

3614: Introduction to Communication Systems

Midterm Exam II

November 6, 2007

SOLUTION

I pledge that I have neither given nor received any assistance on this exam.

(signed)

Name (print)

Student Number

ECE 3614 Midterm II – Test A

1. (20 points) Short answer and multiple choice

1.1 [5 points] Consider a message signal with a peak voltage of 1V and a bandwidth of 25kHz. Rank the following modulation schemes in terms of required bandwidth (least bandwidth = 1, most bandwidth = 4)

- 2 DSBSC
- 1 Single Sideband AM
- 4 FM with $k_f = 5000$
- 3 FM with $k_f = 500$

1.2 [5 points] The Fourier Transform of a periodic signal

- (a) is a series of impulses
- (b) is discrete in frequency
- (c) is the multiplication of an impulse train with the Fourier Transform of one period
- (d) None of the above
- (e) All of the above

1.3 [5 points] Vestigial Sideband modulation has a bandwidth which is

- (a) greater than DSBSC
- (b) the same as LC-AM
- (c) greater than SSB
- (d) All of the above
- (e) None of the above

1.4 [5 points] Single Sideband Modulation

- (a) reduces bandwidth by eliminating the unmodulated carrier from LC-AM.
- (b) reduces bandwidth by also transmitting a phase-shifted version of the message in order to eliminate the lower or upper sideband.
- (c) reduces bandwidth by eliminating the need for a quadrature channel.
- (d) None of the above

2. (30 points) Amplitude Modulation

Consider a message signal

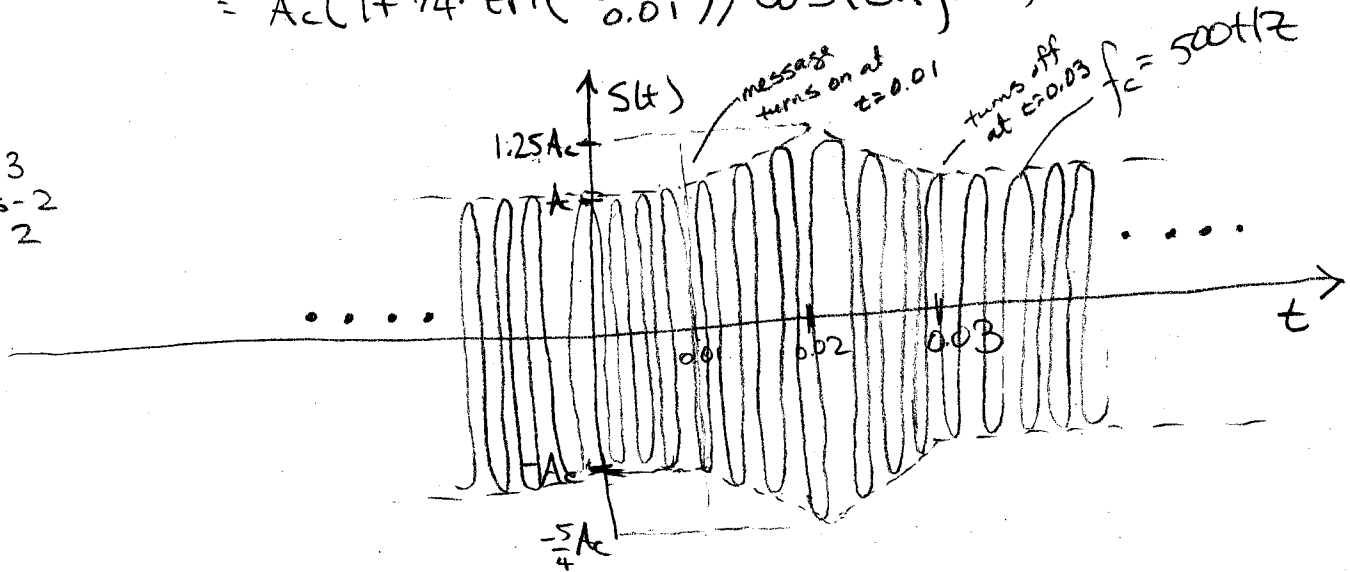
$$m(t) = \text{tri}\left(\frac{t-0.02}{0.01}\right)$$

(a) [8 points] Sketch the time domain signal of a large carrier AM modulated signal if $k_a = 0.25$ and $f_c = 500\text{Hz}$. Clearly label all axes.

