جامعة بيرزيت
BIRZEIT UNIVERSITY

**Faculty of Engineering and Technology**
**Department of Electrical and Computer Engineering**
Communication Systems ENEE 339

Instructor: Dr. Wael Hashlamoun

*Final Exam*
*First Semester 2019-2020*

Date: Sunday January 19, 2020 | Time: 135 minutes
Name: | Student #:

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**Opening Remarks:**

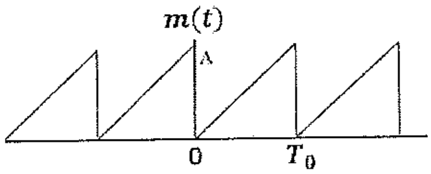
- Calculators are allowed, but mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

**Problem 1: 40 Points + 5 bonus Points (each question is worth 2.5 points)**

When applicable, encircle the correct answer (True or False), or fill in the space, or write down the solution to the following questions:

1. The average power in the periodic signal $m(t)$ is

$m(t) = A\frac{t}{T_0}$




$\langle m(t)^2 \rangle = \frac{1}{T_0}\int_0^{T_0} \left(A\frac{t}{T_0}\right)^2 dt$

$= \frac{A^2}{3}$

   a. $AT_0/2$
   b. $(A)^2T_0/2$
   c. $(A)^2T_0/3$
   d. $A^2/3$

2. The dc value in the Fourier series expansion of the periodic signal $g(t)$, defined over one period $T_0$ is

$$g(t) = \begin{cases} +A, & -T_0/4 \le t \le T_0/4 \\ 0, & otherwise \end{cases}$$

$\langle g(t) \rangle = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} A \, dt$

$= \frac{A}{2}$

   a. $AT_0/2$
   b. $A/2$
   c. $T_0/2$
   d. $(A)^2T_0/2$

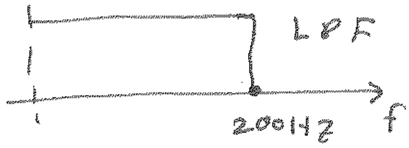
3. The 100% power bandwidth of the signal $x(t) = \frac{1}{2} + \frac{1}{\pi} cos\ 2\pi(120)t - \frac{1}{3\pi} cos\ 2\pi(360)t$ is
   a. 120 Hz
   b. $2\pi(120)$ Hz
   c. 360 Hz
   d. 240 Hz

4. A channel described by the transfer function $H(f) = \frac{1}{1+jf/B}$
   a. Is a distortion-less channel
   b. Introduces only amplitude distortion
   c. Introduces only phase distortion phase
   d. Introduces both amplitude and ~~frequency~~ distortion

5. Let $g(t)$ be a continuous function of time, then one of the following is true
   a. $g(t)\delta(t) = g(0)\delta(t)$
   b. $g(t)\delta(t) = g(0)\delta(0)$
   c. $g(t)\delta(t) = g(0)$
   d. $g(t)\delta(t) = g(t)$

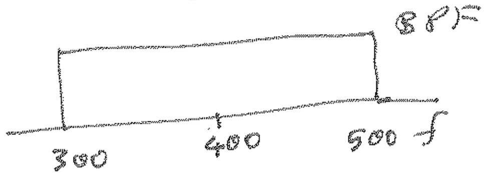
6. The signal $x(t) = \frac{1}{2} + cos\ 2\pi(120)t - \frac{1}{3\pi} cos\ 2\pi(360)t + \frac{1}{5\pi} cos\ 2\pi(600)t$ is passed through an ideal low pass filter with a bandwidth of 200 Hz. Write down the filter output $y(t)$.

$y(t) = \frac{1}{2} + cos\ 2\pi(120)t$




7. The signal $x(t) = \frac{1}{2} + cos\ 2\pi(120)t - \frac{1}{3\pi} cos\ 2\pi(360)t + \frac{1}{5\pi} cos\ 2\pi(600)t$ is passed through an ideal band-pass filter with a bandwidth of 200 Hz and center frequency 400 Hz. Write down the filter output $y(t)$.

