4 Amplitude Modulation (AM)

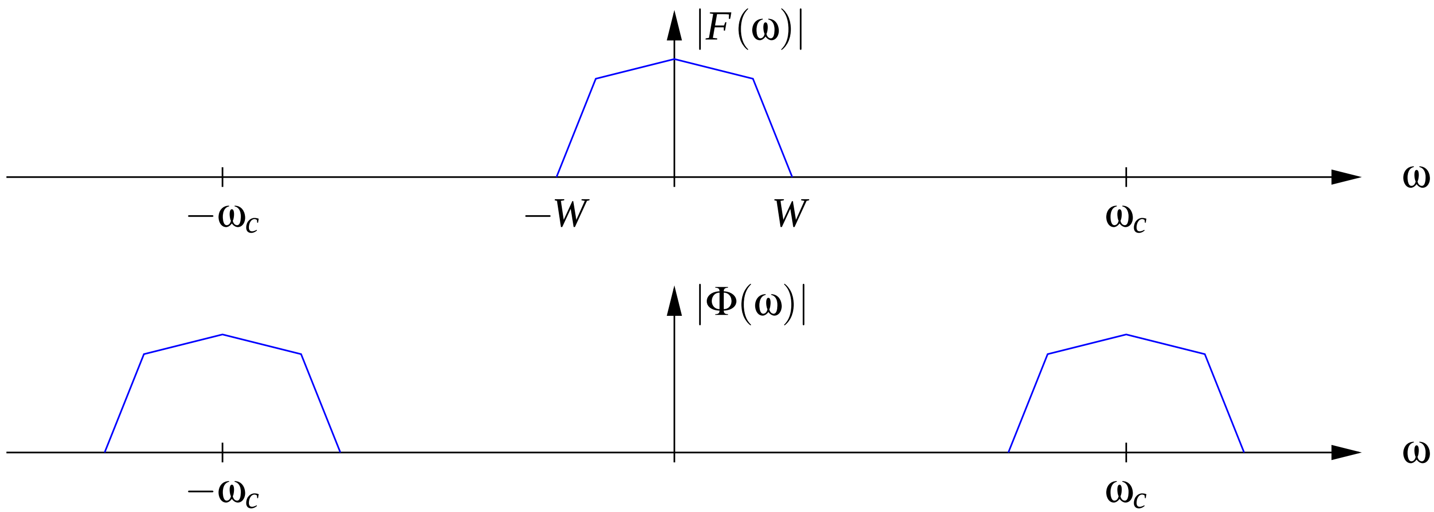
4.1 Introduction

Modulation: Process by which a property or a parameter of a signal is varied in proportion to a second signal.

Amplitude Modulation: The amplitude of a sinusoidal signal with fixed frequency and phase is varied in proportion to a given signal.

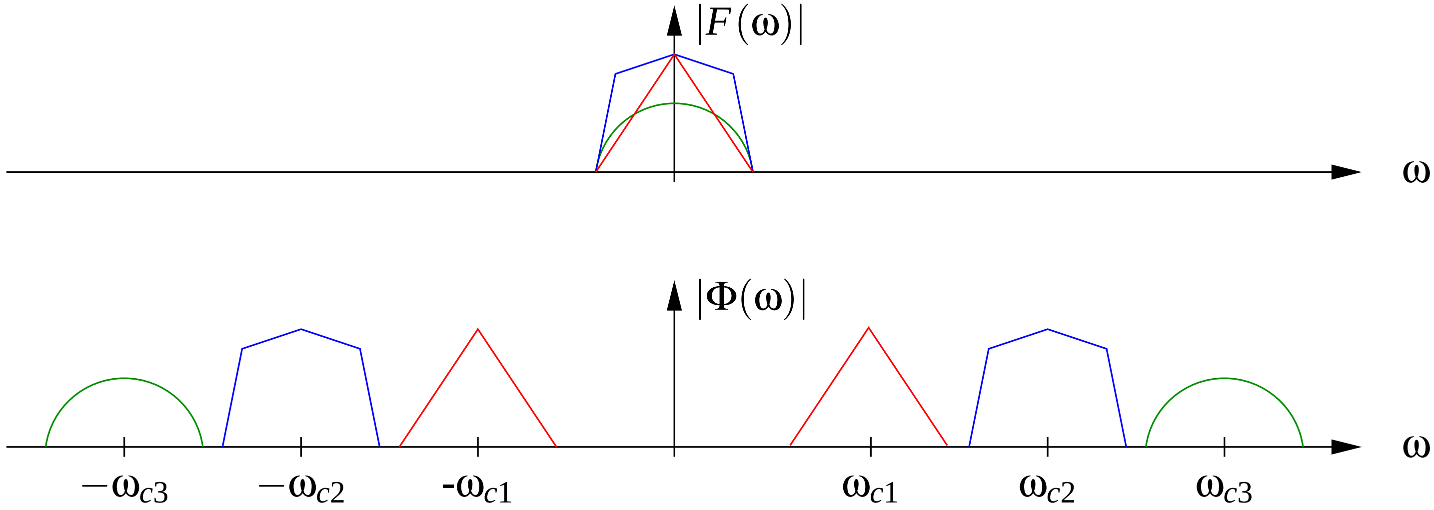
Purpose:

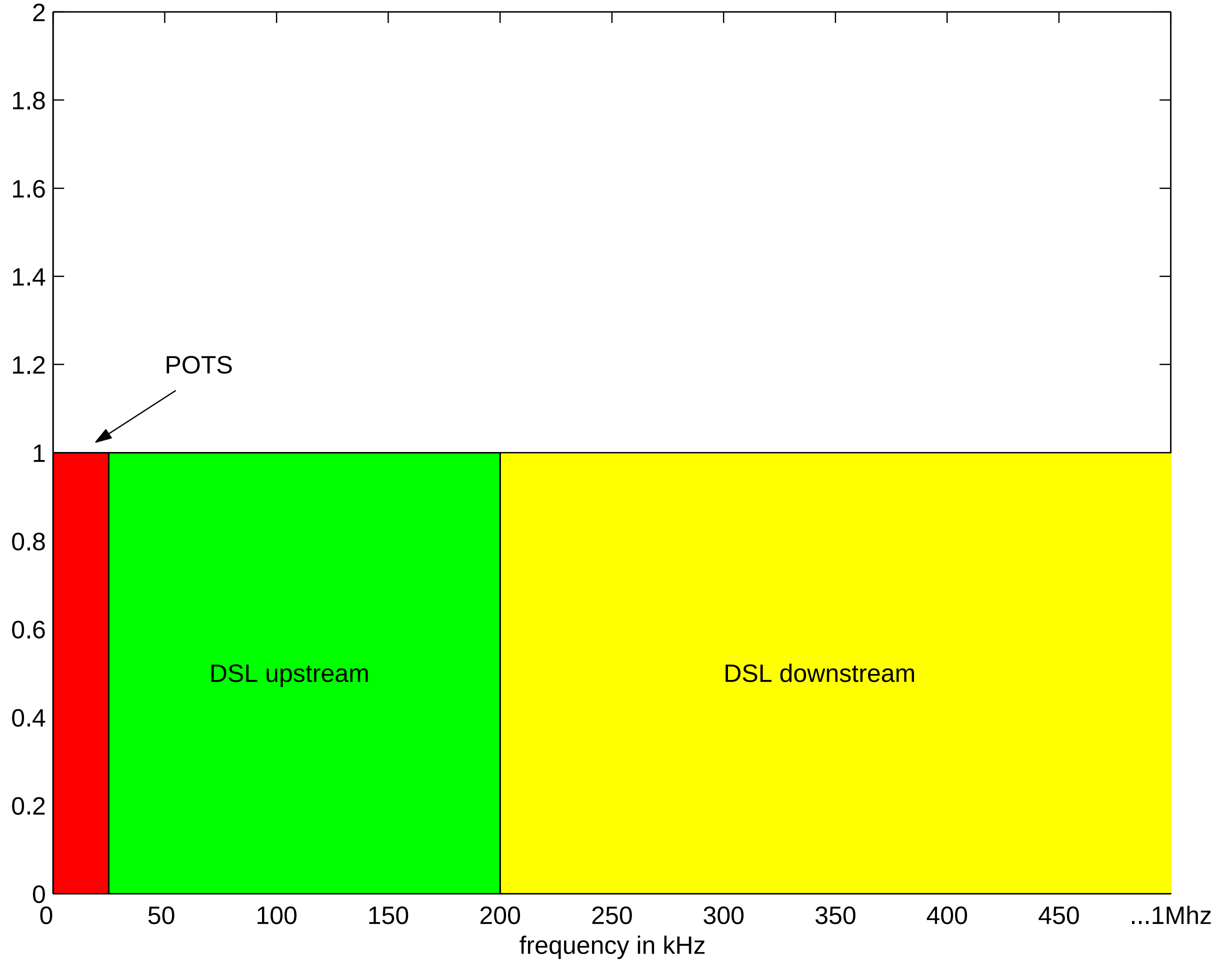
- Adaptation of the information signal to the transmission channel
- Shift of the information signal to an assigned frequency band



- Efficient antenna design: size is at least $1/4^{th}$ of signal wavelength
  $\Rightarrow$ antennas for lowpass signals would be too large ($f = 3$ kHz, $\lambda = 100,000$ m).