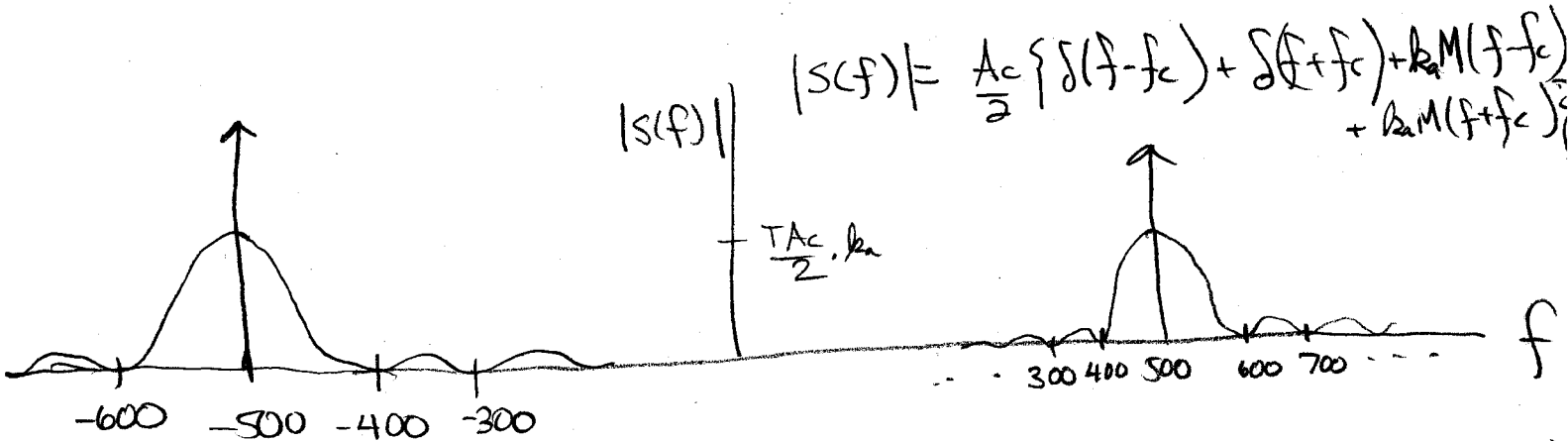
$$s(t) = A_c (1 + k_a m(t)) \cos(2\pi f_c t)$$

$$= A_c \left(1 + \frac{1}{4} \text{tri}\left(\frac{t-0.02}{0.01}\right)\right) \cos(2\pi f_c t)$$

labeling 1
tri-function - 3
correct values - 2
sinusoid - 2



(b) [7 points] Sketch the frequency domain of the transmit signal described in part (a).



