{RMS} = \sqrt{\frac{3}{2\pi} V_m^2 \left(\frac{\pi}{2} + \frac{1}{2} \sin\left(\frac{2\pi}{3}\right)\right)}$$

$$V_{RMS} = 0.841\, V_m \; ; \; I_{RMS} = \frac{V_{RMS}}{R} = 0.841 \frac{V_m}{R}.$$

$$\eta = \frac{(0.827)^2 (V_m^2/R)}{(0.841)^2 (V_m^2/R)} = 96.77\,\%.$$

b. $FF = \frac{V_{RMS}}{V_{DC}} = \frac{0.841\ V_m}{0.827\ V_m} = 101.65\%$.

c. $RF = \sqrt{FF^2 - 1} = \sqrt{(1.0165)^2 - 1} = 18.24\%$.

d. $TUF = \frac{V_{DC} I_{DC}}{}$

$3\text{-}\phi \rightarrow \underline{3 V_s I_s}$

$V_s = V_m/\sqrt{2}$, $I_s = \frac{V_m}{2R}$

$$I_s = \sqrt{\frac{1\times 2}{2\pi} \int_0^{\pi/3} I_m^2 \cos^2(\omega t)\, d(\omega t)}$$

$$I_s = \sqrt{\frac{1}{2\pi} I_m^2 \left(\frac{\pi}{3} + \frac{1}{2}\sin\frac{\pi}{3}\right)} = 0.4854 \frac{V_m}{R}.$$

$$TUF = \frac{(0.827)^2\ V_m^2/R}{3\left(\frac{1}{\sqrt{2}}\right)(0.4854)\frac{V_m^2}{R}} = 66.43\%.$$

e. $PIV = \sqrt{3}\ V_m$

f. $I_{diode,peak} = \frac{V_m}{R} = I_m$

$R = \frac{V_{DC}}{I_{DC}} = \frac{140}{30}\ \Omega$.

$V_m = \frac{V_{DC}}{0.827} = \frac{140}{0.827}$

$I_{diode,\ peak} = \frac{140}{0.827} \cdot \frac{30}{140}$

$= 36.27\ A$.

$I_d = I_m \cdot \frac{1}{\pi} \sin\frac{\pi}{q}$

$I_d = 0.2757\ I_m$

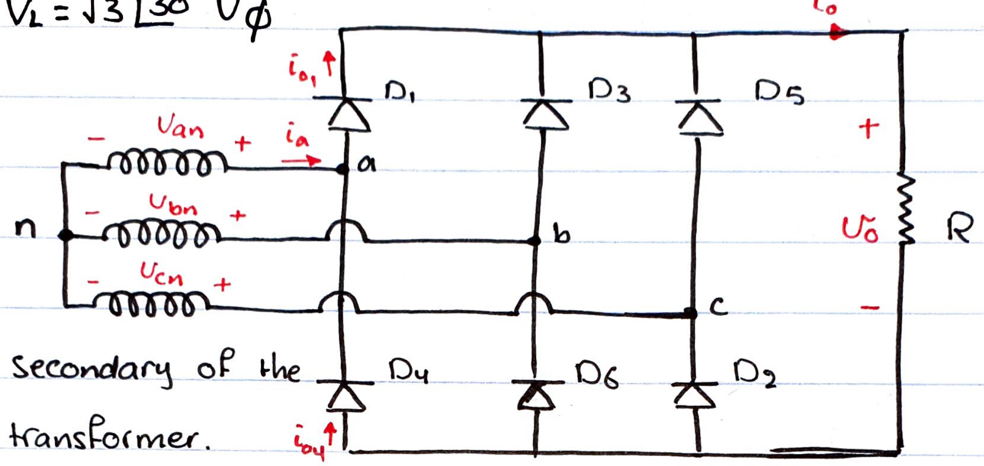
$I_d = 30/3 = 10$

$I_m = 10/0.2757$

$= 36.27\ A$

4. Three Phase Full wave Bridge Rectifier.

$\vec{V_L} = \sqrt{3} \angle 30^\circ \, \vec{V_\phi}$

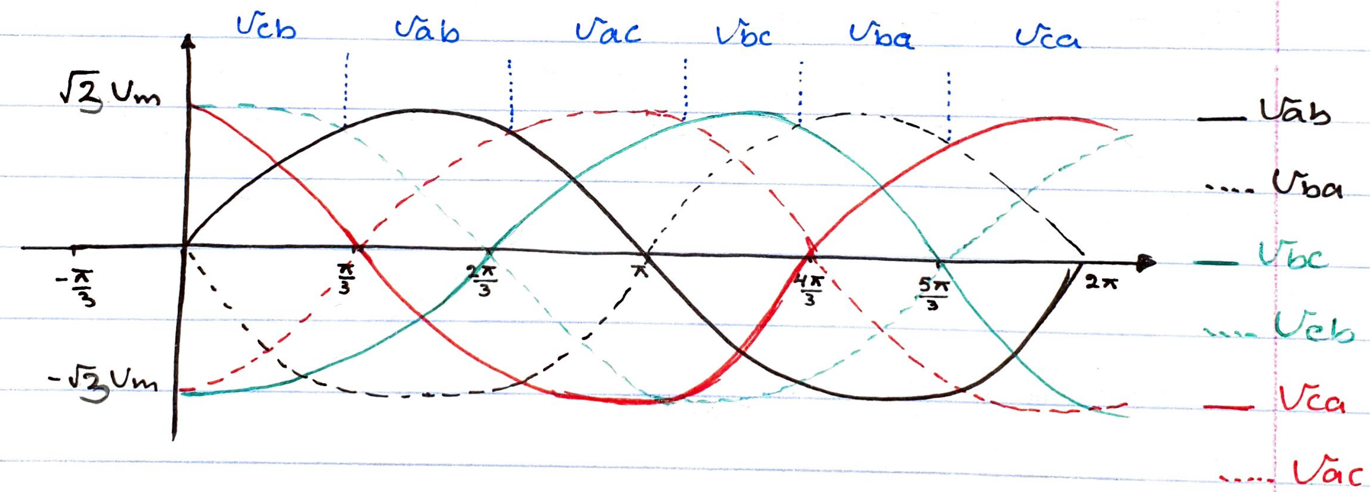


secondary of the transformer.

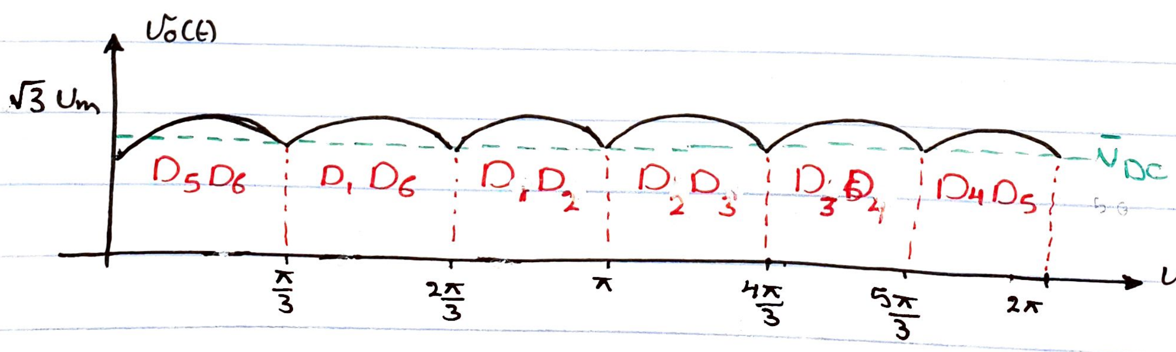
$V_{an} = V_m \sin(\omega t - 30^\circ)$ $\qquad$ $V_{ab} = \sqrt{3}\, V_m \sin(\omega t)$.

$V_{bn} = V_m \sin(\omega t - 150^\circ)$ $\qquad$ $V_{bc} = \sqrt{3}\, V_m \sin(\omega t - 120^\circ)$

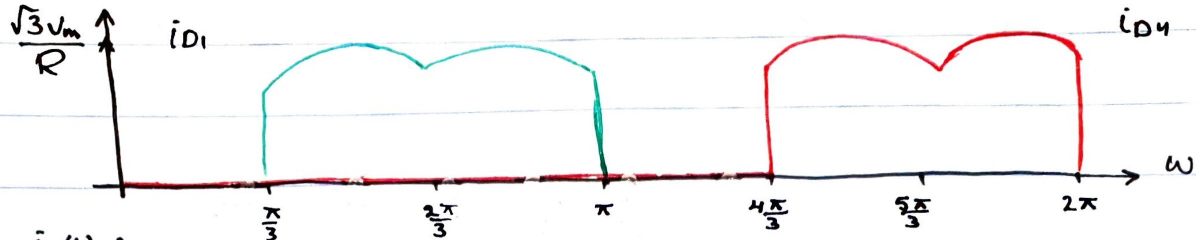
$V_{cn} = V_m \sin(\omega t + 90^\circ)$ $\qquad$ $V_{ca} = \sqrt{3}\, V_m \sin(\omega t + 120^\circ)$.

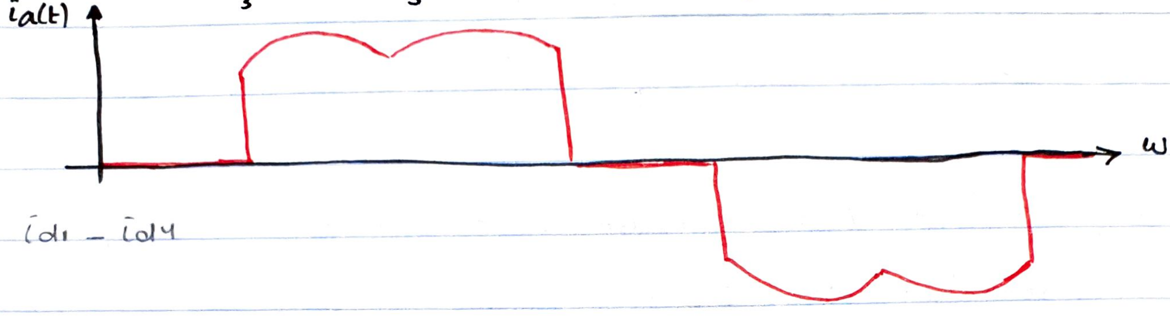


$V_{cb} = V_{cg} - V_{bg}$.



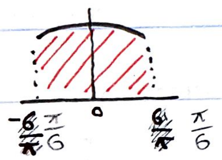
- Each diode conducts for $\frac{2\pi}{3}$.
- Conduction sequence of diodes: D_1D_2, D_2D_3, D_3D_4, D_4D_5, D_5D_6, D_6D_1 ......





$i_a = i_{d1} - i_{d4}$

- The average output voltage :



$$V_{DC} = \frac{1}{2\pi}\int_0^{2\pi} v_o(t)\, d(\omega t)$$

$$V_{DC} = \frac{6}{2\pi}\int_0^{\pi/3} v_{cb}\, d(\omega t) = \frac{6}{2\pi}\int_0^{\pi/3} \sqrt{3}\, V_m \sin(\omega t - 120^\circ)\, d(\omega t).$$

$$V_{DC} = \frac{-6}{2\pi}\int_{-\pi/6}^{\pi/6} \sqrt{3}\, V_m \sin(\omega t - 90^\circ)\, d(\omega t).$$

$$V_{DC} = \frac{6\times 2}{2\pi}\int_0^{\pi/6} \sqrt{3}\, V_m \cos(\omega t)\, d(\omega t).$$

$$V_{DC} = \frac{6}{\pi}\sqrt{3}\, V_m \sin\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{\pi} V_m.$$

- The rms value of the output voltage :

$$V_{RMS} = \sqrt{\frac{6\times 2}{2\pi}\int_0^{\pi/6} (\sqrt{3}\, V_m \cos(\omega t))^2\, d(\omega t)}$$

$$V_{RMS} = \sqrt{\frac{9}{\pi}\int_0^{\pi/6} (1+\cos(2\omega t))\, d(\omega t)}\; V_m.$$

$$V_{RMS} = \sqrt{\frac{9}{\pi}\left[\frac{\pi}{6} + \frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right]}\; V_m = 1.6554\, V_m.$$

- The rms value of the transformer secondary current.

$$I_s = \sqrt{\frac{4\times 2}{2\pi}\int_0^{\pi/6} I_m^2 \cos^2(\omega t)\, d(\omega t)} \quad ; \quad I_m = \frac{\sqrt{3}}{R} V_m$$

$$I_s = \sqrt{\frac{2}{\pi}\left[\frac{\pi}{6} + \frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right]} \cdot I_m \Rightarrow I_s = 0.7804\, I_m.$$

- Example: A 3-ϕ full-wave bridge rectifier, supplying a purly resistive load of R.

1. Draw the circuit.
2. Plot the output voltage and the input corrent. i_a, i_b, i_c
3. Determine:

a. η b. FF c. RF d. TUF e. PIV.

f. Peak current of the diode.

The rectifier deliver $I_{DC} = 60$ A at output voltage of $V_{DC} = 280.7$ V and frequency of 60 Hz.

- a. $\eta = \frac{P_{DC}}{P_{AC}} = \frac{V_{DC}\, I_{DC}}{V_{RMS}\, I_{RMS}}$

$$V_{DC} = \frac{6\times 2}{2\pi}\int_0^{\pi/6} \sqrt{3}\, V_m \cos(\omega t)\, d(\omega t) = \frac{3\sqrt{3}}{\pi} V_m.$$

$$V_{DC} = 1.6542\, V_m, \quad I_{DC} = \frac{1.6542\, V_m}{R}.$$

$$V_{RMS} = \sqrt{\frac{6\times 2}{2\pi}\int_0^{\pi/6} (\sqrt{3}\, V_m \cos(\omega t))^2\, d(\omega t)}$$

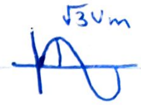
$$V_{RMS} = \sqrt{\frac{9}{\pi}\left[\frac{\pi}{6} + \frac{1}{2}\frac{\sqrt{3}}{2}\right]} \cdot V_m = 1.6554\, V_m.$$

$$I_{RMS} = \frac{1.6554\, V_m}{R}, \quad \eta = \frac{(1.6542)^2}{(1.6554)^2} = 99.86\%.$$

b. $FF = \frac{V_{RMS}}{V_{DC}} = \frac{1.6554}{1.6542} = 1.0007 = 100.07\,\%$.

c. $RF = \sqrt{FF^2 - 1} = \sqrt{(1.0007)^2 - 1} = 3.74\,\%$.

d. $TUF = \frac{V_{DC}\, I_{DC}}{\sqrt{3}\, V_s\, I_s}$, V_s (line to line RMS).



$V_s = \frac{\sqrt{3}\, V_m}{\sqrt{2}}$, $I_s = \sqrt{\frac{4 \times 2}{2\pi} \int_0^{\pi/6} I_m^2 \cos^2(\omega t)\, d(\omega t)}$

$= \sqrt{\frac{2}{\pi}\left(\frac{6}{\pi} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2}\right)}\, I_m = 0.7804\, I_m$.

$TUF = \frac{(1.6542)^2 (V_m^2/R)}{\sqrt{3}\left(\frac{\sqrt{3}}{\sqrt{2}} V_m\right)\left(0.7804 \cdot \frac{\sqrt{3}\, V_m}{R}\right)} = 0.9545$.

e. $PIV = \sqrt{3}\, V_m$

f. $I_{D,\,peak} = \frac{\sqrt{3}\, V_m}{R} = \frac{\sqrt{3}\,(V_{DC}/1.6542)}{(V_{DC}/I_{DC})}$

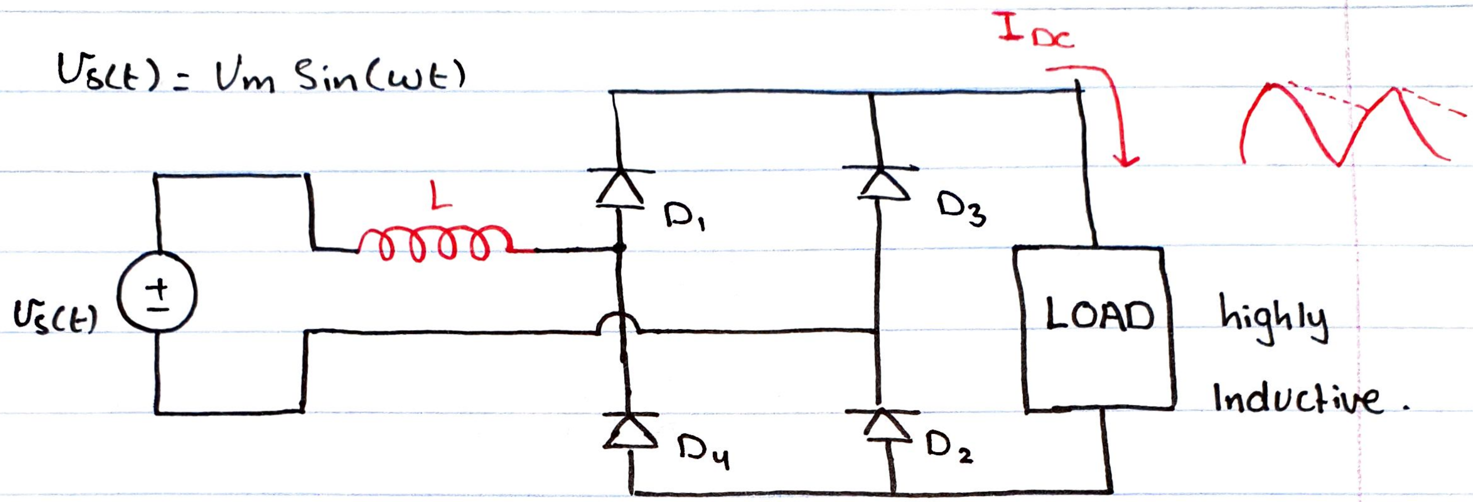
$I_{D,\,peak} = \frac{\sqrt{3}\,(60)}{1.6542} \simeq 62\ A$.

$I_d = I_m \frac{1}{\pi} \sin\frac{\pi}{9}$ $I_d = \frac{66}{3} = 20$.

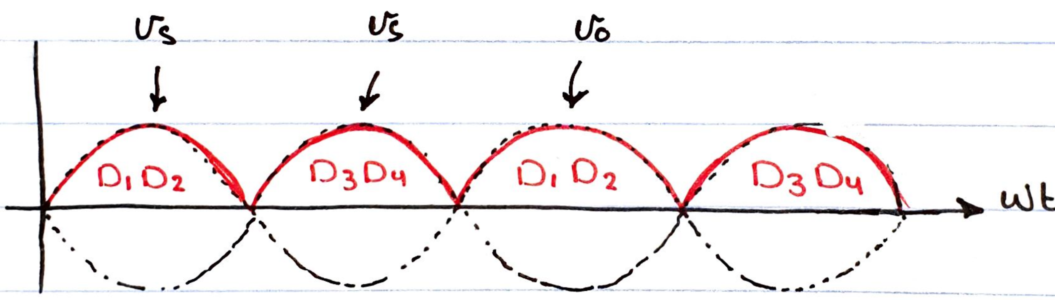
- The effect of load and source inductances.

Consider a 1-Ø full-wave bridge rectifier supplying a highly inductive load.

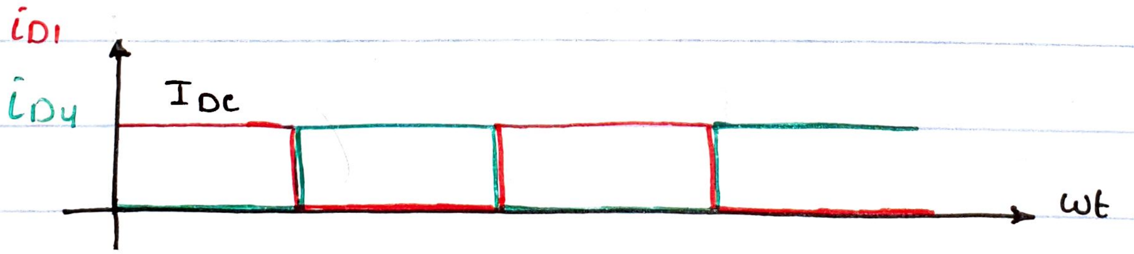
$v_s(t) = V_m \sin(\omega t)$

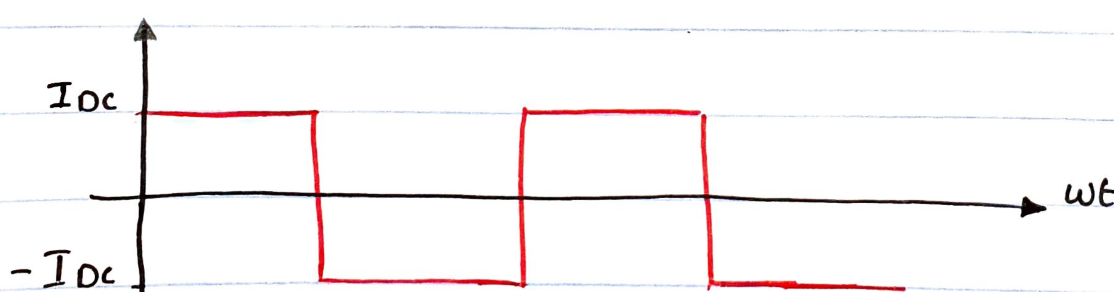


- The highly inductive load causes the load current to be continuous and free ripple. (Constant).

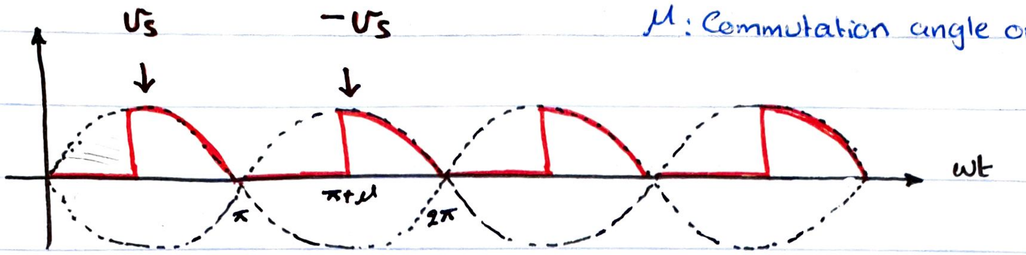


Without L

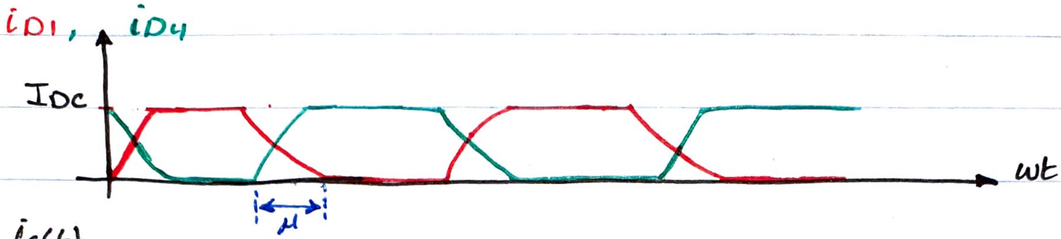


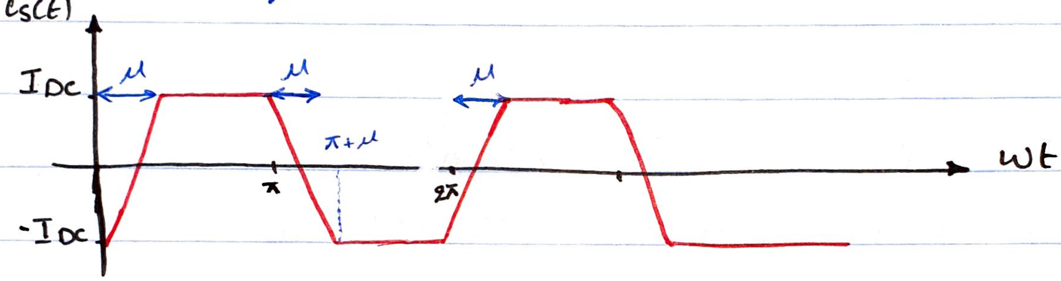


μ : Commutation angle or overloop Period.

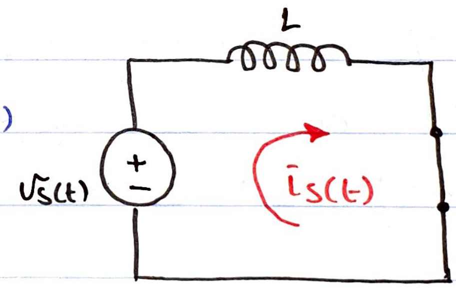


With L





- Equivalent Circuit during overloop period.



$\pi \le \omega t \le \pi + \mu$

$$V_s(t) = L\,\frac{di_s(t)}{dt} = V_m \sin(\omega t)$$

$$\int_{i_s(\pi)}^{i_s(\pi+\mu)} \omega L \, di_s(t) = \int_{\pi}^{\pi+\mu} V_m \sin(\omega t)\, d\omega t$$

$$\omega L(\overset{-I_{DC}}{i_s(\pi+\mu)} - \overset{I_{DC}}{i_s(\pi)}) = V_m(\cos\pi - \cos(\pi+\mu)).$$

$$-2\omega L I_{DC} = V_m(-1+\cos\mu).$$

$$\cos\mu = 1 - \frac{2\omega L I_{DC}}{V_m}.$$

- The average voltage reduction, V_x, due to Commutation is:

$$V_x = V'_{DC} - V_{DC}.$$

V'_{DC} : without L.

V_{DC} : with L.

$$V_x = \frac{2V_m}{\pi} - \frac{2}{2\pi}\int_{\mu}^{\pi} V_m \sin(\omega t)\, d(\omega t)$$

$$V_x = \frac{2V_m}{\pi} + \frac{V_m}{\pi}\left[\cos(\pi) - \cos\mu\right]$$

$$V_x = \frac{V_m}{\pi}\left[1 - \cos\mu\right]$$

$$V_x = \frac{V_m}{\pi}\left(\frac{2\omega L I_{DC}}{V_m}\right) = \frac{2\omega L I_{DC}}{\pi}$$

→ $V_x = 4fL\, I_{DC}$. single phase

where; f is the source frequency.

The average output voltage is given by:

- $V_{DC} = \frac{2V_m}{\pi} - V_x$.

- For 3-ϕ full-wave bridge rectifier:

$V_x = 6fL\, I_{DC}$

$V_{DC} = \frac{3\sqrt{3}\, V_m}{\pi} - V_x$.

- Example: A 3-ϕ bridge rectifier is supplied from Y-connected $\underset{}{\overset{L-L,\ rms}{208}}$ V, 60 Hz source. The load current is 60 A, and has ignored ripple. Calculate the percentage reduction of the output voltage due to commutation, and then calculate the average output voltage if the line inductance per phase is 0.5 mH.

Solution:

$V_x = 6fL\, I_{DC}$

$= 6(60)(0.5)(10^{-3})(60)$

$= 10.8$ V.

$$V_{DC} = \frac{3\sqrt{3}\, V_m}{\pi} - V_x.$$

$$= \frac{3\sqrt{3}\,(\sqrt{2}/\sqrt{3})(208)}{\pi} = 280.7.$$

$$\% = \frac{10.8}{280.7} = 3.85\%. \qquad \frac{V_x}{V_{DC}}$$

$V_{DC} = 280.7 - 10.8$ $\quad V_{DC} - V_x.$

$V_{DC} = 266.9\,V.$

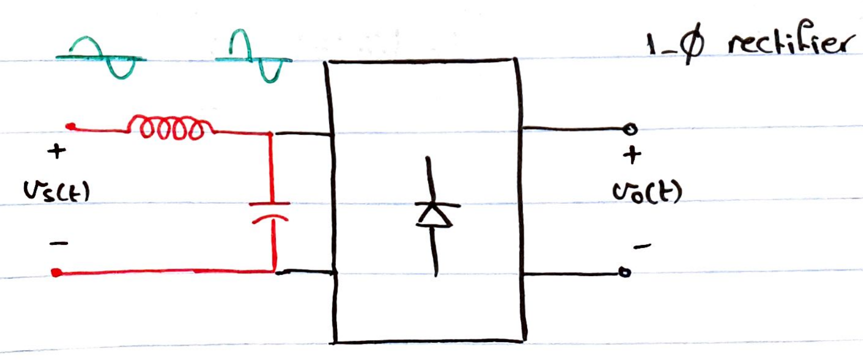
- Rectifier Circuit Design.

- The output of all rectifiers contains harmonics (DC Pulse AC ripple). Therefor, Low pass DC filter are used to reduce the ripple (harmonics).
- Types of DC filters.

1. C-type Capacitor smoothing.
2. L-type Inductor smoothing.
3. LC-type.

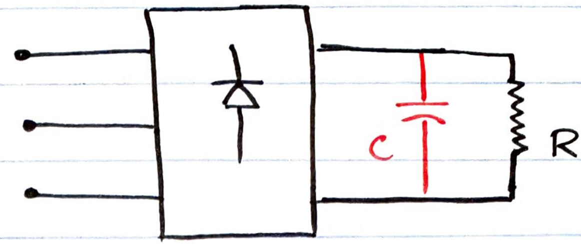
- The input current also contains harmonics, due to rectification process.
- Low pass AC filters are needed to reduce the harmonics from the grid.

LC- filter.

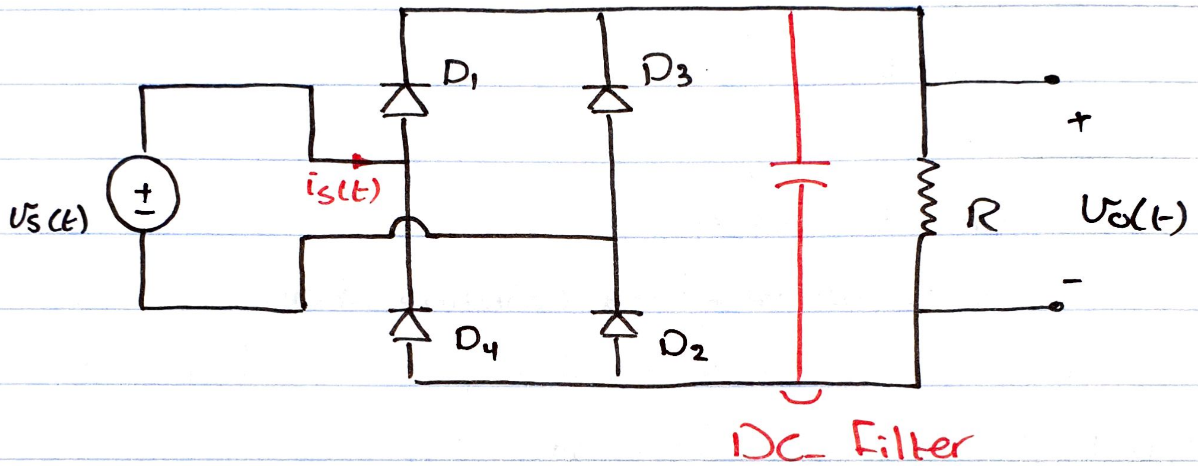


1. C-type Filter.

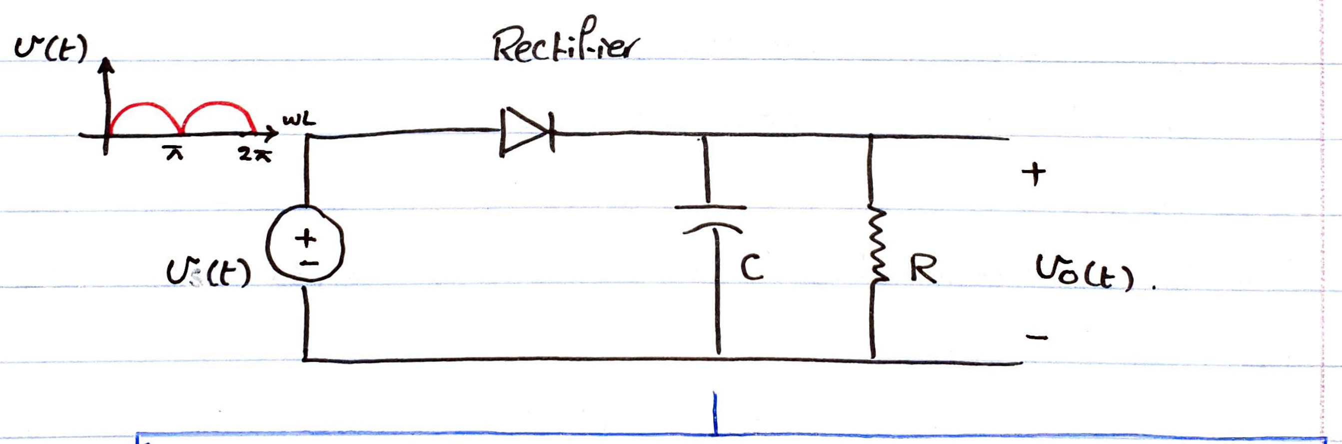
3-ϕ AC Source "Grid"



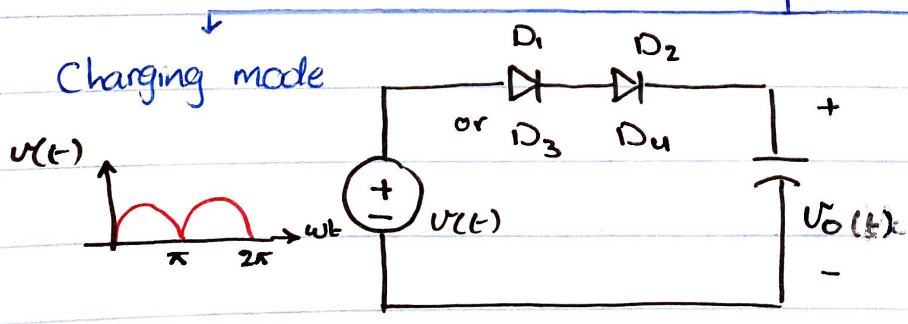
- Consider a 1-ϕ full-wave rectifier.



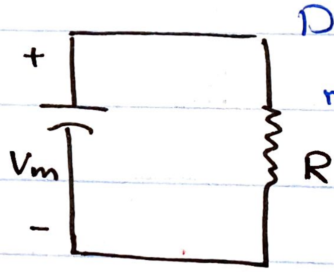
- Circuit model.



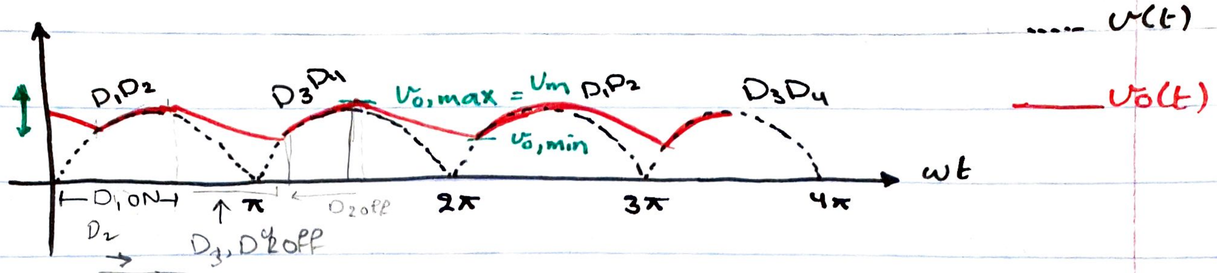
Charging mode

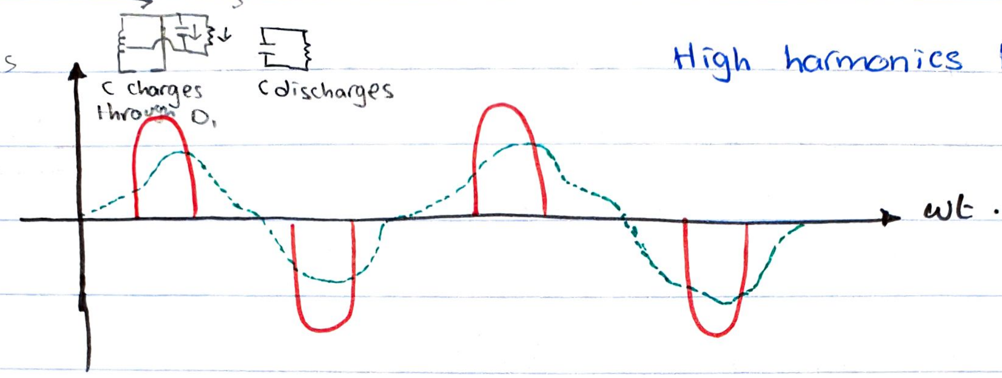


Discharging mode



ΔV_o: Peak to Peak ripple voltage. is



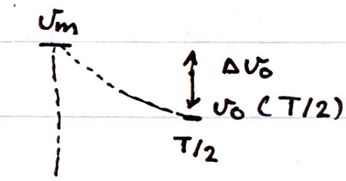


High harmonics Low PF.

- The diode current is given by:

$$i_D = i_C + i_R = \frac{C\,dv_o}{dt} + \frac{v_o}{R} = \omega C\, V_m \cos(\omega t) + \frac{V_m \sin(\omega t)}{R}.$$

- The Output voltage across the capacitor during the discharging mode is:



$$v_o(t) = V_m e^{-t/\tau}, \quad \tau = RC.$$

$$\tau \gg \frac{T}{2} \; ; \quad T: \text{is the period } T = 1/f.$$

Using Taylor expanssion:

$e^{-x} \approx 1 - x$ if x is small.

$$v_o(t) \approx V_m \left(1 - \frac{t}{\tau}\right).$$

$$\Delta V_o = V_{o,max} - V_{o,min}.$$

$$V_{o,max} = V_m.$$

$$V_{o,min} = V_o(T/2) = V_m(1 - T/2\tau).$$

$$\Delta V_o = V_m - V_m(1 - T/2\tau).$$

$$\Delta V_o = \frac{V_m T}{2\tau}.$$

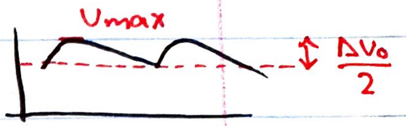
$$\Delta V_o = \frac{V_m}{2fRC}$$

- In general;

$$\Delta V_o = \frac{V_{max}}{f_r RC}$$

where; V_{max} is the maximum value of $v_o(t)$.
f_r is called the ripple frequency.

