ANALOG TO DIGITAL CONVERSION PROBLEM SET SOLUTION

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Problem 1

Question

A baseband signal (t) has a spectral range that extends from 0 to 25 kHz. The signal is sampled at a rate of f_s samples/sec. Find the Nyquist rate for this signal.

Answer

Maximum frequency in (t) is 25 KHz $\Longrightarrow B.W. = 25KHz.$

$$Nyquist\ Rate: f_s = 2 \times B.W. = 50K\ samples/sec.$$

Problem 2

Question

Find the Nyquist rate for the multi-tone signal:

$$m(t) = 2\cos(200\pi t) + 2\cos(300\pi t) + 2\cos(400\pi t)$$

Answer

$m(t)$ is band-limited with bandwidth of 200Hz.

$$Nyquist\ Rate: f_s = 2 \times B.W. = 400\ samples/sec.$$

Problem 3

Question

The Fourier transform of a signal (t) is given as:

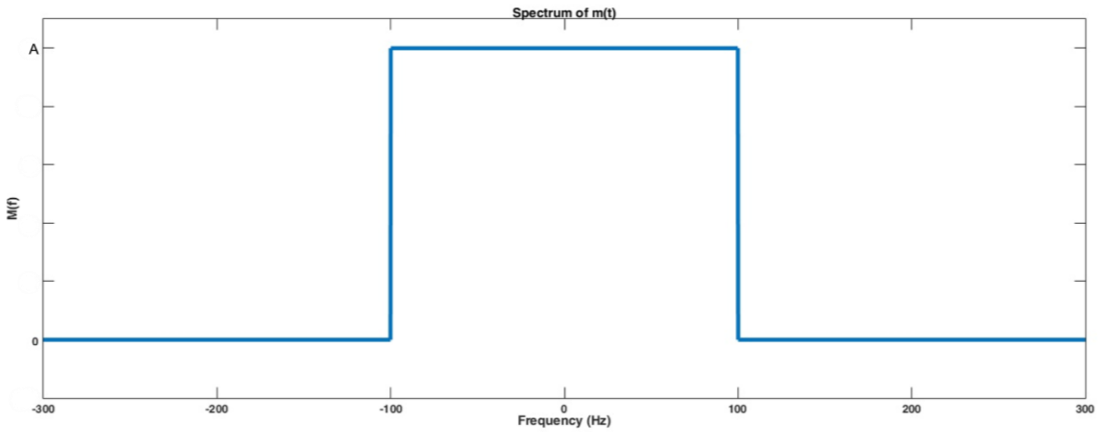
$$M(f) = \begin{cases} A, & -100 \leq f \leq 100 \\ 0, & |f| > 100 \end{cases}$$

The signal is sampled at a rate of 150 Samples/sec.

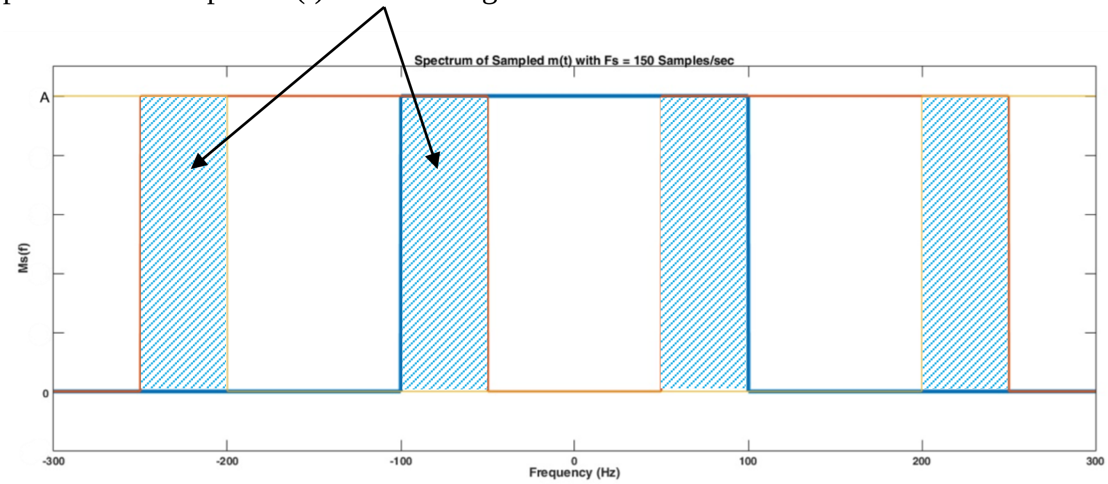
a. Sketch the spectrum of the sampled signal.
b. If the sampled signal is admitted to an ideal low-pass filter with a bandwidth of 100 Hz. Sketch the spectrum of the filter output.
c. Is there a distortion? Explain the reason.

