جامعة بيرزيت

BIRZEIT UNIVERSITY

Faculty of Engineering and Technology

Department of Electrical and Computer Engineering

Modern Communication Systems ENEE 3306

Instructor: Dr. Wael Hashlamoun

Midterm Exam

Second Semester 2017-2018

Date: Sunday 15/4/2018 Time: 75 minutes

Name: Student #:

Opening Remarks:

- Calculators are allowed, however, mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem 1: 25 Points

The signal $m(t)$, with spectrum $M(f)$, given below, is ideally sampled at a rate of f_s samples/sec to generate the signal $m_s(t)$

$$M(f) = \begin{cases} M_0, & -100 \le f \le 100 \\ 0, & otherwise \end{cases}$$

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a. Sketch $M_s(f)$, the spectrum of $m_s(t)$, when $f_s = 180$ samples/sec.

b. If $m_s(t)$ in Part a is passed through an ideal low pass filter with a bandwidth of 100 Hz to produce an output $y_1(t)$.
   - Sketch $Y_1(f)$, the spectrum of the filter output.
   - Is $y_1(t)$ *proportional to* $m(t)$? What does that mean in terms of reconstructing $m(t)$.

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c. Sketch $M_s(f)$, the spectrum of $m_s(t)$, when $f_s = 240$ samples/sec.

d. If $m_s(t)$ in Part c is passed through an ideal low pass filter of bandwidth 110 Hz to produce an output $y_2(t)$.
   - Sketch $Y_2(f)$, the spectrum of the filter output.
   - Is $y_2(t)$ *proportional to* $m(t)$? What does that mean in terms of reconstructing $m(t)$.

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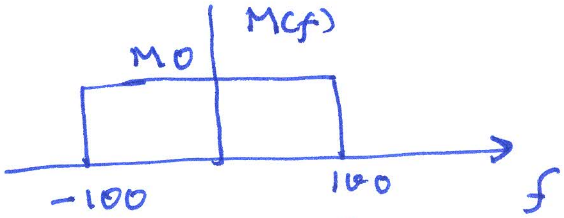
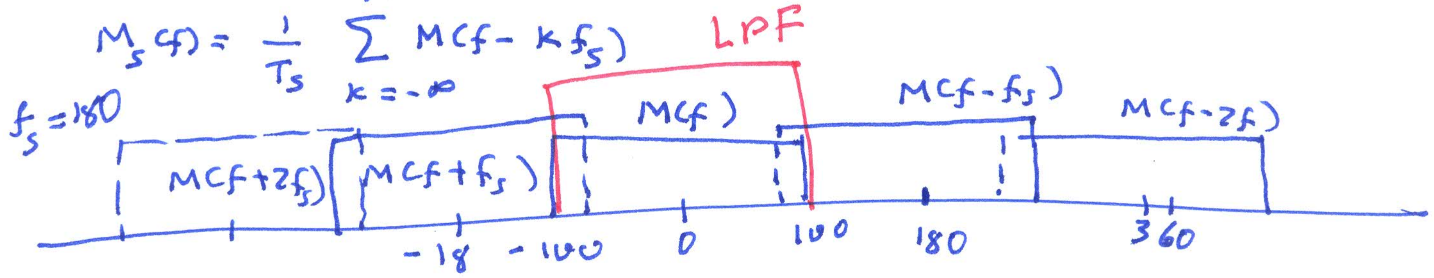
e. Which one of the above two sampling frequencies would you recommend and why?