$y(t) = -\frac{1}{3\pi} cos\ 2\pi(360)t$




8. The carrier $c(t) = 2cos\ 2\pi(1000)t$ along with the message $m(t) = cos\ 2\pi(120)t$ are applied to a single sideband modulator. The modulator output $y(t)$ is
   a. $y(t) = 2cos\ 2\pi(1000)t + cos\ 2\pi(120)t$
   b. $y(t) = 2cos\ [2\pi(1000)t][cos\ 2\pi(120)t]$
   c. $y(t) = 2cos\ 2\pi(1000)t - cos\ 2\pi(120)t$
   d. $y(t) = cos\ 2\pi(1120)t$

9. The carrier $c(t) = 10cos\ 2\pi(1000)t$ along with the message $m(t) = cos\ 2\pi(120)t$ are applied to a frequency modulator with sensitivity $k_f\ V/Hz$. The frequency of the modulator output is
   a. $f(t) = 10\cos(2\pi(1000) + \frac{k_f}{120} sin\ 2\pi(120)t)$ Hz
   b. $f(t) = 1000t + (k_f)120t$ Hz
   c. $f(t) = 1000 + k_f\ cos\ 2\pi(120)t$ Hz
   d. $f(t) = 10[cos\ 2\pi(1000)t][cos\ 2\pi(120)]$ Hz

10. Consider the signal $x(t) = \cos\ 2\pi(100)t\ + \cos\ 2\pi(200)t$. The minimum sampling rate for this signal to be reconstructed from its samples is:
    a. 100 samples/sec
    b. 200 samples/sec
    c. 400 samples/sec
    d. $2\pi(200)$ samples/sec

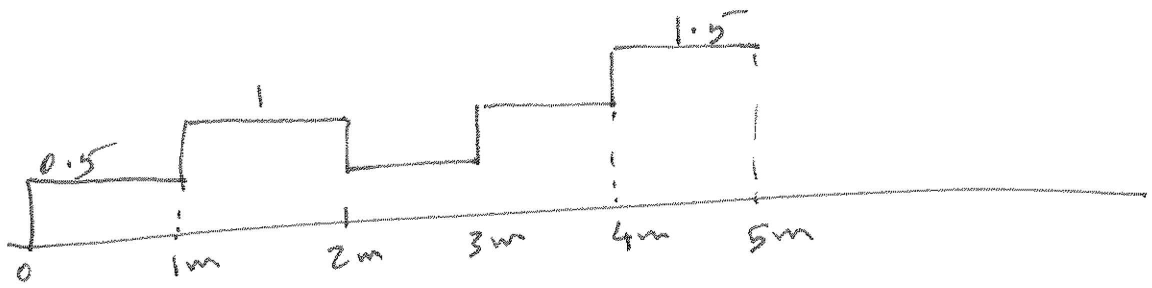
11. The signal $x(t) = 4\cos\ 2\pi(100)t$ is flat-topped sampled at the Nyquist rate to produce the sampled signal $x_s(t)$. The samples are then applied to an ideal low pass filter with bandwidth 100 Hz. Then
    a. The output of the filter is an exact copy of $x(t)$.
    b. The output is distorted due to aliasing.
    c. An equalizer has to follow the low pass filter in order to get $x(t)$ without distortion.
    d. $x(t)$ can be obtained from $x_s(t)$ without the need for the low pass filter.

12. The signal $x(t) = 4\cos\ 2\pi(100)t$ is applied to a PCM system where the sampler operates at a rate of 250 samples/sec. The samples are applied to a 32-level uniform quantizer. The data rate in bits/sec at the binary encoder output is:
    a. 1000 bits/sec
    b. 1250 bits/sec
    c. 1500 bits/sec
    d. 500 bits/sec

13. In binary phase shift keying scheme, the transmitted signals representing binary digits 1 and 0 are:
    a. $s_1(t) = \text{Acos}\ 2\pi(f_1)t\ , s_0(t) = 0$
    b. $s_1(t) = \text{Acos}\ 2\pi(f_1)t\ , -\ \text{Acos}\ 2\pi(f_1)t$
    c. $s_1(t) = \text{Acos}\ 2\pi(f_1)t\ ,\ \text{Acos}\ 2\pi(f_0)t$
    d. $s_1(t) = \text{Acos}\ 2\pi(f_1)t\ +\ \text{Acos}\ 2\pi(f_0)t\ , -\text{Acos}\ 2\pi(f_0)t\ ,$

