Angle Modulation

In this type of modulation, the frequency or phase of carrier is varied in proportion to the amplitude of the modulating signal.

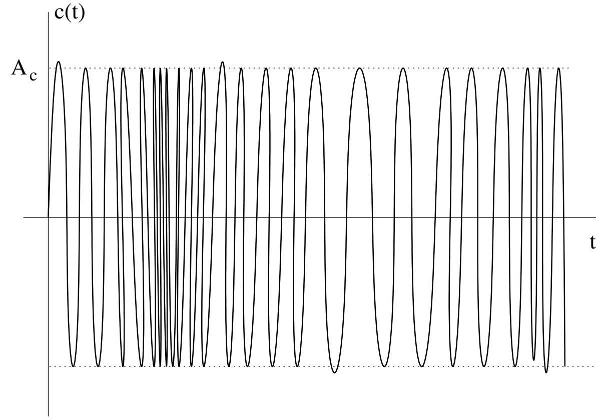

Figure 1: An angle modulated signal

If $s(t) = A_c \cos(\theta_i(t))$ is an angle modulated signal, then

1. Phase modulation:

$$\theta_i(t) = \omega_c t + k_p m(t)$$

where $\omega_c = 2\pi f_c$.

2. Frequency Modulation:

$$\begin{aligned}
\omega_i(t) &= \omega_c + k_f m(t) \\
\theta_i(t) &= \int_0^t \omega_i(t)\, dt \\
&= 2\pi \int_0^t f_i(t)\, dt + \int_0^t k_f m(t)\, dt
\end{aligned}$$

- Phase Modulation If $m(t) = A_m \cos(2\pi f_m t)$ is the message signal, then the phase modulated signal is given by

$$s(t) = A_c \cos(\omega_c t + k_p m(t))$$

Here, k_p is phase sensitivity or phase modulation index.

- Frequency Modulation If $m(t) = A_m \cos(2\pi f_m t)$ is the message signal, then the Frequency modulated signal is given by

$$2\pi f_i(t) = \omega_c + k_f A_m \cos(2\pi f_m t)$$

$$\theta_i(t) = \omega_c t + \frac{k_f A_m}{2\pi f_m} \sin(2\pi f_m t)$$

here, $\frac{k_f A_m}{2\pi}$ is called *frequency deviation* (Δf) and $\frac{\Delta f}{f_m}$ is called *modulation index* (β). The Frequency modulated signal is given by

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

Depending on how small β is FM is either *Narrowband FM*($\beta << 1$) or *Wideband FM*($\beta \approx 1$).

– Narrow-Band FM (NBFM)

In NBFM $\beta << 1$, therefor $s(t)$ reduces as follows:

$$\begin{aligned} s(t) &= A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \\ &= A_c \cos(2\pi f_c t) \cos(\beta \sin(2\pi f_m t)) - \\ & \quad A_c \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t)) \end{aligned}$$

Since, β is very small, the above equation reduces to

$$s(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_m t) \sin(2\pi f_c t)$$

The above equation is similar to AM. Hence, for NBFM the bandwidth is same as that of AM i.e.,

$2 \times message\ bandwidth(2 \times B)$.

A NBFM signal is generated as shown in Figure **??**.

