Problem 2: 20 Points

Consider a digital communication system that transmits one of four signals every T_s seconds over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The signals occur with equal probabilities. Assume $T_s = nT_c$; n an integer. The space bases functions are:

$$\emptyset_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t) \qquad \emptyset_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t)$$

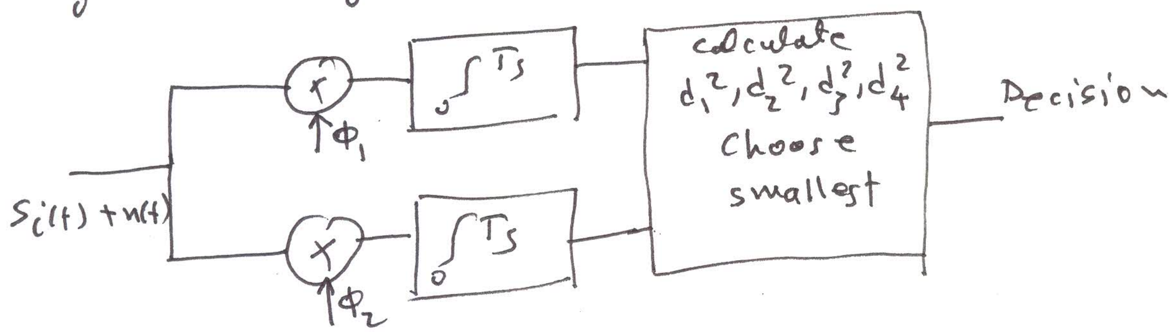
The transmitted signals are given by:

$$s_1(t) = 2\emptyset_1(t) + 2\emptyset_2(t) \qquad s_2(t) = -2\emptyset_1(t) + 2\emptyset_2(t)$$
$$s_3(t) = -2\emptyset_1(t) - 2\emptyset_2(t) \qquad s_4(t) = 2\emptyset_1(t) - 2\emptyset_2(t)$$

4 a. Find the energy in $s_4(t)$.
6 b. Draw the block diagram of the optimum receiver, showing the details of each block
4 c. Find the distance d_{13}^2 between $s_1(t)$ and $s_3(t)$
6 d. Find the average probability of error

a. $E_4 = \int_0^{T_s} s_4(t)^2 dt = \int_0^{T_s} [2\phi_1 + 2\phi_2]^2 dt = 8$

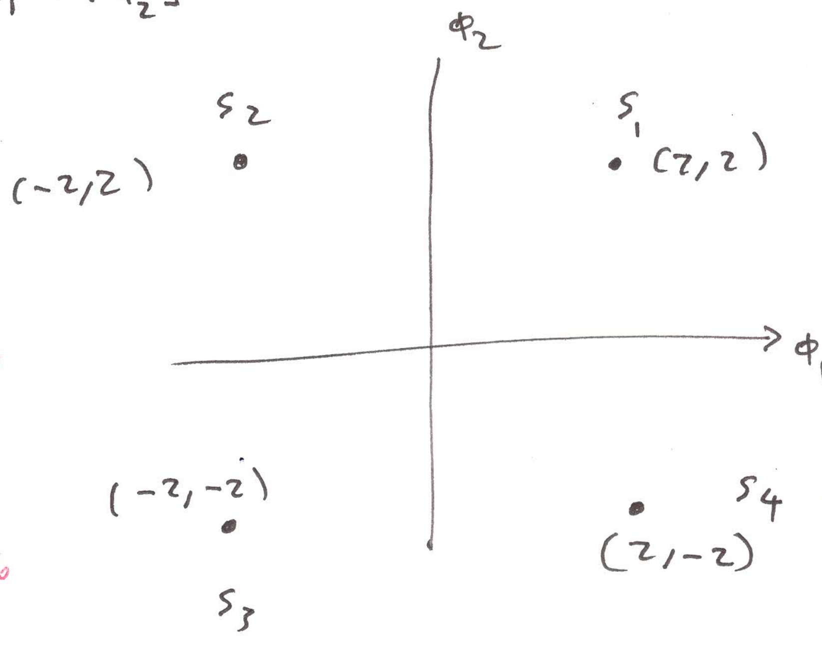
b.



c. $d_{13}^2 = \int_0^{T_s} (s_1 - s_3)^2 dt = \int_0^{T_s} [4\phi_1 + 4\phi_2]^2 dt$

