
Part IV AC Voltage Controllers

An AC Voltage Controller provides an adjustable rms value of the output voltage. In some AC Voltage Controller configurations, the output frequency could also be controlled.

The output of these controllers consists of a fundamental component of the output voltage plus other (undesirable) AC components at higher frequencies called **harmonics**.

Types of AC Voltage Controllers:

- 1) Single Phase Controllers
- 2) Three Phase Controllers

Two main Types of Control can be applied to AC Controllers:

- I) On-Off Control
- II) Phase Angle Control

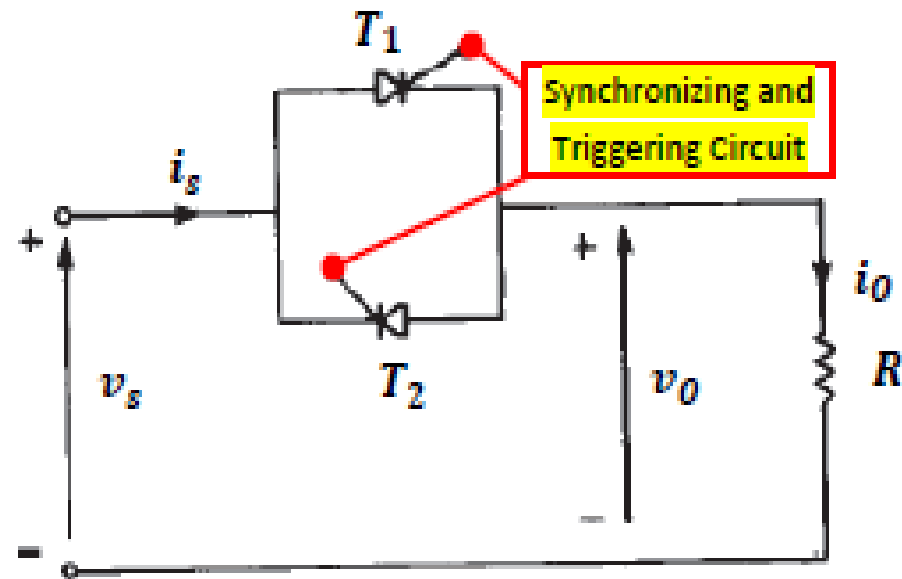
The latter type can be classified as:

- a) Unidirectional or Half-Wave Control
- b) Bidirectional or Full-Wave Control

1) Principle of On-Off Control

- A Single Phase AC Voltage Controller implementing On-Off Control is shown in the Figure below.
- The Thyristor switch is "on" for a particular time t_n and is "off" for another time t_m .

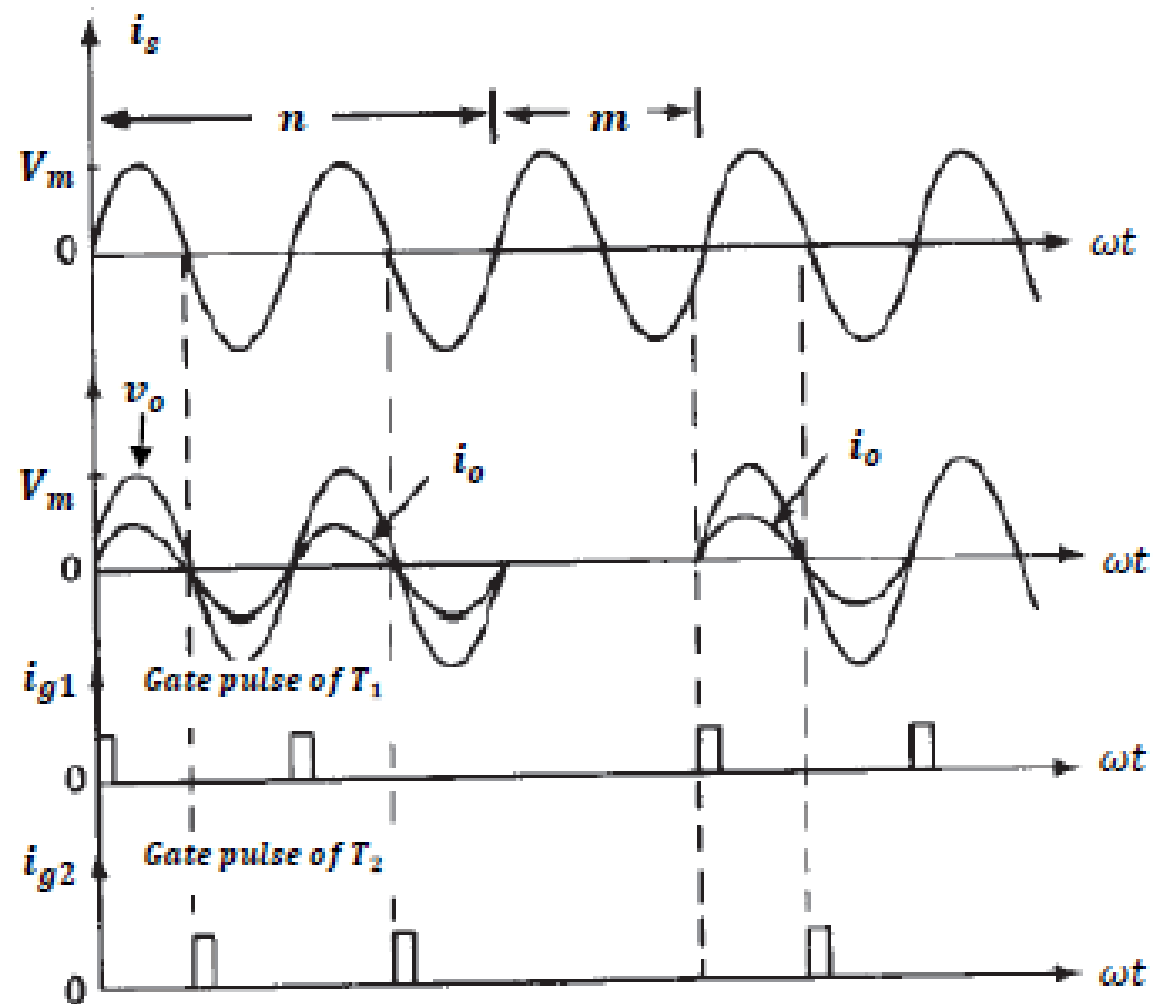
- The Thyristors T_1 and T_2 could be a TRIAC if the latter has the required ratings!



✦ The Synchronizing and Triggering Circuit synchronizes the Thyristors' gate signals with the supply voltage and applies an appropriate pulse of gate current and gate-cathode voltage for a proper turn-on of a particular SCR.

✦ The associated waveforms of this method are shown in the Figure below.

✦ "n" is an integral number of "on" cycles, whilst m is an integral number of "off" cycles.



- Since the load is connected to the supply for “ n ” cycles, and is disconnected for “ m ” cycles, then the rms value of the output voltage is:

$$V_{Orms} = \sqrt{\frac{n}{(n+m)2\pi} \int_0^{2\pi} (V_m \sin \omega t)^2 d\omega t}$$

$$V_{Orms} = \sqrt{\frac{n}{n+m} \sqrt{\frac{(V_m)^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\omega t) d\omega t}}$$

...

$$V_{Orms} = \sqrt{\frac{n}{n+m}} V_s$$

$$V_{Orms} = \sqrt{k} V_s$$

where, $k = \frac{n}{n+m}$, and is called the duty cycle, and V_s is the rms value of the input voltage.

- This type of control is used for applications (loads) with a high mechanical inertia or/and a high thermal time constant; e.g. industrial heating and speed control of motors.

- The output power for the resistive load is:

$$P_o = V_{Orms} I_{Orms} = \sqrt{k} V_s I_{Orms} = R I_{Orms}^2$$

where $I_{Orms} = \frac{V_{Orms}}{R}$, is the rms vale of the output current and equals the input current!

- Assuming lossless controller, then the input power is:

$$P_{in} = P_o = \sqrt{k} V_s I_{Orms}$$

- The input apparent power is:

$$S_{in} = V_s I_{in} = V_s I_{Orms}$$

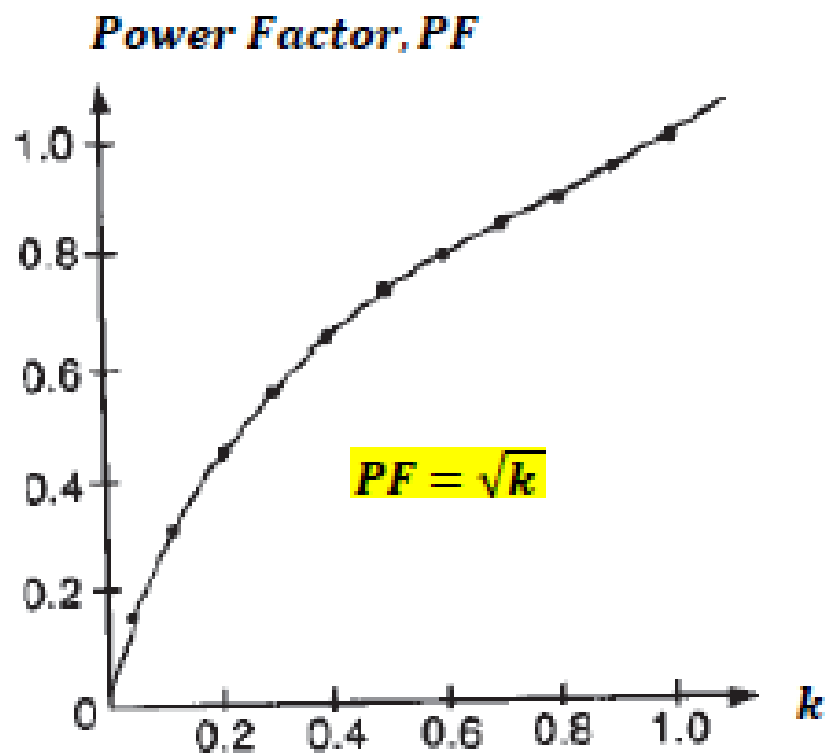
Therefore, the input power Factor is:

$$PF = \frac{P_{in}}{S_{in}} = \frac{\sqrt{k} V_s I_{Orms}}{V_s I_{Orms}} = \sqrt{k} \quad (\text{lagging})$$

$$\Rightarrow PF = \sqrt{\frac{n}{n+m}} \quad (\text{lagging})$$

The input power factor, as a function of the duty cycle, is depicted in the Figure below.