Answer

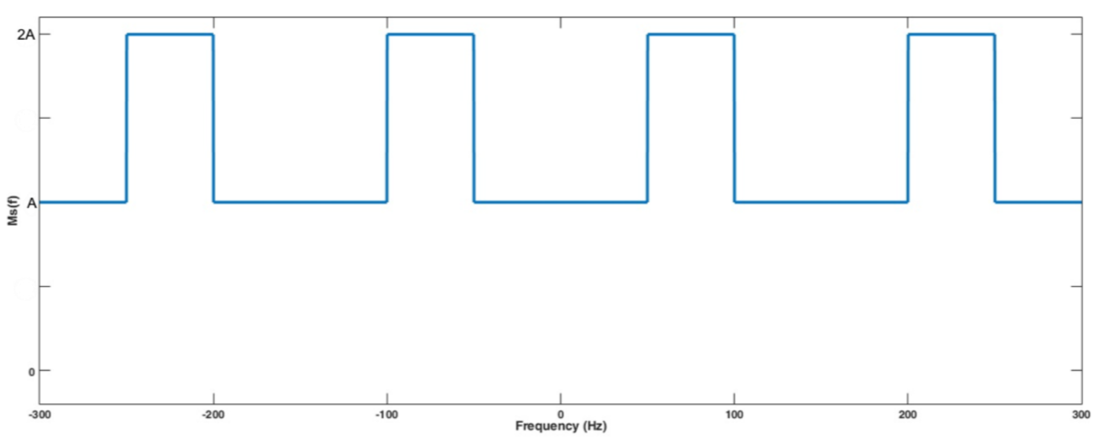
a) Spectrum of m(t)



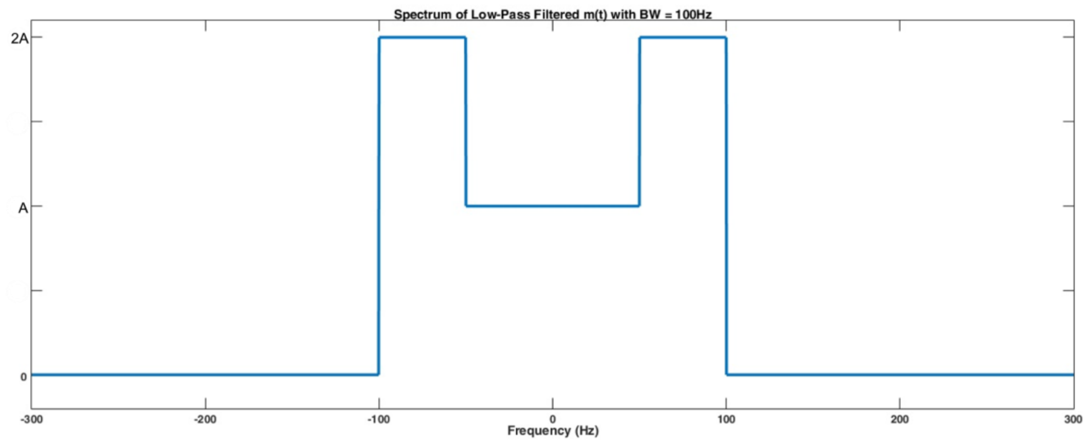
Spectrum of Sampled m(t) Aliasing



By adding amplitudes with each other



b) After applying to a low-pass filter with B.W. of 100Hz.



c) There is a distortion due to aliasing occurred by sampling process which is noticed in the spectrum of sampled signal. Also it can be noticed that the spectrum of the signal after applying to a low-pass filter with bandwidth of 100Hz is not identical to the original spectrum.

