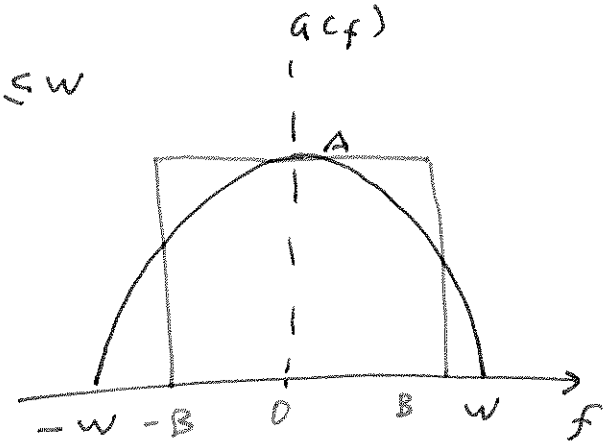
Midterm Exam Communication Systems 29/11/2017
EE 339

Problem 1

$$G(f) = A\cos\left(\frac{\pi f}{2W}\right) \quad -W \le f \le W$$



a. $B.W(\text{absolute}) = W$

b. $2B_{eq}A^2 = \int_{-W}^{W} |G(f)|^2 df$

$$\int_{-W}^{W} |G(f)|^2 df = \int_{-W}^{W} A^2\cos^2\left(\frac{\pi f}{2W}\right) df$$

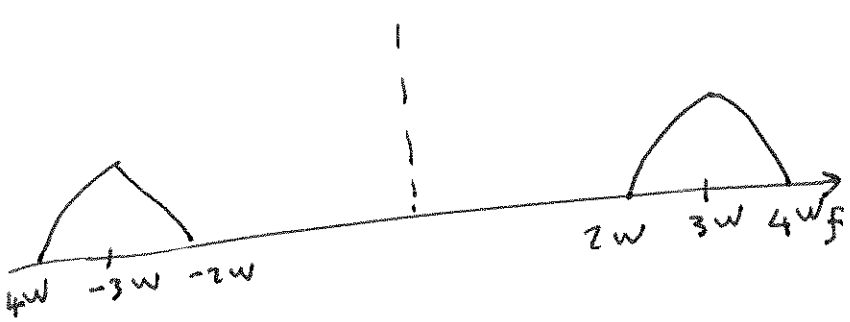
$$= \int_{-W}^{W} \frac{A^2}{2}\left[1+\cos\frac{2\pi f}{2W}\right] df = \frac{A^2}{2}(2W) + \frac{A^2}{2}\int_{-W}^{W}\cos\frac{2\pi f}{2W}\,df \to 0$$

$$2B_{eq}A^2 = \frac{A^2}{2}\,2W \Rightarrow \boxed{B_{eq} = \frac{W}{2}}$$

c. $B_{eq}T_{eq} = \frac{1}{2} \Rightarrow T_{eq} = \frac{1}{2B_{eq}} = \frac{1}{2\cdot\frac{W}{2}} = \frac{1}{W}$

$$\boxed{T_{eq} = \frac{1}{W}}$$

d. $\mathcal{F}\{g(t)\cos 2\pi(3W)t\} = \frac{1}{2}G(f-3W) + \frac{1}{2}G(f+3W)$



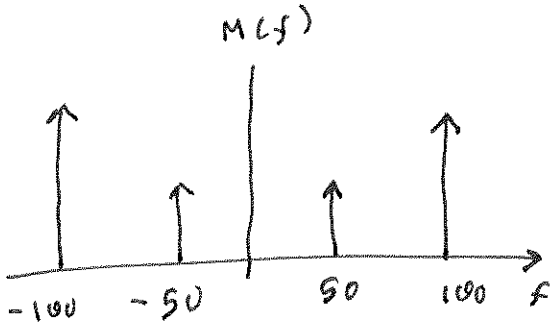
LPF
B.W = W
$\Rightarrow$ output = 0

Problem 2 :

$m(t) = 2 \cos 2\pi(50)t + 4 \cos 2\pi(100)t$

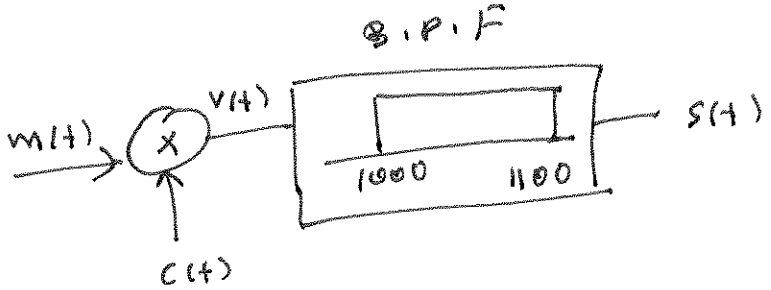
$c(t) = 4 \cos 2\pi(1000)t$

a. $\langle m(t)^2 \rangle = \frac{(2)^2}{2} + \frac{(4)^2}{2} = 10\ W$



b. B.W = 100 Hz .

c.



