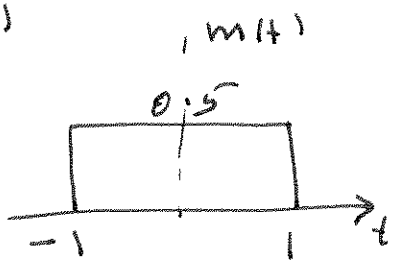
ENEE 339 June 12, 2017

Solution to Final Exam

Problem 1 $m(t) = \begin{cases} 0.5 & -1 < t < 1 \\ 0 & \text{o.w} \end{cases}$



a. $E = \int_{-1}^{1} (m(t))^2 dt = \int_{-1}^{1} (0.5)^2 dt$

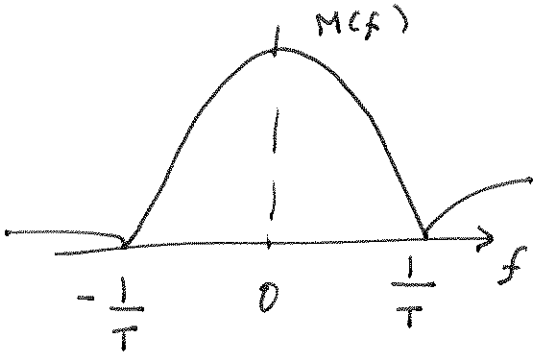
$= 0.5$

b. $m(t) = A\,\text{rect}\left(\frac{t}{T}\right) \Rightarrow M(f) = A\,T\,\text{sinc}\,fT$

$A = 0.5$

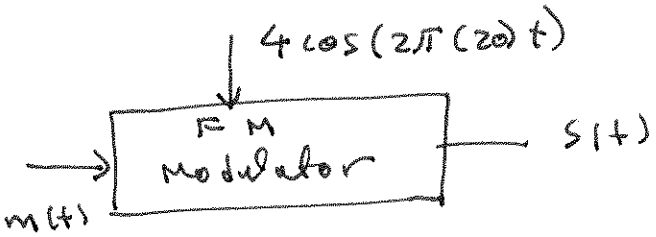
$T = 2$

$\Rightarrow M(f) = (0.5)(2)\,\text{sinc}(2f)$



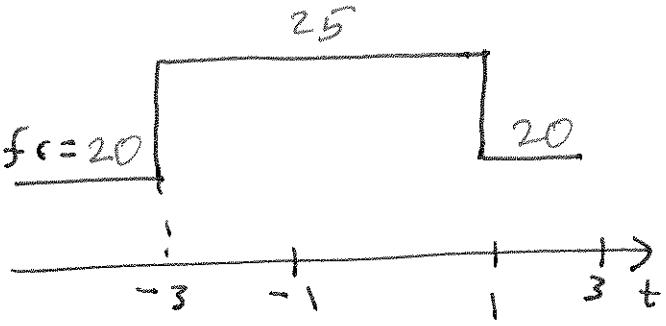
$B.W = \frac{1}{T} = \frac{1}{2}$

c.



