Analog Modulation

Amplitude Modulation Systems

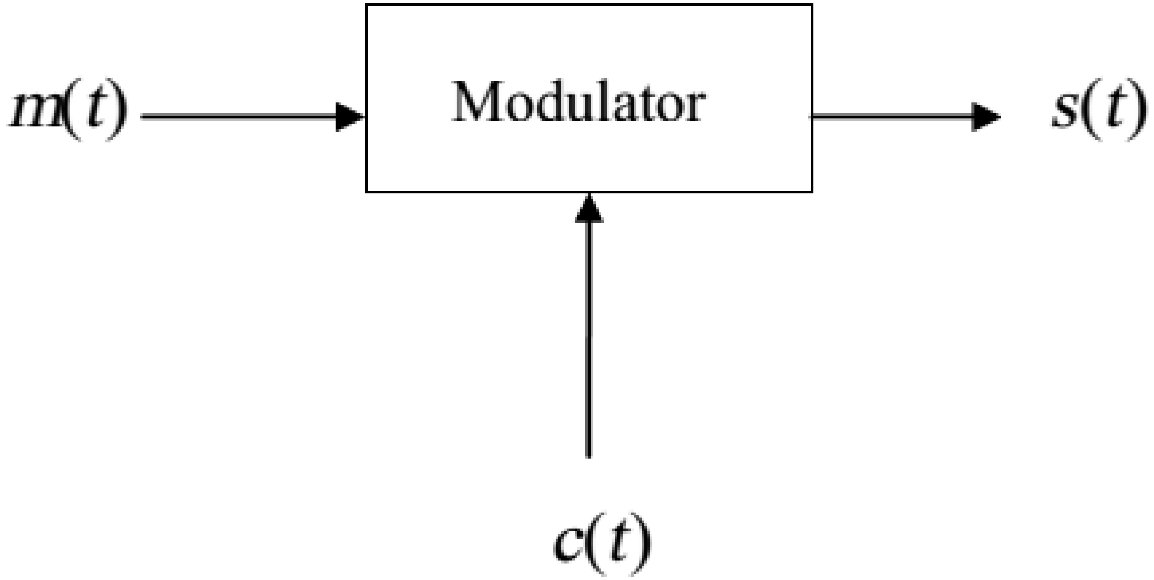
Modulation: is the process by which some characteristic of a carrier $c(t)$ is varied in accordance with a message signal $m(t)$.

Amplitude modulation is defined as the process in which the amplitude of the carrier $c(t)$ is varied linearly with $m(t)$. Four types of amplitude modulation will be considered in this chapter. These are normal amplitude modulation, double sideband suppressed carrier modulation, single sideband modulation, and vestigial sideband modulation.

A common form of the *carrier*, in the case of continuous wave modulation, is a sinusoidal signal of the form

$$c(t) = A_C \cos(2\pi f_C t + \phi)$$

The baseband (message) signal $m(t)$ is referred to as the *modulating signal* and the result of the modulation process is referred to as the *modulated signal* $s(t)$. The following block diagram illustrates the modulation process.



We should point out that modulation is performed at the transmitter and demodulation, which is the process of extracting $m(t)$ from $s(t)$, is performed at the receiver.

Normal Amplitude Modulation

A *normal AM* signal is defined as:

$$s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t$$

where k_a : Amplitude sensitivity (units 1/volt)

Here $s(t)$ can be written in the form:

$$s(t) = A(t)\cos 2\pi f_C t$$

where $A(t) = A_C + A_C k_a\, m(t)$

From which we can observe that $A(t)$ is related linearly to $m(t)$ in a relationship of the form " $y = a + bx$". Therefore, the amplitude of $s(t)$ is linearly related to $m(t)$.

The *envelope* of $s(t)$ is defined as

$$|A(t)| = A_C |1 + k_a m(t)|$$

Notice that the envelope of $s(t)$ has the same shape as $m(t)$ provided that:

1. $|1 + k_a m(t)| \geq 0$ or $|k_a m(t)| \leq 1$

Overmodulation results when $|k_a m(t)| > 1$ resulting in envelope distortion.

2. $f_C >> W$, where W is the highest frequency component in $m(t)$.

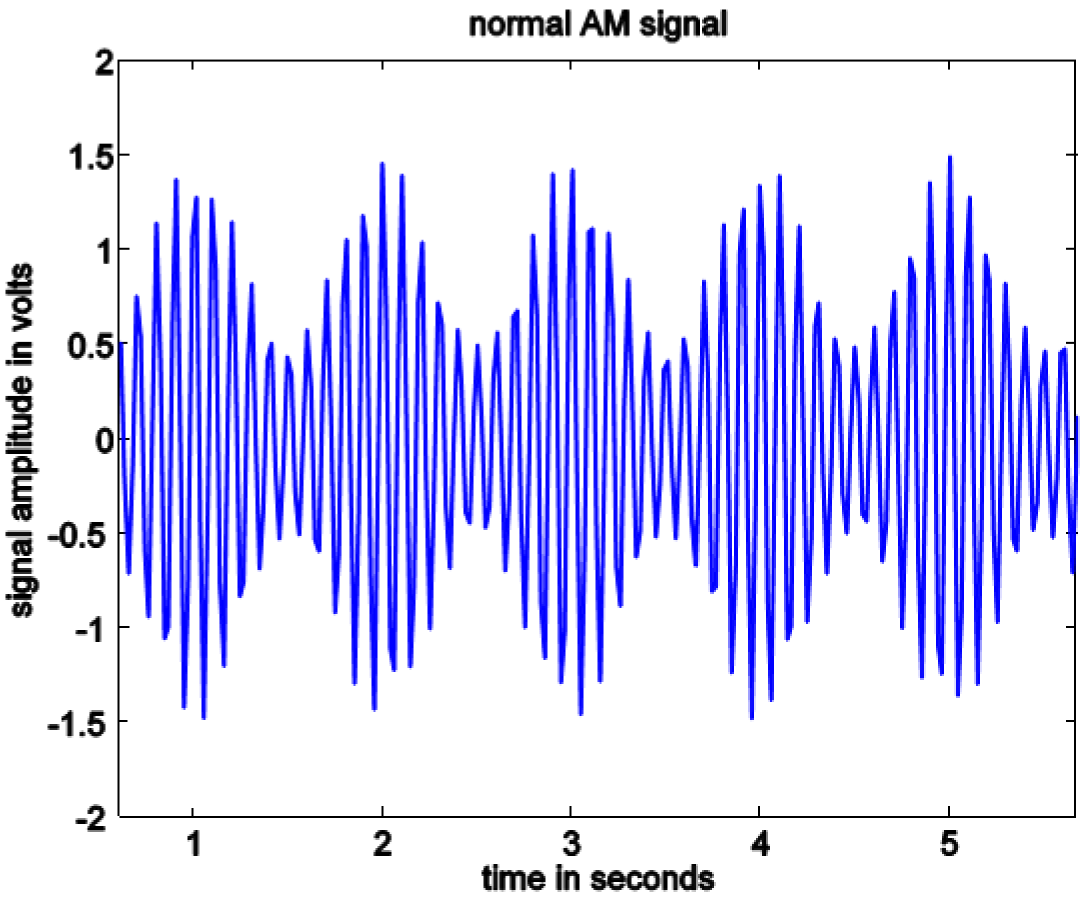
(f_C has to be at least 10 W). This ensures the formation of an envelope that has a shape that resembles the message.

Matlab Demonstration

The figure below shows the normal AM signal $s(t) = (1 + 0.5\cos 2\pi t)\cos 2\pi(10)t$

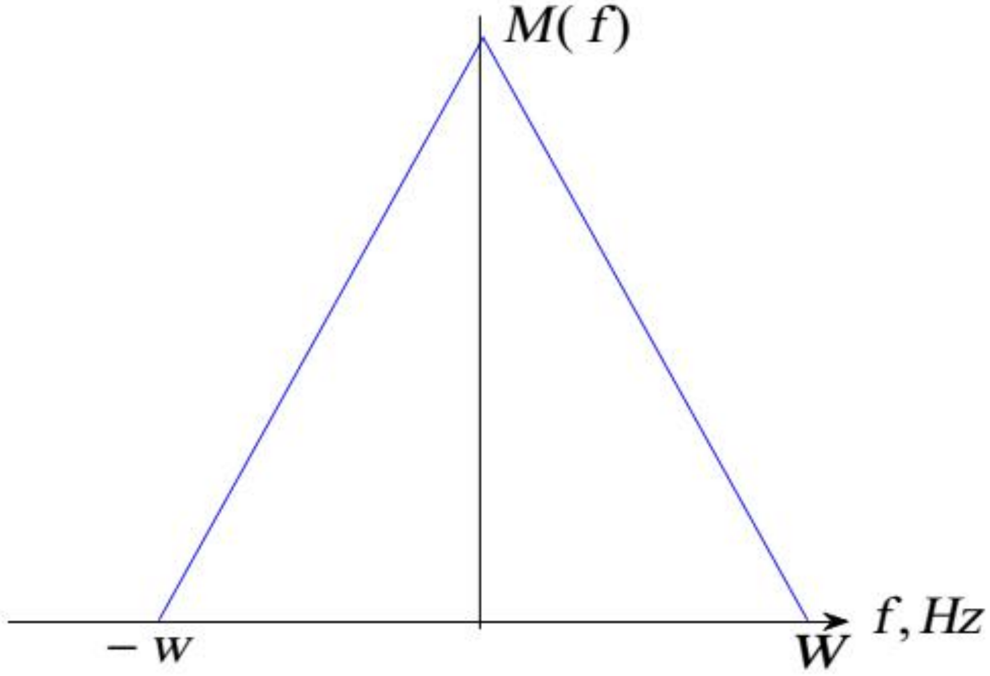
a. Make similar plots for the cases (μ = 0.5, 1, and 1.5)

b. Show the effect of f_C on the envelope. (Take f_C = 4 Hz, and f_C = 25Hz)



Spectrum of the Normal AM Signal

Let the Fourier transform of $m(t)$ be as shown (The B.W of $m(t) = w$ Hz).



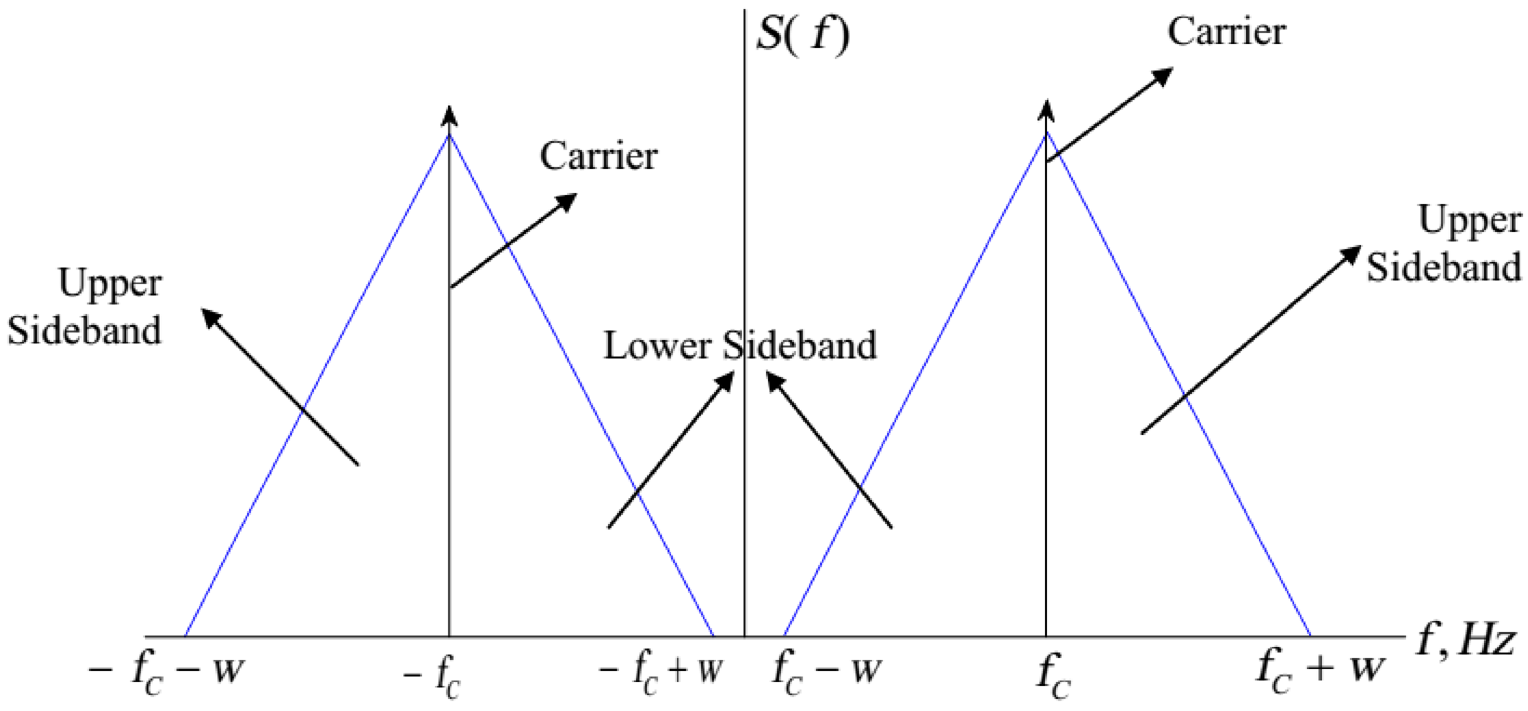
$$s(t) = A_c(1 + k_a m(t))\cos 2\pi f_c t \quad \text{(dc + message)*carrier}$$

$$s(t) = A_c \cos 2\pi f_c t + A_c k_a m(t)\cos 2\pi f_c t \quad \text{(carrier + message*carrier)}$$

Taking the Fourier transform, we get

$$S(f) = \frac{A_c}{2}\delta(f - f_c) + \frac{A_c}{2}\delta(f + f_c) + \frac{A_c k_a}{2} M(f - f_c) + \frac{A_c k_a}{2} M(f + f_c)$$

The spectrum of $s(t)$ is shown below



<u>Remarks</u>

a. $M(f)$ Has been shifted to f_C resulting in a bandpass signal.

b. Two sidebands (upper sideband and lower sideband) and a carrier form the spectrum of $s(t)$.

c. The transmission bandwidth of $s(t)$ is:

$$B.W = (f_C + w) - (f_C - w) = 2w$$

= Twice the message bandwidth.

Power Efficiency

The *power efficiency* of a normal AM signal is defined as:

$$\eta = \frac{power \;\; in \;\; the \; sidebands}{power \; in \; the \; sidebands + power \; in \; the \; carrier}$$

Now, we find the power efficiency of the AM signal for the single tone modulating signal $m(t) = A_m \cos(2\pi f_m t)$. If we denote $\mu = A_m k_a$, then $s(t)$ can be expressed as

$$s(t) = A_C(1 + \mu \cos 2\pi f_m t)\cos 2\pi f_C t$$

$$s(t) = A_C \cos 2\pi f_C t + A_C \mu \cos 2\pi f_C t \cos 2\pi f_m t$$

$$s(t) = A_C \cos 2\pi f_C t + \frac{A_C \mu}{2}\cos 2\pi(f_C + f_m)t + \frac{A_C \mu}{2}\cos 2\pi(f_C - f_m)t$$

Carrier

Upper Sideband

Lower Sideband

$$\text{Power in carrier} = \frac{A_C^{\,2}}{2}$$

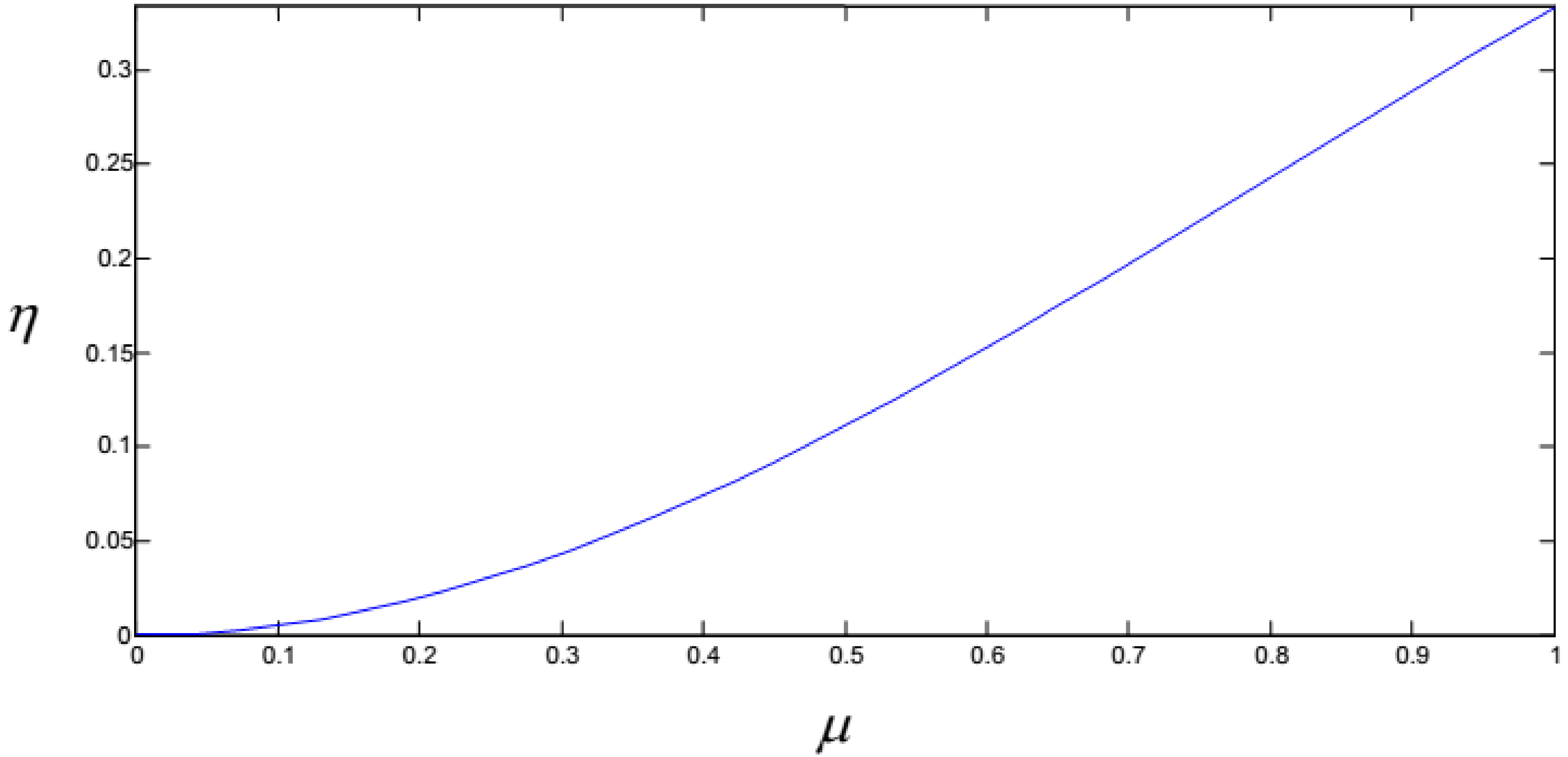
$$\text{Power in sidebands} = \frac{1}{2}\left(\frac{A_C\mu}{2}\right)^2 + \frac{1}{2}\left(\frac{A_C\mu}{2}\right)^2$$

$$= \frac{1}{8}A_C^{\,2}\mu^2 + \frac{1}{8}A_C^{\,2}\mu^2 = \frac{1}{4}A_C^{\,2}\mu^2$$

Therefore,

$$\eta = \frac{\frac{1}{4}A_C^{\,2}\mu^2}{\frac{A_C^{\,2}}{2} + \frac{1}{4}A_C^{\,2}\mu^2} = \frac{\mu^2}{2+\mu^2} \qquad ; \quad 1 \ge \mu \ge 0$$

The following figure shows the relationship between η and μ



The maximum efficiency occurs when $\mu = 1$ (i.e. for 100% modulation. And even then, $\eta = 1/3$). So that 2/3 of the power goes to the carrier.

<u>Remark:</u> Normal AM is not an efficient modulation scheme in terms of the utilization of the transmitted power.