Impulses - 2
labeling - 1
center freq - 1
 $\text{sinc}^2(f)$ - 3

$$M(f) = T \text{sinc}^2(fT) e^{-j2\pi f t_0}$$

$$T = 0.01$$

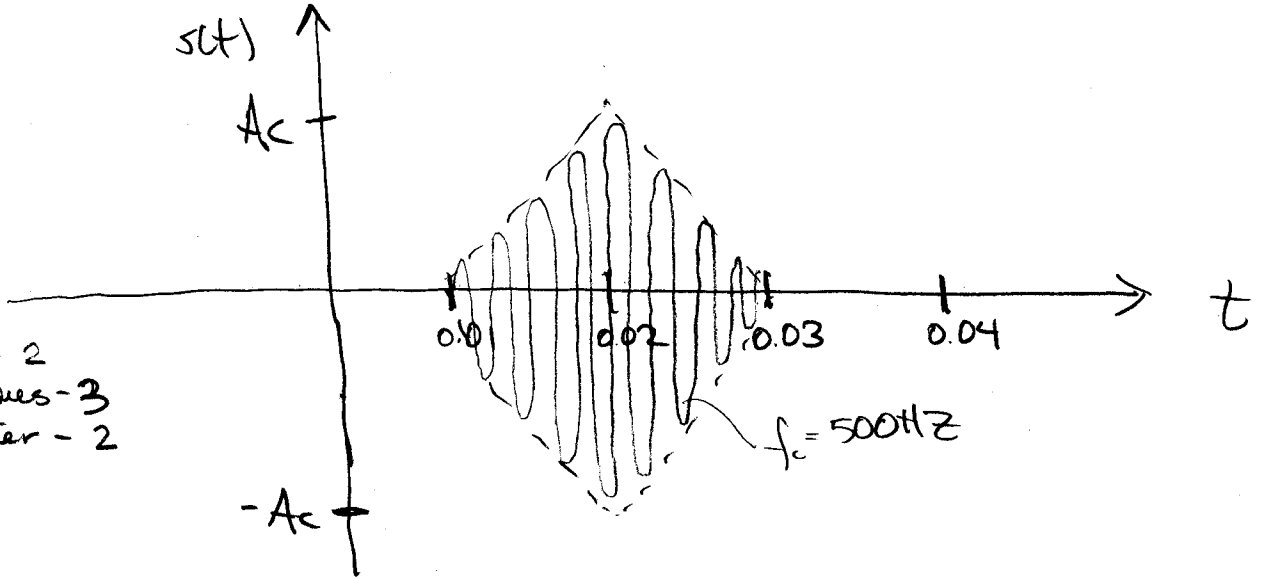
$$t_0 = 0.02$$

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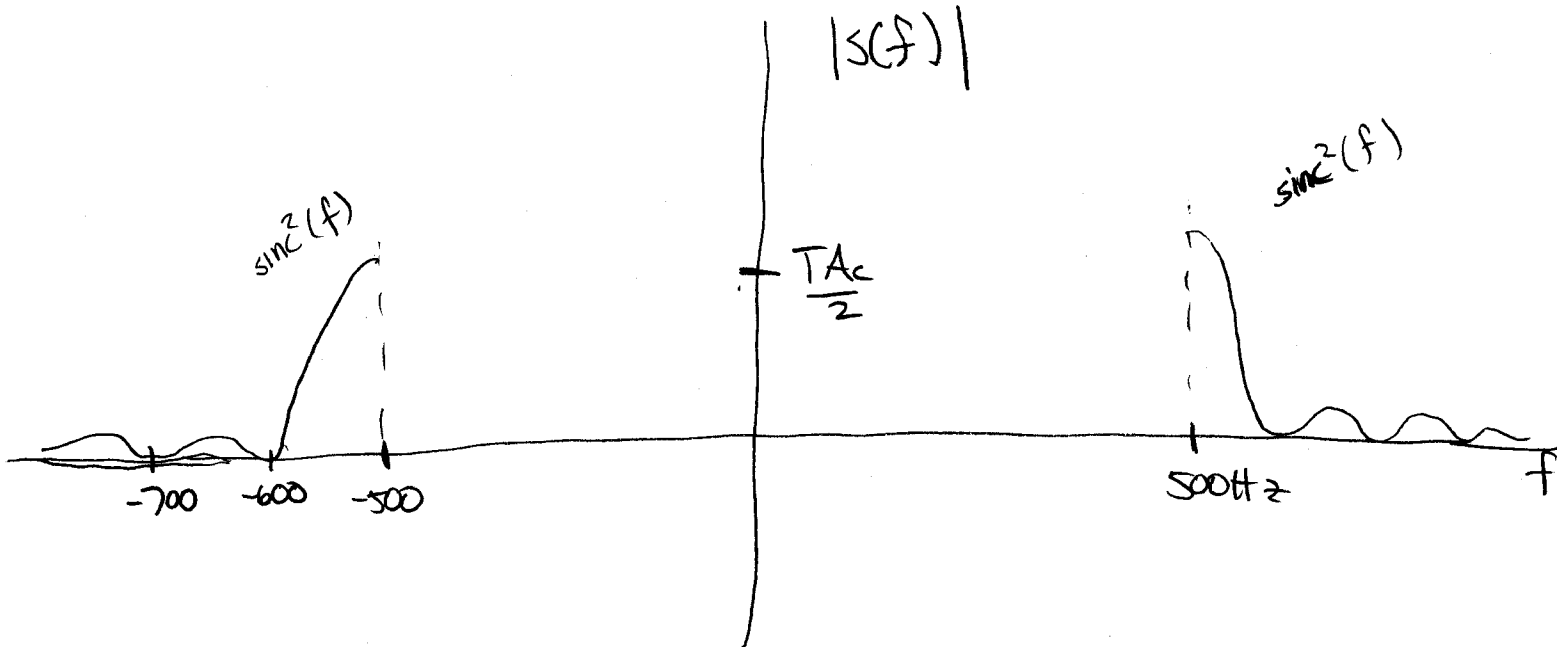
(c) [8 points] Sketch the time domain of a DSB-SC signal when $f_c = 500\text{Hz}$. Clearly label all axes and points.

