



BIRZEIT UNIVERSITY
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
 Communication Systems ENEE 339
 Instructor: Dr. Wael Hashlamoun
Midterm Exam
First Semester 2018-2019

Date: Sunday 18/11/2018

Time: 75 minutes

Name:

Student #:

Opening Remarks:

- Calculators are allowed, but mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem 1: 25 Points

The Fourier transform, $G(f)$, of a signal $g(t)$ is given by:

$$G(f) = \begin{cases} \sqrt{1 - \left(\frac{f}{f_0}\right)^2} & -f_0 \leq f \leq f_0 \\ 0 & |f| > f_0 \end{cases}; f_0 = 1000 \text{ Hz}$$

- 9 a. Find the 3-dB bandwidth of $g(t)$.
 9 b. If $g(t)$ is passed through an ideal low pass filter with bandwidth $f_0/2$ and unity gain, find the energy of the signal at the filter output.
 7 c. The signal $s(t) = g(t)\cos 2\pi(1000t)$ is passed through an ideal low pass filter with bandwidth 1000 Hz, sketch the spectrum of $s(t)$.

a.

$$G(0) = 1$$

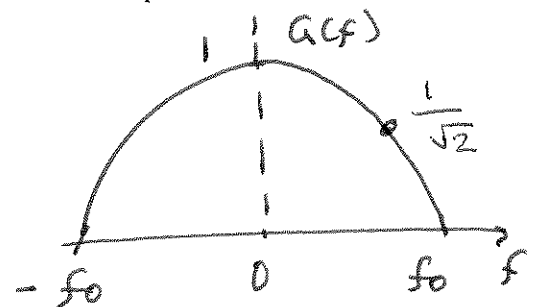
$$-3 = 20 \log \frac{G(B)}{G(0)} = 20 \log \sqrt{1 - \left(\frac{B}{f_0}\right)^2}$$

$$-3 = 10 \log \left(1 - \left(\frac{B}{f_0}\right)^2\right)$$

$$-0.3 = \log \left(1 - \left(\frac{B}{f_0}\right)^2\right) \Rightarrow 10^{-0.3} = 1 - \left(\frac{B}{f_0}\right)^2$$

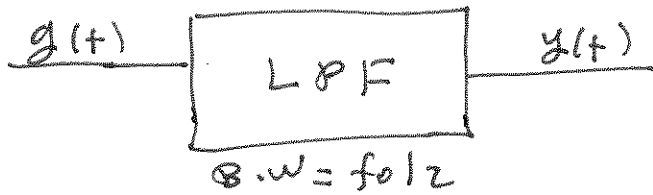
$$\frac{B}{f_0} = \sqrt{1 - 10^{-0.3}} = \sqrt{0.498} = 0.706$$

$$\Rightarrow \boxed{B = 0.706 f_0 = 706 \text{ Hz}}$$



Problem 1

b.



$$Y(f) = A(f) H(f)$$

$$|Y(f)|^2 = |A(f)|^2 |H(f)|^2$$

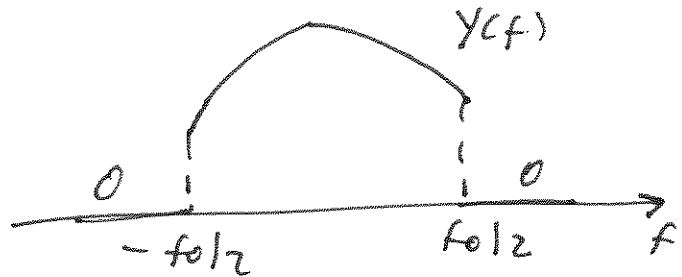
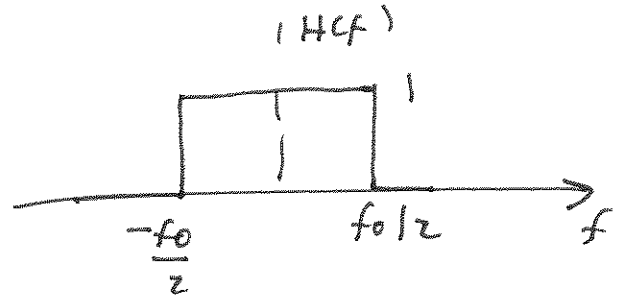
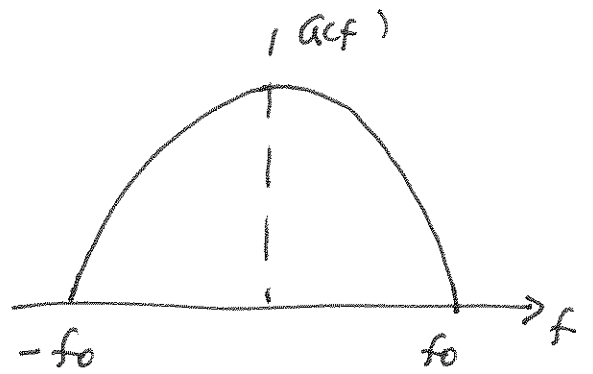
$$E_y = \int_{-f_0/2}^{f_0/2} |A(f)|^2 df$$