- The average output voltage is; V_{DC}.



$$V_{DC} = V_{max} - \frac{\Delta V_o}{2} = V_{max}\left(1 - \frac{1}{2 f_r RC}\right).$$

Type of rectifier.	V_{max}	f_r
1_ϕ half_wave.	V_m	f
1_ϕ full_wave.	V_m	$2f$
3_Phase Star	V_m	$3f$
3_Phase bridge	$\sqrt{3}\,V_m$	$6f$

- If ΔV_o is small, the load current is approximated as

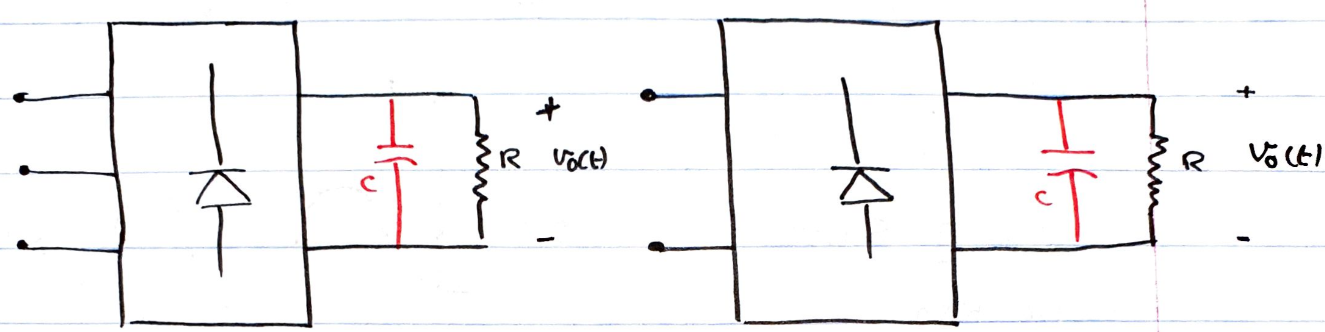
$$I_{DC} = \frac{V_{max}}{R}.$$

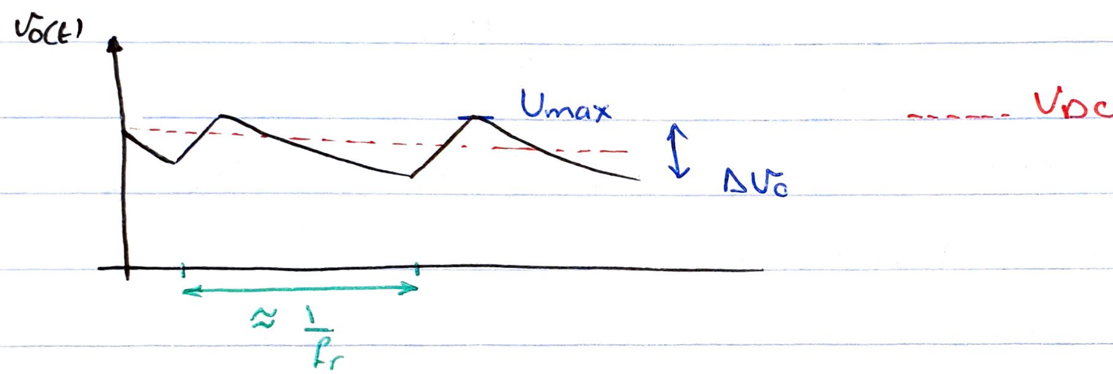
$$\Delta V_o = \frac{I}{f_r C}. \qquad I\uparrow \Rightarrow \Delta V_o \uparrow \Rightarrow V_{DC} \downarrow$$

- Rectifier Circuit Design.

1. C-type Filter

3-ϕ AC - Source   1-ϕ AC - Source.





- Peak to Peak ripple voltage.

$$\Delta V_o = \frac{V_{max}}{f_r RC}$$

- The average Output voltage

$$V_{DC} = V_{max} - \frac{\Delta V_o}{2} = V_{max}\left(1 - \frac{1}{2f_r RC}\right)$$

Since ΔV_o is small, the average load current is approximated as:

$$I_{DC} = \frac{V_{max}}{R}$$

$$\Delta V_o = \frac{I_{DC}}{f_r C}$$

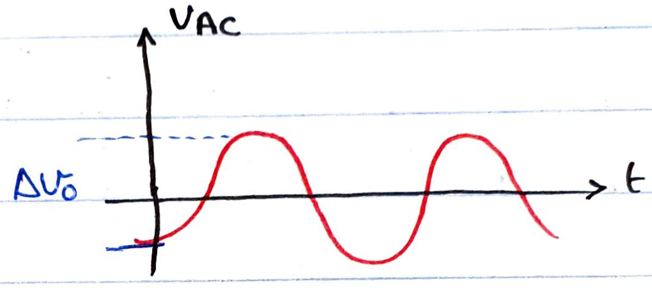
$$V_{DC} = V_{max} - \frac{I_{DC}}{2f_r C}.$$

- Ripple factor:

$$RF = \frac{V_{AC}}{V_{DC}}$$

where ; V_{AC} is RMS value of the AC Component (ripple).

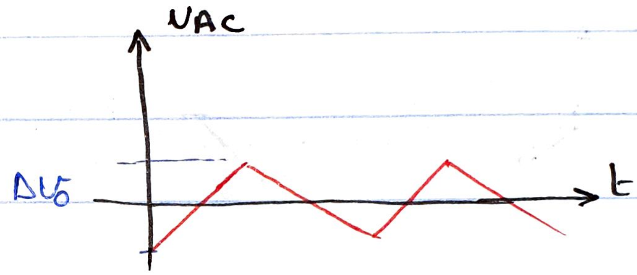
$$V_{AC} = \frac{\Delta V_o}{2\sqrt{2}}$$, by



assuming sinusoidal wave form.

OR:

$$V_{AC} = \frac{\Delta V_o}{2\sqrt{3}}$$, by



assuming triangular waveform.

- $V_{AC} = \frac{\Delta V_o}{2\sqrt{2}}$; It gives more margin for safety.

$$RF = V_{AC} / V_{DC}.$$

$$V_{AC} = \frac{\Delta V_o}{2\sqrt{2}} = \frac{V_{max}}{2\sqrt{2}\, f_r RC}$$

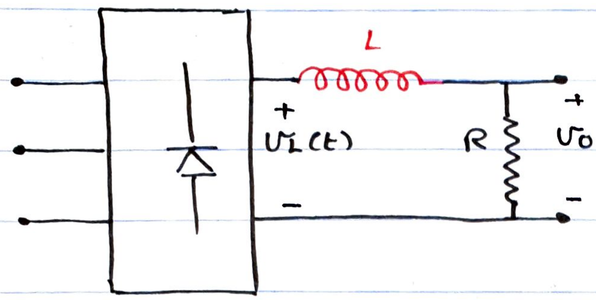
$$V_{DC} = V_{max}\left(1 - \frac{1}{2 f_r RC}\right)$$

$$RF = \frac{1}{2\sqrt{2}\, f_r RC} \cdot \frac{1}{\left(1 - \frac{1}{2 f_r RC}\right)}$$

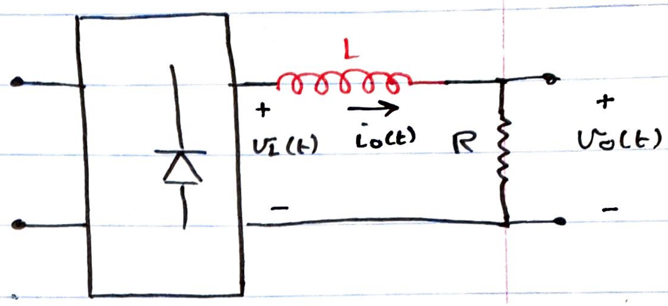
$$RF = \frac{1}{\sqrt{2}} \cdot \frac{1}{(2 f_r RC - 1)}$$

2. L - type Filter.

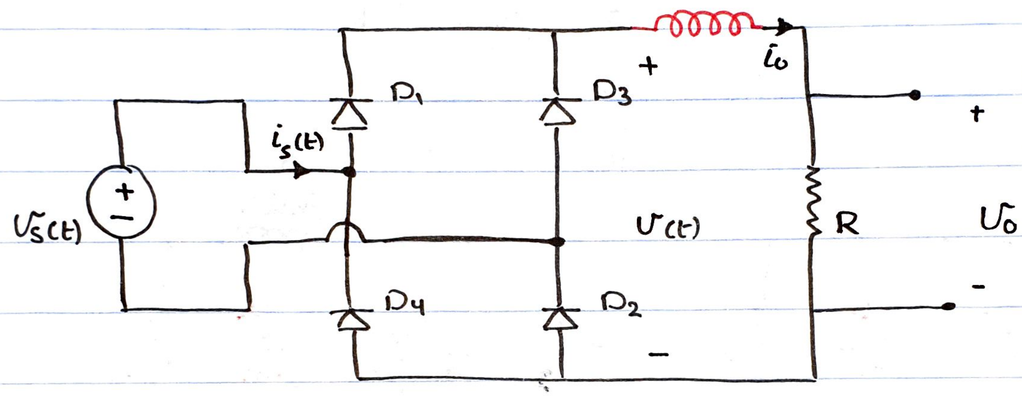
3 - ϕ AC Source.



1 - ϕ AC Source.



Consider a 1 - ϕ Full - wave bridge rectifier:

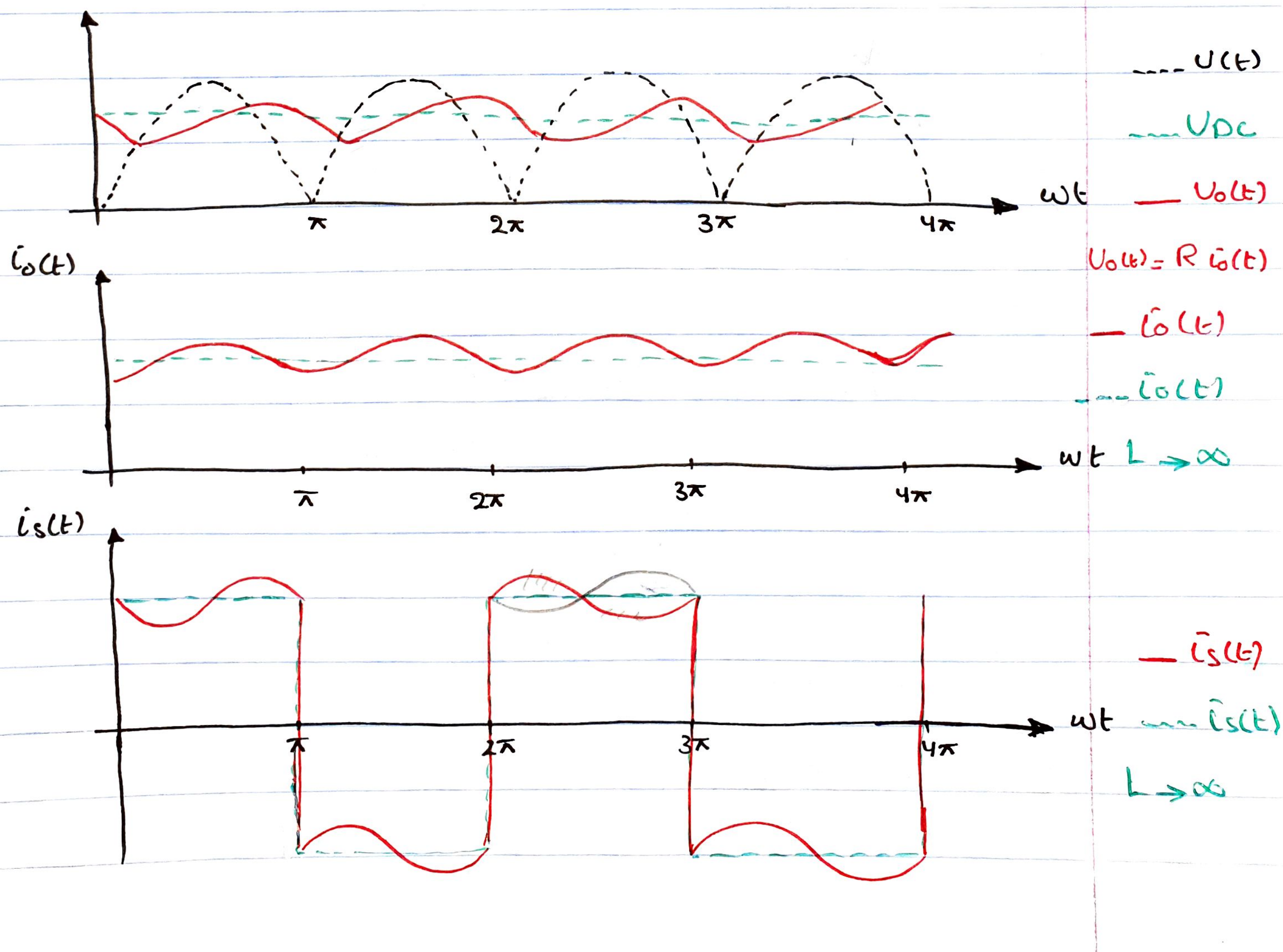


$V_m \sin(\omega t) = R i_o + L \frac{di_o}{dt}$

$i_o = \frac{V_m}{Z} e^{-t/\tau} \sin\beta + \frac{V_m \sin(\omega t - \beta)}{Z}$

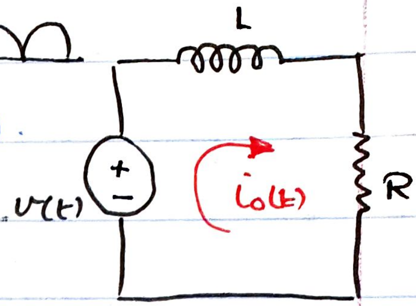
$Z = \sqrt{R^2 + (\omega L)^2}$, $\beta = \tan^{-1} \frac{\omega L}{R}$

$\tau = L/R$.

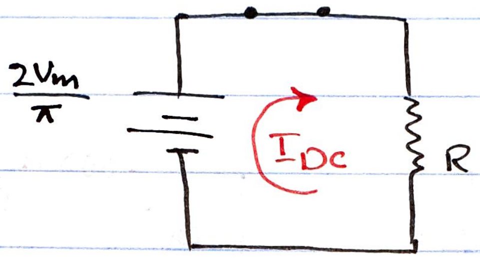


- Fourier Series.

$$v(t) = \frac{2V_m}{\pi} - \sum_{n=2,4,\dots}^{\infty} \frac{4V_m}{\pi(n^2-1)} \cos(n\omega t).$$

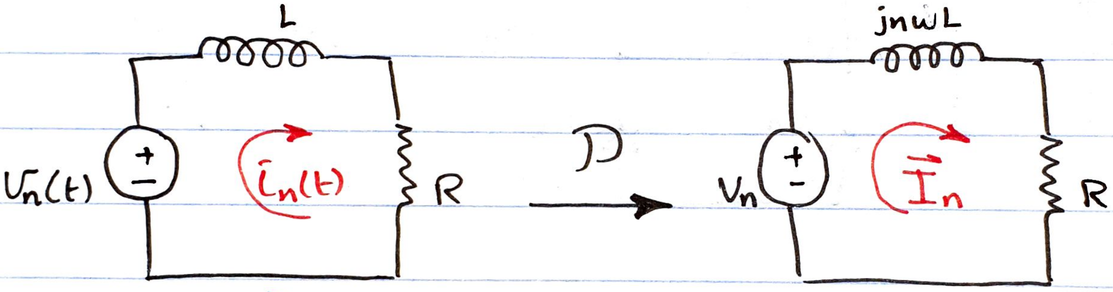


- Equivalent circuit for DC component.



$$I_{DC} = \frac{2V_m}{\pi R}$$

- Equivalent circuit for n^{th} harmonic:



$$\vec{I}_n = \frac{\vec{V}_n}{R + j\omega nL}$$

$$\vec{V}_n = \frac{4V_m}{\pi(n^2-1)} \angle 180^\circ = V_n \angle 180^\circ.$$

$$\vec{I}_n = \frac{V_n \angle 180^\circ}{\sqrt{R^2 + (n\omega L)^2} \angle \tan^{-1}(n\omega L/R)}$$

$$\vec{I}_n = \frac{V_n}{\sqrt{R^2 + (n\omega L)^2}} \angle 180 - \tan^{-1}(n\omega L/R)$$

$$\big\downarrow D^{-1}$$

$$i_n(t) = \frac{-V_n}{\sqrt{R^2 + (n\omega L)^2}} \cos(n\omega t - \theta_n)$$

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right).$$

$$i_o(t) = I_{DC} + \underbrace{i_2(t) + i_4(t) + i_6(t) + \cdots}$$

These harmonic are ignored, since they are attenuated compared with i_2.

$$i_o(t) = I_{DC} - \underbrace{\frac{4V_m}{3\pi} \cdot \frac{1}{\sqrt{R^2 + (2\omega L)^2}}}_{\hat{I}_2} \cos(2\omega t) \quad , \quad I_{DC} = \frac{2V_m}{\pi R}$$

$$= I_{DC} + \hat{I}_2 \cos(2\omega t - 180^\circ).$$

- The RMS value of AC component (ripple) is;

$$I_{AC} = \frac{\hat{I}_2}{\sqrt{2}}$$

$$I_{DC} = \frac{2V_m}{\pi R}$$

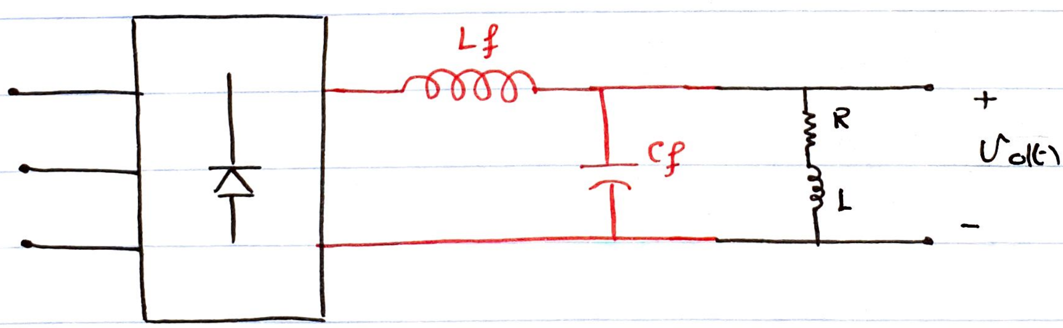
Ripple Factor.

$$RF = \frac{I_{AC}}{I_{DC}} = \frac{\sqrt{2}}{3} \cdot \frac{1}{\sqrt{R^2 + (2\omega L)^2}} \cdot R$$

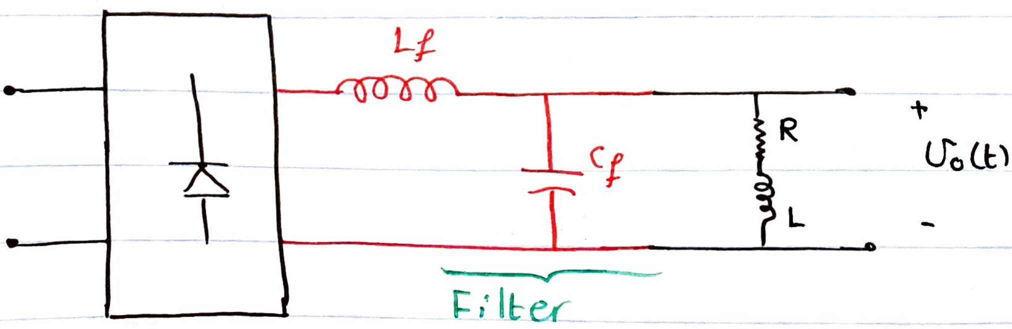
$$\hat{I}_2 = \frac{4V_m}{3\pi}\left(\frac{1}{2\omega L_f}\right)$$

3- LC- type.

3-ϕ AC Source.



1-ϕ AC Source.

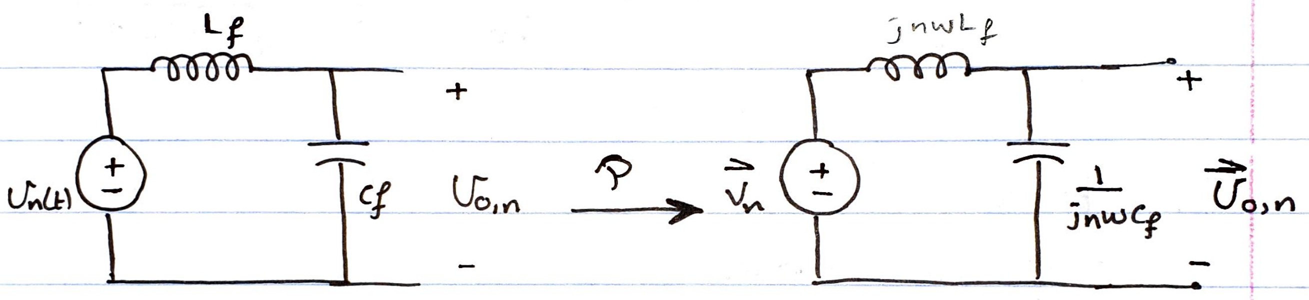


- The following condition must be satisfied to pass the n^{th} harmonic current through the capacitor.

$$\sqrt{R^2 + (n\omega L)^2} \gg \frac{1}{n\omega C_f}.$$

$$\sqrt{R^2 + (n\omega L)^2} = \frac{10}{n\omega C_f} \Rightarrow C_f = \frac{10}{n\omega \sqrt{R^2 + (n\omega L)^2}}$$

- Applying superposition, the equivalent circuit for the n^{th} harmonic is:



$$\vec{V}_{o,n} = \frac{1/jn\omega C_f}{\frac{1}{jn\omega C_f} + jn\omega L_f} \cdot \vec{V}_n$$

$$\vec{V}_{o,n} = \frac{1}{1 - (n\omega)^2 C_f L_f} \vec{V}_n$$

Consider a $1-\phi$ full-wave rectifier.

$$V(t) = \frac{2V_m}{\pi} - \sum_{n=2,4,6}^{\infty} \frac{4V_m}{\pi(n^2-1)} \cos(n\omega t).$$

$$V(t) = V_{DC} + V_2 + \underbrace{V_4 + V_6 + \cdots}_{\text{ignored.}}$$

$$V(t) = \underbrace{\frac{2V_m}{\pi}}_{V_{DC}} - \frac{4V_m}{3\pi}\cos(2\omega t).$$

$$\vec{V}_2 = \frac{-4V_m}{3\pi} \angle 0$$

$$V_{o,2} = \frac{4V_m}{3\pi} \cdot \frac{1}{(2\omega)^2 L_f C_f - 1}$$

$$\Rightarrow V_{AC} = \frac{V_{o,2}}{\sqrt{2}}$$

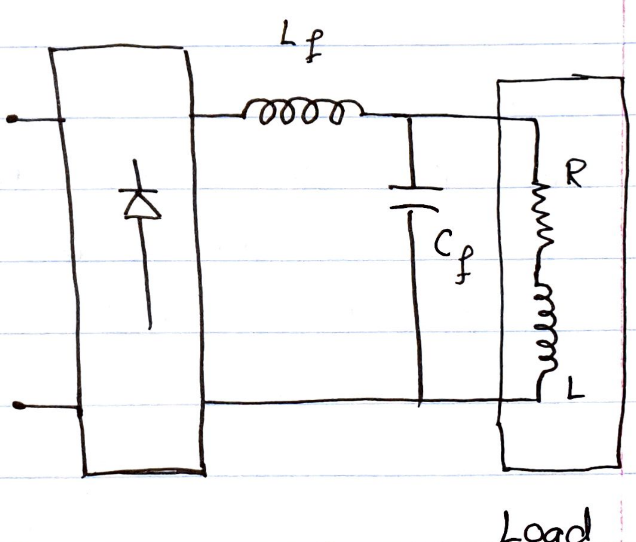
$$V_{DC} = \frac{2V_m}{\pi}$$

$$RF = \frac{\sqrt{2}}{3} \cdot \frac{1}{(2\omega)^2 L_f C_f - 1}$$

- Example:

A LC Filter is used to reduce the ripple Content of the output voltage for a 1-ϕ full wave rectifier. The load resistance, $R = 40\,\Omega$, load inductance, $L = 10\,mH$, and the source frequency is 60 Hz.

Determine the values of L_f & C_f so that the ripple factor of the output voltage is 10% ?



$$\sqrt{R^2 + (n\omega L)^2} = \frac{10}{n\omega C_f}$$

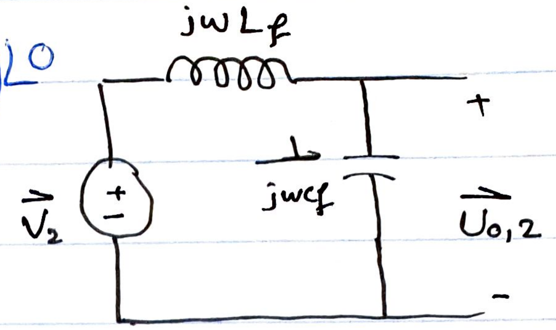
- Consider the domain harmonic (n=2).

$$\sqrt{40^2 + (2(2\pi \times 60))(10 \times 10^{-3})} = \frac{10}{2(2\pi)(60)C_f}$$

Solve for C_f;

$C_f = 426\,\mu F$

$$\vec{V}_{o2} = \frac{1/j\omega C_f}{\frac{1}{j\omega C_f} + j\omega L_f} \cdot \left(-\frac{4V_m}{3\pi}\right)\angle 0$$



$$V_{o2} = \frac{4V_m}{3\pi}\left(\frac{1}{(2\omega)^2 L_f C_f - 1}\right)$$

$$V_{AC} = \frac{4V_m}{\sqrt{2}\,(3\pi)} \; \frac{1}{4\omega^2 L_f C_f - 1}$$

$$RF = \frac{V_{AC}}{V_{DC}} \quad ; \quad V_{DC} = \frac{2V_m}{\pi}$$

$$RF = \frac{2}{\sqrt{2}\,(3)} \; \frac{1}{(4\omega^2 L_f C_f - 1)} = 0.1 \quad ; \quad \omega^2 = (2\pi(60))^2$$

$$C_f = 426\times10^{-6}.$$

$L_f = 30.83\,mH$.

$$RF = \frac{1}{\sqrt{2}\,(2Rf_r C_f - 1)} \quad ; \quad f_r = 2\times 60$$

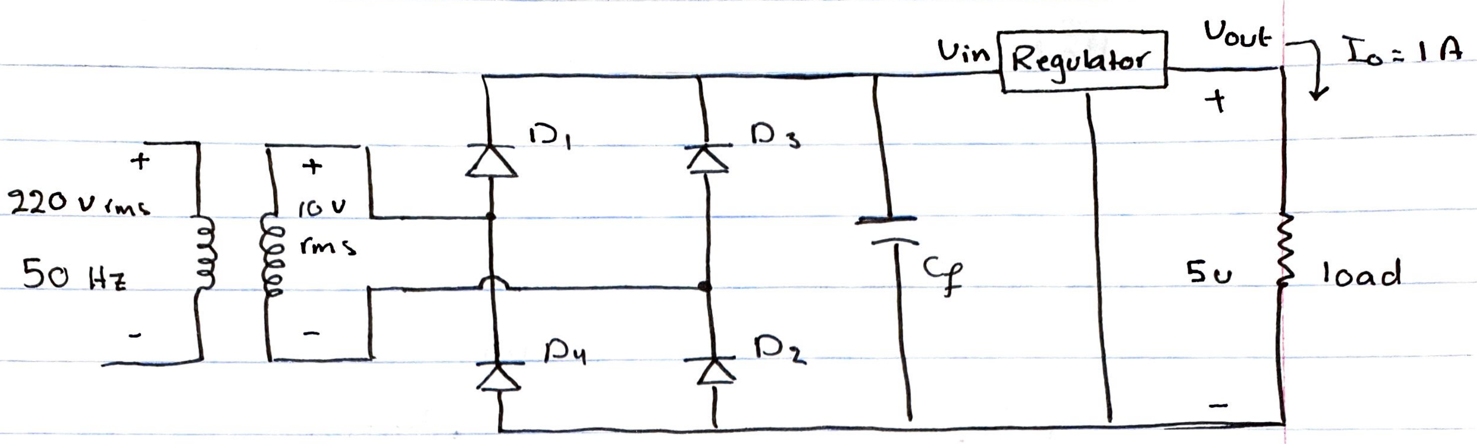
$R = 40$.

$C_f = 840.7\,\mu F$.

- Example:

An output of 5V at 1A is required, the regulator type is LM317 M and is supplied through a 1-ϕ full wave bridge rectifier fed from 10V rms @ 50 Hz transformer secondary. The voltage drop across the regulator must be at least 1.8V. Assuming the diode's voltage drop is 1V.

1. What size of smoothing capacitor is required.



$\Delta V_{in} = V_{max} - V_{min}$

$V_{max} = 10\sqrt{2} - 2 = 12.14\ V.$

Voltage drop across two conducting diode.

$V_{min} = 5 + 1.8 = 6.8\ V.$

$\Delta V_{in} = 12.14 - 6.8 = 5.34\ V.$

$$\Delta V_{in} = \frac{I}{f_r C_f}$$

$$5.34 = \frac{1}{2(50)C_f} \Rightarrow C_f = 1.873\ mF.$$

2. $P_{loss} = P_{in} - P_{out}.$

$= \bar{V}_{in} I - \bar{V}_{out} I$

$$\bar{V}_{in} = V_{max} - \frac{\Delta V_{in}}{2} = 12.14 - \frac{5.34}{2}$$

$\bar{V}_{in} = 9.47\ V.$

$P_{loss} = 9.47(1) - 5(1) = 4.47\ W.$

3. If the RRR is 65 dB, then Calculate the ripple of the output voltage.

$20 \log_{10} X = dB$

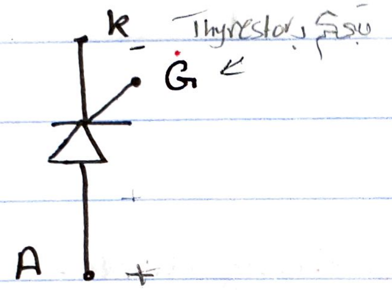
$10^{(dB/20)} = X.$

$$RRR = 65 = 20 \log \frac{\Delta V_{in}}{\Delta V_{out}}, \quad \Delta V_{in} = 5.34$$

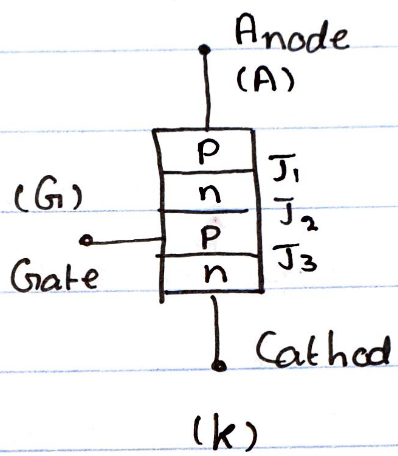
$\Delta V_{out} = 3\ mV.$

- Controlled AC-DC Converters (Controlled Rectifiers).
Silicon Controlled Rectifiers (SCRs).

A thyristor is a 4 layer semiconductor device of PNPN structure with 3pn-junctions.



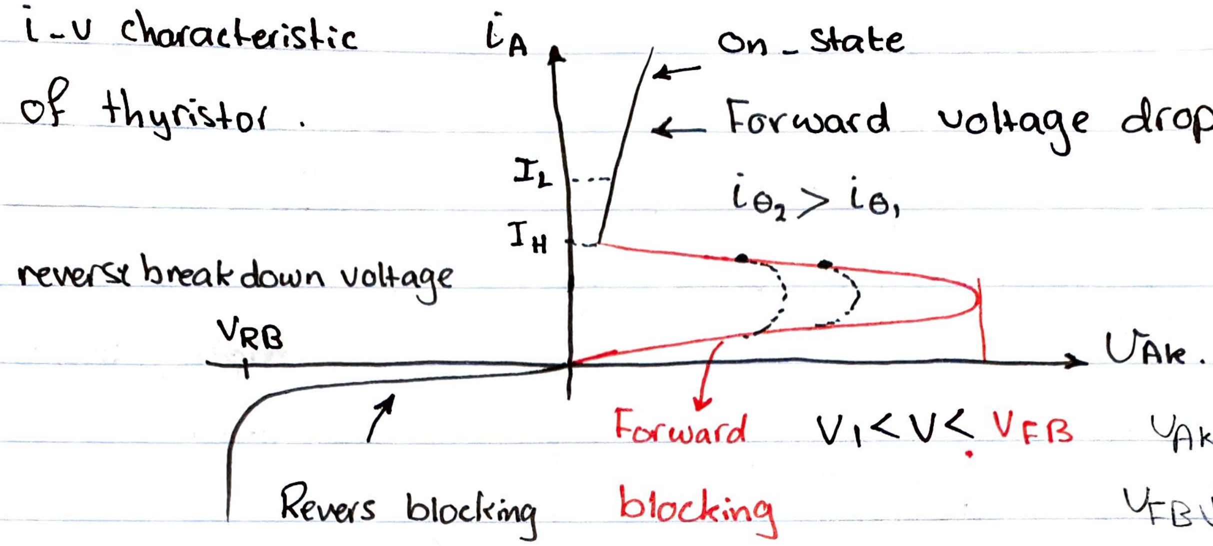
Symbol of thyristor.



- When V_{AK} is positive → J_1, J_3 are forward biased.
  & J_2 is reversed biased.
  → Small leakage current flows.
  → Forward blocking or off-state.
- When $V_{AK} > V_{FB}$, where V_{FB} is the forward breakdown voltage. → J_2 breaks.
  → large forward anode current, I_A flows.
  → Conducting mode or on-state.

- Note:
  - The forward current is limited by an external resistance or impedance.
  - In the conducting mode, there will be a small voltage drop due to the ohmic drop of the leyers (typicaly 1V).

i-v characteristic of thyristor.

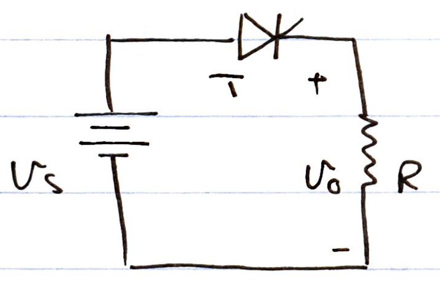


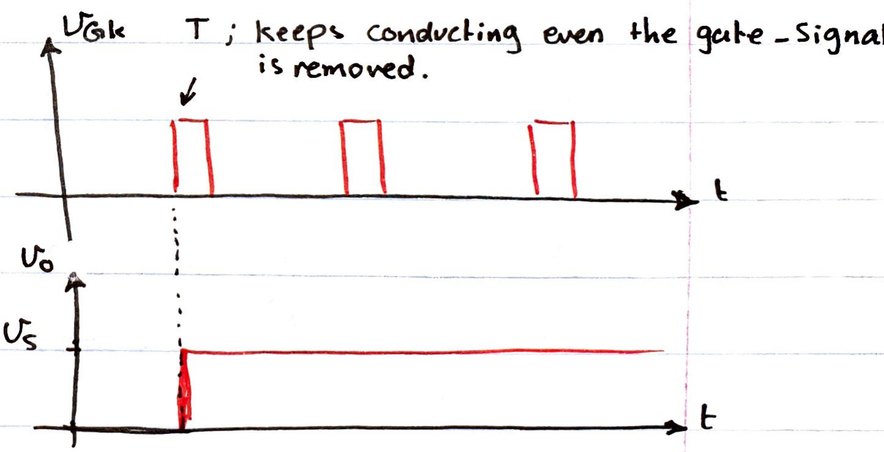
بصورة ما نزيد V_{AK}
لكن بتعدي لـ V_{FB}
حق ما يصير damage to thyrister

- Turn-On.

- The thyristor can be turned on by increasing V_{AK} beyond V_{FB}, such a turn on may damage the thyristor.
- In practice, the thyristor is turned on by applying a positive voltage between the gate and cathode.

  $V_{GK} \approx 3-5$ @ $0.1-0.3$ A for 6 kA device.





- Turn off:

* The thyristor can not be turned off via its gate signal.
- To turn it off, I_A has to fall below I_H.
- In AC circuits, the thyristor is turned off by self commutation (self-turned-off).

  تبديل

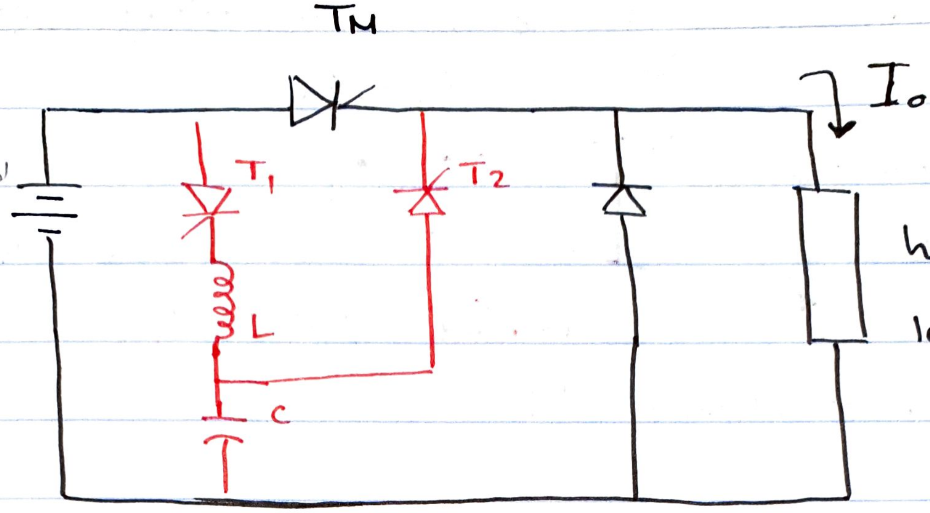
- In the rectifiers & inverters, the thyristors are turned off by line or load commutations.

Resonant Commutation Circuit.

للتقعد Gate قدرها ع

اطفاء Thyrestor نستخدم

Vs هي الدائرة .



highly inductive load

- Assume TM is triggered at $t = t_1$.
- When T_1 is turned on at $t = t_2 > t_1$, A resonanet circuit is formed.
- Applying KVL.

إشتقاق بالنسبة لـ t .

$$V_s = L\frac{di}{dt} + \frac{1}{C}\int i\,dt. \Rightarrow \frac{d^2 i}{dt^2} + \frac{1}{LC}\, i = 0.$$

$$i(t) = A\sin(\omega_o(t-t_2)) + B\cos\omega_o(t-t_2)$$

$$i(t_2) = B = 0.$$

$$i(t) = A\sin(\omega_o(t-t_2))$$

$$\frac{di(t_2)}{dt} = \frac{1}{L}(V_s - \cancel{V_c(t_2)}^{0}) = \omega_o A.$$

$$A = \frac{V_s}{\omega_o L}$$

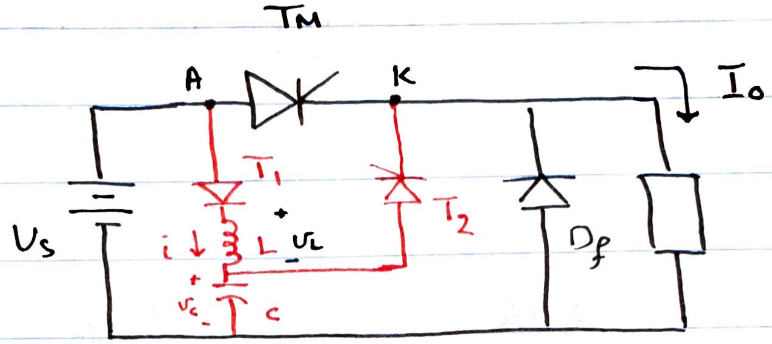
$$i(t) = \frac{V_s}{\omega_o L}\sin(\omega_o(t-t_2)).$$

$$V_c = V_s - L\frac{di(t)}{dt}$$

$$V_c = V_s(1 - \cos\omega_o(t-t_2))$$

- Resonanet Commutation Circuit.

It is Used to turn of the Thyristor DC Circuit.



Higly inductive Load.

- Assume that TM is triggred at $t = t_1$.
- when T_1 is turned on at $t = t_2 > t_1$, a resonanet Circuit is formed.
- KVL in the Circuit.

نشتق $V_s = L\dfrac{di}{dt} + \dfrac{1}{C}\int i\,dt \Rightarrow \dfrac{d^2 i}{dt^2} + \dfrac{1}{LC} i = 0.$

بالنسبة لـ t   نقسم المعادلة ع L

$$i(t) = A\sin(\omega_o(t - t_2)) + B\cos(\omega_o(t - t_2))$$

$$i(t_2) = 0$$

$$\frac{di(t_2)}{dt} = \frac{1}{L}(V_s - \cancel{V_c(t_2)}^{\,0}) = \frac{V_s}{L} = \omega_o A \Rightarrow A = \frac{V_s}{\omega_o L}.$$

$$i(t) = \frac{V_s}{\omega_o L}\sin(\omega_o(t - t_2)).$$

- when; $t = t_2 + \dfrac{\pi}{\omega_o} \Rightarrow i = 0 \Rightarrow T_1$ is turned off.

$$V_c = V_s - L\frac{di}{dt} = V_s(1 - \cos(\omega_o(t - t_2))).$$

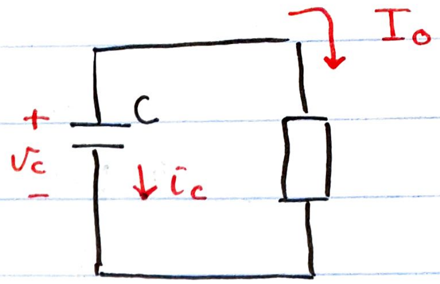
- when; $t = t_2 + \dfrac{\pi}{\omega_o} \Rightarrow V_c = 2V_s.$

The thyristor TM is turned off when T_2 is turned on at $t = t_2$.