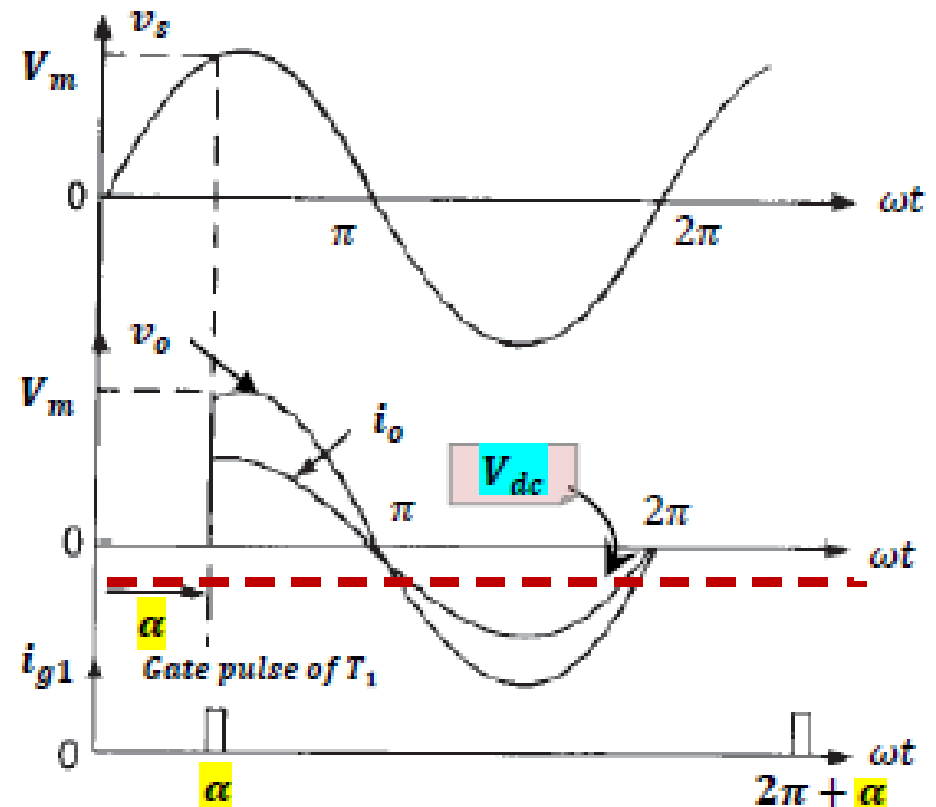
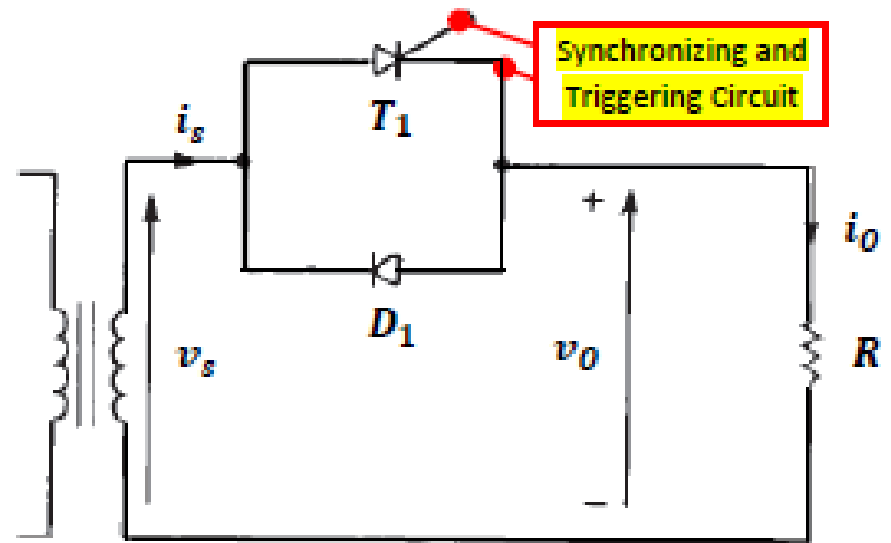
Note that, the input power factor is very low at small duty cycles!



II) Principle of Phase Angle Control

II. 1) Single-Phase Half-Wave Controller

- It is also known as a Unidirectional Controller.
- The circuit topology of a Single-Phase Half-Wave Controller is shown in the Figure next.
- The associated voltage and current waveforms of this controller are shown in the Figure next.



- The **root-mean-square** of the output voltage is:

$$V_{Orms} = \sqrt{\frac{1}{2\pi} \left(\int_{\alpha}^{\pi} (\sqrt{2}V_s \sin \omega t)^2 d\omega t + \int_{\pi}^{2\pi} (\sqrt{2}V_s \sin \omega t)^2 d\omega t \right)}$$

.....

$$V_{Orms} = V_s \sqrt{\frac{1}{2\pi} \left(2\pi - \alpha + \frac{\sin 2\alpha}{2} \right)}; \quad 0 < \alpha < \pi$$

- The **average output voltage** is:

$$V_{dc} = \frac{1}{2\pi} \left(\int_{\alpha}^{\pi} \sqrt{2}V_s \sin \omega t d\omega t + \int_{\pi}^{2\pi} \sqrt{2}V_s \sin \omega t d\omega t \right)$$

...

$$V_{dc} = \frac{\sqrt{2}V_s}{2\pi} (\cos \alpha - 1); \quad 0 < \alpha < \pi$$

Note that, V_{Orms} varies from V_s to $0.707V_s$ by varying α from 0 to π ,

whilst V_{dc} can be varied from 0V to $-\frac{\sqrt{2}V_s}{\pi}$ by varying α from 0 to π .

- The DC current may cause saturation problem to transformer core, hence this method is rarely used.

Example 6-2

A single-phase ac voltage controller in Fig. 6-2a has a resistive load of $R = 10 \Omega$ and the input voltage is $V_s = 120 \text{ V}$, 60 Hz. The delay angle of thyristor T_1 is $\alpha = \pi/2$. Determine the (a) rms value of output voltage, V_o ; (b) input power factor, PF; and (c) average input current.

Solution $R = 10 \Omega$, $V_s = 120 \text{ V}$, $\alpha = \pi/2$, and $V_m = \sqrt{2} \times 120 = 169.7 \text{ V}$.

(a) From Eq. (6-5), the rms value of the output voltage,

$$V_o = 120 \sqrt{\frac{3}{4}} = 103.92 \text{ V}$$

(b) The rms load current,

$$I_o = \frac{V_o}{R} = \frac{103.92}{10} = 10.392 \text{ A}$$

The load power,

$$P_o = I_o^2 R = 10.392^2 \times 10 = 1079.94 \text{ W}$$

Since the input current is the same as the load current, the input volt-ampere rating is

$$\text{VA} = V_s I_s = V_s I_o = 120 \times 10.392 = 1247.04 \text{ VA}$$

The input power factor,

$$\begin{aligned} \text{PF} &= \frac{P_o}{\text{VA}} = \frac{V_o}{V_s} = \left[\frac{1}{2\pi} \left(2\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2} \\ &= \sqrt{\frac{3}{4}} = \frac{1079.94}{1247.04} = 0.866 \text{ (lagging)} \end{aligned} \quad (6-7)$$