14. In binary frequency shift keying scheme, the transmitted signals representing binary digits 1 and 0 are:
    a. $s_1(t) = \text{Acos}\ 2\pi(f_1)t\ , s_0(t) = 0$
    b. $s_1(t) = \text{Acos}\ 2\pi(f_1)t\ , -\ \text{Acos}\ 2\pi(f_1)t$
    c. $s_1(t) = \text{Acos}\ 2\pi(f_1)t\ ,\ \text{Acos}\ 2\pi(f_0)t$
    d. $s_1(t) = \text{Acos}\ 2\pi(f_1)t\ +\ \text{Acos}\ 2\pi(f_0)t\ , -\text{Acos}\ 2\pi(f_0)t\ ,$

15. Reconstruct a staircase signal at the receiver side of a delta demodulator with step size $\Delta=$ 0.5V and sampling time $T_s = 1ms$, when the received data sequence is: 1 1 0 1 1.



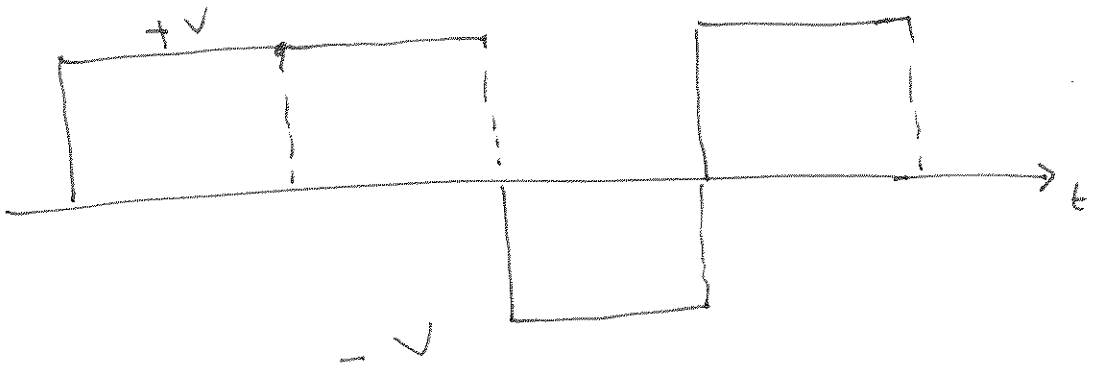

16. Consider a digital communication system, corrupted by AWGN with power spectral density $N_0/2$, that uses $s_1(t)$ to represent digit 1 and $s_2(t)$ to represent digit 0. The bit error probability is minimized when

a. $\int_0^{\tau}(s_1(t) - s_2(t))^2 dt$ is maximum
b. $\int_0^{\tau}(s_1(t) - s_2(t))^2 dt$ is minimum
c. $\int_0^{\tau} s_1(t)s_2(t)dt$ $= 0$
d. $\int_0^{\tau}[s_1(t) + s_2(t)]dt$ $\geq$