$V_A = V_s$, $V_K = 2V_s \Rightarrow V_{AK} = -V_s \Rightarrow$ TM is off.

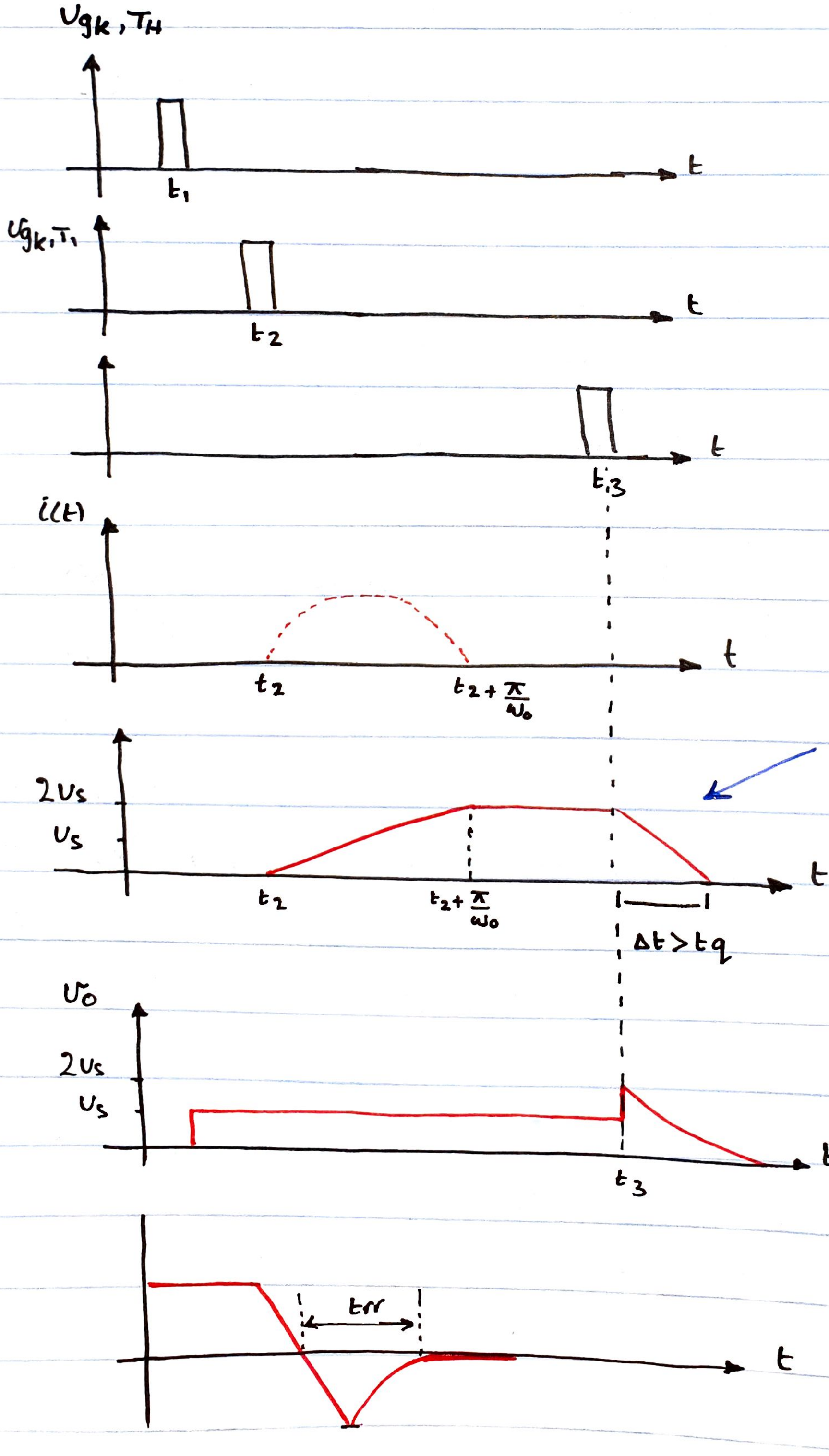
- The capacitor is selected according to:

$$i_c = C\frac{\Delta V_c}{\Delta t} = -I_o$$



$\Delta t \geq t_q$ (turn-off time of the thyristor TM).

$|\Delta V_c| = CV_s - V_{k,final}$, $V_{k,final} \geq V_s$, $|\Delta V_c| \leq V_s$.



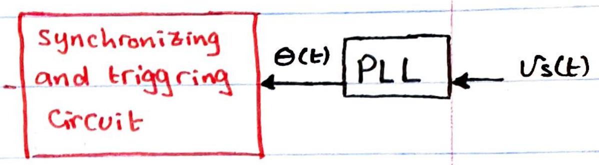
$$-I_o = C\frac{dv}{dt}$$

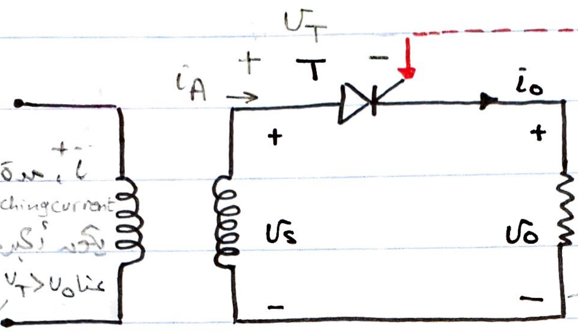
$$-\frac{I_o}{C}\int_{t_3}^{t} dt = \int_{2V_s}^{V_s} dV_c$$

$$V_c = 2V_s - \frac{I_o}{C}(t - t_3)$$

- Controlled Rectifiers.

1_ 1_ϕ Half_wave Controlled Rectifier.





بتشتغل T طالما عليه

i مرة كافية بحيث يطلع الـ i_A

latching current

بيكون أكبر من I_L ← راح يوصل

Forward biased ← $v_T > v_o$ عند

On_state voltage

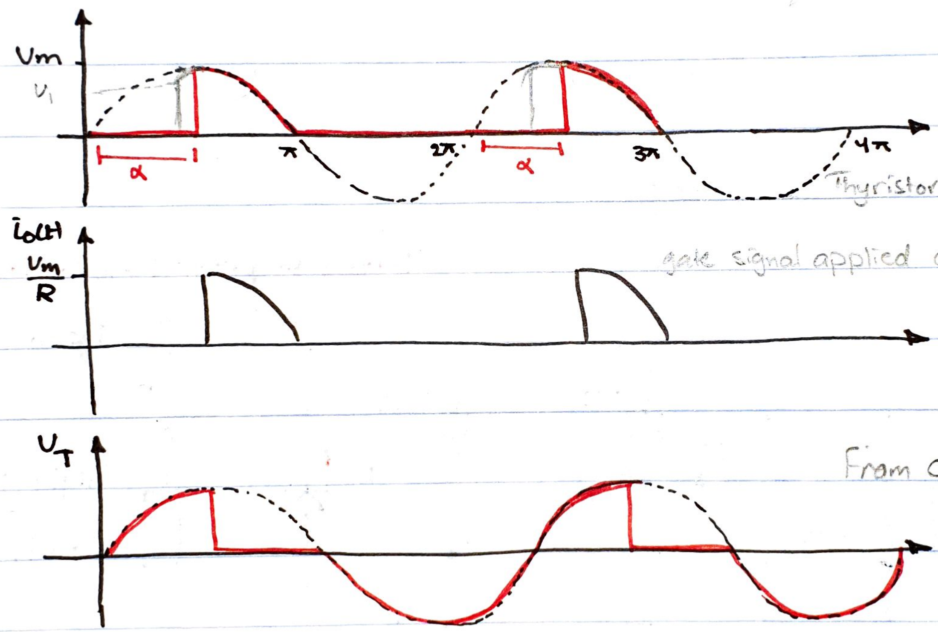
$$X(t) = X_m \sin(\theta(t))$$

$$\theta(t) = (\omega t + \theta_o).$$

الزاوية لتشتغل عندها Thyristor & gate signal

α is Firing angle.

Assume that $v_s(t) = V_m \sin(\omega t)$.



.... $v_s(t)$

—— $v_o(t)$

راح جزء من v عشان بيشتغل Thyristor

عند زاوية α ← gate signal applied at α.

عندما تتغير α ← From 0 to π

V_{dc} varied from $\left[\frac{V_m}{\pi} \rightarrow 0\right]$

1. IF $\alpha = 0$

$\frac{V_m}{2\pi}[1 + \cos(0)]$

maximum. ⇒ $V_{dc} = \frac{V_m}{2\pi}$

2. $\alpha = \pi$

$\frac{V_m}{2\pi}[1 + \cos(\pi)]$

$V_{dc} = 0$.

- $I_{dc} = \frac{V_{dc}}{R}$

- $PIV = V_m$

- The average Output voltage is:

$$V_{DC} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t). \quad , \quad 0 \le \alpha \le \pi$$

$$V_{DC} = \frac{V_m}{2\pi} \cos(\omega t)\Big|_{\pi}^{\alpha}$$

$$V_{DC} = \frac{V_m}{2\pi}(1 + \cos\alpha).$$

- Normalized Output voltage

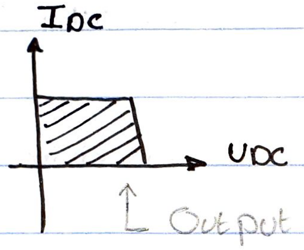
$$V_n = \frac{V_{dc}}{V_m} = 0.5(1 + \cos\alpha)$$

- The RMS value of $v_o(t)$.

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2(\omega t)\, d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi} \frac{1}{2}\left(1-\cos(2\omega t)\right) d\omega t} = \frac{V_m}{2}\sqrt{\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)}$$

$0 < \alpha < \pi$



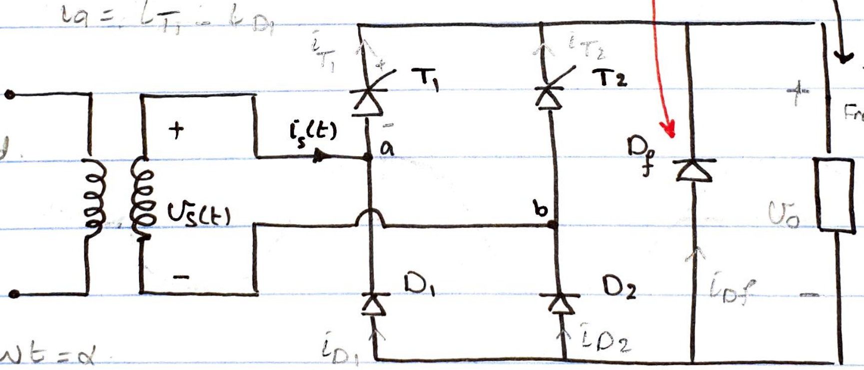
This Converter has only one quadrant of operation.

Output voltage & current have one polarity.

2- $1-\phi$ Semi- Convartor.

It used to produce the conduction losses

$i_a = i_{T_1} = i_{D_1}$



+ve half cycle
T_1 & D_2 Forward.
-ve half cycle
T_2 & D_1 Forward

I_o = Constant.

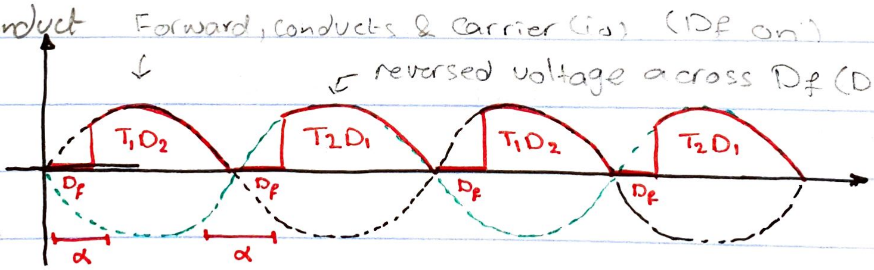
بسبب وجود Load عالي جداً
بكون Free ripple & Continuos
higly inductive load.

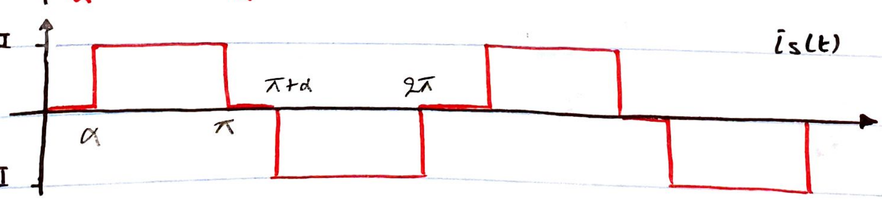
T_1 triggerd at $\omega t = \alpha$

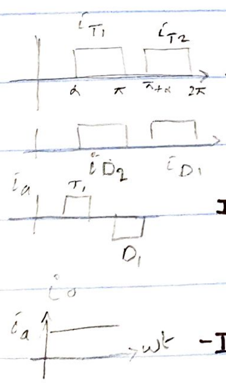
$\alpha < \omega t < \pi$ $v_s(t) = V_m \sin(\omega t)$ 0 to α & π to $\pi+\alpha$ ⇐ يلزم ع الفترة Dm.

T_1 & D_2 Conduct   Forward, conducts & carrier (i_o) (D_f on)   .... $v_s(t)$ or V_{ab}

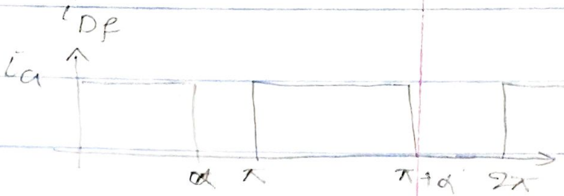
reversed voltage across D_f (D_f OFF)   ..... $V_{ba} = -v_s(t)$







- The average output voltage is:



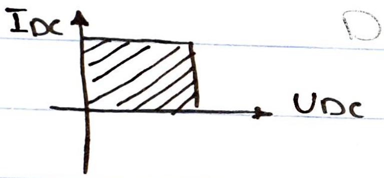
$$V_{DC} = \frac{2}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t)$$

$$V_{DC} = \frac{V_m}{\pi}(1+\cos\alpha) \; ; \quad 0 \le \alpha \le \pi .$$

- The RMS value of $V_o(t)$.

$$V_{RMS} = \sqrt{\frac{2}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2(\omega t)\, d(\omega t)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)}$$

- The Converter has 1-quadrant of operation.



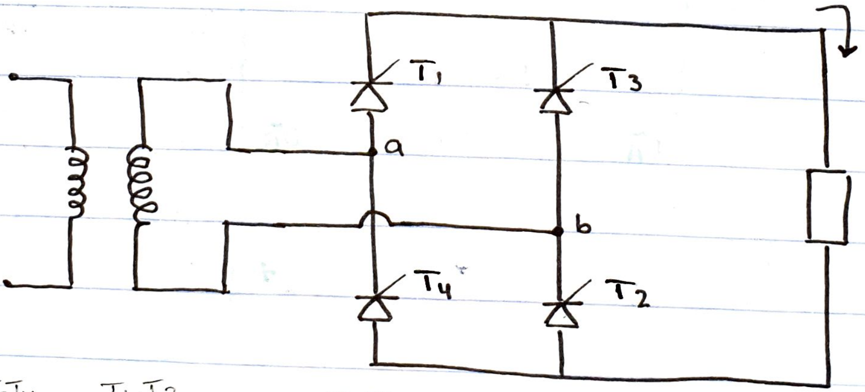
Df [illegible] موجب ⇐ I_{DC}, V_{DC}

Maximum $V_o \Rightarrow V_{dm} = \frac{2V_m}{\pi}$

Normalized $V_o \Rightarrow V_n = \frac{V_{dc}}{V_m} = 0.5(1+\cos\alpha)$.

3- $1\text{-}\phi$ Full wave Converter.

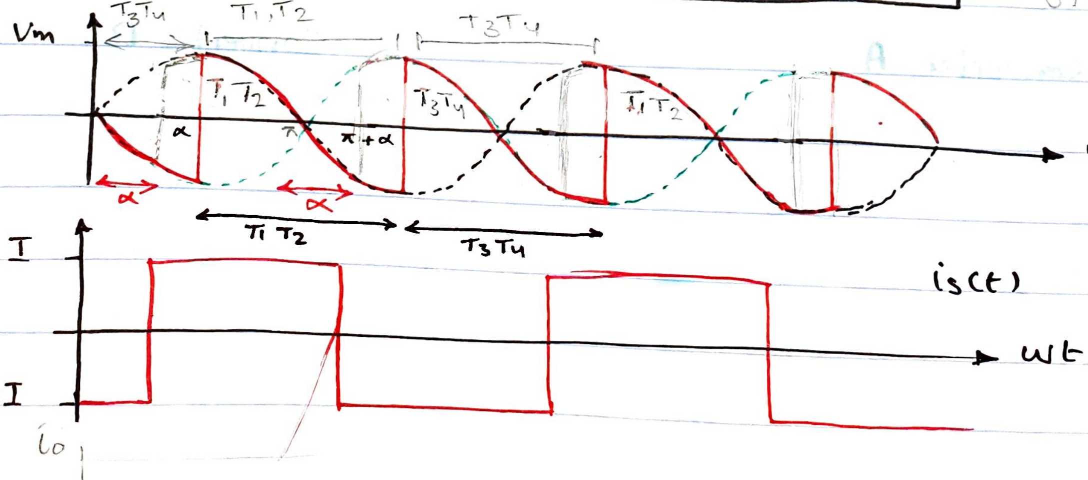
25-10



higly inductive load.

T_1T_2: forward

T_3T_4: forward.

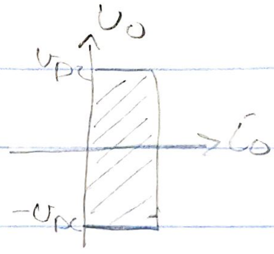


---- Vab.

~ Vba

— Vo

- The average output voltage is:



$$U_{DC} = \frac{2}{2\pi} \int_{\alpha}^{\pi+\alpha} U_m \sin(\omega t)\, d(\omega t)$$

U_o both polarity +ve & -ve

$$U_{DC} = \frac{U_m}{\pi} \cos \omega t \Big|_{\pi+\alpha}^{\alpha} = \frac{U_m}{\pi} (\cos\alpha - \cos(\pi+\alpha)) \quad \text{2 quardants}$$

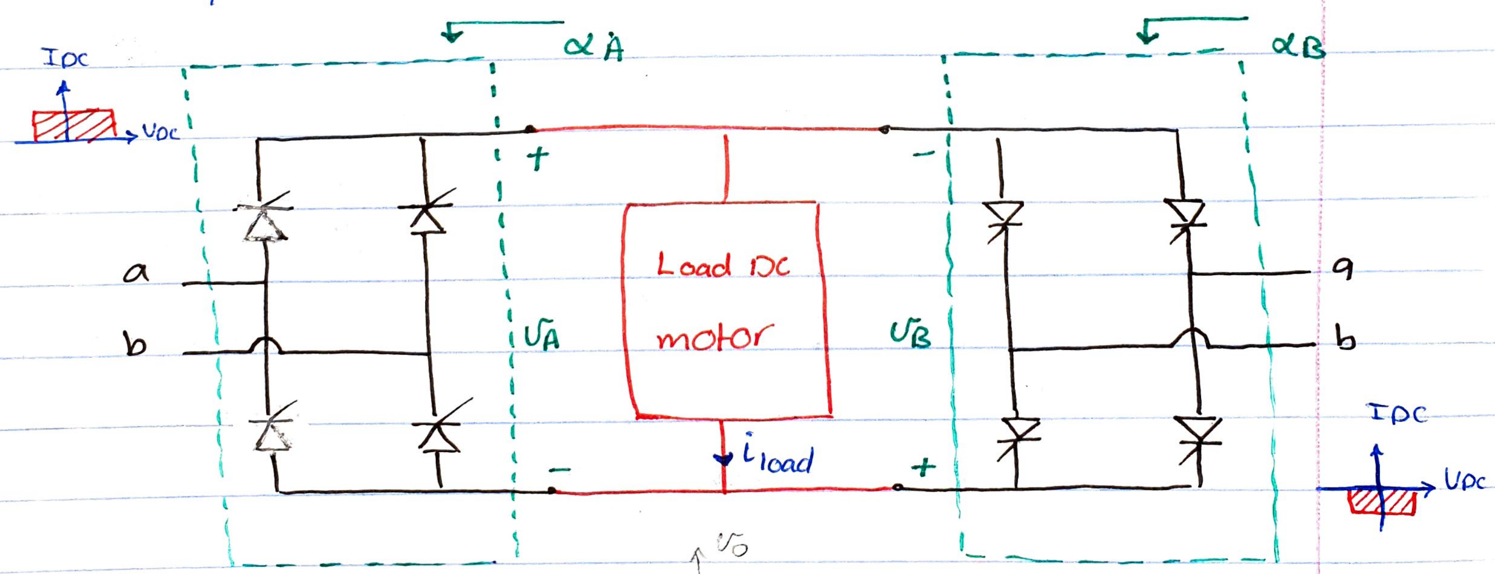
$$U_{DC} = \frac{2U_m}{\pi} \cos\alpha . \qquad 0 < \alpha < \pi .$$

- The average RMS value of $V_o(t)$.

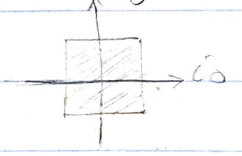
$$V_{RMS} = \frac{U_m}{\sqrt{2}} .$$

- T_1 & T_2 ⇒ still conducting beyond $\omega t = \pi$ even through the input voltage is negative because their anode current are higher than the holding current.

4- 1-Φ Dual Converter.



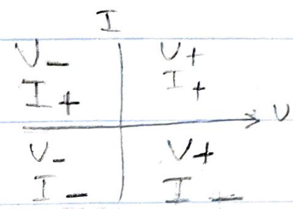
Converter A



Converter B

- Two full-wave converters connected in back-to-back or in anti-parallel to obtain the 4 quadrent operation.
- The converter is controlled using the change-over logic.

$i_{load} > 0$	$v_{load} > 0$	Converter A	Converter B	Q ← الرُّبع
Yes	Yes	ON	OFF	I
Yes	NO	ON	OFF	II
NO	Yes	OFF	ON	IV
NO	NO	OFF	ON	III



- KVL ;

$V_{A,DC} + V_{B,DC} = 0$

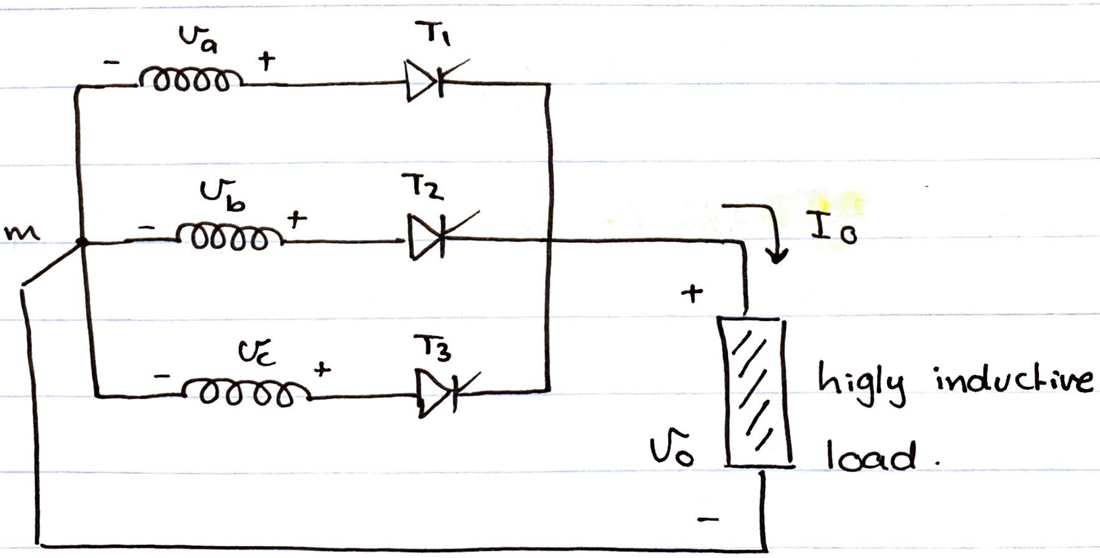
$V_{A,DC} = -V_{B,DC}$

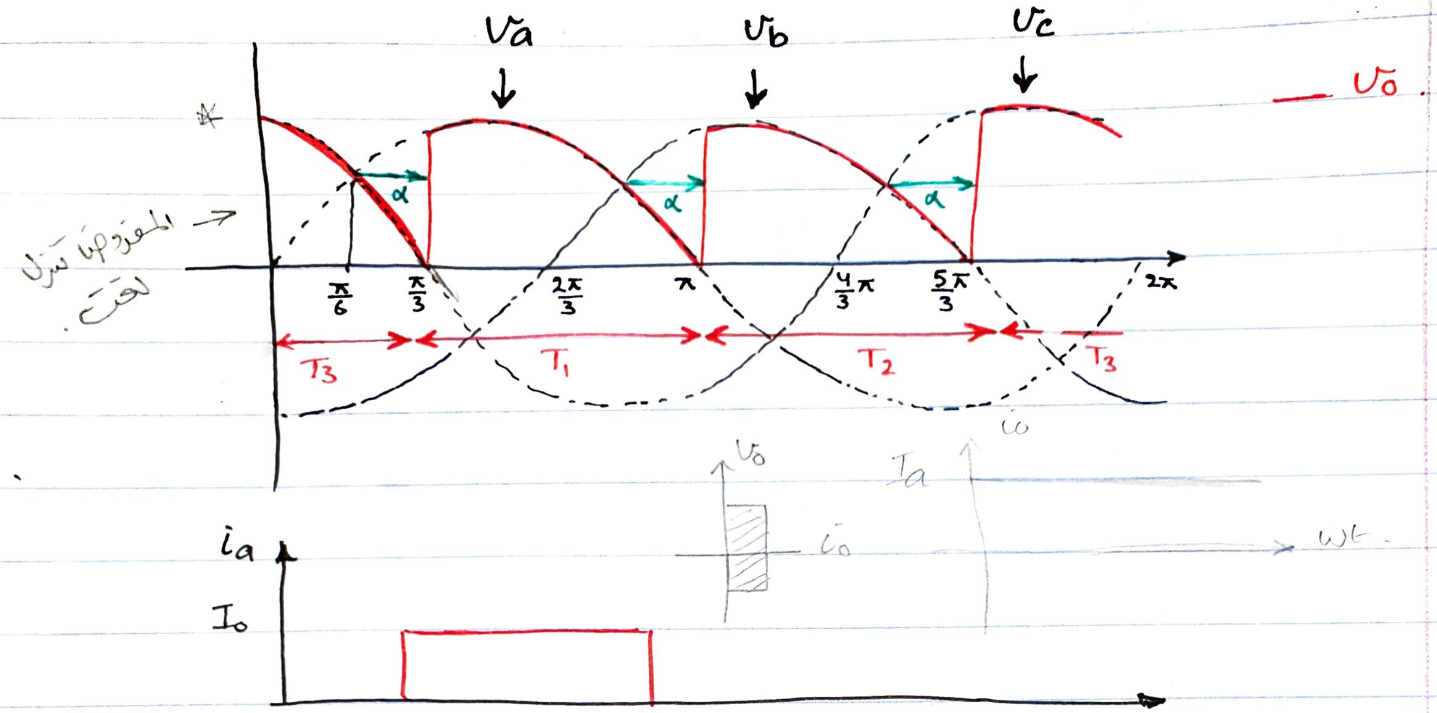
$\frac{2V_m}{\pi} \cos\alpha_A = -\frac{2V_m}{\pi} \cos\alpha_B$

$\cos\alpha_A = -\cos\alpha_B = \cos(\pi - \alpha_B)$

$\alpha_A + \alpha_B = \pi$.

5- 3-ϕ Half - wave Converter.





- The average output voltage.

$$V_{DC} = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin(\omega t)\, d\omega t.$$

$$V_{DC} = \frac{3V_m}{2\pi}\left[\cos\left(\frac{\pi}{6}+\alpha\right) - \cos\left(\frac{5\pi}{6}+\alpha\right)\right]$$

$$= \frac{3V_m}{2\pi}\left[\frac{\sqrt{3}}{2}\cos\alpha - \frac{1}{2}\sin\alpha - \left(\frac{\sqrt{3}}{2}\cos\alpha - \frac{1}{2}\sin\alpha\right)\right]$$

- q phase star rectifier

$V_{dc} = \frac{q}{\pi} V_m \sin\left(\frac{\pi}{q}\right)$

with $\Rightarrow q = 3$!

$V_{dc} \Rightarrow O$

$\rightarrow \frac{3V_m}{\pi}\frac{\sqrt{3}}{2}$.

$$V_{DC} = \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha. \quad ; \quad 0 \le \alpha \le \pi, \text{ highly inductive}$$

$$0 \le \alpha \le \frac{\pi}{6}, \text{ purly resistive.}$$

- The RMS value of output voltage.

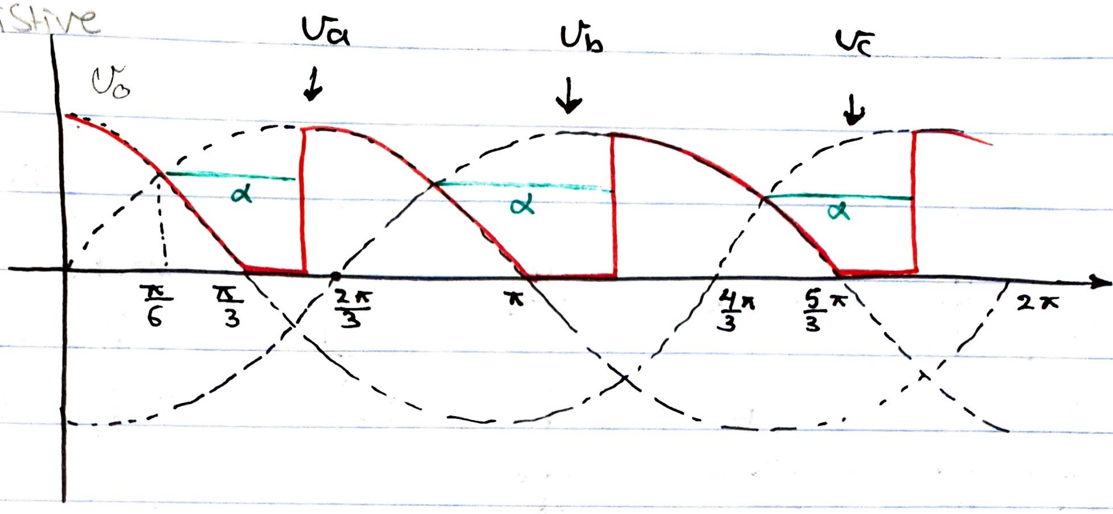
$$V_{RMS} = \sqrt{\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2(\omega t)\, d(\omega t)} \quad , \sin^2 x = \frac{1}{2}[1-\cos 2x].$$

$V_o(max)$ at $\alpha = 0. \Rightarrow V_{dm} = \frac{3\sqrt{3}}{2\pi} V_m$

$$V_{rms} = \sqrt{3}\, V_m \sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi}\cos 2\alpha}$$

$V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha.$

Purly resistive



For 60°

- The average output voltage.

$$V_{DC} = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi} V_m \sin(\omega t)\, d\omega t = \underbrace{\frac{3V_m}{2\pi}\left[\cos\left(\frac{\pi}{6}+\alpha\right)+1\right]}_{\text{Purely resistive load, } \frac{\pi}{6} \le \alpha \le \pi}$$

- The RMS value of $v_o(t)$.

$$V_{RMS} = \sqrt{\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi} V_m^2 \sin^2(\omega t)\, d\omega t} = \sqrt{3}\,V_m\sqrt{\frac{5}{24} - \frac{\alpha}{4\pi} + \frac{1}{8\pi}\sin\left(\frac{\pi}{3}+2\alpha\right)}$$

- Note:

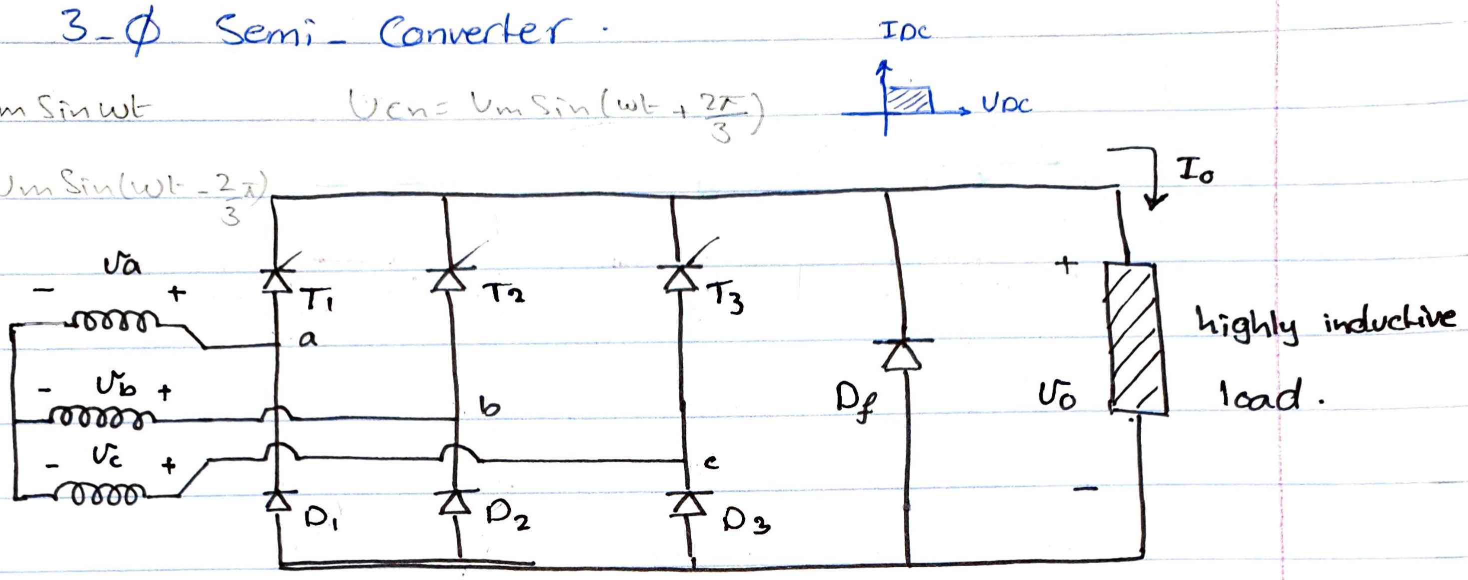
When the load is purely resistive, and $\alpha \ge \frac{\pi}{6}$, $v_o(t)$ can never have negative parts.

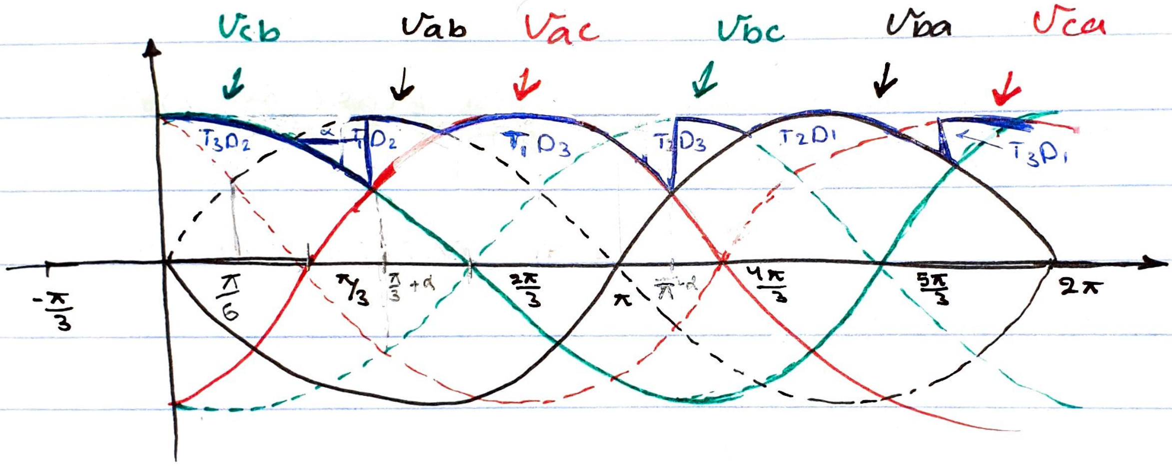
6- 3-ϕ Semi- Converter.

$V_{an} = V_m \sin \omega t$

$V_{cn} = V_m \sin(\omega t + \frac{2\pi}{3})$

$V_{bn} = V_m \sin(\omega t - \frac{2\pi}{3})$





Mode (I)

$0 \leq \alpha \leq \frac{\pi}{3}$ $\quad V_{DC(I)} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}} V_{ab}\, d(\omega t) + \int_{\frac{2\pi}{3}}^{\pi+\alpha} V_{ac}\, d(\omega t)\right]$

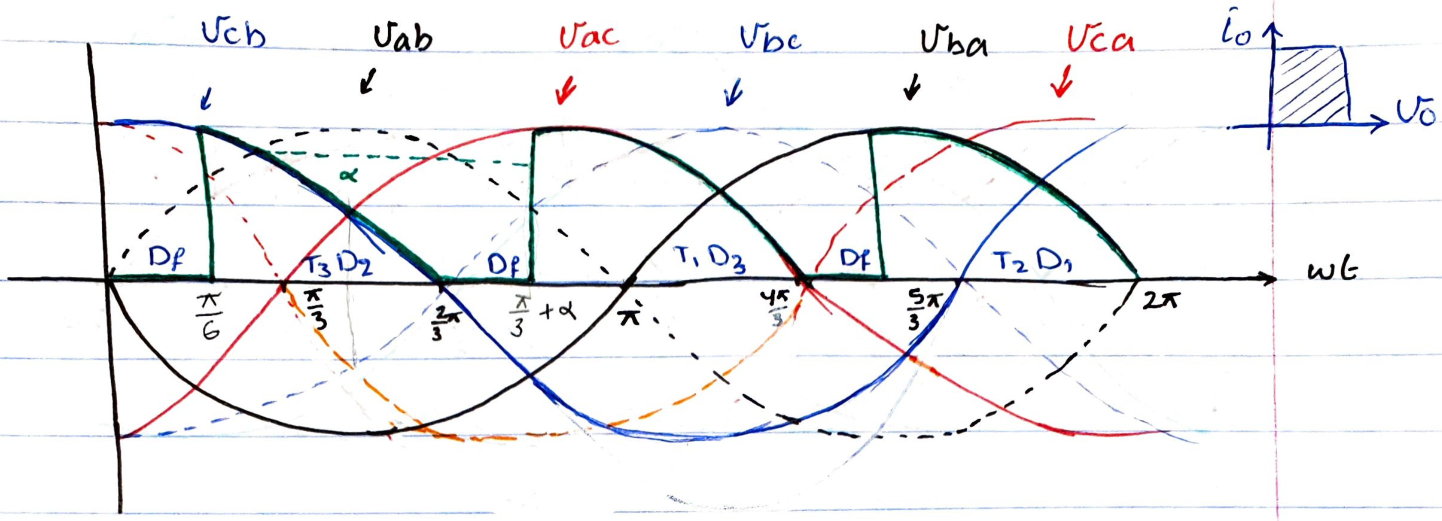
$V_{ab} = \sqrt{3} V_m \sin(\omega t)$.