(c) From Eq. (6-6), the average output voltage,

$$V_{dc} = -120 \times \frac{\sqrt{2}}{2\pi} = -27 \text{ V}$$

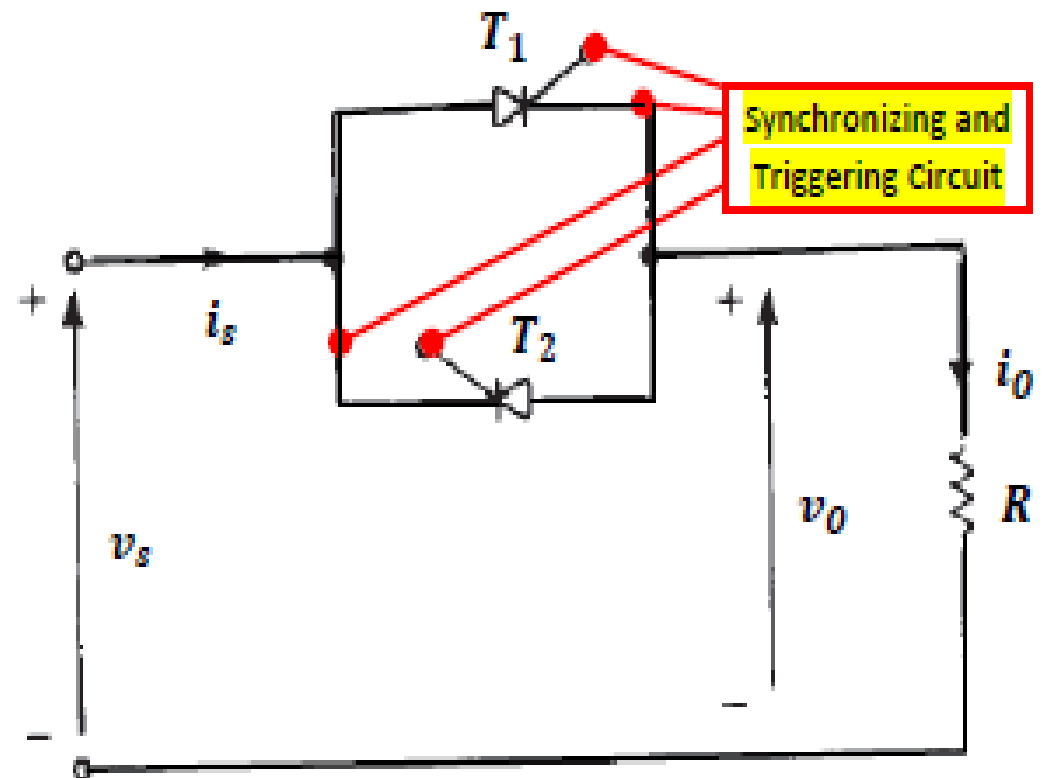
and the average input current,

$$I_D = \frac{V_{dc}}{R} = -\frac{27}{10} = -2.7 \text{ A}$$

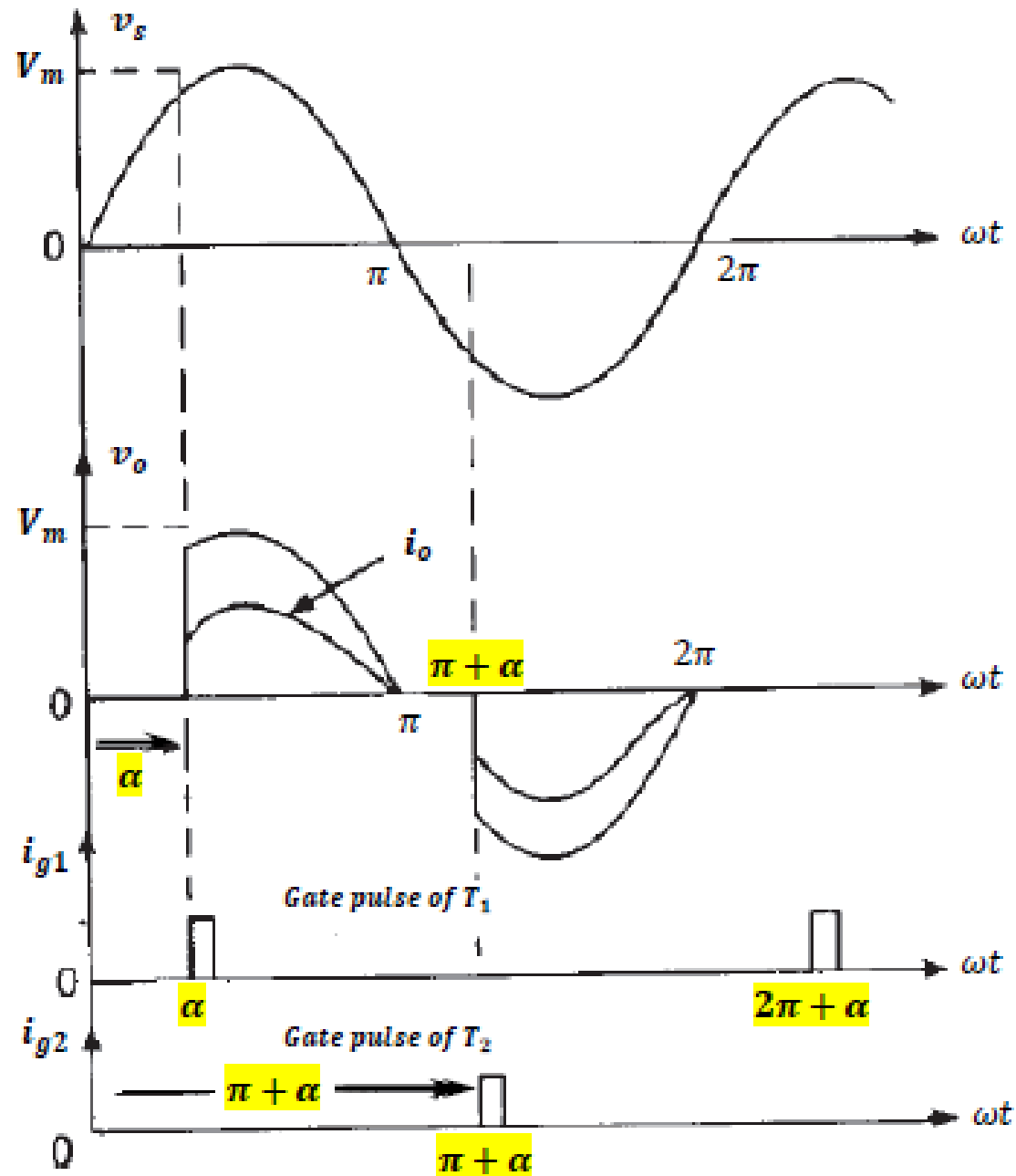
Note. The negative sign of I_D signifies that the input current during the positive half-cycle is less than that during the negative half-cycle. If there is an input transformer, the transformer core may be saturated. The unidirectional control is not normally used in practice.

II. 2) Single-Phase Full-Wave Controller

- ❖ It is known as a Bidirectional Controller
- ❖ The circuit topology of a Single-Phase Full-Wave Controller is shown in the Figure next.
- ❖ T_1 controls the power flow during the positive half-cycle, whilst T_2 controls the power flow during the negative half-cycle.



❖ The associated voltage and current waveforms of this controller are shown in the Figure below.



- ❖ The **root-mean-square** of the output voltage is:

$$V_{Orms} = \sqrt{\frac{2}{2\pi} \int_{\alpha}^{\pi} (\sqrt{2}V_s \sin \omega t)^2 d\omega t}$$

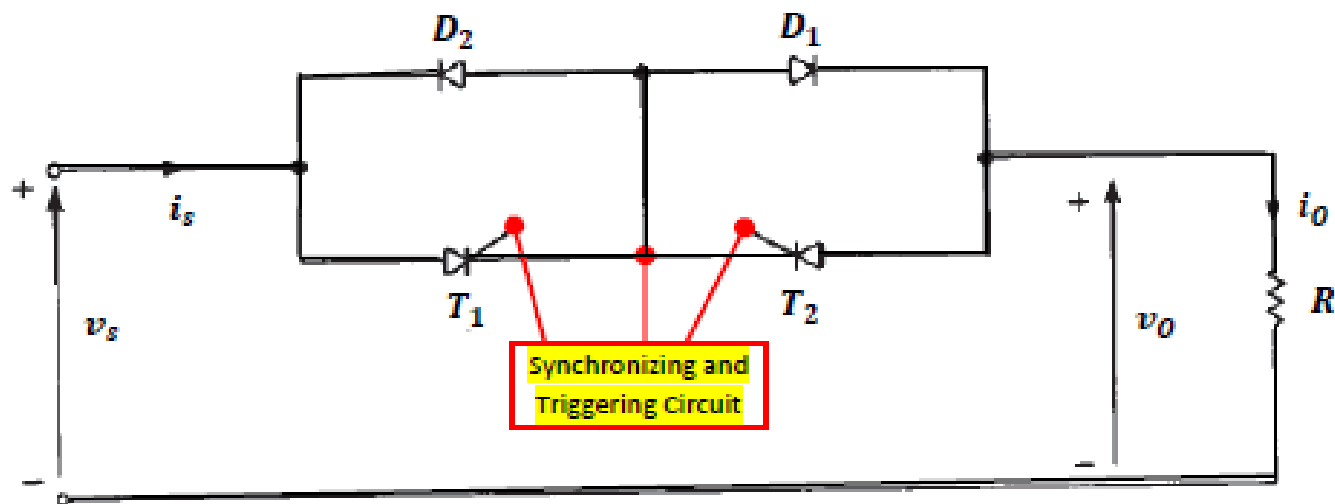
$$V_{Orms} = \sqrt{\frac{4V_s^2}{4\pi} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) d\omega t}$$

....

$$V_{Orms} = V_s \sqrt{\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right)}; \quad 0 < \alpha < \pi$$

Note that, V_{Orms} varies from V_s to 0 by varying α from 0 to π ; a wider range of control!

- ❖ Because of the waveform symmetry in positive and half cycles, the **average output voltage** is zero. Therefore, there is no DC saturation problem to transformer core.
- ❖ Note that, the gate-circuits for T_1 and T_2 must be isolated; two isolations are required!
- ❖ By the connection below, the Thyristors have a common cathode; only one isolation is required!
- ❖ However, the conduction losses are increased, as there are two conducting devices at the same time!



Example 6-3

A single-phase full-wave ac voltage controller in Fig. 6-3a has a resistive load of $R = 10 \Omega$ and the input voltage is $V_s = 120 \text{ V}$, 60 Hz. The delay angles of thyristors T_1 and T_2 are equal: $\alpha_1 = \alpha_2 = \alpha = \pi/2$. Determine the (a) rms output voltage, V_o ; (b) input power factor, PF; (c) average current of thyristors, I_A ; and (d) rms current of thyristors, I_R .