Problem 4

Question

Find the Nyquist rate for the signal (t)=$Asinc(1000t)$.

Answer

Find Fourier transform:

$$M(f) = \frac{A}{1000} rect\left(\frac{f}{1000}\right)$$

Hence, $m(t)$ is band-limited with bandwidth of $\frac{1000}{2} = 500$Hz.

$$Nyquist\ Rate: f_s = 2 \times B.W. = 1000\ samples/sec.$$

Problem 5

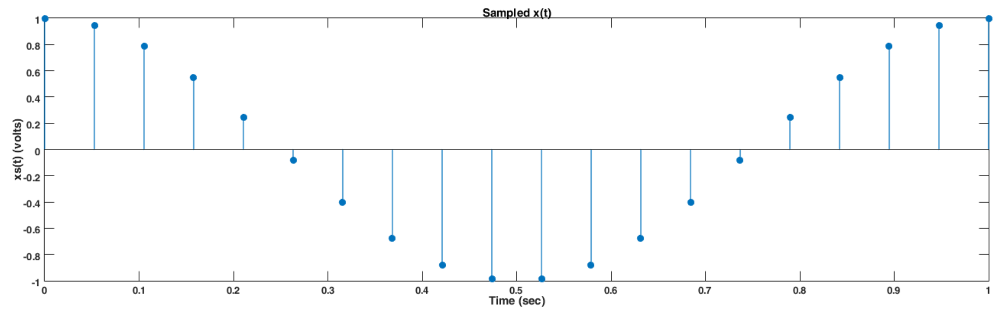
Question

The signal x(t) = cos($2\pi t$) is uniformly sampled at a rate of 20 samples/sec. The samples are applied to an 8-level uniform quantizer with a dynamic range (-1, 1) V and a step size of 0.25 V.

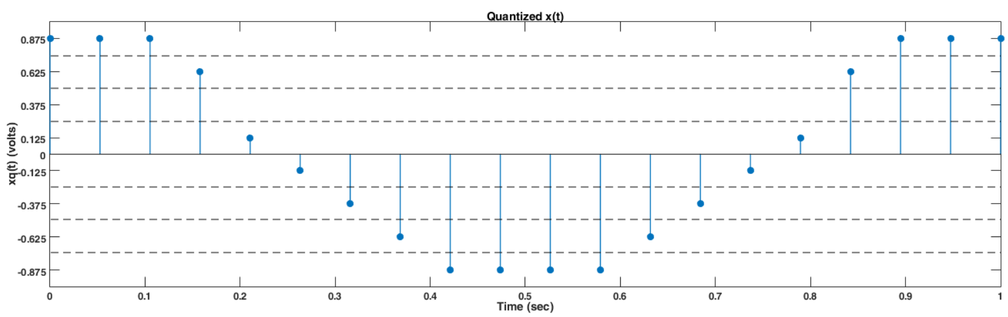
a. Plot the sampled signal over one cycle of the message.
b. Plot the quantizer output over one cycle of the message.
c. Repeat Part b if the signal applied to the quantizer is g(t) = 0.25 cos($2\pi t$).
d. Comment on the results of Parts b and c.

Answer

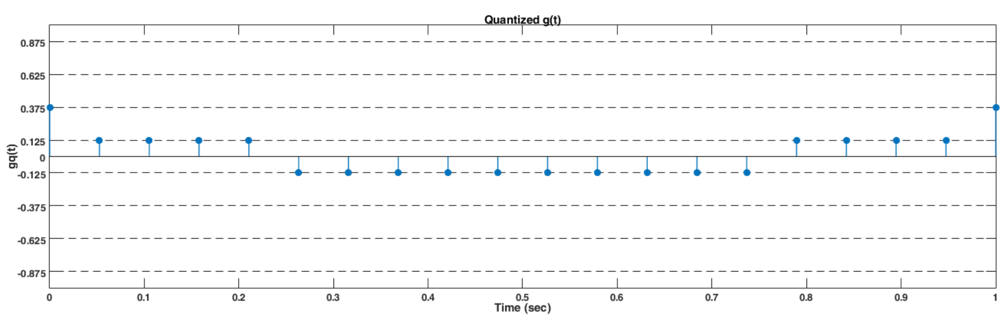
a) Sampled signal



b) Quantized signal



c) Quantized g(t)