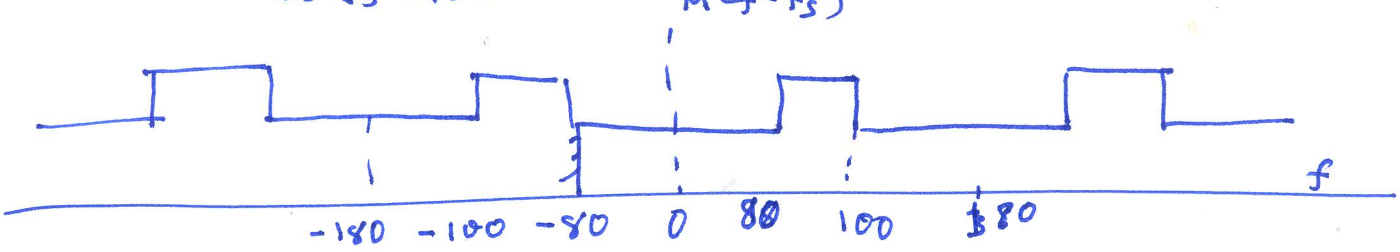


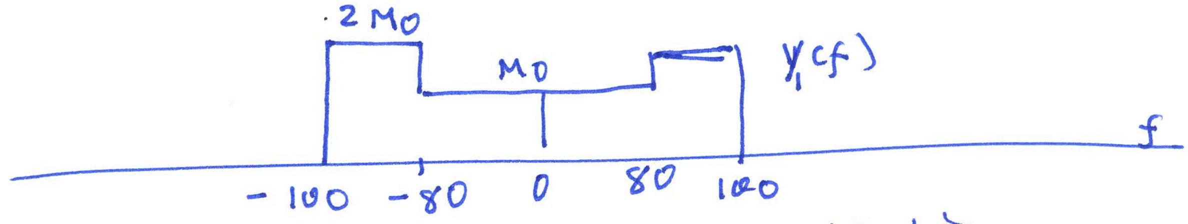
$$M_s(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} M(f - k f_s)$$