Exercise:

a. Show that for the general AM signal $s(t)=A_C\ 1+k_a m(t)\ \cos(2\pi f_c t)$, the power efficiency is given by $\eta=\dfrac{\frac{1}{2}A_C^{\ 2}\left\langle k_a^{\ 2}m(t)^2\right\rangle}{\frac{A_C^{\ 2}}{2}+\frac{1}{2}A_C^{\ 2}\left\langle k_a^{\ 2}m(t)^2\right\rangle}=\dfrac{\left\langle k_a^{\ 2}m(t)^2\right\rangle}{1+\left\langle k_a^{\ 2}m(t)^2\right\rangle}$, where $\left\langle k_a^{\ 2}m(t)^2\right\rangle$ is the average power in $k_a m(t)$

b. Apply the above formula for the single tone modulated signal $s(t)=A_C(1+\mu\cos 2\pi f_m t)\cos 2\pi f_C t$

AM Modulation Index

Consider the AM signal

$$s(t) = A_C(1 + k_a m(t))\cos 2\pi f_C t = A(t)\cos 2\pi f_C t$$

The envelope of $s(t)$ is defined as:

$$|A(t)| = A_C|1 + k_a m(t)|$$

To avoid distortion, the following condition must hold

$$|1 + k_a m(t)| \geq 0 \qquad \text{or} \qquad |k_a m(t)| \leq 1$$

The modulation index of an AM signal is defined as:

$$Modulation\ \ Index\ (M.I) = \frac{|A(t)|_{\max} - |A(t)|_{\min}}{|A(t)|_{\max} + |A(t)|_{\min}}$$

Example: (single tone modulation)

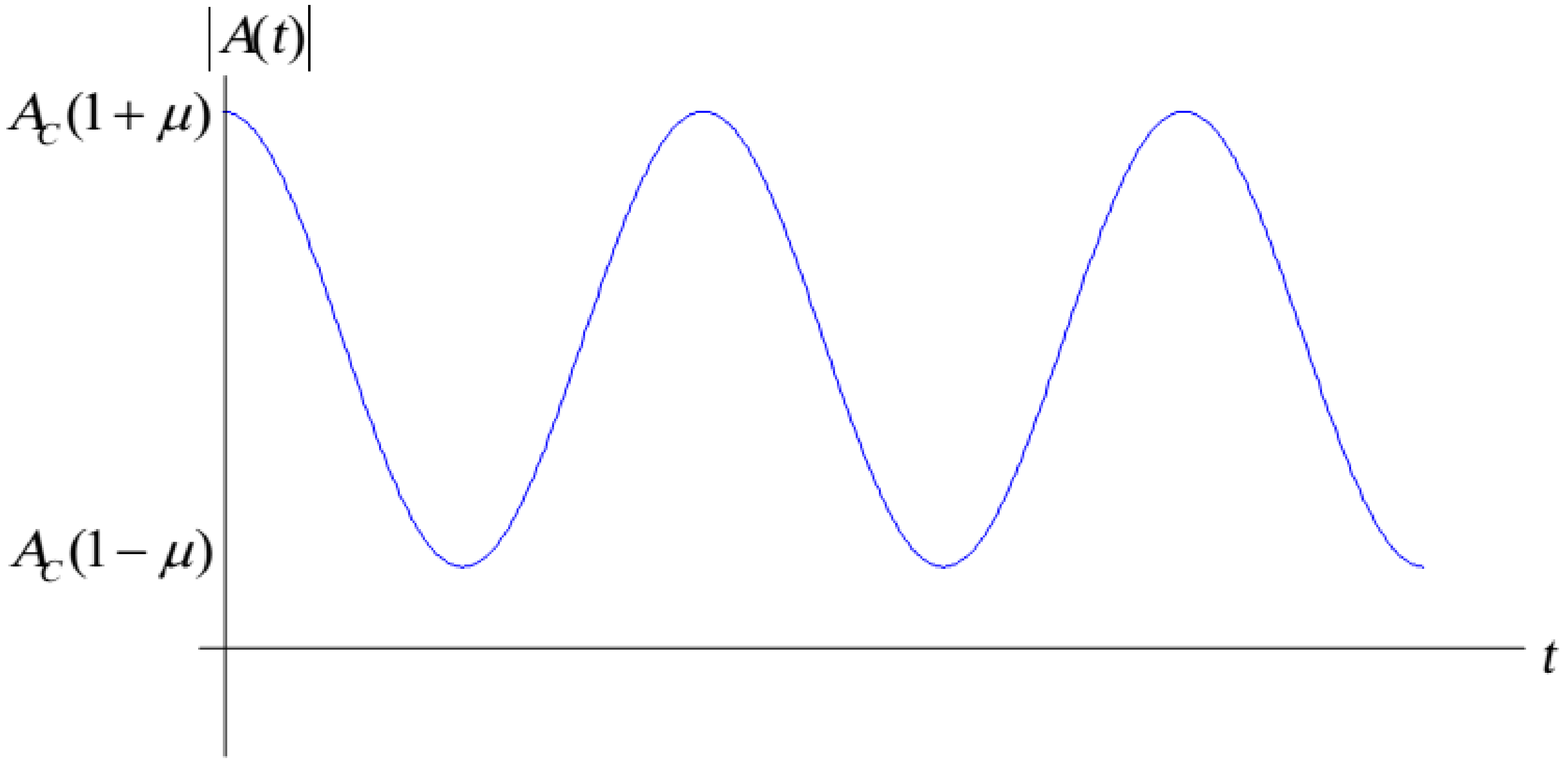
Let $m(t) = A_m \cos 2\pi f_m t$

$$then,\ s(t) = A_C(1 + k_a A_m \cos 2\pi f_m t)\cos 2\pi f_C t$$

$$= A_C(1 + \mu \cos 2\pi f_m t)\cos 2\pi f_C t \qquad where,\ \mu = k_a A_m$$

To avoid distortion $k_a A_m = \mu < 1$

The envelope $|A(t)| = A_C|(1 + \mu \cos 2\pi f_m t)|$ is plotted below



$$|A(t)|_{\max} = A_C(1+\mu), \qquad |A(t)|_{\min} = A_C(1-\mu)$$

So,

$$M.I = \frac{A_C(1+\mu) - A_C(1-\mu)}{A_C(1+\mu) + A_C(1-\mu)} = \frac{2A_C\mu}{2A_C} = \mu$$

Therefore, the modulation index is μ.

Overmodulation

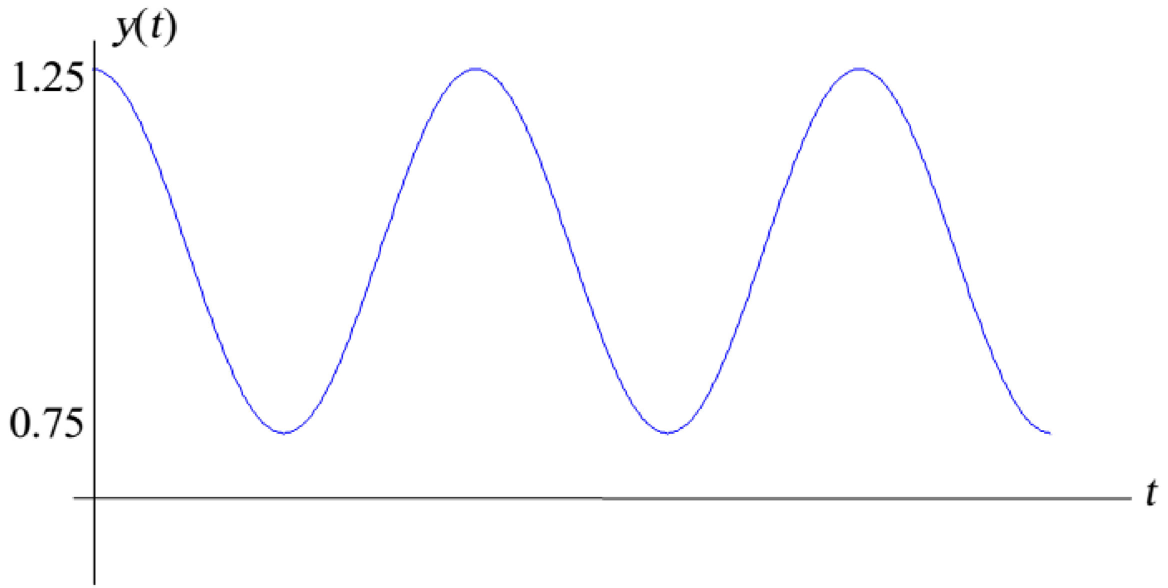
When the modulation index $\mu > 1$, an ideal envelope detector cannot be used to extract $m(t)$ and *distortio*n takes place.

Example: Let $s(t) = A_C(1+\mu\cos 2\pi f_m t)\cos 2\pi f_C t$ be applied to an ideal envelope detector, sketch the demodulated signal for $\mu = 0.25, 1.0, \text{ and } 1.25$.

As was mentioned before, the output of the envelope detector is $y(t) = A_C|1+\mu\cos 2\pi f_m t|$

Case1 : ($\mu = 0.25$)

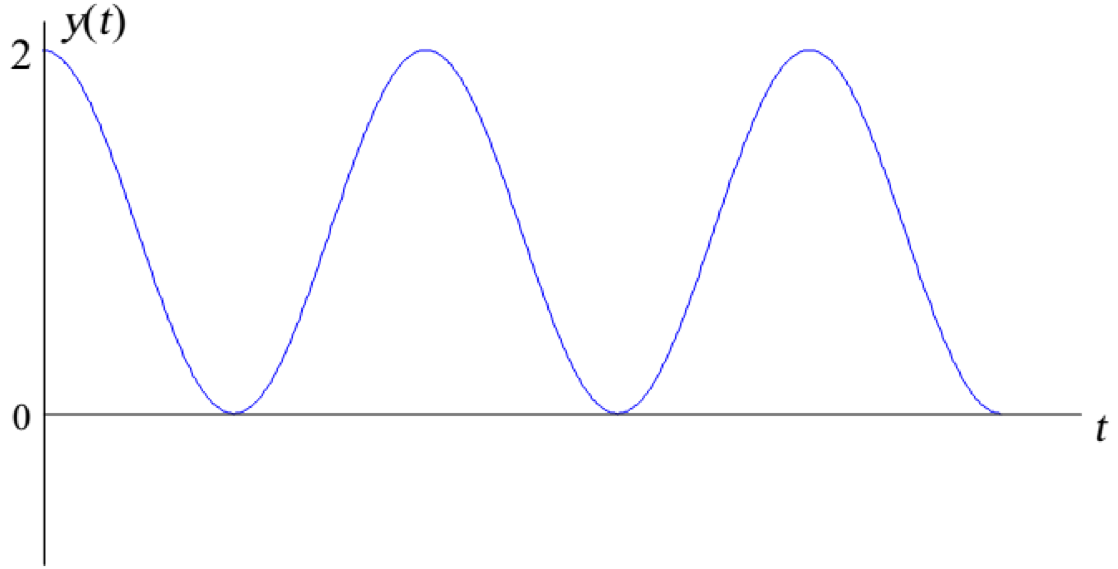
$$y(t) = A_C|1 + 0.25\cos 2\pi f_m t|$$



Here, $m(t)$ can be extracted without distortion.

<u>Case2:</u> (μ = 1.0)

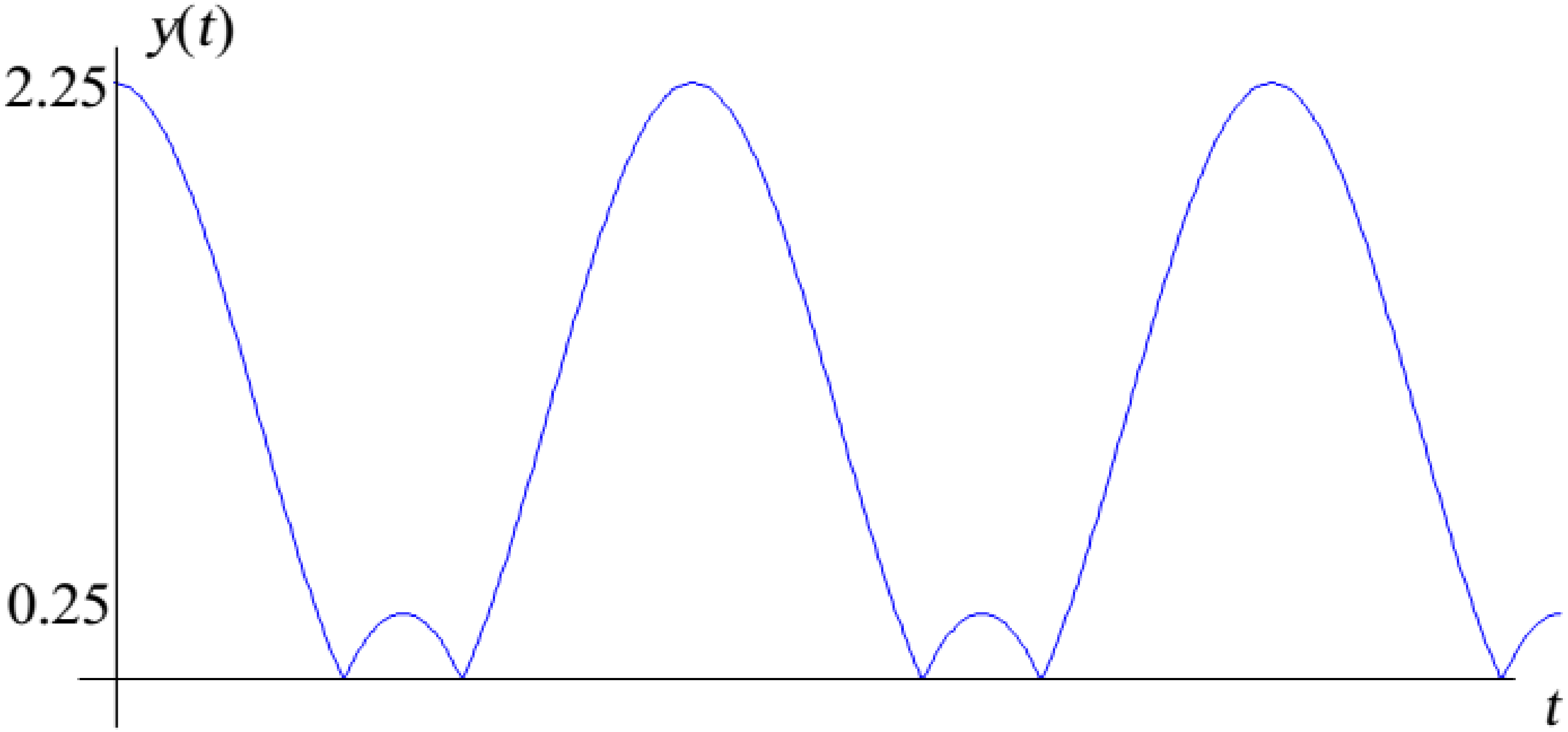
$$y(t) = A_c \left| 1 + \cos 2\pi f_m t \right|$$



Here again, $m(t)$ can be extracted without distortion.

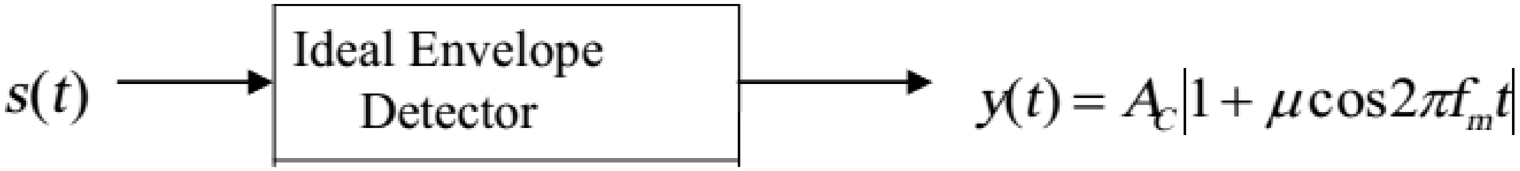
Case3: (μ = 1.25)

$$y(t) = A_c\left|1 + 1.25\cos 2\pi f_m t\right|$$



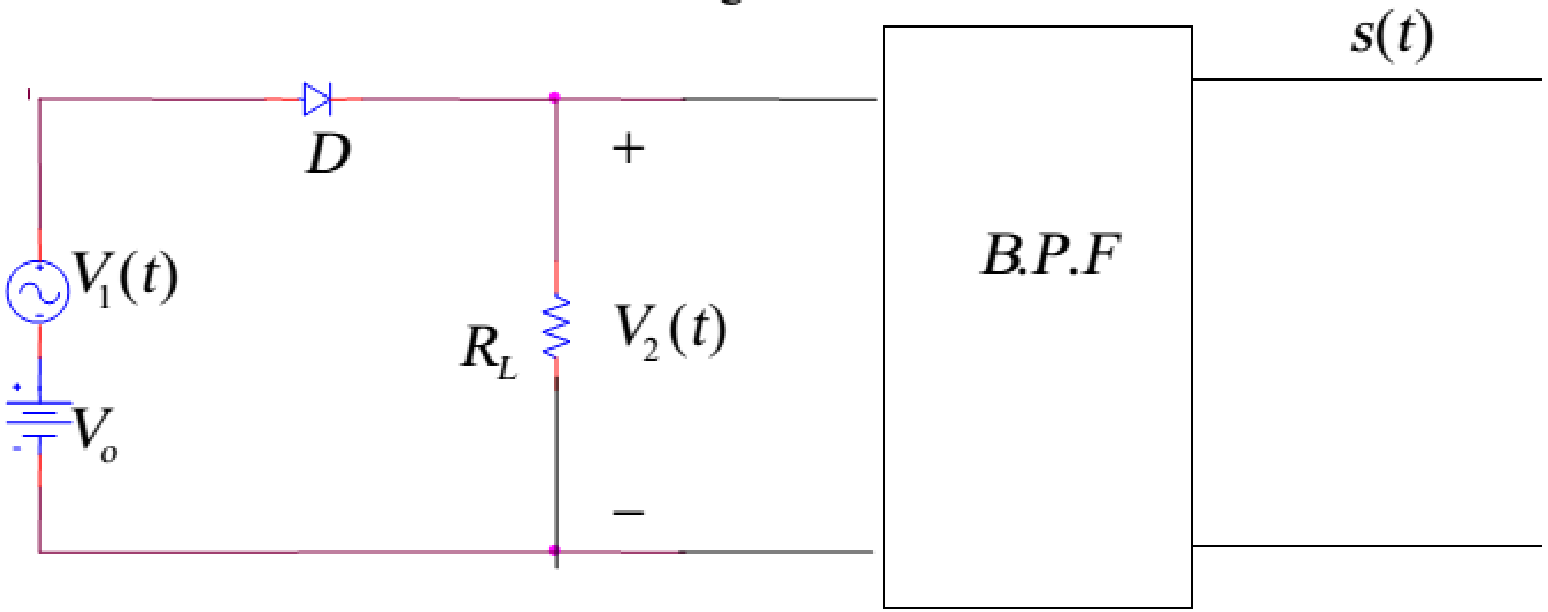
Here, $m(t)$ cannot be recovered without distortion.

Note: the block diagram that illustrates the envelope detector process is shown below



Generation of Normal AM: (Square Law Modulator)

Consider the circuit shown in the figure.



For small variations of $V_1(t)$ around a suitable operating point, $V_2(t)$ can be expressed as:

$$V_2 = \alpha_1 V_1 + \alpha_2 V_1^2 \;; \qquad \text{Where } \alpha_1 \text{ and } \alpha_2 \text{ are constants.}$$

Let $V_1(t) = m(t) + A_C' \cos 2\pi f_C t$

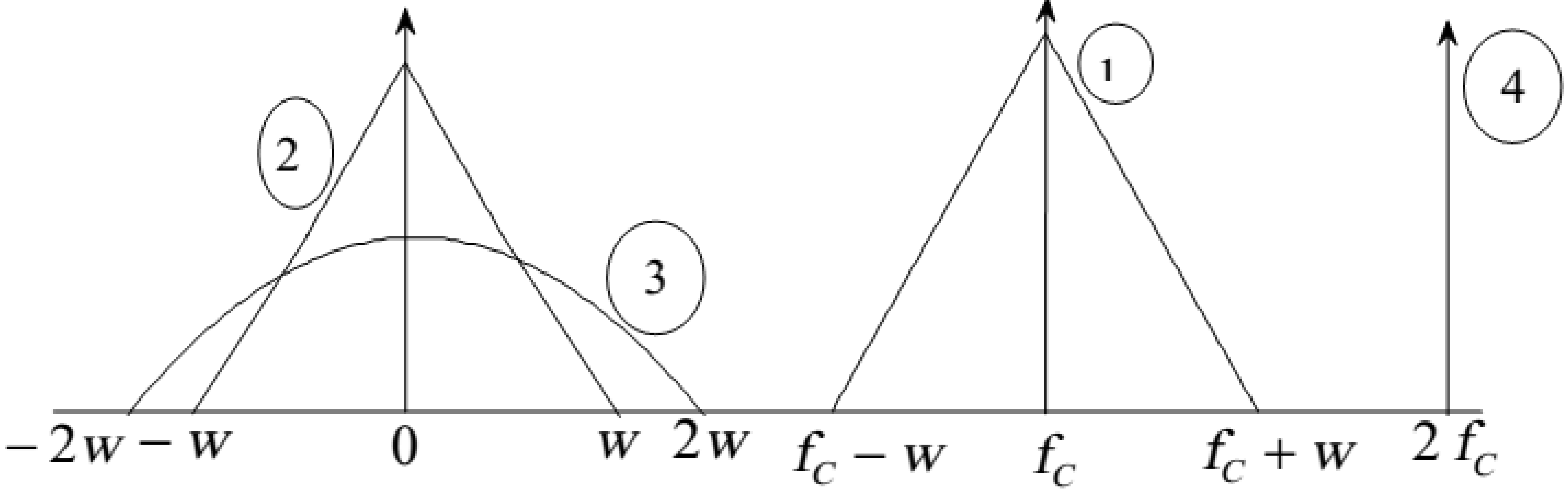
Substituting $V_1(t)$ into the nonlinear characteristics and arranging terms, we get

$$V_2(t) = \alpha_1 A_C' \left[1 + \frac{2\alpha_2}{\alpha_1} m(t) \right] \cos 2\pi f_C t + \alpha_1 m(t) + \alpha_2 m(t)^2 + \alpha_2 A_C'^2 \cos^2(2\pi f_C t)$$

$$V_2(t) = (1) + (2) + (3) + (4)$$

The first term is the desired AM signal obtained by passing $V_2(t)$ through a bandpass filter.

$$s(t) = \alpha_1 A_C' \left[1 + \frac{2\alpha_2}{\alpha_1} m(t) \right] \cos 2\pi f_C t$$



2
3
1
4
−2w
−w
0
w
2w
fc − w
fc
fc + w
2 fc

Note: the numbers shown in above figure represent the number of term in $V_2(f)$.

(1) = The desired normal AM signal

(2) = $M(f)$

(3) = $M(f) * M(f)$

(4) = The cosine square term amounts to a term at $2f_c$ and a DC term.