- Simultaneous transmission of several information signals (e.g. radio broadcasting)

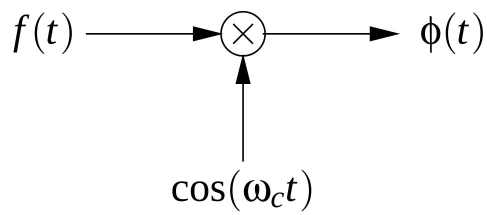




POTS
DSL upstream
DSL downstream
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 50 100 150 200 250 300 350 400 450 ...1Mhz
frequency in kHz

4.2 Double-Sideband Suppressed Carrier AM (DSB-SC)

4.2.1 Modulation



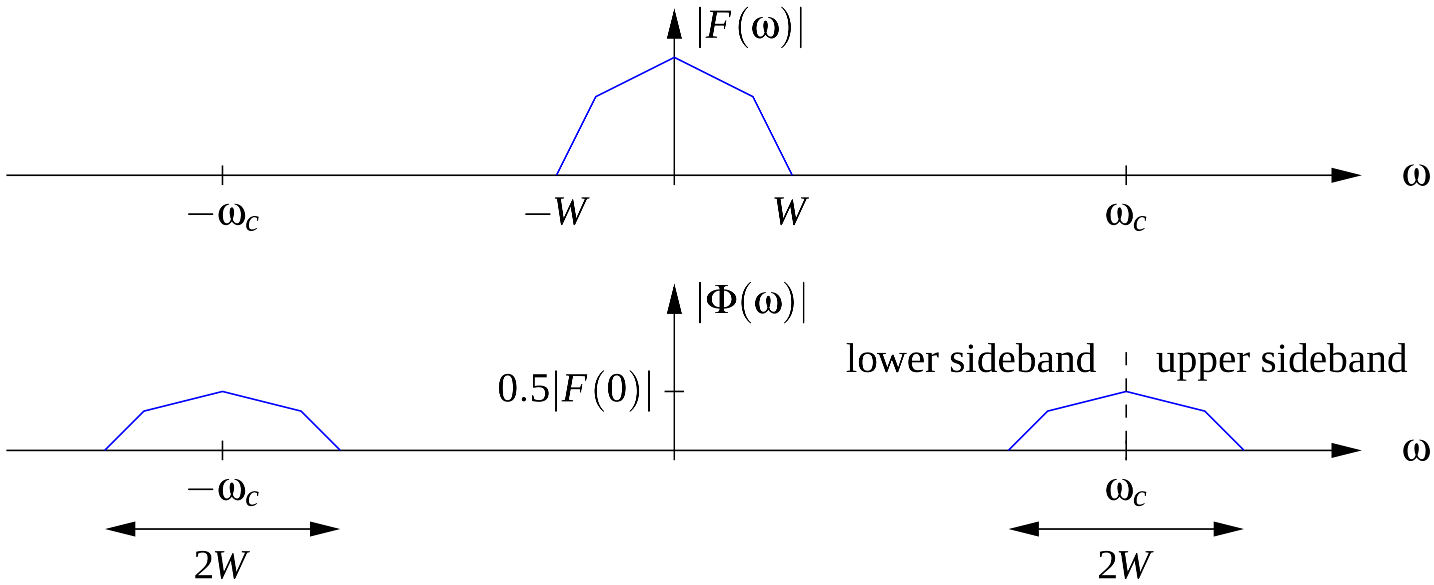
Generation of DSB-SC modulated signal:

$$\phi(t) = f(t)\cos(\omega_c t)$$

$\phi(t)$: modulated transmit signal
$f(t)$: modulating signal, real valued
$\cos(\omega_c t)$: carrier signal, ω_c: carrier frequency in rad/sec

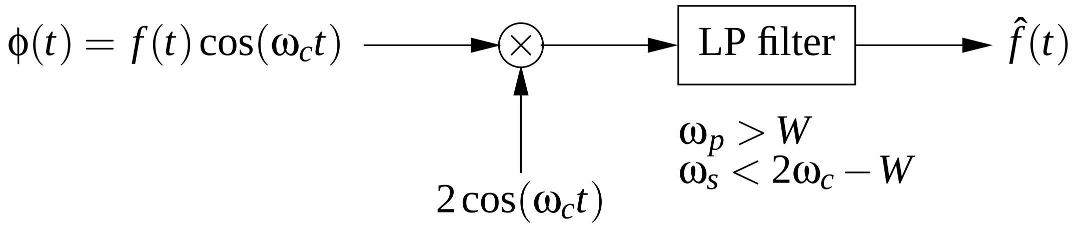
Spectrum of DSB-SC modulated signal:

$$\phi(t) = f(t)\cos(\omega_c t) \quad \circ\!\!-\!\!\bullet \quad \Phi(\omega) = \frac{1}{2}F(\omega - \omega_c) + \frac{1}{2}F(\omega + \omega_c)$$



- Carrier frequency has to be larger than twice the bandwidth $\omega \geq 2W$.
- Bandwidth of the modulated signal $\phi(t)$ is twice as large as the bandwidth of the modulating signal $f(t)$.
- No separate carrier is present in $\phi(t)$.
- Upper sideband: spectral content for positive frequencies above ω_c.
  Lower sideband: spectral content for positive frequencies below ω_c.
- Information in upper and lower sideband are redundant since $\Phi(\omega_c+\omega) = \Phi^*(\omega_c-\omega)$, or equivalently: $|\Phi(\omega_c + \omega)| = |\Phi(\omega_c - \omega)|$ and $\angle\Phi(\omega_c + \omega) = -\angle\Phi(\omega_c - \omega)$

4.2.2 Demodulation



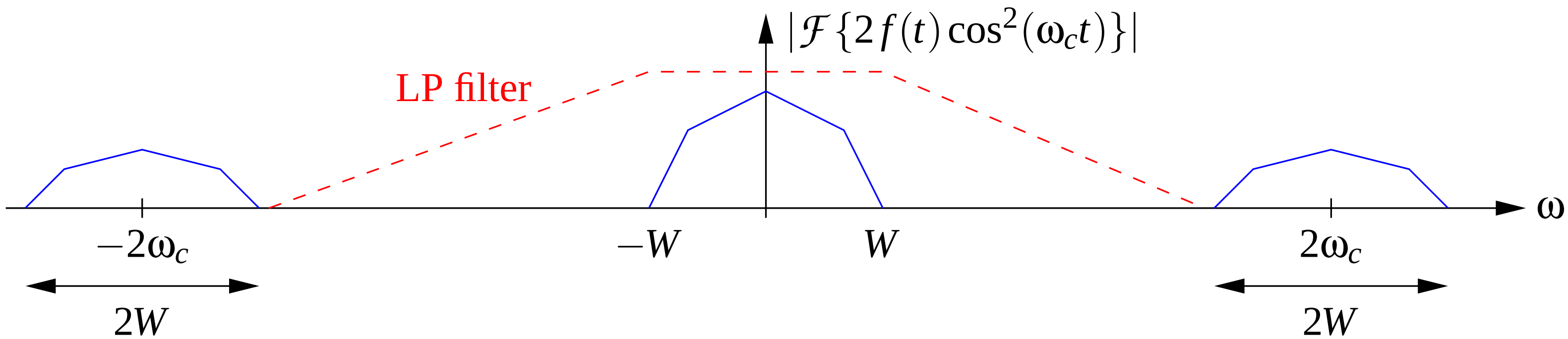
Before lowpass filtering:

$$\phi(t)\, 2\cos(\omega_c t) = 2f(t)\cos^2(\omega_c t) = f(t)\,(1 + \cos(2\omega_c t))$$

$$\mathcal{F}\{\phi(t)\, 2\cos(\omega_c t)\} = F(\omega) + \frac{1}{2}F(\omega - 2\omega_c) + \frac{1}{2}F(\omega + 2\omega_c)$$