$V_{ac} = -\sqrt{3} V_m \sin(\omega t + 120)$.

$$V_{DC(I)} = \frac{3\sqrt{2}}{2\pi} V_m (1 + \cos\alpha).$$

$$V_{RMS(I)} = \sqrt{\frac{3}{2\pi}\left[\int_{\frac{\pi}{3}+\alpha}^{2\pi/3} (V_{ab})^2\, d(\omega t) + \int_{2\pi/3}^{\pi+\alpha} V_{ac}^2\, d(\omega t)\right]}$$

$$V_{rms} = \sqrt{3} V_m \sqrt{\frac{3}{4\pi}\left(\frac{2\pi}{3} + \sqrt{3}(\cos\alpha)^2\right)}$$

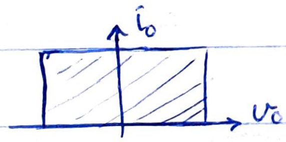


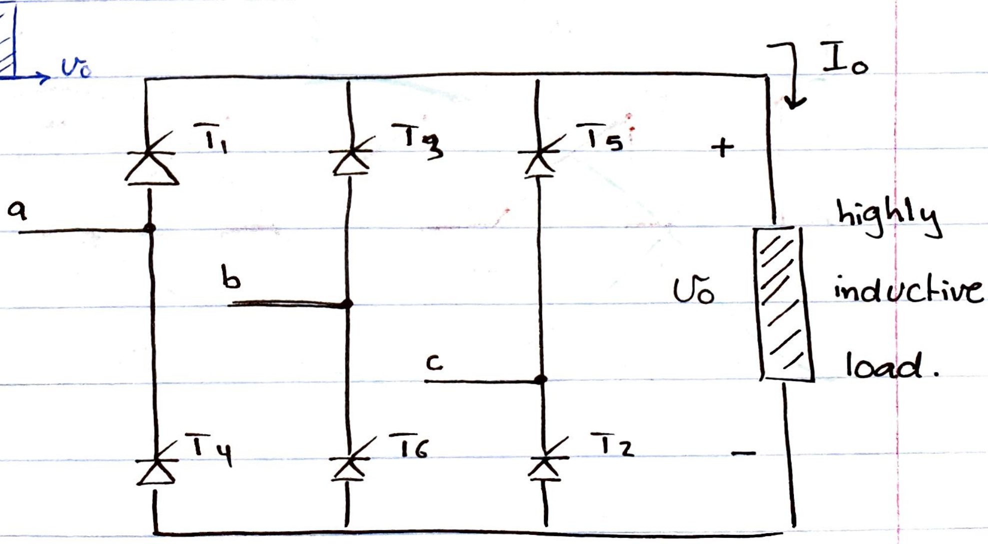
Mode (II) : $\frac{\pi}{3} \leq \alpha \leq \pi$.

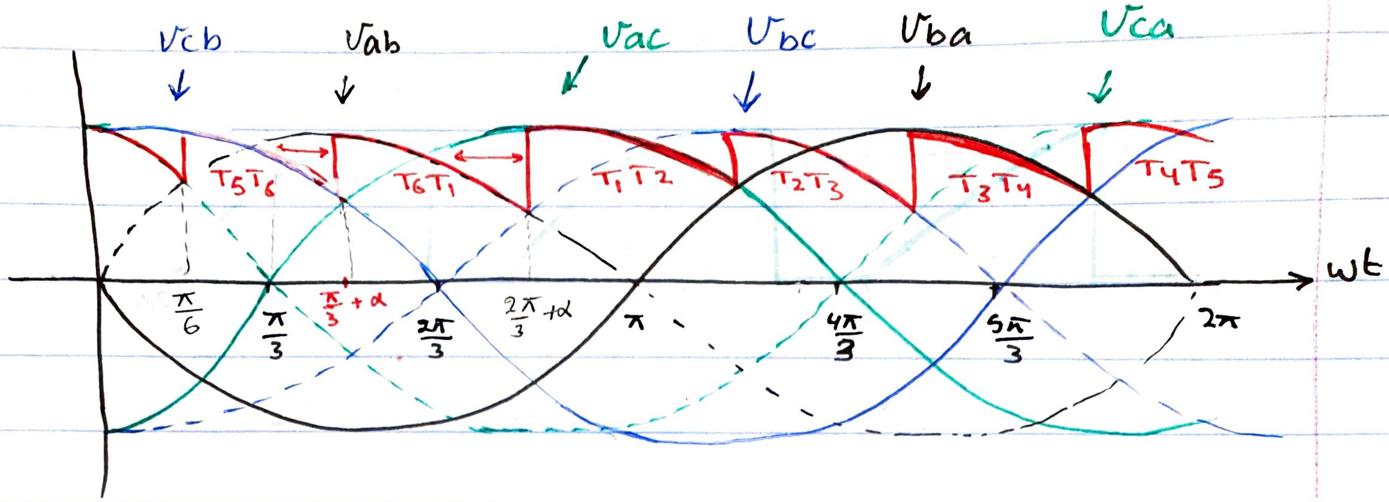
$$V_{DC_{(II)}} = \frac{3}{2\pi} \int_{\frac{\pi}{3}+\alpha}^{\frac{4\pi}{3}} V_{ac} \, d(\omega t) = \frac{3\sqrt{3}}{2\pi} (1 + \cos\alpha) = V_{DC_{(I)}}$$

$$V_{RMS} = \sqrt{\frac{3}{2\pi} \int_{\frac{\pi}{3}+\alpha}^{4\pi/3} (V_{ac})^2 \, d(\omega t)}$$

7_ 3_Φ Full_ Converter.





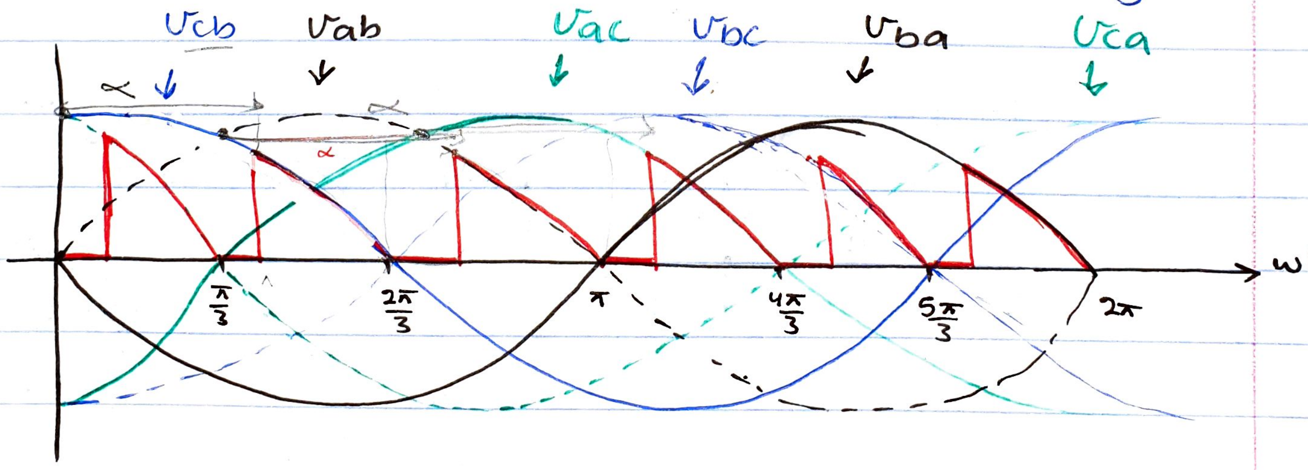


$$V_{DC} = \frac{6}{2\pi}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_{ab}\, d(\omega t) \quad , \quad V_{ab} = \sqrt{3}\, V_m \sin(\omega t).$$

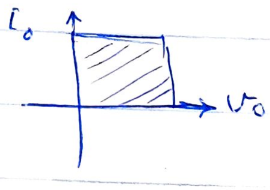
$$V_{DC} = \frac{3\sqrt{3}}{\pi} V_m \cos\alpha .$$

$$V_{RMS} = \sqrt{\frac{6}{2\pi}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} (V_{ab})^2\, d(\omega t)} = \sqrt{3}\, V_m \sqrt{\left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right)}$$

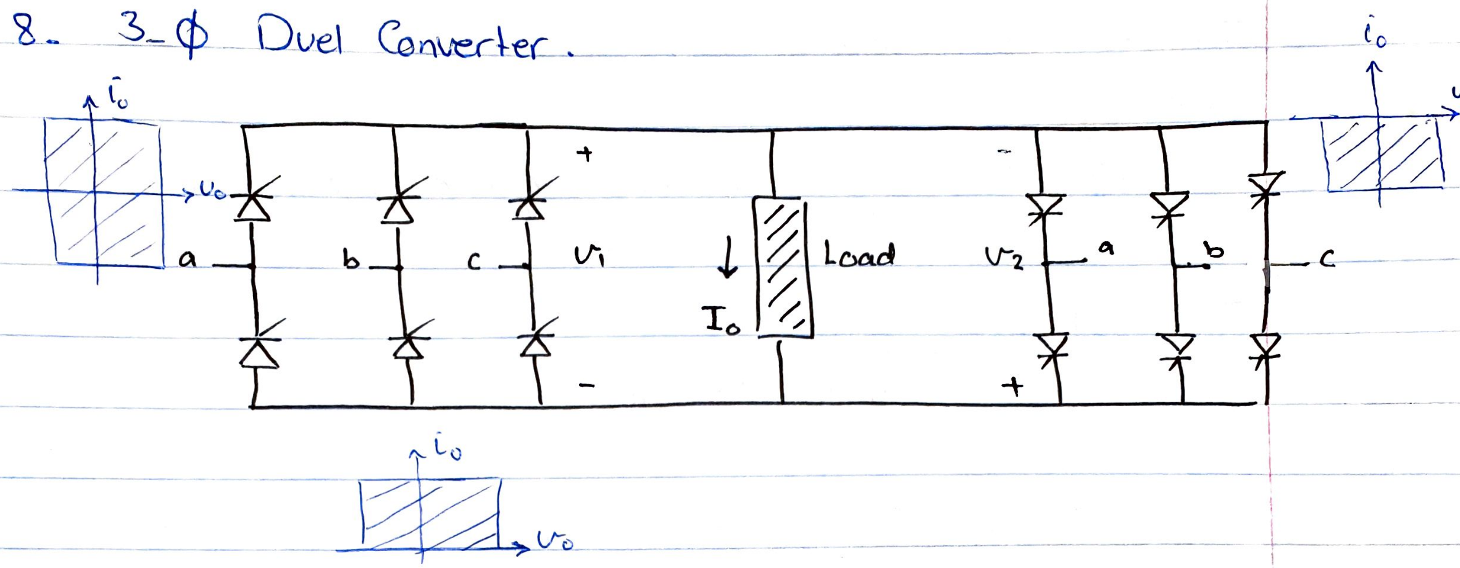
?! Special Case: If the load is resistor & $\alpha \geq \frac{\pi}{3}$.



$$V_{DC} = \frac{6}{3\pi}\int_{\frac{\pi}{3}+\alpha}^{\pi} V_{ab}\, d(\omega t) = \frac{3\sqrt{3}}{\pi} V_m \left(\cos\left(\frac{\pi}{3}+\alpha\right) + 1\right).$$



8. 3_ϕ Duel Converter.



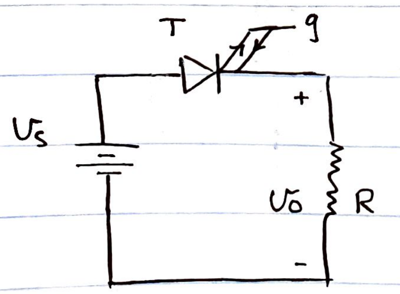
8 - 11

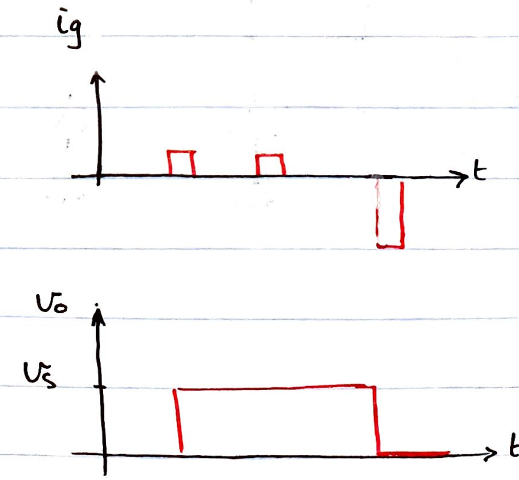
- Gate Turn-off Thyristors (GTOs).

- It is a special thyristor that can be turned off via its gate. It is self-turned off, but it has higher on-state voltage than normal thyristor.
(e.g, 3.4 v for 1.2 kv @ 0.5 kA device).

Symbol 

- It has the same layers structure of thyristor, but it is modified by adding n^+ (heavy doped n-type) at the anode to operate as a sink to holes at turn off.





- Turn on → Short pulse of small positive current applied to the gate.
- Turn off → short pulse of large negative current applied to the gate.

$$I_g \approx \left(\frac{1}{5} - \frac{1}{3}\right) \underbrace{I_A}_{\text{Anode current.}}$$

- Power Factor Connection (PFC).

The idea is to improve the PF of the controlled rectifiers or to use the controlled rectifiers as PFC equipment.

- Control strategies.

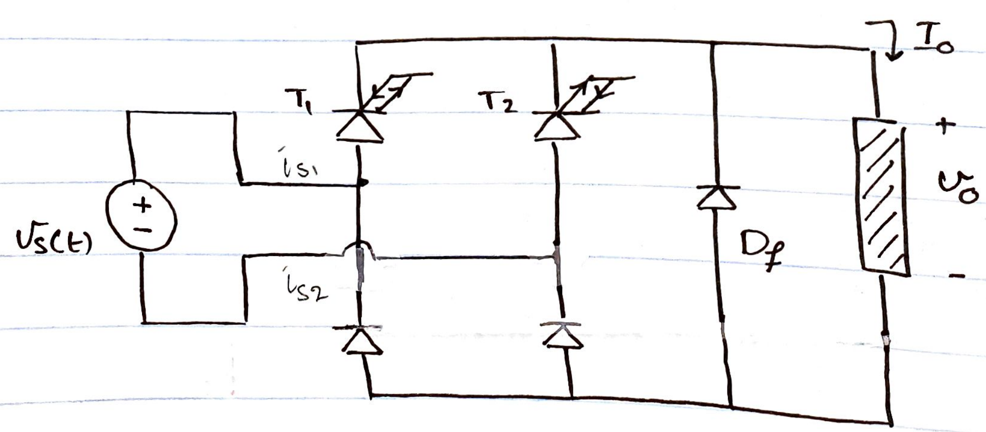
1. Extinction Angle Control (EAC)

1.1 EAC for 1-ϕ semi-converter.

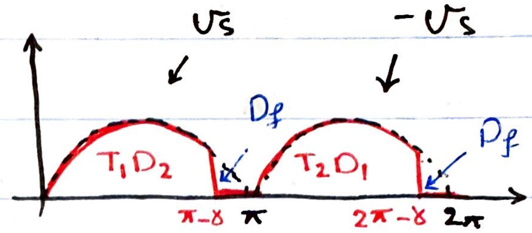
1.2 EAC for 1-ϕ Full-Converter.

2. Symmetrical Angle Control.

1.1 EAC - 1-ϕ semi Converter.

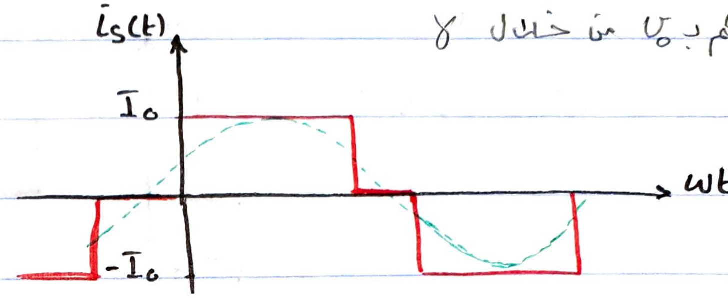


	ON	OFF
T_1	0	$\pi - \gamma$
T_2	π	$2\pi - \gamma$



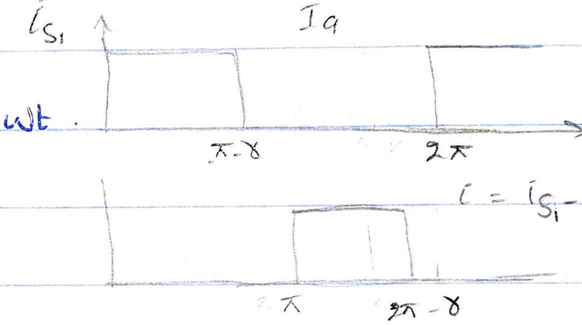
— $v_o(t)$.

نتحكم بـ v_o من خلال γ



---- $i_{s_1}(t)$ Main Component of $i_1(t)$

$i_{s_1}(t)$ leads $v_s(t)$ → Generation of reactive power.



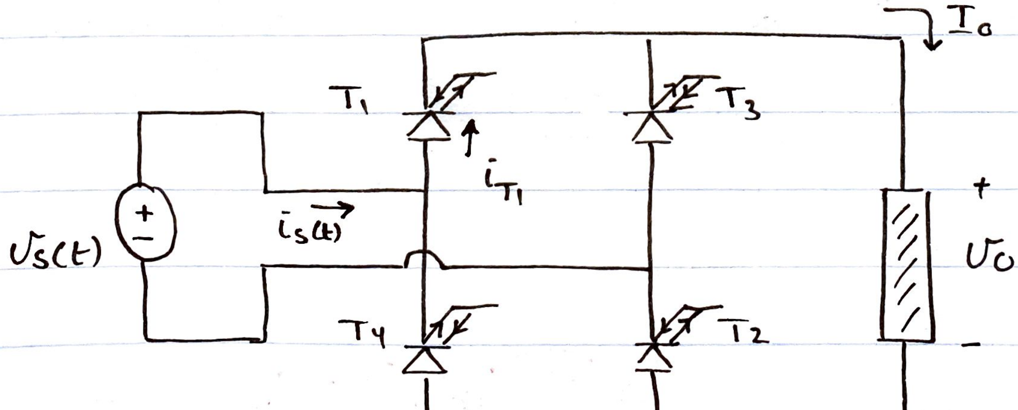
$i = i_{s_1} - i_{s_2}$ ← load current

$$V_{DC} = \frac{2}{2\pi}\int_0^{\pi-\gamma} V_m \sin(\omega t)\, d\omega t.$$

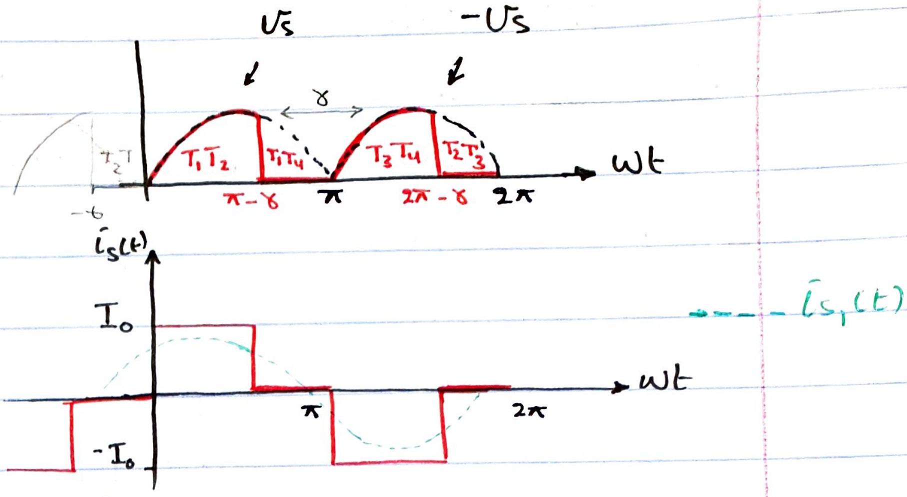
$$V_{DC} = \frac{V_m}{\pi}(1+\cos\gamma).$$

$$V_{RMS} = \sqrt{\frac{2}{2\pi}\int_0^{\pi-\gamma} V_m^2 \sin^2(\omega t)\, d(\omega t)}$$

1.2. EAC - 1-ϕ Full Converter. (Static VAR Compensator).



	ON	OFF
T_1	0	π
T_2	$-\gamma$	$\pi-\gamma$
T_3	π	2π
T_4	$\pi-\gamma$	$2\pi-\gamma$

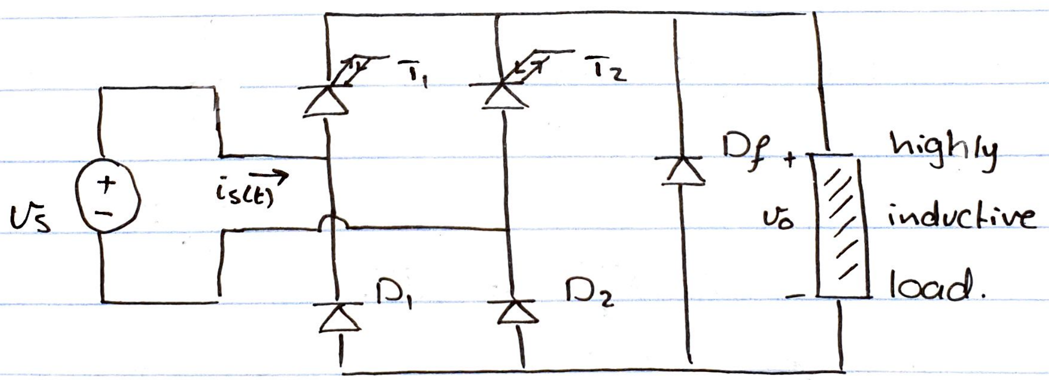


$i_{s_1}(t)$ leads $V_s(t)$ → Generation of reactive power.

$$V_{DC} = \frac{V_m}{\pi}(1+\cos\gamma).$$

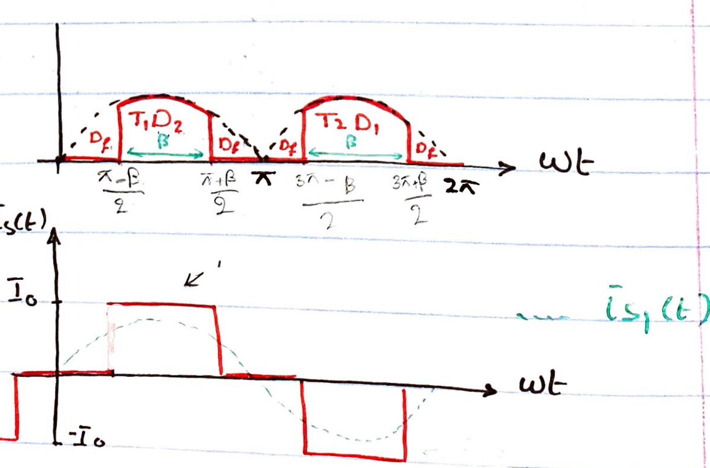
$$V_{RMS} = \sqrt{\frac{2}{2\pi}\int_0^{\pi-\gamma} V_m^2 \sin^2(\omega t)\, d\omega t}$$

2- Symmetrical Angle Control.



	ON	OFF
T_1	$\frac{\pi-\beta}{2}$	$\frac{\pi+\beta}{2}$
T_2	$\frac{3\pi-\beta}{2}$	$\frac{3\pi+\beta}{2}$

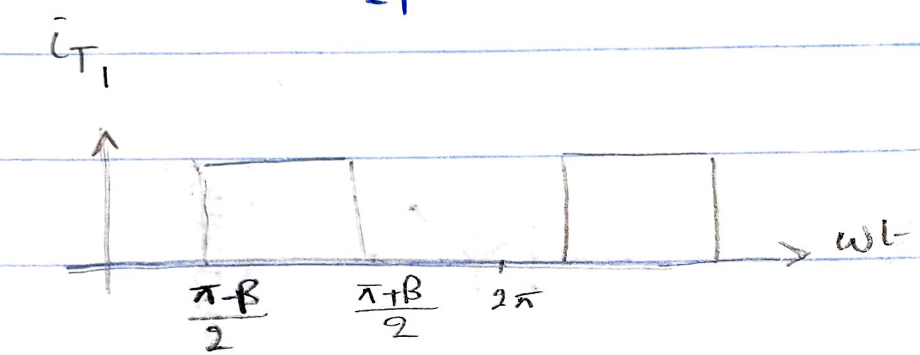
β: Conduction angle.

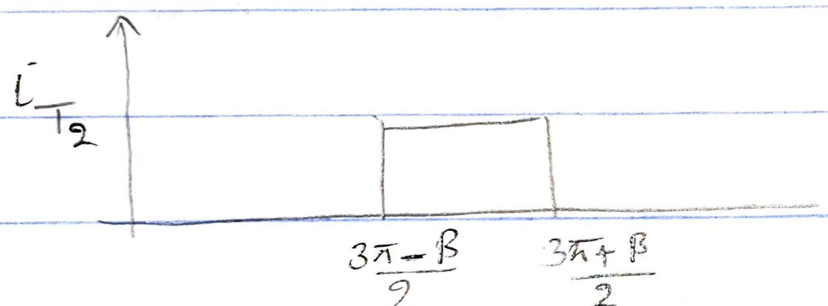


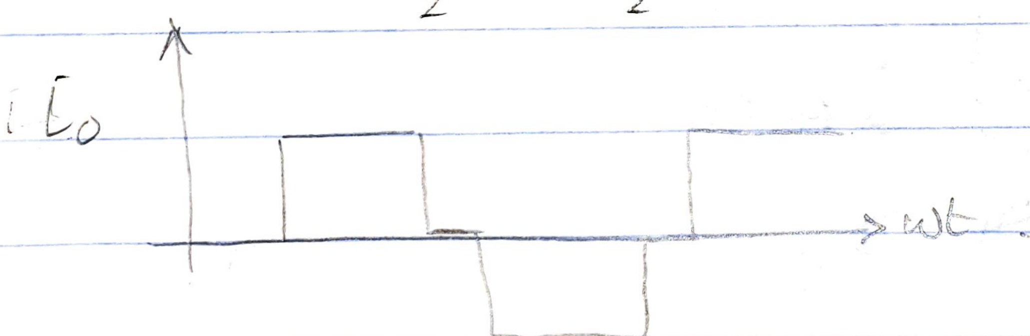
$i_{s_1}(t)$ is in phase with $v_s(t)$.

$\Rightarrow$ Displacment factor $= \cos(\Theta_{v_s} \cancel{-} \Theta_{i_{s_1}}) = 1$

$\Rightarrow PF = \dfrac{I_{s_1}}{I_1} \times \cos(\Theta_{v_s} \cancel{-} \Theta_{i_{s_1}})$



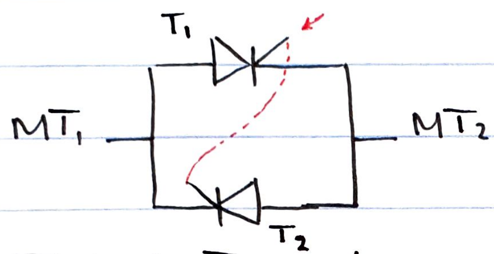


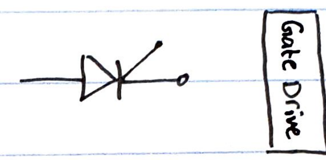


- Chapter 4: AC voltage Controller.

TRIAC → Triode for Alternating current.

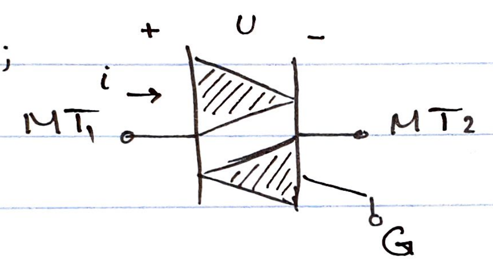
It is equivelant to two anti-parallel thyristors.



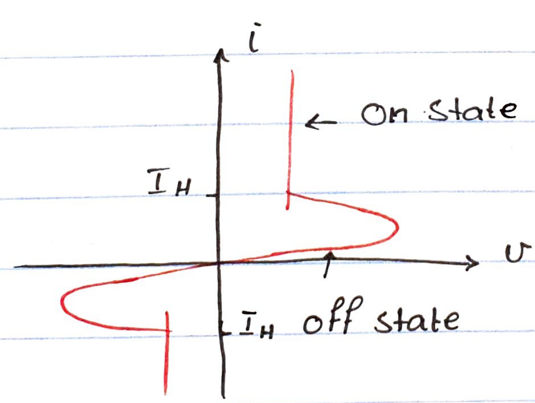


MT: Main Terminal.

- Symbol ;



i_v charachtaristic



- Types of AC Controller
- 1-ϕ Controllers.
- 3-ϕ Controllers.

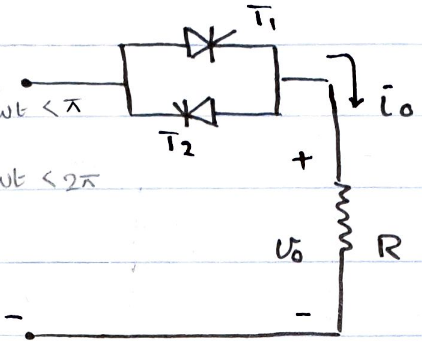
- Control Methods
- ON_OFF Control.
- Phase angle Control → Undirectional Half-wave
  → Bidirectional Full-wave.

- AC Voltage Controllers.

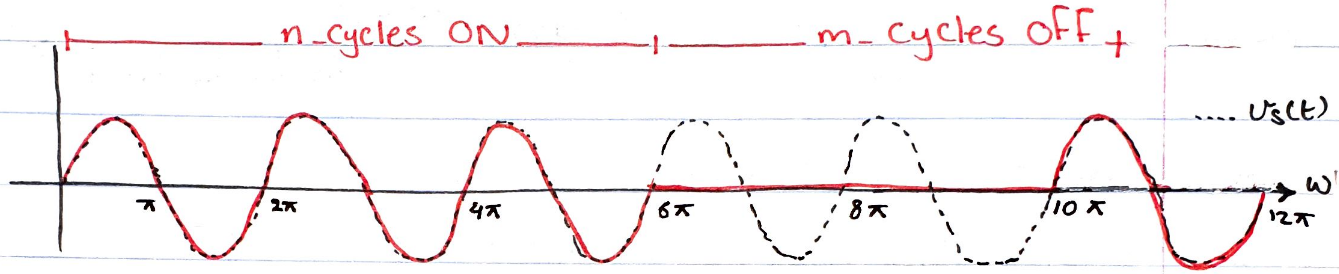
1- On-Off Control

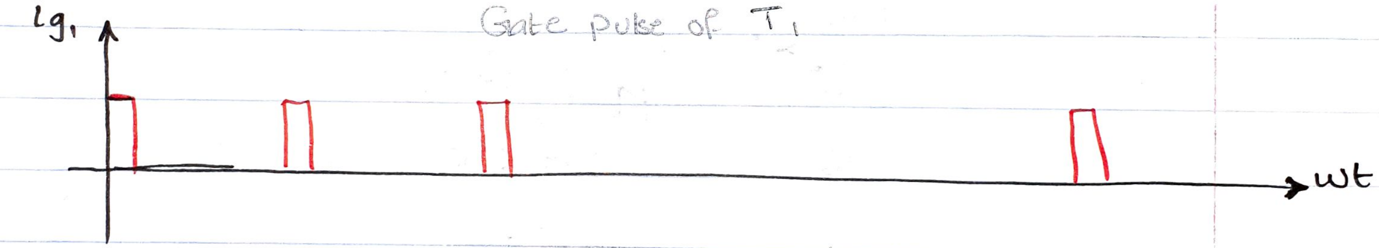
T_1: Forward baised $\alpha < \omega t < \pi$

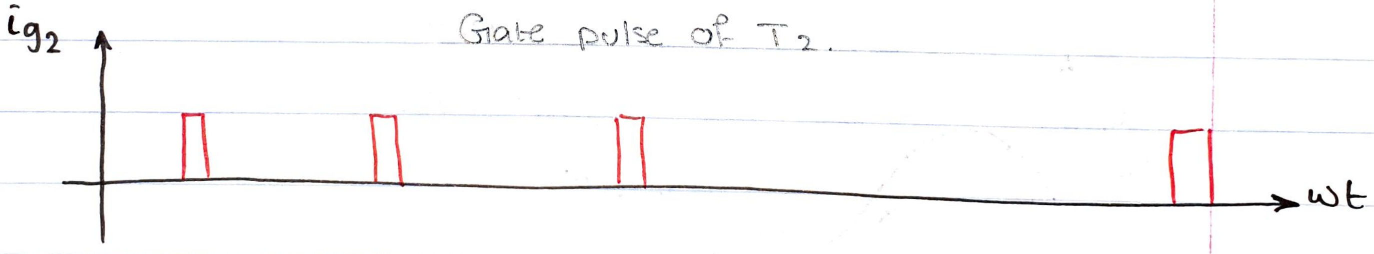
T_2: Forward baised $\pi < \omega t < 2\pi$



$v_s = V_m \sin(\omega t)$.







$$V_{RMS} = \sqrt{\frac{n}{(m+n)(2\pi)} \int_0^{2\pi} V_m^2 \sin^2(\omega t)\, d(\omega t)} = \sqrt{\frac{n}{n+m}}\, \frac{V_m}{\sqrt{2}}$$

$V_{RMS} = \sqrt{k}\, V_s$, where; V_s is the RMS vaine of $v_s(t)$.

k is the duty cycle $k = \frac{n}{n+m}$

n: number of "On-Cycle".

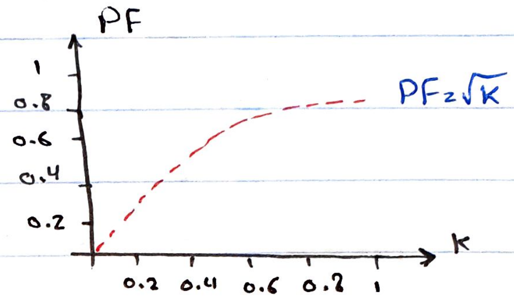
m: number of "OFF-Cycle".

- Input Power Factor.

$$PF = \frac{P_s}{S_s}$$

$$P_s = \frac{V_{RMS}^2}{R} \quad \text{" lossless Converter "}.$$

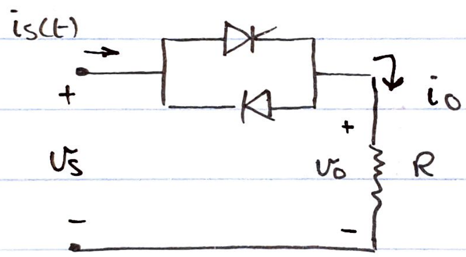
$$S_s = V_s I_s \quad , \quad I_s = \frac{V_{RMS}}{R}$$

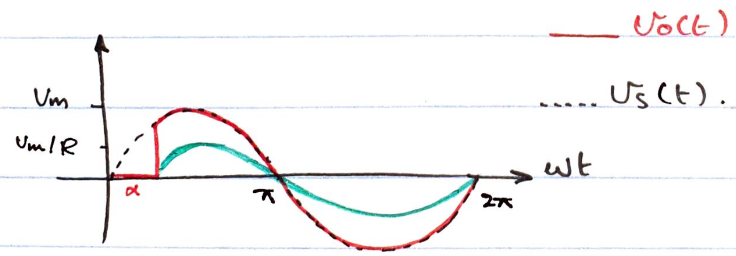


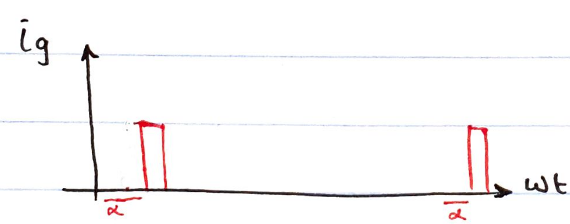
$$PF = \frac{(\cancel{V}_{RMS}^2 / R)}{\left(\frac{\cancel{V}_{RMS}^2}{\sqrt{K}} / R\right)} = \sqrt{K} \quad \text{"lagging"}$$

2. Phases Angle Control

2.1 1-ϕ half-wave (unidirectional) Control.







$$V_{RMS} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{2\pi} V_m^2 \sin^2(\omega t)\, d(\omega t)}$$

$$V_{RMS} = \sqrt{\frac{V_m^2}{4\pi} \int_{\alpha}^{2\pi} (1 - \cos(2\omega t))\, d\omega t} \qquad \sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$V_{RMS} = \sqrt{\frac{V_m^2}{4\pi}\left[(2\pi - \alpha) - \frac{1}{2}\sin(2\omega t)\right]_{\alpha}^{2\pi}}$$

$$V_{RMS} = \sqrt{\frac{V_m^2}{4\pi}\left[2\pi - \alpha + \frac{\sin(2\alpha)}{2}\right]}$$

$$= V_s\sqrt{\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)} \quad ; V_s = \frac{V_m}{\sqrt{2}}.$$

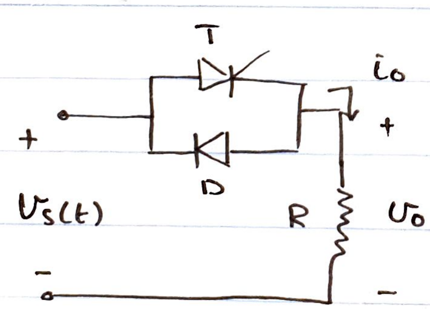
$$V_{DC} = \frac{1}{2\pi}\int_{\alpha}^{2\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(\cos\alpha - 1) \quad , V_m = \sqrt{2}\, V_s.$$

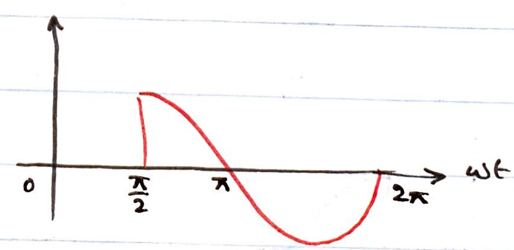
The DC component may cause a saturation problem in the transformer core.