Limitations of this technique:

a. Variations of $V_1(t)$ should be small to justify the second order approximation of the nonlinear characteristic.

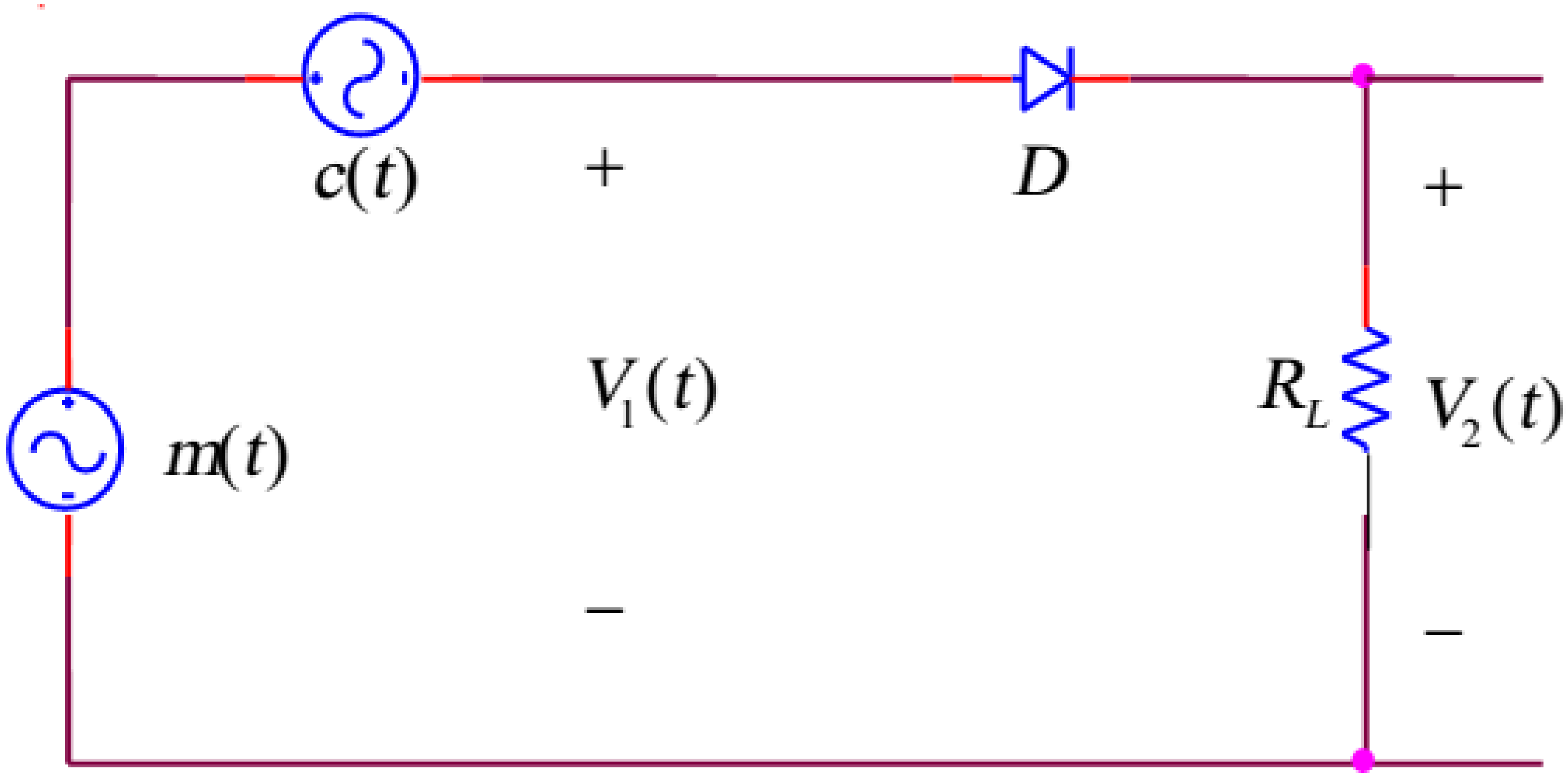
b. The bandwidth of the filter should be such that $f_C - w > 2w \Rightarrow f_C \geq 3w$

When $f_C >> w$, a bandpass filter with reasonable edge could be used.

When f_C is of the order $3w$, a filter with sharp edges should be used.

Generation of Normal AM: (The switching Modulator)

Assume that the carrier $c(t)$ is large in amplitude so that the diode –shown in the figure below- acts like an ideal switch.



When $m(t)$ is small compared with $|c(t)|$, then

$$V_2(t) = \begin{cases} m(t) + A_C \cos\omega_C t & ; \quad c(t) > 0 \\ 0 & ; \quad c(t) < 0 \end{cases}$$

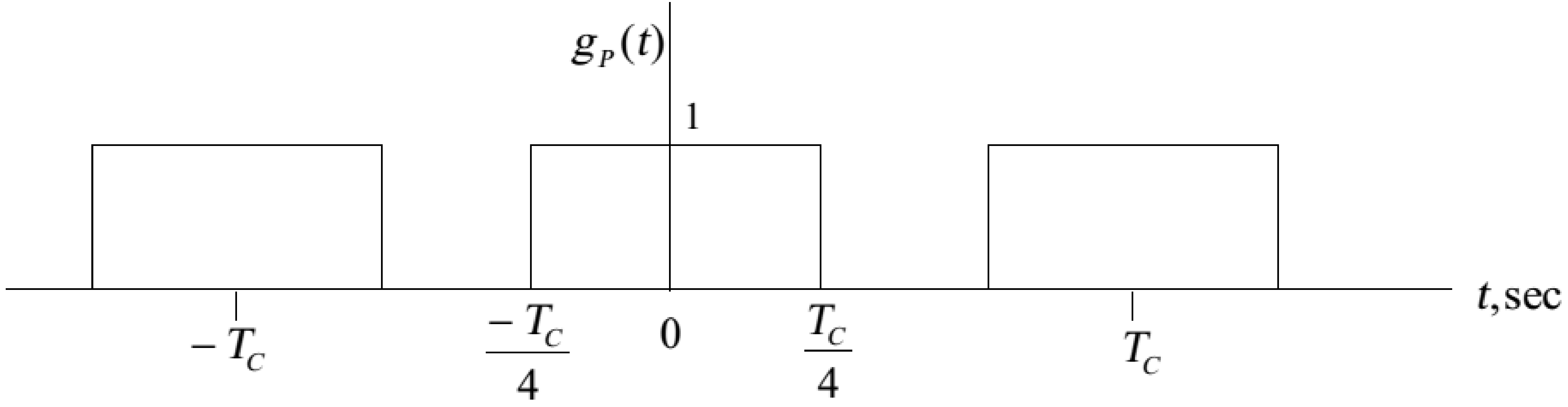
i.e. diode opens and closes at a rate of f_C (times/sec) (frequency of $c(t)$, the carrier)

$$V_2(t) = [A_C \cos\omega_C t + m(t)] g_P(t)$$

Where $g_P(t)$: is the periodic switching function.

$$g_P(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_C t - \frac{1}{3}\cos 3\omega_C t + \frac{1}{5}\cos 5\omega_C t + \ldots\right)$$

Expanding $g_P(t)$ in a Fourier series, we get

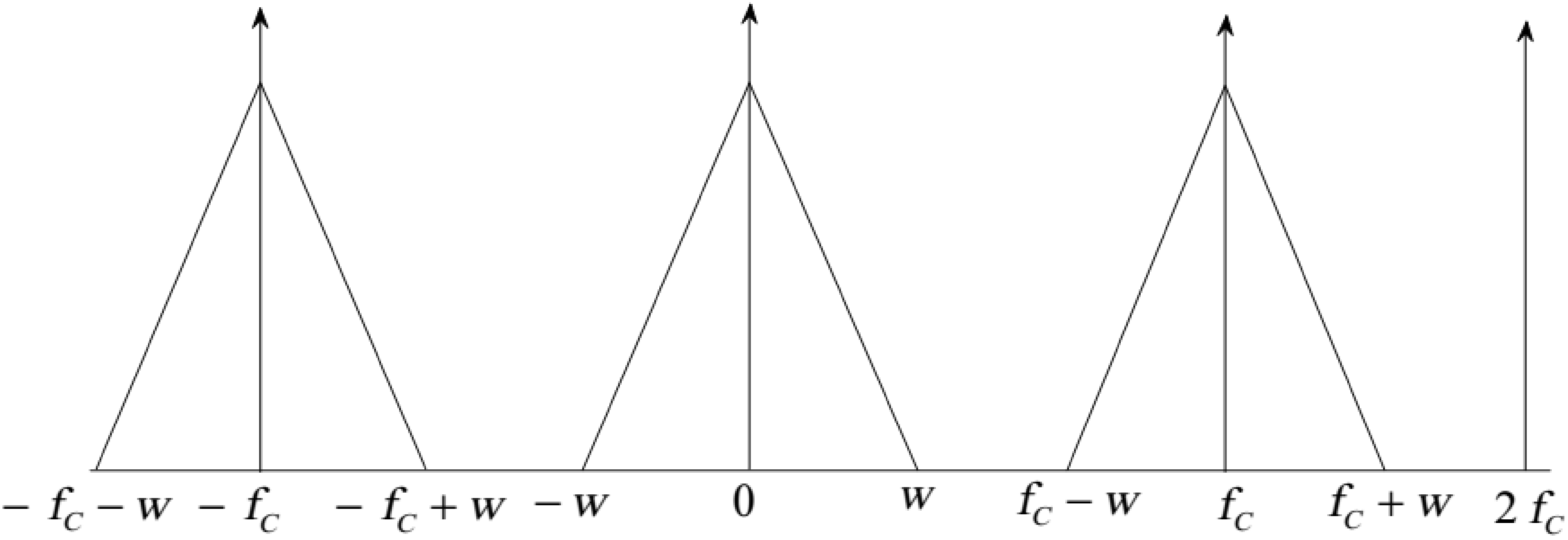


$$V_2(t) = [A_C \cos\omega_C t + m(t)]\left(\frac{1}{2}\right) + \left(\frac{2}{\pi}\cos\omega_C t\right)(A_C \cos\omega_C t + m(t)) - \left(\frac{2}{3\pi}\cos 3\omega_C t\right)$$
$$(m(t) + A_C \cos\omega_C t) + \ldots$$

⇨

$$V_2(t) = \frac{m(t)}{2} + \frac{A_C}{2}\cos\omega_C t + \frac{2}{\pi} m(t)\cos\omega_C t + \frac{A_C}{\pi} + \frac{A_C}{\pi}\cos 2\omega_C t + \frac{2}{3\pi} m(t)\cos 3\omega_C t$$

$$+ \frac{2}{3\pi} A_C \cos 2\omega_C t + \ldots$$



With a bandpass filter centered at f_C with a bandwidth of $2w$; the filter passes the second term (a carrier) and the third term (a carrier multiplied by the message). we get:

$$s(t) = \frac{A_C}{2}\cos\omega_C t + \frac{2}{\pi}m(t)\cos\omega_C t$$

$$s(t) = \frac{A_C}{2}\left(1 + \frac{4}{\pi A_C}m(t)\right)\cos\omega_C t \ ; \qquad \text{Desired AM signal.}$$

$$\text{Modulation Index} = M.I = \frac{4}{\pi A_C}\left|m(t)\right|_{\max}$$

<u>Demodulation of AM signal</u>: (The Envelope Detector)

The ideal envelope detector is one which responds to the envelope of the signal, but insensitive to phase variation. If

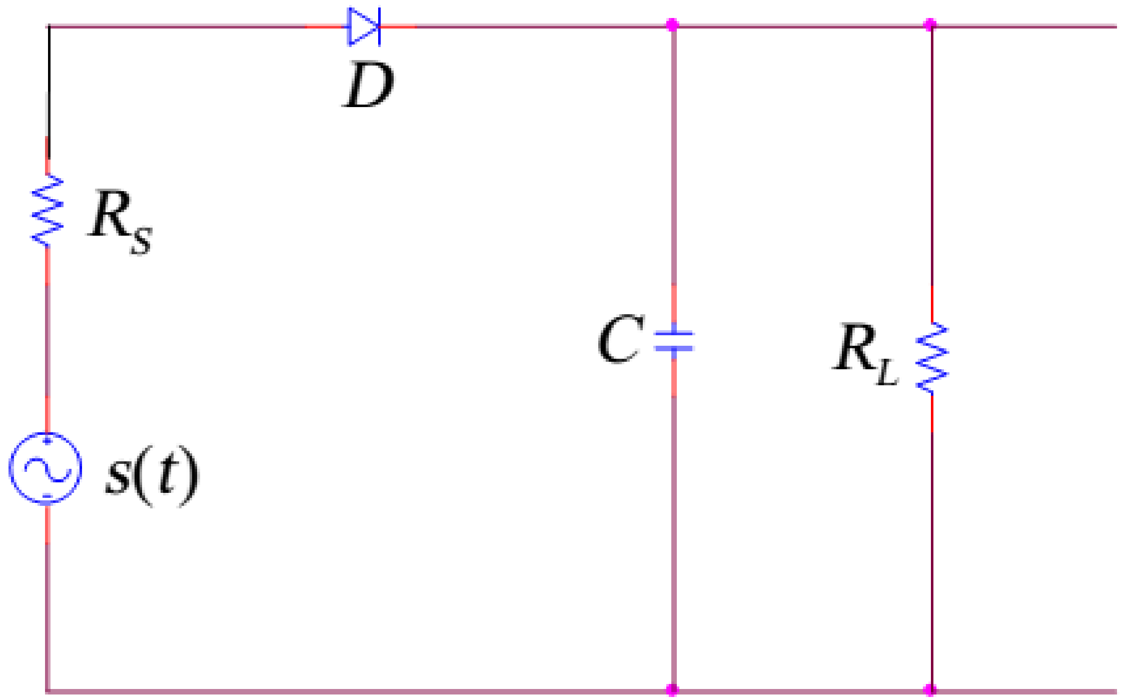
$$s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t$$

Then, the output of the ideal envelope detector is

$$y(t) = A_C\left|1 + k_a m(t)\right|$$

A simple practical envelope detector

It consists of a diode and resistor-capacitor filter.

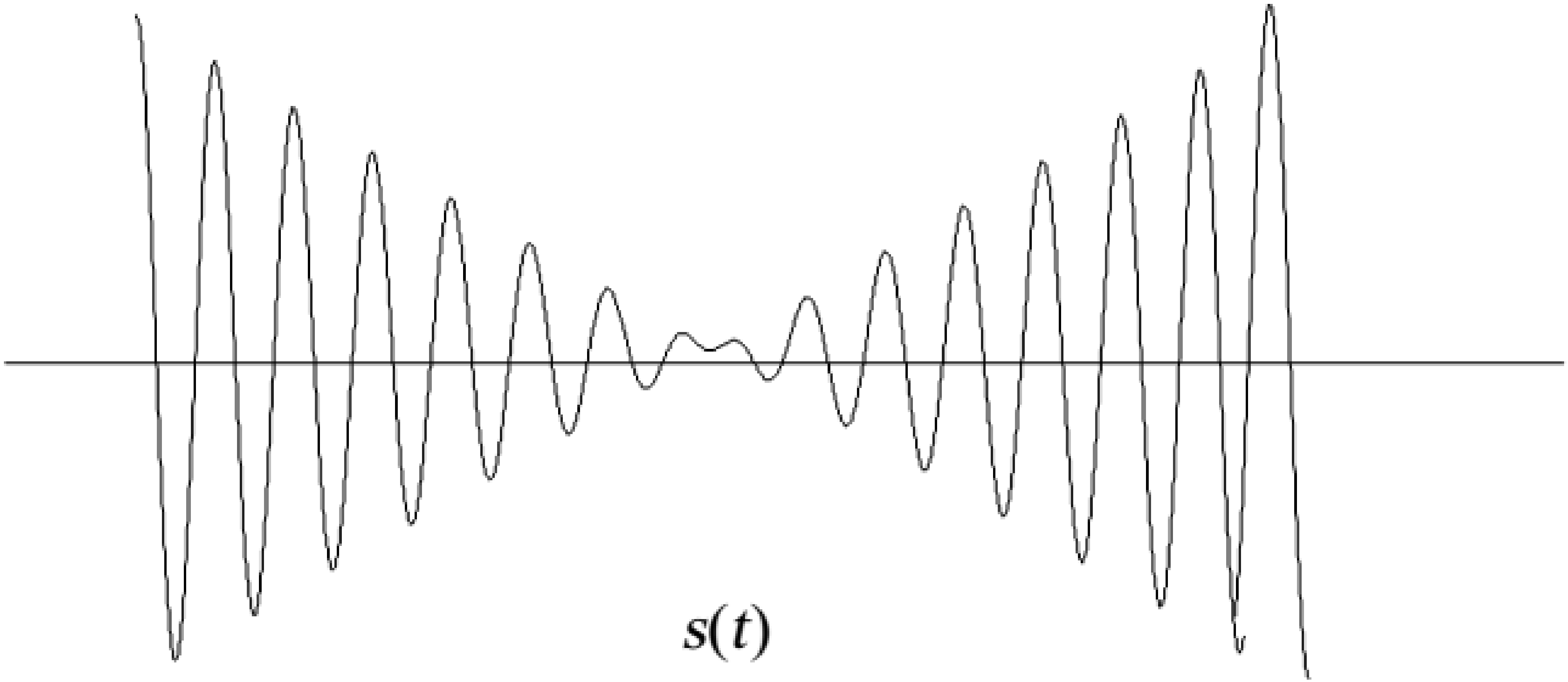


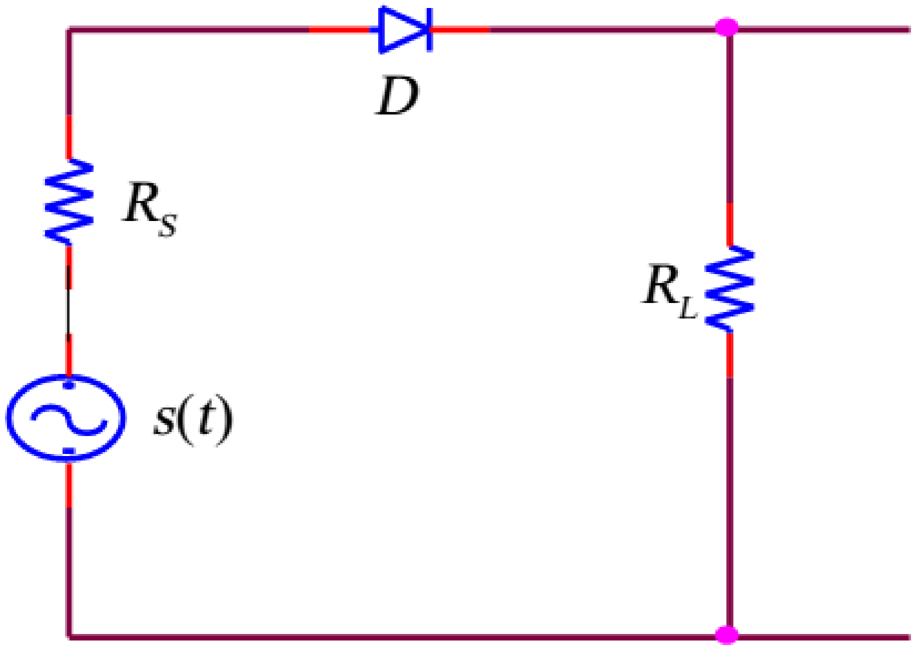
During the positive half cycle of the input, the diode is forward biased and C charges rapidly to the peak value of the input. When $s(t)$ falls below the maximum value, the diode becomes reverse biased and C discharges slowly through R_L. To follow the envelope of $s(t)$, the circuit time constant should be chosen such that :

$$\frac{1}{f_C} << R_L C << \frac{1}{w}$$

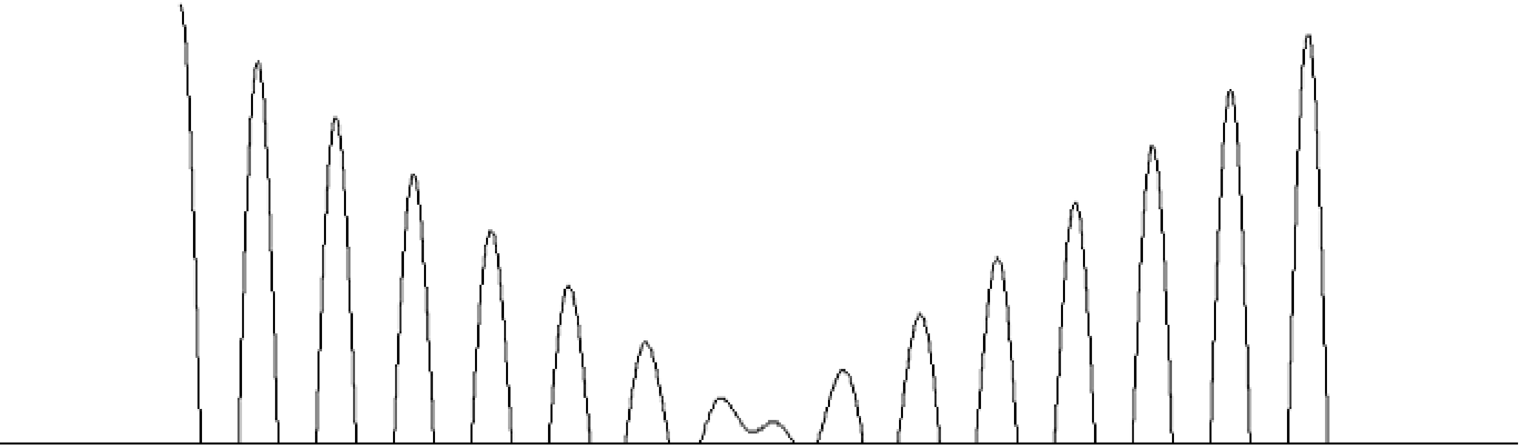
where:

w is the message B.W and f_c is the carrier frequency





Half – Wave Rectifier

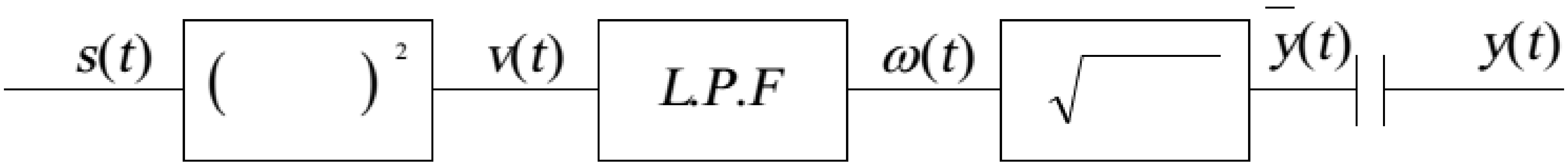


Output of half wave rectifier (without C)

When C is added, the output follows the envelope of $s(t)$. The circuit output (with C connected) follows a curve that connects the tips of the positive half cycles.