$$s(t) = A_c m(t) \cos(2\pi f_c t)$$

turn-on/off - 2
tri/ peaks / time values - 3
on/off carrier - 2
label - 1



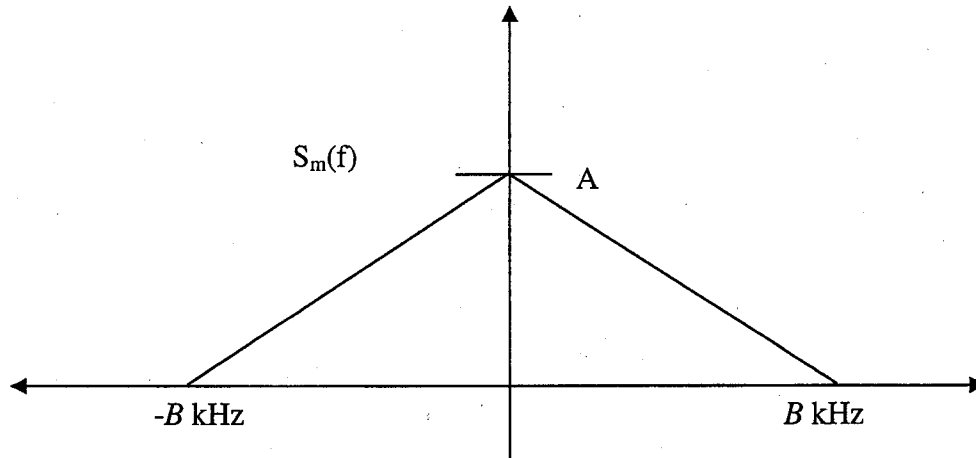
(d) [7 points] Sketch the frequency domain of a SSB signal when $f_c = 500\text{Hz}$.



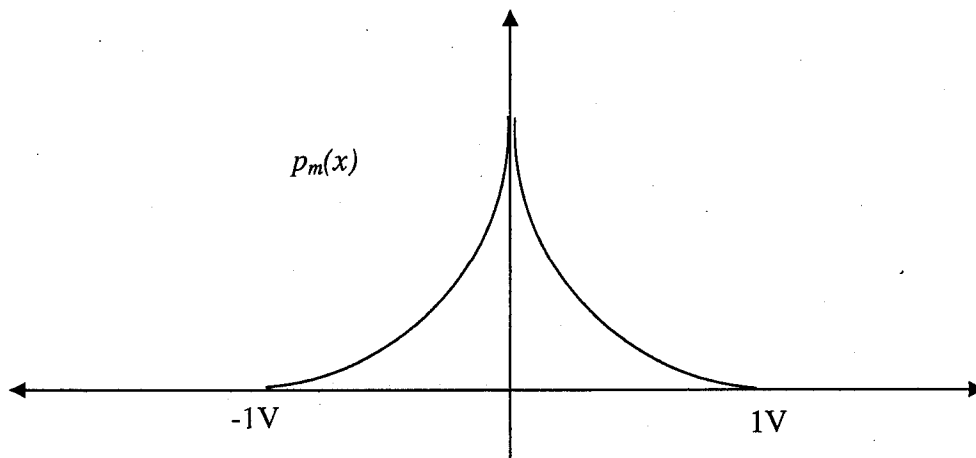
No impulse - 1
labeling - 1
Upper - 1
sinc^2 shape - 3
f_c - 1

3. (30 points) Frequency Modulation

Consider a message signal with power spectral density



and probability density function $f_m(x) = ke^{-|x|}$ where $k = \frac{1}{2(1+e^{-2})}$ which is plotted below:

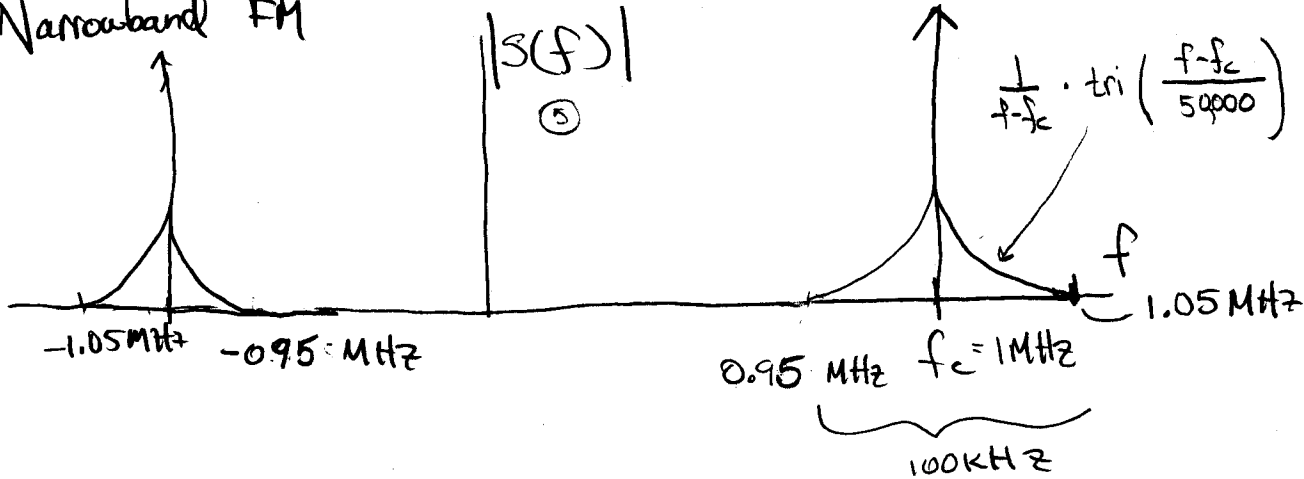


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(a) [10 points] Plot the Power spectral density, ^{or spectrum} of an FM signal if $f_c = 1\text{MHz}$ and $k_f = 200$, and $B = 50$

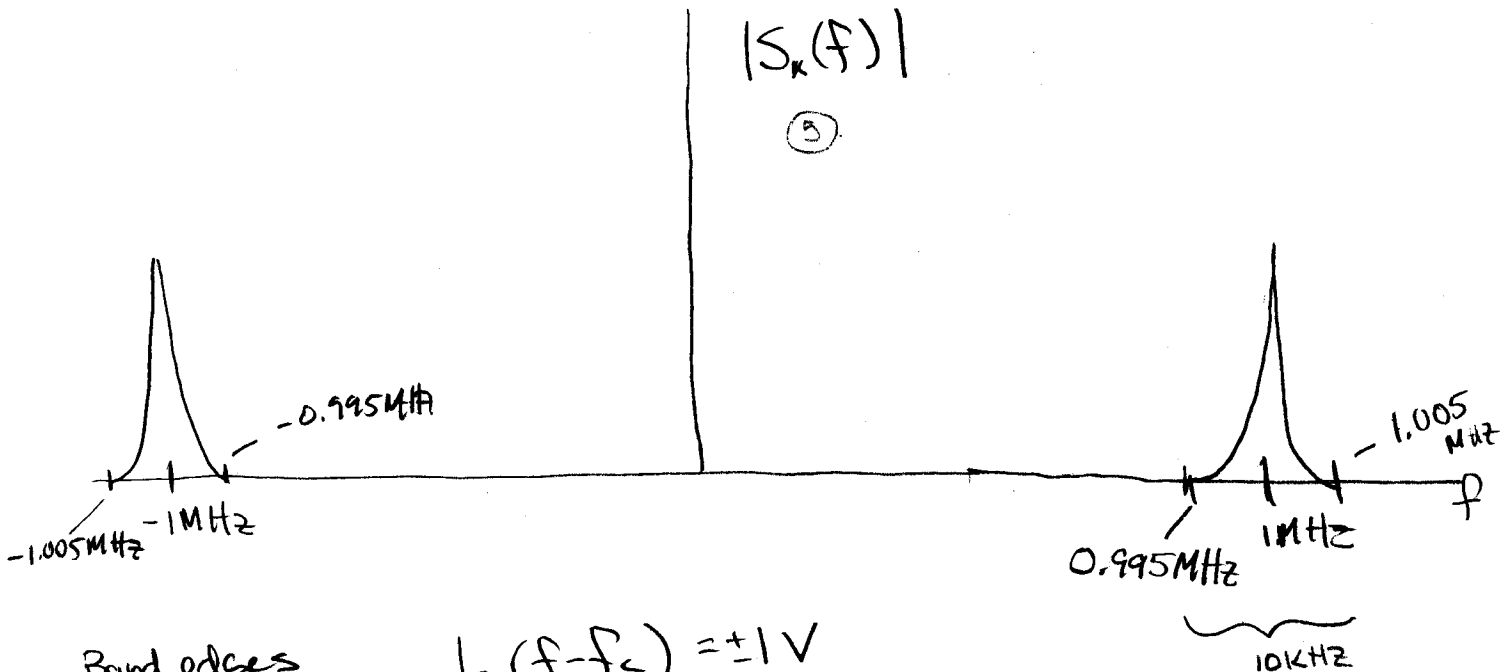
③ $D = \frac{\Delta f}{W} = \frac{k_f \cdot V_p}{W} = \frac{200 \cdot 1}{50,000} = 4 \times 10^{-3}$ N.B. approx