- Example: A 1-ϕ AC voltage controller (Half-wave controller) has a resistive load of $R = 10\,\Omega$, and input voltage is $V_s = 120$ V, 60 Hz. The delay angle of the thyristor is $\frac{\pi}{2}$. Determine;

a- V_{RMS}, b- Average input current. c- input P.F.

a-





$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_{\pi/2}^{2\pi} V_m^2 \sin^2(\omega t)\, d\omega t}.$$

$$V_{RMS} = \sqrt{\frac{V_m^2}{4\pi}\left[\frac{3\pi}{2} + \frac{\sin(2(\pi/2))}{2}\right]} = \sqrt{\frac{3}{8}}\,(120\sqrt{2})$$

$$= 103.92 \text{ V}.$$

b- $I_{DC,S} = \frac{1}{2\pi} \int_{\pi/2}^{2\pi} \frac{V_m}{R} \sin(\omega t)\, d(\omega t)$.

$$I_{DC,S} = \frac{V_m}{2\pi R}\left[\cos\frac{\pi}{2} - \cos(2\pi)\right] = \frac{-120\sqrt{2}}{2\pi(10)} \text{ A}$$

c- $PF = \frac{P_s}{S_s}$

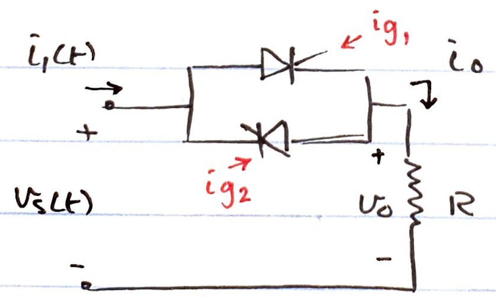
$$P_s = \frac{V_{RMS}^2}{R} = \frac{(103.92)^2}{10} = 1079.94 \text{ W}.$$

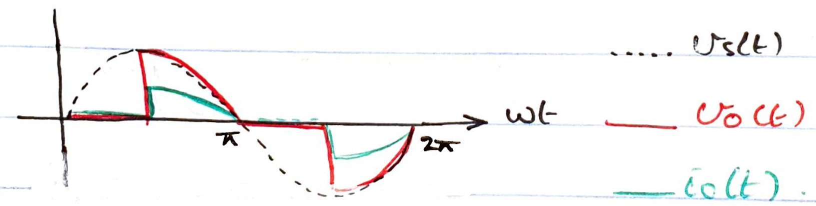
$$S_s = V_s I_s = 120\left(\frac{V_{RMS}}{10}\right) = 12(103.92)$$

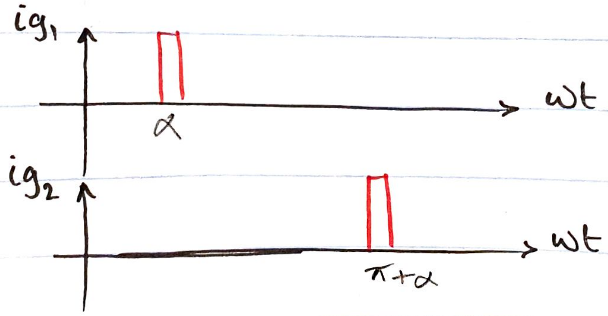
$$S_s = 1247.04 \text{ VA}.$$

$$PF = \frac{1079.94}{1247.04} = 0.866 \text{ lagging}.$$

2-2 $1\text{-}\phi$ Full wave (Bidirectional) control.







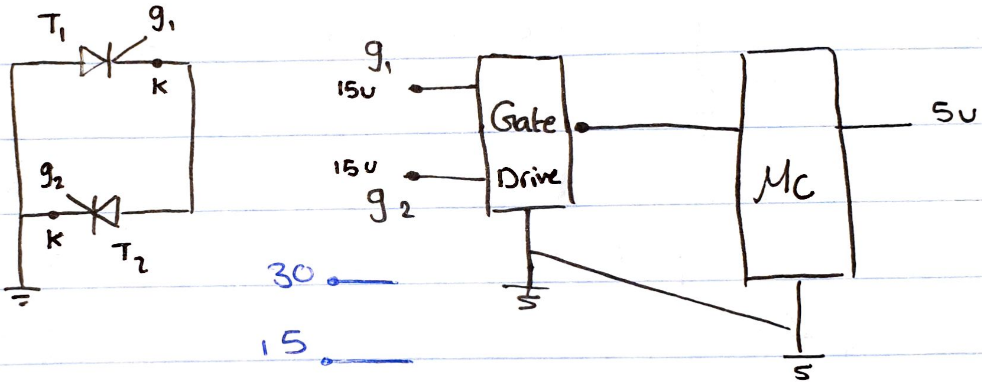
- The RMS value of output voltage:

$$V_{RMS} = \sqrt{\frac{2}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2(\omega t)\, d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi} (1-\cos(2\omega t))\, d(\omega t)}$$

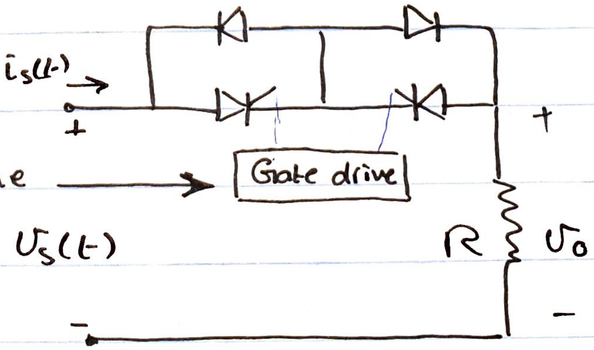
$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[\pi - \alpha - \frac{\sin(2\omega t)}{2}\right]_{\alpha}^{\pi}}$$

$$= V_s\sqrt{\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]}$$



The gate drive circuits must be isolated using pulse transformers or opto couplers.

2 conducting devices at the same time → conduction losses ↑



- This circuit is used to solve the issue of isolation.

- Example: A 1-ϕ Full wave AC voltage Controller has a resistive load of $R = 10\,\Omega$, and the input voltage is 120 V, 60 Hz. The delay angle of thyristors T_1 & T_2 are $\frac{\pi}{2}$ & $\frac{3\pi}{2}$. Determine;

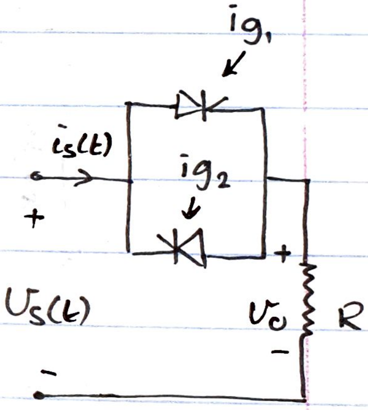
a- V_{RMS}. b- Average current of thyristors.

c- RMS current of thyristors. d- Input PF.

Solution.

$$I_{av} = \frac{1}{2\pi}\int_{\frac{\pi}{2}}^{\pi} I_m \sin(\omega t) = 2.7\ A. \quad , \quad I_m = \frac{120\sqrt{2}}{10}$$

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_{\frac{\pi}{2}}^{\pi} I_m^2 \sin^2(\omega t)} = 6\ A.$$



$$V_{RMS} = \sqrt{\frac{2}{2\pi}\int_{\frac{\pi}{2}}^{\pi} V_m^2 \cdot \sin^2(\omega t)\, d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/2}^{\pi} (1-\cos 2\omega t)\, d\omega t}$$

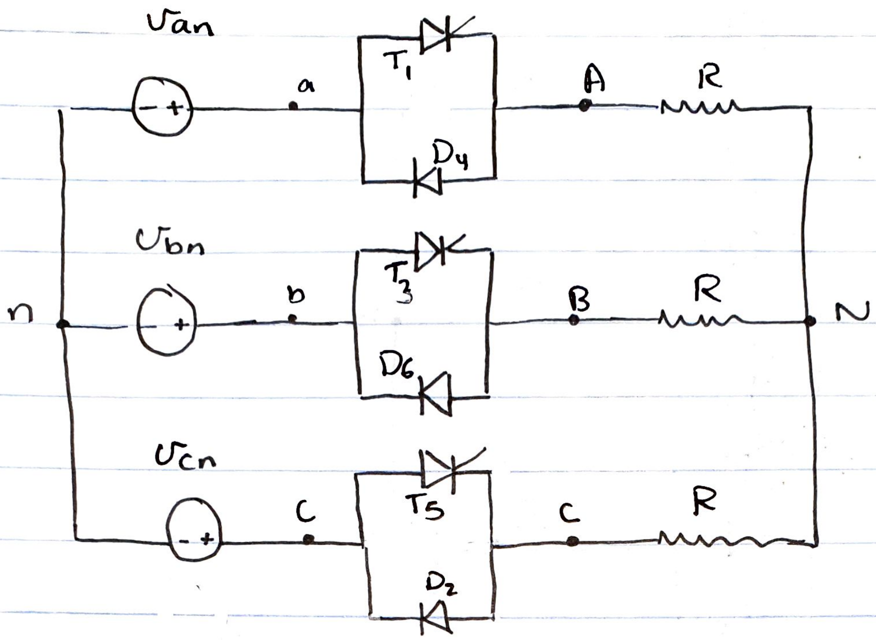
$$= V_s\sqrt{\frac{1}{\pi}\left[\pi - \frac{\pi}{2} + \frac{\sin\pi}{2}\right.} = 84.85\ V.$$

$$PF = \frac{P_s}{S_s} \quad , \quad P_s = \frac{V_{rms}^2}{R} = \frac{(84.85)^2}{10} = 720\ W.$$

$$S_s = V_s I_s = 120 \cdot \frac{84.85}{10} = 8.485\ W$$

$$PF = \frac{720}{8.485} = 84.86.$$

2-3 3-ϕ Half-wave (Unidirectional) AC controller.



(Look up table)

- Modes of oparations.

• Mode I $0 \le \alpha \le 60°$

Possible Combinations of conductions devices.

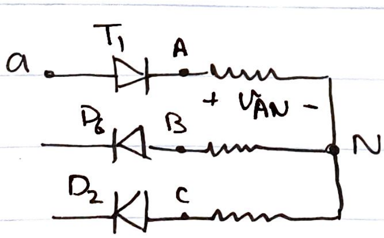
1- 2 Thyristors & 1 diode.
2- 1 Thyristor & 2 diodes.
3- 1 Thyristor & 1 diode.

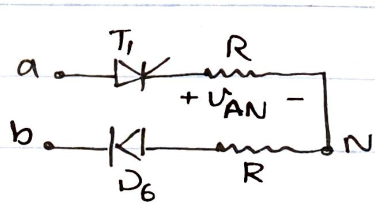
• Mode II $60 \le \alpha \le 120°$

1- 1 Thyristor & 2 diodes.
2- 1 Thyristor & 1 diode.

• Mode III $180° \le \alpha \le 210°$.

1- 1 Thyristor & 1 diode.



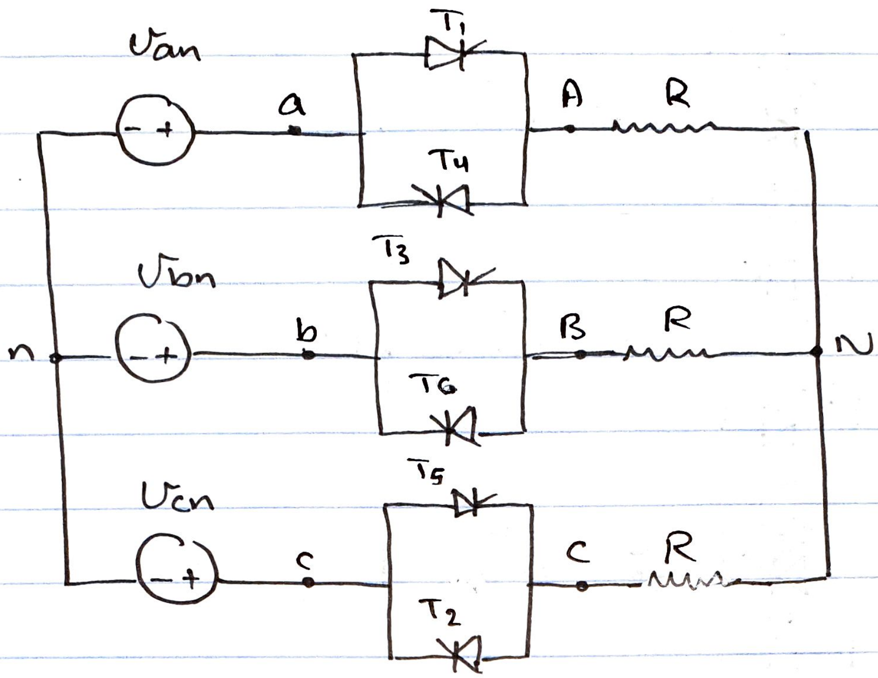


$v_{AN} = v_{an}$

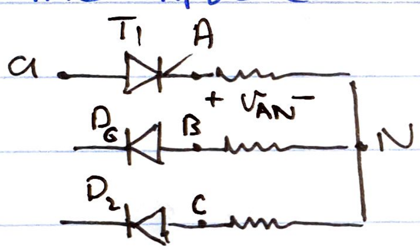
$v_{AN} = \frac{v_{ab}}{2}$.

$v_{bn} = v_{BN}$, $v_{cn} = v_{CN}$

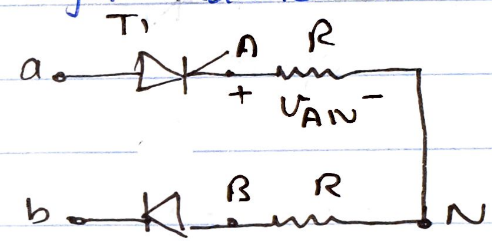
2.4 - 3-Φ Full-wave (Bidirectional AC Controller.



The input current has high harmonics.



$V_{AN} = V_{an}$



$V_{AN} = \frac{V_{ab}}{2}$

- This input current has DC component & high harmonics.

- Modes of Operations.
- Mode I $0 \leq \alpha \leq 60^\circ$

1- 3 Thyristors.
2- 2 Thyristors.

- Mode II $60^\circ \leq \alpha \leq 90^\circ$

1- 2 Thyristors.
2- 1 Thyristor

- Mode III $90^\circ \leq \alpha \leq 150^\circ$

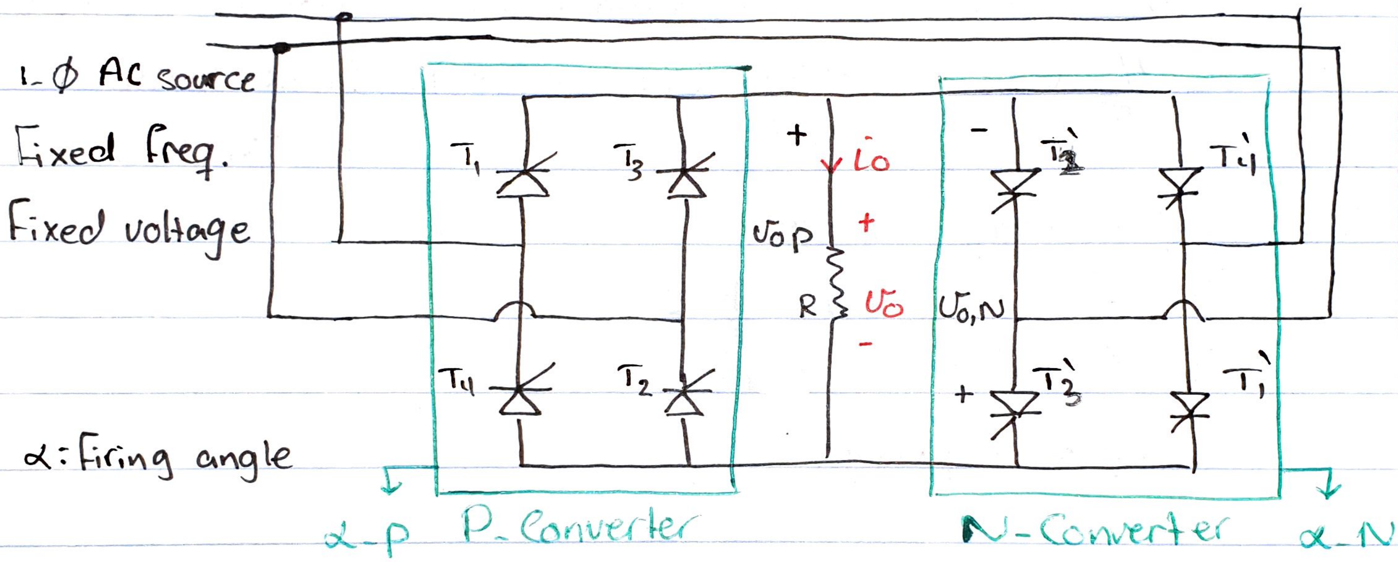
1- 1 Thyristor On.

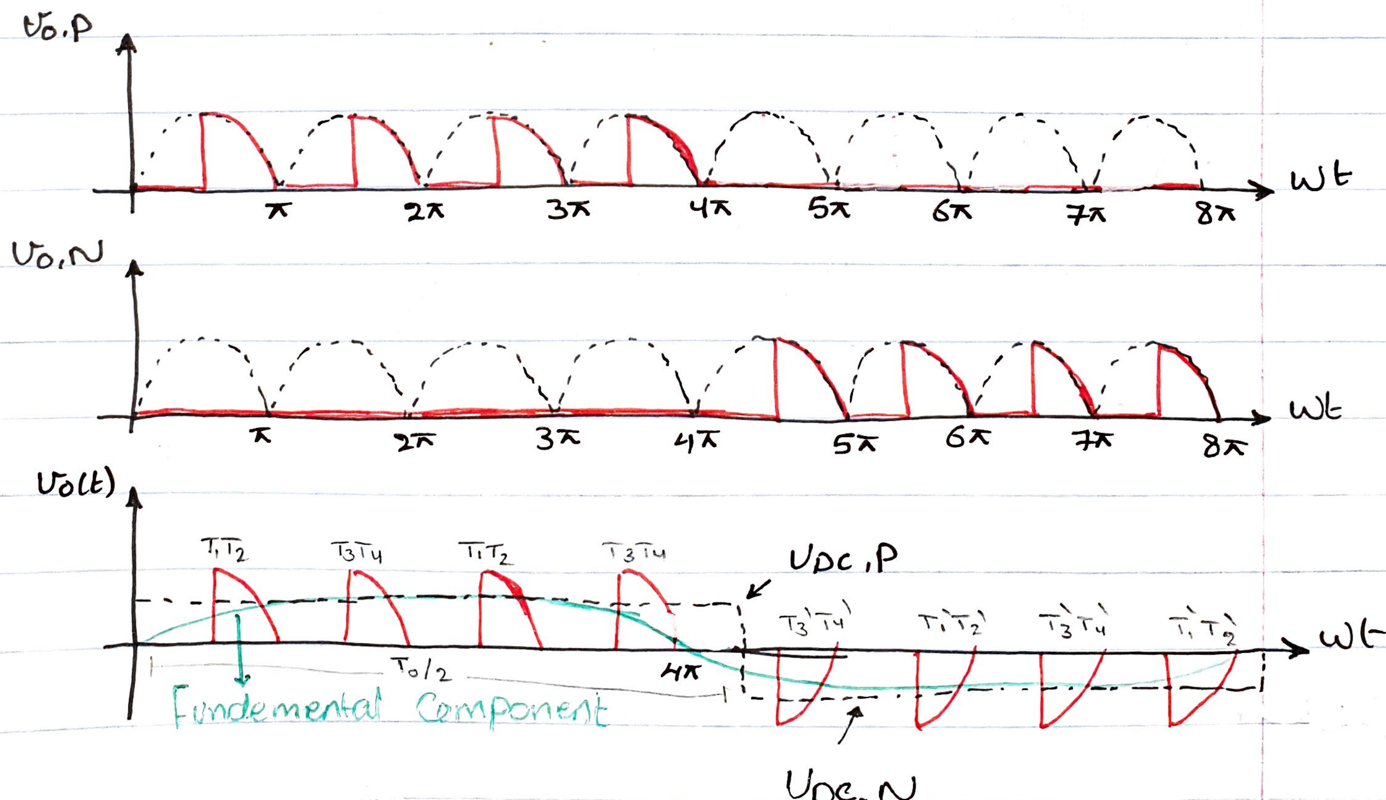
- Cycle Converters.

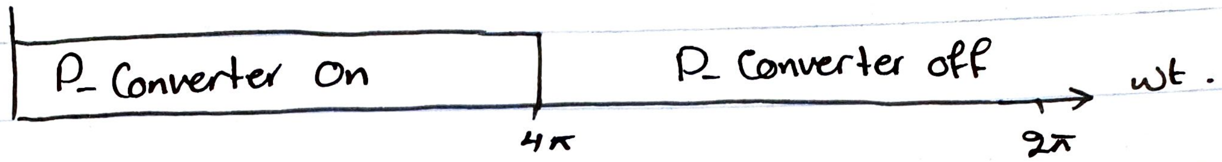
$1\text{-}\phi$, $3\text{-}\phi$ (AC Power) — Fixed voltage, Fixed frequency $\Rightarrow$ cycle converters $\Rightarrow$ $1\text{-}\phi$, $3\text{-}\phi$ (AC Power) — Variable voltage, variable frequency (VVVF).

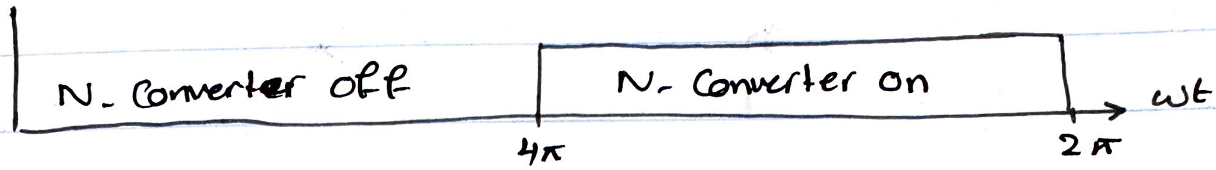
f_o = Friction of input freq. $\simeq < \frac{1}{3} f_i$

1- $1\text{-}\phi$ / $1\text{-}\phi$ Cycle converter.









$V_{DC,P} = |V_{DC,N}| = \frac{V_m}{\pi}(1+\cos\alpha).$

$V_o(t) = V_{o,1}(t) + \text{Harmonics}.$

* $V_{o,1} = \hat{V}_{o,1}\sin(\omega_o t).$ ⇒ Fundemental Component of $V_o(t)$

* $\hat{V}_{o1} = \frac{4}{\pi}\cdot V_{DC} = \frac{4V_m}{\pi^2}(1+\cos 2\alpha)$ ⇒ Peak value of $V_{o1}(t)$.

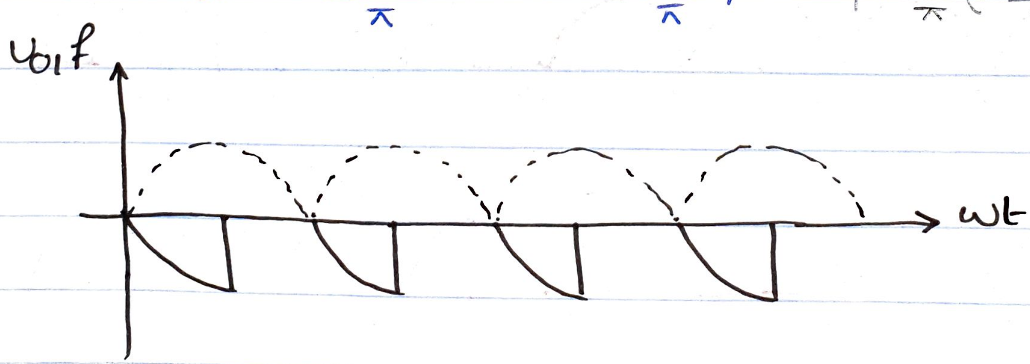
- The Output Frequency is:

$f_o = \frac{T}{T_o} f = \frac{2\pi}{8\pi}(f) = \frac{1}{4} f.$

f ↑ Input Frequency.

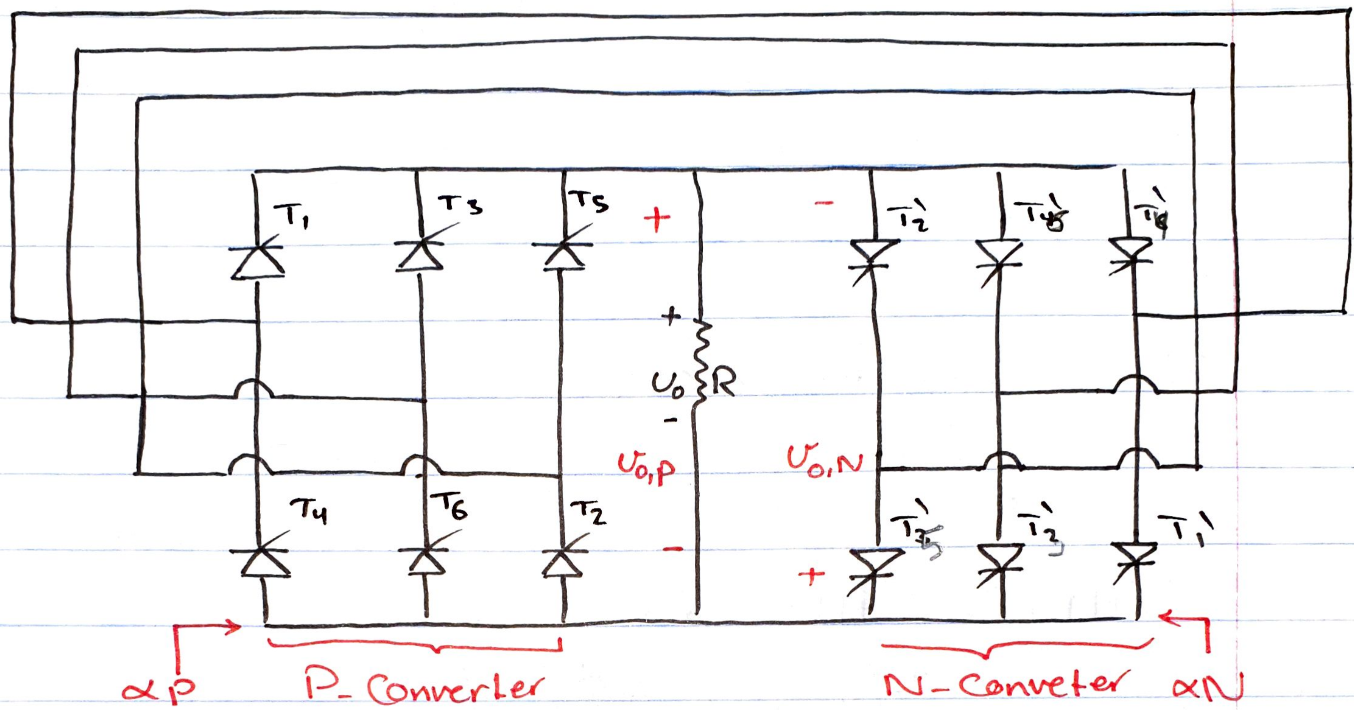
- Note: If the load highly inductive;

$\hat{V}_{o1} = \frac{2V_m}{\pi}\cos\alpha\left(\frac{4}{\pi}\right)$ $\hat{V}_{o1} = \frac{4}{\pi}\left(\frac{2V_m}{\pi}\cos\alpha\right)$



- If a low Filter was used at the output, then the load will have only the Fundemental Component applied across it.

2- 3-ϕ/1-ϕ CycloConverters. It used in higher power applications.



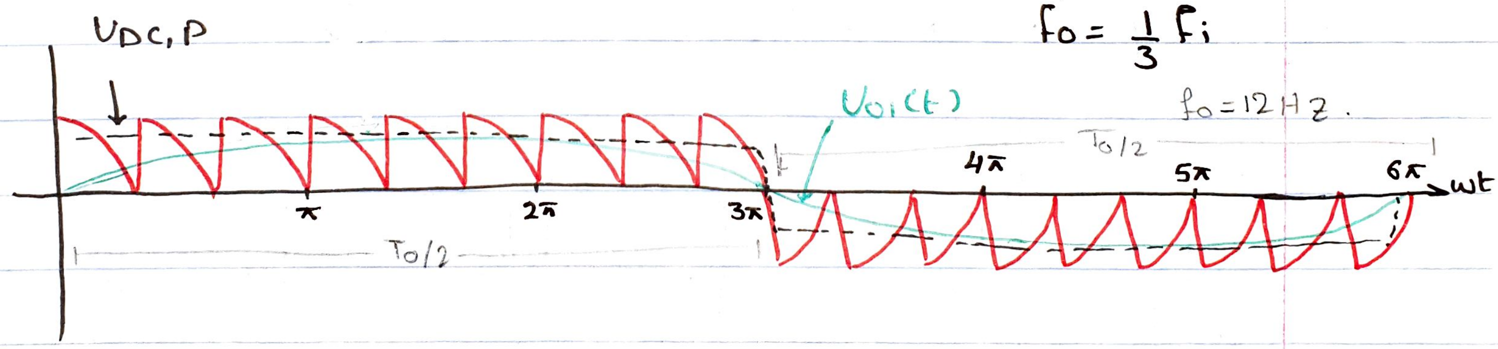
$\alpha P = \alpha N = \alpha$ $\qquad f_o = 6\pi$

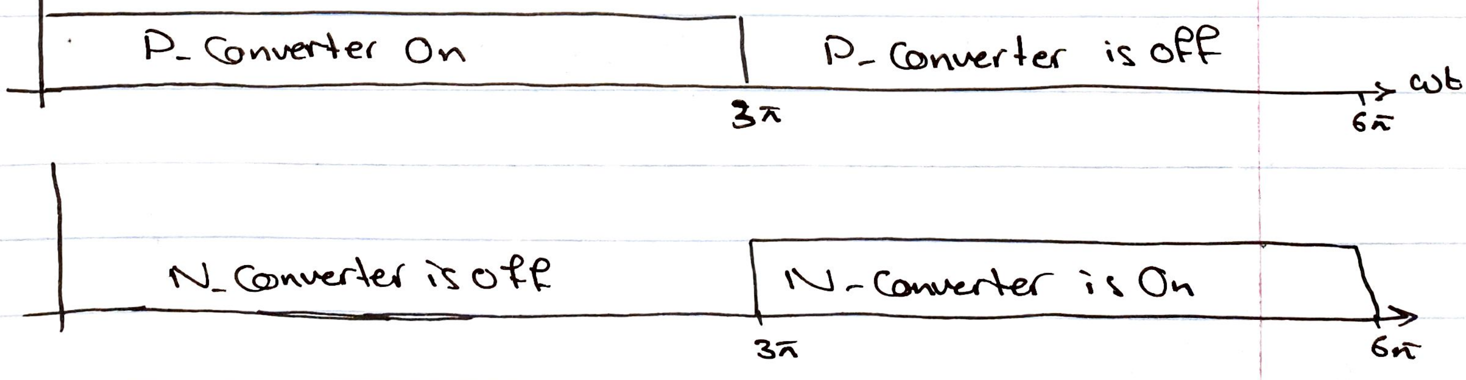
$V_{\alpha,P} = |V_{DC,N}| = V_{DC}$ $\qquad \alpha = \pi/3$.

harmonic distortion قل ← Pulses كلما زاد عدد

$f_o = \frac{1}{3} f_i$

$f_o = 12Hz$.





- $$U_{DC} = U_{DC,P} = |U_{DC,N}| = \begin{cases} \frac{3\sqrt{3}}{\pi} U_m \cos\alpha , & 0 \le \alpha \le \frac{\pi}{3} \\ \frac{3\sqrt{3}}{\pi} U_m \left(1 + \cos\left(\alpha + \frac{\pi}{3}\right)\right), & \alpha \ge \frac{\pi}{3} \end{cases}$$

$V_o(t) = V_{o1}(t) +$ Harmonics. - IF the load highly inductive-

$V_{o1}(t) = \hat{V}_{o1} \sin(\omega t)$. $\quad \hat{V}_{o1} = \frac{4}{\pi}\left(\frac{3\sqrt{3}}{\pi}\cos\alpha\right) \; 0 \le \alpha \le \pi$

$\hat{V}_{o1} = \frac{4}{\pi} V_{DC}$.

- The Output frequency:

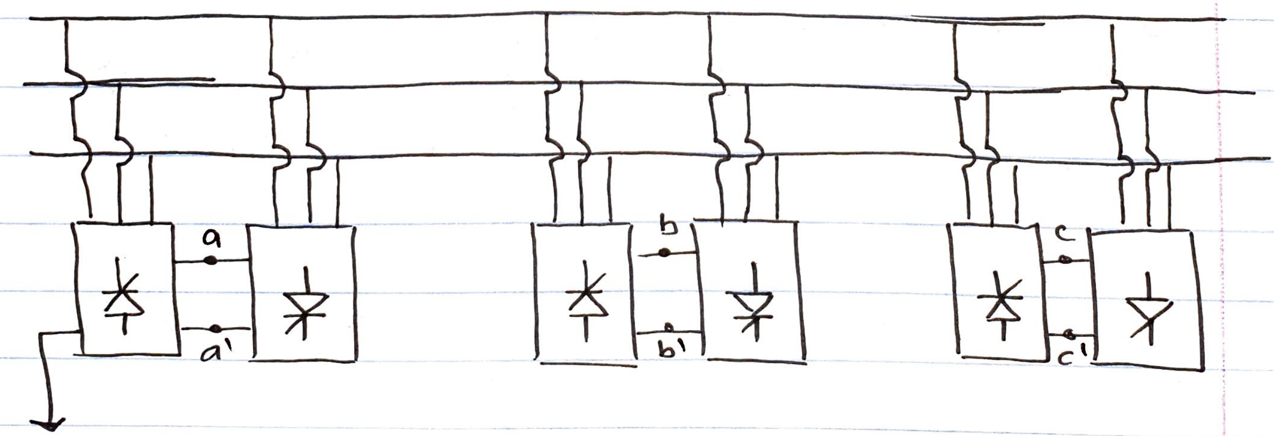
$f_o = \frac{T}{T_o} f = \frac{2\pi}{6\pi} f = \frac{1}{3} f$, $T = 2\pi$

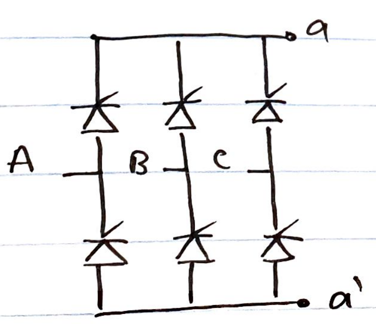
$T_o =$ variable.

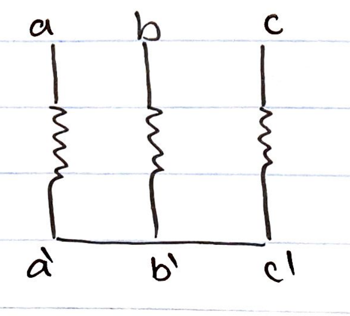
THD	> THD	> THD
$1\text{-}\phi / 1\text{-}\phi$	$2\text{-}\phi / 1\text{-}\phi$ Half wave	$2\text{-}\phi / 1\text{-}\phi$ Full wave.

3- $3\text{-}\phi / 3\text{-}\phi$ cycloconverters. (36 SCRs are needed).

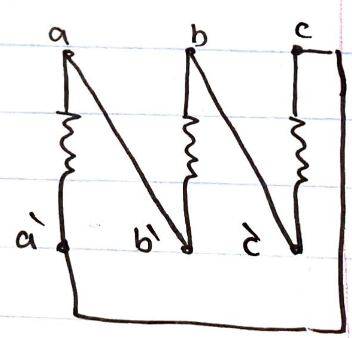
3.1- $3\text{-}\phi$ Full_wave / $3\text{-}\phi$ CycloConverter.





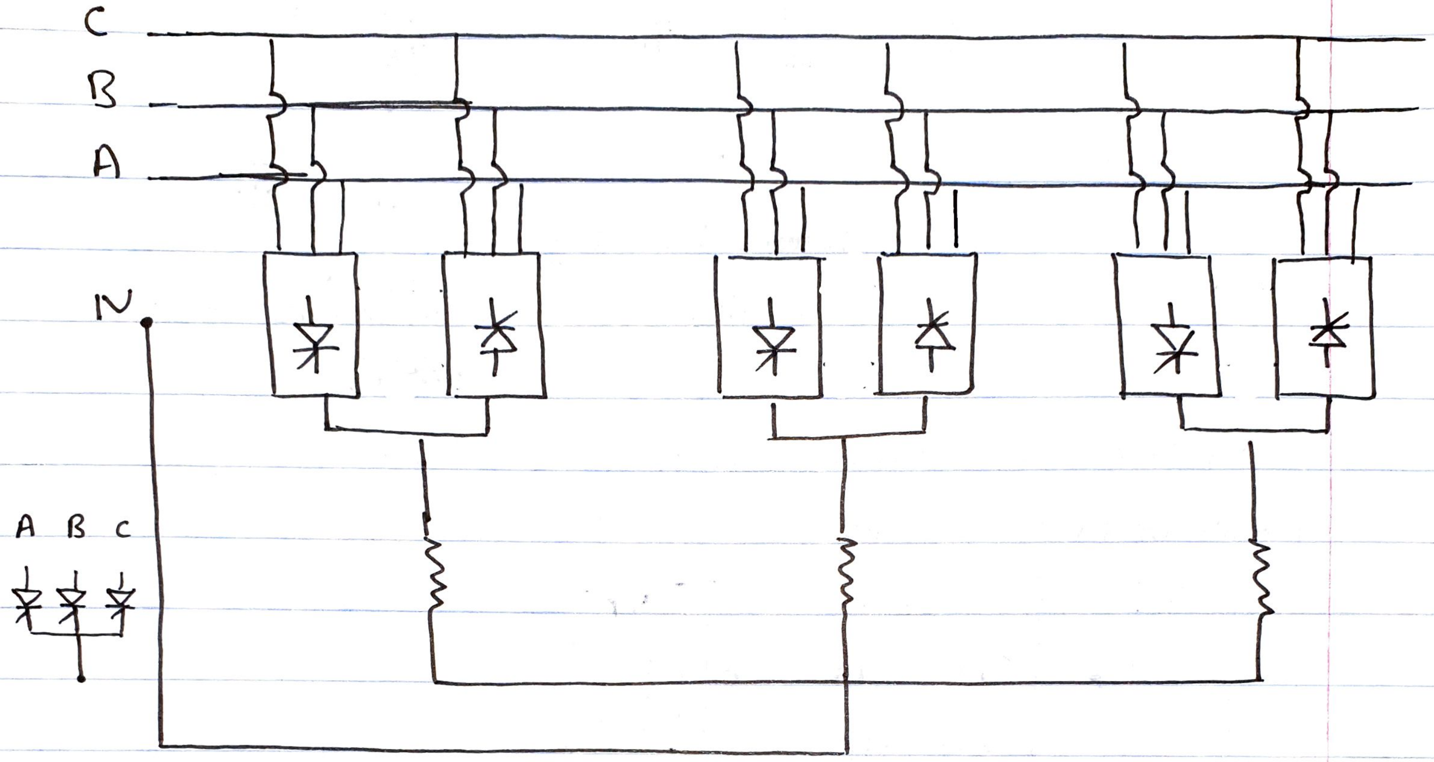


Y_Connected load

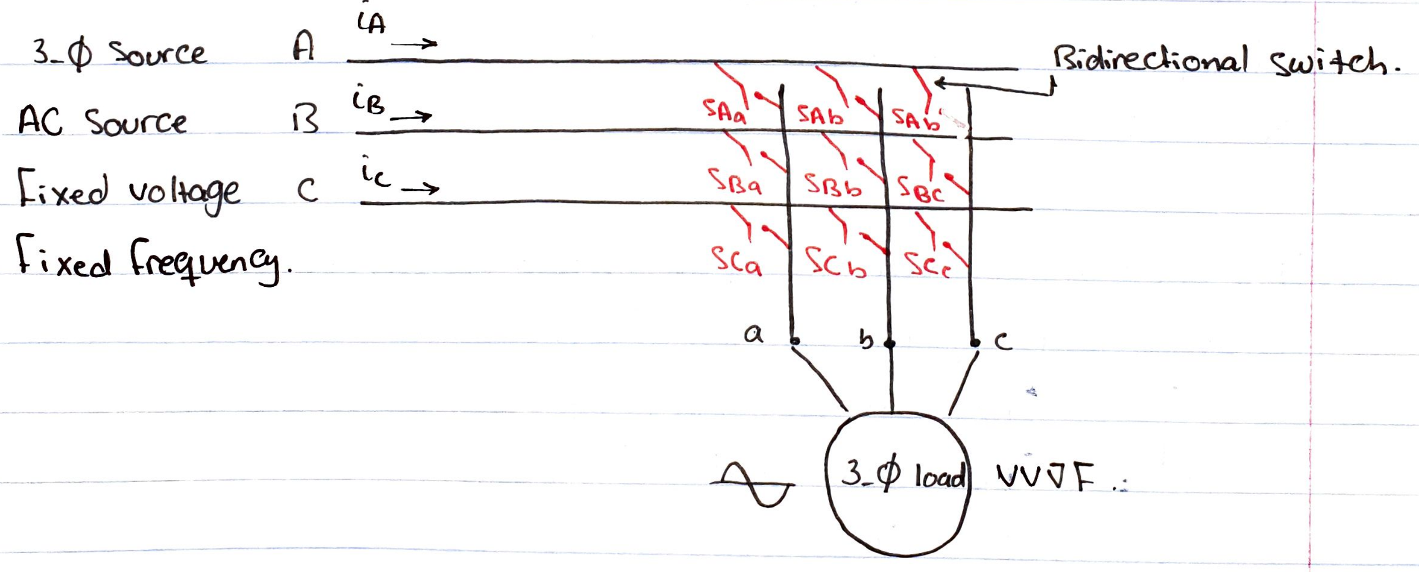


D_Connected load

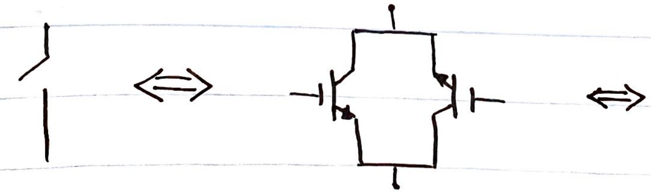
3.2. 3_ϕ half wave / 3_ϕ Cyclo Converter.