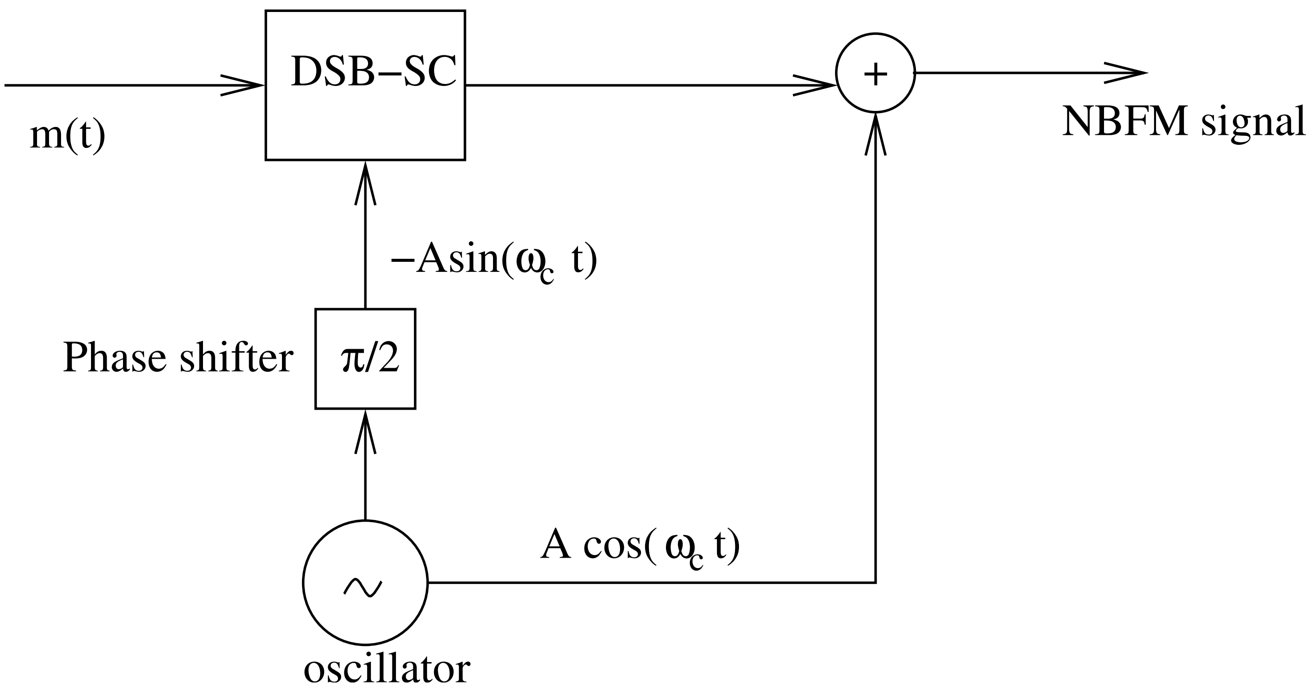

Figure 2: Generation of NBFM signal

– Wide-Band FM (WBFM)

A WBFM signal has theoritically infinite bandwidth.
Spectrum calculation of WBFM signal is a tedious process.
For, practical applications however the Bandwidth of a WBFM signal is calculated as follows:
Let $m(t)$ be bandlimited to BHz and sampled adequately at $2B$Hz. If time period $T = 1/2B$ is too small, the signal can be approximated by sequence of pulses as shown in Figure **??**