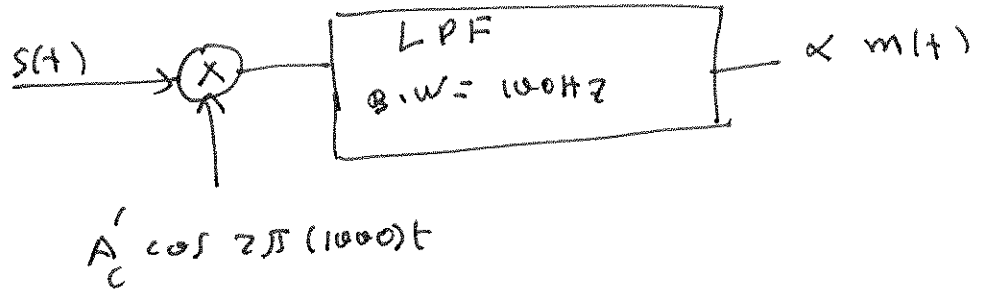
$V(t) = 4 \cos 2\pi(1000)t \left[2 \cos 2\pi(50)t + 4 \cos 2\pi(100)t \right]$

$= 4 \cos 2\pi(1050)t + 4 \cos 2\pi(950)t$

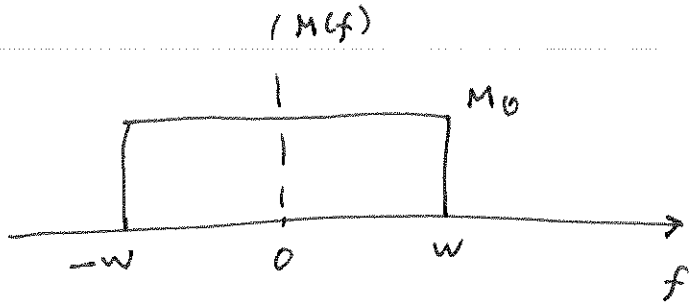
$+ 8 \cos 2\pi 1100\, t + 8 \cos 2\pi(900)t$

$\Rightarrow \boxed{S(t) = 4 \cos 2\pi(1050)t + 8 \cos 2\pi(1100)t}$

d.



Problem 3



$c(t) = 10 \cos 2\pi(10{,}000)t$

$m(t) = M_0(2w)\,\text{sinc}(2wt)$

a. From Tables

$A\,\text{rect}\left(\frac{t}{T}\right) \rightarrow AT\,\text{sinc}(fT)$

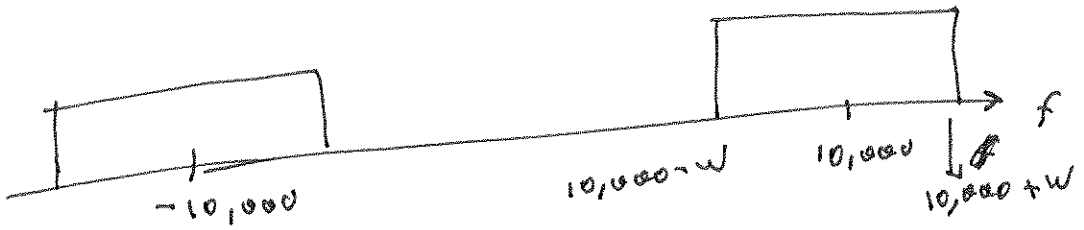
$A\,\text{rect}\left(\frac{f}{T}\right) \leftarrow AT\,\text{sinc}(tT)$

$M_0\,\text{rect}\left(\frac{f}{2w}\right) \leftarrow M_0(2w)\,\text{sinc}\left(t\,\overleftarrow{T}\,2w\right)$

$$\boxed{m(t) = M_0(2w)\,\text{sinc}(2wt)}$$

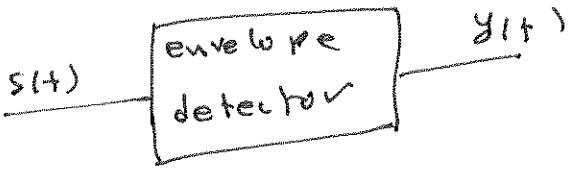
b. $s(t) = m(t)\ 10 \cos 2\pi(10{,}000)t$