d) It is noticed from the quantized x(t) signal that all quantization values were used and the new signal is similar to the original one. Conversely, the quantized g(t) signal is similar to a square wave which is not a precise representation of the original cosine wave, and this occurred due to applying the small signal g(t) to a wide dynamic range quantizer.

Problem 6

Question

The signal (t)=cos($1000\pi t$) is to transmitted using a PCM system (a system composed of a sampler, quantizer, and binary encoder).

a. If sampling is done at the Nyquist rate and a uniform quantizer with 32 levels is employed, what is the resulting data rate in bits/sec and the resulting SQNR.

b. Find the SQNR if the signal is sampled at 1.5 times the Nyquist rate.

Answer

a) Sampling done at Nyquist Rate $\Longrightarrow f_s = 2 \times B.W. = 1000\ samples/sec.$

Quantizer of 32 levels means that each sample is represented by $n = \log_2 32 = 5bits.$

$$Bit\ Rate = n \times f_s = 5\ Kbps.$$

$$SQNR = 6.02n + 1.76 = 31.86dB$$

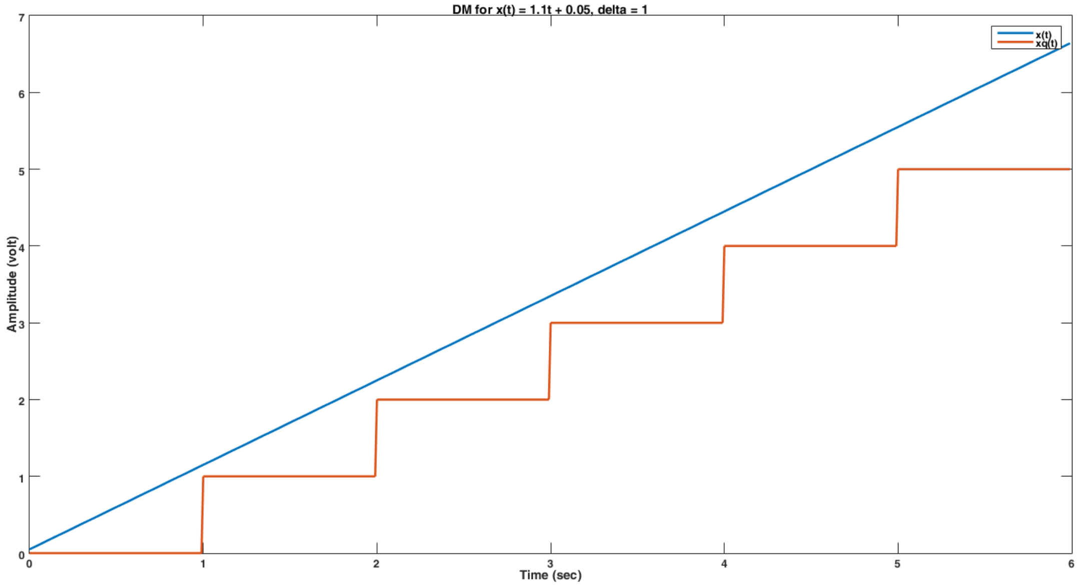
b) Since SQNR doesn't depend on sampling frequency it’s the same as part a.

Problem 7

Question

Draw the output of the DM given that the input corresponds to $x(t) = 1.1t + 0.05$ when the input is sampled at t = 0, 1, 2, 3, 4, 5, … and Δ=1.