$$= 2 \int_0^{f_0/2} \left(1 - \left(\frac{f}{f_0}\right)^2\right) df$$

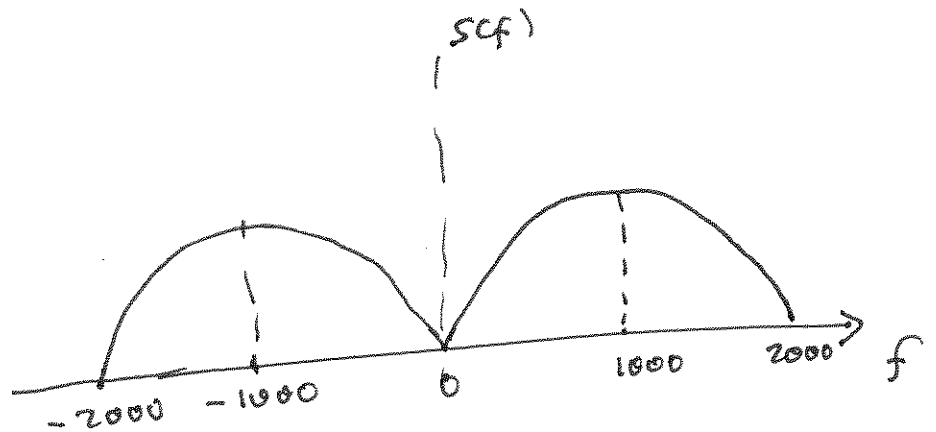
$$= 2 \left[f - \frac{f^3}{3f_0^2} \right]_0^{f_0/2}$$

$$= 2 \left[\frac{f_0}{2} - \frac{f_0^3}{24f_0^2} \right] = 2 \left[\frac{f_0}{2} - \frac{f_0}{24} \right]$$

$$= f_0 - \frac{f_0}{12} = \boxed{\frac{11}{12} f_0 = \frac{11}{12} \times 1000 = 916.6}$$



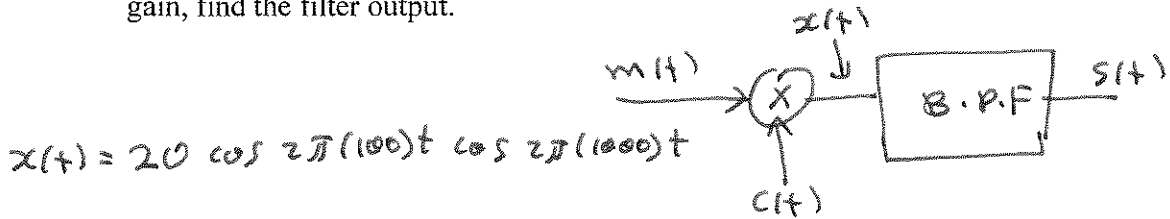
c.



Problem 2: 25 Points

The message $m(t) = 2 \cos(2\pi 100t)$ along with the carrier $c(t) = 10 \cos(2\pi 1000t)$ are applied to an upper single sideband modulator, which uses the frequency discrimination method, to generate the modulated signal $s(t)$.

- 6 a. Find the average power in $m(t)$.
- 6 b. If $c(t)$ is applied to an ideal envelope detector, sketch the signal observed at its output.
- 6 c. Find the time-domain expression for the modulated signal $s(t)$.
- 7 d. If $s(t)$ is applied to a coherent demodulator consisting of a multiplier, which uses the signal $c'(t) = \cos(2\pi 1000t + \phi)$, followed by a low pass filter with bandwidth 120 Hz and unity gain, find the filter output.