$f_s = 180$



For $-100 < f < 100$

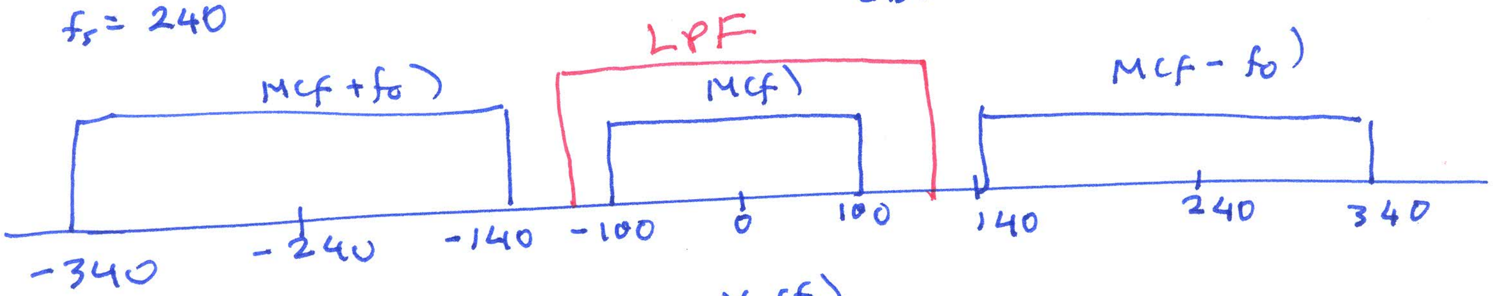


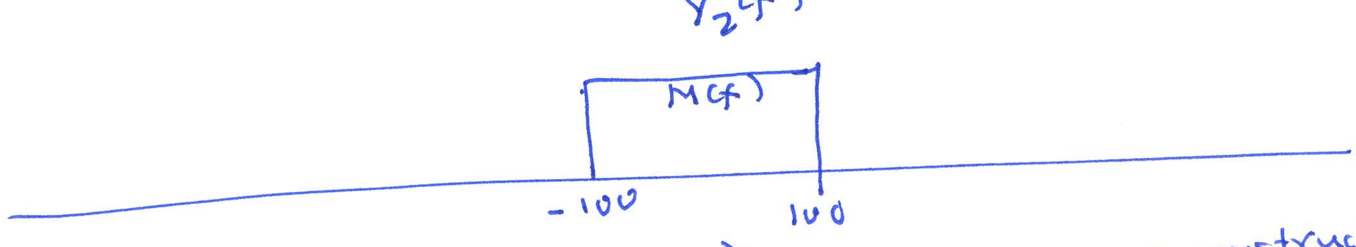


$Y_1(f) \neq K_1 M(f) \Rightarrow$ Aliasing $\Rightarrow$ distortion

$\Rightarrow$ m(t) cannot be reconstructed without distortion

$f_s = 240$





$Y_2(f) = K_2 M(f) \Rightarrow$ m(t) can be reconstructed from $m_s(t)$ without distortion

$\Rightarrow$ second case: $f_s \geq 2W$

$240 = f_s \geq 2(100) = 200$

$\Rightarrow$ select second sampling rate since it is higher than the nyquist rate

$\Rightarrow$ No distortion.

Problem 2: 25 Points

The signal $m(t) = \cos(2\pi(150)t)$ is to be transmitted using a PCM system (a system composed of a sampler, quantizer, and a binary encoder).

a. If sampling is done at the Nyquist rate and a uniform quantizer with 32 levels and a dynamic range between (-1, 1) is employed,

- What is the resulting data rate in bits/sec
- What is the resulting SQNR?
- If the encoder output is modulated using binary phase shift keying, find the 90% modulated signal bandwidth.
- If the encoder output is converted into polar non-return to zero format, find the 90% modulated signal bandwidth.

b. Find the SQNR if the signal is sampled at 1.2 times the Nyquist rate.

c. Find the data rate in bits per second if a nonuniform quantizer with 32 levels is used.

a. 6 $R_b = f_s \log_2 M = 2(150) \times 5 = 1500$ bits/sec

7 $SQNR = \dfrac{\langle m^2(t) \rangle}{\Delta^2/12} = \dfrac{A_m^2/2}{[2/32]^2/12} = \dfrac{0.5}{3.255 \times 10^{-4}} = 1536$

$A_m = 1$

$\Delta = \dfrac{m_{max} - (m)_{min}}{M} = \dfrac{2}{32} = 0.0625$

3 BPSK $B.W = 2R_b = 3{,}000$ Hz

3 PNRZ $B.W = R_b = 1500$ Hz

3 b. $SQNR = \dfrac{A_m^2/2}{\Delta^2/12} = 1536$ (same as above)

3 c. $R_b = f_s \log_2 M = 1500$ bits/sec

Problem 3: 25 Points

Consider the signal $x(t) = 4\sin(2\pi t)$.

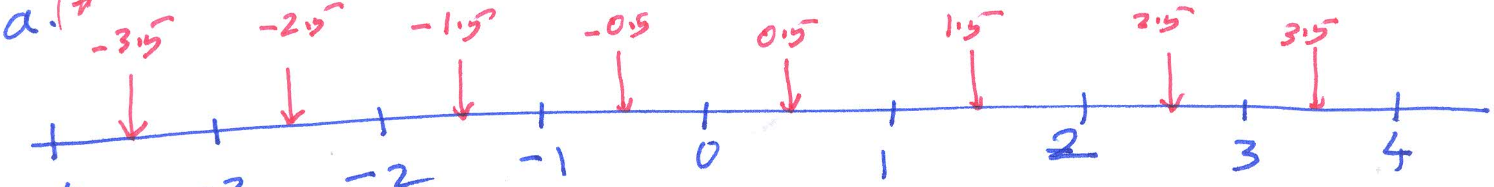
8 a. Design an 8-level uniform quantizer with a dynamic range (-4, 4) V, i.e., find the thresholds and representation values.

7.5 b. One sample is taken from the signal $x(t)$ at time t=0.05 and applied to the uniform quantizer of Part a. Find the received signal value corresponding to this sample.