After lowpass filtering:

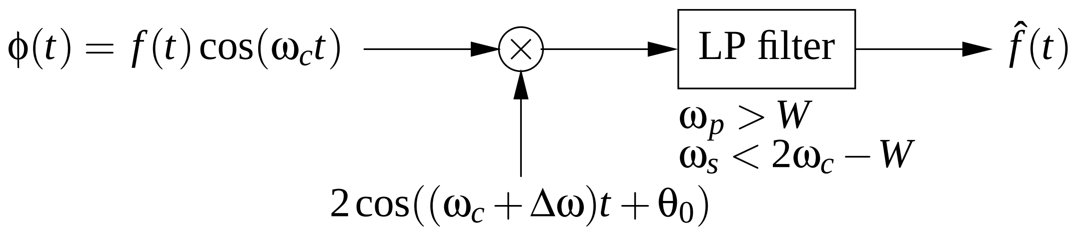
$$\hat{F}(\omega) = F(\omega)$$



The oscillators at the transmitter and receiver have to be synchronized, i.e. the carrier frequency ω_c as well as the phase must be identical (**coherent demodulation**).

Influence of Frequency and Phase Offset:

The oscillator at the receiver has a constant phase offset of θ_0 as well as a slightly different carrier frequency of $\omega_c + \Delta\omega$ when compared to the one at the transmitter.



Before lowpass filtering:

$$\phi(t)\, 2\cos((\omega_c + \Delta\omega)t + \theta_0) = 2f(t)\cos(\omega_c t)\cos((\omega_c + \Delta\omega)t + \theta_0)$$

$$= f(t)\cos((2\omega_c + \Delta\omega)t + \theta_0) + f(t)\cos(\Delta\omega t + \theta_0)$$

After lowpass filtering:

$$\hat{f}(t) = f(t)\cos(\Delta\omega t + \theta_0)$$
$$= \frac{1}{2}f(t)\exp(j\Delta\omega t)\exp(j\theta) + \frac{1}{2}f(t)\exp(-j\Delta\omega t)\exp(-j\theta)$$

$$\hat{F}(\omega) = \frac{1}{2}\exp(j\theta)F(\omega - \Delta\omega) + \frac{1}{2}\exp(-j\theta)F(\omega + \Delta\omega)$$

Phase error only (i.e. $\Delta\omega = 0$):

$$\hat{f}(t) = f(t)\cos(\theta_0) \qquad \circ\!\!-\!\!\bullet \qquad \hat{F}(\omega) = F(\omega)\cos(\theta_0)$$

$\Rightarrow$ The recovered signal is scaled by a constant. For $\theta_0 = \pm 90^\circ$ we have $\hat{f}(t) = 0$.

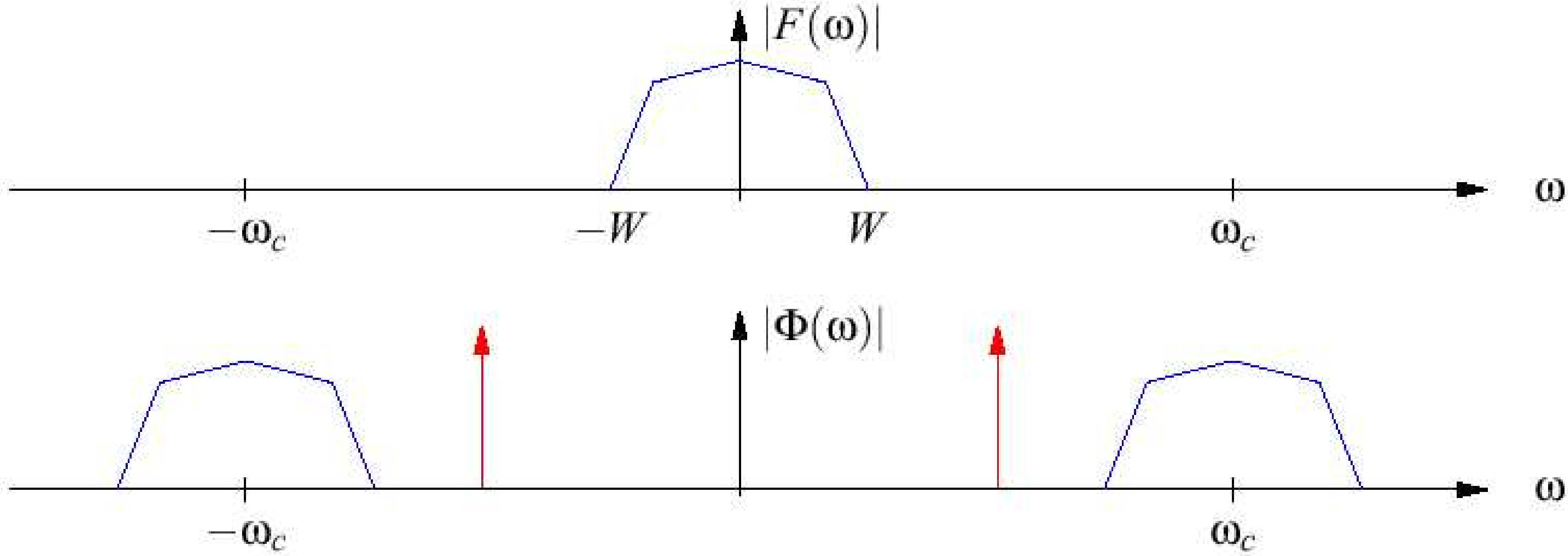
Frequency error only (i.e. $\theta_0 = 0$):

$$\hat{f}(t) = f(t)\cos(\Delta\omega t) \qquad \circ\!\!-\!\!\bullet \qquad \hat{F}(\omega) = \frac{1}{2}F(\omega - \Delta\omega) + \frac{1}{2}F(\omega + \Delta\omega)$$

$\Rightarrow$ The recovered signal is still modulated by a cosine signal of low frequency $\Delta\omega$.

Pilot Carrier

- send a sinusoidal tone whose frequency and phase is proportional to ω_c



- sent outside the passband of the modulate signal
- Receiver detects the tone, translates to correct frequency(doubling) and demodulates

Example - Commercial Stereo FM Stations

Transmitter

- need to transmit left(L) and right(R) as well as (L+R) for monophonic
- (L+R) occupies $0 - 15kHz$
- so does (L-R), so shift up using DSB-SC with $\omega_c = 38kHz$
- place pilot tone at 19kHz

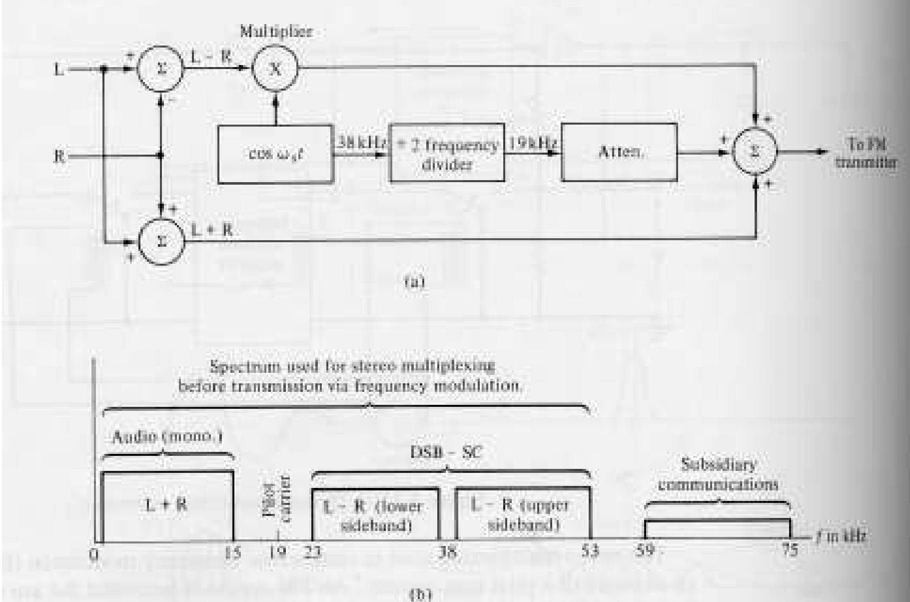


Figure 5.12 (a) Block diagram and (b) spectrum of a stereo-multiplex system.