17. The energy in the signal $s(t) = A\cos(2\pi f_c t)$, $0 \leq t \leq \tau$, $T_c = n\tau$, n is an integer, is

a. $A^2/2$
b. $A^2$
c. $A^2\tau/2$
d. $A^2\tau$

18. Sketch the polar non-return to zero binary encoding for the bit stream 1 1 0 1



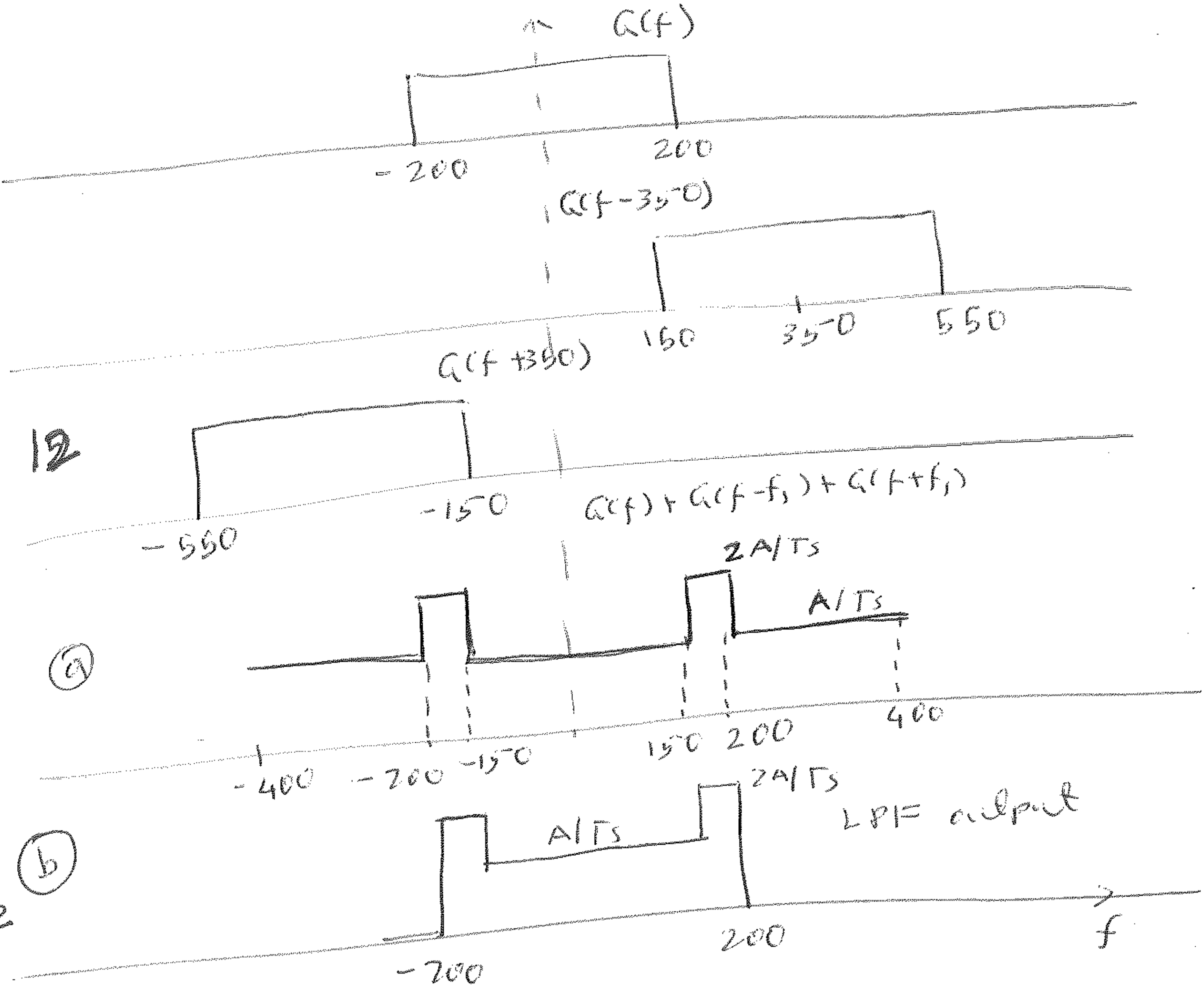
**Problem 2: 20 Points**
The Fourier transform, $G(f)$, of a signal $g(t)$ is given as:

$$G(f) = \begin{cases} A, & -200 \le f \le 200 \\ 0, & |f| > 200 \end{cases}$$

The signal $g(t)$ is ideally sampled at a rate of 350 samples/sec to produce the samples signal $g_s(t)$.

a. Find and sketch $G_s(f)$, the Fourier transform of $g_s(t)$ for $-400 \le f \le 400$
b. If $g_s(t)$ is applied to an ideal low pass filter with a bandwidth of 200 Hz, sketch the Fourier transform of the signal appearing at the output of the filter.
c. Based on the results of Part b, do you think that $g(t)$ can be recovered from $g_s(t)$ without distortion? Explain why.

$$G_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f - nf_s)$$




4 (c) since $f_s < 2(200) = 400 \Rightarrow$ Distortion
$\Rightarrow g(t)$ cannot be recovered.