② - Narrowband FM



(b) [10 points] Plot the Power spectral density of an FM signal if $f_c = 1\text{MHz}$ and $k_f = 5000$ and $B = 1$.

③ $D = \frac{5000 \cdot 1}{1000} = 5$ Wideband approx. ②



Band edges \rightarrow

$$\frac{1}{k_f} (f - f_c) = \pm 1 \text{ V}$$

$$f - f_c = \pm 1 \text{ V} \cdot 5000 \text{ Hz/V}$$

$$f = \pm 5 \text{ kHz} + 1 \text{ MHz}$$

(c) [5 points] How does the bandwidth of the signal plotted in part (a) compare to Carson's Rule?

$$\begin{aligned} \text{Carson's Rule} &\rightarrow B = 2(\Delta f + W) && \textcircled{2} \\ &= 2(200 + 50,000) \\ &= 100.4 \text{ kHz} && \textcircled{1} \end{aligned}$$

$$\begin{aligned} \text{Figure} & \quad 2 \times 50 \text{ kHz} = 100 \text{ kHz} && \textcircled{2} \\ & \quad 0.4\% \text{ difference} \end{aligned}$$

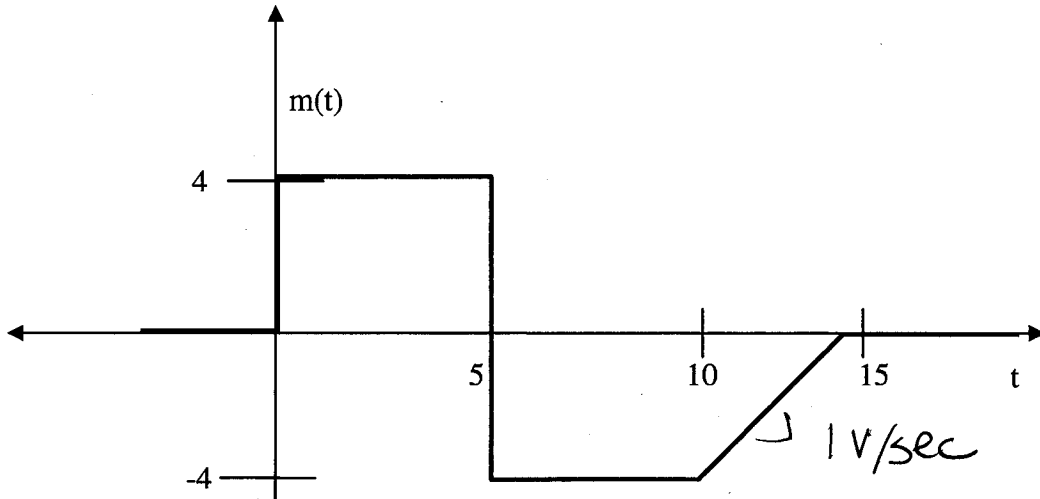
(d) [5 points] How does the bandwidth of the signal plotted in part (b) compare to Carson's Rule?

$$\begin{aligned} \text{Carson's Rule} & \quad B = 2(\Delta f + W) \\ & \quad = 2(5000 + 1000) \\ & \quad = 12 \text{ kHz} \end{aligned}$$