Answer

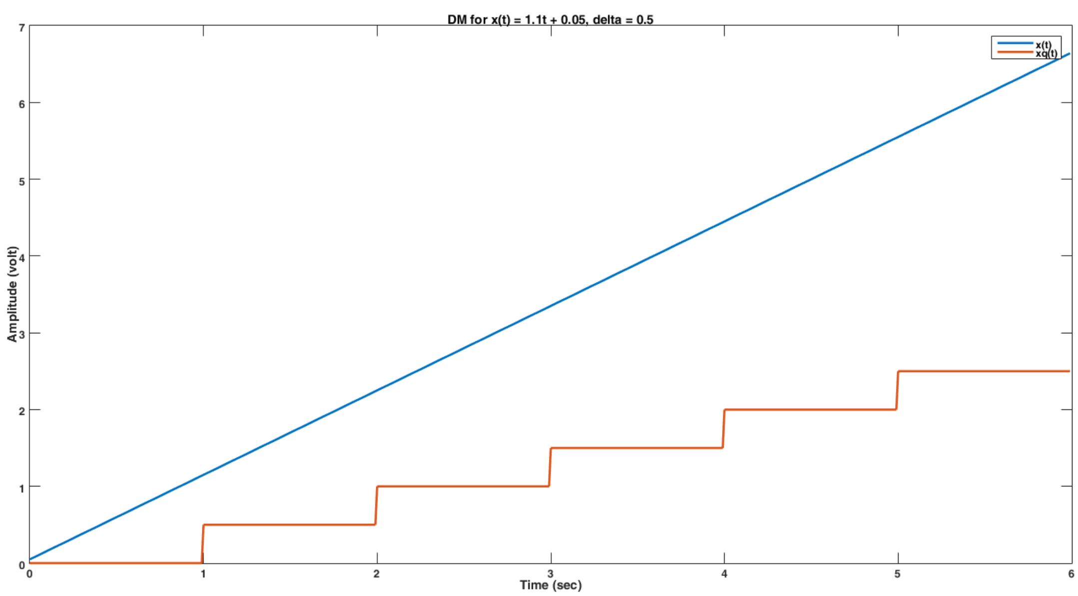


Problem 8

Question

Repeat Problem 7, if $\Delta = 0.5$.

Answer

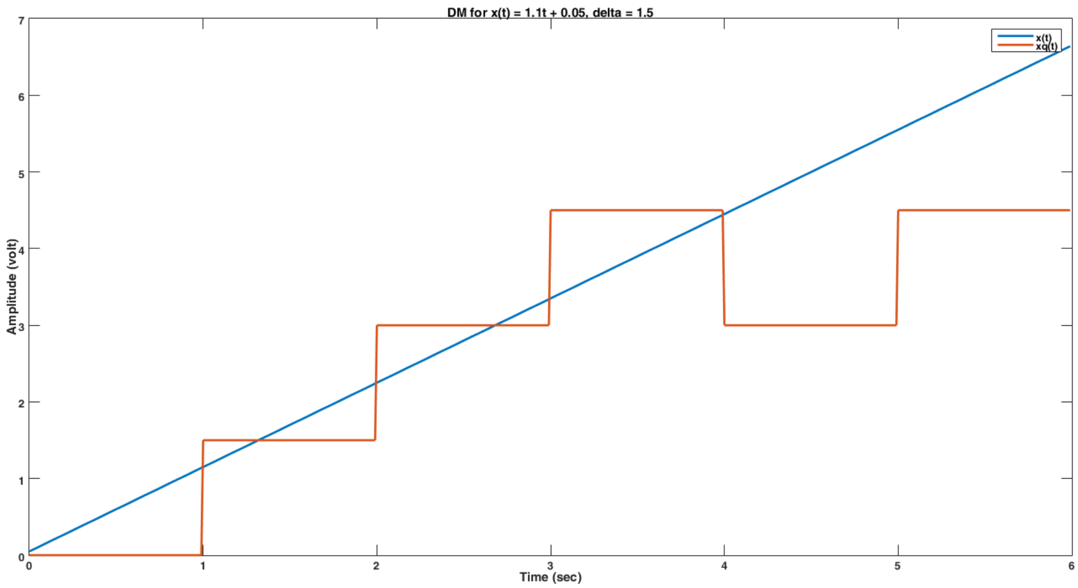


Problem 9

Question

Repeat Problem 7, if Δ=1.5. Comment on the results of Problems 7, 8, and 9.

