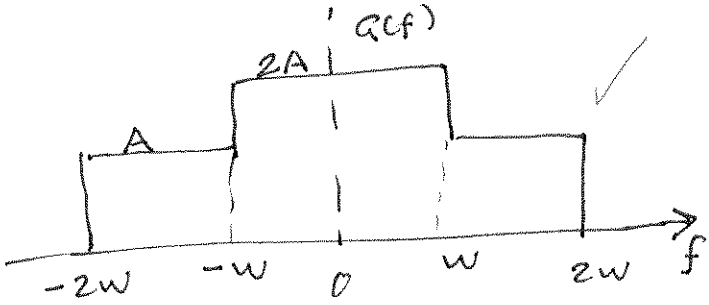
ENEE 339

Solution to Midterm April 23, 2017

Problem 1

a. $B.W = 2W$

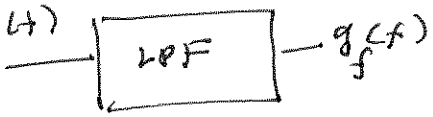


b. $E_g = 2\int_0^W (2A)^2 df + 2\int_W^{2W} (A)^2 df$

$E_g = 10 A^2 W$

c. $E' = 2\int_0^W (2A)^2 df + 2\int_W^{3W/2} (A)^2 df$

$E' = 9 A^2 W$



d. $G(f) = A\,\text{rect}\left(\frac{f}{4W}\right) + A\,\text{rect}\left(\frac{f}{2W}\right)$ (1)

From Table $\text{rect}\left(\frac{t}{T}\right) \rightarrow T\,\text{sinc}\,fT$

$\text{sinc}\,2Wt \rightarrow \frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$

$\Rightarrow 2W\,\text{sinc}\,2Wt \rightarrow \text{rect}\left(\frac{f}{2W}\right)$ (2)

using (2), (1) becomes in the time domain

$$g(t) = A(4W)\,\text{sinc}\,4Wt + A(2W)\,\text{sinc}\,2Wt$$

Problem 2

$m(t) = 2\cos 2\pi(40)t + 4\cos 2\pi(80)t$

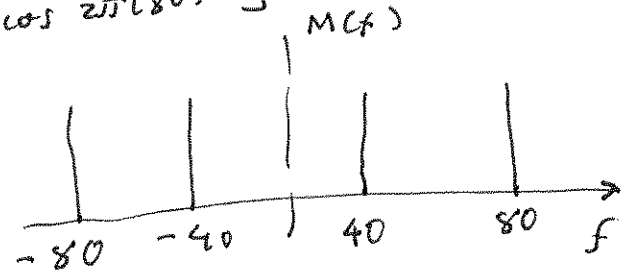
$c(t) = 4\cos 2\pi(1000)t$

a. $\langle m(t)^2 \rangle = \frac{(2)^2}{2} + \frac{(4)^2}{2}$; terms are orthogonal

$= 2 + 8 = 10\ W$

b. $s(t) = A_c\, m(t) \cos 2\pi f_c t$

$s(t) = 5\left[2\cos 2\pi(40)t + 4\cos 2\pi(80)t\right] \cos 2\pi(1000)t$

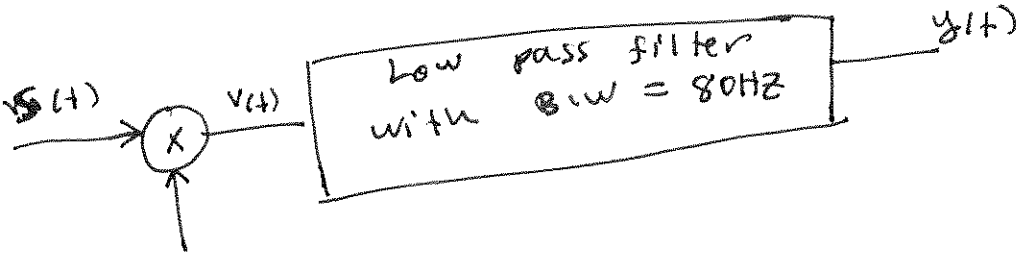


b. B.W $= 2W$

$= 2(80)$

$= 160\ Hz$

c.