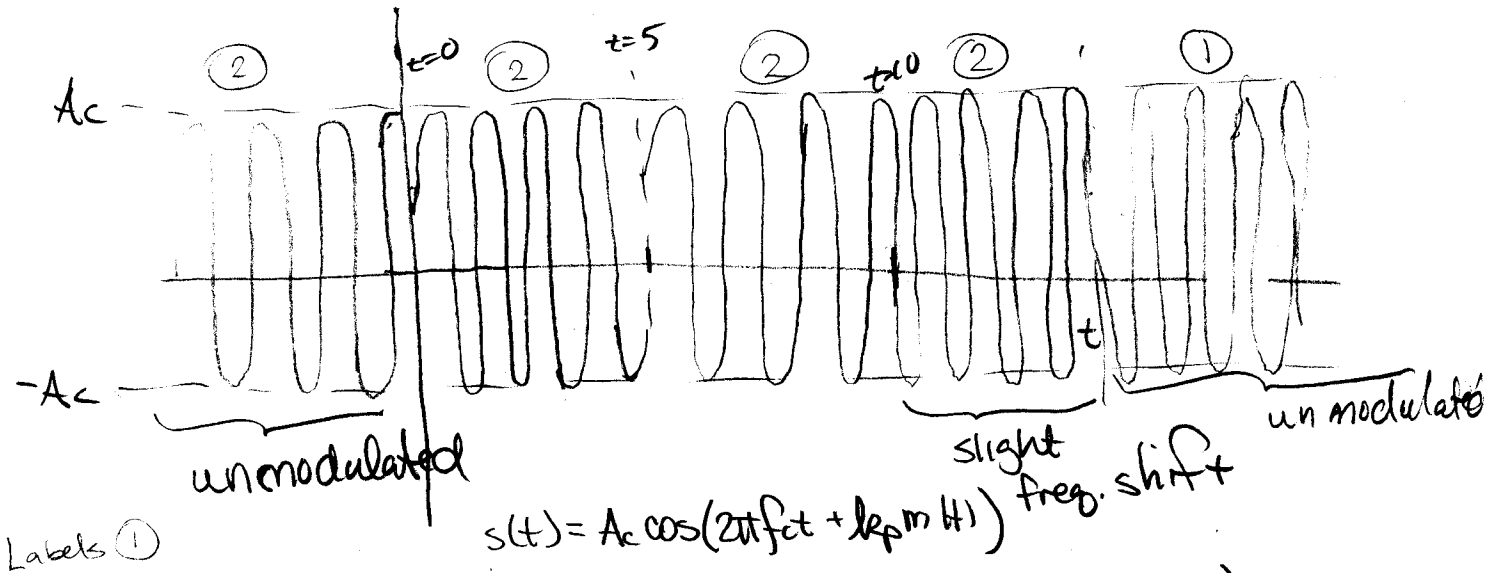
$$\begin{aligned} \text{Figure} & \quad 2 \times 5 \text{ kHz} = 10 \text{ kHz} \\ & \quad 20\% \text{ difference} \end{aligned}$$

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4. (20 points) Consider the following message signal