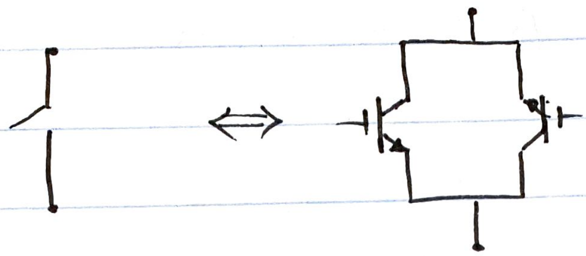


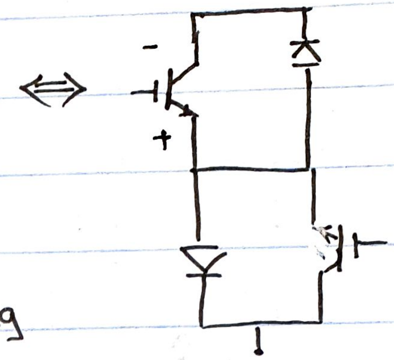
- Matrix Converter. "AC voltage Regulator". 9 bidirectional switch



- The switch must be able to support a voltage of either polarity and must be able to conduct the current in either direction.







Revers Blocking IGBT

Non-revers blocking IGBT

- The relationship between the output voltages & input phase voltages is determined by the states of a switches.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} S_{Aa} & S_{Ba} & S_{Ca} \\ S_{Ab} & S_{Bb} & S_{Cb} \\ S_{Ac} & S_{Bc} & S_{Cc} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

في كل صف من المصفوفة
يجب أن يكون مفتاح
واحد On.

$V_{abc} = V_{ABC}$.

- Note: To avoid applying short circuit between the lines of input phases, only one switch in each row of the matrix S is ON.

- The relation between the input current & output currents is determined by the states of switches.

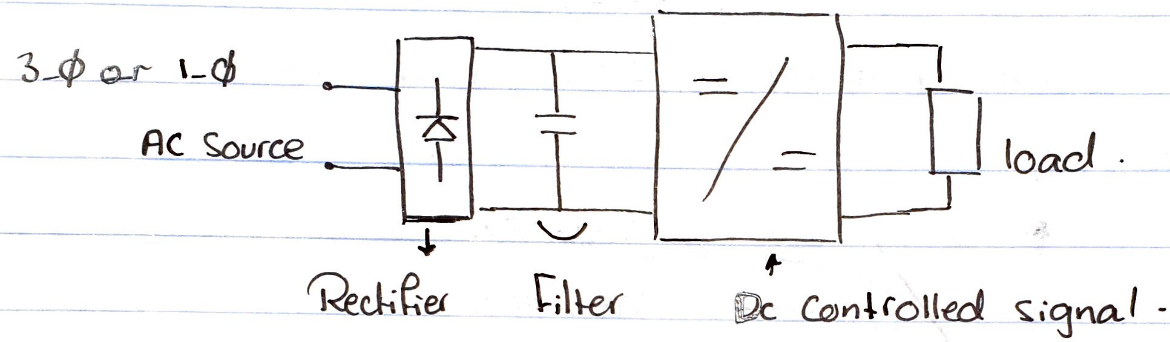
$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} S_{Aa} & S_{Ba} & S_{Ca} \\ S_{Ab} & S_{Bb} & S_{Cb} \\ S_{Ac} & S_{Bc} & S_{Cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

- The Converter must be Controlled in away to avoide discontinuous load current.
- The Converter is Controlled using SPWM technique to produce VVVF.
- THD << THD cycloconverter.

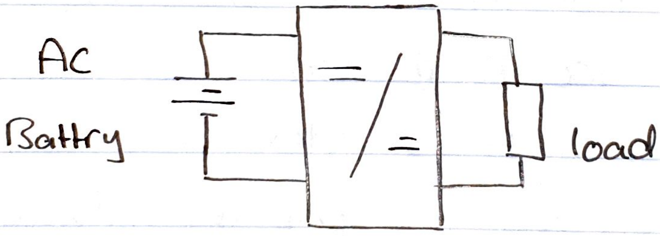
- Chapter 5: DC-DC Converters (Chopper Circuit). Switch Mode Converters.

- Applications.
- Switch Mode power supplies.
- DC Machine Drives. DC motor

1. Rectifired AC signal via Uncontrolled rectifiers.



2. DC Battry "Electrics Cars".

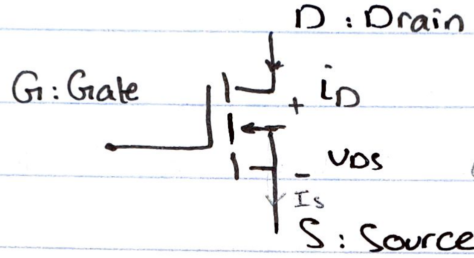


دوائر كهربائية تقوم بتقطيع الجهد
المستمر (DC) عبر الحمل، ويكون تردد
التقطيع قليل عند استخدام Thyristor
ويصل لـ 1 kHz وعند استخدام
Power transistor يصل لـ 10 kHz

- Types of DC-DC converters.
1. Step-down (Buck) Converter.
2. Step-up (Boost) Converter.
3. Step-up/down (Buck-boost) Converter.
4. Half bridge Chopper cell.
5. Full-bridge Chopper cell.

- MOSFET.

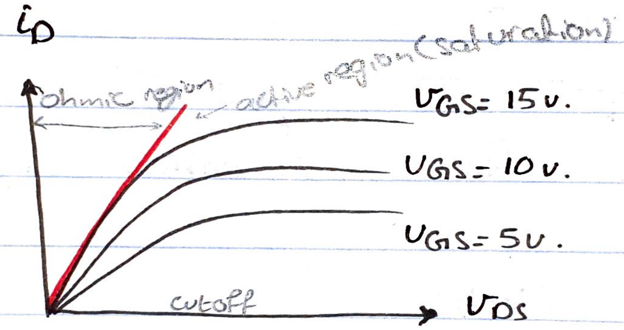
- Metal Oxide Semi Conductor Field Effect Transistor.
- It is a voltage Controlled device.
- Fast switching device (0 kHz - 1 MHz).
- Symbol



(n_channel).

- We need low gate voltage to turn on the device.
- i_v Curve.

$V_{DS,on} = R_{on}\, i_{DS,on}$



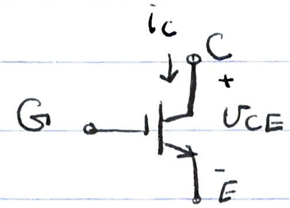
- It operates like resister when its conducting.
- It is availabed at 100 A @ 100 - 200 V.
  10 A @ 1 kV.
- It has no reverse blocking capability. Therefor, it Comes with built in Untiparalle diode.

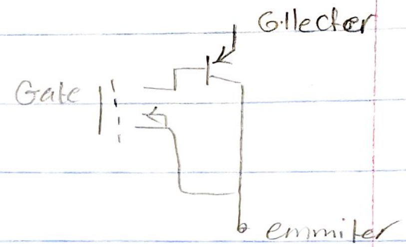
- IGBT سؤال فاينل

- Insulated Gate Bipolar Transistor.
- It is voltage Controlled device.
- It Combines the best Feature of MOSFET & BJT.

MOSFET	BJT.
- Fast switching. ≤ 20 kHz	- low voltage drop.
- low V_{GE} turn it on	- High current density.
- Small gate Control Power.	1.2 kV _ 600 A , 600 V _ 1.2 kA
	low on-state voltage drop.

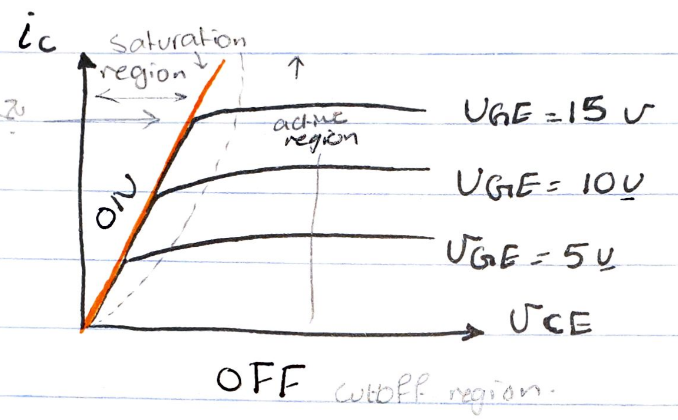
- Symbol:





- i - v curve.

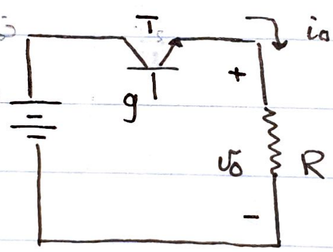
تزيد قليلة.



1. Step-down (buck) Converter.

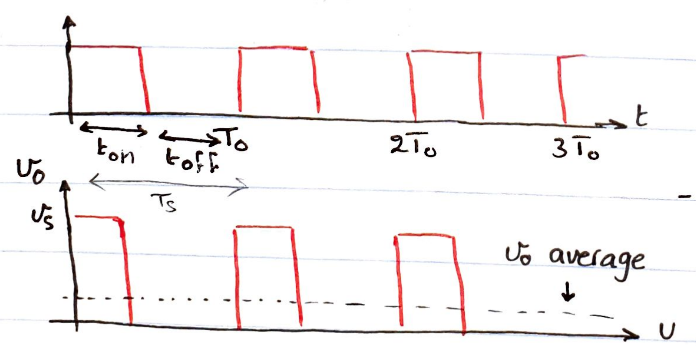
When Ts on (مغلقة) ⇒ فولتية المصدر (V_s) رح تنتقل لفولتية الحمل (R)

When Ts off ⇒ ما في تيار

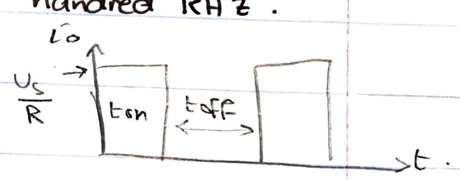


- T_s: Switching time (period)
- $f_s = \frac{1}{T_s}$: switching frequency
  Chopping freq.
  or Carrier freq.

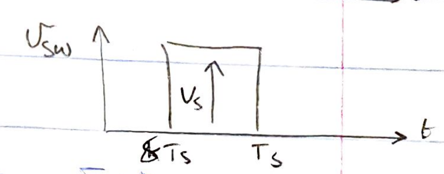
gate signal



- A few KHz ≤ f_s ≤ A few handred KHz.



- The average output voltage:



$$V_o = \frac{1}{T_s}\int_0^{T_s} V_o(t)\,dt = \frac{t_{on}}{t_s} V_s \Rightarrow V_o = \delta V_s$$

$$I_o = \frac{\delta}{R} V_s.$$

$\delta = \frac{t_{on}}{T_s}$ "duty cycle", $0 \le \delta \le 1$

$0 \le V_o \le V_s$

transfer Function:

$$T = \frac{V_o}{V_s} = \delta$$

- The rms value of $V_o(t)$ is.

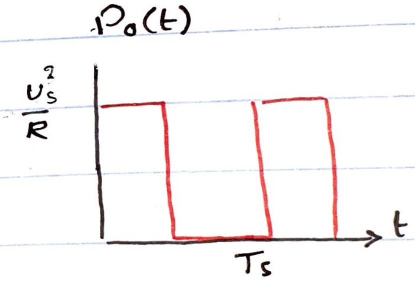
$$V_{RMS} = \sqrt{\frac{1}{T_s}\int_0^{T_s} V_o^2\,dt} = \sqrt{\frac{1}{T_s}\int_0^{t_{on}} V_s^2}$$

$$V_{RMS} = \sqrt{\frac{t_{on}}{T_s} V_s^2} = \sqrt{\delta}\, V_s$$

- Effective Resistor (source). مافي ضياع في الطاقة. ⇒ $P_{out} = P_{in}$.

Assume a loss less chopper, $P_{out} = P_{in} = \bar{i}_{in} V_{in}$.

$$P_{out} = \frac{1}{T_s}\int_0^{t_{on}} \frac{V_s^2}{R}\,dt = \delta\frac{V_s^2}{R}$$

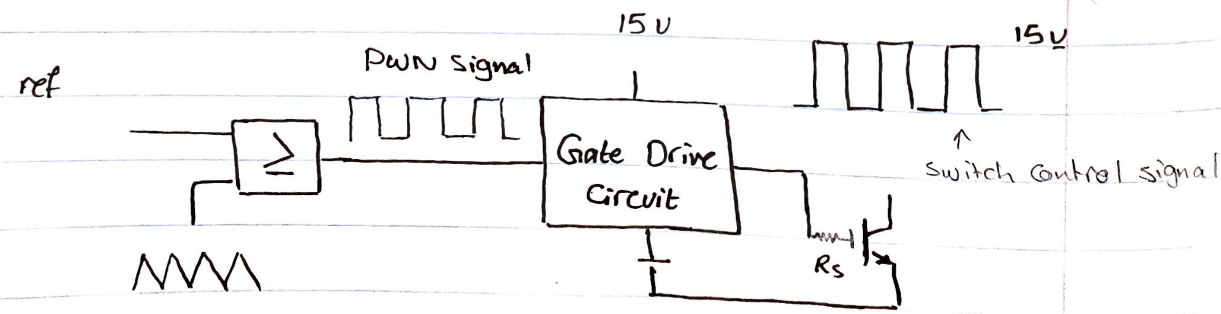


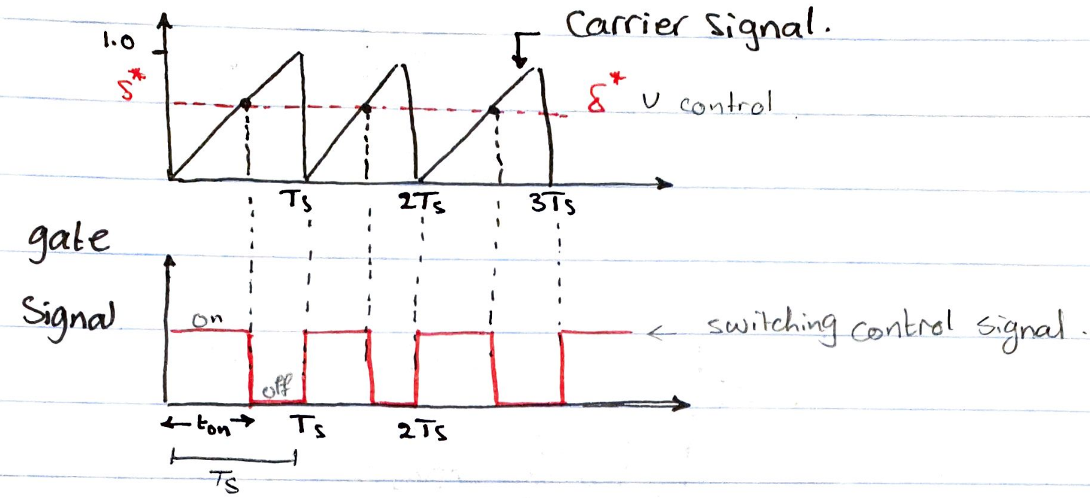
$$P_{in} = \frac{V_s^2}{R_{eq}} = P_{out} = \frac{V_s^2}{R/\delta}$$

$$R_{eq} = \frac{R}{\delta}, \quad P_{in} = \delta\frac{V_s^2}{R_{eq}}$$

- Duty cycle Control. t_{on}, t_{off} متغيرات T_s : ثابتة ← PWM ⇒ t_{on} تغير زمن التوصيل

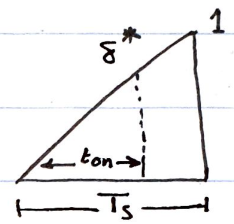
The duty cycle is controlled using pulse width modulation scheme, which is achived by comparing constant voltage [reference voltage or reference δ] with carrier signal [sawtooth or triangular].





From symmetrical triangls.

$$\frac{\delta^*}{1} = \frac{t_{on}}{T_s} \Rightarrow \delta^* = \frac{t_{on}}{T_s}$$



- Example: DC Chopper (Buck) has a resistive load of $10\,\Omega$ & the input Voltage is 220 V. When the switch is On, its Voltage is 2 V (V_{ch}), & the chopping frequency is 1 kHz. If the duty Cycle is 50%, determine the;

a. Average output voltage. b. RMS output voltage.
c. Chopper efficiency d. effective input resistive.

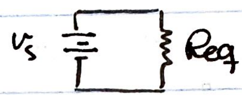
a. $V_{O\,AV} = \delta (V_s')$ where ; $V_s' = 220 - 2 = 218$.
$V_o = (0.5)(218) = 109\ V$.

b. $V_{o,rms} = \sqrt{\delta} \times V_s'$
$= \sqrt{0.5}\ (220-2) = 154.15\ V$.

c. $P_{out} = \frac{V_s'^2}{R} \cdot \delta = \frac{218^2}{10} \times 0.5 = 2376.2\ w$. $\eta = \frac{P_{out}}{P_{in}} = 99.09\%$

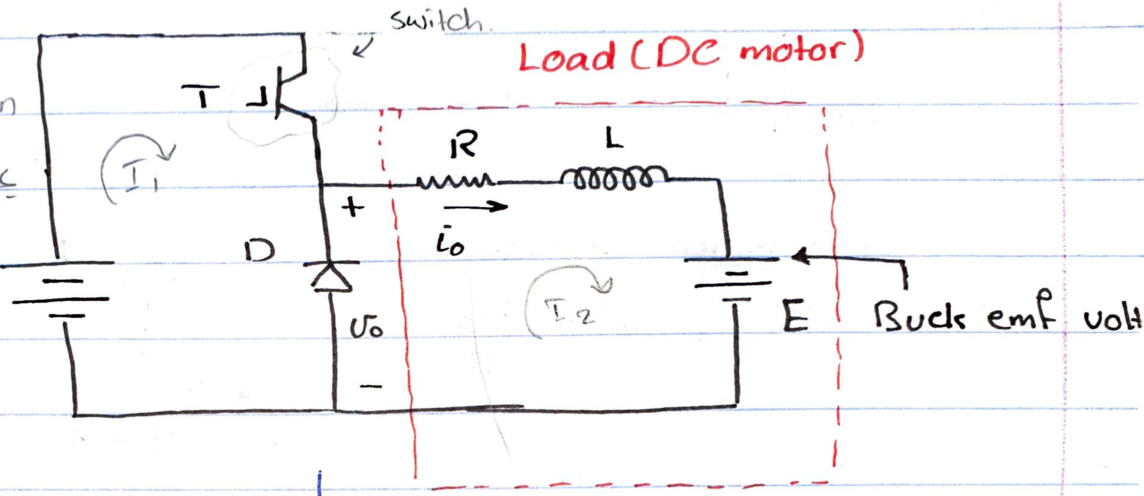
$P_{in} = \frac{V_s}{R_{eq}} \cdot I_{Load}\ \delta = V_s\ I_o = \frac{220\ (220-2)\ (0.5)}{10}$
$= 2398\ w$

d- $R_{eq} = \frac{R}{\delta} = \frac{10}{0.5} = 20\,\Omega.$



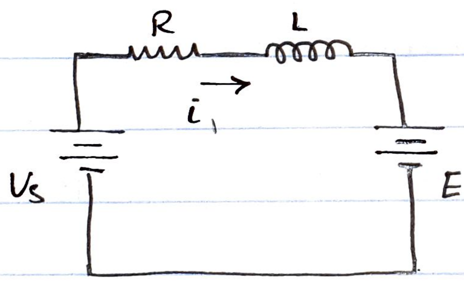
- Step-down Chopper with RL Load. (Buck)

لما يكون T ← off يكون open ← نعطي Df تخلي التيار يجري في كل السيركيت.



Mode I

T is ON & L is charged (short)

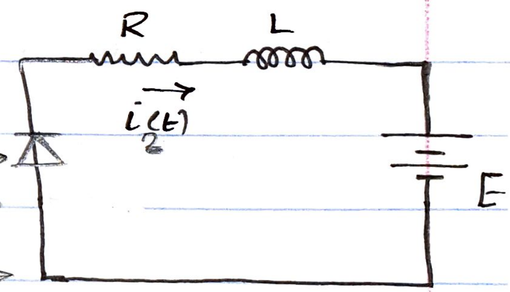


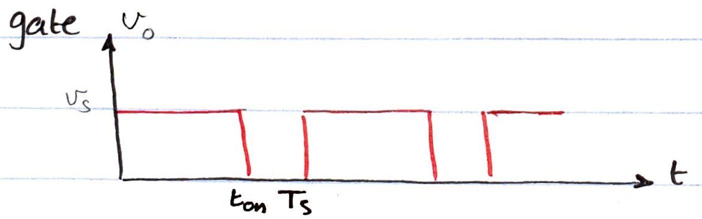
Mode II

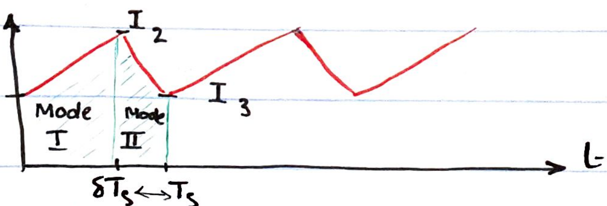
T is off & L is discha

بس لما يشتغل ← بصير SC

الفولتية تساوي صفر ←







ΔI

ripple current

Peak-to-Peak current. $(1-\delta)T_s$

$\Delta I = I_2 - I_1$

Peak to Peak current.

- Mode I : $0 \le t \le \delta T_s$

$-V_s + Ri + L\frac{di}{dt} + E = 0.$

L - Short

$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}.$

$$i(t) = \frac{V_1 - E}{R} + \left(I_1 - \left(\frac{U_s - E}{R}\right)\right) e^{-t/\tau} \quad ; \quad \tau = \frac{L}{R}$$

$$I_2 = i(\delta T_s) = \frac{U_s - E}{R} + \left(I_1 - \left(\frac{U_s - E}{R}\right)\right) e^{-\delta T_s/\tau}$$

- Mode II : $\delta T_s \le t \le T_s$.

$$Ri + L\frac{di}{dt} + E = 0.$$

$$i(t) = \frac{-E}{R} + \left(I_s - \left(\frac{-E}{R}\right)\right) e^{-(t-\delta T_s)/\tau}$$

$$I_1 = I_s = i(T_s) = \frac{-E}{R} + \left(I_1 + \frac{E}{R}\right) e^{-\frac{(t-\delta T_s)}{\tau}}$$

$$\Delta I = I_2 - I_1 = \frac{U_s}{R}\left[\frac{1 + e^{-T_s/\tau} - e^{-\delta T_s/\tau} - e^{-(1-\delta)T_s/\tau}}{1 - e^{-T_s/\tau}}\right]$$

Maximum ΔI

$$\frac{\partial \Delta I}{\partial \delta} = 0 \Rightarrow \frac{T_s}{\tau} e^{-\delta T_s/\tau} - \frac{T_s}{\tau} e^{-(1-\delta)T_s/\tau} = 0$$

$$\delta = 0.5$$

$$\Delta I_{,max} = \Delta I\Big|_{\delta=0.5} = \tanh\left(\frac{T_s}{4\tau}\right) \cdot \frac{U_s}{R} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\Delta I_{,max} = \frac{U_s}{R}\tanh\left(\frac{T_s}{4\tau}\right)$$

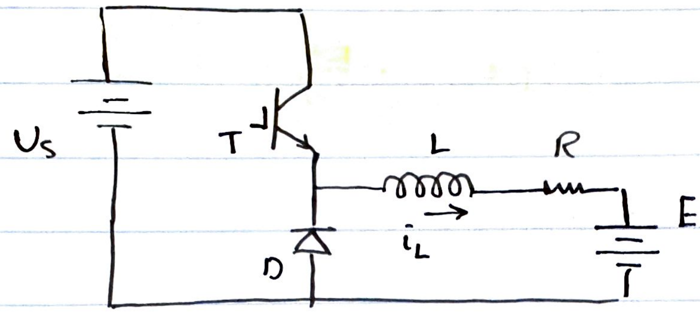
For highly inductive load. $\tanh(x) \approx x$.
where x is small

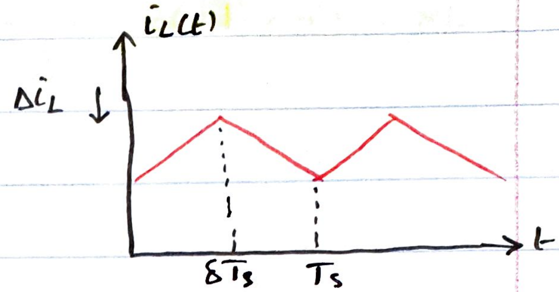
$$\Delta I_{,max} = \frac{U_s}{R}\left(\frac{T_s}{4\tau}\right)$$

$$\Delta I_{,max} = \frac{U_s T_s}{4L}$$

$$\Delta I_{,max} = \frac{U_s}{4 f_s L} \quad \text{.......} \ (*)$$

- Buck Converter



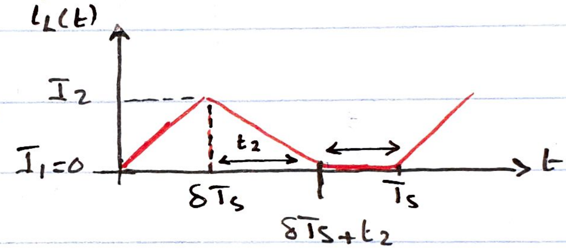


$$\Delta i_L = \frac{U_s}{4 f_s L}$$

- Discontinuous inductor (load) current.

- Mode I ; $0 \le t \le \delta T_s$

$$U_s = L\frac{di}{dt} + Ri + E$$



$$i(t) = \frac{U_s - E}{R} + \left(0 - \frac{U_s - E}{R}\right)e^{-t/\tau}$$

$$i(t) = \frac{U_s - E}{R}\left(1 - e^{-t/\tau}\right)$$

$$I_2 = i(\delta T_s) = \left(\frac{U_s - E}{R}\right)\left(1 - e^{-\delta T_s/\tau}\right)$$

- Mode II ;

$$L\frac{di}{dt} + Ri + E = 0$$

$$i(t) = \frac{-E}{R} + \left(I_s + \frac{E}{R}\right)e^{\frac{-(t - \delta T_s)}{\tau}}$$

$$i(t) = 0 = \frac{-E}{R} + \left(I_2 + \frac{E}{R}\right)e^{-t_2/\tau}$$

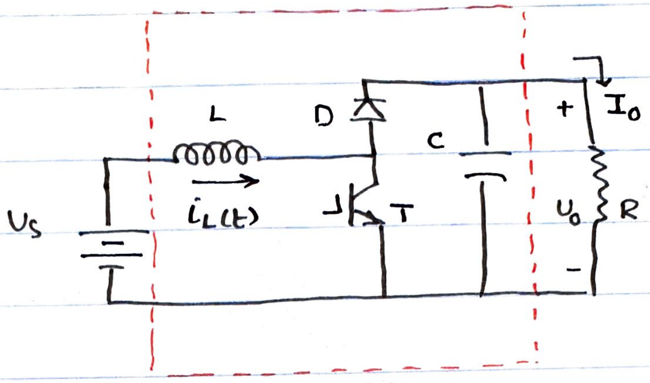
Since the current is discontinuous, then the current is zero at the end of mode II $\Rightarrow I_3 = I_1 = 0$

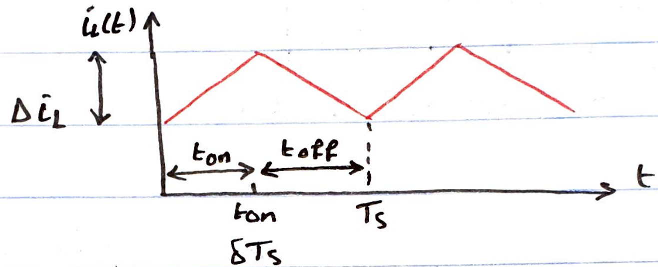
$$\frac{E}{R} = \left(I_2 + \frac{E}{R}\right)e^{-t_2/\tau}$$

$$e^{t_2/\tau} = \left(\frac{RI_2}{E} + 1\right) \quad , \quad t_2 = \tau \ln\left[\frac{RI_2}{E} + 1\right]$$

To make the current discontinuous ; $t_2 \le (1 - \delta) T_s$.

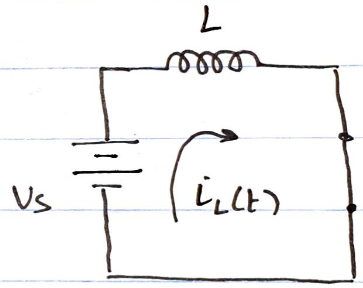
2. Step-up (Boost) Converter





- When T is ON (short) & D is OFF (open)

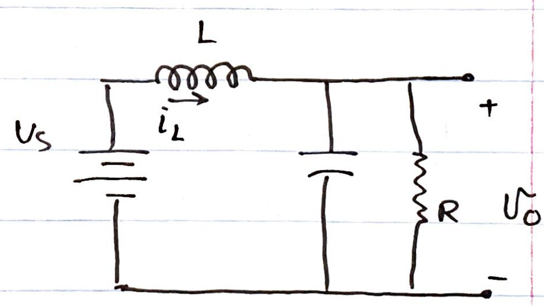
$$U_s = L\frac{di_L}{dt} = L\frac{\Delta i_L}{dt}$$



$$L\Delta i_L = t_{on} U_s \quad \text{.......} \ (1)$$

- When T is off & D is on (short).

$$U_s = L\frac{di}{dt} + U_o.$$



$$U_s = -L\frac{\Delta i_L}{t_{off}} + U_o.$$

$$L\Delta i_L = (U_o - U_s)\, t_{off} \quad \text{.....} \ (2).$$

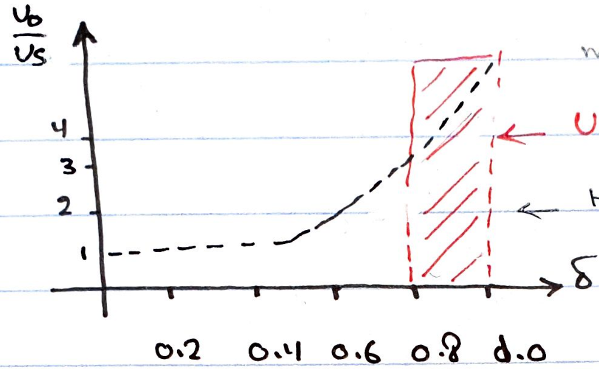
$$t_{on} U_s = (U_o - U_s)\, t_{off} \qquad (1) = (2)$$

$$(t_{off} + t_{on})\, U_s = t_{off}\, U_o.$$

$$T_s U_s = t_{off}\, U_o.$$

$$\frac{V_o}{V_s} = \frac{T_s}{t_{off}} = \frac{T_s}{T_s - t_{on}}$$

$$\frac{V_o}{V_s} = \frac{1}{1-\delta} \; ; \; 0 < \delta = \frac{t_{on}}{T_s} < 1 .$$

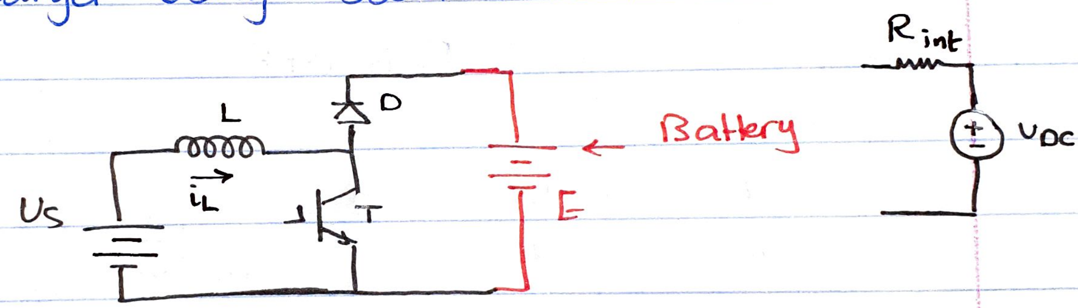


must be avoid.

unstable region.

Huge difference in V_o.

- Battery Charger Using Boost Converter.

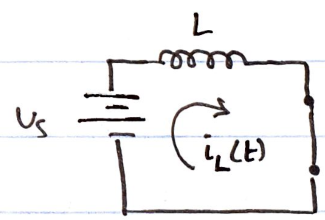


- Mode I (T is ON).

$$V_s = L \frac{di_L}{dt} \Rightarrow i_L = \frac{V_s}{L} t + I_1$$

$$\frac{di_L}{dt} > 0 \Rightarrow V_s > 0$$

$$i_1(t) = \frac{V_s}{L} t + I_1$$



S → Closed, D → off

L → Charged.

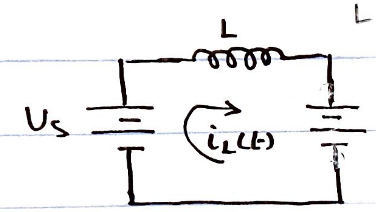
- Mode II (T is OFF).

$$V_s = L \frac{di_L}{dt} + E$$

$$\frac{V_s - E}{L} = \frac{di}{dt} < 0$$

$$i_2(t) = \frac{V_s - E}{L} t' + I_2 -$$

$$V_s < 0 .$$



S → Open, D → on

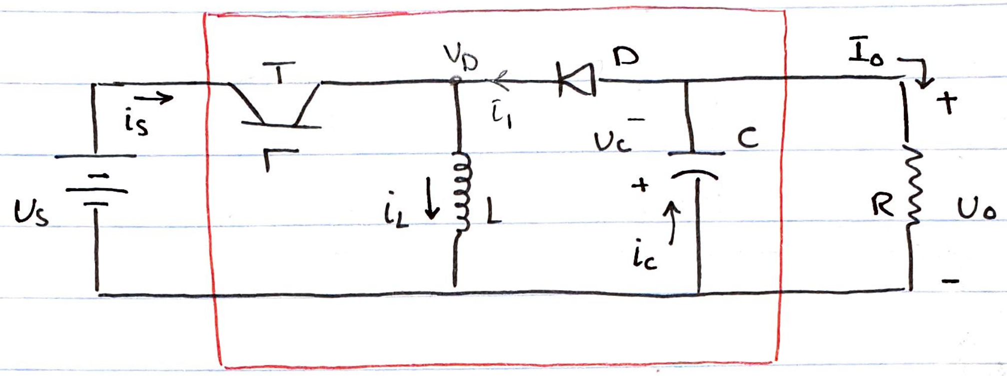
L → discharged.

Combined Condition

$$0 < V_s < E$$

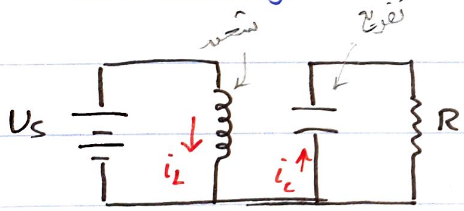
بقول بنجح ليمس $E > V_s$ ⇐ وهاي فكرة Boost Converter

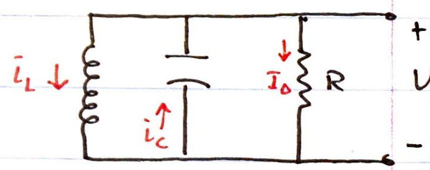
3. Step up down (Buck_Boost) DC_DC Converter.



Buck_Boost

Mode I	Mode II
T is On, D is OFF ← open	T is OFF, D is ON ← closed
L is Charged.	L is discharged.
C is discharged.	C is charged.





Mode I:

$$U_s = L\frac{\Delta i_L}{t_{on}} \leftarrow I_2 - I_1$$

$$L\Delta i_L = U_s\, t_{on} \;\ldots\ldots (1).$$

$$\Delta i_L = \frac{U_s}{L} t_{on}.$$

Mode II:

$$V_o = -L\frac{\Delta i_L}{t_{off}}$$

$$L\Delta i_L = -t_{off}\, U_s \;\ldots\ldots (2).$$

$$\Delta i = \frac{-U_s\, t_{off}}{L}.$$

$(1) = (2) \Rightarrow -t_{off} V_o = t_{on} U_s$

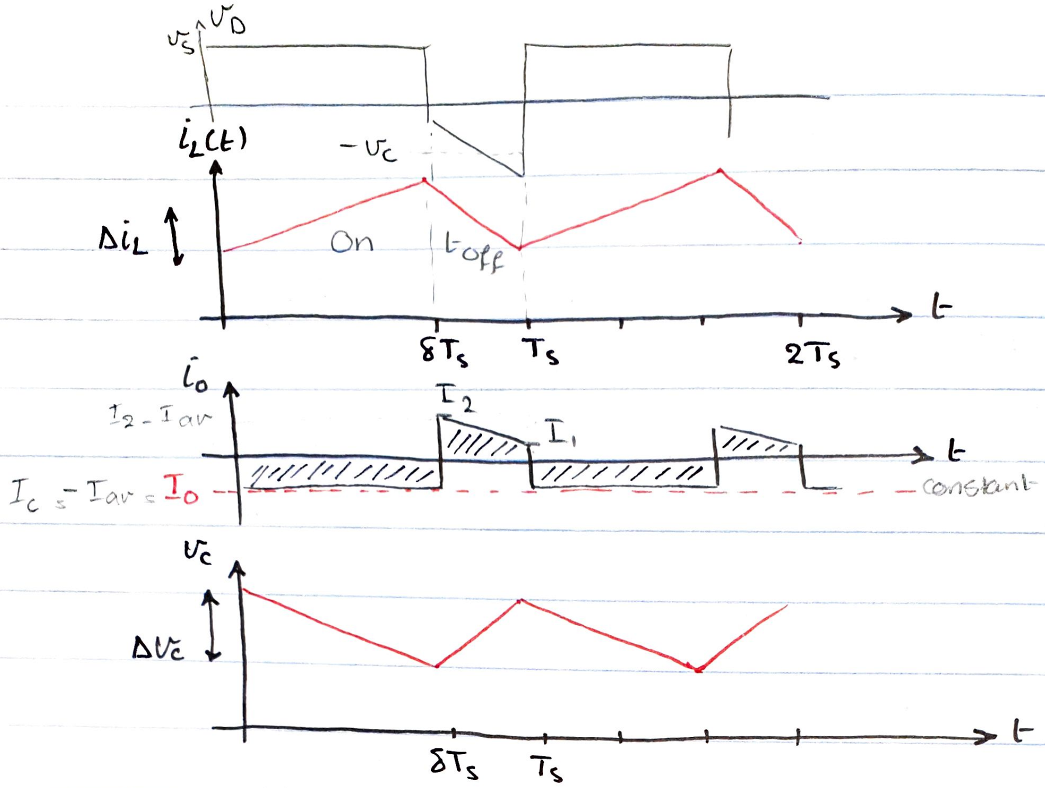
$$\frac{U_o}{U_s} = \frac{-t_{on}}{t_{off}} = \frac{-t_{on}}{T_s - t_{on}} = \frac{-\delta}{1-\delta}$$ $U_D = U_s$. (Mode I)

$$\frac{V_o}{U_s} = \frac{-\delta}{1-\delta}.$$

$\delta < 0.5$, $|U_o| < U_s$ (Buck).