7.5 c. One sample is taken from the signal $x(t)$ at time t=0.05 and applied to a μ-law companding system with $\mu = 255$ (a compressor followed by the uniform quantizer of Part a and an expander). Find the received signal value corresponding to this sample.

2 d. Compare the amount of distortion incurred on the sample by the two quantizers.

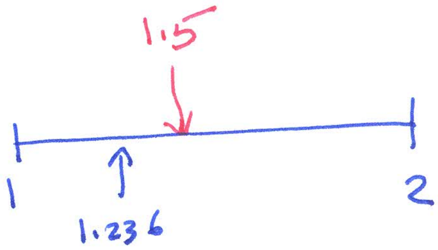
a.



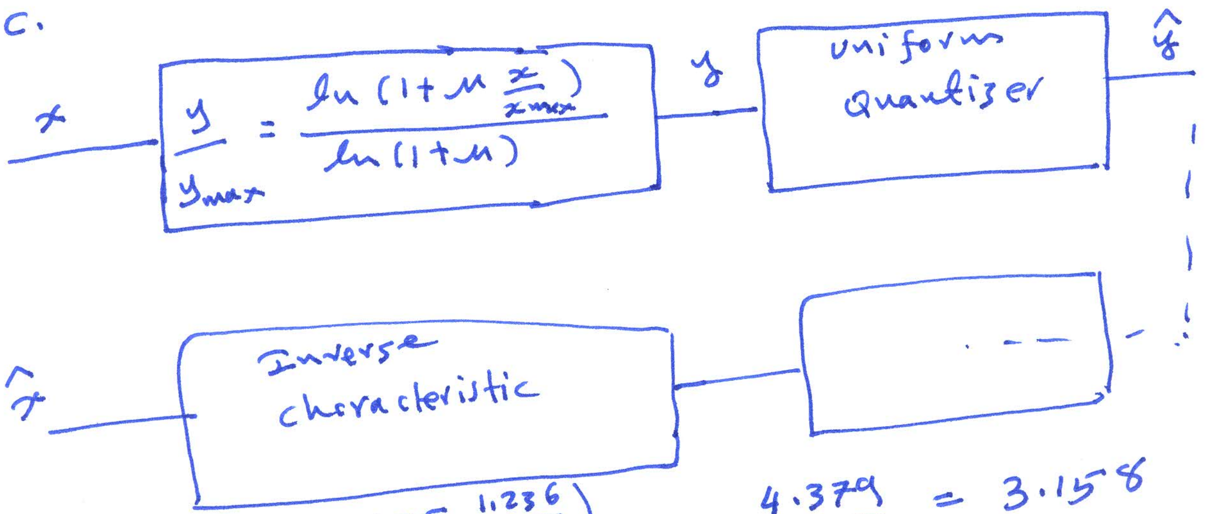
b. $x(0.05) = 4\sin(2\pi * 0.05) = 1.236$

$\boxed{\hat{x} = 1.5} \Rightarrow e = |x - \hat{x}| = |1.236 - 1.5|$

$= 0.264$



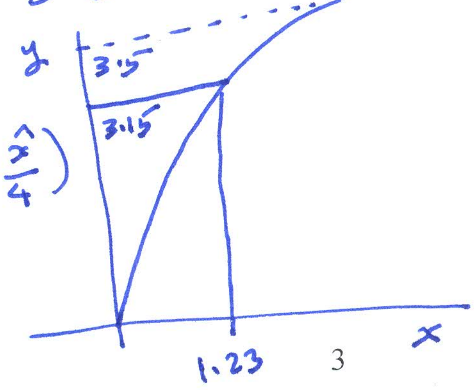
c.



$$y = 4\,\frac{\ln\left(1+255\,\frac{1.236}{4}\right)}{\ln(1+255)} = 4 * \frac{4.379}{5.545} = 3.158$$