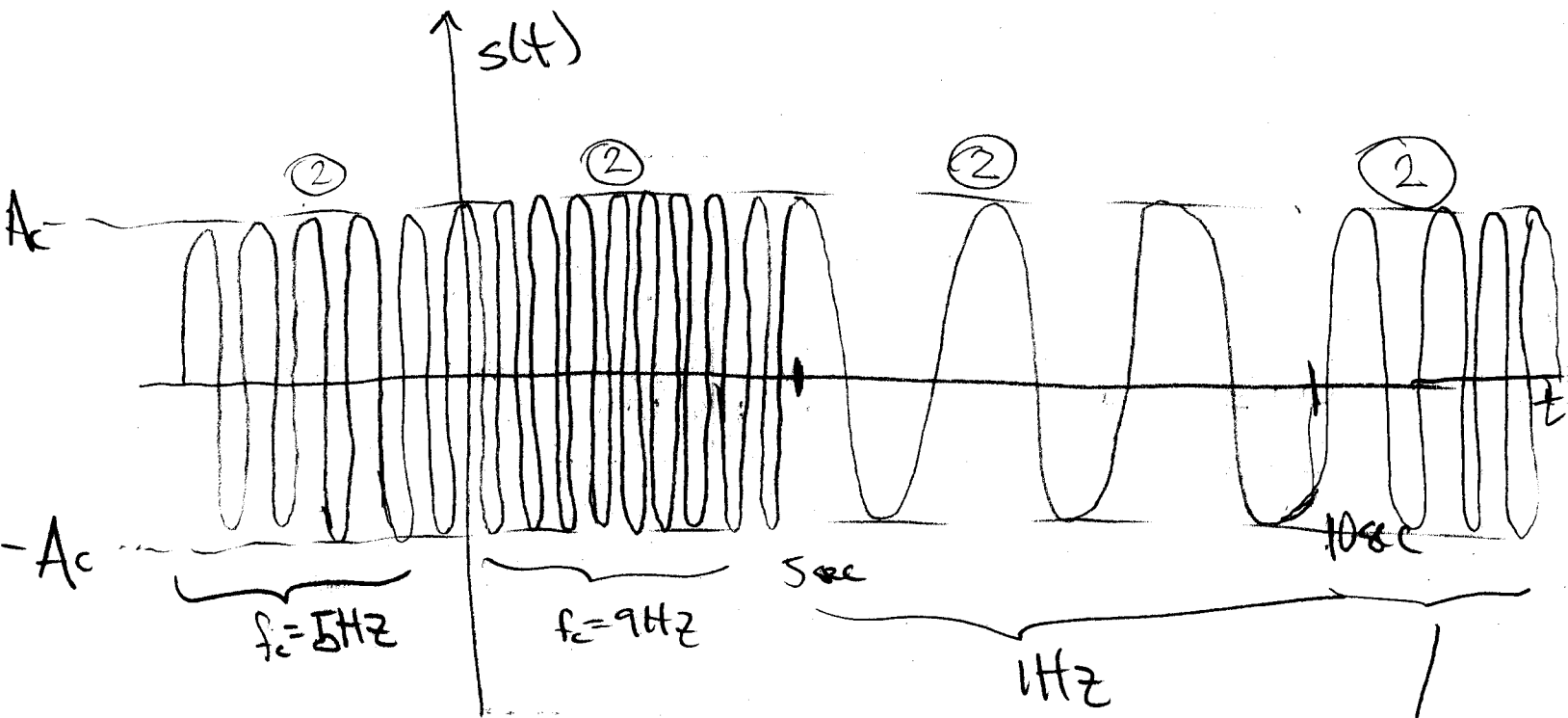
(a) [10 points] Write an expression for the time domain signal and plot the time domain signal for a phase modulated signal when $f_c = 1\text{Hz}$ and $k_p = \pi/10$ rad/V



$t < 0$	$m(t) = 0$	$s(t) = A_c \cos(2\pi f_c t)$
$0 < t \leq 5$	$m(t) = 4$	$s(t) = A_c \cos(2\pi f_c t + 4\pi/10)$
$5 < t \leq 10$	$m(t) = -4$	$s(t) = A_c \cos(2\pi f_c t - 4\pi/10)$
$10 \leq t \leq 14$	$m(t) = t - 14$	$s(t) = A_c \cos(2\pi f_c t + (\pi/10) \cdot (t - 14))$ $= A_c \cos(2\pi(f_c + 1/20)t - 14\pi/10)$
$t > 14$	$m(t) = 0$	$s(t) = A_c \cos(2\pi f_c t)$

(b) [10 points] ~~Write an expression for the time domain signal and~~ plot the time domain signal for a frequency modulated signal when $f_c = 5\text{Hz}$ and $k_f = 1\text{Hz/V}$

$$1\text{Hz/V}(\pm 4\text{V}) = \pm 4\text{Hz}$$



freq ramps up from 1Hz to 5Hz

$$\begin{aligned}
 t < 0 & \quad m(t) = 0 & \quad s(t) = A_c \cos(2\pi \cdot 5t) \\
 0 \leq t \leq 5 & \quad m(t) = 4 & \quad s(t) = A_c \cos(2\pi \cdot 9t) \\
 5 < t \leq 10 & \quad m(t) = -4 & \quad s(t) = A_c \cos(2\pi t) \\
 10 < t \leq 14 & \quad m(t) = t - 14 & \quad s(t) = A_c \cos(2\pi \cdot 5t + 2\pi(-4 + t)t) \\
 & & & = A_c \cos(2\pi(1+t)t) \\
 t > 14 & \quad m(t) = 0 & \quad s(t) = A_c \cos(2\pi \cdot 5t)
 \end{aligned}$$