Problem 2: 20 Points

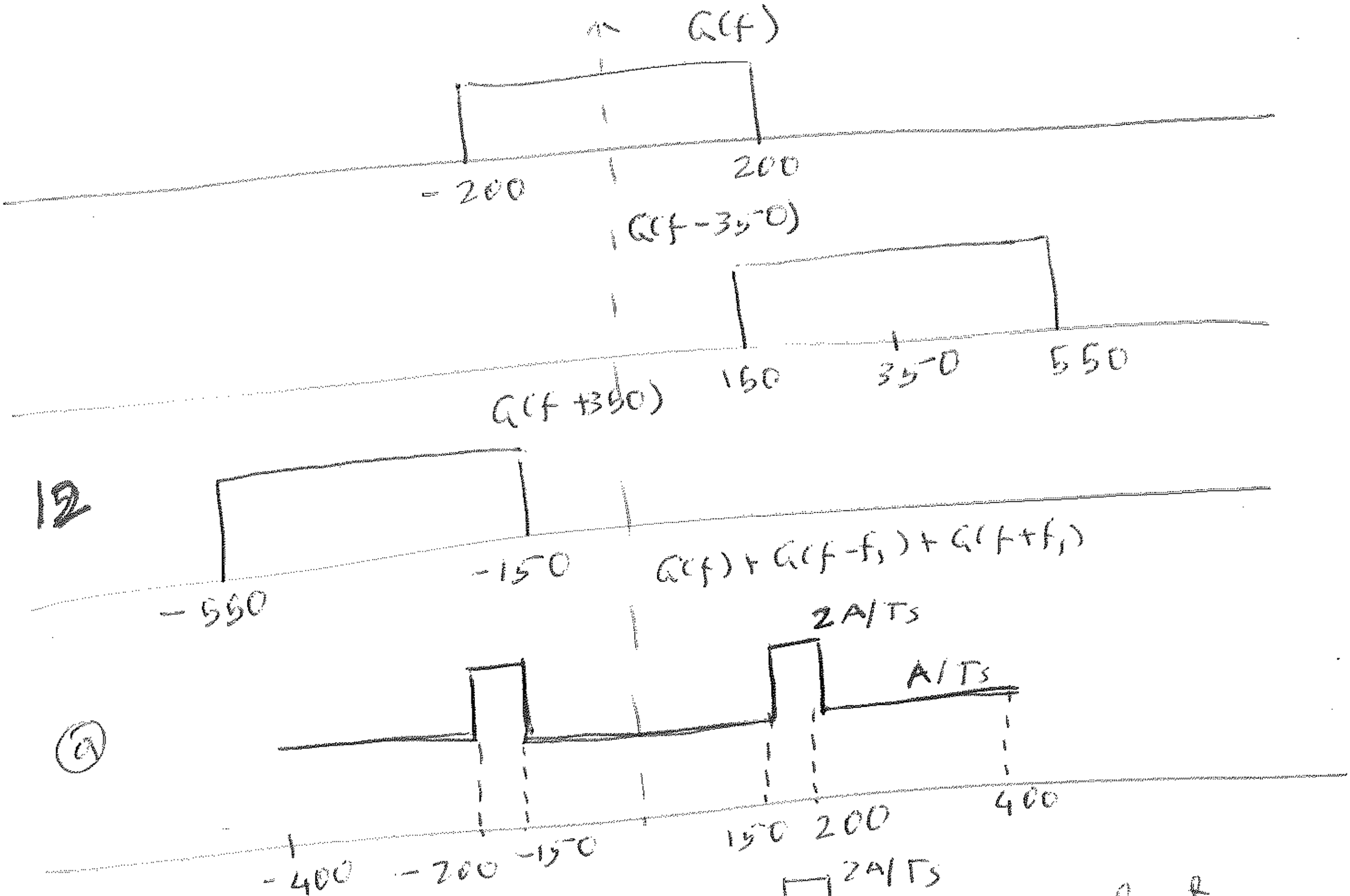
The Fourier transform, $G(f)$, of a signal $g(t)$ is given as:

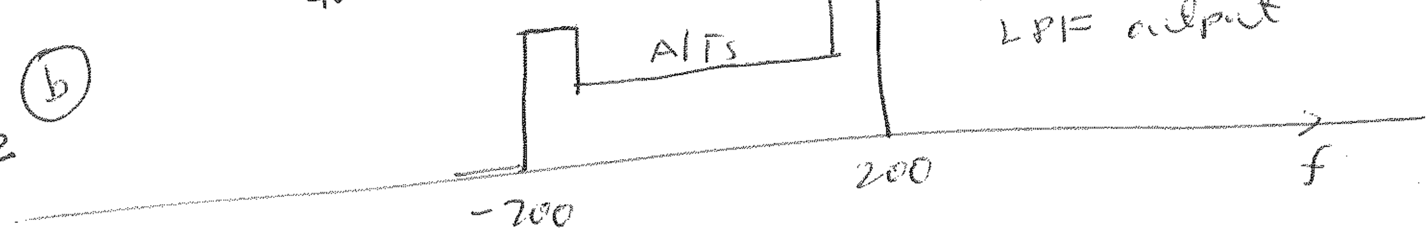
$$G(f) = \begin{cases} A, & -200 \le f \le 200 \\ 0, & |f| > 200 \end{cases}$$

The signal $g(t)$ is ideally sampled at a rate of 350 samples/sec to produce the samples signal $g_s(t)$.

a. Find and sketch $G_s(f)$, the Fourier transform of $g_s(t)$ for $-400 \le f \le 400$
b. If $g_s(t)$ is applied to an ideal low pass filter with a bandwidth of 200 Hz, sketch the Fourier transform of the signal appearing at the output of the filter.
c. Based on the results of Part b, do you think that $g(t)$ can be recovered from $g_s(t)$ without distortion? Explain why.

$$G_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f - nf_s)$$





4 (c) since $f_s < 2(200) = 400 \Rightarrow$ Distortion

$\Rightarrow g(t)$ cannot be recovered.

Problem 3: 18 Points

The signal $x(t) = 4cos(2\pi f_0 t)$ is applied to a uniform quantizer with L quantization levels and a dynamic range $(-4, 4)$ V. Find the minimum value of L that will achieve a signal to quantization noise ratio $SQNR \geq 1000$.

4 $\Delta = \frac{4-(-4)}{L} = \frac{8}{L}$

4 $\langle x(t)^2 \rangle = \frac{A_m^2}{2} = \frac{(4)^2}{2} = 8$; average signal power

4 quantization noise $= \frac{\Delta^2}{12}$

$SQNR = \frac{\langle x(t)^2 \rangle}{\Delta^2/12} = \frac{8}{(8/L)^2/12} = \frac{8 \times 12 \times L^2}{64}$

$SQNR = \frac{3}{2} L^2 \geq 1000$

6

$L^2 \geq \frac{2000}{3}$

$L \geq \sqrt{\frac{2000}{3}}$

$L \geq 26$

Problem 4: 22 Points

The binary digital communication signaling scheme, discussed in class, employs the following two equally probable signals $s_1(t)$ and $s_2(t) = -s_1(t)$ to represent binary logic 1 and 0 respectively over a channel corrupted by AWGN with power spectral density $N_0/2$ W/Hz. Here,

$$s_1(t) = \begin{cases} A\dfrac{t}{\tau}, & 0 \le t \le \tau \end{cases}$$

where τ is the binary symbol duration.

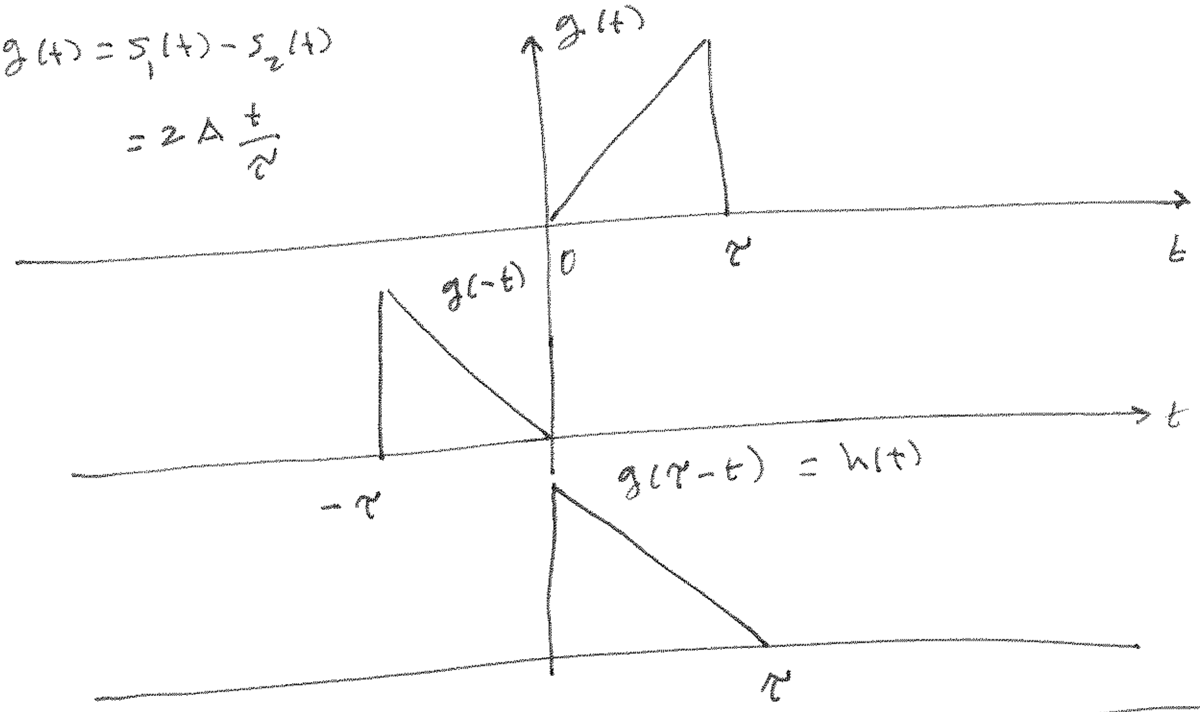
8 a. Find and sketch the impulse response, $h(t)$, of the matched filter, designed to minimize the probability of error.