$\Rightarrow \boxed{\hat{y} = 3.5}$

$$\frac{3.5}{4} = \frac{\ln\left(1+255\,\frac{\hat{x}}{4}\right)}{\ln(1+255)} \Rightarrow 4.85 = \ln\left(1+255\,\frac{\hat{x}}{4}\right)$$



$\Rightarrow \boxed{\hat{x} = 1.98}$

$|e| = |x - \hat{x}| = |1.236 - 1.98| = \boxed{0.744}$

Problem 4: 25 Points

A binary digital signaling scheme employs the following two equally probable signals $s_1(t)$ and $s_2(t)$ to represent binary logic 1 and 0, respectively, over a channel corrupted by AWGN with power spectral density $N_0/2$ W/Hz:

$$s_1(t) = A\cos\left(\frac{\pi t}{\tau}\right), \quad 0 \le t \le \tau$$

$$s_2(t) = B\cos\left(\frac{\pi t}{\tau}\right), \quad 0 \le t \le \tau$$

a. Find the energy, E_1, in $s_1(t)$ and the energy, E_2, in $s_2(t)$.
b. Find the average probability of error of the optimum receiver.
c. What is the probability of error when $A = B$?
d. Find the relationship between A and B such that the probability of error is minimized.
e. Draw the optimum receiver, implemented in terms of a correlator, for the general case when $A \neq B$, indicating the parameters of the main receiver units.

Good Luck

a. $E_1 = \int_0^{\tau} s_1(t)^2 dt = \int_0^{\tau} A^2 \cos^2\left(\frac{\pi t}{\tau}\right) dt = \frac{A^2}{2}\int_0^{\tau}\left[1 + \cos\left(\frac{2\pi}{\tau} t\right)\right]dt = \frac{A^2 \tau}{2}$

5

$E_2 = \int_0^{\tau} s_2(t)^2 dt = \frac{B^2 \tau}{2}$

b. $\int_0^{\tau} (s_1(t) - s_2(t))^2 dt = \int_0^{\tau} (A-B)^2 \cos^2 \frac{\pi t}{\tau}\, dt = \frac{(A-B)^2 \tau}{2}$

6

$P_b = Q\left(\sqrt{\frac{\int_0^{\tau}(s_1 - s_2)^2 dt}{2N_0}}\right) = Q\left(\sqrt{\frac{(A-B)^2 \tau}{4 N_0}}\right)$