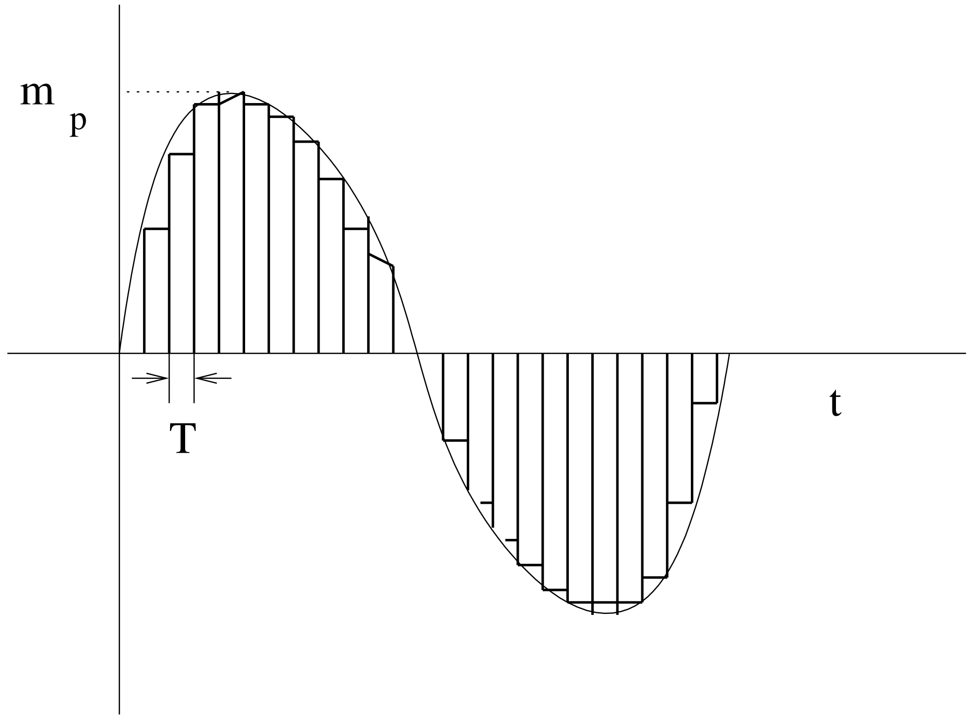

Figure 3: Approximation of message signal

If tone modulation is considered, and the peak amplitude of the sinusoid is m_p, the minimum and maximum frequency deviations will be $\omega_c - k_f m_p$ and $\omega_c + k_f m_p$ respectively. The spread of pulses in frequency domain will be $\frac{2\pi}{T} = 4\pi B$

as shown in Figure **??**

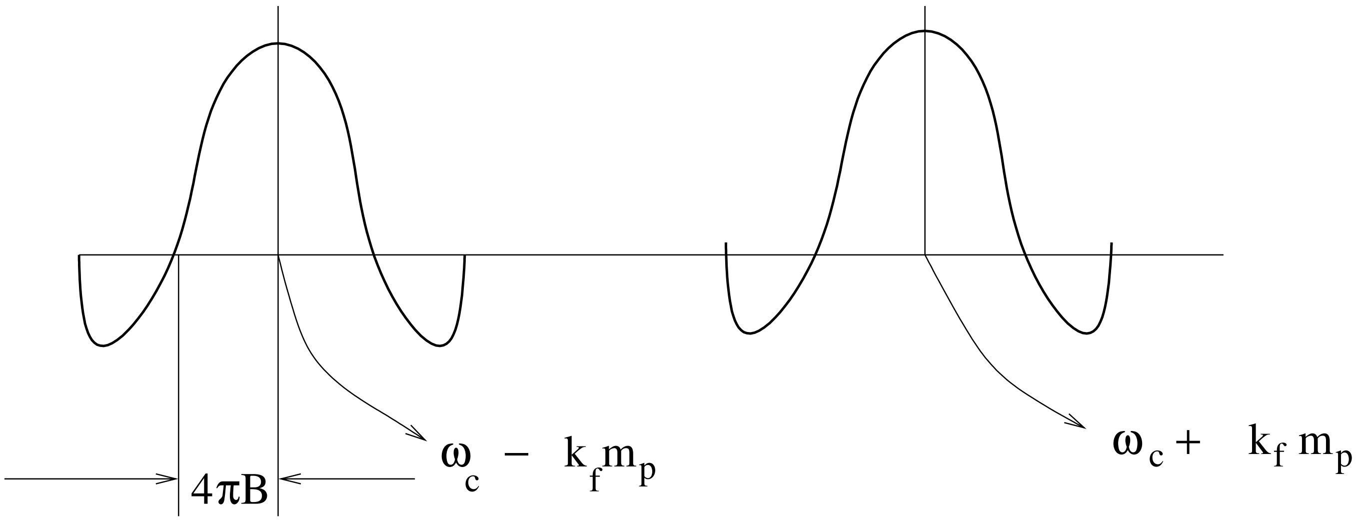

Figure 4: Bandwidth calculation of WBFM signal

Therefore, total BW is $2k_f m_p + 8\pi B$ and if frequency deviation is considered

$$
\begin{aligned}
BW_{fm} &= \frac{1}{2\pi}(2k_f m_p + 8\pi B) \\
BW_{fm} &= 2(\Delta f + 2B)
\end{aligned}
$$

* The bandwidth obtained is higher than the actual value. This is due to the staircase approximation of $m(t)$.
* The bandwidth needs to be readjusted. For NBFM, k_f is very small an d hence Δf is very small compared to B. This implies

$$B_{fm} \approx 4B$$

  But the bandwidth for NBFM is the same as that of AM which is $2B$
* A better bandwidth estimate is therefore:

$$\begin{aligned} BW_{fm} &= 2(\Delta f + B) \\ BW_{fm} &= 2(\frac{k_f m_p}{2\pi} + B) \end{aligned}$$