**Problem 3: 18 Points**

The signal $x(t) = 4cos(2\pi f_0 t)$ is applied to a uniform quantizer with L quantization levels and a dynamic range $(-4, 4)$ V. Find the minimum value of L that will achieve a signal to quantization noise ratio $SQNR \geq 1000$.

4 $\Delta = \frac{4-(-4)}{L}$ ; $= \frac{8}{L}$

4 $\langle x(t)^2 \rangle = \frac{A_m^2}{2} = \frac{(4)^2}{2} = 8$ ; average signal power

4 quantization noise $= \frac{\Delta^2}{12}$

$SQNR = \frac{\langle x(t)^2 \rangle}{\Delta^2/12} = \frac{8}{(8/L)^2/12} = \frac{8 \times 12 \times L^2}{64}$

$SQNR = \frac{3}{2} L^2 \geq 1000$

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$L^2 \geq \frac{2000}{3}$

$L \geq \sqrt{\frac{2000}{3}}$

$L \geq 26$

**Problem 4: 22 Points**

The binary digital communication signaling scheme, discussed in class, employs the following two equally probable signals $s_1(t)$ and $s_2(t) = -s_1(t)$ to represent binary logic 1 and 0 respectively over a channel corrupted by AWGN with power spectral density $N_0/2$ W/Hz. Here,

$$s_1(t) = \begin{cases} A\dfrac{t}{\tau}, & 0 \le t \le \tau \end{cases}$$

where $\tau$ is the binary symbol duration.

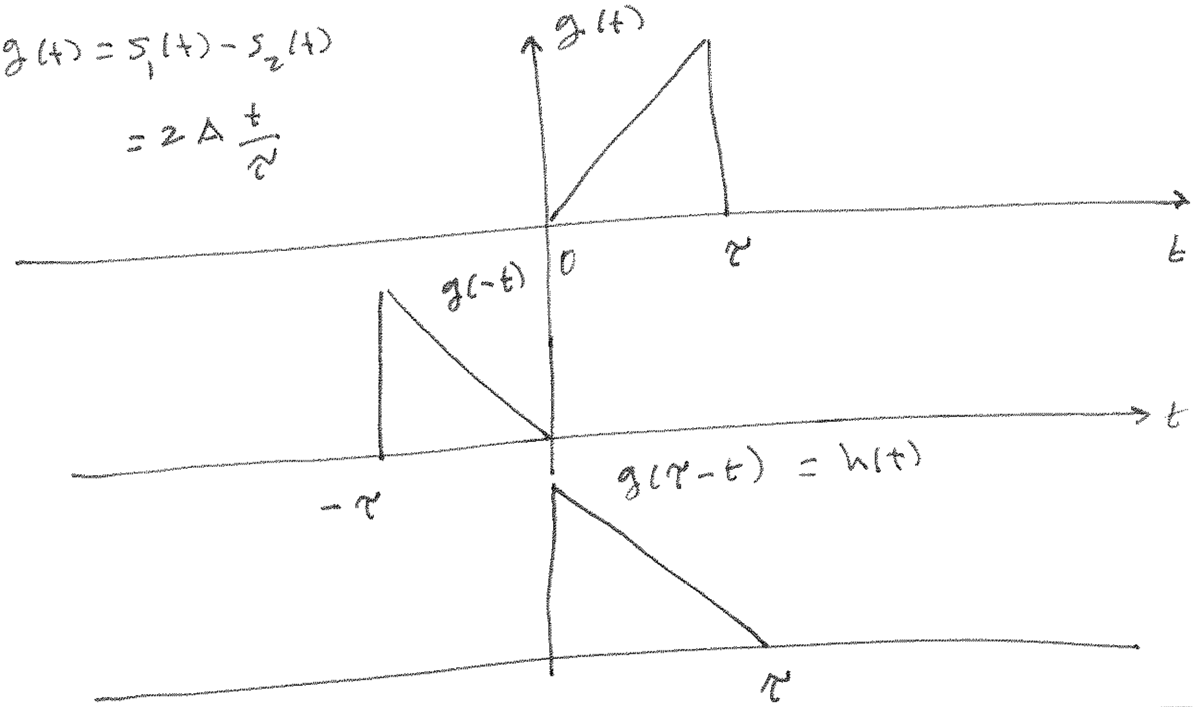
6 a. Find and sketch the impulse response, $h(t)$, of the matched filter, designed to minimize the probability of error.