$d_{13}^2 = 16 + 16 = 32$

$d_{13} = \sqrt{32} = 4\sqrt{2}$



d. $P_s = 2Q\left(\frac{d_{12}}{\sqrt{2N_0}}\right)$ 3/6

$P_s = 2Q\left(\frac{4}{\sqrt{2N_0}}\right)$ 3/6

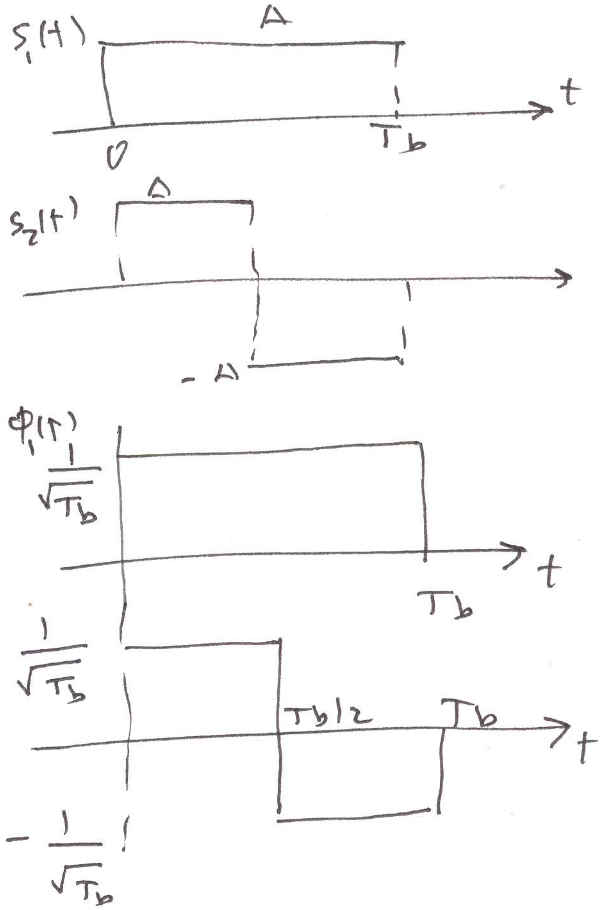
$= 2Q\left(\sqrt{\frac{8}{N_0}}\right)$

Problem 3:

Consider a binary digital communication system that transmits one of two possible symbols every T_b seconds over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The signals occur with equal probabilities. The transmitted signals are:

$$s_1(t) = \begin{cases} A, & 0 \le t \le T_b \\ 0, & otherwise \end{cases} \qquad s_2(t) = \begin{cases} A, & 0 \le t \le T_b/2 \\ -A, & T_b/2 \le t \le T_b \end{cases}$$

5 a. Are $s_1(t)$ and $s_2(t)$ orthogonal? Prove your answer
5 b. Make use of the result of Part a to find the set of bases functions for the signal space.
5 c. Find and sketch the signal space representation of the signals.
5 d. Find the system average probability of error



a.

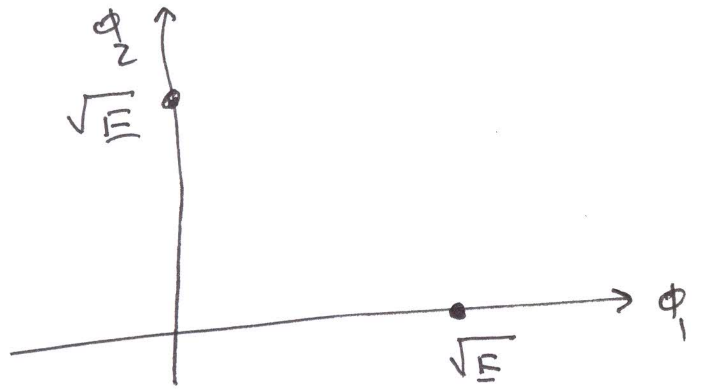
$$\int_0^{T_b} s_1(t)\, s_2(t)\, dt = 0$$

$\Rightarrow$

b. $$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \frac{s_1(t)}{\sqrt{A^2 T_b}} = \frac{A}{\sqrt{A^2 T_b}}$$

$$\phi_2(t) = \frac{s_2(t)}{\sqrt{E_2}}$$

c.



$E_1 = A^2 T_b$, $E_2 = A^2 T_b$

d. $$P_b = Q\left(\frac{d_{12}}{\sqrt{2N_0}}\right) = Q\left(\frac{\sqrt{2E}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

Problem 4: 20 Points

In a 4-station CDMA system, the binary chip sequences (signature waveforms) assigned for users A, B, C, and D are