  This is also called *Carson's Rule*

– Demodulation of FM signals

Let $\Phi_{fm}(t)$ be an FM signal.

$$\Phi_{fm}(t) = \left[A\cos(\omega_c t + k_f \int_0^t m(\alpha)\, d\alpha)\right]$$

This signal is passed through a differentiator to get

$$\dot{\Phi}_{fm}(t) = A\left(\omega_c + k_f m(t)\right)\sin\left(\omega_c t + k_f \int_0^t m(\alpha)\, d\alpha\right)$$

If we observe the above equation carefully, it is both amplitude and frequency modulated.

Hence, to recover the original signal back an envelope detector can be used. The envelope takes the form (see Figure **??**):

$$Envelope = A\left(\omega_c + k_f m(t)\right)$$

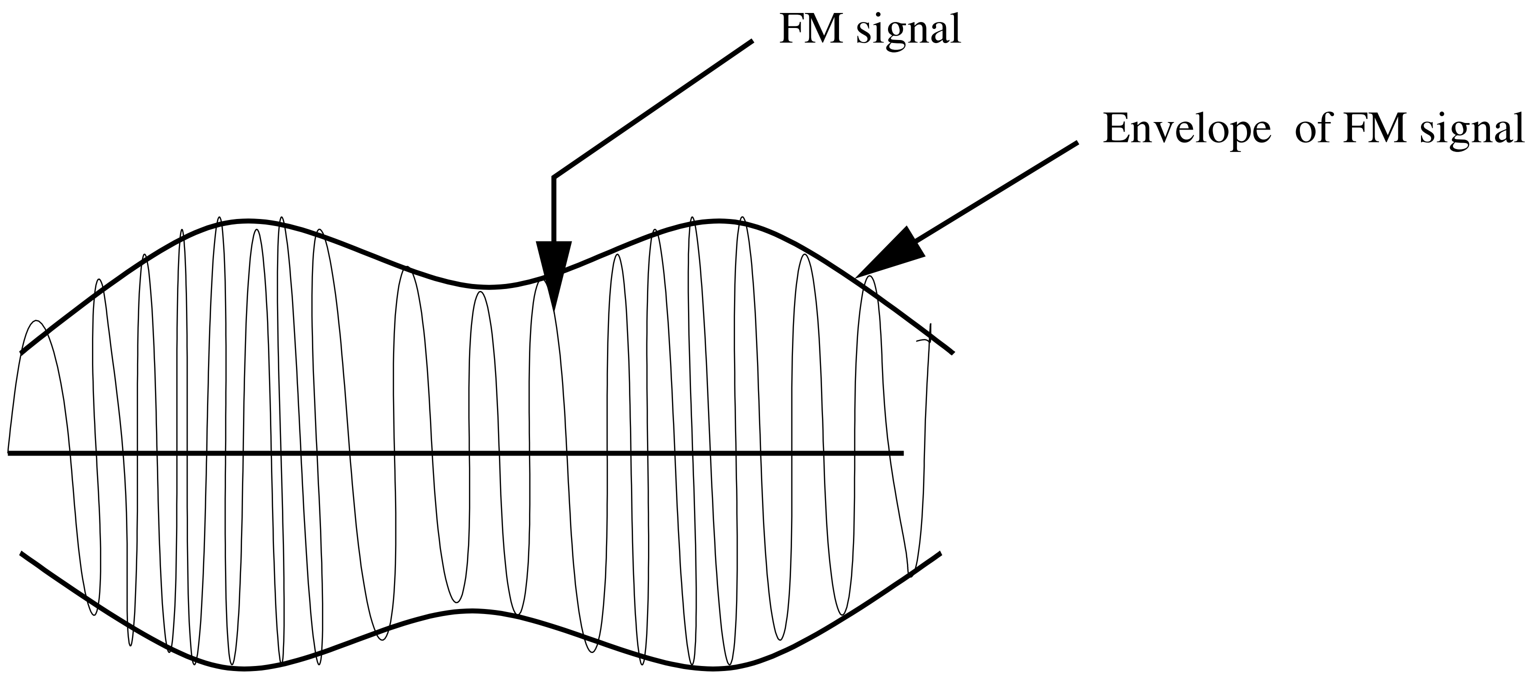

Figure 5: FM signal - both Amplitude and Frequency Modulation

The block diagram of the demodulator is shown in Figure **??**

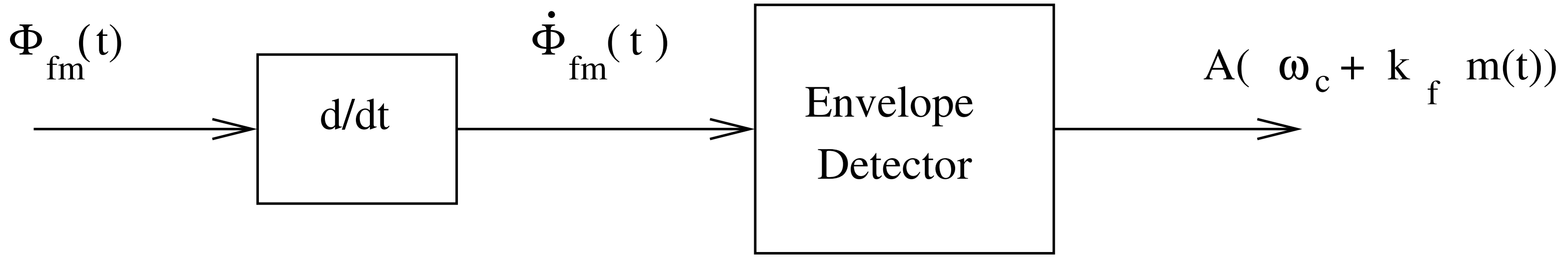

Figure 6: Demodulation of an FM signal

- The analysis for *Phase Modulation* is identical.
  - Analysis of bandwidth in PM

$$
\begin{aligned}
\omega_i &= \omega_c + k_p m'(t) \\
m'_p &= [m'(t)]_{max} \\
\Delta\omega &= k_p m_p \\
BW_{pm} &= 2(\Delta f + B) \\
BW_{pm} &= 2(\frac{k_p m'_p}{2\pi} + B)
\end{aligned}
$$