Solution $R = 10 \Omega$, $V_s = 120 \text{ V}$, $\alpha = \pi/2$, and $V_m = \sqrt{2} \times 120 = 169.7 \text{ V}$.

(a) From Eq. (6-8), the rms output voltage,

$$V_o = \frac{120}{\sqrt{2}} = 84.85 \text{ V}$$

(b) The rms value of load current, $I_o = V_o/R = 84.85/10 = 8.485 \text{ A}$ and the load power, $P_o = I_o^2 R = 8.485^2 \times 10 = 719.95 \text{ W}$. Since the input current is the same as the load current, the input volt-ampere rating,

$$\text{VA} = V_s I_s = V_s I_o = 120 \times 8.485 = 1018.2 \text{ W}$$

The input power factor,

$$\begin{aligned} \text{PF} &= \frac{P_o}{\text{VA}} = \frac{V_o}{V_s} = \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2} \\ &= \frac{1}{\sqrt{2}} = \frac{719.95}{1018.2} = 0.707 \text{ (lagging)} \end{aligned} \quad (6-9)$$

(c) The average thyristor current,

$$\begin{aligned} I_A &= \frac{1}{2\pi R} \int_{\alpha}^{\pi} \sqrt{2} V_s \sin \omega t \, d(\omega t) \\ &= \frac{\sqrt{2} V_s}{2\pi R} (\cos \alpha + 1) \\ &= \sqrt{2} \times \frac{120}{2\pi \times 10} = 2.7 \text{ A} \end{aligned} \tag{6-10}$$

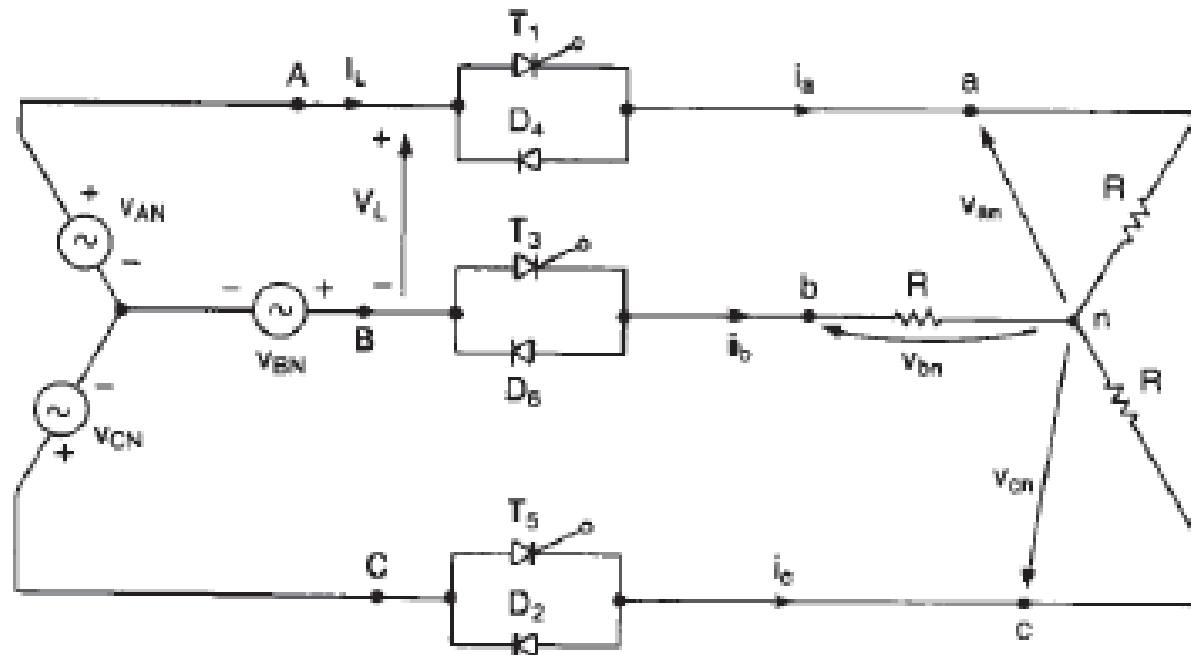
(d) The rms value of the thyristor current,

$$\begin{aligned} I_R &= \left[\frac{1}{2\pi R^2} \int_{\alpha}^{\pi} 2V_s^2 \sin^2 \omega t \, d(\omega t) \right]^{1/2} \\ &= \left[\frac{2V_s^2}{4\pi R^2} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) \, d(\omega t) \right]^{1/2} \\ &= \frac{V_s}{\sqrt{2} R} \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2} \\ &= \frac{120}{2 \times 10} = 6 \text{ A} \end{aligned} \tag{6-11}$$

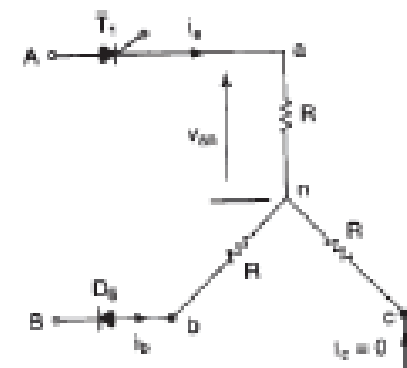
Three Phase Controllers

A- Three-Phase Half-Wave (Unidirectional) Controllers

➤ The circuit topology for Unidirectional Three-Phase Controller is shown in the Figure below.



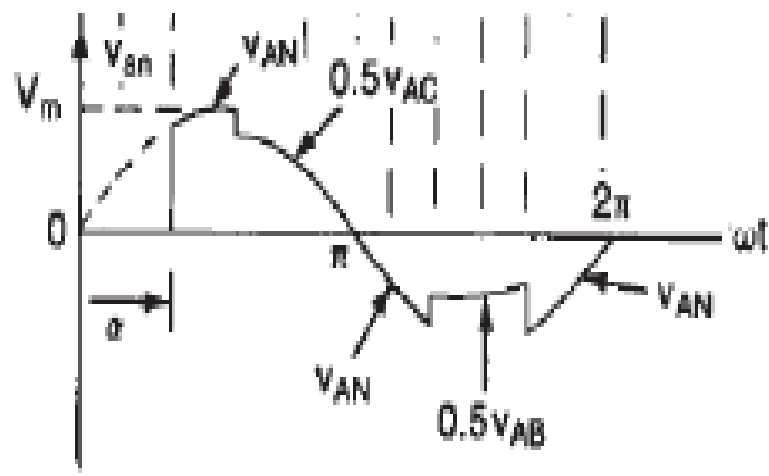
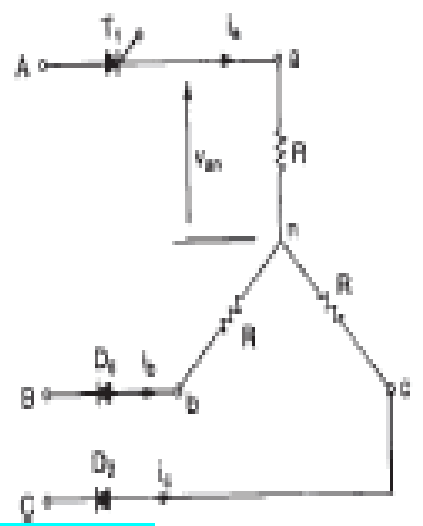
- At least one SCR must be conducting to allow the power flow to the load.
- The power flow to the load is controlled by T_1 , T_3 and T_5 . The diodes D_2 , D_4 and D_6 provide the return current path.
- Depending on the value of α and voltage level, when two devices are conducting, the load phase voltage is $\frac{V_L}{2}$.



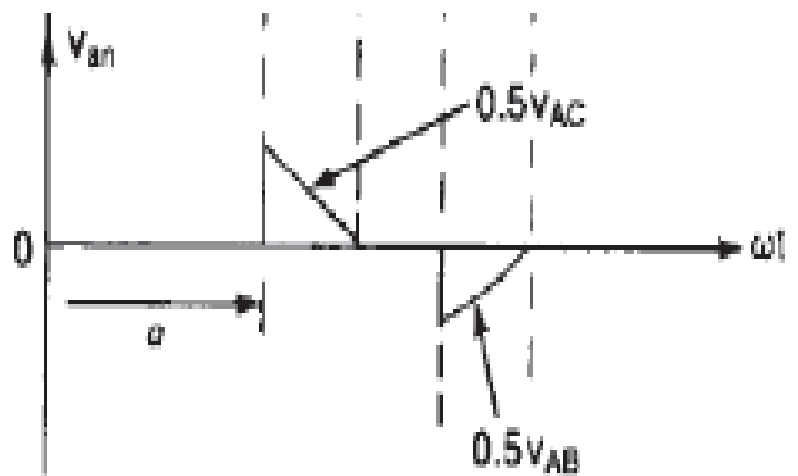
➤ Depending on the value of α and the voltage level, when three devices are conducting; normal three phase voltages are applied to the load.

➤ Examples of the output voltage for different values of the firing angle are shown below.

➤ Due to the asymmetrical nature of the output voltage waveform, the input current may contain a DC component. Besides, the harmonic content is high, therefore this controller is rarely used.