$\delta > 0.5$, $|U_o| > U_s$ (Boost).

$\delta = 0.5$, $U_o = -U_s$ (Inversion).



- Input/Output Currents relationship.

 Assuming lossless converter.

$$P_{out} = P_{in} \quad -P_{in}$$

$$V_o I_o = V_s I_s \; ; \; I_s : \text{average input current.}$$

$$\frac{I_s}{I_o} = \frac{V_o}{V_s} = \frac{-\delta}{1-\delta} = \frac{\delta}{1-\delta}$$

- Peak-to-Peak ripple current Δi_L:

• Mode I ; (Inductor current ripple).

$$V_s = \frac{L\,\Delta i_L}{t_{on}} \Rightarrow \Delta i_L = \frac{t_{on}}{L} V_s \; ; \; t_{on} : \delta T_s .$$

$$\Delta i_L = \frac{\delta T_s}{L} V_s .$$

$$\Delta i_L = \frac{\delta V_s}{f_s L} \Leftarrow$$

- Capacitor voltage ripple.

$$i_c = C \frac{\Delta v_c}{t_{on}} = I_o$$

$$\Delta v_c = \frac{t_{on}}{C} I_o \; ; \; t_{on} : \delta T_s$$

$$\Delta v_c = \frac{\delta I_o}{f_s C} \Leftarrow$$

- Example: A buck-boost has an input voltage of $V_s = 12\,V$. The duty cycle, $\delta = 0.25$ & the switching frequency is 25 kHz. The inductance, $L = 150\,\mu H$, and filter capacitance, $C = 220\,\mu F$. The average load current, $I_o = 1.25\,A$.

Determine ;

a- Average Output Voltage.

b- Peak-to-Peak Output voltage ripple.

c- Peak-to-Peak inductor ripple current.

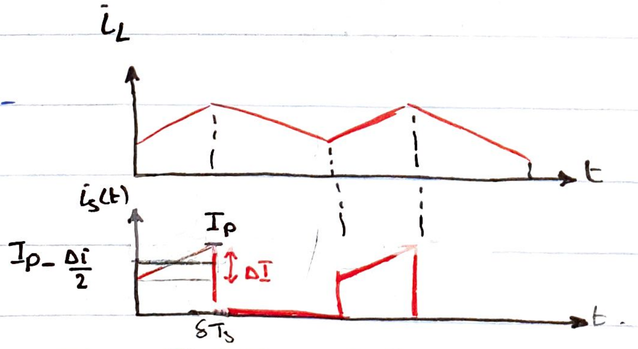
d- Peak Current of switch.

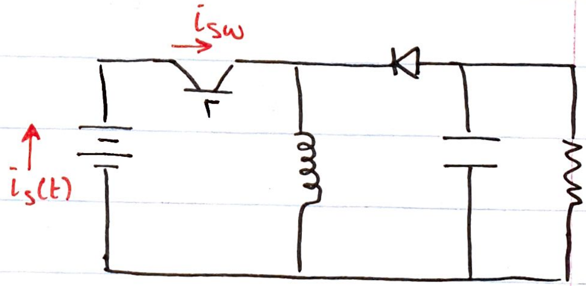
a- $\dfrac{V_o}{V_s} = \dfrac{-\delta}{1-\delta} = \dfrac{-0.25}{1-0.25} \Rightarrow V_o = \dfrac{-0.25}{1-0.25} \cdot V_s = -4\,V$ (Buck)

b- $\Delta V_c = \dfrac{\delta I_o}{f_s C} = \dfrac{0.25\,(1.25)}{25\times10^{3}\,(220)\times10^{-6}} = 56.8\,mV$.

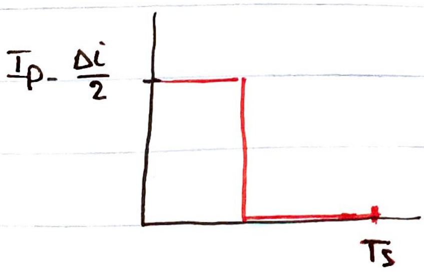
c- $\Delta i_L = \dfrac{\delta V_s}{f_s L} = \dfrac{0.25\,(12)}{25\times10^{3}\,(150)\times10^{-6}} = 0.8\,A$.

d-





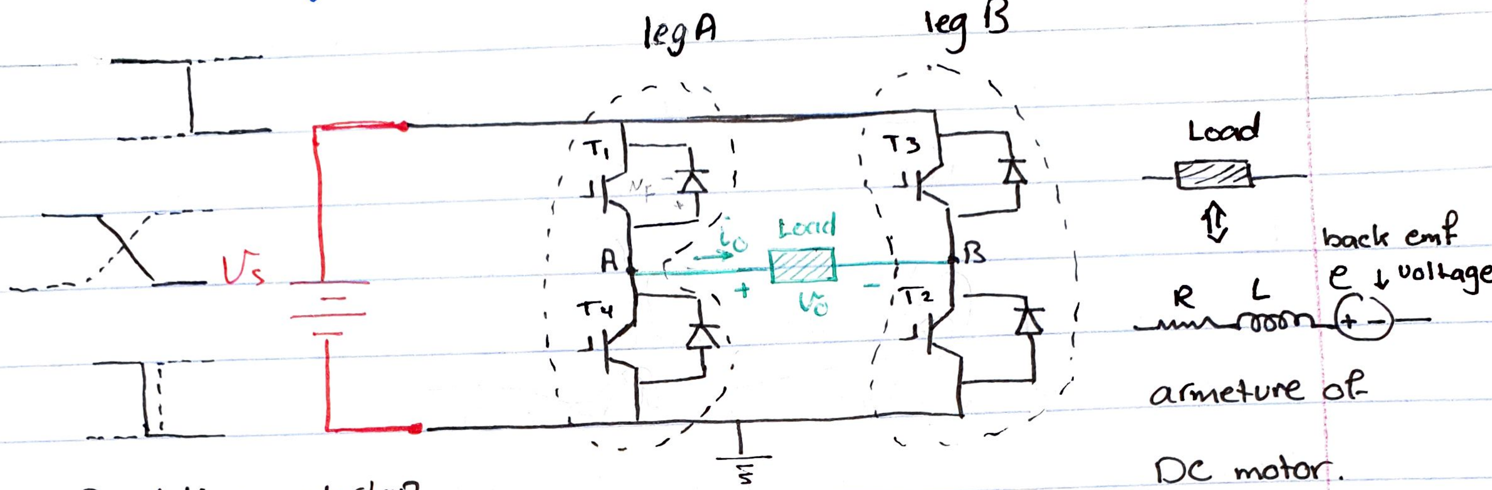
$I_s = \delta\left(I_p - \dfrac{\Delta i}{2}\right) \Rightarrow I_p = \dfrac{I_s}{\delta} + \dfrac{\Delta i}{2}$ $\qquad I_s = \dfrac{+\delta\, I_o}{1-\delta}$



$= \dfrac{0.4167}{0.25} + 0.4 \qquad = \dfrac{+0.25\,(1.25)}{0.75}$

$I_p = 2.067\,A. \qquad = 0.4167\,A.$

4 : Full - bridge DC/DC Converter.



Dead time underlap

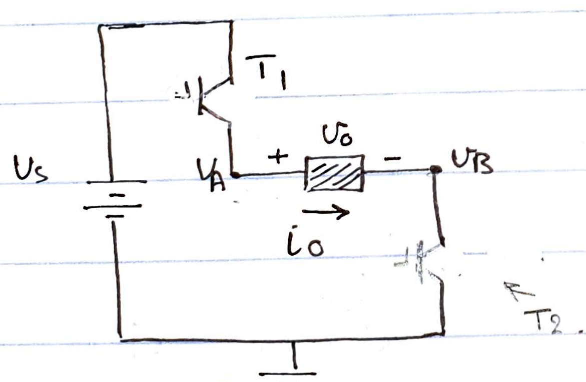
period (1_5) μsec

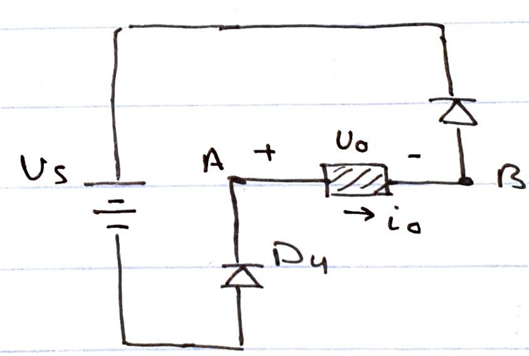
امكانية اطفاء واحد من Switches وتشغيل الثاني

- Note: A dead time (Underlap period) must be applied for each leg to avoid shorting the DC link.

خوف تلف حدوث short circuit & DC motor

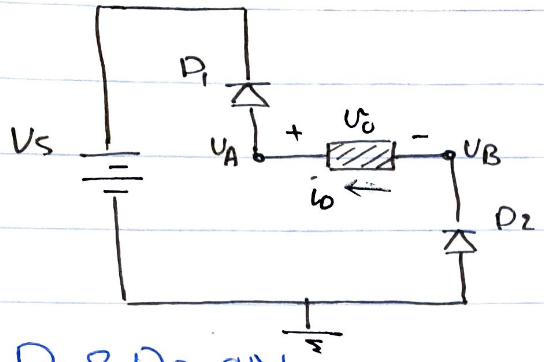
$i_o > 0$

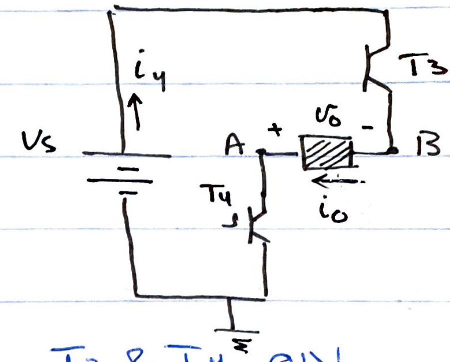




T_3, T_4 off ← open	D_2, D_4 ON.
T_1, T_2 ON.	D_1, D_2 OFF
D_1, D_2, D_3, D_4 off.	T_1, T_2, T_3, T_4 off
$V_A = V_s$, $V_B = 0$.	$V_A = 0$, $V_B = V_s$
$V_o = V_A - V_B = V_s$	$V_o = -V_s$
i_o builds up	i_o decays.

$i_o < 0$.





D_1 & D_2 ON	T_3 & T_4 ON
D_3 & D_4 OFF	T_1 & T_2 OFF
T_1, T_2, T_3, T_4 OFF	D_1, D_2, D_3, D_4 OFF
$V_A = V_S$, $V_B = 0$.	$V_A = 0$, $V_B = V_S$
$V_o = V_A - V_B = V_S$.	$V_o = -V_S$
i_o decays.	i_o builds up.

- Possible switching states.

$V-$ $I+$	$V+$ $I+$
$V-$ $I-$	$V+$ $I-$

Quadrant.	i_o	V_o	T_1	T_2	T_3	T_4	D_1	D_2	D_3	D_4	
	+	+	1	1	0	0	0	0	0	0	
I	+	osc	1	0	0	0	0	0	1	0	I
	+	0	0	1	0	0	0	0	0	1	
	-	+	0	0	0	0	1	1	0	0	
IV	-	0	0	0	0	1	0	1	0	0	IV
	-	0	0	0	1	0	1	0	0	0	
	-	-	0	0	1	1	0	0	0	0	
III	-	0	0	0	0	1	0	1	0	0	III
	-	0	0	0	1	0	1	0	0	0	
	+	-	0	0	0	0	0	0	1	1	
II	+	0	1	0	0	0	0	0	1	0	II
	+	0	0	1	0	0	0	0	0	1	

- The average value of v_A is V_A

$V_A = \delta_A V_s$

where δ_A is the duty cycle of leg A or T_1

The average value of v_B is V_B

$V_B = \delta_B V_s$

↳ duty cycle of leg B or T_3

Switching stratiges :

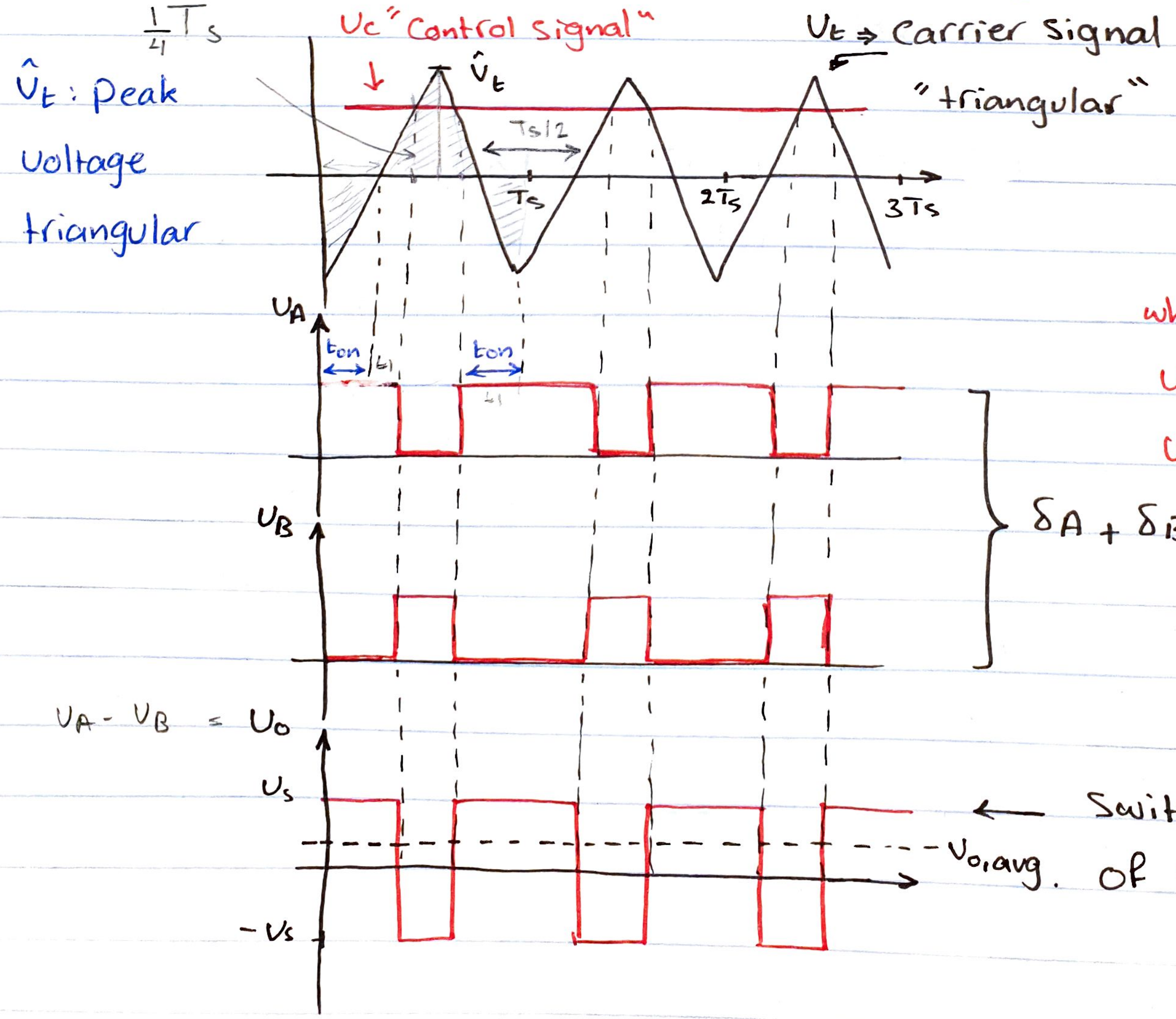
1. PWM with bipolar switching

$(T_1 \& T_2)$ & $(T_3 \& T_4)$ are related as pairs

2. PWM with unpolar switching

Each leg is operated independently.

1. Output voltage with bipolar switching.



Switching function

when $V_c \geq V_t$, $T_1 \& T_2$ ON

$v_A = V_s$, $T_3 \& T_4$ off

$v_B = 0$

when $V_c \leq V_t$ $T_1 \& T_2$ off

$v_A = 0$ $T_3 \& T_4$ ON

$v_B = V_s$

$\delta_A + \delta_B = 1$

Switching frequency of $V_o = f_s$

$V_o = V_A - V_B$

$V_o = (\delta A - \delta B) V_s$ $\qquad\qquad \delta A + \delta B = 1$

$V_o = (\delta A - (1 - \delta A)) V_s$

$V_o = (2\delta A - 1) V_s$

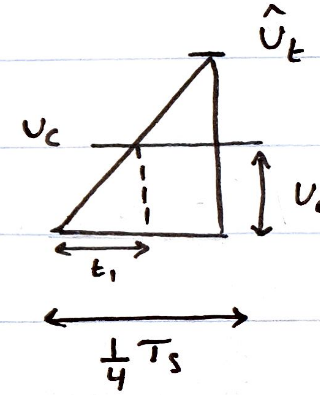
$0 \leq \delta A \leq 1 \Rightarrow -V_s \leq V_o \leq V_s$

- Note: when $\delta A = 0.5 \Rightarrow V_o = 0$

$\Rightarrow$ V_o square wave.

$\Rightarrow$ The converter operater as $1\text{-}\phi$ inverter in square-wave mode.

- $\frac{1}{4} T_s$ from the Output voltage with bipolar switching.



$t_{on} = t_1 + t_1 + \frac{T_s}{2} = 2t_1 + \frac{T_s}{2}$

$\frac{t_1}{\left(\frac{T_s}{4}\right)} = \frac{V_c}{\hat{V}_t} \Rightarrow t_1 = \frac{T_s}{4} \frac{V_c}{\hat{V}_t}$

$\frac{V_c}{V_o} = \frac{\hat{V}_t}{V_s}$

$t_{on} = \frac{T_s}{2} \frac{V_c}{\hat{V}_t} + \frac{T_s}{2}$

$\delta A = \frac{t_{on}}{T_s} = \frac{1}{2}\left(1 + \frac{V_c}{\hat{V}_t}\right)$

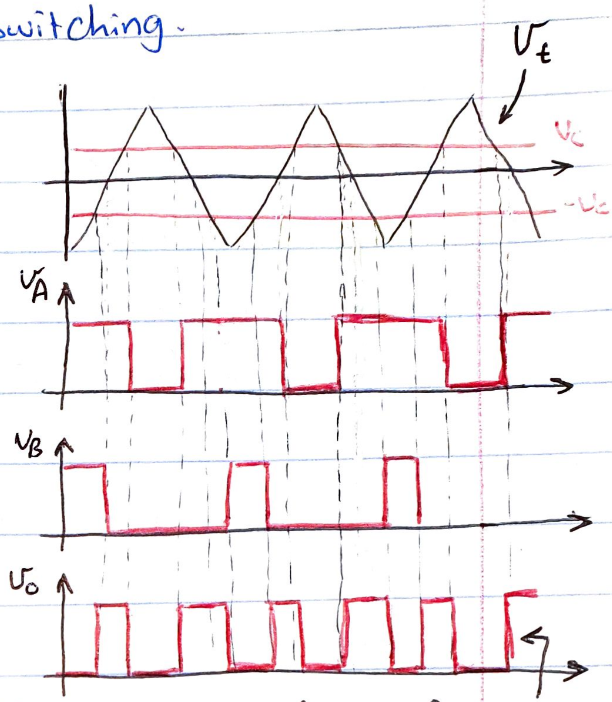
$V_c = (2\delta A - 1)\hat{V}_t$

2. Output Voltage with unpolar switching.

Switching Function.

leg A:
- when $V_c \geq V_t \Rightarrow$ T_1 is ON
  - $V_A = V_s$ — T_4 is OFF
- when $V_c \leq V_t$ — T_1 is OFF
  - $V_A = 0$ — T_4 is ON

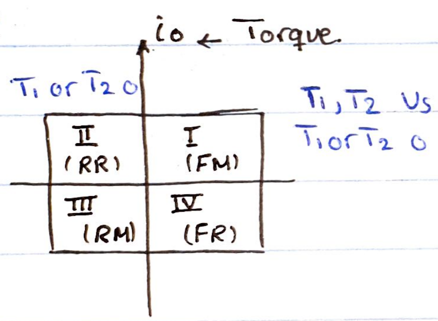
leg B:
- when $-V_c \geq V_t \Rightarrow$ T_3 is ON
  - $V_B = V_s$ — T_2 is OFF
- when $-V_c \leq V_t$ — T_3 is OFF
  - $V_B = 0$. — T_2 is ON



effective switching freq. $= 2f_s \Rightarrow$ lower THD

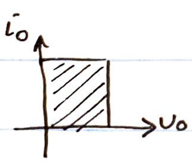
- Classifications of chopper circuits "Buck Converter".

Quadrans of operation (DC machine)

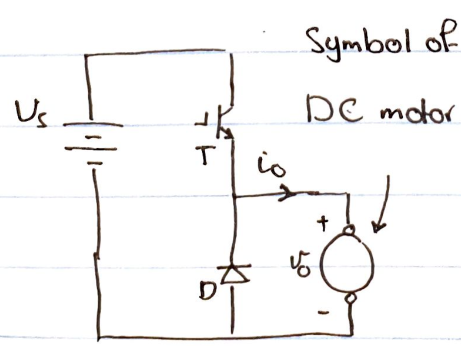


Quadrant	operation.
I	Forward Motoring (FM).
IV	Forward Regeneration (FR).
III	Revers Motoring (RM).
II.	Revers Regeneration (RR).

- Class A.

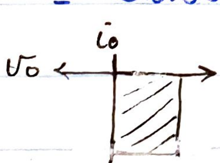


Forward motoring.

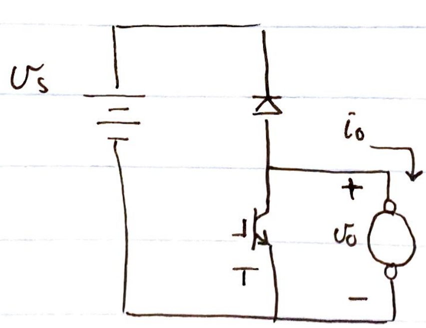


Power Flow: Source $\rightarrow$ load.

- Class B

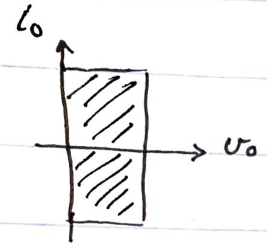


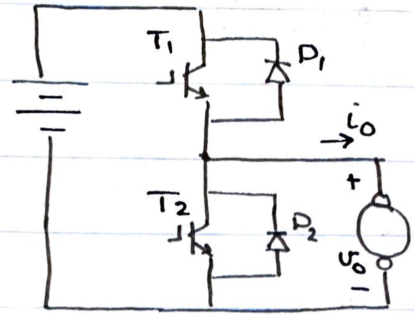
Forward regeneration.



Power Flow: source $\leftarrow$ load

- Class C :





Half_bridge

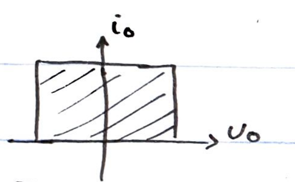
DC. DC Converter.

Power Flow: Source ⇌ Loa

Forward motoring &

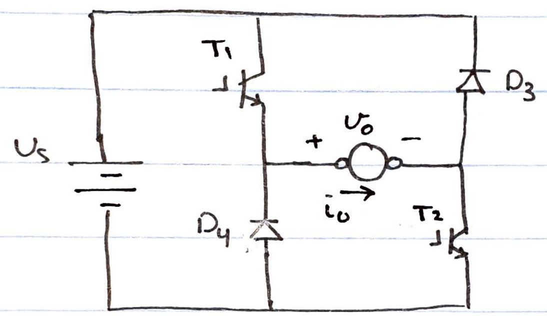
Forward regeneration.

- Class D.



FM

RR



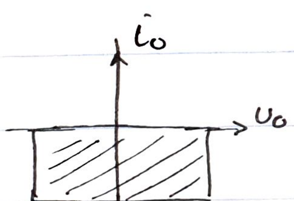
شغل T_2 ← (−) V_o مع (−) V_s

شغل T_1 ← (+) V_o مع (+) V_s

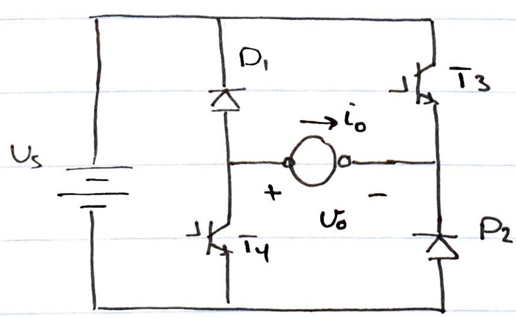
بدي $V_o = 0$ ⇐ بطفي T_1 or T_2

Power Flow: Source ⇌ Load.

- Class E

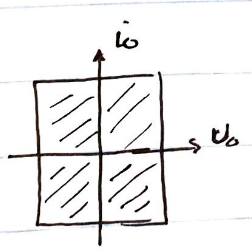


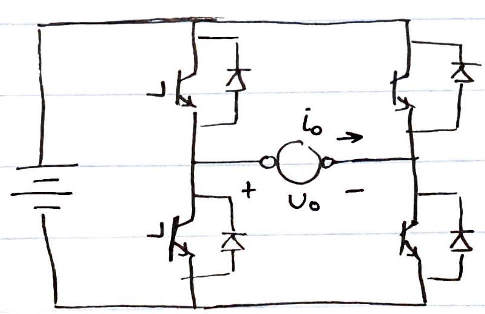
RM & FR



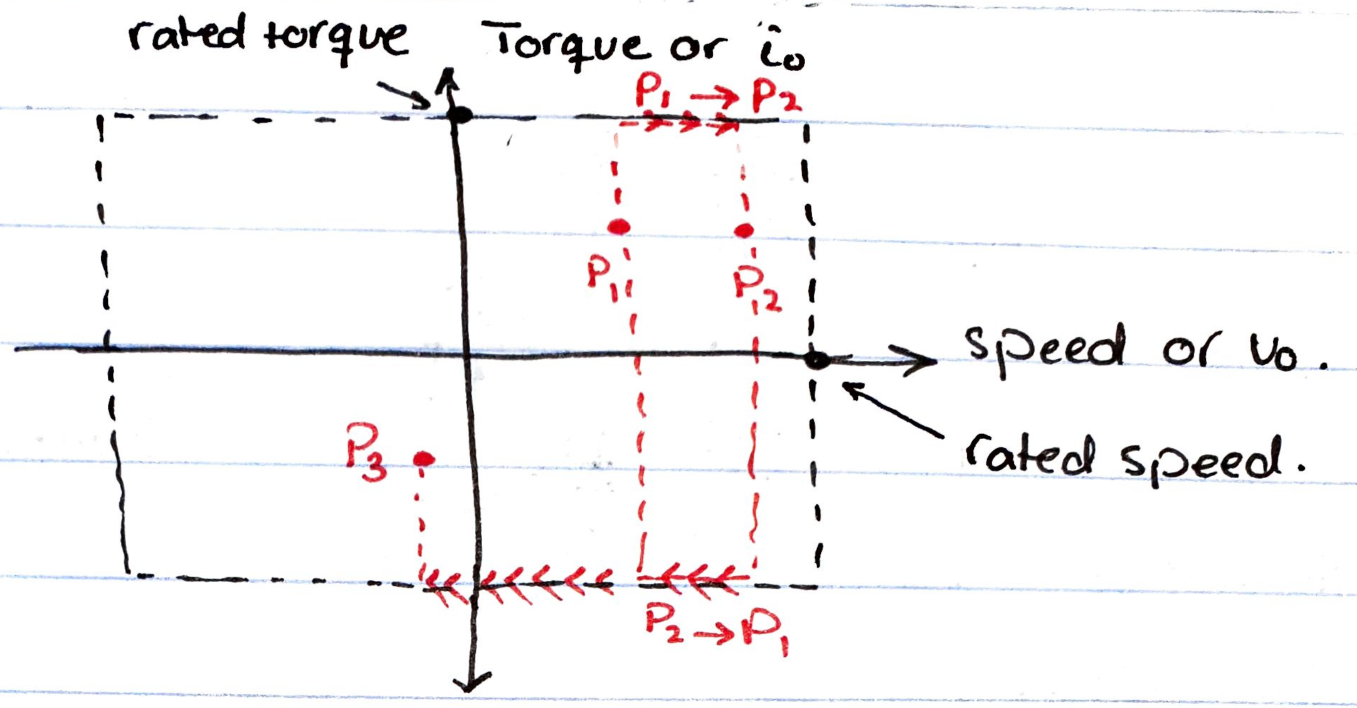
Power Flow: Source ⇌ Load.

- Class F.





\- Example.

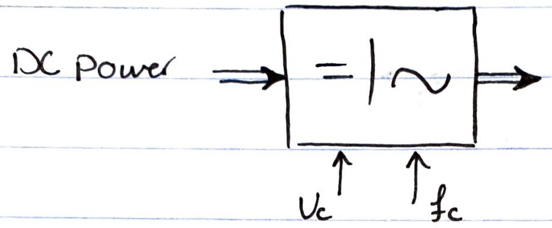


Rated / Nominal — أي اشي مصمم يشتغل على هذه القيم للأبد

Maximum Rated / Nominal — (أقصى قيمة) قيم يشتغل عليها لفترات قصيرة

- Chapter 6: DC_AC Converters "Inverters".

The Function of the inverter is to convert the DC power into AC power with variable voltage & variable frequency (VVVF).



$v_o(t) = v_{o_1} +$ Harmonics.

$v_o(t)$: Output voltage.

v_{o_1} : Fundemental Component.

- Types of inverters:

1. Voltage Source Inverter (VSI).
2. Current Source Inverter (CSI).

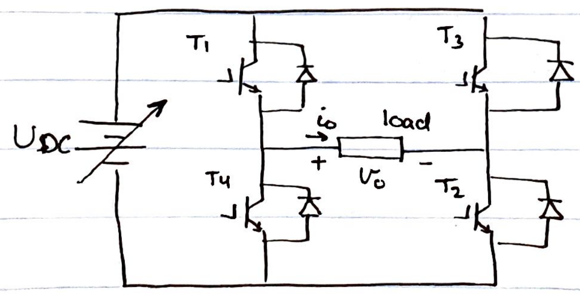
- Applications.

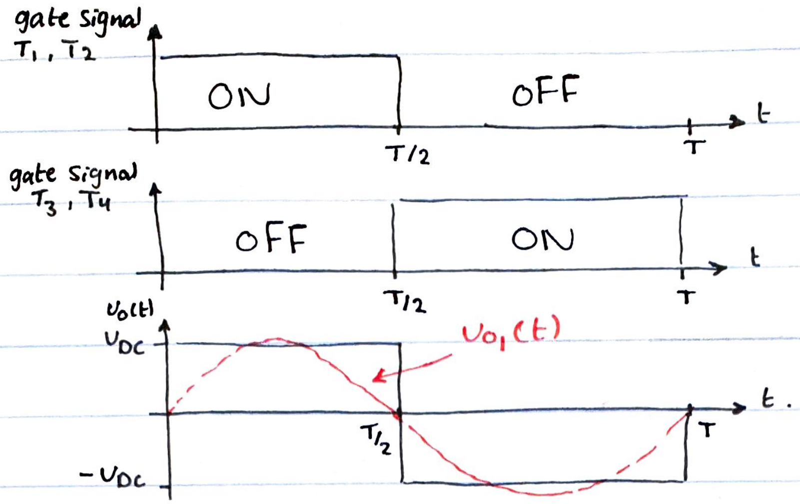
- AC Machine Drive.
- Renewable Energy PV & wide tourbines.

- Modulation Schemes.

a. Square_wave.
b. Sinusoidal Pulse width Modulation (SPWM).
c. Delta or Hysteresis Modulation.
d. Space Vector Modulation (SVM).

1.1 - $1-\phi$ Voltage Source Inverter Full_bridge "square wave".

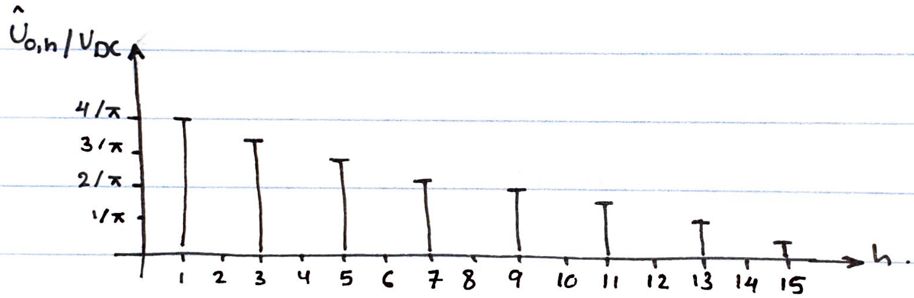




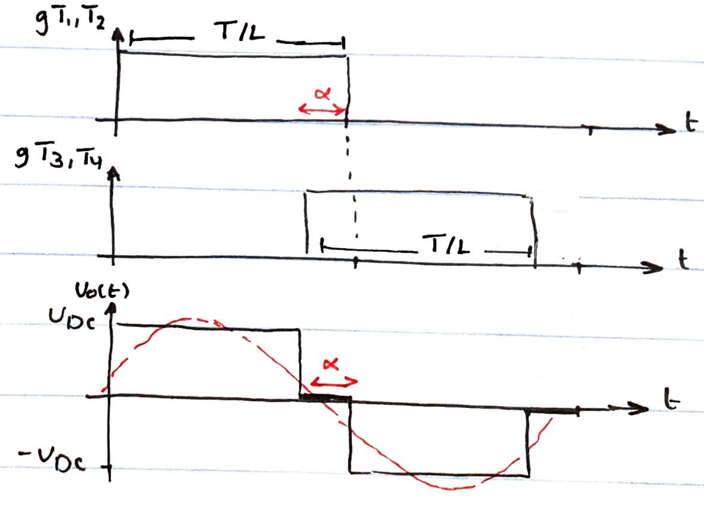
- Using Fourier series, the magnitude of the Fundemental at the output and its harmonics are:

$$\hat{V}_{o,h} = \frac{4}{\pi h} V_{DC}.$$

where ; h is the harmonic number. h = odd (integer).



- Note: The frequency of $V_{o1}(t)$ is controlled by changing ($T = 1/f$) of the gate signal.
- The voltage is controlled by varying V_{DC}. This can be done by using uncontrolled rectifiers with buck converters or controlled rectifie



Single phase VSI Voltage Cancellation Method for square wave mode.

- Using Fourier Series,

$$\hat{V}_{o,h} = \frac{4}{\pi h} V_{DC} \sin(\beta h).$$

$$\beta = 90^\circ - \frac{\alpha}{2}.$$

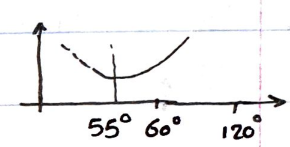
- Note:

1. If $\alpha = 60^\circ \Rightarrow \beta = 60^\circ \Rightarrow \hat{V}_{o,h} = \frac{4}{\pi h} \sin(60 h) \Rightarrow$ No $\underbrace{\text{triplen harmonics}}_{3,9,15,21,27,\ldots}$

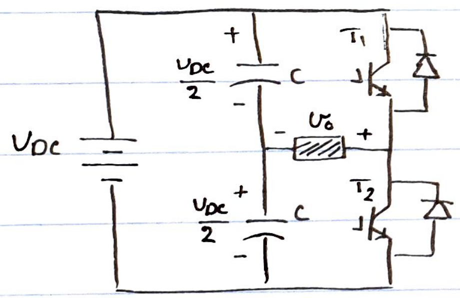
$h = 3 \times$ odd number

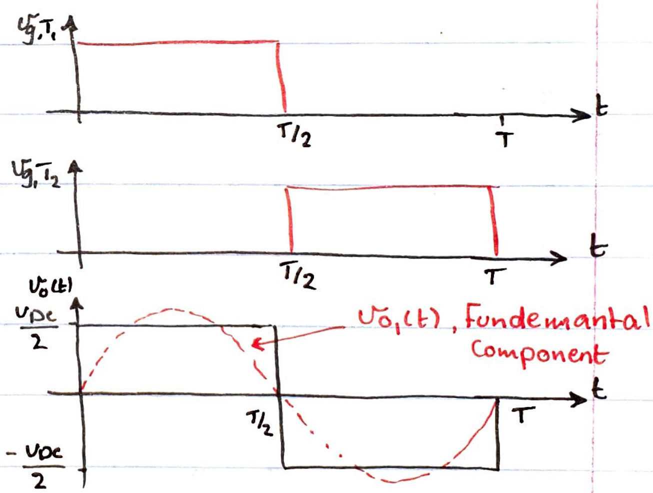
$\hat{V}_{o,3} = 0$, $\hat{V}_{o,9} = 0$, $\hat{V}_{o,15} = 0$, ...

2. THD_{min} is achieved when $\alpha = 55^\circ$



1.2. 1-Φ VSI "Half-bridge" square wave Mode.

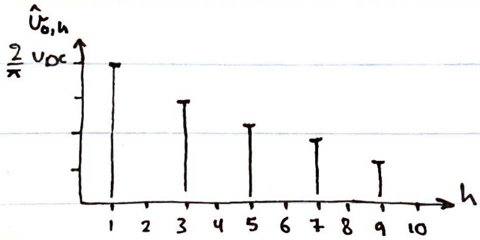




- Using Fourier series, the peak value of the fundemantal of the Output and it's harmonics are:

$$\hat{V}_{o,h} = \frac{4}{\pi} \cdot \frac{V_{DC}}{2} \cdot \frac{1}{h}, \quad \hat{V}_{o,1} = \frac{4}{\pi} \cdot \frac{V_{DC}}{2} = \frac{2}{\pi} V_{DC}.$$

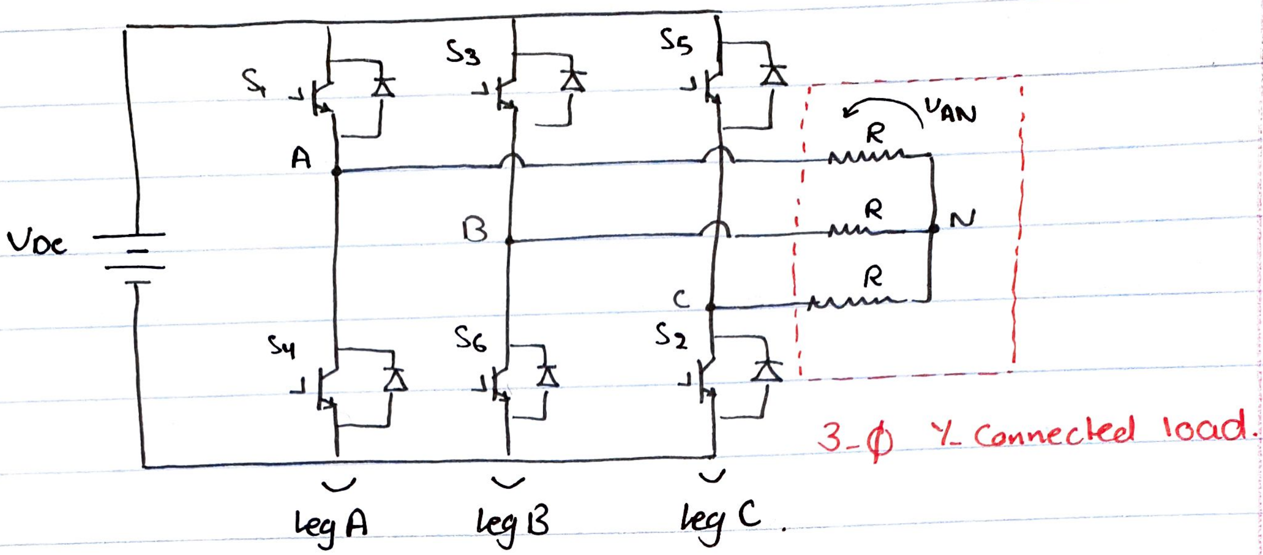
where h is the harmonic order (odd).