– The difference between FM and PM is that the bandwidth is independent of signal bandwidth in FM while it is strongly dependent on signal bandwidth in PM. [a]

[a] owing to the bandwidth being dependent on the peak of the *derivative of m(t)* rather than $m(t)$ itself

Angle Modulation: An Example

- An angle-modulated signal with carrier frequency $\omega_c = 2\pi \times 10^6$ is described by the equation:

$$\phi_{EM}(t) = 12\cos(\omega_c t + 5\sin 1500t + 10\sin 2000\pi t)$$

1. Determine the power of the modulating signal.
2. What is Δf?
3. What is β?
4. Determine $\Delta\phi$, the phase deviation.
5. Estimate the bandwidth of $\phi_E M(t)$?

1. $P = 12^2/2 = 72\ units$
2. Frequency deviation Δf, we need to estimate the instantaneous frequency:

$$\omega_i = \frac{d}{dt}\theta(t) = \omega_c + 7,500\cos 1500t + 20,000\pi t$$

The deviation of the carrier is $\Delta\omega = 7,500\cos 1500t + 20,000\pi t$. When the two sinusoids add in phase, the maximum value will be $7,500 + 20,000\pi$ Hence $\Delta f = \frac{\Delta\omega}{2\pi} = 11,193.66Hz$

3. $\beta = \frac{\Delta f}{B} = \frac{11,193.66}{1000} = 11.193$
4. The angle $\theta(t) = \omega_c t + 5\sin 1500t + 10\sin 2000\pi t)$. The maximum angle deviation is 15, which is the phase deviation.
5. $B_{EM} = 2(\Delta f + B) = 24,387.32\ Hz$