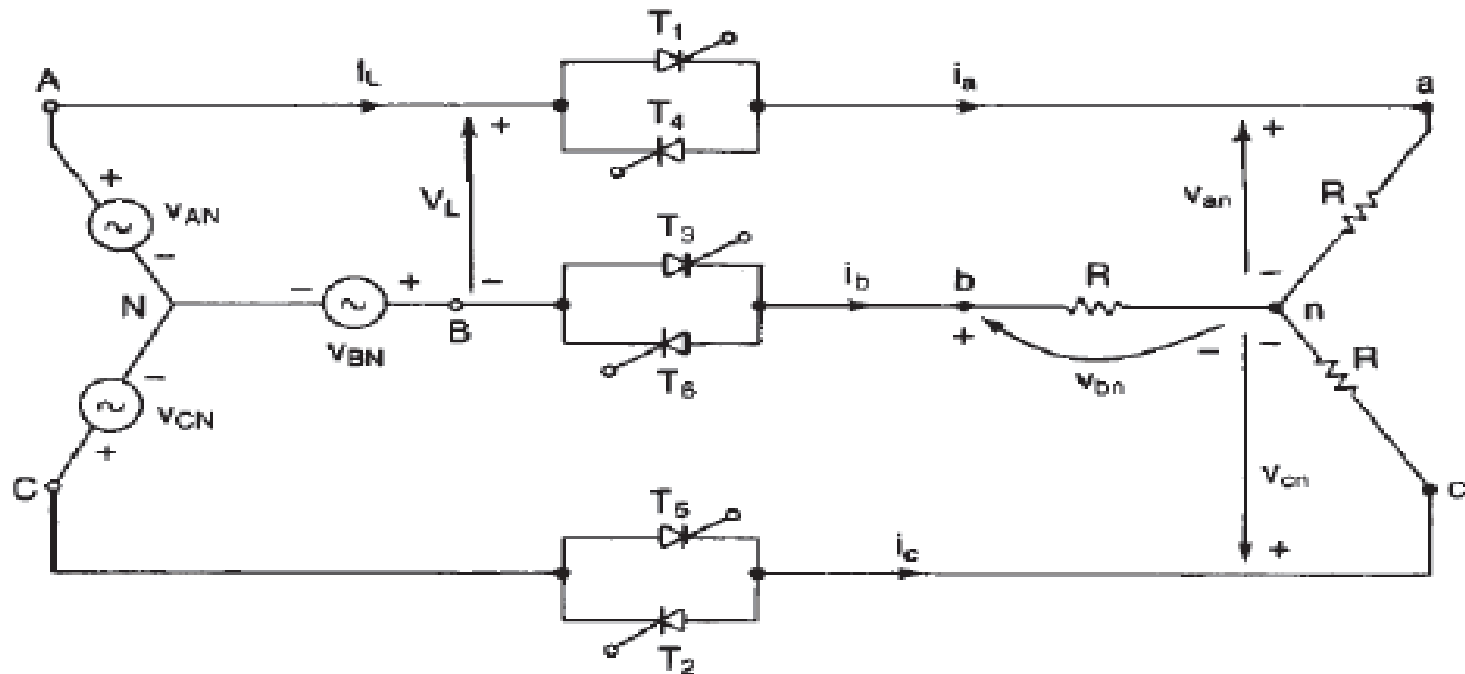
(a) For $\alpha = 60^\circ$



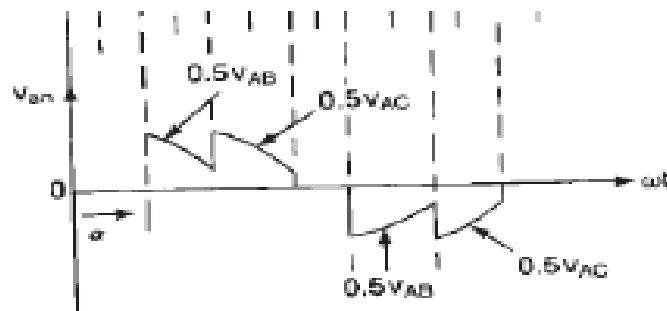
(b) For $\alpha = 150^\circ$

B- Three-Phase Full-Wave Controllers

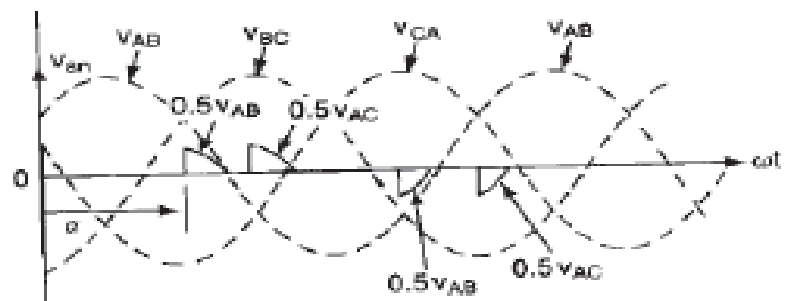
- They are also known as Bidirectional Three-Phase Controllers.
- The circuit topology of the Full-Wave Controller is shown in the Figure below.
- The number of conducting Thyristors at any time depends on the value of the firing angle.



- Examples of the output voltage are shown in the Figure below.



(a) For $\alpha = 60^\circ$



(b) For $\alpha = 120^\circ$

- Note that, the AC Voltage Controllers provide a variable output voltage, but the frequency of the output is fixed. In addition, the harmonic content is high, especially at low output voltage range.

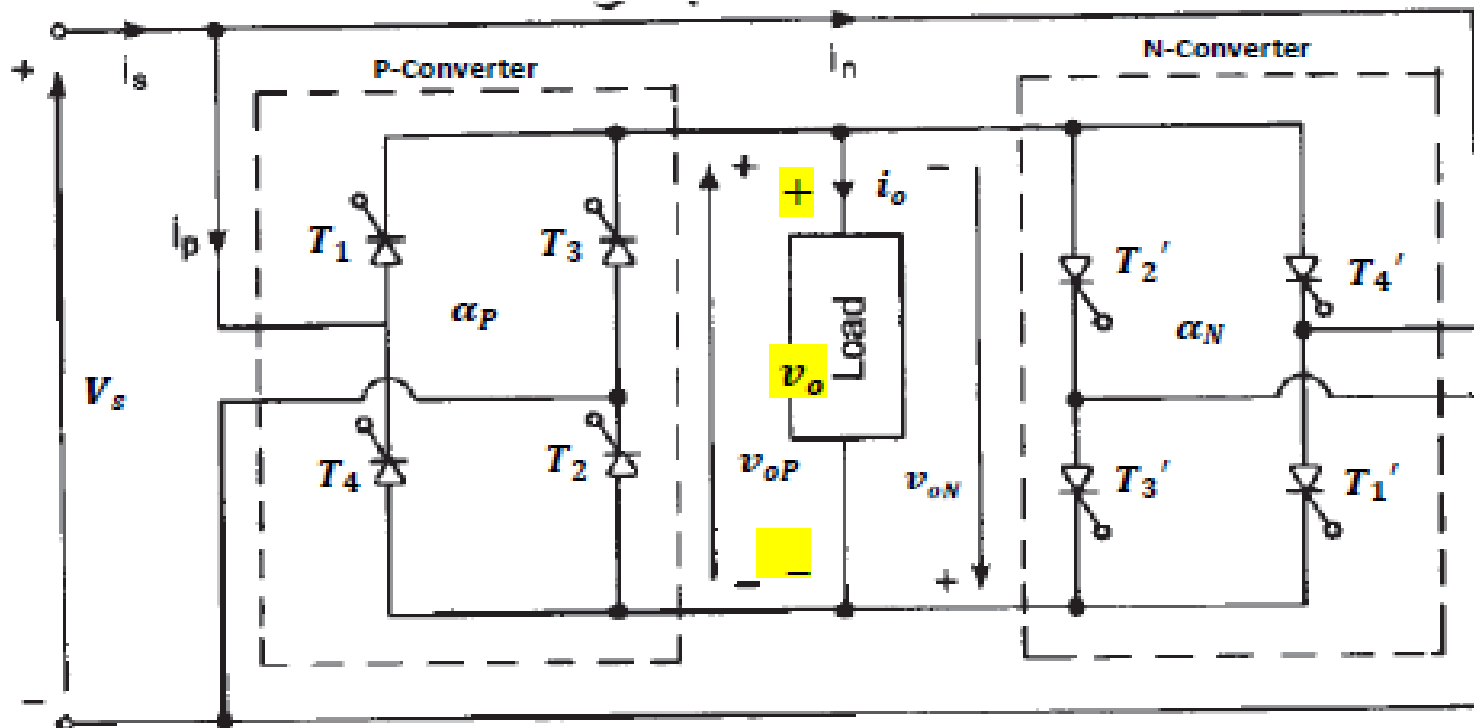
Exercise: show modification on the above circuit to allow the controller to reverse the direction of rotation of an AC motor.

Cycloconverters

- ✦ They convert AC power at a fixed voltage and frequency to AC power at a Variable Voltage and Variable Frequency (VVVF). The output frequency is usually fractions of the input frequency ($\sim < \frac{1}{3}$ the source frequency).
- ✦ Thus, their major applications are low speed AC motor drives.