Receiver

- narrow bandpass filter at 19kHz and then double to 38kHz
- after demodulation using pilot tone, we have

$$\text{Left channel} = (L + R) + (L - R) = 2L$$
$$\text{Right channel} = (L + R) - (L - R) = 2R$$

Phase Locked Loop(PLL)

- Pilot Tone Problem -BP filters drift in tuning, bad at rejecting noise
- Solution: Phase Locked Loop(PLL)

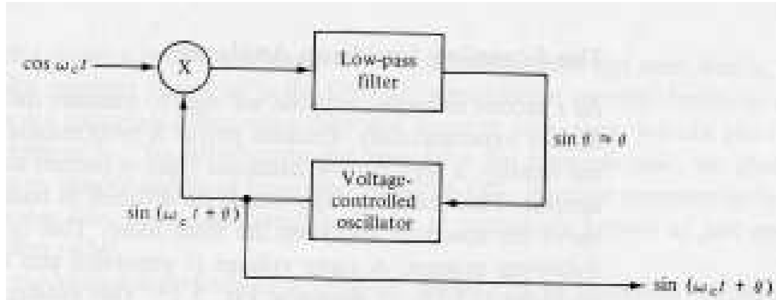


Figure 5.13 The phase-locked loop.

- Operation when Voltage Controlled Oscillator(VCO) frequency(ω_{VCO}) is close to ω_c
  - low-frequency component of output is proportional to magnitude and sign of phase difference
  - this voltage adjusts ω_{VCO} to keep phase difference a minimum
- Bandwidth of PLL determined by LPF
  - Small BW $\Rightarrow$ good noise rejection but receiver may never lock
  - Large BW $\Rightarrow$ good lock but bad noise rejection

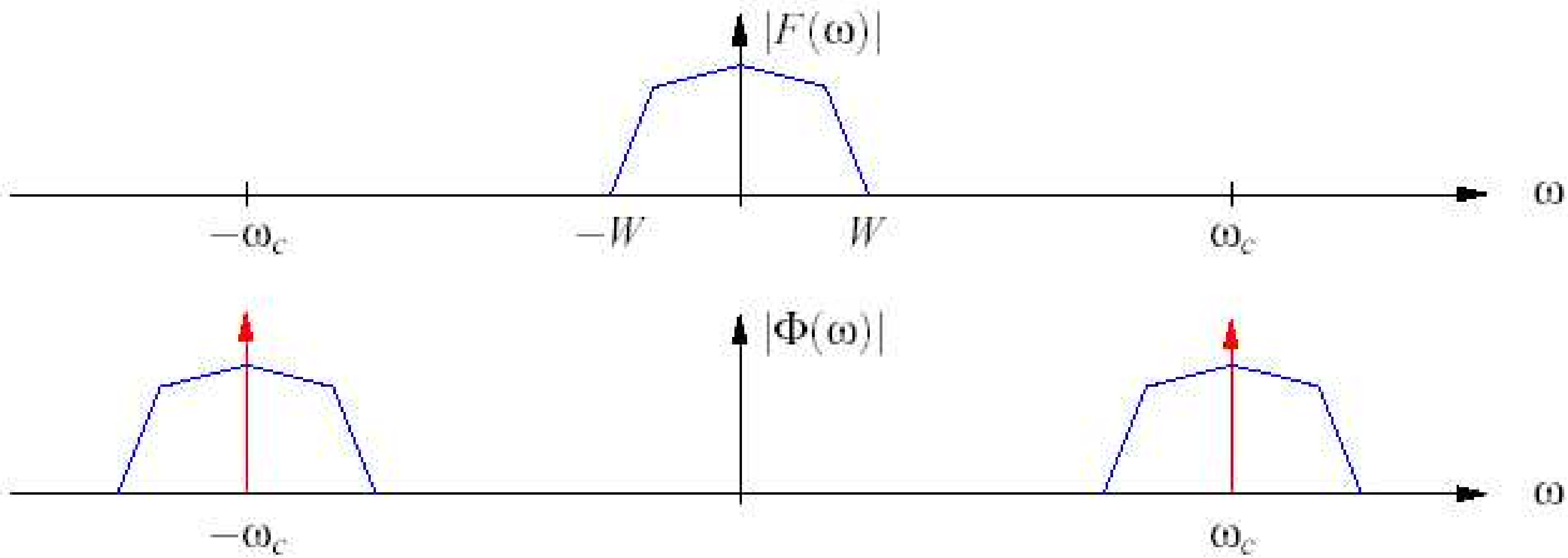
4.3 Double-Sideband Large Carrier AM

4.3.1 Modulation

- Reduces complexity of receiver
- Since this type of AM is used in commercial broadcast stations, usually termed AM
- Similar to DSB-SC, except that we incorporate the carrier
  - carrier must be larger than the rest of the signal
  - ruins low-frequency response of the system, so must not require frequency response down to 0.

$$\phi_{AM} = f(t)\cos(\omega_c t) + A\cos(\omega_c t)$$

$$\Phi_{AM}(\omega) = \frac{1}{2}F(\omega+\omega_c) + \frac{1}{2}F(\omega-\omega_c) + \pi A\delta(\omega+\omega_c) + \pi A\delta(\omega-\omega_c)$$



- if A is large enough signal recovery is done with envelope detection

$$[A + f(t)] \geq 0 \quad \text{for all } t$$

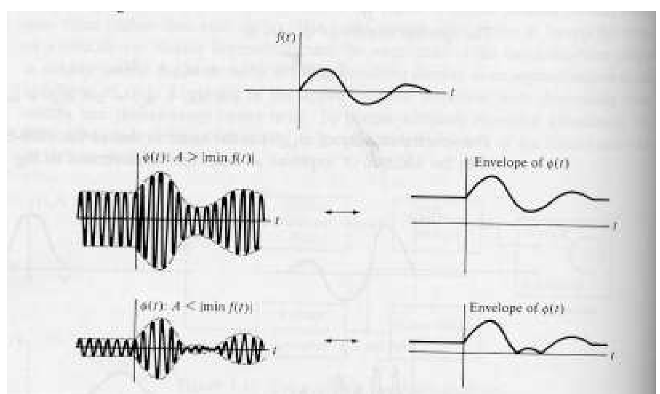


Figure 5.17 Importance of sufficient carrier in DSB-LC waveform.

- Let $f(t) = \cos(\omega_m t)$, we define m to control the amount of modulation

$$m = \frac{\text{peak DSB-SC amplitude}}{\text{peak carrier amplitude}}$$

$$\begin{aligned}\phi(t) &= A\cos(\omega_c t) + mA\cos(\omega_m t)\cos(\omega_c t) \\ &= A[1 + m\cos(\omega_m t)]\cos(\omega_c t)\end{aligned}$$

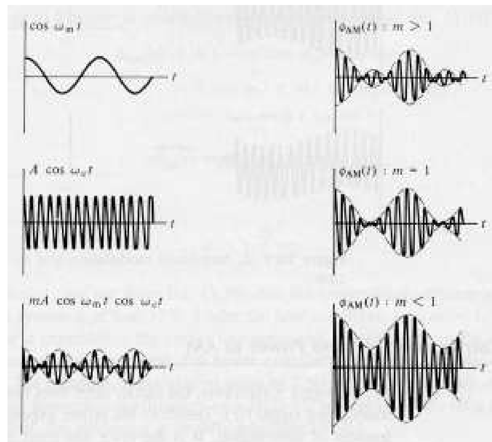


Figure 5.18 Effects of varying modulation indexes.