$A_c' \cos 2\pi f_c t$

$A_c' \cos 2\pi(1000)t$

Analysis: $v(t) = A_c' \cos 2\pi f_c t\ s(t)$

$= A_c A_c' \cos 2\pi f_c t \cos 2\pi f_c t\ m(t)$

$= \frac{A_c A_c'}{2} m(t) \cos^2 2\pi f_c t$

$= \frac{A_c A_c'}{2} m(t)\left[1 + \cos 4\pi f_c t\right]$

$\Rightarrow \quad y(t) = \frac{A_c A_c'}{2} m(t).$

Problem 3

$$s(t) = A_c \cos 2\pi f_c t \,(1 + k_a m(t)) \quad ; \quad \begin{matrix} A_c = 10 \,, f_c = 10000 \\ k_a = 0.2 \end{matrix}$$

$$s(t) = A_c [1 + 0.2 \times 0.3 \cos 2\pi (500) t] \cos 2\pi f_c t$$

$$s(t) = A_c [1 + 0.6 \cos 2\pi (500) t] \cos 2\pi f_c t$$

a. M.I. $= 0.6$

b. $s(t) = 10 \cos 2\pi f_c t + 6 \cos 2\pi f_c t \cos 2\pi f_m t$

$$s(t) = 10 \cos 2\pi f_c t + 3 \cos 2\pi (f_c + 500) t + 3 \cos 2\pi (f_c - 500) t$$

carrier — sidebands

$$P_{av}(\text{carrier}) = \frac{A_c^2}{2} = \frac{10^2}{2} = 50$$

$$P_{av}(2 \text{ sidebands}) = \left(\frac{3^2}{2}\right) \times 2 = 9 \; ; \text{ each with } 4.5 \text{ watt}$$

c. power efficiency $= \dfrac{\text{power in sideband}}{\text{total transmitted power}}$

$$= \frac{9}{50 + 9} = \frac{9}{59}$$

$$= 0.152$$

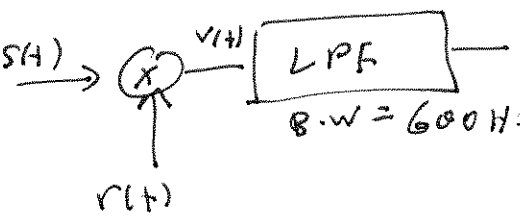
Also, power efficiency $= \dfrac{\mu^2}{2 + \mu^2} = \dfrac{(0.36)^2}{2 + (0.36)^2}$

$= 0.152$; (formula derived in class)

d. $y(t) = A_c [1 + \mu \cos \omega_m t] \cos \omega_c t \cdot \cos \omega_c t$

$$= \frac{A_c [1 + \mu \cos \omega_m t]}{2} [1 + \cos 2\omega_c t]$$

$$\Rightarrow y(t) = \frac{A_c}{2} [1 + \mu \cos \omega_m t]$$



Problem 4:

$$s(t) = 10 \cos\left(2\pi(10000)t + 1.2 \sin 2\pi(200)t\right)$$

a. $$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\left(2\pi(10000)t + 1.2 \sin 2\pi(200)t\right)$$

$$= f_c + \frac{1}{2\pi} \times 1.2\,(2\pi(200)) \cos 2\pi(200)t$$

$$= f_c + 240 \cos 2\pi(200)t \qquad (1)$$

b. peak frequency deviation $= 240$ from (1)

Also, $\beta = \frac{\Delta f}{f_m} \Rightarrow \Delta f = \beta f_m = (1.2)(200)$

$= 240$ Hz.

c. $s(t) = (10) J_0(1.2) \cos 2\pi f_c t$ $\equiv 6.711$

$+ (10) J_1(1.2) \cos 2\pi(f_c + f_m)t$ $\equiv 4.983$

$+ (10) J_{-1}(1.2) \cos 2\pi(f_c - f_m)t$ $= 4.983$

$+ (10) J_2(1.2) \cos 2\pi(f_c + 2f_m)t$ $= 0.1593$

$+ (10) J_{-2}(1.2) \cos 2\pi(f_c - 2f_m)t$ $= 0.1593$

Total average power $= \frac{(10)^2}{2} = 50$

carrier	$f_c + f_m$ $(f_c - f_m)$	$f_c + 2f_m$ / $(f_c - 2f_m)$
$\frac{(6.711)^2}{2}$	$2 \times \frac{(4.983)^2}{2}$	$2 \times \frac{(1.593)^2}{2}$
22.5	24.83	2.537

$\Rightarrow$ 94.6% not enough

$\Rightarrow$ B.W $= 2 \times (2f_m)$

$= 4 f_m$

$= 4(200)$

$= 800$ Hz