6 b. Find the optimum threshold used by the threshold detector at the receiver.

6 c. Find the system average probability of error. Leave your answer in terms of the Q function.

Good Luck

$$g(t) = s_1(t) - s_2(t) = 2A\frac{t}{\tau}$$




$$\lambda^* = \frac{1}{2}(E_1 - E_2) = 0 \quad ; \quad P_b = Q\left(\sqrt{\frac{\int^{\tau}(s_1-s_2)^2\,dt}{2N_0}}\right)$$

$$\int_0^{\tau}(s_1-s_2)^2\,dt = \int_0^{\tau}\left(\frac{2At}{\tau}\right)^2 dt = \frac{4A^2}{\tau^2}\int_0^{\tau} t^2\,dt$$

$$= \frac{4A^2}{\tau^2}\cdot\frac{\tau^3}{3} = \frac{4}{3}A^2\tau$$

$$P_b = Q\left(\sqrt{\frac{4A^2\tau}{6N_0}}\right) = Q\left(\sqrt{\frac{2A^2\tau}{3N_0}}\right)$$

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**Angle Sum and Difference Formulas**

$$\sin(\theta\pm\varphi)=\sin\theta\cos\varphi\pm\cos\theta\sin\varphi$$

$$\cos(\theta\pm\varphi)=\cos\theta\cos\varphi\mp\sin\theta\sin\varphi$$

$$\tan(\theta\pm\varphi)=\frac{\tan\theta\pm\tan\varphi}{1\mp\tan\theta\tan\varphi}$$

**Sum-to-Product Formulas**

$$\sin\theta+\sin\varphi=2\sin\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)$$

$$\sin\theta-\sin\varphi=2\cos\left(\frac{\theta+\varphi}{2}\right)\sin\left(\frac{\theta-\varphi}{2}\right)$$

$$\cos\theta+\cos\varphi=2\cos\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)$$

$$\cos\theta-\cos\varphi=-2\sin\left(\frac{\theta+\varphi}{2}\right)\sin\left(\frac{\theta-\varphi}{2}\right)$$

**Product-to-Sum Formulas**

$$\sin\theta\sin\varphi=\frac{1}{2}\left[\cos(\theta-\varphi)-\cos(\theta+\varphi)\right]$$

$$\cos\theta\cos\varphi=\frac{1}{2}\left[\cos(\theta-\varphi)+\cos(\theta+\varphi)\right]$$

$$\sin\theta\cos\varphi=\frac{1}{2}\left[\sin(\theta+\varphi)+\sin(\theta-\varphi)\right]$$

$$\cos\theta\sin\varphi=\frac{1}{2}\left[\sin(\theta+\varphi)-\sin(\theta-\varphi)\right]$$