$f_i(t) = f_c + K_f\, m(t)$

$= 20 + (10)\, m(t)$

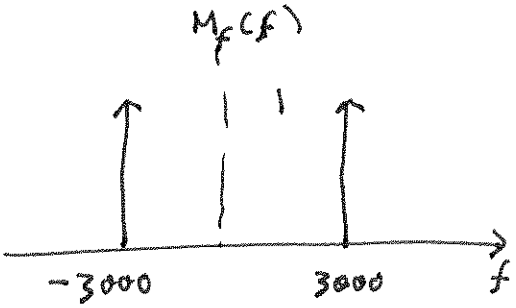


Problem 2

$m(t) = 2\cos(2\pi(3000)t) + 4\cos(2\pi 16000 t)$

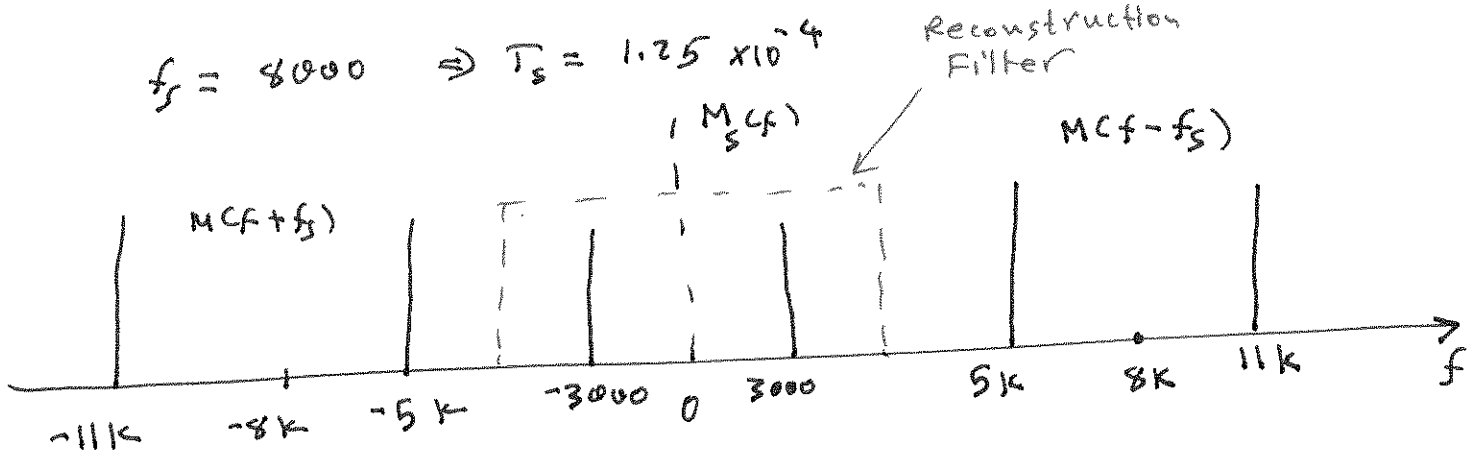
a. $m_f(t) = 2\cos(2\pi(3000)t)$

b. $M_f(f) = \delta(f-3000) + \delta(f+3000)$



c. $M_s(f) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} M_f(f - kf_s)$

$f_s = 8000 \Rightarrow T_s = 1.25 \times 10^{-4}$



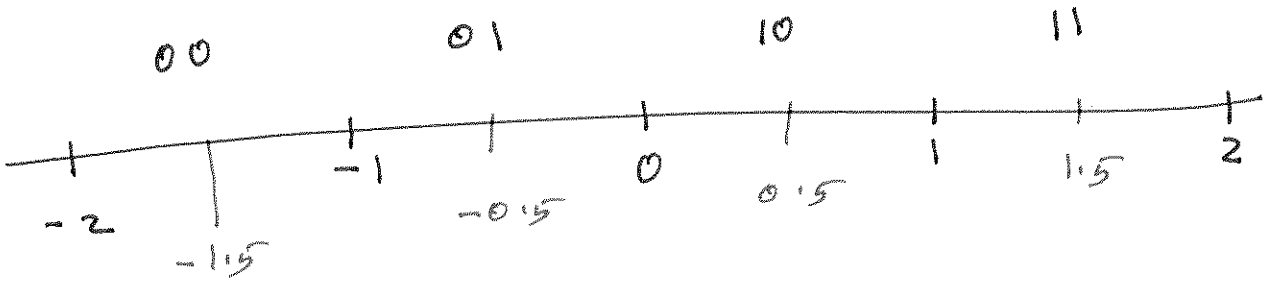
d. Reconstruction Filter: Low pass filter

e. $3000 < B.W < 5000$

f. $m_R(t) = f_s\, m_f(t)$

$m_R(t) = (8000)(2)\cos 2\pi(3000)t$

Problem 3



$f_s = 2 \times 4000 = 8000$ Hz $\Rightarrow T_s = 0.125$ ms

a. Time	0	0.125	0.25	0.375	0.5
sampled values (part a.)	1.25	-1.45	0.9	-0.45	-0.2
Quantized values (part b.)	1.5	-1.5	0.5	-0.5	-0.5
Binary Representation (part c)	11	00	10	01	01

d. $R_b = (f_s)$(# of bits/ quantized sample)

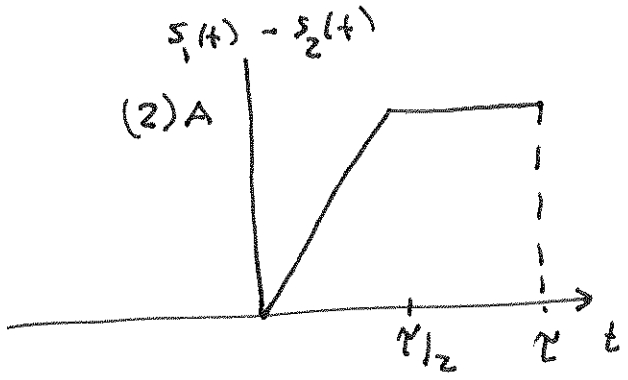
$= 2 \times 4000 \times 2 = 16{,}000$ bits/sec

e. $B.W = 2 \times R_b$

$= 2 \times 16{,}000 = 32{,}000$ Hz

Problem 4

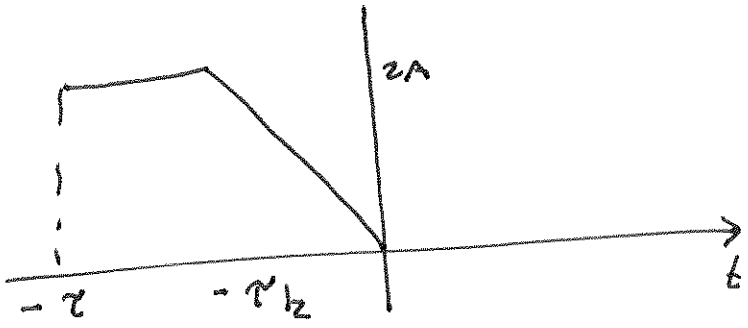
a. $h(t) = s_1(\tau - t) - s_2(\tau - t)$