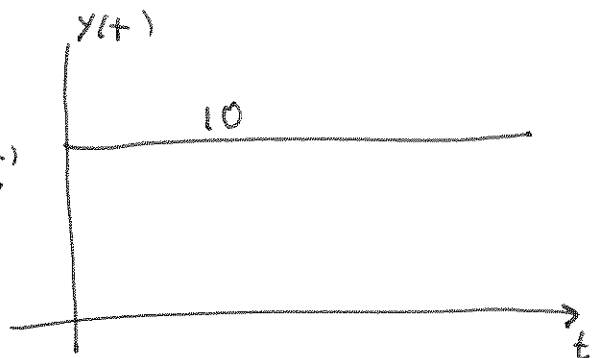
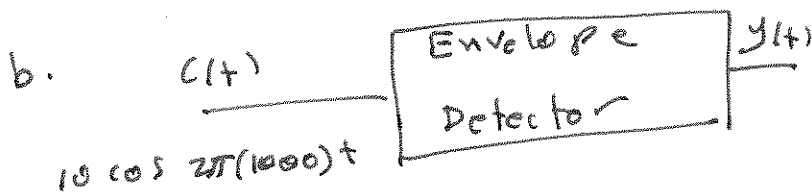
$$x(t) = 20 \cos 2\pi(100)t \cos 2\pi(1000)t$$

$$x(t) = 10 \cos 2\pi(1100)t + 10 \cos 2\pi(900)t$$

The B.P.F. removes lower band

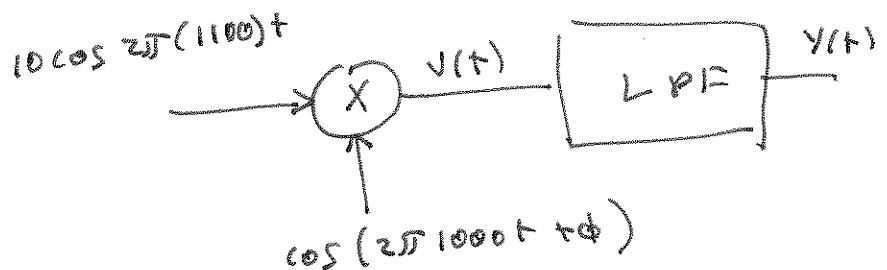
⇒
c. $s(t) = 10 \cos 2\pi(1100)t$

a. $\langle m(t)^2 \rangle = \frac{(2)^2}{2} = 2$



d.

$$\begin{aligned} v(t) &= 10 \cos 2\pi(1100)t \cdot \cos(2\pi 1000t + \phi) \\ &= 5 \cos(2\pi 2100t + \phi) + 5 \cos(2\pi 100t + \phi) \end{aligned}$$



$$y(t) = 5 \cos(2\pi 100t + \phi)$$

$$y(t) = 5 \cos \phi \cos 2\pi(100)t - 5 \sin \phi \sin 2\pi(100)t$$

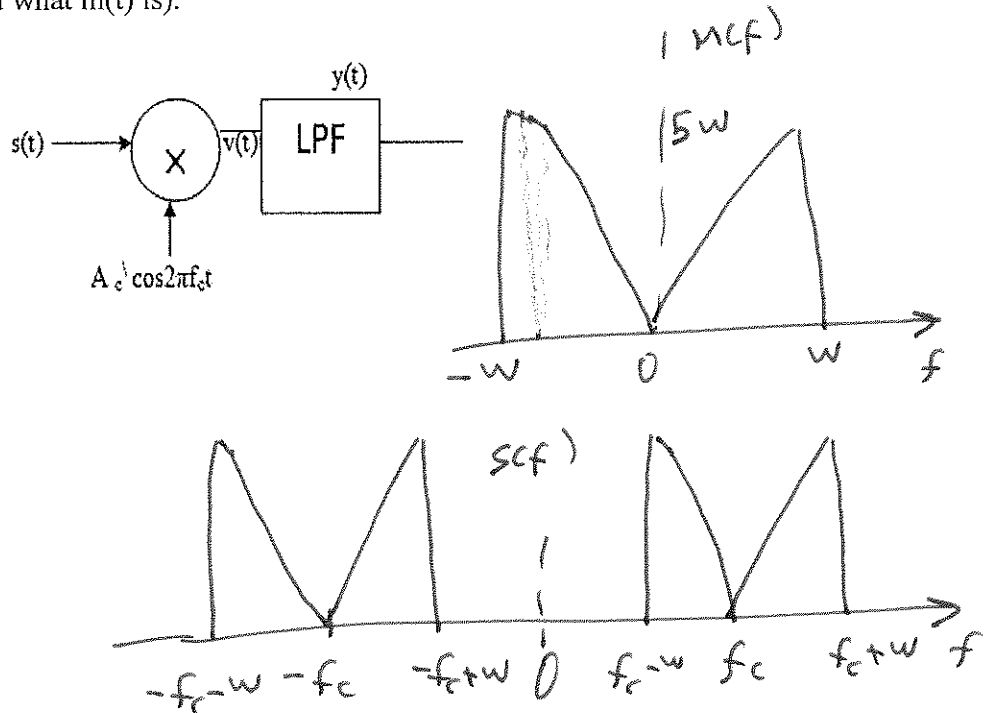
Problem 3: 25 Points

The Fourier transform of a message $m(t)$ is given as:

$$M(f) = \begin{cases} 5|f| & -W \leq f \leq W \\ 0 & |f| > W \end{cases}; W = 1000$$