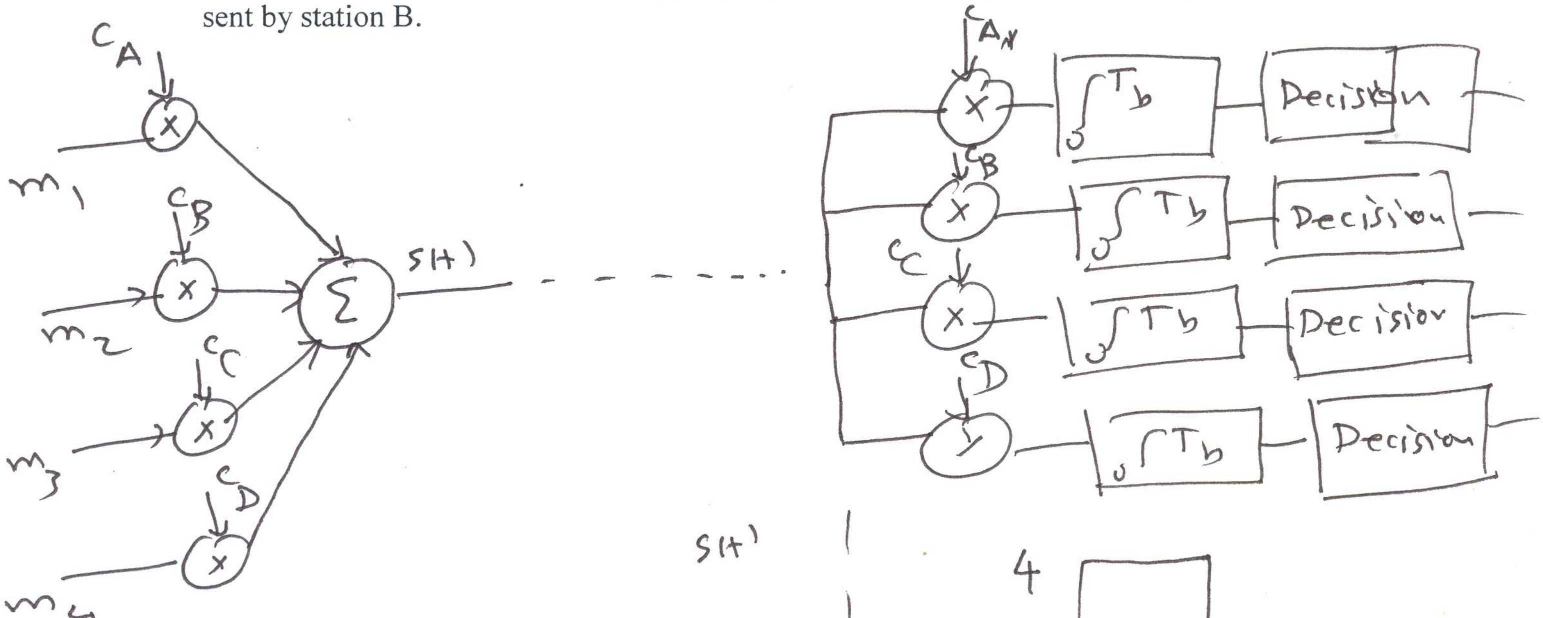
A = {+1 +1 +1 +1} B = {+1 -1 +1 -1} C = {-1 -1 +1 +1} D = {-1 +1 +1 -1}.

The chip duration = T_c and the bit duration = T_b

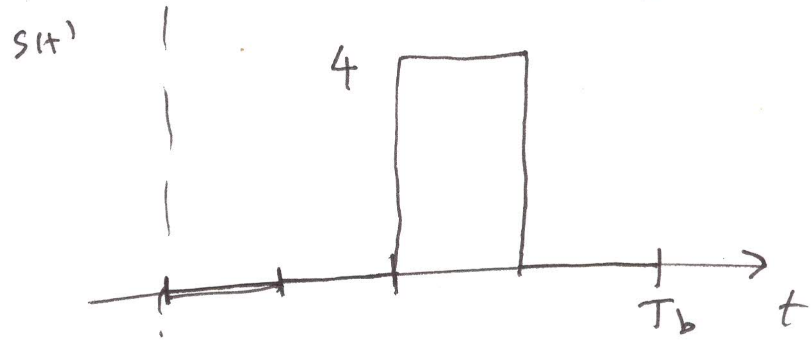
8 a. Draw the block diagram of the transmitter and the receiver, showing the details of each block.

6 b. Find and sketch the transmitted signal for $0 \le t \le T_b$ when each one of the four stations transmits digit 1.

6 c. If the receiver observes the following chip signal [+2 -2 +2 +2] for $0 \le t \le T_b$, find the bit sent by station B.



b.



c.

$$\int_0^{T_b} s(t)\, C_B(t)\,dt \Rightarrow$$

$$= 2\left(\frac{3T_b}{4}\right) - 2\left(\frac{T_b}{4}\right)$$

$$= \frac{6T_b - 2T_b}{4} = \frac{4T_b}{4}$$

$$= T_b > 0$$

$\Rightarrow$ digit 1