- percentage of modulation for DSB-LC signal with sinusoidal modulation

$$\%\text{mod} = \frac{A(1+m) - A(1-m)}{A(1+m) + A(1-m)} \times 100\% = m \times 100\%$$

- we call m the modulation index
- in order to detect the signal with no distortion we require $m \leq 1$

4.3.2 Carrier and Sideband Power in AM

- carrier provides no information so it is just wasted power
- for an AM signal $\phi_{AM}(t) = A\cos(\omega_c t) + f(t)\cos(\omega_c t)$ the power is

$$\begin{aligned}\overline{\phi_{AM}^2(t)} &= A^2\overline{\cos^2(\omega_c t)} + \overline{f^2(t)\cos^2(\omega_c t)} + 2A\overline{f(t)\cos^2(\omega_c t)} \\ &= A^2\overline{\cos^2(\omega_c t)} + \overline{f^2(t)\cos^2(\omega_c t)} \\ &= A^2/2 + \overline{f^2(t)}/2\end{aligned}$$

- so we can express the total power as,

$$P_t = P_c + P_s = \frac{1}{2}A^2 + \frac{1}{2}\overline{f^2(t)}$$

so that the fraction of the total power contained in the sidebands is

$$\mu = \frac{P_s}{P_t} = \frac{\overline{f^2(t)}}{A^2 + \overline{f^2(t)}}$$

- so when $f(t) = \cos(\omega_m t)$ we get

$$\overline{\phi_{AM}^2(t)} = \frac{1}{2}A^2 + (\frac{1}{2})(\frac{1}{2})m^2A^2$$

$$\mu = \frac{m^2}{2+m^2}$$

- so for best case, i.e., $m = 1$, 67% of the total power is wasted with the carrier

4.3.3 Demodulation

- the price we pay for wasted power is a tradeoff for simple receiver design
- receiver is simply an envelope detector

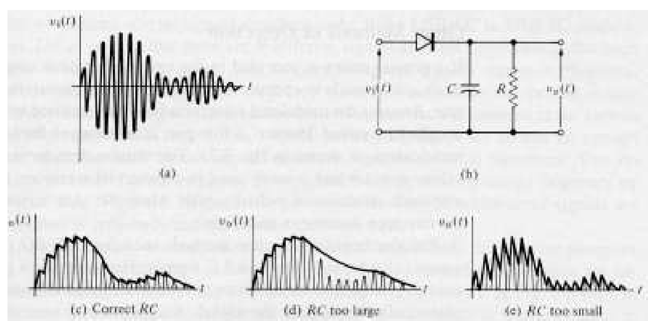
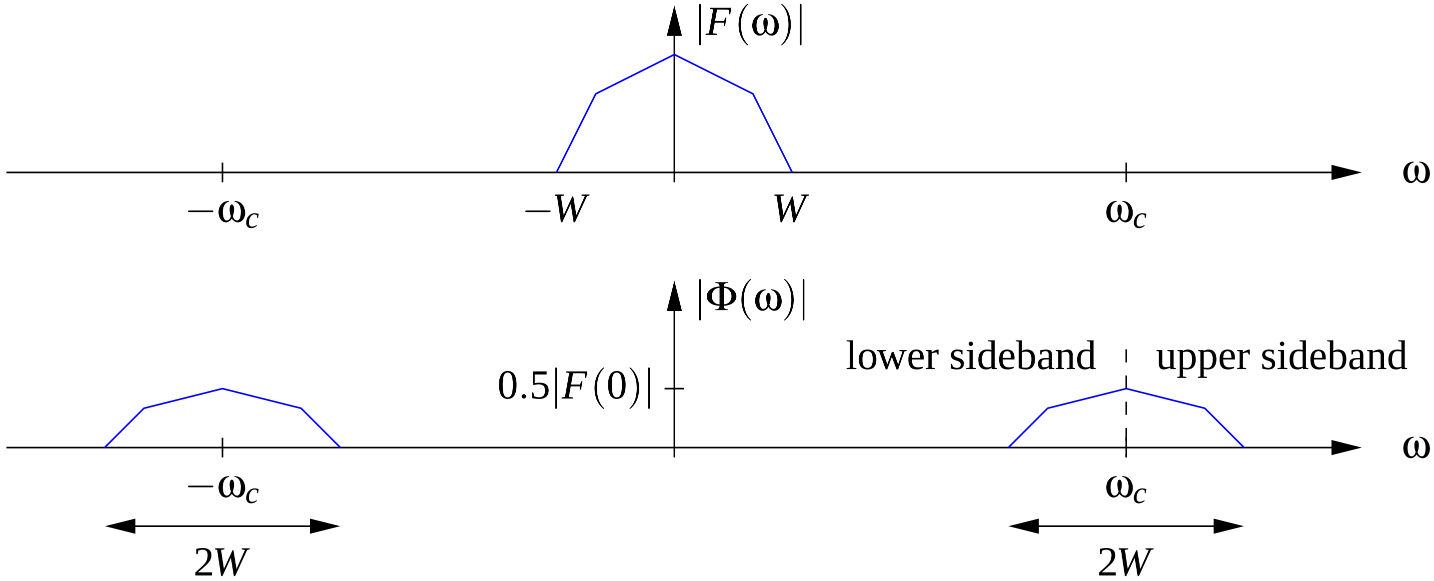


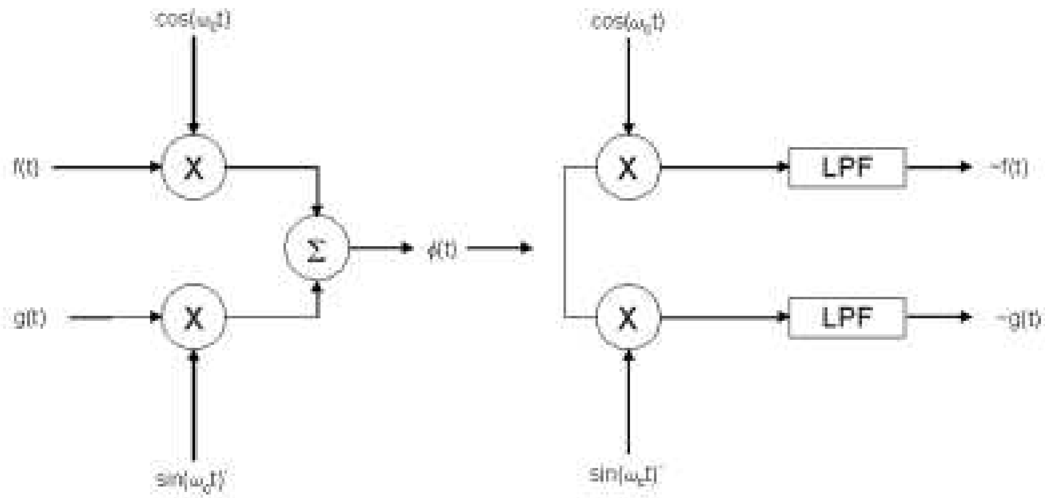
Figure 5.22 The envelope detector.

4.4 Quadrature AM (IQ)



- for real signal $f(t)$, $F(\omega) = F^*(-\omega)$
- using this symmetry we can transmit two signals that form a complex signal with same bandwidth
- we use two sinusoidal carriers, each exactly 90° out of phase

  remember, $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$
- transmitted over the same frequency band,



$$
\begin{aligned}
\phi(t) &= f(t)\cos(\omega_c t) + g(t)\sin(\omega_c t) \\
\phi(t)\cdot\cos(\omega_c t) &= f(t)\cos^2(\omega_c t) + g(t)\sin(\omega_c t)\cos(\omega_c t) \\
&= \frac{1}{2}f(t) + \frac{1}{2}f(t)\cos(2\omega_c t) + \frac{1}{2}f(t)\sin(2\omega_c t) \\
\phi(t)\cdot\sin(\omega_c t) &= f(t)\cos(\omega_c t)\sin(\omega_c t) + g(t)\sin^2(\omega_c t) \\
&= \frac{1}{2}f(t)\sin(2\omega_c t) + \frac{1}{2}g(t) - \frac{1}{2}\cos(2\omega_c t)
\end{aligned}
$$