4 c. $P_b = Q(0) = 0.5$

4 d. maximize : $(A-B)^2$

Let $g = (A-B)^2 \Rightarrow$

g is max when $\boxed{B = -A}$

5 e-

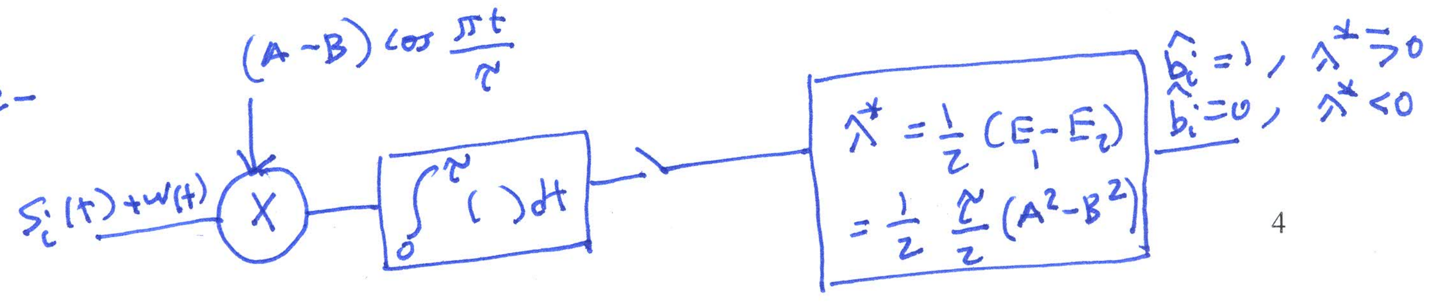


TABLE 4-2
Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$	Comments on Derivation
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc} \tau f$	Direct evaluation
2.	$2W \operatorname{sinc} 2Wt$	$\Pi\left(\frac{f}{2W}\right)$	Duality with pair 1, Example 4-7
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2 \tau f$	Convolution using pair 1
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$	Direct evaluation
5.	$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$	Differentiation of pair 4 with respect to α
6.	$\exp(-\alpha\lvert t\rvert),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$	Direct evaluation
7.	$e^{-\pi(t/\tau)^2}$	$\tau e^{-\pi(f\tau)^2}$	Direct evaluation
8.	$\delta(t)$	1	Example 4-9
9.	1	$\delta(f)$	Duality with pair 7
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$	Shift and pair 7
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$	Duality with pair 9
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$	Exponential representation of cos and sin and pair 10
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$	
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$	Integration and pair 7
15.	$\operatorname{sgn} t$	$(j\pi f)^{-1}$	Pair 8 and pair 13 with superposition
16.	$\frac{1}{\pi t}$	$-j \operatorname{sgn}(f)$	Duality with pair 14
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda}\, d\lambda$	$-j \operatorname{sgn}(f) X(f)$	Convolution and pair 15
18.	$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s),\ f_s = T_s^{-1}$	Example 4-10

TABLE A6.4 ***Trigonometric Identities***

$$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$$
$$\cos\theta = \tfrac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$$
$$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$
$$\cos^2\theta = \tfrac{1}{2}[1 + \cos(2\theta)]$$
$$\sin^2\theta = \tfrac{1}{2}[1 - \cos(2\theta)]$$
$$2\sin\theta\cos\theta = \sin(2\theta)$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

Table of Standard Integrals

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

2. $\int \frac{dx}{x} = \ln|x| + C$

3. $\int e^x\,dx = e^x + C$

4. $\int \sin x\,dx = -\cos x + C$

5. $\int \cos x\,dx = \sin x + C$

6. $\int \tan x\,dx = -\ln|\cos x| + C$

7. $\int \cot x\,dx = \ln|\sin x| + C$

8. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

9. $\int \sec^2 x\,dx = \tan x + C$

10. $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$

11. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

12. $\int \operatorname{cosec} x\,dx = \ln|\operatorname{cosec} x - \cot x| + C$

13. $\int \sinh x\,dx = \cosh x + C$

14. $\int \cosh x\,dx = \sinh x + C$

15. $\int \tanh x\,dx = \ln\cosh x + C$

16. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \quad (|x| < a)$

17. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C = \ln\left(x + \sqrt{x^2 + a^2}\right) + C'$

18. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C = \ln\left(x + \sqrt{x^2 - a^2}\right) + C' \quad (x > a)$

Q-Function Table

z	$Q(z)$	z	$Q(z)$
0.0	0.50000	2.0	0.02275
0.1	0.46017	2.1	0.01786
0.2	0.42074	2.2	0.01390
0.3	0.38209	2.3	0.01072
0.4	0.34458	2.4	0.00820
0.5	0.30854	2.5	0.00621
0.6	0.27425	2.6	0.00466
0.7	0.24196	2.7	0.00347
0.8	0.21186	2.8	0.00256
0.9	0.18406	2.9	0.00187
1.0	0.15866	3.0	0.00135
1.1	0.13567	3.1	0.00097
1.2	0.11507	3.2	0.00069
1.3	0.09680	3.3	0.00048
1.4	0.08076	3.4	0.00034
1.5	0.06681	3.5	0.00023
1.6	0.05480	3.6	0.00016
1.7	0.04457	3.7	0.00011
1.8	0.03593	3.8	0.00007
1.9	0.02872	3.9	0.00005