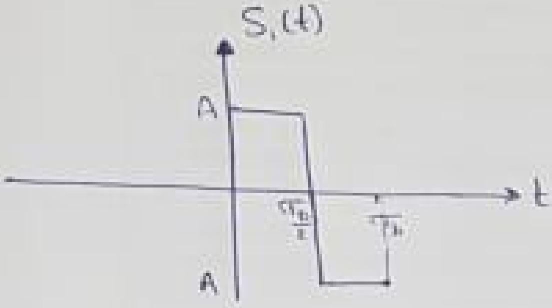
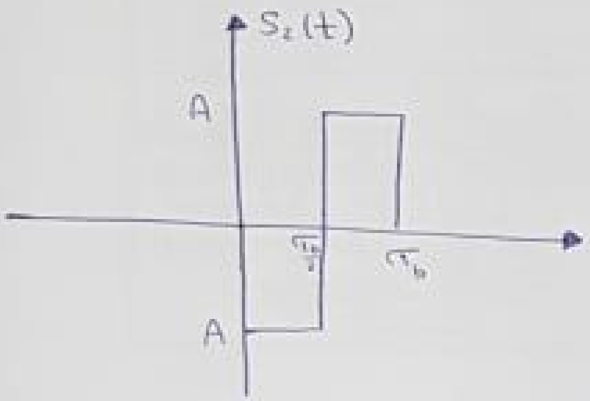
Problem N1

$$S_1(t) = \begin{cases} A & 0 \leq t \leq \frac{T_b}{2} \longrightarrow 1 \\ -A & \frac{T_b}{2} \leq t \leq T_b \longrightarrow 0 \end{cases}$$



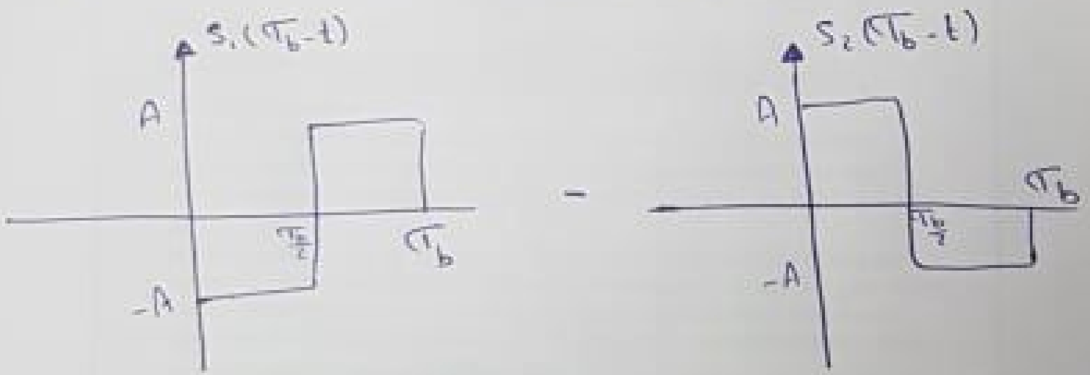
$P = 1/2$



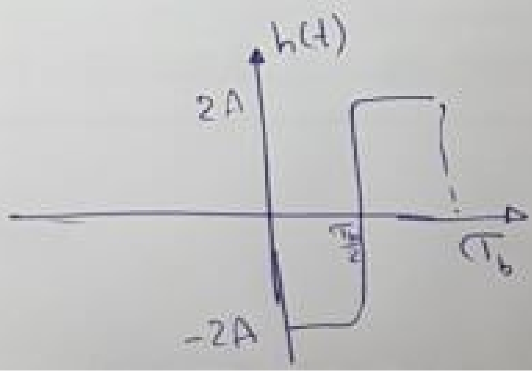
$P = 1/2$

① Impulse response:

$$h(t) = S_1(T_b - t) - S_2(T_b - t)$$



$h(t) =$



b) Average probability of error :

$$P_b = Q\left(\sqrt{\frac{\int_0^{T_b}(S_1 - S_2)^2 dt}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{\int_0^{\frac{T_b}{2}} 4A^2 dt + \int_{\frac{T_b}{2}}^{T_b} 4A^2 dt}{2N_0}}\right)$$