4.5 Single-Sideband AM (SSB)

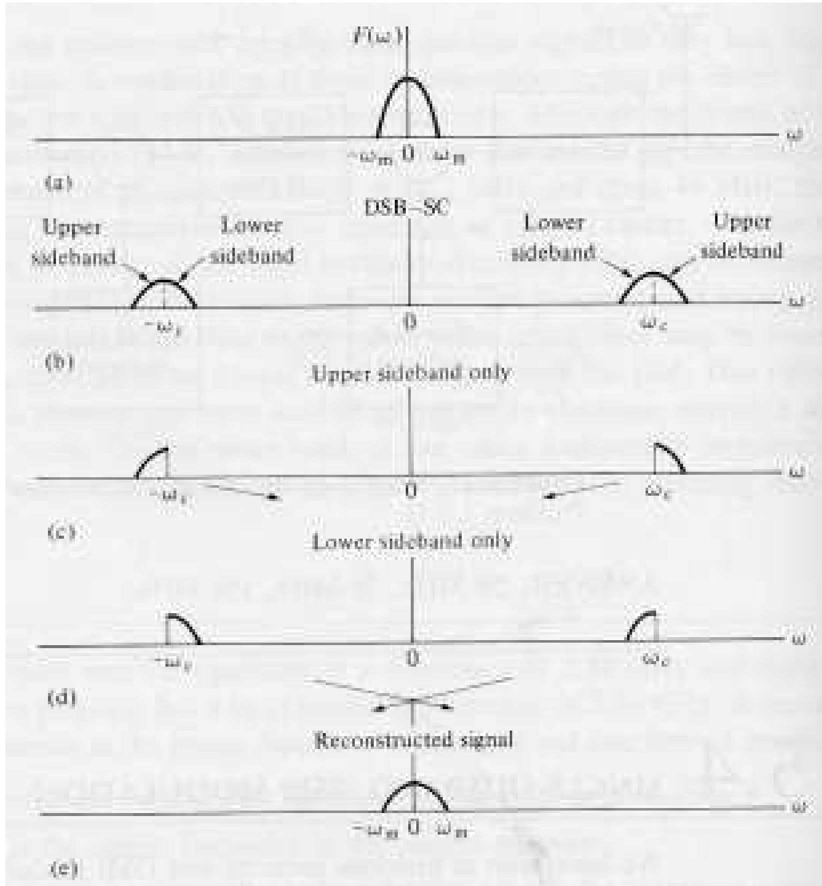


Figure 5.28 Spectra of DSB and SSB signals.

- remember for real $f(t)$, $F(-\omega) = F^*(\omega)$
- a single sideband contains entire information of the signal
- let's just transmit the upper/lower sideband.

4.5.1 Modulation

- one way is to generate DSB signal, and then suppress one sideband with filtering
- hard to do in practice, can't get ideal filters
- assume no low-frequency information $\Rightarrow$ no components around ω_c
- use heterodyning(frequency shifting), only need to design on sideband filter
- another way is the use of phasing
- assume a complex, single-frequency signal, $f(t) = e^{j\omega_m t}$ with carrier signal $f(t) = e^{j\omega_c t}$
- multiplying we get $\phi(t) = f(t)e^{j\omega_c t} = e^{j\omega_m t}e^{j\omega_c t}$
- using the frequency-translation property of the Fourier Transform, our spectrum becomes

$$\Phi(\omega) = 2\pi\delta(\omega - (\omega_c + \omega_m))$$

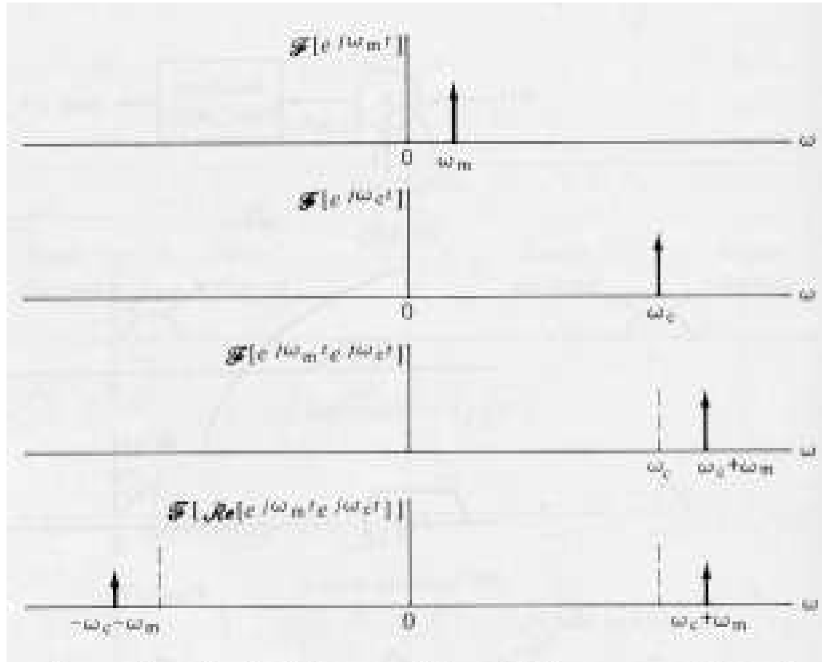


Figure 5.30 Analytical generation of SSB—upper sideband.

- to make the signal $\phi(t)$ realizable, we take the $\mathbb{R}\{\phi(t)\}$

$$\begin{aligned}\mathbb{R}\{\phi(t)\} &= \mathbb{R}\{e^{j\omega_m t}\}\mathbb{R}\{e^{j\omega_c t}\} - \mathbb{I}\{e^{j\omega_m t}\}\mathbb{I}\{e^{j\omega_c t}\} \\ &= \cos(\omega_m t)\cos(\omega_c t) - \sin(\omega_m t)\sin(\omega_c t)\end{aligned}$$

- So the upper side band is

$$\phi_{SSB_+}(t) = \cos(\omega_m t)\cos(\omega_c t) - \sin(\omega_m t)\sin(\omega_c t)$$

- likewise the lower sideband is

$$\phi_{SSB\_}(t) = \cos(\omega_m t)\cos(\omega_c t) + \sin(\omega_m t)\sin(\omega_c t)$$

- in general we write,

$$\phi_{SSB_{\mp}}(t) = f(t)\cos(\omega_c t)\ \pm\ \hat{f}(t)\sin(\omega_c t)$$

where $\hat{f}(t)$ is $f(t)$ shifted by 90°

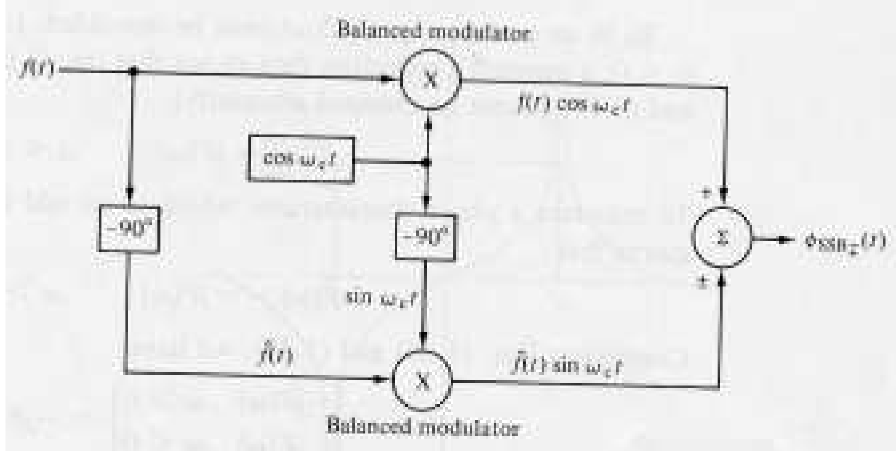


Figure 5.31 Phase-shift method of generating SSB.

4.5.2 Demodulation

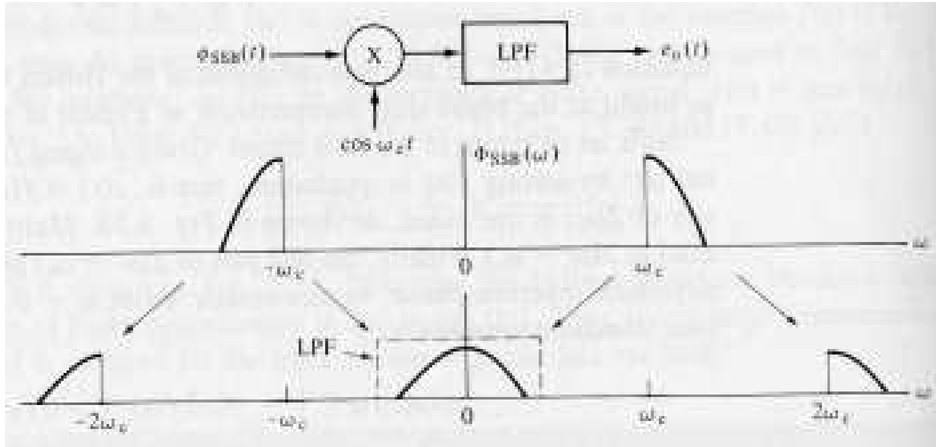


Figure 5.34 Synchronous demodulation of an SSB signal.