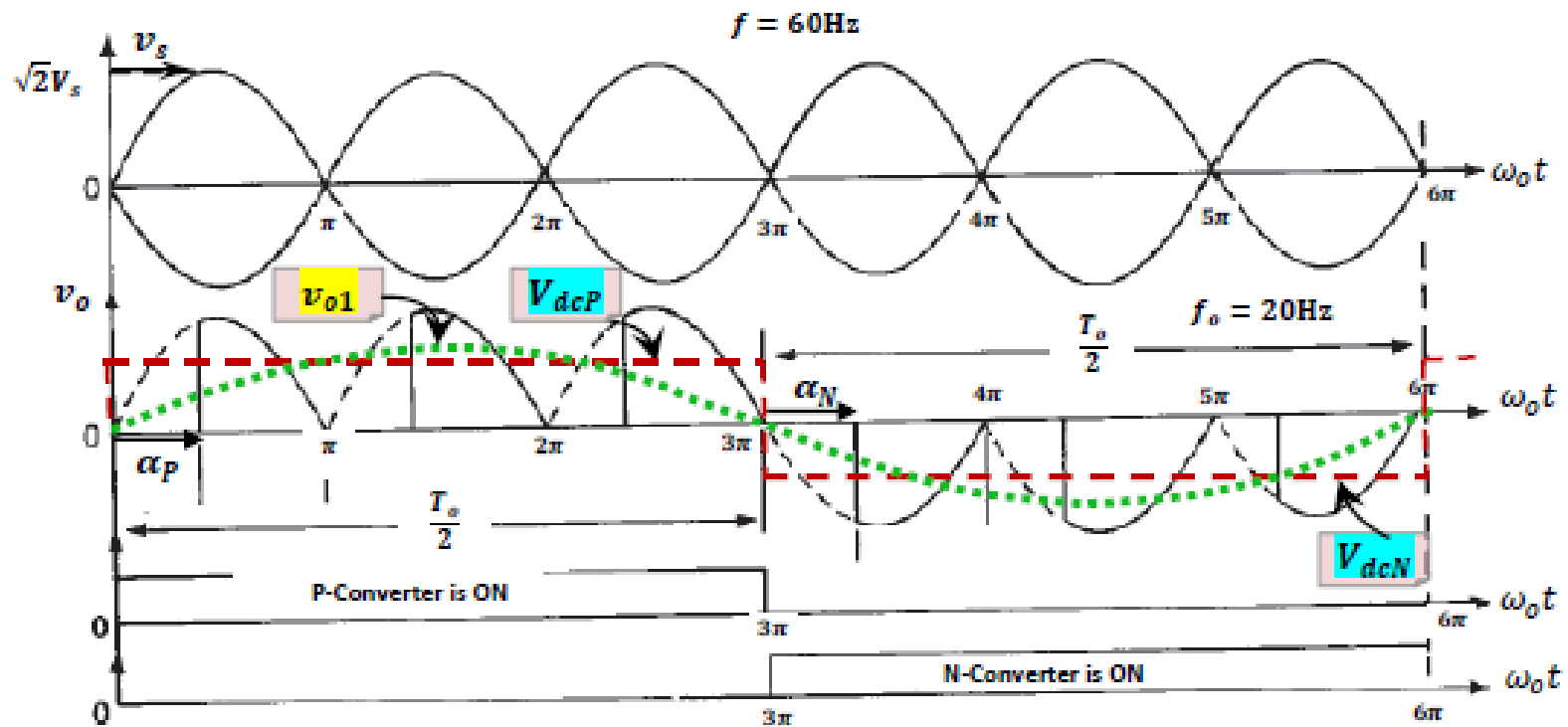
1) Single-Phase/Single-Phase Cycloconverters

- ❖ The circuit topology of a Single-Phase/Single-Phase Cycloconverter is shown in the Figure below.



- ❖ Two Single-Phase Controlled Rectifiers (converters) are operated such that their average output voltages are equal and opposite to each other.
- ❖ If α_P is the delay angle of Thyristors (T_1 and T_2) in the Positive Converter (P-Converter), and α_N is the delay angle of Thyristors (T_1' and T_2') in the Negative Converter (N-Converter), then: $\alpha_P = \alpha_N = \alpha$, and $V_{dcP} = |V_{dcN}|$.

- ❖ The associated waveforms (for one period of the output voltage) of this type of Cycloconverter, supplying a resistive load, are shown in the Figure below.



- ❖ For a resistive load, the average output voltage of the P-Converter is:

$$V_{dcP} = \frac{V_m}{\pi} (1 + \cos \alpha_P)$$

where, V_m is the peak phase voltage.

- ❖ For a resistive load, the average output voltage of the N-Converter is:

$$V_{dcN} = \frac{V_m}{\pi} (1 + \cos \alpha_N)$$

◆ The combined outputs of the two converters produce an effective square wave across the load, whose DC level is $\frac{V_m}{\pi}(1 + \cos \alpha)$;

◆ The fundamental component of the output has a peak value of:

$$\hat{V}_{o1} = \frac{4}{\pi} \left(\frac{V_m}{\pi} (1 + \cos \alpha) \right); \quad 0 < \alpha < \pi$$

◆ If the load is highly inductive, then the DC level of any converter is $\frac{2V_m}{\pi} \cos \alpha$, and the fundamental component of the output has a peak value of:

$$\hat{V}_{o1} = \frac{4}{\pi} \left(\frac{2V_m}{\pi} \cos \alpha \right)$$

◆ If a low pass filter was used at the output, then the load will have only the fundamental component applied across it!

◆ It is obvious that, in either case of load type, the peak value of the fundamental component at the output voltage (and hence its rms value) can be varied by varying the firing angle of each converter, bearing in mind that $\alpha_P = \alpha_N = \alpha$. (always!)

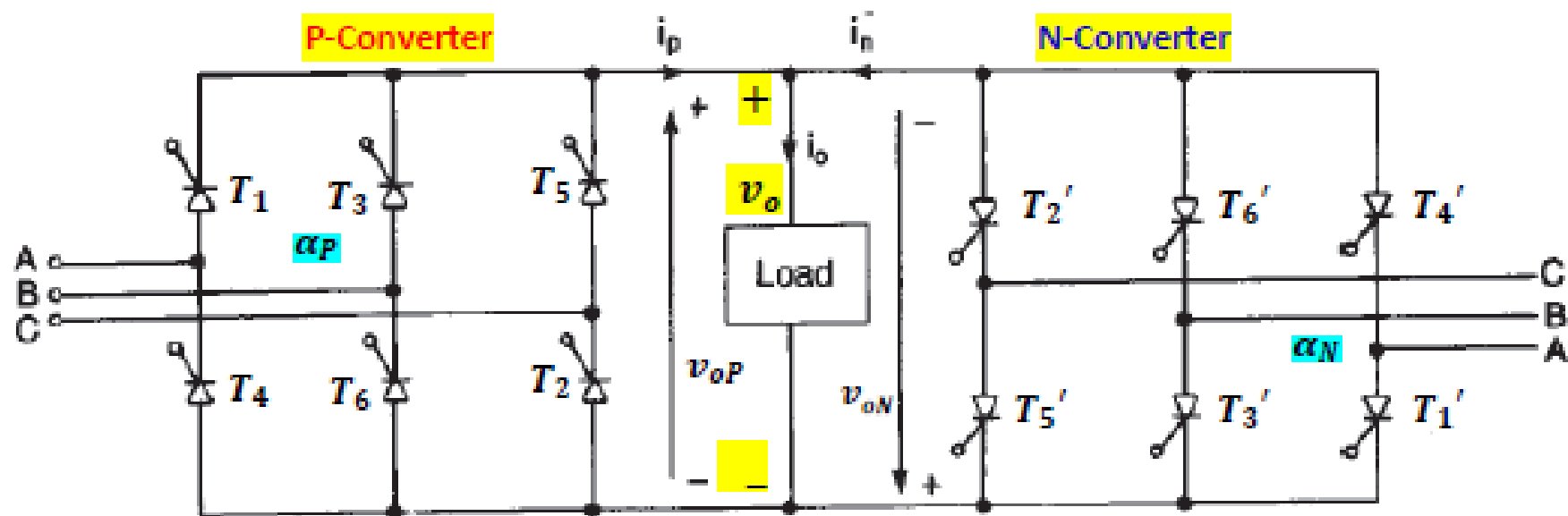
◆ The frequency of the fundamental component of the output voltage is: $f_o = \frac{1}{T_o}$, and can be controlled by adjusting the control voltage of each converter; $\left(f_o = \frac{T}{T_o} f \right)$. In this case,

$$f_o = \frac{2\pi}{6\pi} 60 = 20\text{Hz!}$$

◆ Note that, this type of controller has a Variable Voltage (controlled by the delay angles) and a Variable Frequency (controlled by the period during which each converter is operated) at its output. Hence, **VVVF Drive!**