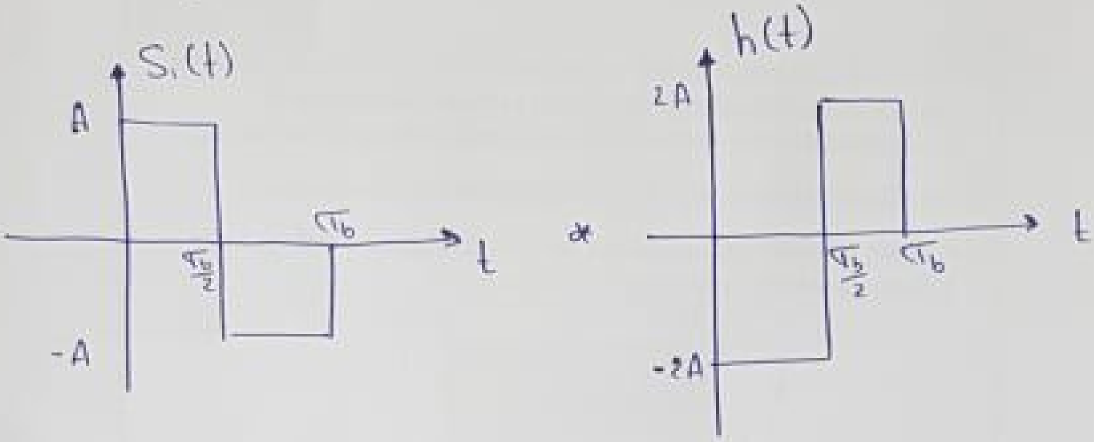
$$= Q\left(\sqrt{\frac{\left(\cancel{\frac{4A^2T_b}{2}}\right) + \left(4A^2T_b - \cancel{\frac{4A^2T_b}{2}}\right)}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{2A^2T_b}{N_0}}\right)$$

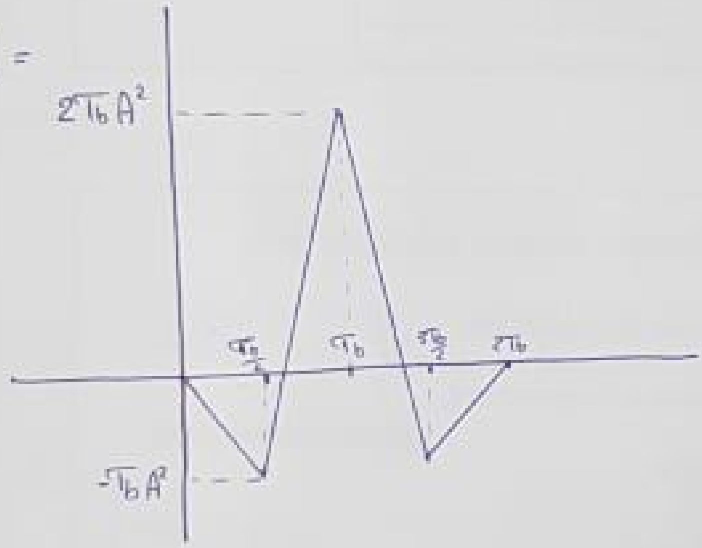
c)

$$S_1 \longrightarrow \boxed{\text{Matched Filter}} \longrightarrow y(t)$$

output of the matched filter is obtained by convolving $h(t)$ by $S_1(t)$:



$y(t) =$



$$y(T_b) = 2T_bA^2$$

Problem #2

FSK: $S_1(t) = A\cos(2\pi f_1 t) \quad 0 \le t \le T_b \longrightarrow 1$

$S_2(t) = A\cos(2\pi f_2 t) \quad 0 \le t \le T_b \longrightarrow 0$

PSK: $S_1(t) = B\cos(2\pi f_c t) \quad 0 \le t \le T_b \longrightarrow 1$

$S_2(t) = -B\cos(2\pi f_c t) \quad 0 \le t \le T_b \longrightarrow 0$

$$P_{b(FSK)} = P_{b(BSK)}$$

$$P_{b(BSK)} = P_{b(FSK)} = Q\left(\sqrt{\frac{Ed}{2N_0}}\right)$$

So lets Find (Ed) For FSK & BSK:

FSK:

$$Ed = \int_0^{T_b} \left(A\cos(2\pi f_1 t) - A\cos(2\pi f_2 t)\right)^2 dt$$

$$= A^2 T_b$$

$$P_{b\,FSK} = Q\left(\sqrt{\frac{A^2 T_b}{2N_0}}\right)$$

BSK :

$$E_d = \int_0^{T_b} \left(B\cos(2\pi f_c t) + B\cos(2\pi f_c t)\right)^2 dt$$

$$= 2B^2 T_b$$

$$P_{b\,BSK} = Q\left(\sqrt{\frac{B^2 T_b}{N_0}}\right)$$

To Find the Relationship between A & B , since They have the same probability of error:

$$P_{b\,FSK} = P_{b\,BSK}$$

$$\sqrt{\frac{A^2 \cancel{T_b}}{2\cancel{N_0}}} = \sqrt{\frac{B^2 \cancel{T_b}}{\cancel{N_0}}}$$

$$\frac{A}{\sqrt{2}} = B \longrightarrow \boxed{A = \sqrt{2}\,B}$$