Example: (Demodulation of AM signal)

Let $s(t) = (1 + k_a m(t))\cos\omega_C t$ be applied to the scheme shown below, find $y(t)$.



$$v(t) = s(t)^2 = [1 + k_a m(t)]^2 \cos^2 \omega_C t$$

$$= \frac{1}{2}[1 + k_a m(t)]^2 + \frac{1}{2}[1 + k_a m(t)]^2 \cos 2\omega_C t$$

The filter suppresses the second term and passes only the first term, hence

$$\omega(t) = \frac{1}{2}\left(1 + k_a m(t)\right)^2$$

$$\bar{y}(t) = \sqrt{\omega(t)} = \frac{1}{\sqrt{2}}\left(1 + k_a m(t)\right)$$

$$y(t) = \frac{1}{\sqrt{2}} k_a m(t)$$

Note that the dc term is blocked by capacitor.

Concluding remarks about AM:

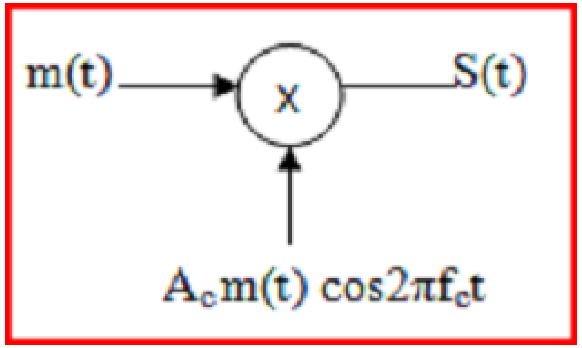
i. Modulation is accomplished using a nonlinear device.
ii. Demodulation is accomplished using a simple envelope detector.
iii. AM is wasteful of power; most power resides in the carrier (not in the sidebands).
iv. The transmission B.W = twice message B.W

Double Sideband Suppressed Carrier Modulation (DSB-SC)

A DSB-SC signals is an AM signals that has the form:

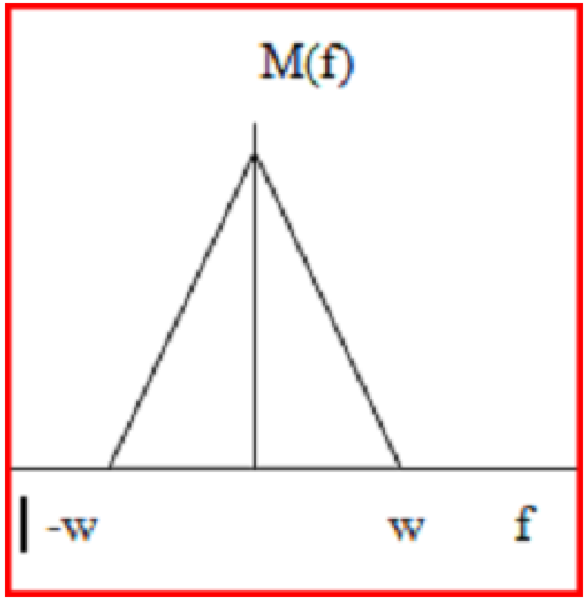
$s(t) = A_c m(t) \cos 2\pi f_c t.$

where $f_c >> w$, w is the baseband signal's band width .



The spectrum of s(t) is:

$$S(f)= \frac{Ac}{2} M(f- f_C)+ \frac{Ac}{2} M(f+ f_c)$$



Remarks:

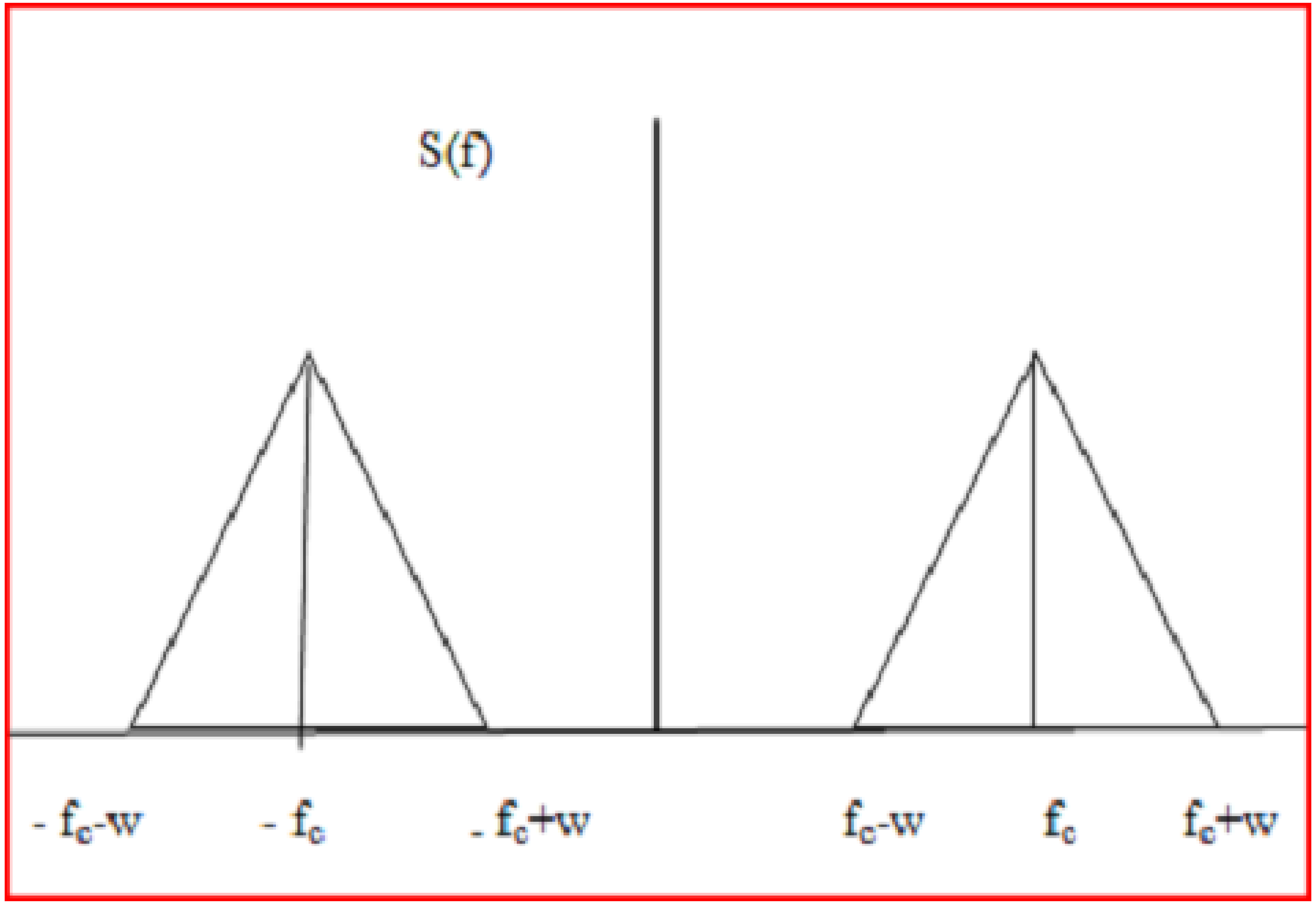
1. No impulses are present in the spectrum at $\pm f_c$ and, hence, no carriers is transmitted.

2. The transmission B.W of s(t)=2w. (same of AM).

3. power efficiency

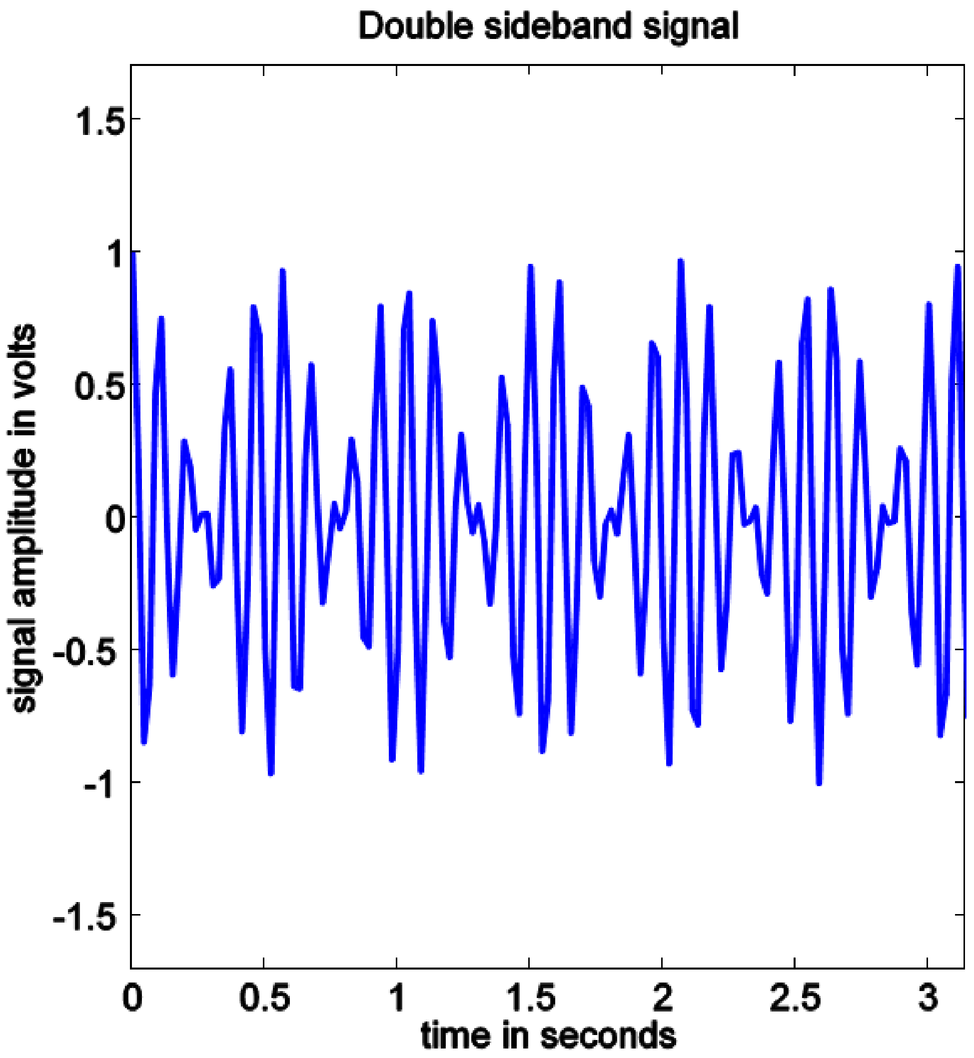
$$=\frac{\textit{power in the side bands}}{\textit{total transmitted power}}=100\%.$$

This is a power efficient modulation scheme.



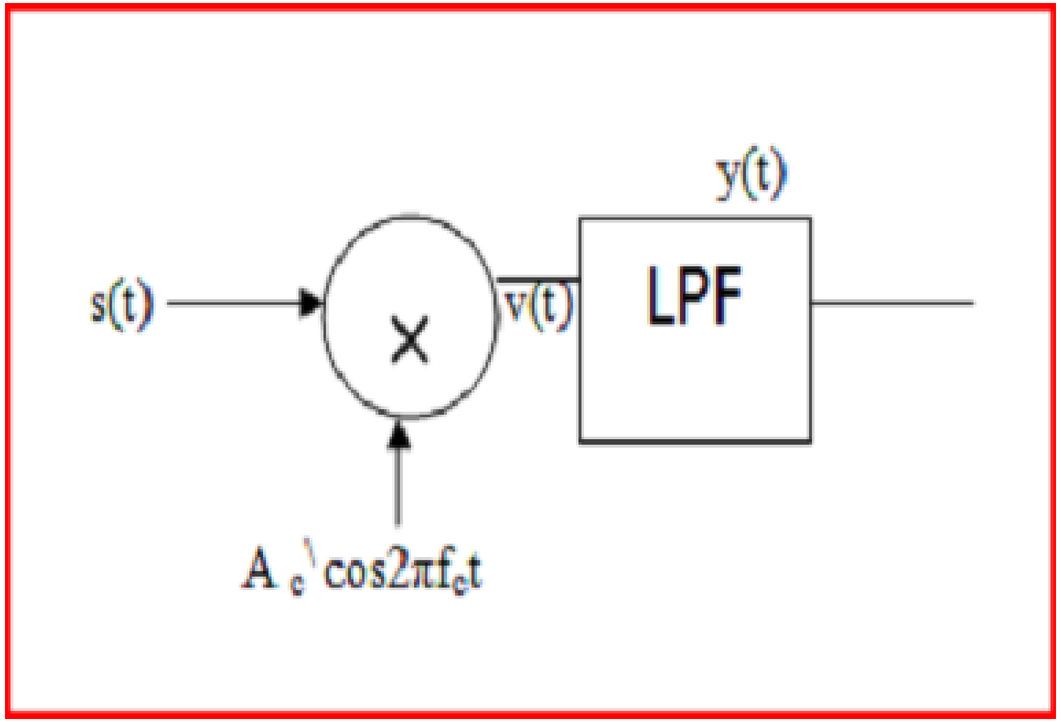
4. Coherent detector is required to extract m(t) from s(t) (will be demonstrated shortly) No envelope detection is used.

5. **Computer simulation**: The next figure shows a DSB-SC signal when m(t)=cos2πt and c(t)=cos2π(10)t. You can easily see that m(t) cannot be recovered using envelope detection.



Demodulation of a DSB-SC signals

A DSB-SC signal is demodulated using what is known as *coherent demodulation*. This means that the modulated signal s(t) is multiplied by a locally generated signal at the receiver which has the same frequency and phase as the carrier c(t) at transmitting side.



a. **Perfect coherent demodulation.**

Let $c(t)= A_c \cos 2\pi f_c t$, $c^{\backslash}(t)= A_c^{\backslash} \cos(2\pi f_c t)$

Mixing the received signal with the version of the carrier at the receiving side, we get

$$v(t)=s(t)\ A_c^{\backslash} \cos 2\pi f_c t = A_c\ A_c^{\backslash} m(t) \cos^2 2\pi f_c t$$

$$= \frac{Ac\ A\ c\backslash}{2} m(t)[1 + \cos 2\,(2\pi f_c t)]$$

$$= \frac{Ac\ A\ c\backslash}{2} m(t) + \frac{Ac\ A\ c\backslash}{2} m(t) \cos 2(2\pi f_c t)$$

Proportional to m(t) high frequency signals at $2f_c$ (A DSB-SC)

The high frequency component can be eliminated using the LPF. The output is

$$y(t) = \frac{Ac\ A\ c\backslash}{2} m(t) = Km(t),$$

Therefore, m(t) has been recovered from s(t) without distortion, i.e., a distortion less system.

b. Effect of carrier noncoherence on demodulated signal

Here we consider two cases.

Case 1: A constant phase difference between c(t) and $c'(t)$

Let $c(t) = A_c \cos 2\pi f_c t$, $c'(t) = A_c' \cos(2\pi f_c t + \emptyset)$

We use the demodulator considered above

$$v(t) = A_c m(t) \cos 2\pi f_c t \,.\, A_c' \cos(2\pi f_c t + \emptyset)$$

$$= \frac{A_c A_c'}{2} m(t) [\cos(4\pi f_c t + \emptyset) + \cos \emptyset]$$

$$= \frac{A_c A_c'}{2} m(t) \cos(4\pi f_c t + \emptyset) + \frac{A_c A_c'}{2} m(t) \cos \emptyset$$

↑ high frequency term ↑ low frequency term

The output of the low pass filter is:

$$y(t) = \frac{A_c A_c'}{2} m(t) \cos \emptyset$$

For $0 < \emptyset < \frac{\pi}{2}$, $0 < \cos \emptyset < 1$, and y(t) suffer from an attenuation due to Ø.

However, for $\emptyset = \frac{\pi}{2}$, $\cos \emptyset = 0$ and $y(t) = 0$, signal disappears. The disappearance of a message component at the demodulator output is called *quadrature null effect.*

This highlights the importance of maintaining synchronism between the transmitting and receiving carrier signals $c^{\backslash}(t)$ and c(t).

Case 2: Constant frequency difference between c(t) and $c^{\backslash}(t)$

$$\text{Let } c(t)= A_c\cos 2\pi f_c t \qquad ,C^{\backslash}(t)= A_c^{\backslash}\cos(2\pi f_c+\Delta f)t$$

In an analysis similar to case a, we get

$$v(t)= A_c m(t)\cos 2\pi f_c t \,.\, A_c^{\backslash}\cos(2\pi f_c+\Delta f)t$$

$$=\frac{Ac\,A\,c\backslash}{2}m(t)[\cos\ (4\pi f_c t+2\pi\Delta ft)+\cos 2\pi\Delta ft]$$

After low pass filtering,

$$y(t)= \frac{Ac\,A\,c\backslash}{2}m(t)\cos 2\pi\Delta ft$$

So the demodulated signals appears as if double side band modulated on a carrier with magnitude Δf. As can be observed, this is not a distortionless transmission.

Example: Let $m(t)=\cos 2\pi(1000)t$ and let $\Delta f=100HZ$.

From the analysis in case 2 above,

$$y(t)=\frac{Ac\ A\ c\backslash}{2}\cos 2\pi(1000)t\ \cos 2\pi(100)t$$

$$=\frac{Ac\ A\ c\backslash}{4}[\cos\ 2\pi(1100)t+\cos\ 2\pi(900)t]$$

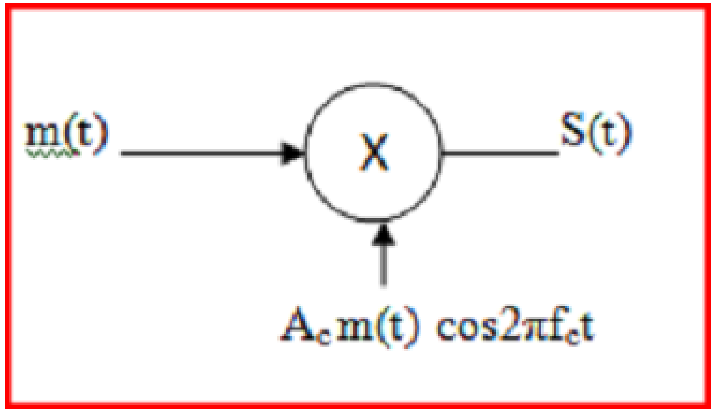
The original message was a signal with a frequency of =1000Hz, while the output consists of a signal two frequencies at $f_1=1100Hz$ and $f_2=900Hz$.

$\Rightarrow$ Distortion

Exercise: Use Matlab to plot both m(t) and y(t) and see the distortion caused by the lack of synchronization between the transmitting and receiving oscillators.

Generation of DSB-SC

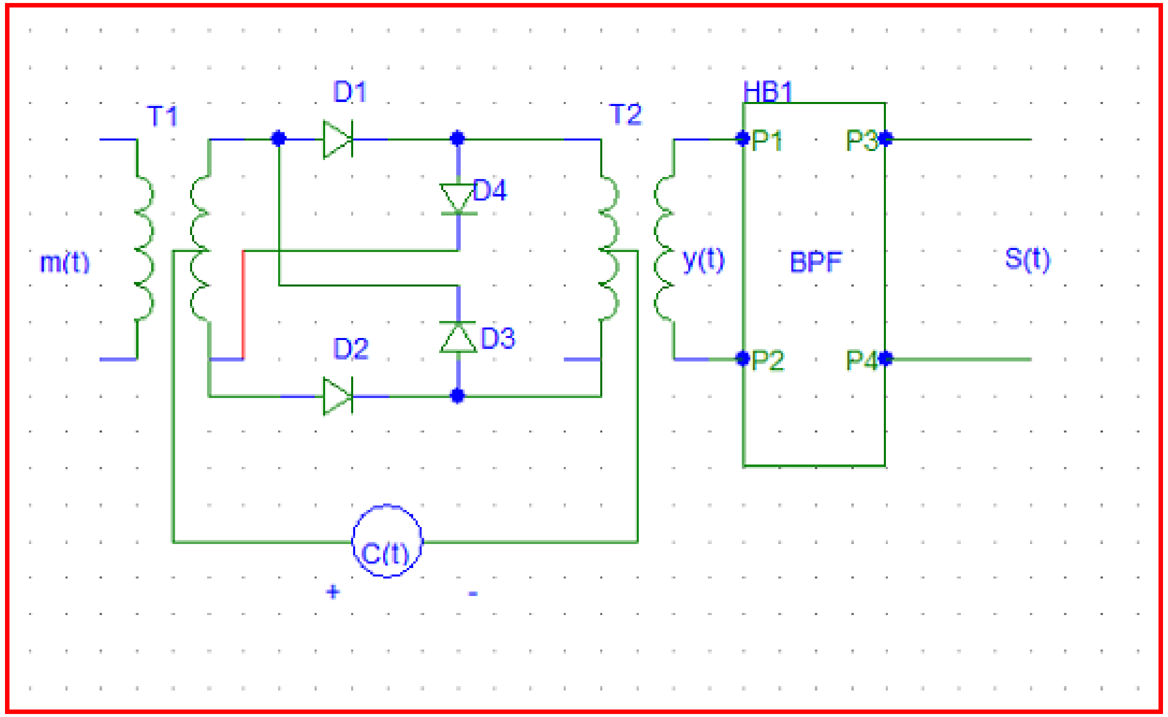
a. **Product modulator** : It multiplies the message signal m(t) with the carrier c(t). This technique is usually applicable when low power levels are possible and over a limited carrier frequency range.



b .**Ring modulator**:

consider the scheme shown in the figure.