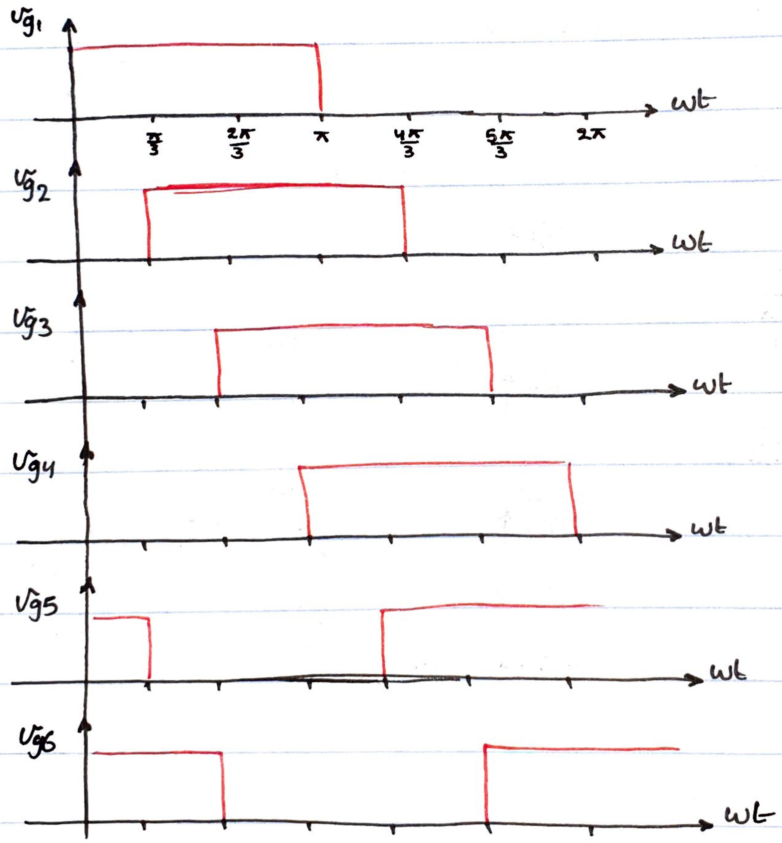
Frequency spectrum.

$$THD_V = \sqrt{\frac{\sum_{h=3,5,\ldots} \hat{V}_{o,h}^2}{\hat{V}_{o,1}^2}}$$

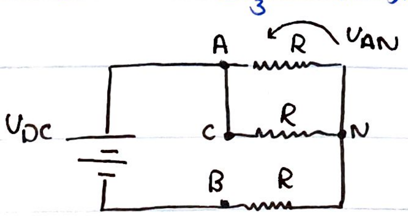
1-3- 3-ϕ VSI "Square wave Mode".



- gate Signal.

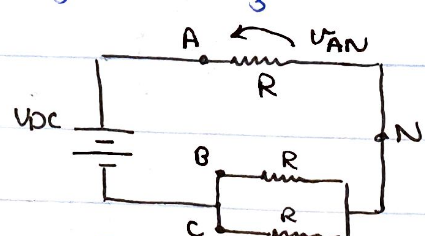


- $0 \le \omega t \le \frac{\pi}{3}$. S_1, S_5, S_6 are ON



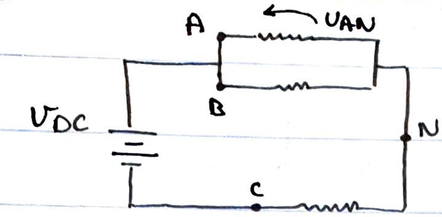
$$V_{AN} = \frac{R/2}{R/2 + R} V_{DC} \Rightarrow V_{AN} = \frac{V_{DC}}{3}.$$

- $\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$, S_1, S_2 & S_6 are ON.



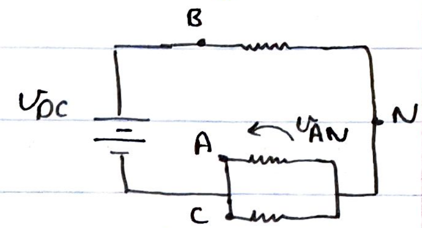
$$V_{AN} = \frac{R}{R + R/2} \cdot V_{DC} \Rightarrow V_{AN} = \frac{2}{3} V_{DC}$$

- $\frac{2}{3}\pi \le \omega t \le \pi$ S_1, S_2 & S_3 are ON



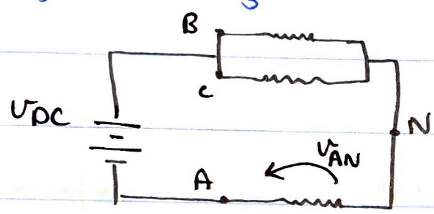
$$V_{AN} = \frac{R/2}{R/2 + R} \cdot V_{DC} \Rightarrow V_{AN} = \frac{V_{DC}}{3}$$

- $\pi \le \omega t \le \frac{4\pi}{3}$, S_2, S_3 & S_4 are ON



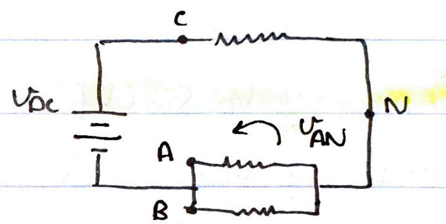
$$V_{AN} = \frac{R/2}{R/2 + R}(-V_{DC}) \Rightarrow V_{AN} = \frac{-V_{DC}}{3}$$

- $\frac{4\pi}{3} \le \omega t \le \frac{5\pi}{3}$, S_3, S_4 & S_5 are ON



$$V_{AN} = \frac{R}{R_{/2} + R}(-V_{DC}) \Rightarrow V_{AN} = \frac{-2}{3} V_{DC}$$

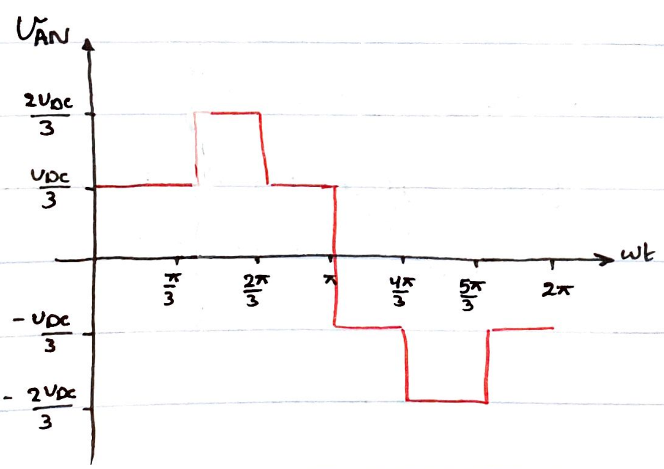
- $\frac{5\pi}{3} \le \omega t \le 2\pi$, S_4, S_5 & S_6 are ON

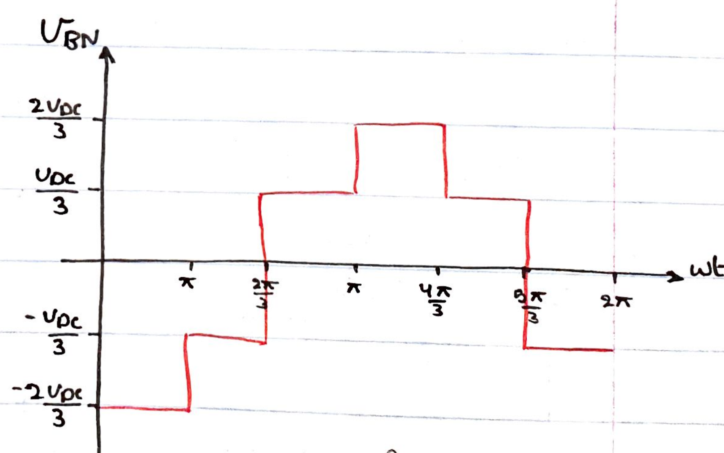


$$V_{AN} = \frac{R/2}{R/2 + R}(-V_{DC}) \Rightarrow V_{AN} = \frac{-1}{3} V_{DC}$$

-

State	Conducting Period	Conducting Devices	V_{AN}
1	$0 \le \omega t \le \frac{\pi}{3}$	S_1, S_5, S_6	$V_{DC}/3$
2	$\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$	S_1, S_2, S_6	$2V_{DC}/3$
3	$\frac{2\pi}{3} \le \omega t \le \pi$	S_1, S_2, S_3	$V_{DC}/3$
4	$\pi \le \omega t \le \frac{4\pi}{3}$	S_2, S_3, S_4	$-V_{DC}/3$
5	$\frac{4\pi}{3} \le \omega t \le \frac{5\pi}{3}$	S_3, S_4, S_5	$-2V_{DC}/3$
6	$\frac{5\pi}{3} \le \omega t \le 2\pi$	S_4, S_5, S_6	$-V_{DC}/3$

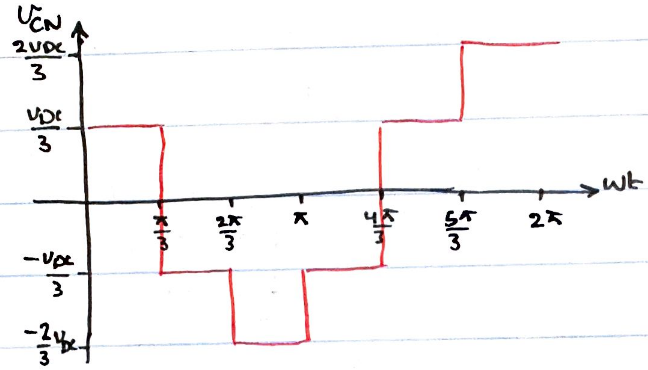


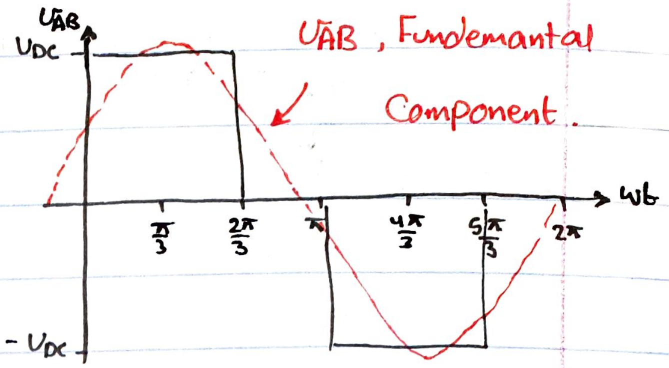


صار shift 60°

وبعدها بنكملها

V_{AB}: Line-to-Line voltage.





- Using Fourier series, the peak value of the fundemantal at line-line output & it's harmonics are:

$\hat{V}_{LL,h} = \frac{4}{\pi} V_{DC} \sin(h\beta)$; $\beta = 90 - \frac{\pi}{2}$, h is odd.

$\hat{V}_{LL,h} = \frac{4}{\pi} V_{DC} \sin(60h)$, α is 60°.

$v_{LL,h} = \frac{\sqrt{3}}{h}\left(\frac{1}{\pi}\frac{V_{DC}}{2}\right)$, $h = 6m \mp 1$

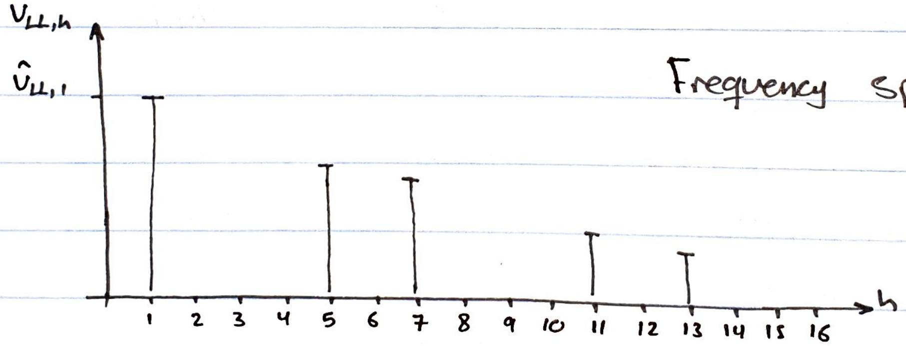
$\hat{V}_{LL,1} = \frac{2\sqrt{3}}{\pi} V_{DC} \Rightarrow V_{LL,1,RMS} = \frac{\hat{V}_{LL,1}}{\sqrt{2}}$

Y 400V 2A 50Hz

$\hat{V}_{LL,1}$: Peak of the main component.

Δ 220V $2\sqrt{3}$A 50Hz.

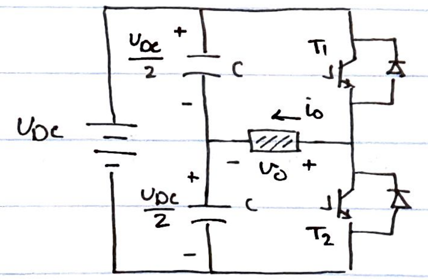
$\hat{V}_{LL,1,RMS} = \frac{\sqrt{6}}{\pi} V_{DC}$.

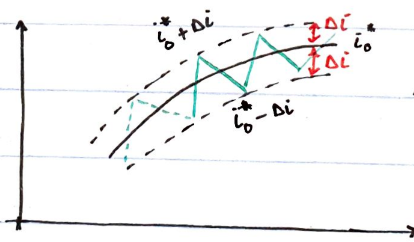


Frequency spectrum.

2. Hysteresis or Delta Modulation.

"Instantenous Current Controller".



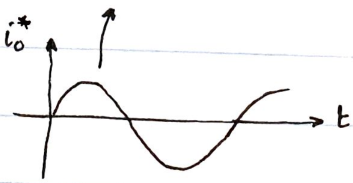


Δi: Hysteresis band.

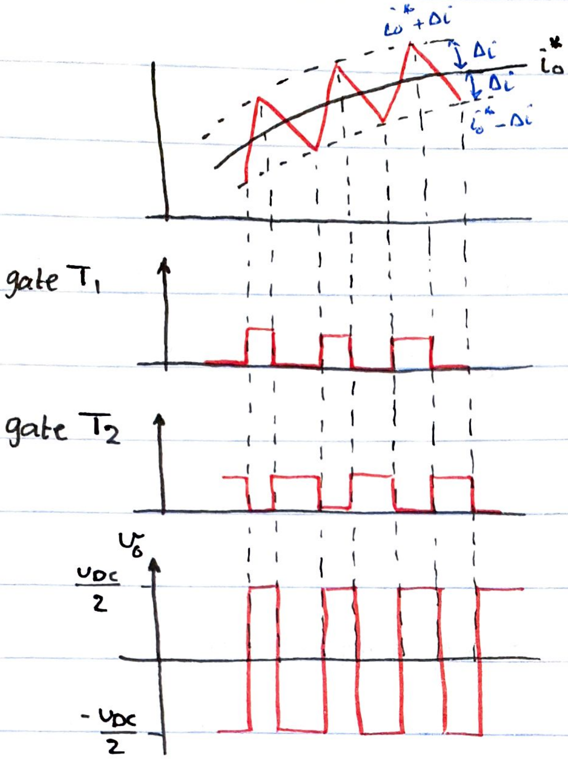
Δi = (2-5) % rated current

reference current i_o^*

Actual current i_o



$i_o^* = I_m^* \sin(\omega t + \theta_c)$



Δi : Hysteresis Band,

$\Delta i = (2-5)\%$ rated current.

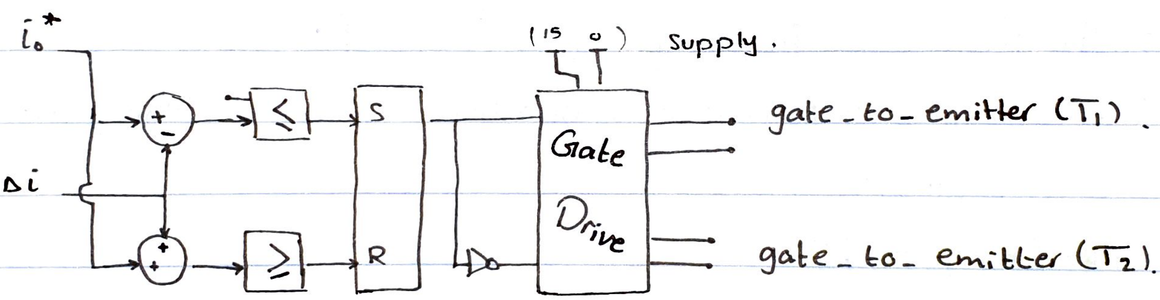
- Switching Function.

IF $i_o < i_o^* - \Delta i$ set $v_o = \frac{V_{DC}}{2}$, T_1 is ON.

IF $i_o \geq i_o^* - \Delta i$ Reset $v_o = -\frac{V_{DC}}{2}$, T_2 is ON.

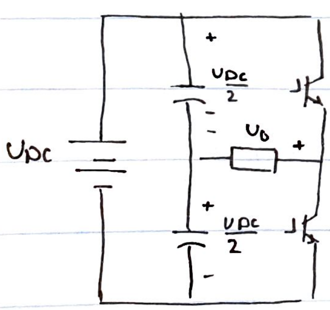
- Implementation of hystresis Current Controller

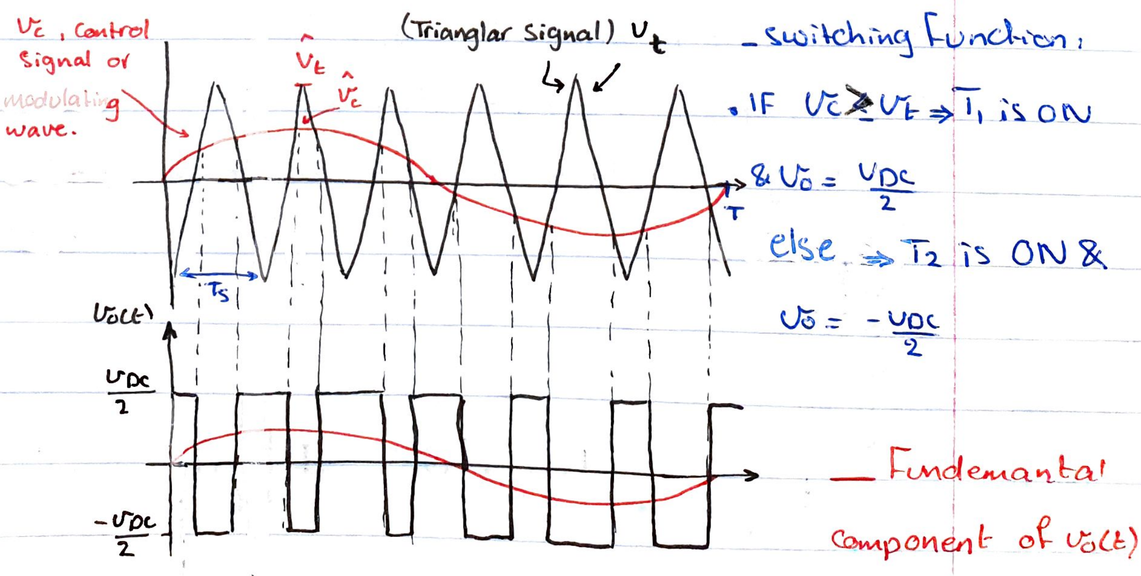
- Implementation of hystresis current Controller.



3. Sinusoidal pulse Width Modulation (SPWM).

3.1. 1-ϕ Half-bridge VSI (SPWM).





- switching function:

- IF $V_c \geq V_t \Rightarrow T_1$ is ON & $v_o = \frac{V_{DC}}{2}$

else $\Rightarrow T_2$ is ON & $v_o = -\frac{V_{DC}}{2}$

- The output voltage is given by (using Fourier series).

$v_o(t) = v_{o,1}(t) + \text{Harmonics}$.

$v_{o,1}(t) = M \frac{V_{DC}}{2}$.

where; M is called the modulation index and it is defined as the ratio between the peak value of V_t, $\hat{V}_t$ and the peak value of the modulating wave, $\hat{V}_c$.

$$M = \frac{\hat{V}_c}{\hat{V}_t}$$

T_s: Switching Period. $f_s = \frac{1}{T_s}$: Switching frequency or carrier frequency.

T: period of modulating wave.

$\omega_1 = 2\pi f_1 = 2\pi / T$, f_1: Frequency of modulating wave.

- Frequency modulating; m_f: It is the ratio of switching frequency to the fundemantal frequency, $m_f = f_s / f_1$. [Integer].

- The harmonics of $v_o(t)$ appear of sidebands of switching frequency and its multiples.

$$f_h = (n m_f \mp k) f_1$$

n & m are integer; For even values of k, n is odd.

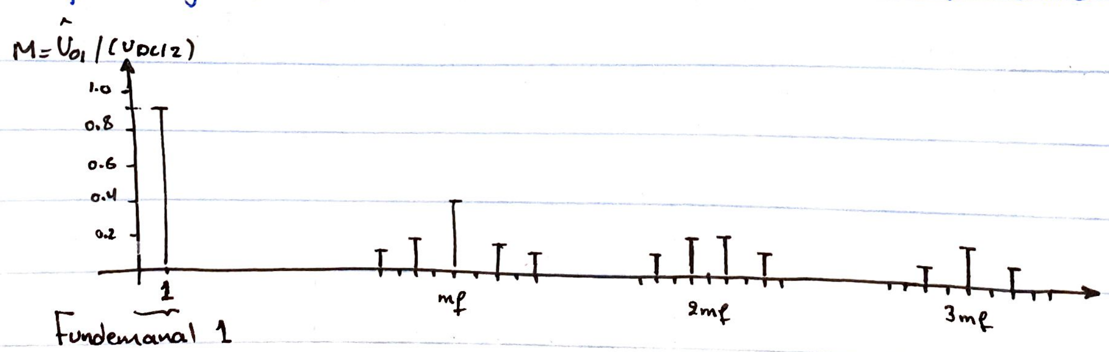
For odd values of k, n is even.

m_f, $m_f \mp 2$, $m_f \mp 4$, $m_f \mp 6$, ...

$2m_f \mp 1$, $2m_f \mp 3$, $2m_f \mp 5$, $2m_f \mp 7$, ...

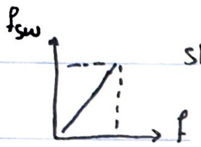
$3m_f$, $3m_f \mp 2$, $3m_f \mp 4$, $3m_f \mp 6$, ...

- m_f: Integer, & odd number to elemenate the even harmonics.



- Notes:

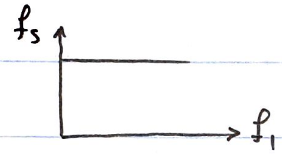
1. m_f should be odd integer to eleminate the even harmonics from $v_o(t)$.
2. When m_f is small ($m_f \leq 21$), the switching frequency must be changed with load frequency, f_1, to keep m_f constant ($m_f = f_s/f_1$ = constant).



slop = $m_f = \frac{f_s}{f_1}$

Synchronous SPWM

3. For large value of m_f ($m_f \geq 21$) the synchronous SPWM is not important.



4. If m_f is high number, the THD is small. However, the switching losses of the converter will become high (efficiency ↓).

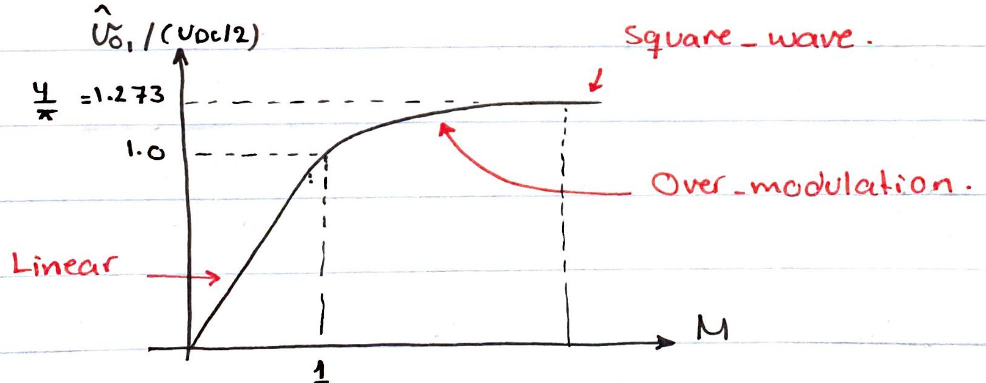
- Over-modulation:

When $M \leq 1$, the relation between $\hat{V}_{o1}$ & $V_{DC}/2$ is linear relationship.

$\hat{V}_{o1} = M \frac{V_{DC}}{2}$.

When $1 \leq M \leq 3.24$, the THD of $v_o(t)$ is high. The region is called overmodulation region.

When $M \geq 3.24$, we will get square wave output voltage.

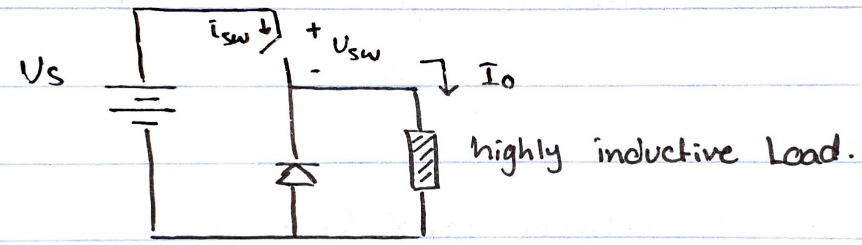


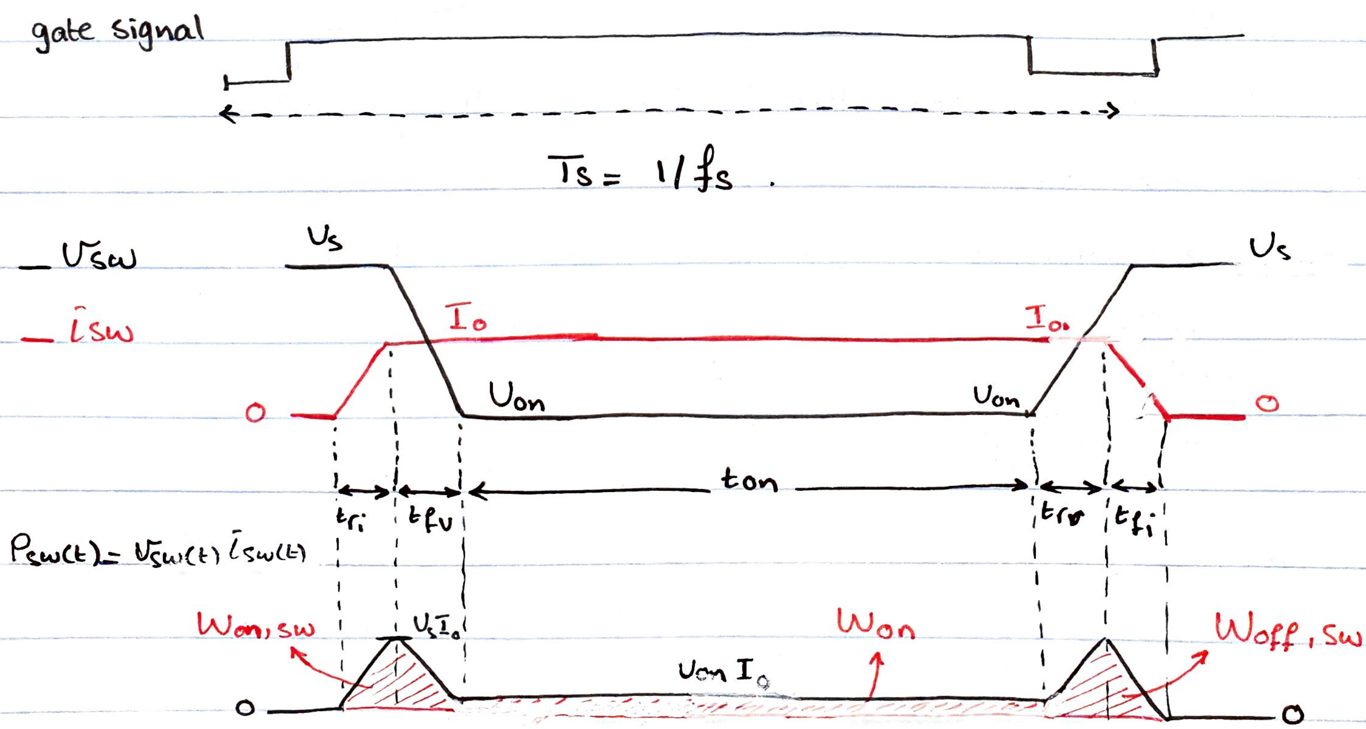
- Switching and Conduction in a Controllable switch.

- Real devices do not have ideal Charactaristic and dissipate Power when used.
- Too much power dissipated may lead to the destruction of the device.
- Sources of power dissipation.

- Switching losses — Conduction Losses.

- Power dissipation in a Chopper switch with inductive load.





- $t_{sw,on} = t_{ri} + t_{fv}$ ↓ Crossover turn-on time , $t_{sw,off} = t_{rv} + t_{fi}$ ↓ Crossover turn-off time.

$W_{on,sw} = \frac{1}{2} t_{sw,on} U_s I_o$.

$W_{off,sw} = \frac{1}{2} t_{sw,off} U_s I_o$.

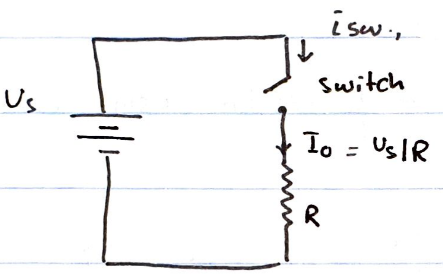
W_{sw} (Switching energy losses) $= \frac{1}{2} U_s I_o (t_{sw,on}\ t_{sw,off})$

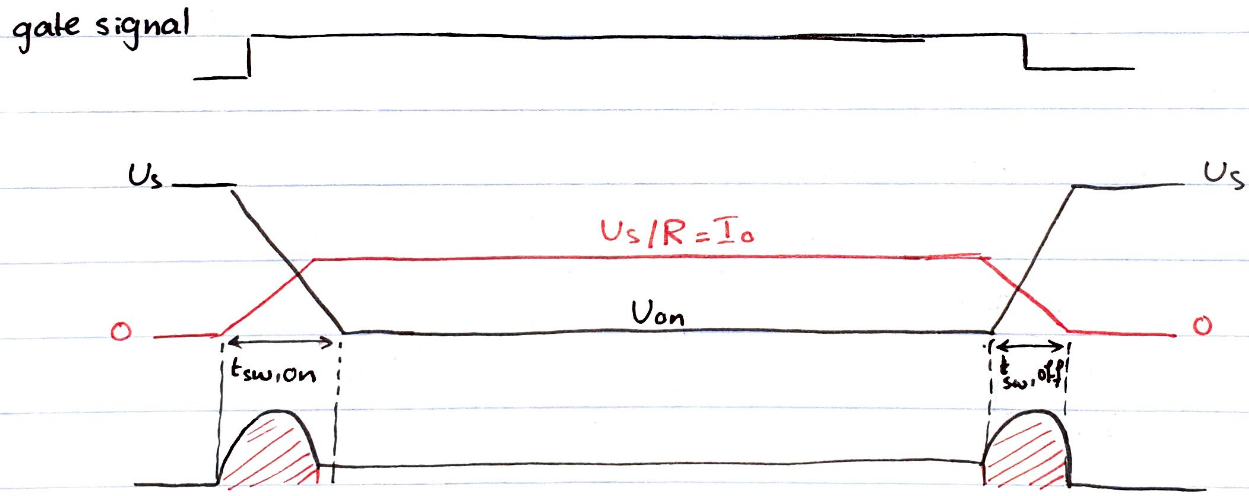
W_{on} (Conduction energy losses) $= V_{on} I_o t_{on}$

P_{on} (Conduction power losses) $= V_{on} I_o t_{on} f_s$.

P_{sw} (Switching power losses) $= \frac{W_{sw}}{T_s} \doteq \frac{1}{2} U_s I_o f_s (t_{sw,on} + t_{sw,off})$.

- Power dissipation in a chopper switch with resistive load.





$0 \leq t \leq t_{sw,on}$

$i_{sw} = I_o \frac{t}{t_{sw,on}}$, $V_{sw} = U_s \left(1 - \frac{t}{t_{sw,on}}\right)$

$$W_{on,sw} = \int_0^{t_{sw,on}} V_{sw}\, i_{sw}\, dt.$$

$$W_{on,sw} = \frac{I_o U_s}{t_{sw,on}} \left(\frac{t_{sw,on}^2}{2} - \frac{t_{sw,on}^2}{3} \right) = I_o U_s \left(\frac{t_{sw,on}}{2} - \frac{t_{sw,on}}{3} \right)$$

$$W_{on,sw} = \frac{1}{6} U_s I_o t_{sw,on}.$$

- $0 \le t \le t_{sw,off}$.

$$i_{sw} = I_o \left(1 - \frac{t}{t_{sw,off}}\right)$$

$$v_{sw} = V_s \frac{t}{t_{sw,off}}$$

$$W_{off,sw} = \int_0^{t_{sw,off}} v_{sw}\, i_{sw}\, dt.$$

$$W_{off,sw} = \frac{1}{6} V_s I_o t_{sw,off}.$$

- $$W_{sw} = W_{sw,on} + W_{sw,off} = \frac{1}{6} V_s I_o (t_{sw,on} + t_{sw,off})$$

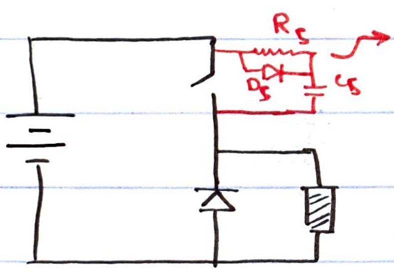
- $$P_{sw} = \frac{W_{sw}}{T_s} = \frac{1}{6} V_s I_o f_s (t_{sw,on} + t_{sw,off}) = \frac{1}{3} P_{sw}\Big|_{\text{inductive load}}.$$

- Note: Switching losses can be reduced using snubber circuit.

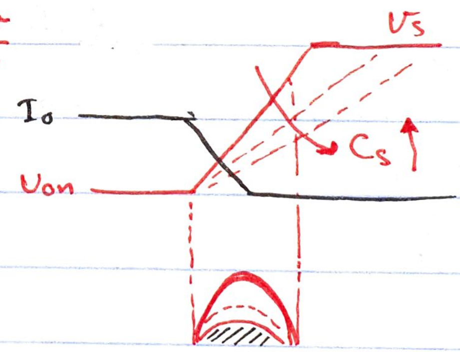
• Turn-off snubber circuit.

$i = C_s \frac{dv}{dt}$

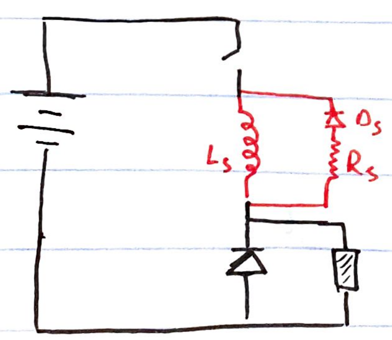
$\tau \ll T_s$

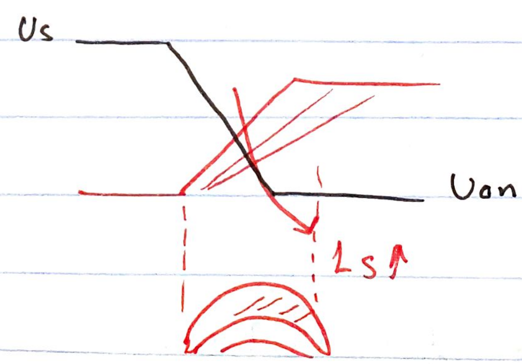


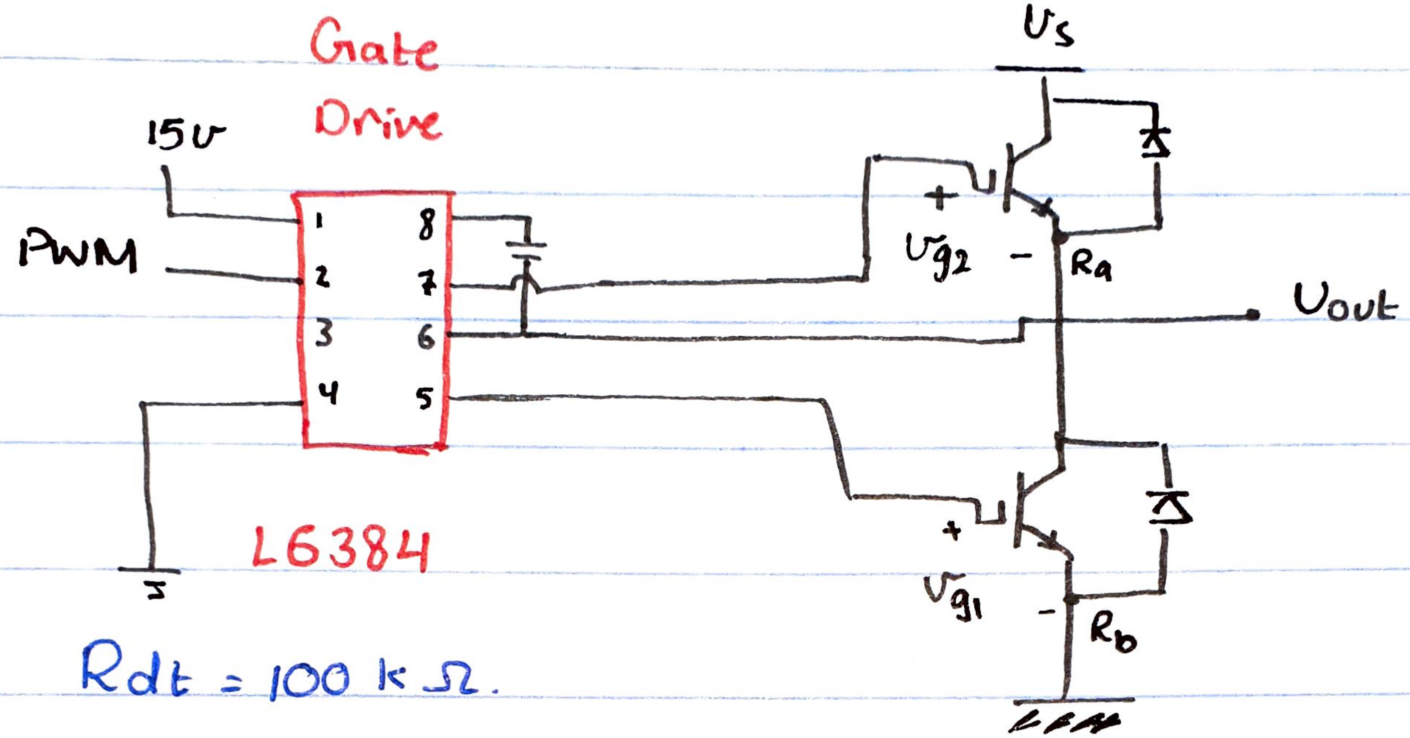
→ To reduce $\frac{dv}{dt}$



• Turn-on snubber circuit.







$R_{dt} = 100\ k\Omega$.

Dead time $= 1\ \mu sec$.

- Function of gate drive.

Level shiffting.

Dead time.

Voltage amplification.