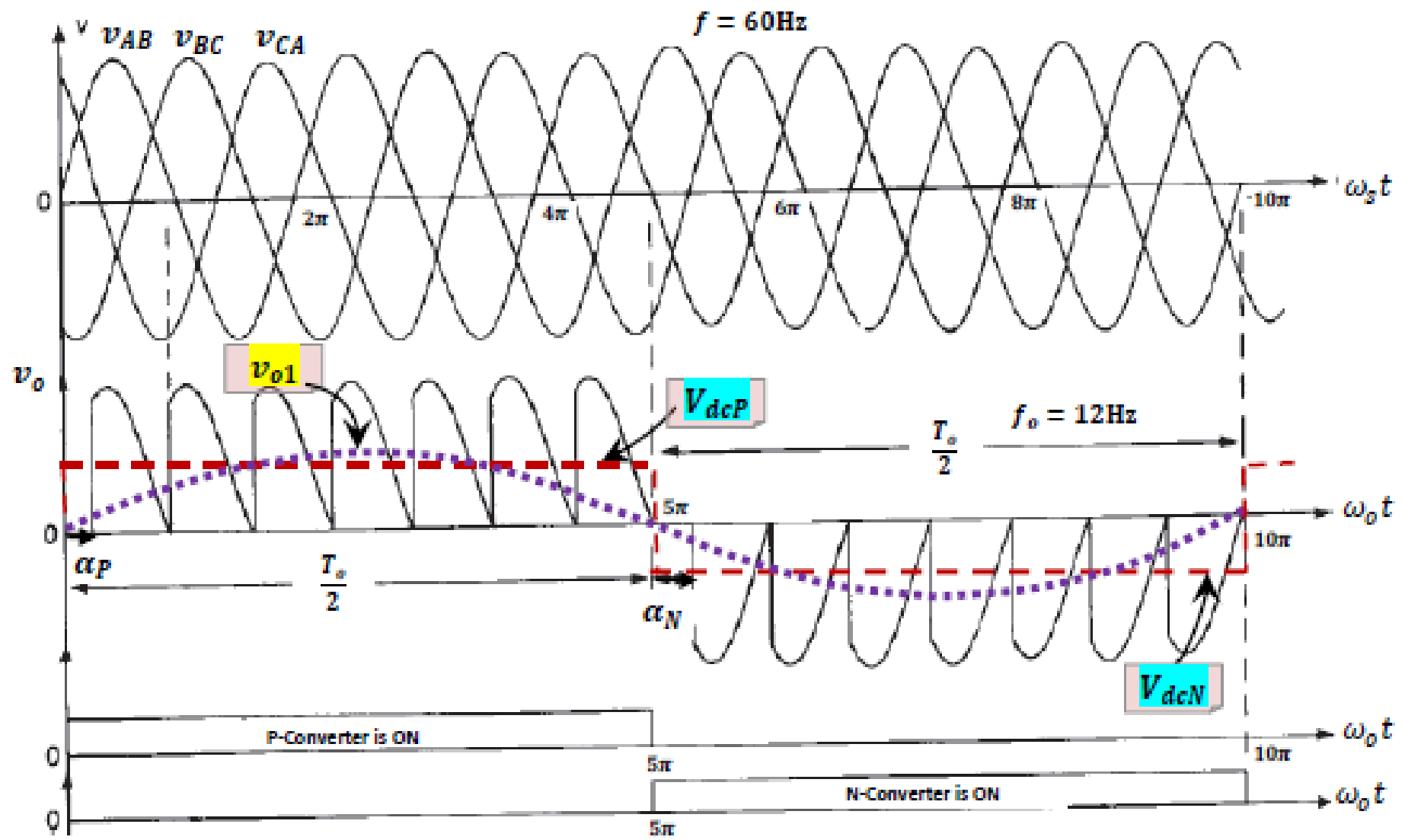
II. Three-Phase/Single-Phase Cycloconverters

- The circuit topology of a Three-Phase/Single-Phase Cycloconverter is shown in the Figure below.



- It is used in higher power applications!
- Two Three-Phase Controlled Rectifiers (converters) are operated such that their average output voltages are equal and opposite to each other.
- If α_P is the delay angle of Thyristor (T_1) in the Positive Converter (P-Converter), and α_N is the delay angle of Thyristor (T_1') in the Negative Converter (N-Converter), then: $\alpha_P = \alpha_N = \alpha$, and $V_{dcP} = |V_{dcN}|$.

The associated voltage waveforms of a Three Phase/Single Phase Cycloconverter for one period of the output voltage are shown in the Figure below. The waveforms are illustrative and not exact as there should be 30 pulses in 5 cycles of the input voltage.



- Similar to Single Phase/Single Phase Cycloconverter, for a resistive load, the average output voltage of the P-Converter is:

$$V_{dcP} = \begin{cases} \frac{3\sqrt{3}V_m}{\pi} \cos \alpha; & \alpha \leq \frac{\pi}{3} \\ \frac{3\sqrt{3}V_m}{\pi} \left(1 + \cos\left(\alpha + \frac{\pi}{3}\right)\right); & \frac{\pi}{3} < \alpha < \frac{4\pi}{6} \end{cases}$$

where, V_m is the peak phase voltage.

- For a resistive load, the average output voltage of the N-Converter is:

$$V_{dcN} = \begin{cases} \frac{3\sqrt{3}V_m}{\pi} \cos \alpha; & \alpha \leq \frac{\pi}{3} \\ \frac{3\sqrt{3}V_m}{\pi} \left(1 + \cos\left(\alpha + \frac{\pi}{3}\right)\right); & \frac{\pi}{3} < \alpha < \frac{4\pi}{6} \end{cases}$$

- The combined outputs of the two converters produce an effective square wave across the load,

whose DC level is, $V_{dc} = \begin{cases} \frac{3\sqrt{3}V_m}{\pi} \cos \alpha; & \alpha \leq \frac{\pi}{3} \\ \frac{3\sqrt{3}V_m}{\pi} \left(1 + \cos\left(\alpha + \frac{\pi}{3}\right)\right); & \frac{\pi}{3} < \alpha < \frac{4\pi}{6} \end{cases}$

- The fundamental component of the output has a peak value of:

$$\hat{V}_{o1} = \frac{4}{\pi} (V_{dc})$$

$$\hat{V}_{o1} = \frac{4}{\pi} \begin{cases} \frac{3\sqrt{3}V_m}{\pi} \cos \alpha; & \alpha \leq \frac{\pi}{3} \\ \frac{3\sqrt{3}V_m}{\pi} \left(1 + \cos\left(\alpha + \frac{\pi}{3}\right)\right); & \frac{\pi}{3} < \alpha < \frac{4\pi}{6} \end{cases}$$

- ◆ If the load is highly inductive, then the DC level of any converter is $\frac{3\sqrt{3}V_m}{\pi} \cos \alpha$, and the fundamental component of the output has a peak value of:

$$\widehat{V}_{o1} = \frac{4}{\pi} \left(\frac{3\sqrt{3}V_m}{\pi} \cos \alpha \right); \quad 0 \leq \alpha < \pi$$

- ◆ If a low pass filter was used at the output, then the load will have only the fundamental component applied across it!
- ◆ It is obvious that, in either case of load, the peak value of the fundamental component at the output voltage (and hence its rms value) can be varied by varying the firing angle of each converter, bearing in mind that $\alpha_P = \alpha_N = \alpha$. (always!)
- ◆ The frequency of the fundamental component of the output voltage is: $f_o = \frac{1}{T_o}$, and can be controlled by adjusting the control voltage of each converter; $\left(f_o = \frac{T}{T_o} f \right)$. In this case,

$$f_o = \frac{2\pi}{10\pi} 60 = 12\text{Hz!}$$

- ◆ Note that, this type of controller has a Variable Voltage (controlled by the delay angles) and a Variable Frequency (controlled by the period during which each converter is operated) at its output. Hence, a **VVVF Drive** results!

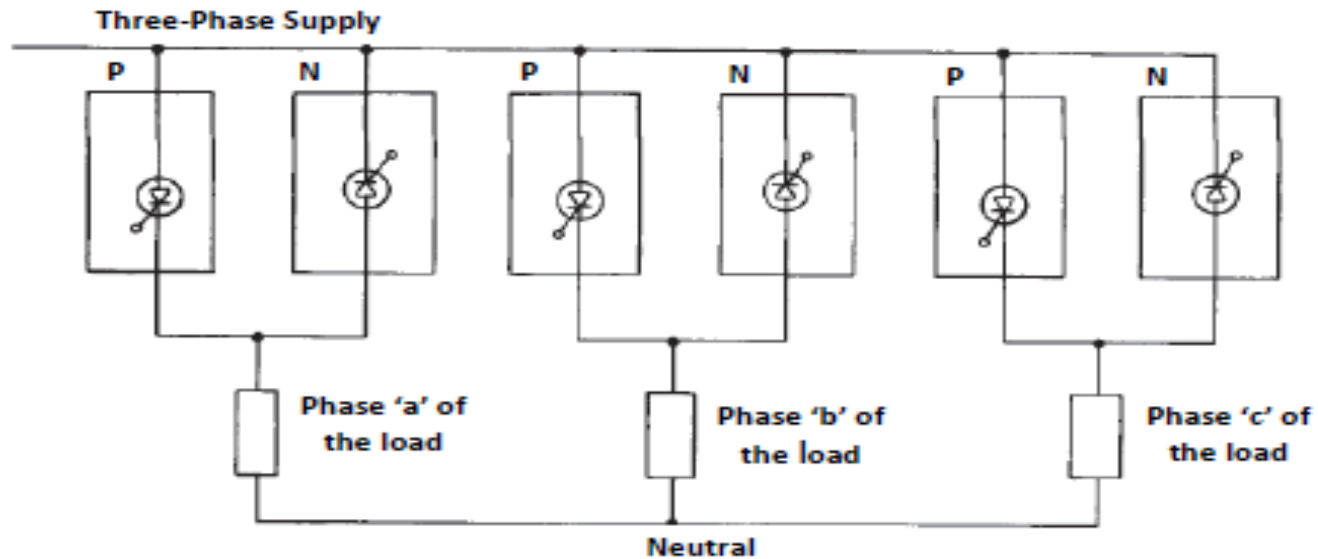
III. Three-Phase/Three-Phase Cycloconverters

a) Full-Wave Three-Phase/Three-Phase Cycloconverter

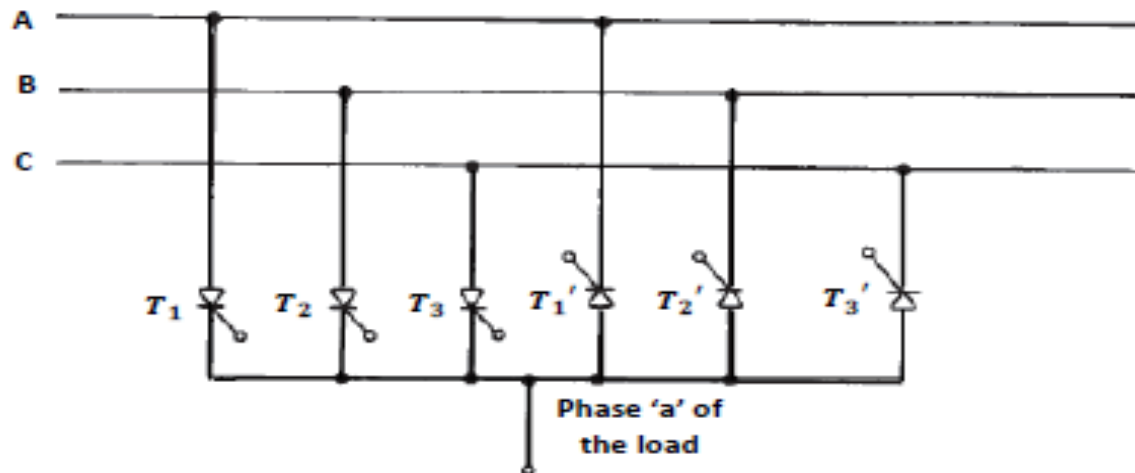
- If '3' Three-Phase/Single-Phase Cycloconverters are used (with appropriate control voltages applied to each converter), then a Full-Wave Three-Phase/Three-Phase Cycloconverter results. The loads in this case have to be connected as WYE or Delta.
- The controlled voltages of the converters have to be shifted by 120° for each phase!
- In this type of converter 36 SCRs are needed!