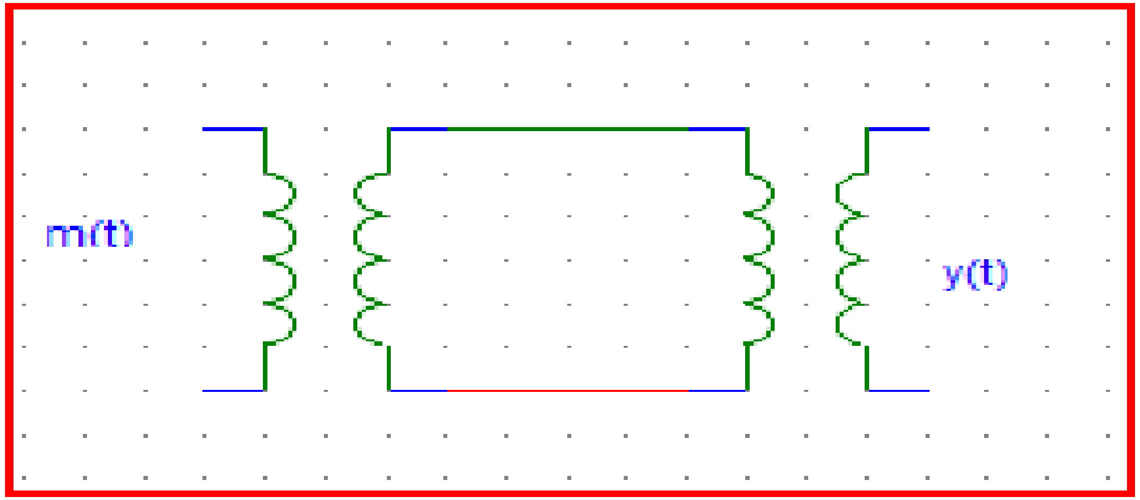
Let c(t)>> m(t). Here the carrier c(t) control the behavior of the diodes .



During the positive half cycle of c(t), c(t) > 0, and D1 and D2 are ON while D3 and D4 are OFF.

$y(t) = m(t)$

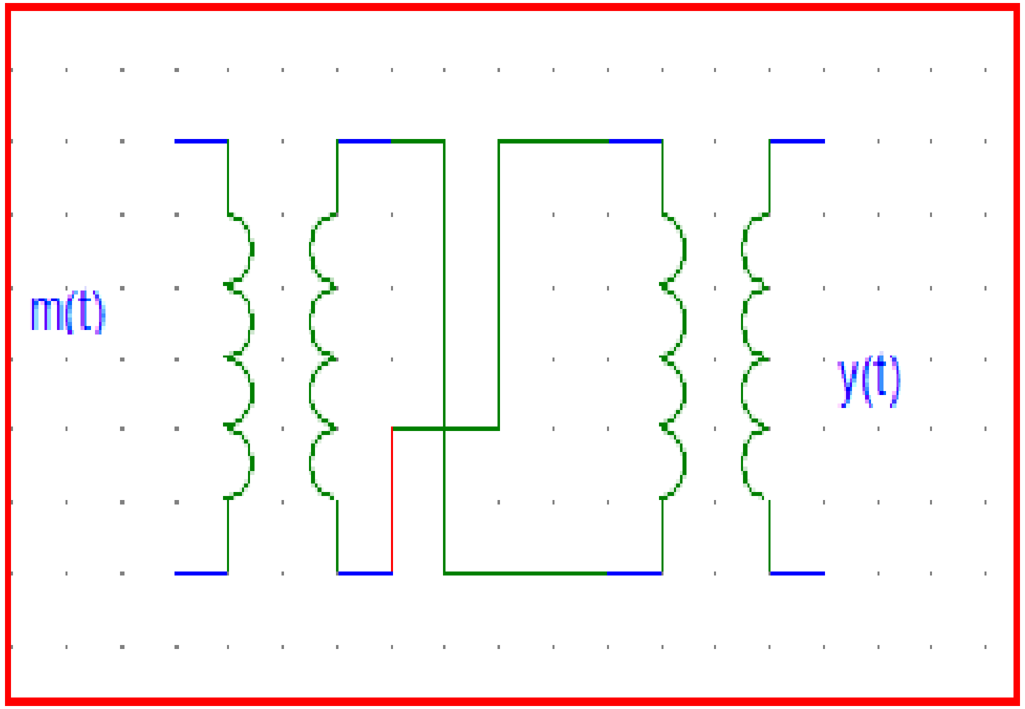
and the circuit appears like this



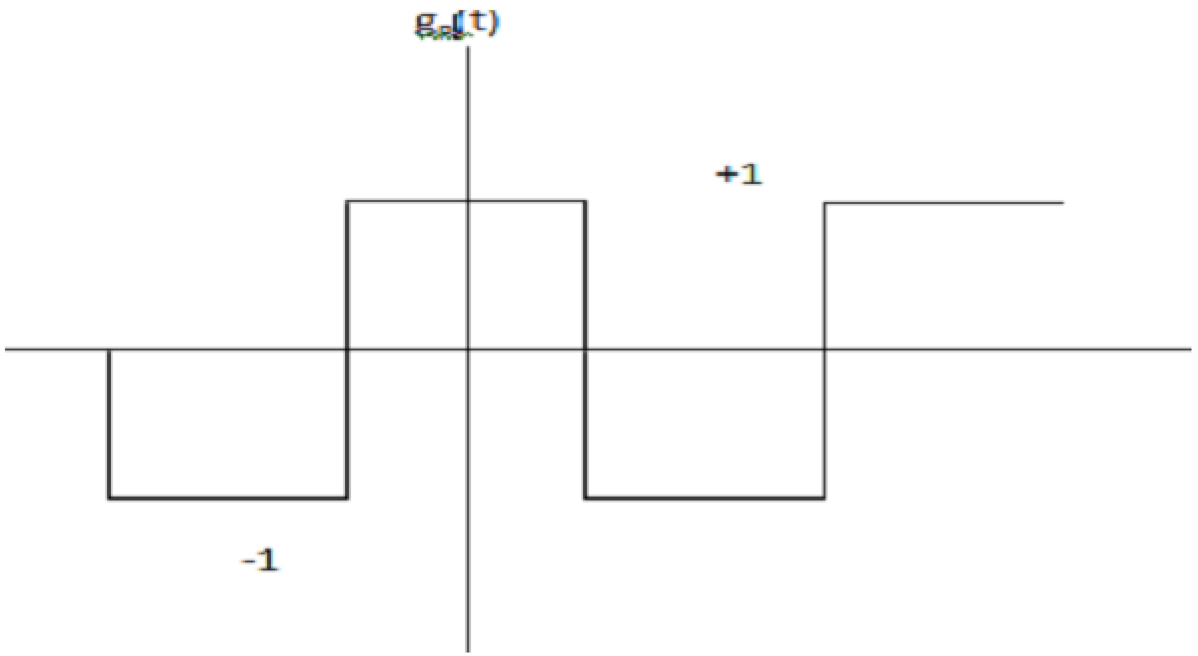
During the negative half cycle of c(t), c(t) < 0 and D3 and D4 are ON while and D1 and D2 are OFF

$y(t) = -m(t)$

and the circuit appears like this



So m(t) is multiplied by +1 during the +ve half cycle of c(t) and m(t) multiplied by -1 during the -ve half cycle of c(t). Mathematically, y(t) hehaves as if multiplied by the switching function $g_p(t)$ where $g_p(t)$ is the square periodic function with period $T_c=\frac{1}{fc}$, where f_c the period of c(t). By expanding $g_p(t)$ in a Fourier series, we get



$$y(t) = m(t)[\frac{4}{\pi}\cos 2\pi f_c t - \frac{4}{3\pi}\cos 3(2\pi f_c t) + \frac{4}{5\pi}\cos 5(2\pi f_c t)]$$

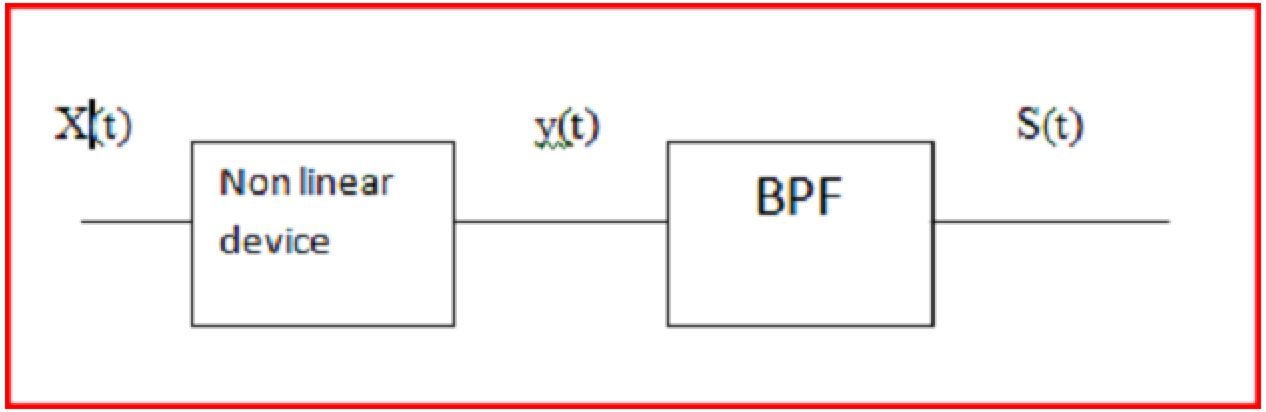
$$= m(t)\frac{4}{\pi}\cos 2\pi f_c t - m(t)\frac{4}{3\pi}\cos 3(2\pi f_c t) + m(t)\frac{4}{5\pi}\cos 5(2\pi f_c t)$$

When y(t) passes through the BPF, the only component that appears at the output is the desired DSB-SC signal, which is

$$s(t) = m(t)\frac{4}{\pi}\cos 2\pi f_c t$$

C. Nonlinear characteristic

Consider the scheme shown in the figure



Let the non linear characteristic be of the form

$$y(t)=a_0x(t)+a_1x^3(t)$$

Let $x(t)= A\cos 2\pi f_c t+m(t)$, ($m(t)$ is the message signals)

$$y= a_0(A\cos 2\pi f_c t+m(t))+a_1 (A\cos 2\pi f_c t+m(t))^3$$

$$= a_0 A\cos 2\pi f_c t+ a_0 m(t)+ a_1 A^3\cos^3 2\pi f_c t+ a_1 m(t)^3+3 a_1 A^2 m(t)\cos^2 2\pi f_c t$$

$$+3A a_1\cos 2\pi f_c t$$

After some algebraic manipulations, a DSB-SC term appear in x(t) along with other undesirable terms. The band pass filter will admit the desired signal, which is

$$s(t)=\frac{3(A)^2 a1}{2} m(t) \cos(2)2\pi f_c t,$$

Note that the carrier frequency=$2f_c$ in this case.

Carriers recovery for coherent demodulation

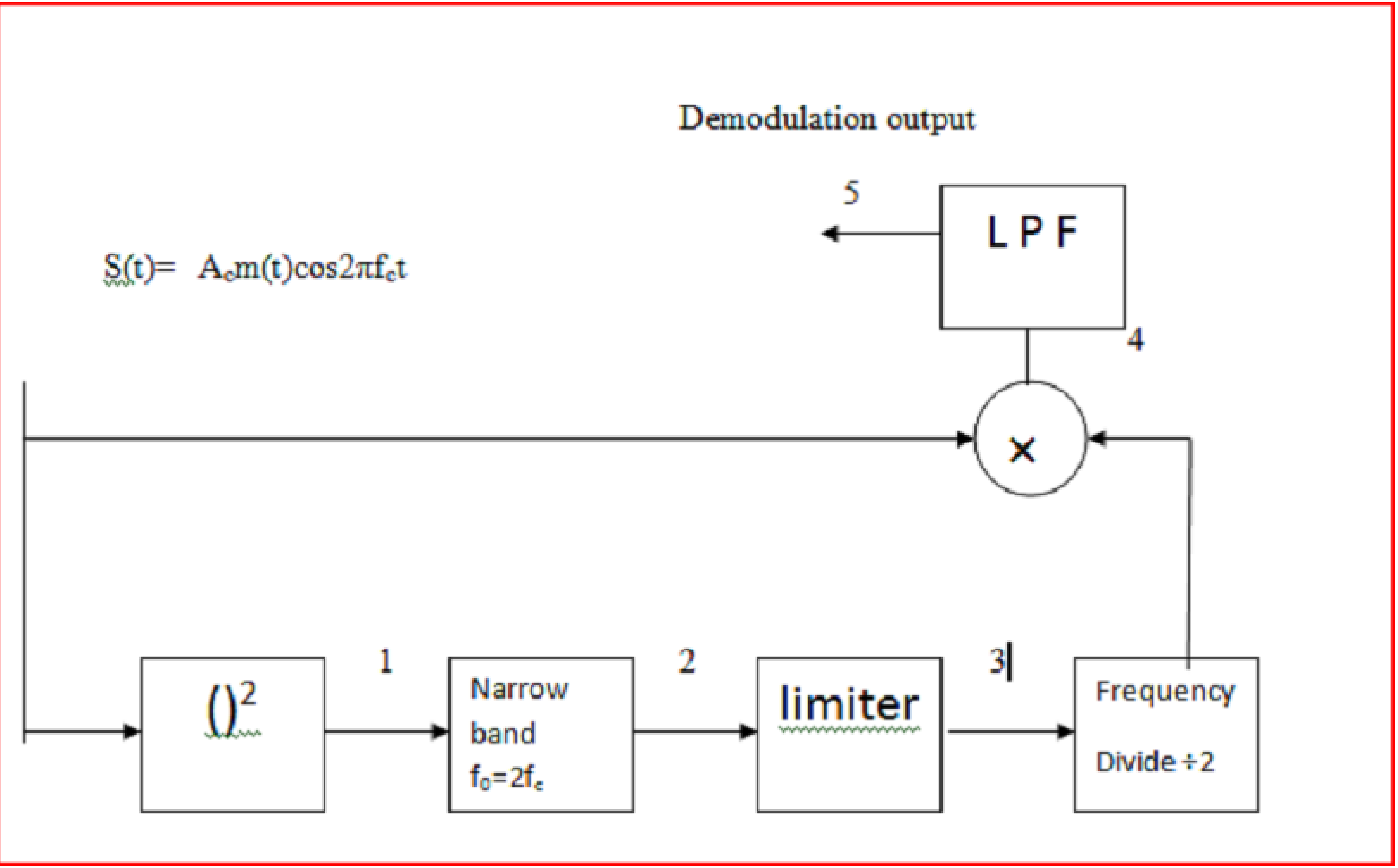
We consider briefly two circuits that are used to extract the carriers f_c from the incoming DSB-SC signal. we recall that demodulation of DSB-SC signal requires the availability of a signal with the same frequency and phase as the carrier c(t) at the transmitter

a. **Squaring loop :**

The basic elements of squaring loop are shown in the figure below. The incoming signal is the DSB-SC signal:

$$s(t)=A_c m(t)\cos 2\pi f_c t.$$

In the figure, we mark five signals that appear at the output of the five blocks. In summary these signals are:


Demodulation output
5
L P F
4
$S(t) = A_c m(t) \cos 2\pi f_c t$
×
1
2
3
$()^2$
Narrow
band
$f_0 = 2f_c$
limiter
Frequency
Divide ÷2

1- $(A_c m(t)\cos 2\pi f_c t)^2 = (\frac{Ac}{2}m(t))^2(1+\cos 2w_c t)$

$= (\frac{Ac}{2}m(t))^2 + (\frac{Ac}{2}m(t))^2\ (\cos 2w_c t)$

↑ Low pass term ↑ band pass around $2f_c$

2- $(\frac{Ac}{2}m(t))^2\ (\cos 2w_c t) = K\cos 2w_c t$ (when the BPF is of narrow B.W)

3- $K\cos 2w_c t$ (The limiter removes any variation in amplitude but keeps the frequency unchanged). The frequency divider produces a signal $K\cos w_c t$.

4- $(A_c m(t)\cos 2\pi f_c t)\ K\cos w_c t$

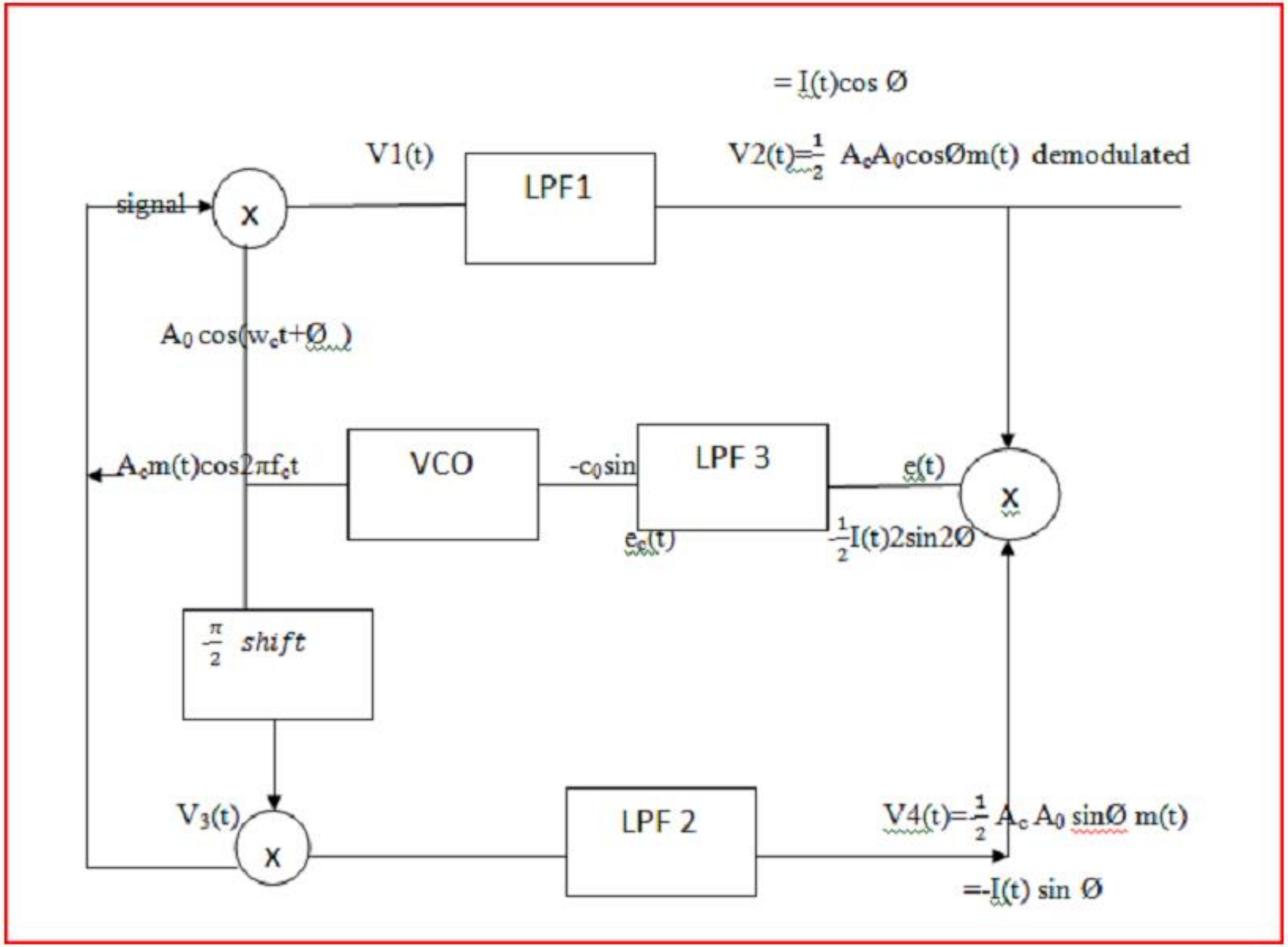
$= A_c K m(t)\cos^2 w_c t$

$\frac{A_c(K)}{2}m(t) + \frac{A_c(K)}{2}m(t)\ \cos 2w_c t$

5- $A_c\frac{(K)}{2}m(t)$

Therefore, demodulation has been achieved, even though the receiver does not have a copy of the carrier, but was able to generate its own version of the carrier via this loop.