**Double Angle Formulas**

$$\sin(2\theta)=2\sin\theta\cos\theta$$

$$\cos(2\theta)=\cos^2\theta-\sin^2\theta$$

$$=2\cos^2\theta-1$$

$$=1-2\sin^2\theta$$

$$\tan(2\theta)=\frac{2\tan\theta}{1-\tan^2\theta}$$

**Half Angle Formulas**

$$\sin^2\theta=\frac{1-\cos(2\theta)}{2}$$

$$\cos^2\theta=\frac{1+\cos(2\theta)}{2}$$

$$\tan^2\theta=\frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

**Basic Identities**

$$\sin\theta=\frac{1}{\csc\theta}\qquad\cos\theta=\frac{1}{\sec\theta}$$

$$\tan\theta=\frac{1}{\cot\theta}\qquad\cot\theta=\frac{1}{\tan\theta}$$

$$\csc\theta=\frac{1}{\sin\theta}\qquad\sec\theta=\frac{1}{\cos\theta}$$

**Periodicity**

$$\sin(\theta+2\pi)=\sin\theta$$

$$\cos(\theta+2\pi)=\cos\theta$$

$$\tan(\theta+\pi)=-\tan\theta$$

**Pythagorean Identities**

$$\sin^2\theta+\cos^2\theta=1$$

$$\sec^2\theta-\tan^2\theta=1$$

$$\csc^2\theta-\cot^2\theta=1$$

**Co-Function Identities**

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta\qquad\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta\qquad\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta\qquad\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$$

**Even or Odd Symmetry**

$$\sin(-\theta)=-\sin\theta\qquad\cos(-\theta)=\cos\theta$$

$$\tan(-\theta)=-\tan\theta\qquad\cot(-\theta)=-\cot\theta$$

$$\csc(-\theta)=-\csc\theta\qquad\sec(-\theta)=\sec\theta$$

$$\exp(\pm j\theta)=\cos\theta\pm j\sin\theta$$

$$\cos\theta=\tfrac{1}{2}[\exp(j\theta)+\exp(-j\theta)]$$

$$\sin\theta=\frac{1}{2j}[\exp(j\theta)-\exp(-j\theta)]$$

**TABLE A6.2** *Fourier-Transform Pairs*

| Time Function | Fourier Transform |
|---|---|
| $\mathrm{rect}\left(\frac{t}{T}\right)$ | $T\,\mathrm{sinc}(fT)$ |
| $\mathrm{sinc}(2Wt)$ | $\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$ |
| $\exp(-at)u(t),\quad a>0$ | $\frac{1}{a+j2\pi f}$ |
| $\exp(-a\lvert t\rvert),\quad a>0$ | $\frac{2a}{a^2+(2\pi f)^2}$ |
| $\exp(-\pi t^2)$ | $\exp(-\pi f^2)$ |
| $\begin{cases}1-\frac{\lvert t\rvert}{T}, & \lvert t\rvert<T\\ 0, & \lvert t\rvert\ge T\end{cases}$ | $T\,\mathrm{sinc}^2(fT)$ |
| $\delta(t)$ | $1$ |
| $1$ | $\delta(f)$ |
| $\delta(t-t_0)$ | $\exp(-j2\pi f t_0)$ |
| $\exp(j2\pi f_c t)$ | $\delta(f-f_c)$ |
| $\cos(2\pi f_c t)$ | $\frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$ |
| $\sin(2\pi f_c t)$ | $\frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)]$ |
| $\mathrm{sgn}(t)$ | $\frac{1}{j\pi f}$ |
| $\frac{1}{\pi t}$ | $-j\,\mathrm{sgn}(f)$ |
| $u(t)$ | $\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$ |
| $\sum_{i=-\infty}^{\infty}\delta(t-iT_0)$ | $\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_0}\right)$ |

## Table of Common Integrals

$$\int k\,dx=x+C$$

$$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C\text{ for }n\neq-1$$

$$\int\frac{1}{x}\,dx=\ln|x|+C$$

$$\int e^x\,dx=e^x+C$$

$$\int a^x\,dx=\frac{a^x}{\ln a}+C$$

$$\int\sin x\,dx=-\cos x+C$$

$$\int\cos x\,dx=\sin x+C$$

$$\int\sec^2x\,dx=\tan x+C$$

$$\int\csc^2x\,dx=-\cot x+C$$

$$\int\csc^2x\,dx=-\cot x+C$$

$$\int\sec x\tan x\,dx=\sec x+C$$

$$\int\csc x\cot x\,dx=-\csc x+C$$

$$\int\tan x\,dx=\ln|\sec x|+C$$

$$\int\cot x\,dx=\ln|\sin x|+C$$

$$\int\sec x\,dx=\ln|\sec x+\tan x|+C$$

$$\int\csc x\,dx=\ln|\csc x-\cot x|+C$$

$$\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\int\frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$