b. $\lambda^* = \frac{1}{2}(E_1 - E_2)$

since $E_1 = E_2$

$\Rightarrow \lambda^* = 0$



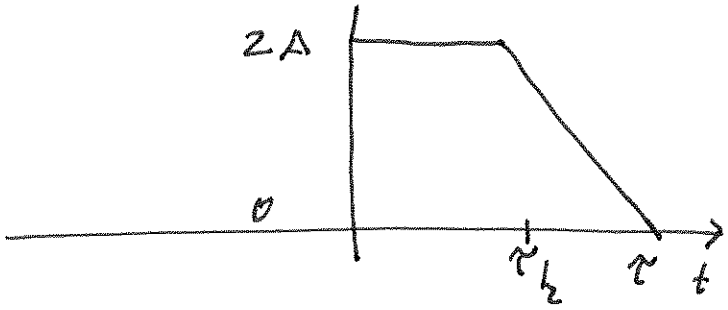
Find E_1

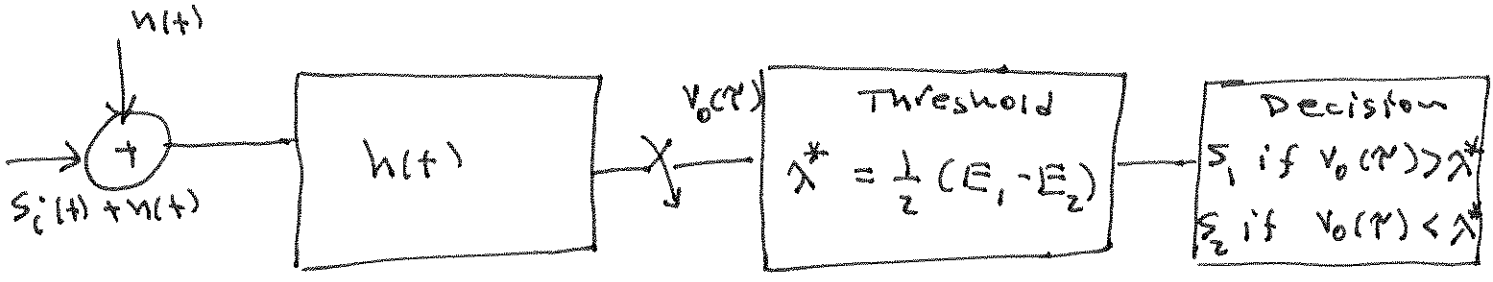
$$E_1 = \int_0^{\tau/2} \left(A\frac{2t}{\tau}\right)^2 dt + \int_{\tau/2}^{\tau} A^2\, dt$$

$$= \frac{1}{6}A^2\tau + A^2\frac{\tau}{2}$$

$$E_1 = \frac{2}{3}A^2\tau$$

$h(t) = s_1(\tau - t) - s_2(\tau - t)$





Problem 5

a. $s_1(t) = 4\cos 2\pi f_1 t \qquad 0 < t \le \tau$

$s_2(t) = 4\cos 2\pi f_2 t \qquad 0 < t \le \tau$

a. $P(E) = Q\left(\sqrt{\dfrac{\int_0^\tau (s_1(t) - s_2(t))^2}{2N_0}}\right)$

$$\int_0^\tau (s_1(t) - s_2(t))^2 dt = \int_0^\tau s_1(t)^2 dt + \int_0^\tau s_2(t)^2 dt - 2\int_0^\tau s_1(t)s_2(t) dt$$

($\int_0^\tau s_1(t)s_2(t)dt \approx 0$)

$$= \frac{A^2\tau}{2} + \frac{A^2\tau}{2} = A^2\tau$$

$$P(E) = Q\left(\sqrt{\frac{A^2\tau}{2N_0}}\right) = Q\left(\sqrt{\frac{8\tau}{N_0}}\right)$$

b. $P(E) = Q\left(\sqrt{\dfrac{8\tau}{0.001}}\right) \le 8.8417 \times 10^{-5}$

$$Q(x) \le 8.8417 \times 10^{-5} \Rightarrow x = 3.75$$

$$3.75 = \sqrt{\frac{8\tau}{0.001}} \Rightarrow 14.0625 = \frac{8\tau}{0.001}$$

$$\tau = 1.7578 \times 10^{-3} \Rightarrow \boxed{R_b \le 569 \text{ bits/sec}}$$

c.

$B.W = (f_1 - f_2) + 2R_b$

$= [(8-2) + 2]K$

$\boxed{B.W = 8\ KHz}$