b) Half-Wave Three-Phase/Three-Phase Cycloconverter

- The circuit topology of such a Cycloconverter is shown in the Figure below.
- In this type of converter only 18 SCRs are needed!

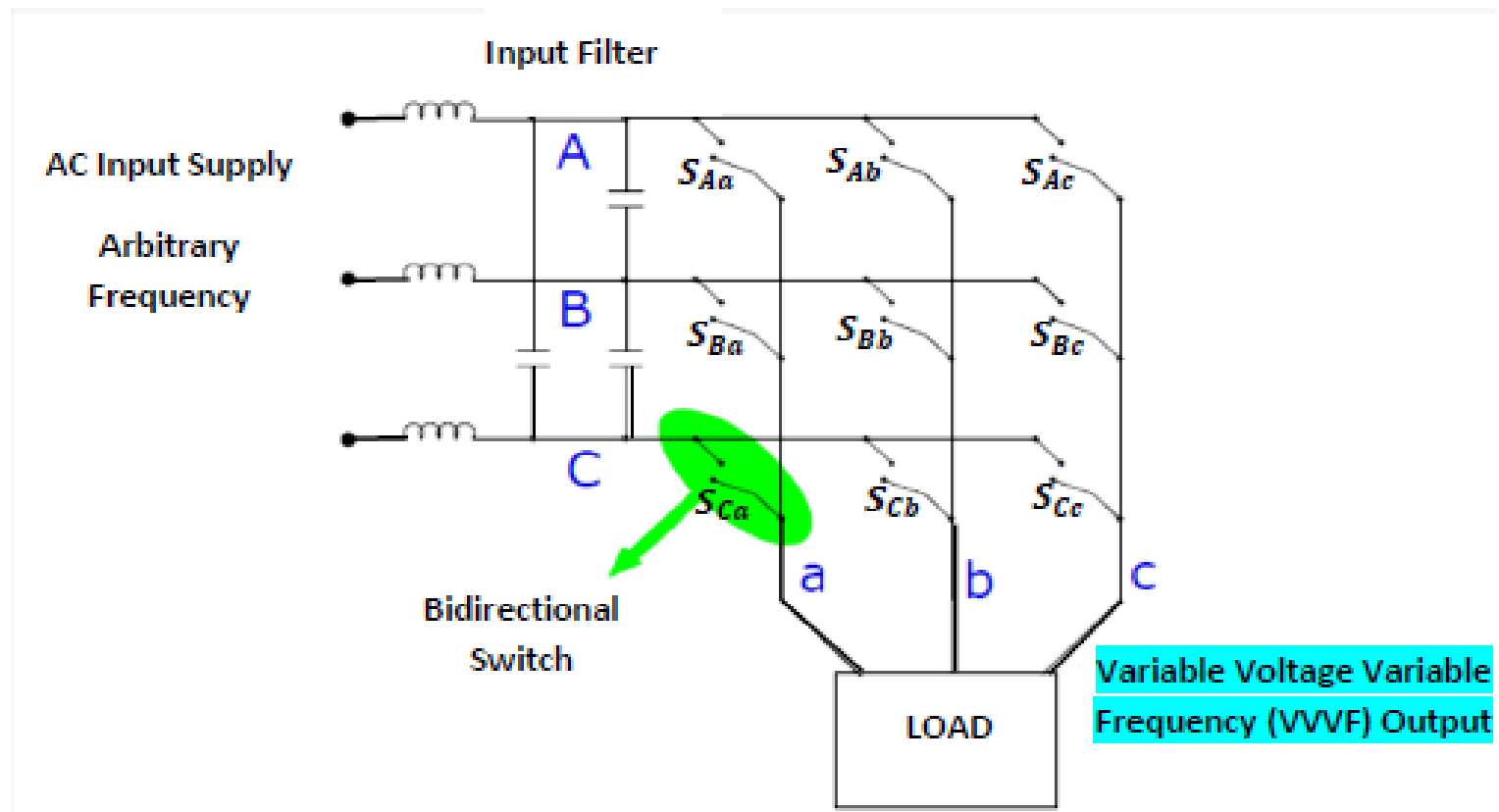


- Each phase of the output can be connected to the three inputs, as shown in the Figure below.



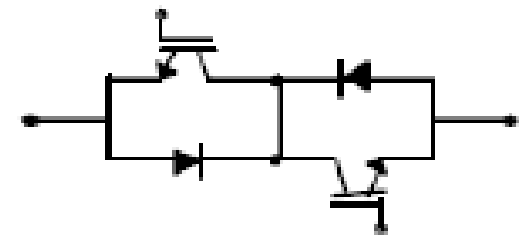
Matrix Converters

- A Matrix converter is fairly a new converter topology, which was first proposed in the beginning of the 1980's. It is a newer form of the Cycloconverter!
- A Matrix Converter consists of **nine bidirectional switches** connecting any of the three input voltages to any of the three output phases directly as shown in the Figure below. The inputs are of voltage source characteristics and the outputs are of current source characteristics (because most loads are of an inductive nature).

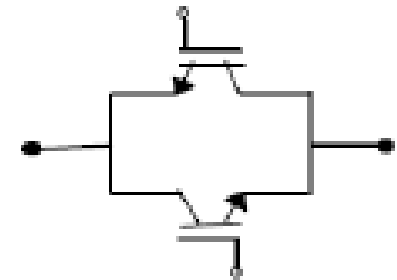


- The switches in the Matrix Converter are bidirectional switches; that is, they must be able to support a voltage of either polarity, and be able to conduct a current in either direction.

- An example of a bidirectional switch constructed from non-reverse blocking IGBTs and diodes is shown in the Figure next.



- If reverse blocking IGBTs are available, then the bidirectional switch will be as shown in the Figure next.



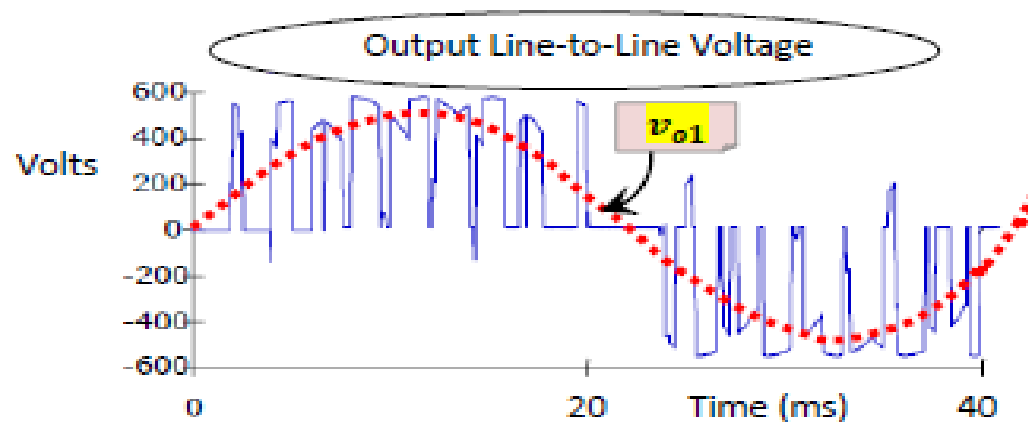
- Any input phase can be connected to any output phase at any time, depending on the control. The output depends on the state of switches;

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} S_{Aa} & S_{Ba} & S_{Ca} \\ S_{Ab} & S_{Bb} & S_{Cb} \\ S_{Ac} & S_{Bc} & S_{Cc} \end{bmatrix} \begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix}$$

where, V_{an} , V_{bn} and V_{cn} , are the output phase – load neutral voltages!

- However, no two switches from the same phase can be connected at the same time, such that to avoid applying a short circuit across the input phases. This makes the current commutation and switches control very complex in such a converter.

- These converters are controlled by Pulse Width Modulation (PWM) techniques to produce a **Variable Voltage Variable Frequency (VVVF)** three-phase output (even for output frequencies higher than the input frequency).
- An example of an output voltage waveform at 25Hz (obtained from 50Hz sources) is shown in the Figure below.



- The outputs of these converters have lower harmonic content compared to those in the outputs of Cycloconverters, as clearly seen in the frequency spectrum of the Figure below, at which the switching frequency $f_s = 2kHz$. Here, the switching frequency is reduced deliberately for illustration purposes!