This message is applied to a double sideband suppressed carrier modulator along with the carrier $c(t) = 10 \cos(2\pi(f_c)t)$ to produce the modulated signal $s(t)$

- 6 a. Find and sketch $S(f)$, the Fourier transform of $s(t)$, for an arbitrary value of f_c .
- 6 b. Find the transmission bandwidth of $s(t)$.
- 6 c. Find the minimum required value of f_c in terms of W .
- 7 d. Show that the receiver in the figure below can demodulate $m(t)$ from $s(t)$ without distortion (you do not have to find what $m(t)$ is).



$s(t) = 10 \cos 2\pi f_c t m(t)$

$S(f) = 5 M(f - f_c) + 5 M(f + f_c)$

a. \longrightarrow

b.

$B.W = 2W$

$B.W = 2(1000) = 2000 \text{ Hz}$

c. $f_c - W \geq 0 \Rightarrow f_c \geq W = 1000 \text{ Hz}$

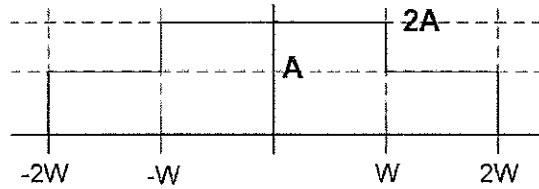
d. $v(t) = 10 (\cos 2\pi f_c t) m(t) \cdot A_c \cos 2\pi f_c t$
 $= 5A_c m(t) [\cos 4\pi f_c t + 1]$

$\Rightarrow y(t) = 5A_c m(t)$

Problem 4: 25 Points

Consider the message signal $g(t)$, shown in the figure below

$$g(t) = \begin{cases} 2A & -W \leq t \leq W \\ A & W \leq |t| \leq 2W \\ 0 & |t| > 2W \end{cases}; A = 1, W = 1$$



$g(t)$ is applied to an FM modulator with sensitivity $k_f = 20 \text{ Hz/V}$ to produce an FM signal $s(t)$. The unmodulated carrier frequency is 1000 Hz.

- Use the time-bandwidth relationship to find the equivalent rectangular bandwidth of $g(t)$.
- Find and plot the instantaneous frequency of $s(t)$ versus time.
- Find the peak frequency deviation of $s(t)$.
- Find the time domain representation for the modulated signal $s(t)$ for all time t .

$$a. T_{eq} = \frac{\left(\int_{-\infty}^{\infty} |g(t)| dt \right)^2}{\int_{-\infty}^{\infty} |g(t)|^2 dt}$$

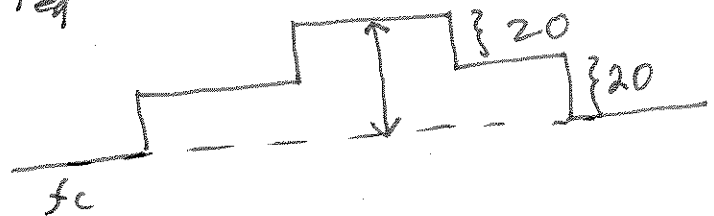
$$\int_{-2W}^{2W} |g(t)| dt = 2AW + 2W(2A) = 6AW = 6$$

$$\int_{-2W}^{2W} |g(t)|^2 dt = (A^2)(2W) + 4A^2(2W) = 10A^2W = 10$$

$$\Rightarrow T_{eq} = \frac{(6)^2}{10} = \frac{36}{10} = 3.6 \frac{A^2W^2}{A^2W} = 3.6W$$

$$B_{eq} T_{eq} = \frac{1}{2} \Rightarrow B_{eq} = \frac{1}{2 \cdot 3.6} = \frac{1}{7.2} = 0.138$$

b. \longrightarrow



c. $\Delta f_{max} = 40 \text{ Hz}$



$$d. s(t) = \begin{cases} A_c \cos 2\pi f_c t & t < -2W, t \geq 2W \\ A_c \cos 2\pi(f_c + 20)t & -2W \leq t < -W, W \leq t \leq 2W \\ A_c \cos 2\pi(f_c + 40)t & -W \leq t \leq W \end{cases}$$