Costas Loop:



= I(t)cos Ø
V1(t)
LPF1
V2(t)=1/2 AcA0cosØm(t) demodulated
signal
x
A0 cos(wct+Ø)
Acm(t)cos2πfct
VCO
-c0sin
LPF 3
e(t)
x
ec(t)
1/2 I(t)2sin2Ø
π/2 shift
V3(t)
x
LPF 2
V4(t)=1/2 Ac A0 sinØ m(t)
=-I(t) sin Ø

The VCO: is an oscillator that produces a signal whose frequency is proportional to the voltage $e_c(t)$.

When $e_c(t) = 0$, the frequency of the oscillator is called the free running frequency. Let this frequency = f_c (the incoming carrier frequency)

When there is a phase difference Ø, we have

$$V_1(t)= A_cA_0m(t)\cos(w_ct)\cos(w_ct+Ø)$$

$$V_2(t)=\frac{AcA0}{2}m(t)\cos Ø$$

$$V_4(t)=\text{Low pass }\{A_0A_cm(t)\cos(w_ct)\sin(w_ct+Ø)\}$$

$$V_4(t)=\frac{AcA0}{2}m(t)\sin Ø \text{ after LPF 2}$$

$$e(t) = \frac{AcA0}{2} m(t)^2 (\sin\varnothing)(\cos\varnothing)$$

$$= \frac{AcA0}{24} m(t)^2 \sin 2\varnothing$$

When the B.W of LPF3 is very narrow, the output can be approximated as:

$$e_c(t) = c_0 \sin 2\varnothing$$

This is the feedback signal that is applied to the VCO. Ideally, when $\varnothing=0$, $e_c(t)=0$ and VCO frequency (and phase) are equal to the frequency of the input signal s(t).

If the phase difference $\varnothing$ between the incoming s(t) and the VCO output increases, then $e_c(t)$ increases forcing the frequency of the VCO to decrease so that it remains in synchronism with the input phase. (Recall that the frequency of the VCO decreases if its input voltage increases; the slope of the VCO characteristics is negative).

Single Sideband Modulation

In this type of modulation, only one of the two sidebands of DSB-SC is retained while the other sideband is suppressed. This mean that B.W of the SSB signal is one half that of DSB-SC. The saving in the bandwidth comes at the expense of increasing modulation complexity.

The time-domain representation of a SSB signal is

$$s(t)=A_c m(t)\cos w_c t \pm A_c \hat{m}(t) \sin w_c t$$

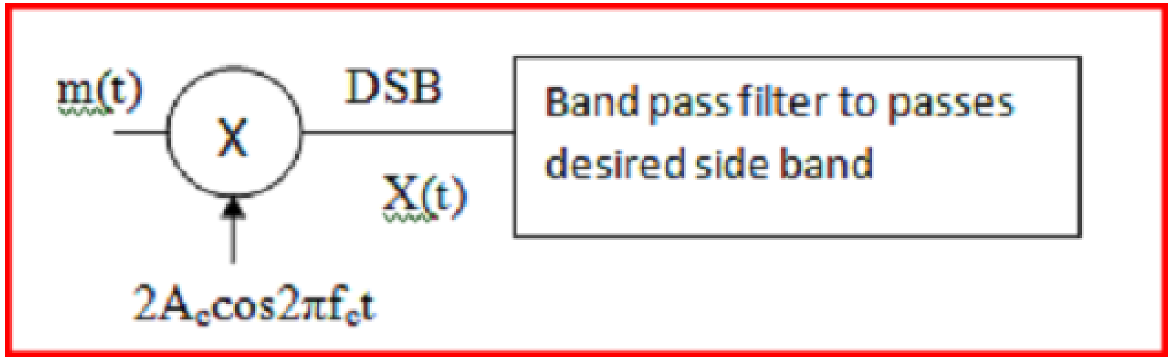
$\hat{m}(t)$: Hilbert transform of m(t)

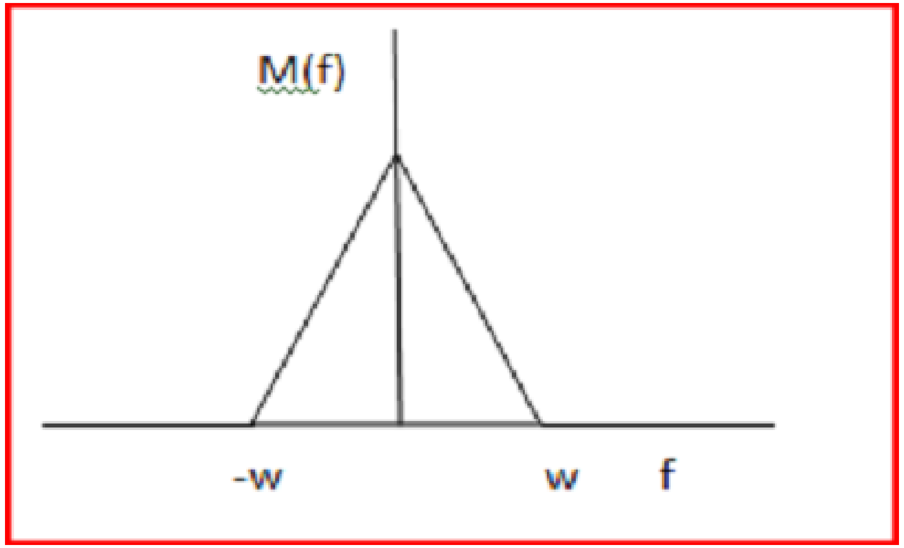
-sign: upper sideband is retained

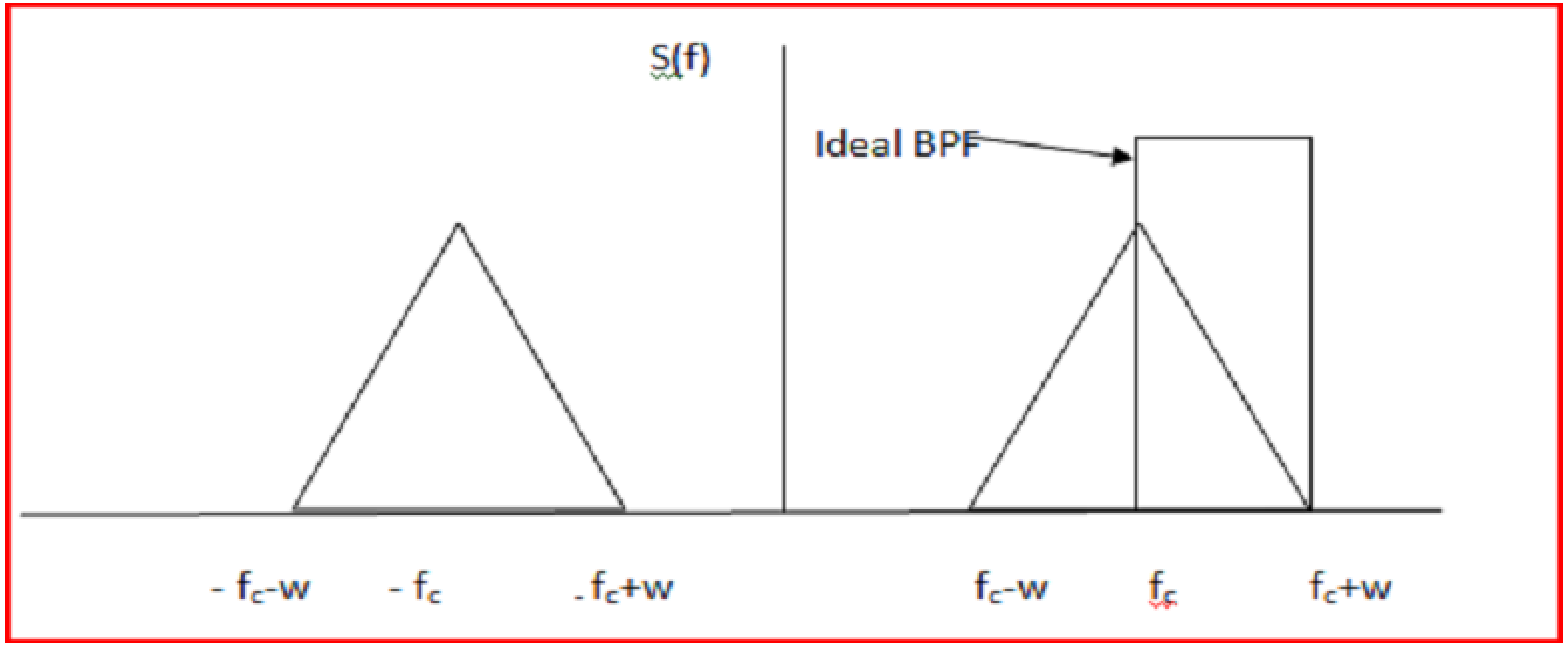
+sign: lower sideband is retained

Generation of SSB: Filtering Method (Frequency Discrimination Method)

A DSB-SC signal $X(t)=2A_c m(t) \cos w_c t$ is generated first. A band pass filter with appropriate B.W and center frequency is used to pass the desired side band only



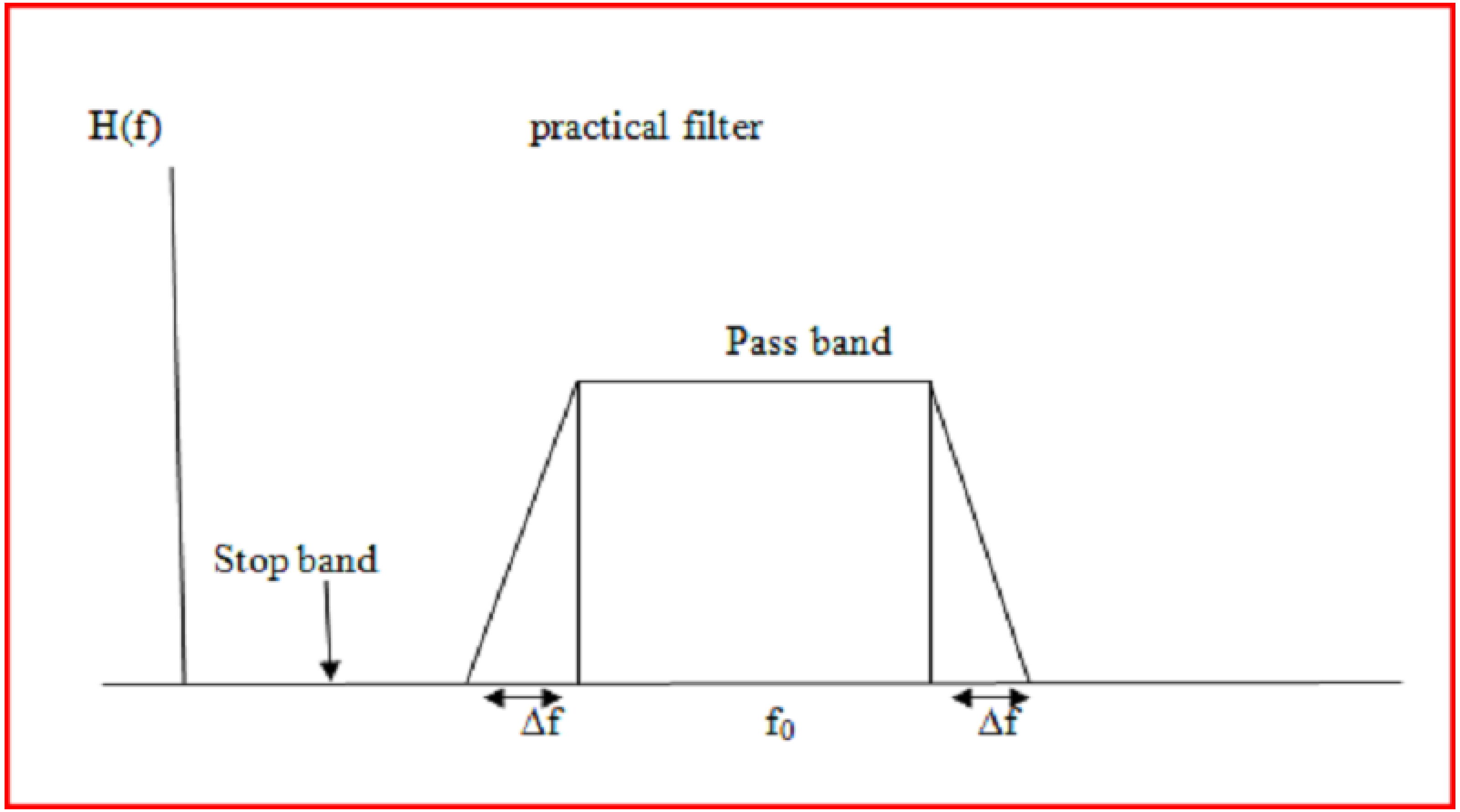



S(f)
Ideal BPF
- fc-w
- fc
- fc+w
fc-w
fc
fc+w


S(f)
- fc-w
- fc
fc
fc+w

The band pass filter must satisfy two conditions:

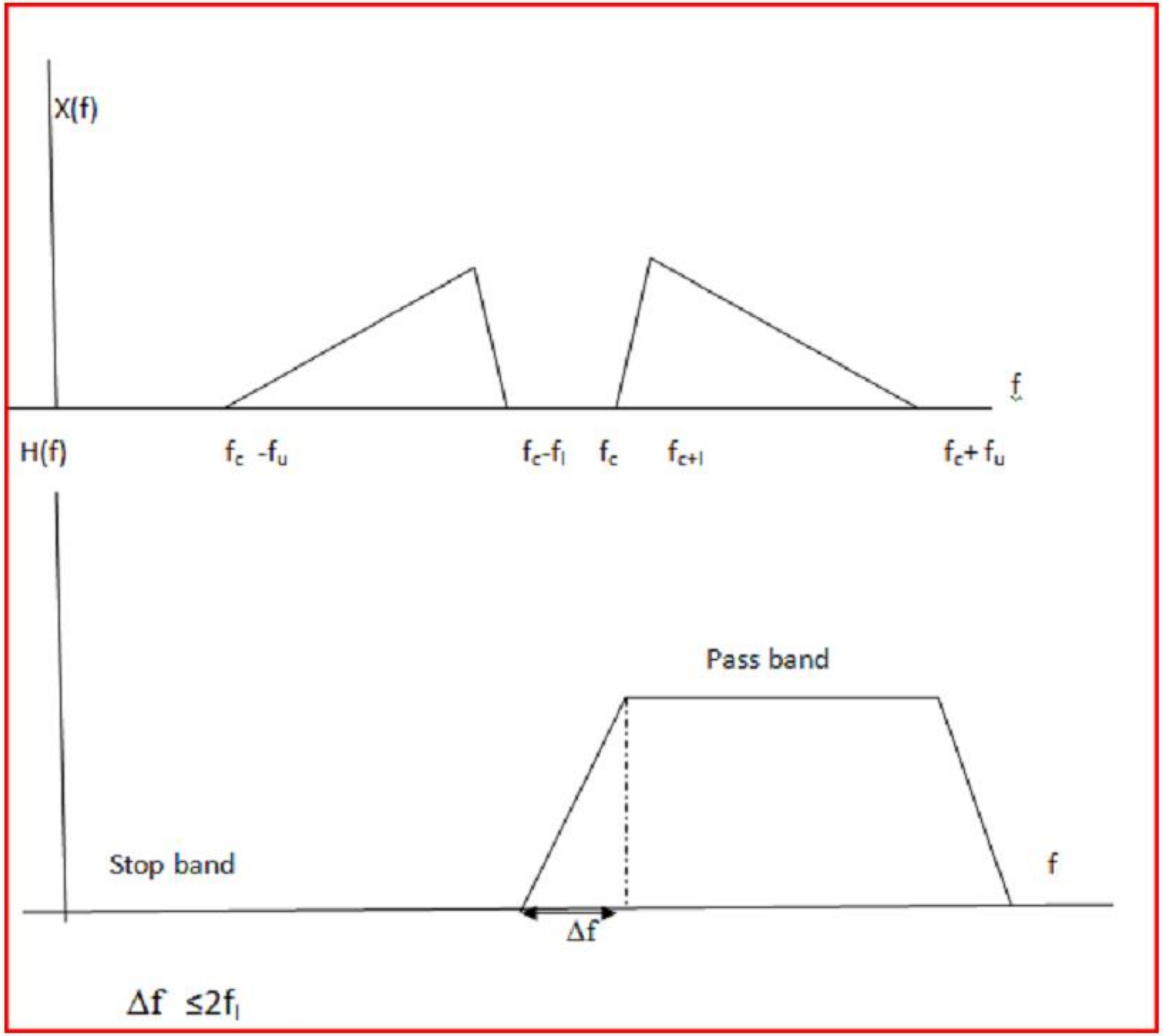
a. The pass band of the filter must occupy the same frequency range as the desired sideband.
b. The width of the transition band of the filter separating the pass band and the stop band must be at least 1% of the center frequency of the filter. i.e., $0.01f_0 \leq \Delta f$. This is sort of a rule of thumb for realizable filters on the relationship between the transition band and the center frequency.


H(f)
practical filter
Pass band
Stop band
Δf
f0
Δf

Two remarks should be considered when generating a SSB signal.

1. Ideal filter do not exist in practice meaning that a complete elimination of the undesired side band is not possible. The consequence of this is that either part of the undesired side band is passed or the desired one will be highly attenuated. SSB modulation is suitable for signals with low frequency components that are not rich in terms of their power content.

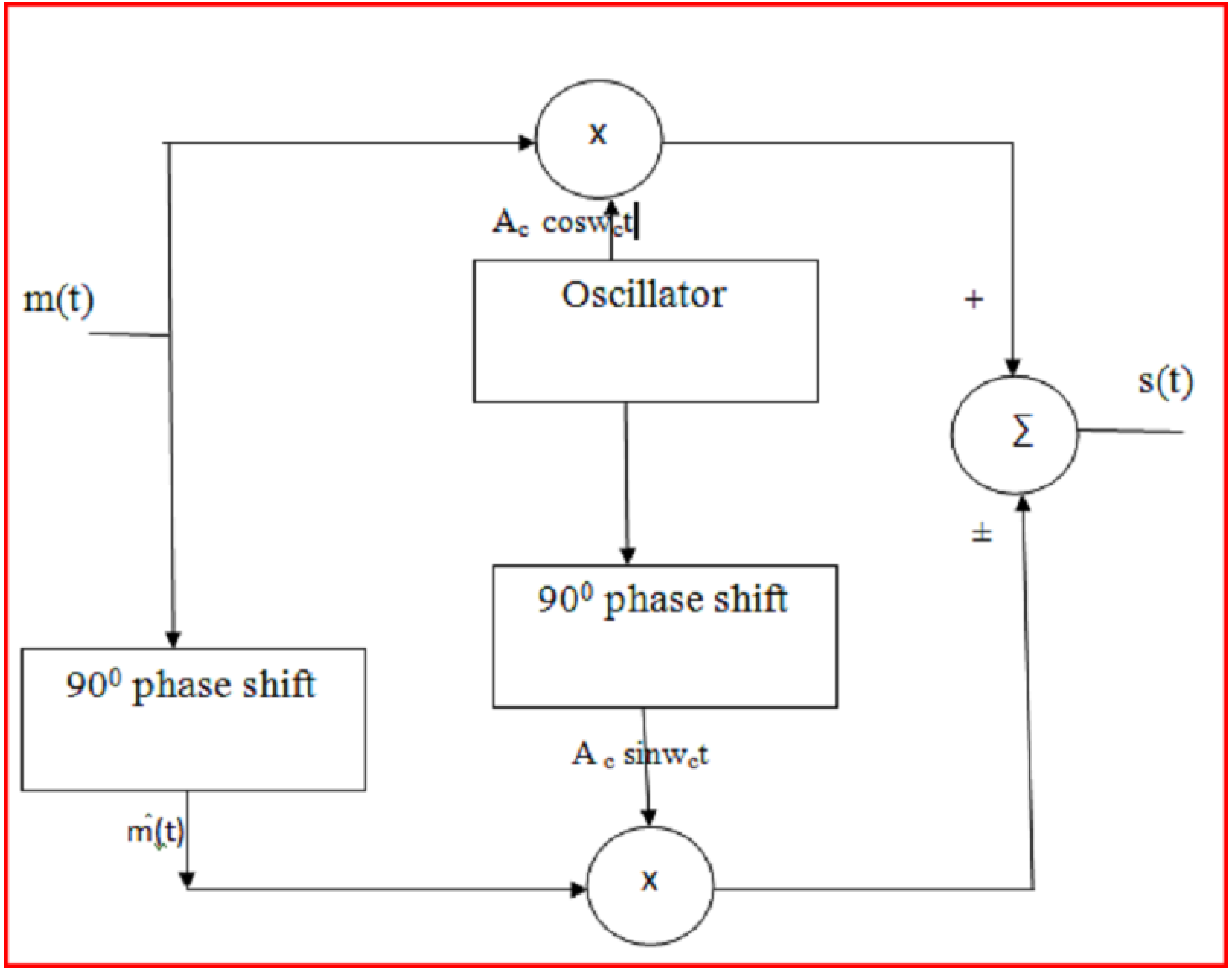
2. The width of the transition band of the filter should be at most twice the lowest frequency components of the message signal so that a reasonable separation of the two side band is possible.


X(f)
f
H(f)
f_c -f_u
f_c-f_l
f_c
f_{c+l}
f_c+ f_u
Pass band
Stop band
f
Δf
Δf ≤2f_l

Generation of SSB Signal: Phase Discrimination Method .