6 b. Find the optimum threshold used by the threshold detector at the receiver.

8 c. Find the system average probability of error. Leave your answer in terms of the Q function.

Good Luck

$$g(t) = s_1(t) - s_2(t) = 2A\frac{t}{\tau}$$



$$\lambda^* = \frac{1}{2}(E_1 - E_2) = 0 \;;\; P_b = Q\left(\sqrt{\frac{\int_0^\tau (s_1 - s_2)^2\,dt}{2N_0}}\right)$$

$$\int_0^\tau (s_1 - s_2)^2\,dt = \int_0^\tau \left(\frac{2At}{\tau}\right)^2 dt = \frac{4A^2}{\tau^2}\int_0^\tau t^2\,dt$$

$$= \frac{4A^2}{\tau^2}\cdot\frac{\tau^3}{3} = \frac{4}{3}A^2\tau$$

$$P_b = Q\left(\sqrt{\frac{4A^2\tau}{6N_0}}\right) = Q\left(\sqrt{\frac{2A^2\tau}{3N_0}}\right)$$ 7

Angle Sum and Difference Formulas

$$\sin(\theta \pm \varphi) = \sin\theta\cos\varphi \pm \cos\theta\sin\varphi$$

$$\cos(\theta \pm \varphi) = \cos\theta\cos\varphi \mp \sin\theta\sin\varphi$$

$$\tan(\theta \pm \varphi) = \frac{\tan\theta \pm \tan\varphi}{1 \mp \tan\theta\tan\varphi}$$

Sum-to-Product Formulas

$$\sin\theta + \sin\varphi = 2\sin\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)$$

$$\sin\theta - \sin\varphi = 2\cos\left(\frac{\theta+\varphi}{2}\right)\sin\left(\frac{\theta-\varphi}{2}\right)$$

$$\cos\theta + \cos\varphi = 2\cos\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)$$

$$\cos\theta - \cos\varphi = -2\sin\left(\frac{\theta+\varphi}{2}\right)\sin\left(\frac{\theta-\varphi}{2}\right)$$

Product-to-Sum Formulas

$$\sin\theta\sin\varphi = \frac{1}{2}\left[\cos(\theta-\varphi) - \cos(\theta+\varphi)\right]$$

$$\cos\theta\cos\varphi = \frac{1}{2}\left[\cos(\theta-\varphi) + \cos(\theta+\varphi)\right]$$

$$\sin\theta\cos\varphi = \frac{1}{2}\left[\sin(\theta+\varphi) + \sin(\theta-\varphi)\right]$$

$$\cos\theta\sin\varphi = \frac{1}{2}\left[\sin(\theta+\varphi) - \sin(\theta-\varphi)\right]$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Half Angle Formulas

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Basic Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

Periodicity

$$\sin(\theta + 2\pi) = \sin\theta$$

$$\cos(\theta + 2\pi) = \cos\theta$$

$$\tan(\theta + \pi) = -\tan\theta$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\csc^2\theta - \cot^2\theta = 1$$

Co-Function Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

Even or Odd Symmetry

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta$$

$$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \tfrac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$$

$$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$$

TABLE A6.2 *Fourier-Transform Pairs*

Time Function	*Fourier Transform*
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
$\mathrm{sinc}(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$	$T\,\mathrm{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Table of Common Integrals

$$\int k\,dx = x + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \text{ for } n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = \ln|\csc x - \cot x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$