Answer



The plots above show that when Δ = 1, the DM output is the closest to the original signal, where when Δ = 0.5 the output doesn't follow closely the actual curve, which makes slope overload distortion. Also increasing Δ to 1.5 makes higher granular noise to the output.

Problem 10

Question

Design a 16-level uniform quantizer for an input signal with a dynamic range of $(-10,+10)V$.

a. Find the quantizer output and the quantization error for an input sample of 1.2 V.

b. Find the binary representation corresponding to the sample -2.63 assuming natural binary encoding is used.

c. Find the average SQNR.

Answer

$$x_{max} = 10V, L = 16 \Longrightarrow step\ size = \frac{2x_{max}}{L} = 1.25V$$

Threshold Values = {-10, -8.75, -7.5, -6.25, -5, -3.75, -2.5, -1.25, 0, 1.25, 2.5, 3.75, 5, 6.25, 7.5, 8.75, 10}

a) Since $0 < 1.2 < 1.25$, quantizer output for 1.2V = $\frac{1.25+0}{2} = 0.75$

Quantization error = 1.2 – 0.75 = 0.45V

b) For 16-level quantizer, 4-bit binary representation is used

Representation of -2.63 is 0101.

c) $SQNR = 6.02n + 1.76 = 6.02(4) + 1.76 = 25.84\ dB.$

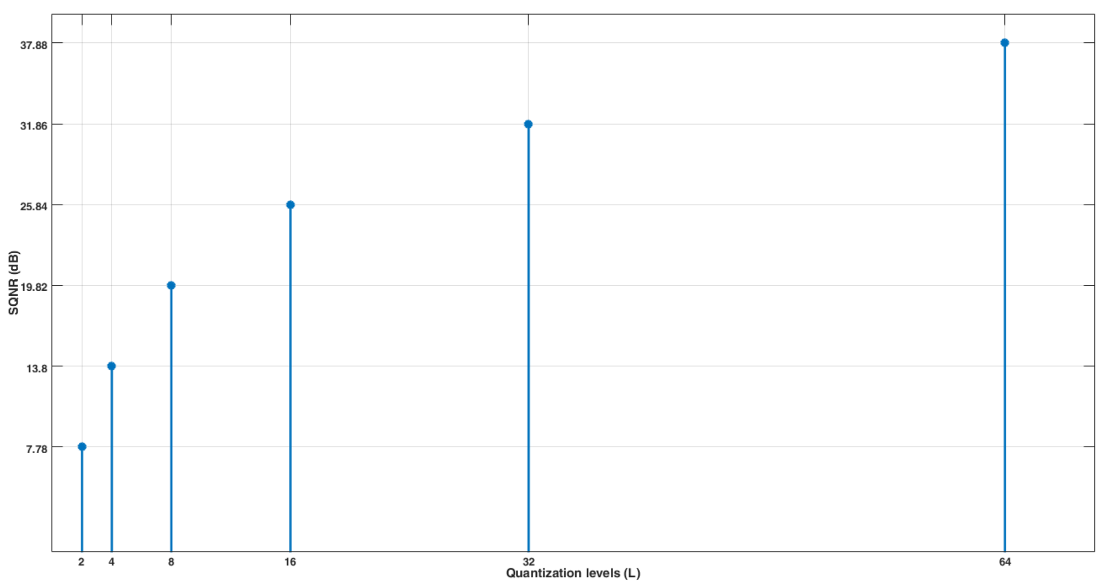
Problem 11

Question

Plot the SQNR, in dB, derived in class versus L (number of quantization levels) for L=2, 4, 8, 16, 32, 64. What are your conclusions?