Synchronous detection, analogous to DSB-SC

Influence of Frequency and Phase Offset:

The oscillator at the receiver has a constant phase offset of θ as well as a slightly different carrier frequency offset of $\Delta\omega$ giving

$$\phi_d(t) = \cos[(\omega_c + \Delta\omega)t + \theta]$$

Before lowpass filtering:

$$\begin{aligned}\phi_{SSB_{\mp}}(t)\phi_d(t) &= [f(t)\cos(\omega_c t) \pm \hat{f}(t)\sin(\omega_c t)]\cos[(\omega_c + \Delta\omega)t + \theta] \\ &= \frac{1}{2}f(t)\{\cos[(\Delta\omega)t + \theta] + \cos[(2\omega_c + \Delta\omega)t + \theta]\} \\ &= \pm\frac{1}{2}\hat{f}(t)\{\sin[(\Delta\omega)t + \theta] - \sin[(2\omega_c + \Delta\omega)t + \theta]\}\end{aligned}$$

After lowpass filtering:

$$e_o(t) = \frac{1}{2}f(t)\cos[(\Delta\omega)t + \theta] \mp \frac{1}{2}\hat{f}(t)\sin[(\Delta\omega)t + \theta]$$

Phase error only (i.e. $\Delta\omega = 0$):

$$e_o(t) = \frac{1}{2}[f(t)\cos\theta \mp \hat{f}(t)\sin\theta]$$

To understand this better we re-write the above equation as

$$e_o(t) = \frac{1}{2}\mathbb{R}\{[f(t) \pm j\hat{f}(t)]e^{j\theta}]\}$$

$\Rightarrow$ So phase error in the receiver oscillator results in phase distortion.

Frequency error only (i.e. $\theta = 0$):

$$e_0(t) = \frac{1}{2}[f(t)\cos(\Delta\omega)t \mp \hat{f}(t)\sin(\Delta\omega)t]$$

or

$$e_o(t) = \frac{1}{2}\mathbb{R}\{[f(t) \pm j\hat{f}(t)]e^{j\Delta\omega t}\}$$

$\Rightarrow$ Demodulated signal contains spectral shifts and phase distortions.

4.6 Vestigial-Sideband AM (VSB)

- compromise between DSB and SSB.
- partial suppression of one sideband

$$\Phi_{VSB}(\omega) = [\frac{1}{2}F(\omega-\omega_c) + \frac{1}{2}F(\omega+\omega_c)]H_V(\omega)$$

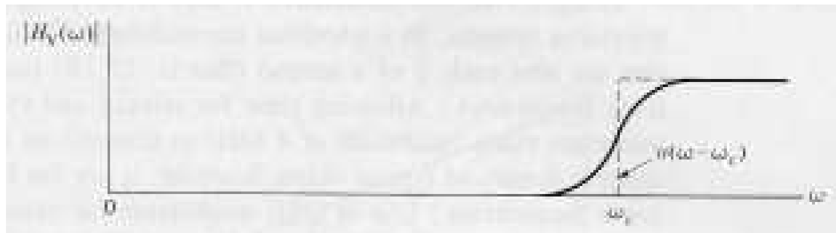


Figure 5.36 VSB filtering.

- after synchronous detection we have

$$E_o(\omega) = \frac{1}{4}F(\omega)H_V(\omega+\omega_c) + \frac{1}{4}F(\omega)H_V(\omega-\omega_c)$$

$$= \frac{1}{4}F(\omega)[H_V(\omega+\omega_c) + H_V(\omega-\omega_c)]$$

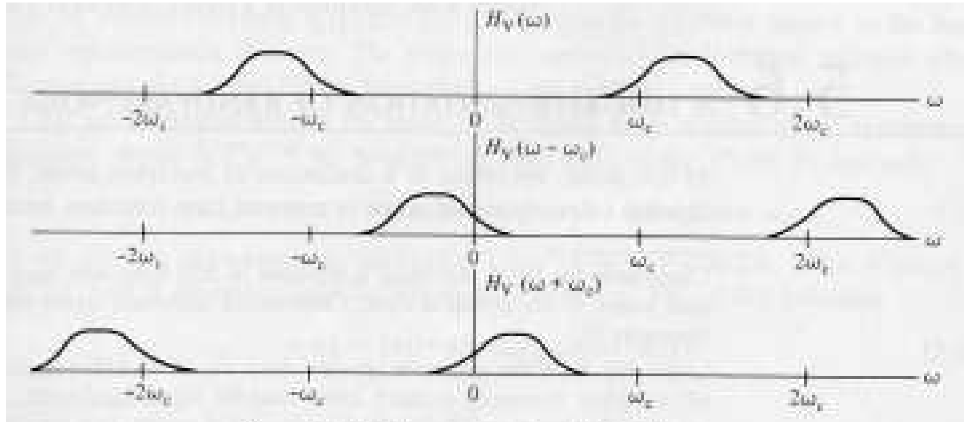
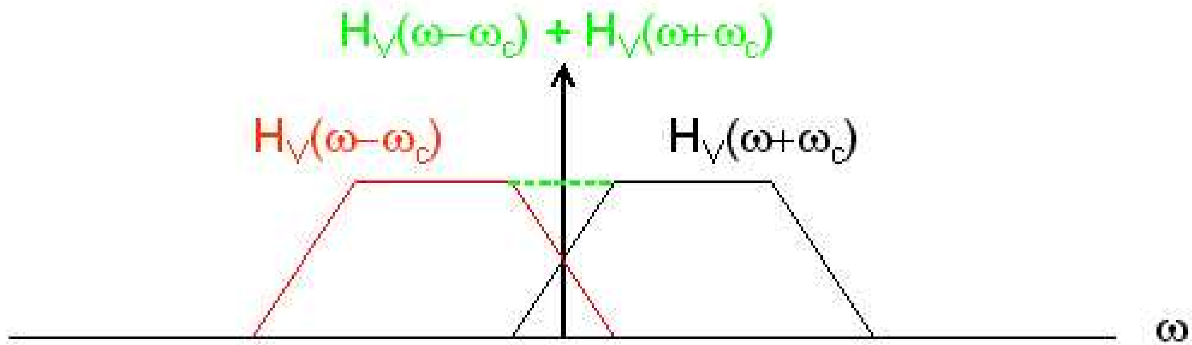


Figure 5.37 Characteristics of VSB filtering.



thus for reproduction of $f(t)$ we require

$$[H_V(\omega - \omega_c) + H_V(\omega + \omega_c)]_{LP} = \text{constant}$$

- magnitude can be satisfied, but phase requirements are hard to satisfy
- use when phase is not important

4.6.1 Video Transmission in Commercial TV Systems

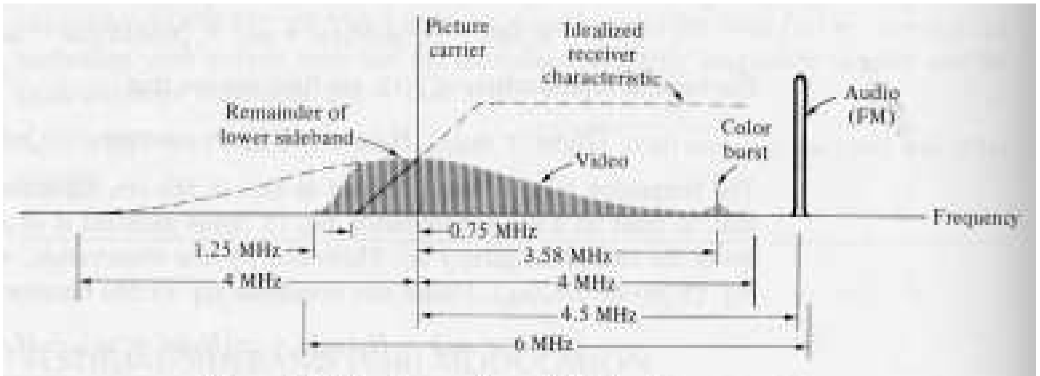
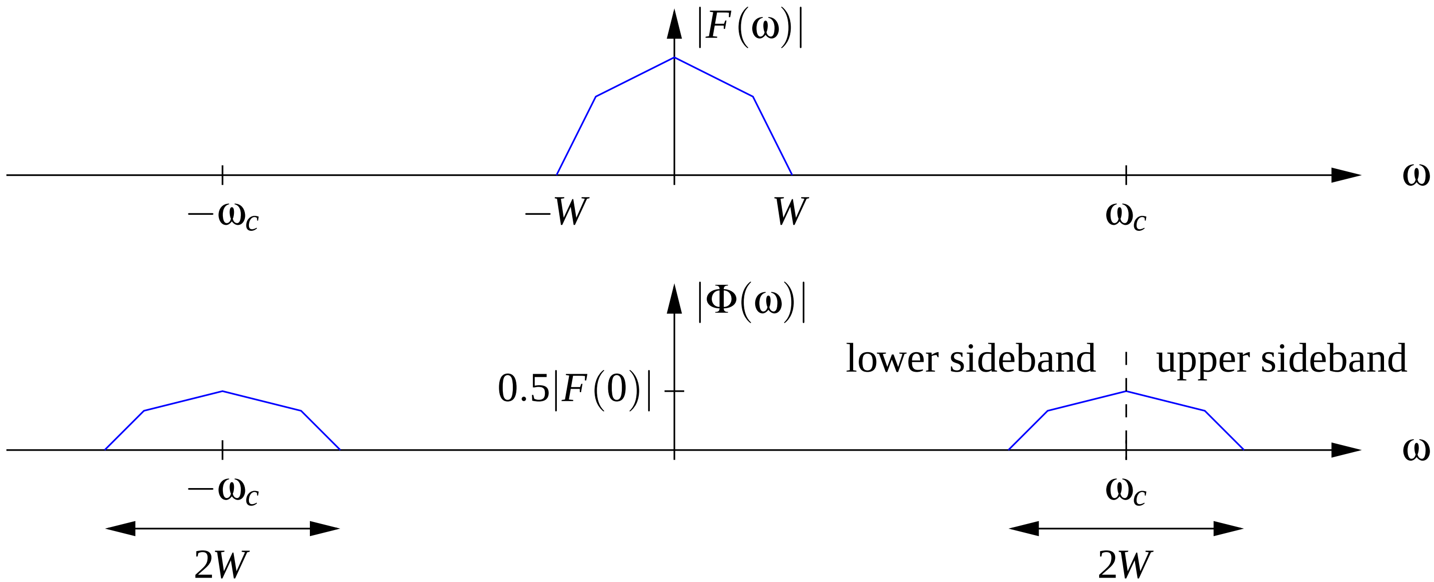