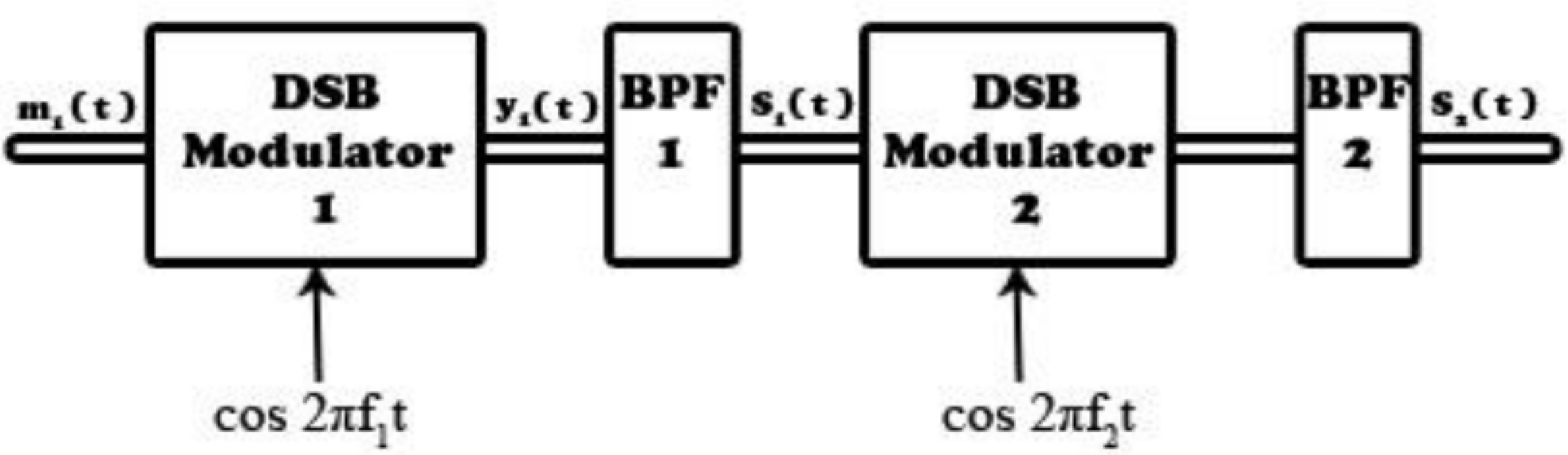
The method is based on the time –domain representation of the SSB signal

$$s(t) = A_c m(t) \cos w_c t \pm A_c \hat{m}(t) \sin w_c t$$


x
A c coswct
m(t)
Oscillator
+
s(t)
Σ
±
90⁰ phase shift
90⁰ phase shift
A c sinwct
m̂(t)
x

Two- stage generation of SSB signal

When the conditions on the filter cannot be met in a single-stage SSB system, a two-stage scheme is used instead where less stringent conditions on the filters can be imposed. The block diagram illustrates this procedure.



$m_1(t)$ is the base band signal with a gap in its spectrum extending over $(0, f_l)$.

$y_1(t)$: is a DSB-SC signal on a carrier frequency f_1.

BPF_1 selects the upper side band of $y_1(t)$. The parameters of the filter are f_{01} (center frequency) and the transition band length Δf_1.

We must maintain that

$\Delta f_1 \geq 0.01 \, f_{01}$ and $\Delta f_1 \leq 2 \, f_I$

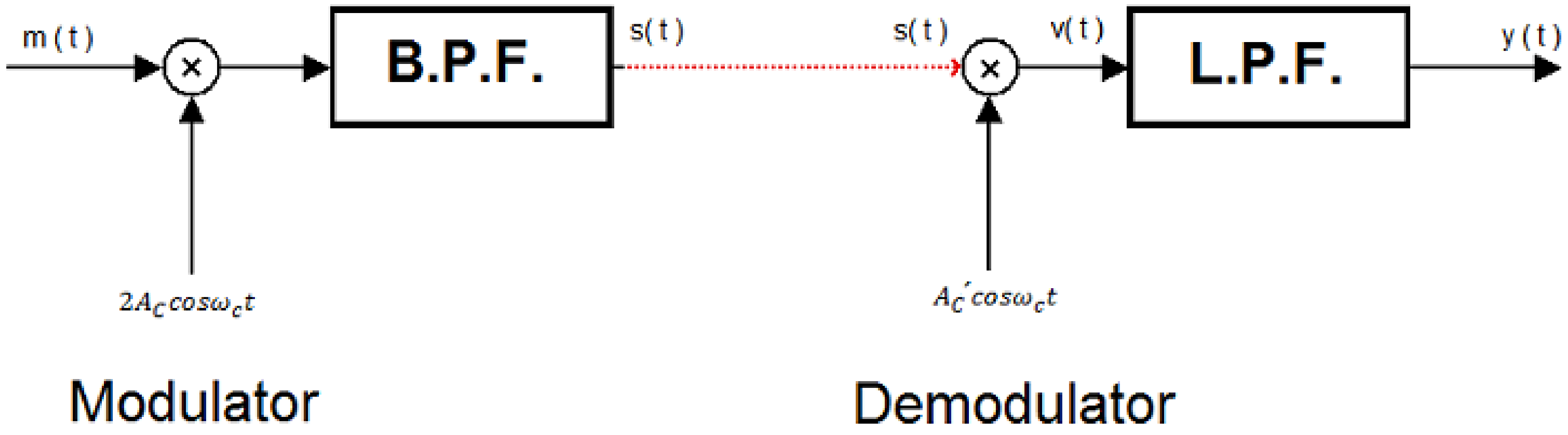
$s_1(t)$ is a single side band signal. The frequency gap of this signal extends over $(0, f_1 + f_L)$. The second modulator views this signal as the baseband signal to be modulated on a carrier with frequency f_2.

The second modulator generates a DSB signal. The second BPF with center frequency f_{02} and transition band Δf_2 selects the upper side band. Again, we maintain that

$\Delta f_2 \geq 0.01 \, f_{02}$ and $\Delta f_2 \leq 2 \, (f_1 + f_I)$

Demodulation of SSB: Time-Domain Analysis

A SSB signal can be demodulated using coherent demodulation (oscillator at the receiver should have the same frequency and phase as those of transmitter carrier) as shown in the figure:



Let the received signal be the upper single sideband

$$s(t) = A_C m(t) cos\omega_c t - A_C \hat{m}(t) sin\omega_c t$$

At the receiver this is mixed with the carrier signal. The result is

$$v(t) = s(t)A_C' cos\omega_c t$$

$$= A_C' [A_C m(t)cos\omega_c t - A_C \hat{m}(t)sin\omega_c t]cos\omega_c t$$

$$= A_C A_C' m(t)cos2\omega_c t - A_C A_C' \hat{m}(t)\, sin\omega_c t\, cos\omega_c t$$

$$= \frac{A_C A_C'}{2} m(t) + \frac{A_C A_C'}{2} m(t)cos2\omega_c t - \frac{A_C A_C'}{2} \hat{m}(t)\, sin2\omega_c t$$

The low pass filter admits only the first terms. The output is:

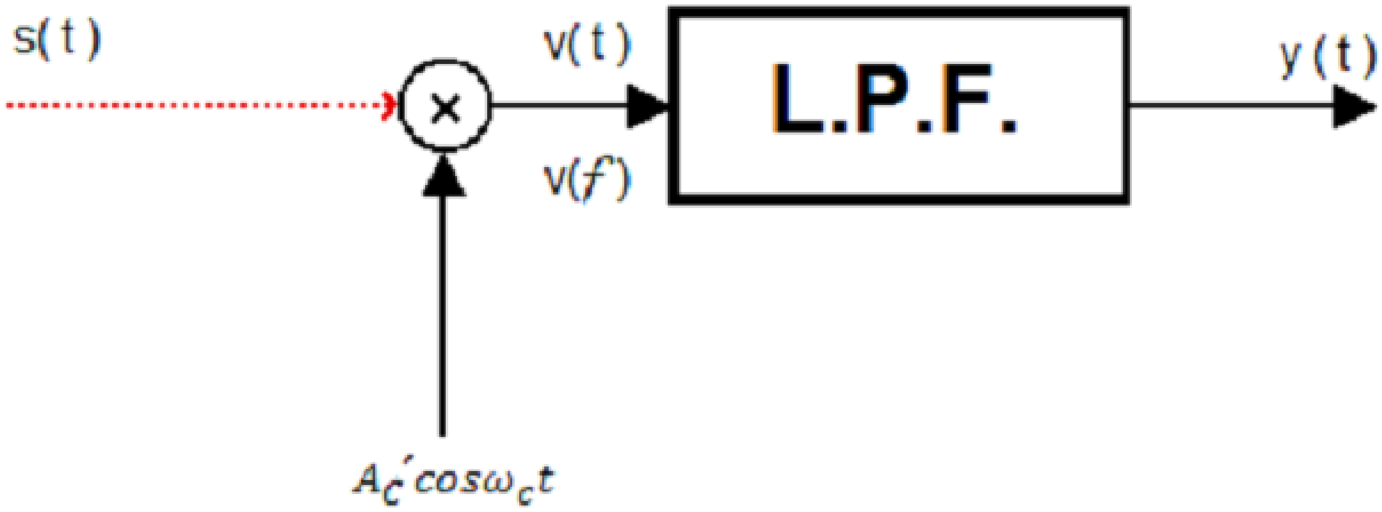
$$y(t) = \frac{A_C A_C'}{2} m(t)$$

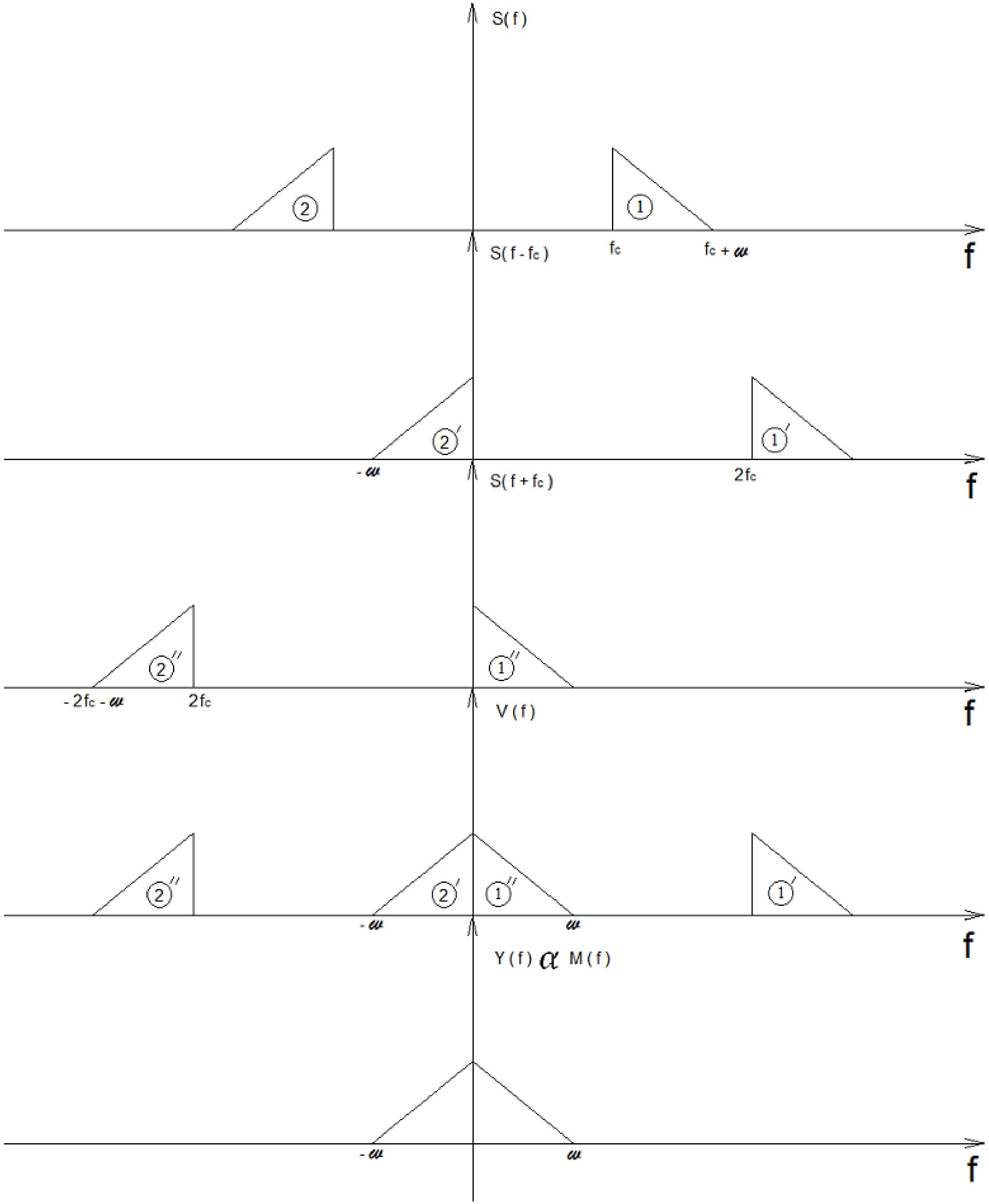
The following steps demonstrate the demodulation process viewed in the frequency domain .

$$V(f) = \frac{A_c'}{2} S(f - f_c) + \frac{A_c'}{2} S(f + f_c)$$

$$Y(f) = Low\ pass\ \left\{ \frac{A_c'}{2} S(f - f_c) + \frac{A_c'}{2} S(f + f_c) \right\}$$

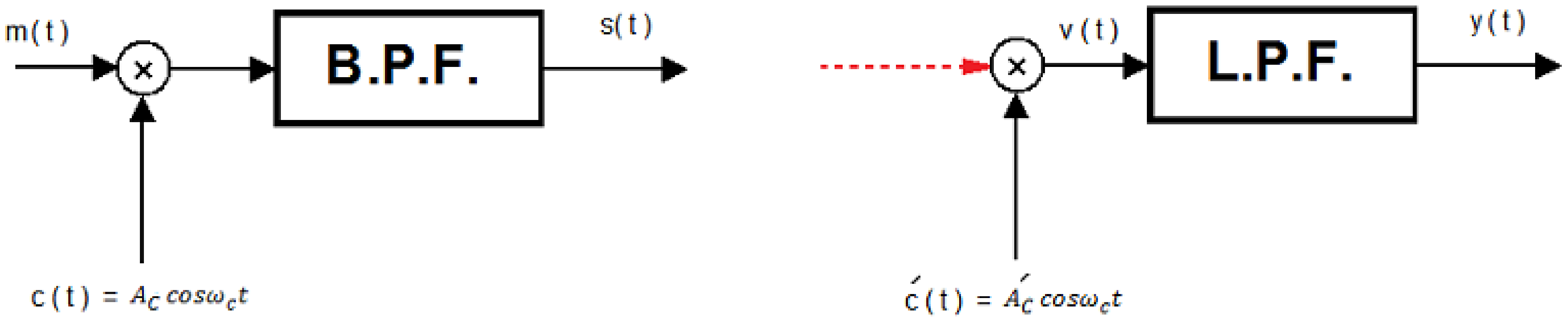
<u>Demodulation of SSB signal : Why one side band is enough ?: A frequency-domain perspective</u>




S(f)
2
1
fc
fc + ω
f
S(f - fc)
2′
1′
- ω
2fc
f
S(f + fc)
2″
1″
- 2fc - ω
2fc
f
V(f)
2″
2′
1″
1′
- ω
ω
f
Y(f) α M(f)
- ω
ω
f

Demodulation of SSB : Coherent Demodulation

a. perfect coherent



when $c(t) = A_c \cos \omega_c t$, $\acute{c}(t) = \acute{A}_c \cos \omega_c t$,

we have perfect coherence and

$$y(t) = \frac{A_c \acute{A}_c}{2} m(t)$$

b. **Constant phase difference**

The local oscillator takes the form

$\acute{c}(t) = \acute{A}_c \cos(\omega_c t + \emptyset);$

$v(t) = [A_c m(t) \cos \omega_c t - A_c \widehat{m}(t) \sin \omega_c t] \acute{A}_c \cos(\omega_c t + \emptyset)$

$= A_c \acute{A}_c m(t) \cos \omega_c t \cos(\omega_c t + \emptyset) - A_c \acute{A}_c \widehat{m}(t) \sin \omega_c t \cos(\omega_c t + \emptyset)$

$= \frac{A_c \acute{A}_c}{2} m(t) \cos(2\omega_c t + \emptyset) + \frac{A_c \acute{A}_c}{2} m(t) \cos(\emptyset)$

$- \frac{A_c \acute{A}_c}{2} \widehat{m}(t) \cos(2\omega_c t + \emptyset) - \frac{A_c \acute{A}_c}{2} \widehat{m}(t) \sin(\emptyset)$

➔ $y(t) = \frac{A_c \acute{A}_c}{2} m(t) \cos(\emptyset) - \frac{A_c \acute{A}_c}{2} \widehat{m}(t) \sin(\emptyset)$

Note that there is a distortion due to the appearance of the Hilbert transform of the message signal at the output.

c. $\acute{c}(t) = \acute{A}_c \cos\ 2\pi(f_c + \Delta f)t;$ Constant frequency shift

$v(t) = [A_c m(t) \cos \omega_c t - A_c \widehat{m}(t) \sin \omega_c t] \acute{A}_c \cos\ 2\pi(f_c + \Delta f)t$

$$= \frac{A_c \acute{A}_c}{2} m(t)[\cos(2\omega_c + \Delta\omega)t + \cos 2\pi\Delta f t]$$

$$- \frac{A_c \acute{A}_c}{2} \widehat{m}(t)[\sin(2\omega_c + \Delta\omega)t + \sin 2\pi\Delta f t]$$

➔ $$y(t) = \frac{A_c \acute{A}_c}{2} m(t) \cos 2\pi\Delta f t + \frac{A_c \acute{A}_c}{2} \widehat{m}(t) \sin 2\pi\Delta f t$$

Once again we have distortion and m(t) appears as if single sideband modulated on a carrier frequency = Δ f.

Example :

Let m(t) = $cos2\pi(1000)t$, $\Delta f =$ 100Hz and let s (t) be an upper sideband signal . Then ,

$$y(t) = \frac{A_c \acute{A}_c}{2} cos2\pi(1000)t\ cos2\pi(100)t + \frac{A_c \acute{A}_c}{2} sin2\pi(1000)t\ \ sin2\pi(100)t$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$y(t) = \cos 2\pi(900)t \neq \cos 2\pi(1000)t$$

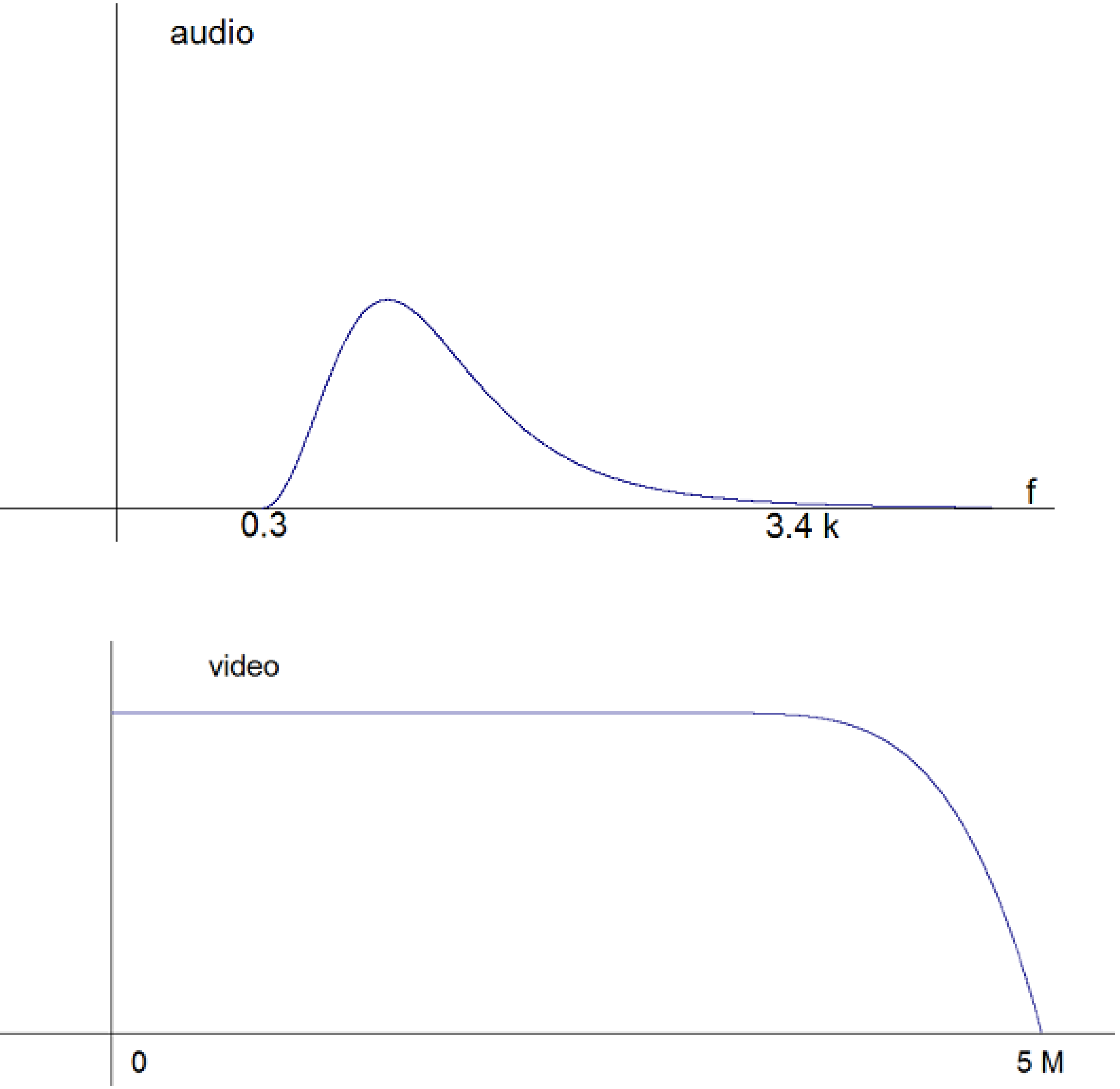
→ Distortion

So, a message component with f = 1000Hz appears as a 900Hz component at the demodulator output.