Answer

$$SQNR = 6.02n + 1.76 = 6.02 \log_2 L + 1.76$$



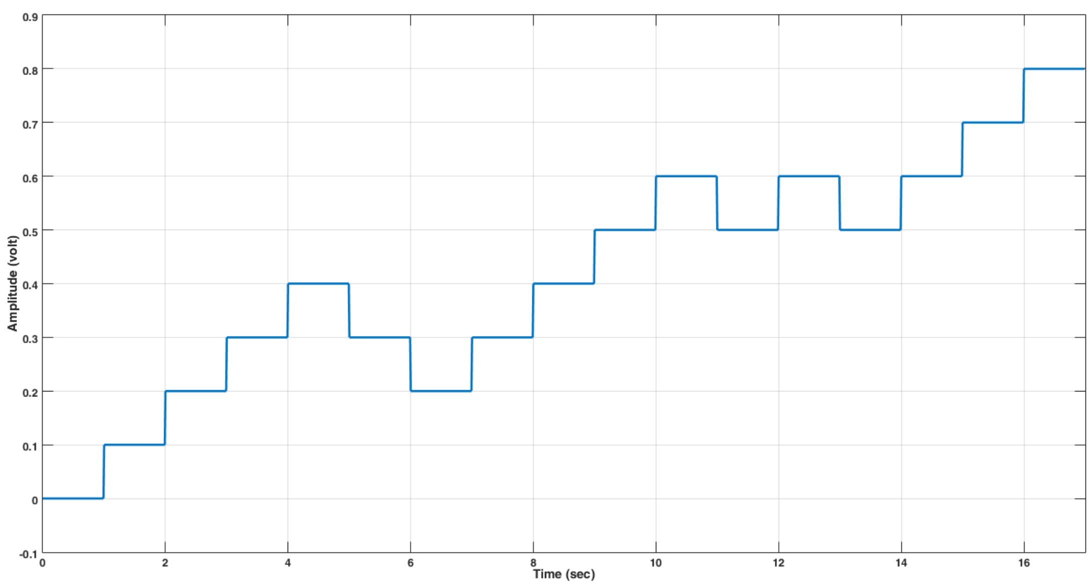
It's noticed that SQNR in dB increases logarithmically in respect to the increasing of quantization level, which means that increasing quantization levels too much doesn't make a big difference on SQNR.

Problem 12

Question

Reconstruct a staircase signal at the receiver side of a delta demodulator with Δ = 0.1V, when the received data sequence is 1 1 1 1 0 0 1 1 1 1 0 1 0 1 1 1.

Answer

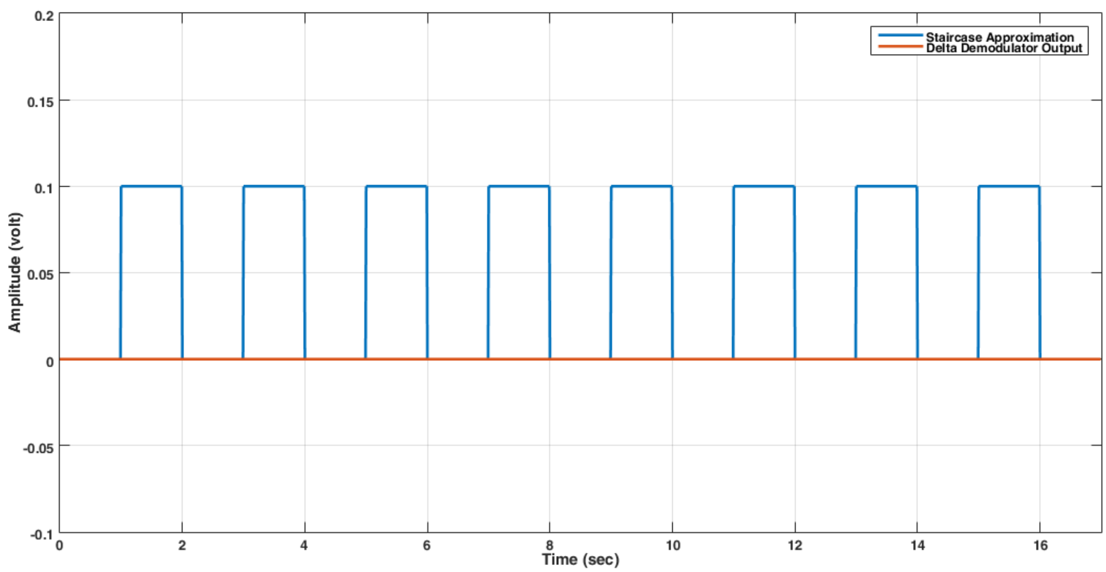


Problem 13

Question

Reconstruct a staircase signal at the receiver side of a delta demodulator with Δ = 0.1V, when the received data sequence is 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0. Comment on the nature of the message signal.