Figure 5.38 Spectrum of a public television transmission.

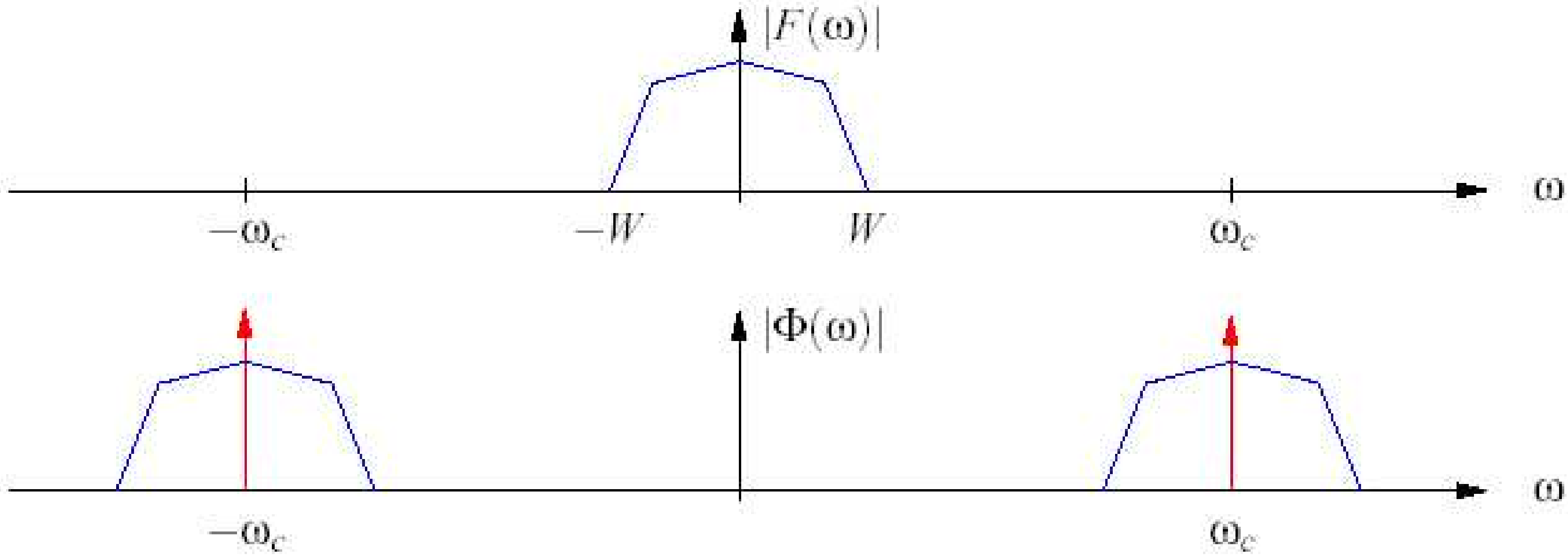
- video requires $4MHz$ bandwidth to transmit
- so DSB would require $8MHz$ per channel
- use VSB to decrease the needed bandwidth to $5MHz$

4.7 Summary



Double Sideband-Suppressed Carrier(DSB-SC)

- spectrum at ω_c is a copy of baseband spectrum with scaling factor of $1/2$
- information is sidebands is redundant
- for coherent detection, we must have same frequency and phase of carrier signal
- detection can be done with pilot tone, PLL



Double Sideband-Large Carrier(DSB-LC)

- same as DSB-SC, with an addition of a carrier term
- detection is a simple envelope detector
- Wastes, at best case, 67% of the power in the carrier term
- frequency response at low-frequencies are ruined

Quadrature Amplitude Modulation(QAM)

- efficient utilization of bandwidth

- forms a complex signal with two sinusoidal carriers of same frequency, 90° out of phase

Single Sideband Modulation(SSB)

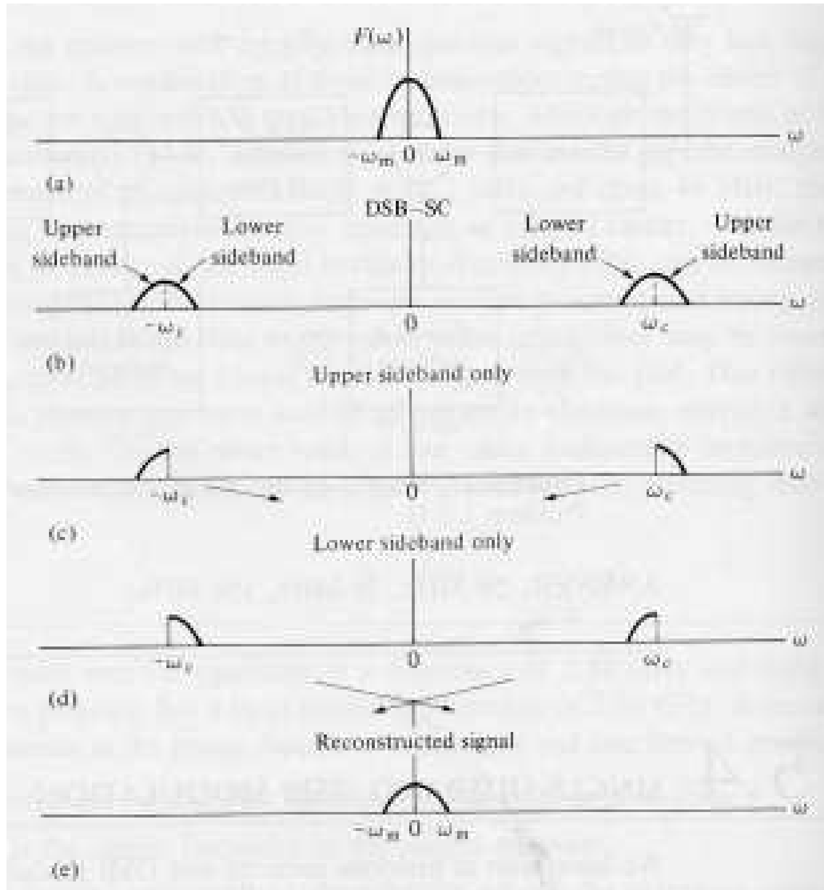


Figure 5.28 Spectra of DSB and SSB signals.

- suppress either upper or lower sideband for more efficient bandwidth utilization
- generated by filtering DSB-SC

- can also use phasing to cancel the "negative" frequencies
- can use either suppressed carrier, pilot tone, or large carrier AM also

Vestigial Sideband(VSB)

- compromises DSB and SSB
- transmitter and receiver filters must be complementary, i.e., they must add to a constant at baseband
- phase must not be important