c. $S(f) = 5\left[M(f-10{,}000) + M(f+10{,}000)\right]$



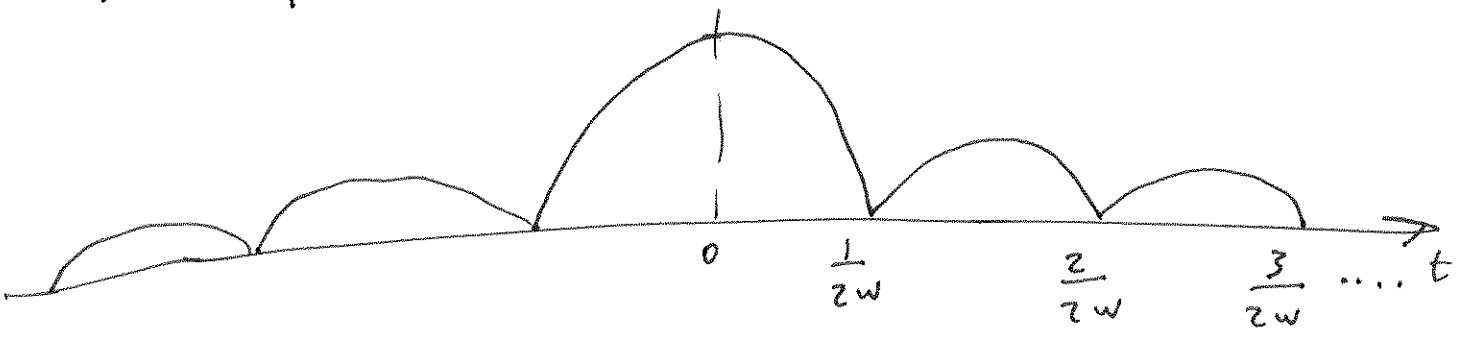
d. $B.W = 2w$

e. $y(t) = |s(t)|$



$LP\left(10 \cos 2\pi(10{,}000)t\ (2wM_0)\,\text{sinc}(2wt)\right|$

$y(t) = |m(t)| = 2wM_0\,|\text{sinc}\,2wt|$



Problem 4

$$s(t) = 10 \cos\left[2\pi(10{,}000)t + 1.6 \sin 2\pi(100)t\right]$$

$$K_f = 10 \text{ Hz/V}.$$

a. $$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\left[2\pi(10{,}000)t + 1.6 \sin 2\pi(100)t\right]$$

$$= 10{,}000 + \frac{1}{2\pi}\cdot(1.6)(2\pi)(100)\cos 2\pi(100)t$$

$$\boxed{f_i'(t) = 10{,}000 + (1.6)(100)\cos 2\pi(100)t}$$

b. $$f_i'(t) = f_c + K_f\, m(t)$$

$$10\, m(t) = (1.6)(100)\cos 2\pi(100)t \Rightarrow$$

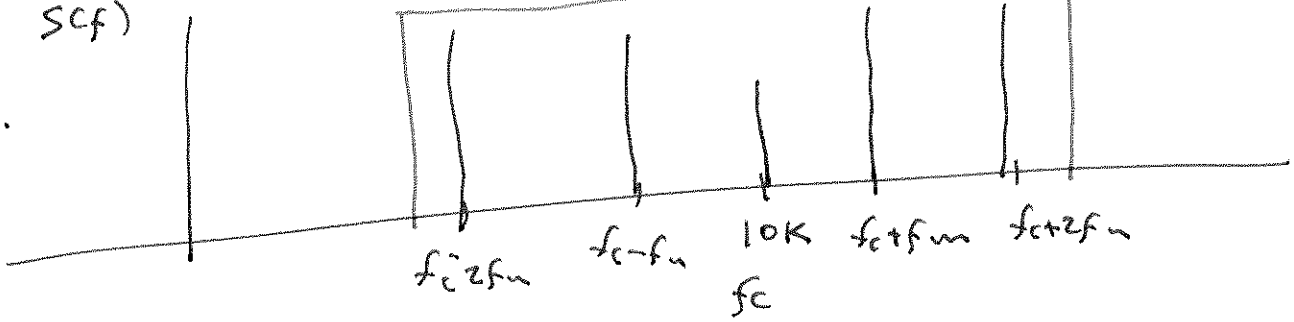
$$\boxed{m(t) = 16 \cos 2\pi(100)t}$$

c. $\Delta f = (1.6)(100) = 160$ Hz ; Also, $\beta = \frac{\Delta f}{f_m} \Rightarrow f_m = \beta f_m$

$$\boxed{\Delta f = 160 \text{ Hz}}$$

B.W $= 500 = 5 f_m$

d.



filter output consists of 5 terms $f_m = 100$

$$g(t) = A_c\left[J_0(1.6)\cos 2\pi f_c t + J_1(1.6)\cos 2\pi(f_c + f_m)t\right.$$

$$+ J_{-1}(1.6)\cos 2\pi(f_c - f_m)t$$

$$+ J_2(1.6)\cos 2\pi(f_c + 2f_m)t$$

$$\left. + J_{-2}(1.6)\cos 2\pi(f_c - 2f_m)t\right]$$

$$\frac{\langle g(t)^2\rangle}{\langle s(t)^2\rangle} = \frac{\frac{A_c^2}{2}\left[(0.4554)^2 + 2(0.5669)^2 + 2(0.2570)^2\right]}{(A_c^2/2)} = 0.9890$$