Answer

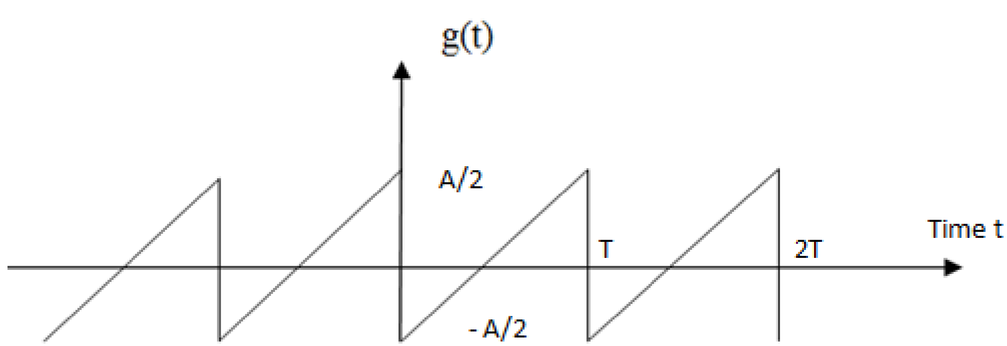


It's noticed that the staircase approximation is a square wave which can't be used to approximate the original signal due to wide sampling period Ts. The output of a delta demodulator will be zero due to low-pass filtering for the oscillating.

Problem 14

Question

The triangular signal g(t), shown in the figure below, is applied to an 8-level uniform quantizer with a range of (-A/2, A/2). Find the SQNR. The noise error can still be assumed to be a uniform random variable over $(-\frac{\Delta}{2}, \frac{\Delta}{2})$.



Answer

$$SQNR = \frac{P_g}{D}, D: Quantization\ Error$$

$$P_g = \frac{1}{T}\int_0^T |g(t)|^2 dt = \frac{1}{T}\int_0^T \left(\frac{A}{T}t - \frac{A}{2}\right)^2 dt = \frac{A^2}{12}, \qquad D = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} e^2 de = \frac{\Delta^2}{12}$$

$$SQNR = \frac{A^2}{\Delta^2}, \qquad Here\ \Delta = \frac{2\left(\frac{A}{2}\right)}{L} = \frac{A}{2^n} \Longrightarrow SQNR = 4^n$$

$$in\ dB, SQNR = 10\log 4^n = 6.02n = 6.02 \times 3 = 18.06\ dB.$$

Problem 15

Question

Design a 7-level uniform quantizer for the input signal with a dynamic range of ±10V. Note that the quantized values include the zero level (the quantized values are 0, $\pm\frac{\Delta}{2}$, $\pm\frac{3\Delta}{2}$, $\pm\frac{5\Delta}{2}$).

Answer

The last quantized value is $\frac{5\Delta}{2} \Longrightarrow \frac{5\Delta}{2} + \frac{\Delta}{2} = 3\Delta = x_{max} \Longrightarrow \Delta = \frac{10}{3}$.

$$y(t) = \begin{cases} -\frac{25}{3}, & -10 < t < -\frac{20}{3} \\ -5, & -\frac{20}{3} < t < -\frac{10}{3} \\ -\frac{5}{3}, & -\frac{10}{3} < t < 0 \\ 0, & t = 0 \\ \frac{5}{3}, & 0 < t < \frac{10}{3} \\ 5, & \frac{10}{3} < t < \frac{20}{3} \\ \frac{25}{3}, & \frac{20}{3} < t < 10 \end{cases}$$