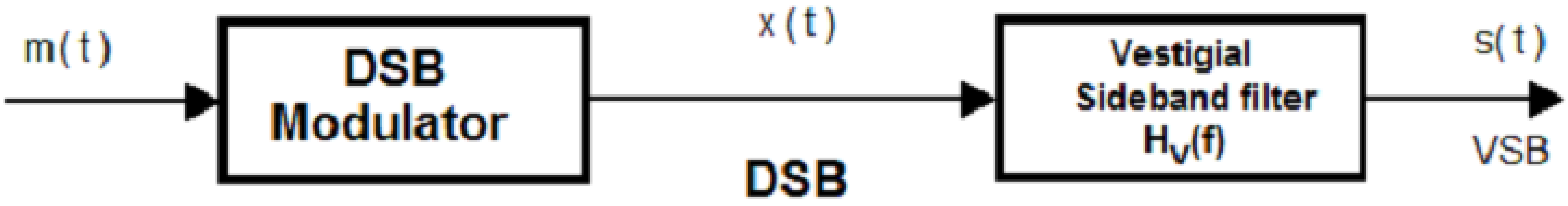
Again , distortion results as a result of failing to synchronize the transmitter and receiver carrier frequencies.

Vestigial sideband (VSB) modulation :

- This type of modulation finds applications in the transmission of video signal .
- Unlike the audio signal , the video signal is rich in low frequency components around the zero frequency .
- The B.W of a video signal is about 5MHz.
- If a video signal is to be transmitted using DSB, it requires a 10 MHz B.W ; too large .
- If a video signal is to be transmitted using SSB (B.W = 5MHz) distortion will results due to the inability to suppress one of the sidebands completely using practical filters.
- A compromise between DSB and SSB was proposed called vestigial sideband modulation .
- Here, a DSB-SC signal is first generated The DSB is applied to a band pass filter (called a *vestigial filter*) that has an asymmetrical frequency response about (-+fc).
- The filter allows one of the sidebands to pass almost without attenuation , while a trace or a vestige of the second sideband is allowed to pass (most of the second sideband is attenuated)
- A typical spectral density of an audio and a video signal is shown below.


audio
f
0.3
3.4 k
video
0
5 M

Generation of a VSB : Filtering method

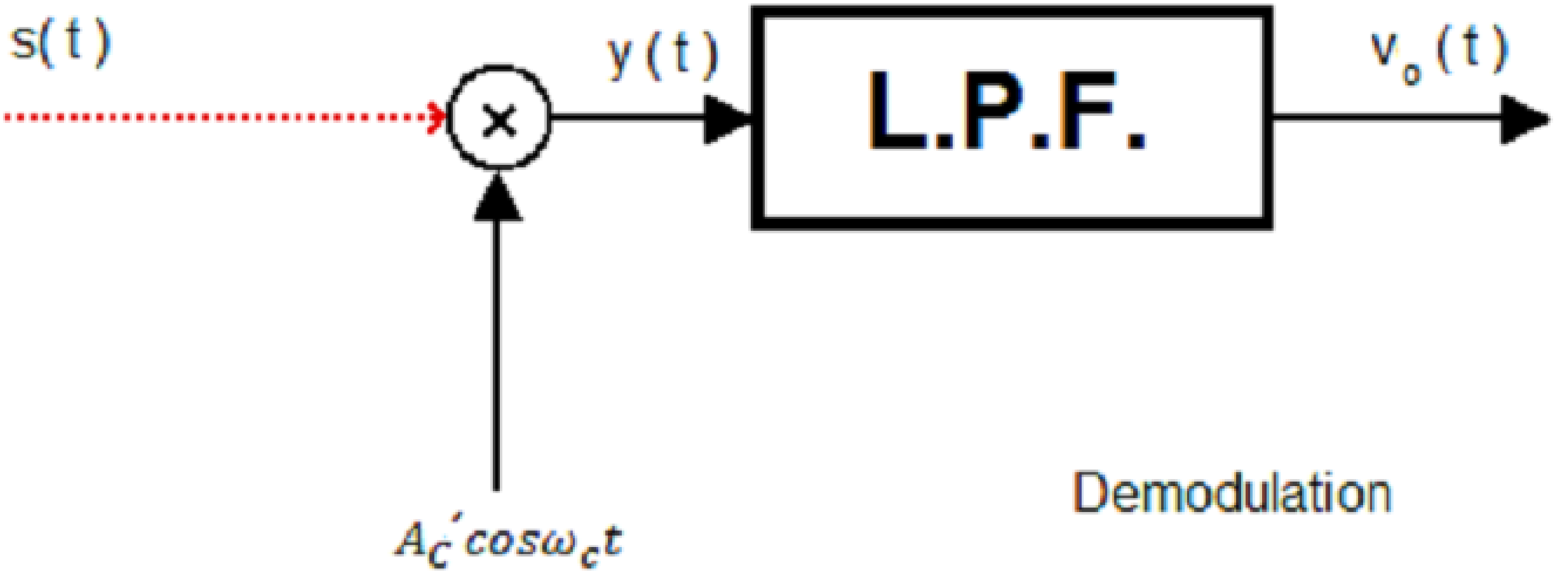


Let $H_v(f)$ be the transfer function of the vestigial filter. We need to find a condition on the characteristic of the filter such that the demodulated signal at the receiver is proportional to the message signal. Now we proceed to find such a condition.

$$x(t) = A_c m(t) \cos \omega_c t \; ; \qquad \text{A DSB-SC signal}$$

$$S(f) = X(f) H_v(f) \; ; \qquad \text{The Fourier transform of the filter output.}$$

$$= \frac{A_c}{2} \{M(f - f_c) + M(f + f_c)\} H_v(f) \; ; \qquad \text{VSB signal}$$



Demodulation

The objective is to specify a condition on $H_v(f)$ such that $V_0(t)$ is an exact replica of m(t) .

$$y(t) = \acute{A}_c s(t)\cos\omega_c t$$

$$Y(f) = \frac{\acute{A}_c}{2}\{S(f - f_c) + S(f + f_c)\}$$

$$= \frac{A_c\acute{A}_c}{4}\{M(f - 2f_c) + M(f)\}H_v(f - f_c)$$

$$+ \frac{A_c\acute{A}_c}{4}\{M(f + 2f_c) + M(f)\}H_v(f + f_c)$$

$$Vo(f) = \frac{A_c Á_c}{4}\{H_v(f - f_c) + H_v(f + f_c)\}M(f)$$

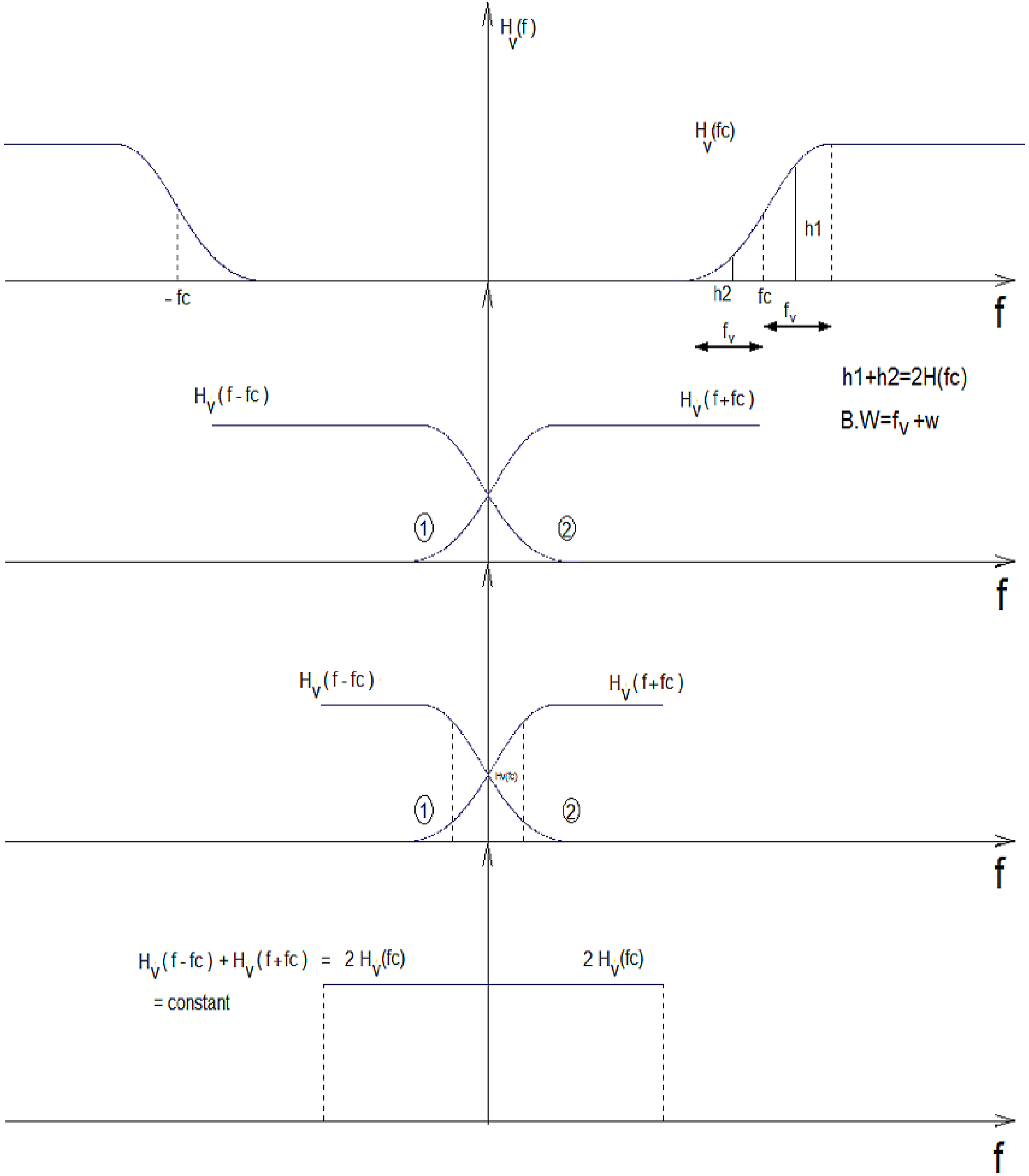
In order for $V_0(f)$ to be proportional to M(f) , we require that

$$H_v(f - f_c) + H_v(f + f_c) = constant = 2H_v(f_c)$$

When this condition is imposed on the filter, the output becomes

$$V_o(f) = \frac{A_c Á_c}{2} H_v(f_c)M(f)$$

$$v_o(t) = \frac{A_c Á_c}{2} H_v(f_c)m(t)$$



1. B.W = $W + f_v$; f_v is the size of the vestige .
2. VSB can be demodulated using coherent demodulation .

Generation of VSB: phase discrimination method

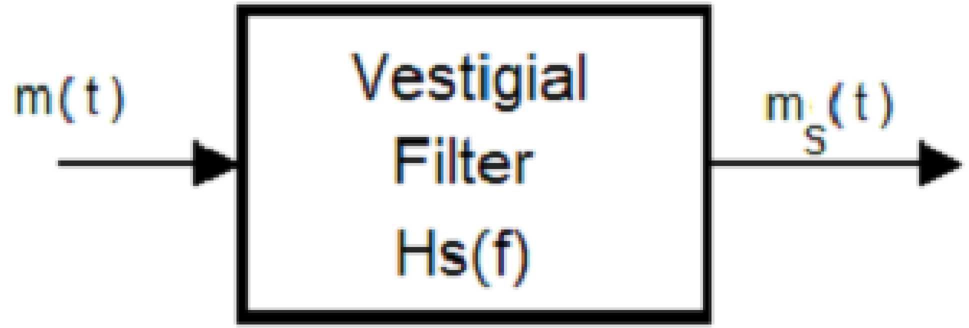
The time-domain representation of a VSB signal is

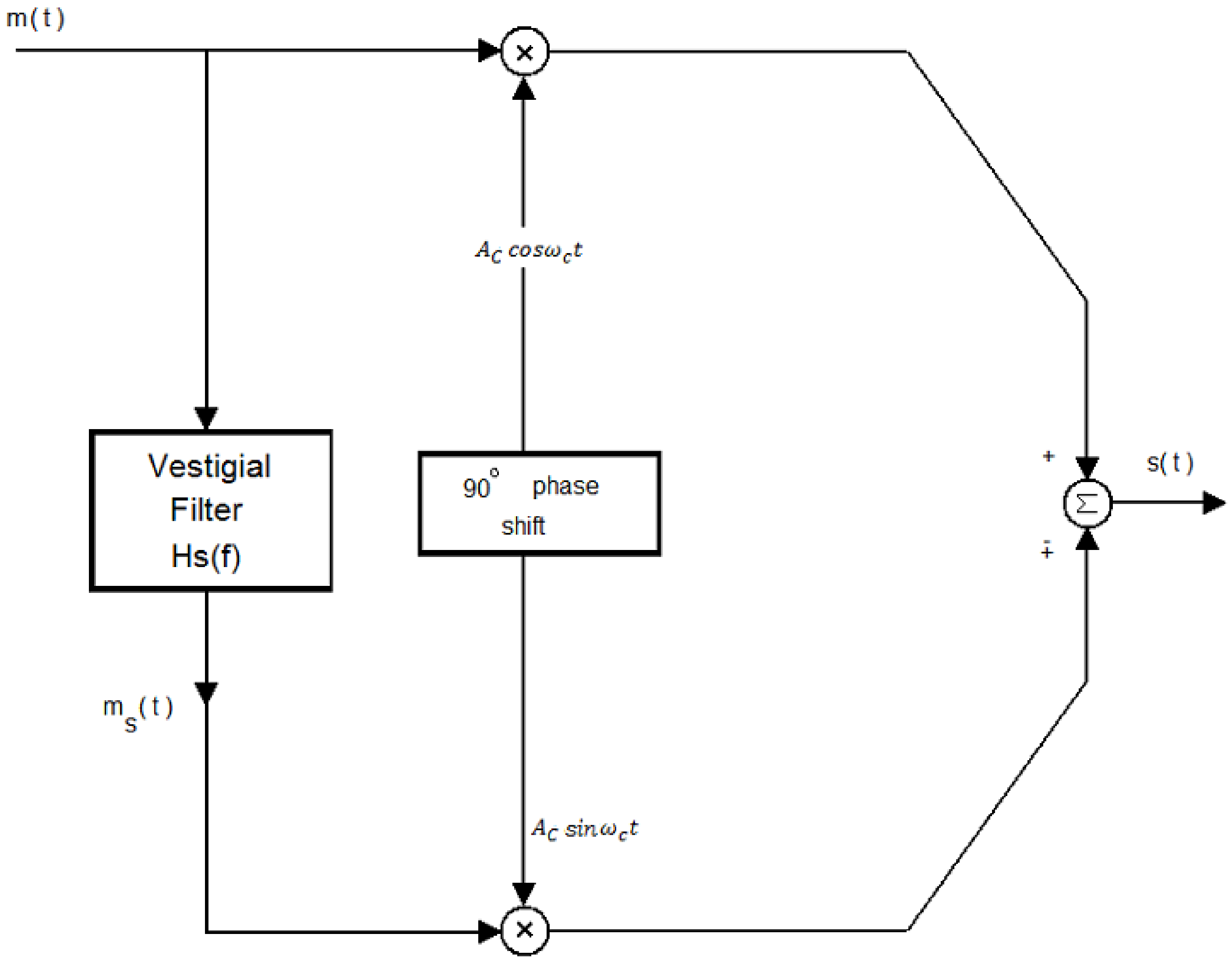
$$s(t) = A_c m(t) \cos \omega_c t \mp A_c m_s(t) \sin \omega_c t$$

Where $m_s(t)$ is the response of a vestigial filter (in the base band spectrum) to the message m(t). Using the time-domain representation ,

the following scheme may be used

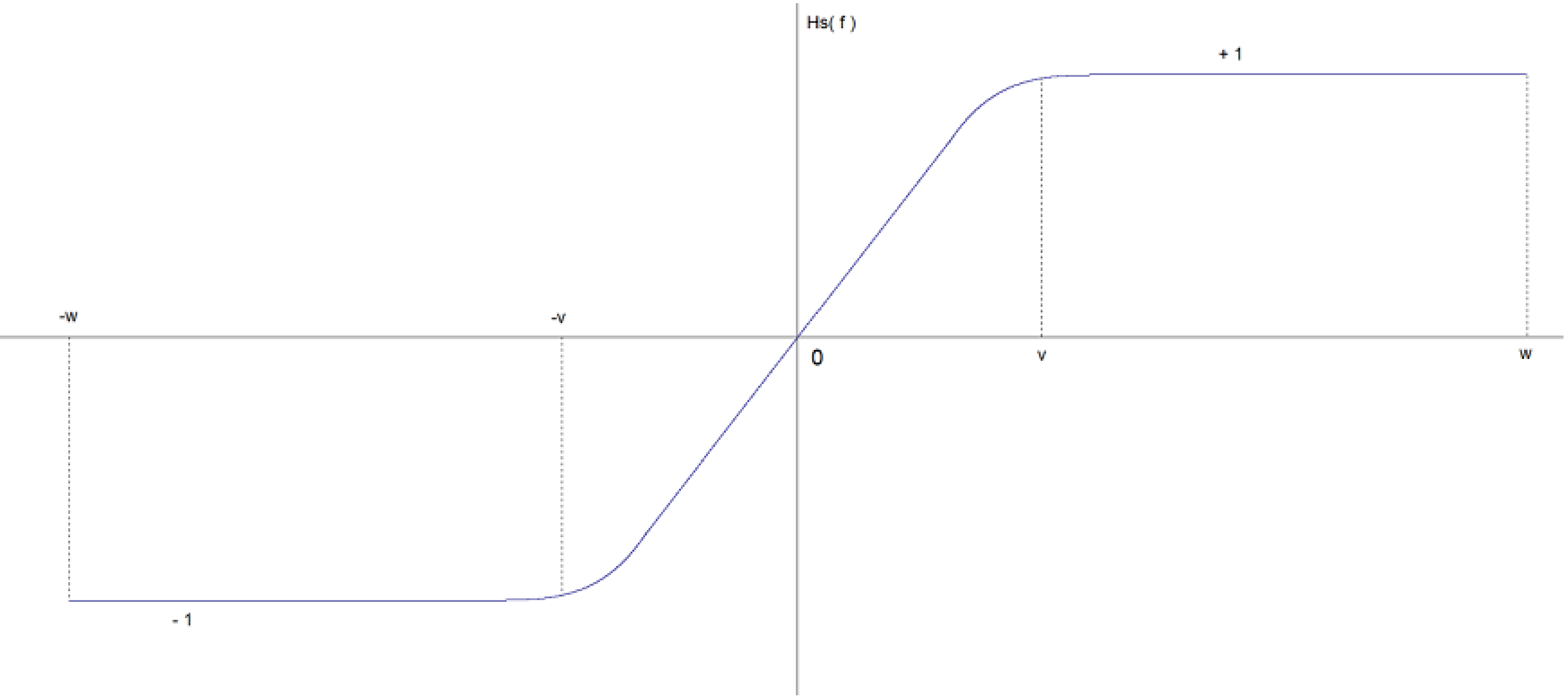
to generate a VSB signal





The – sign means that most of the upper sideband is admitted

+ sign means that most of the lower sideband is admitted

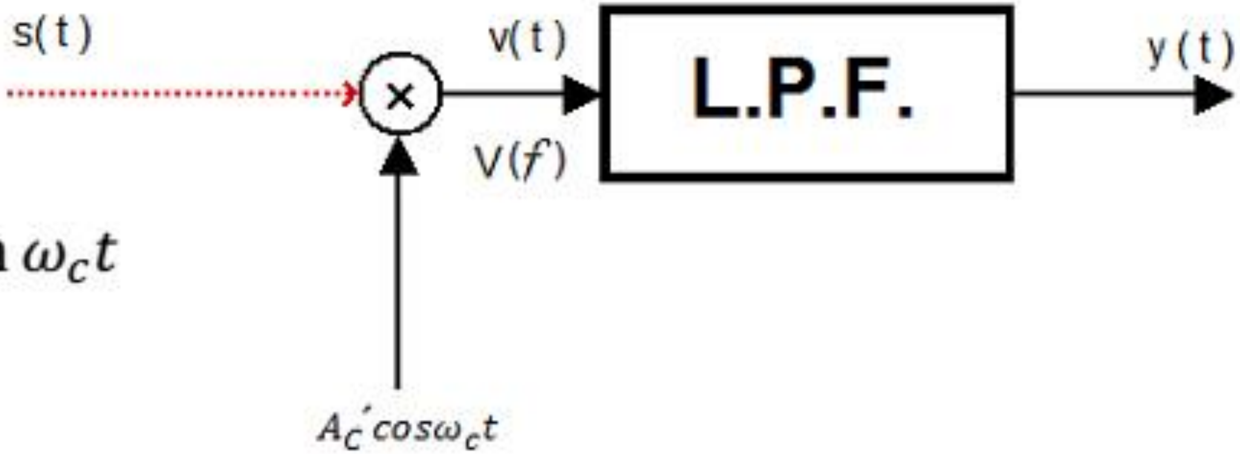


The transfer function HS(f) of the low pass filter is related to the band pass characteristic by:

$$H_s(f) = Low\ pass\ \{H_v(f + f_c) - H_v(f - f_c)\}$$

Coherent Detection of VSB : Time Domain Analysis

Let the received VSB signal be given as:



$$s(t) = A_c m(t)\cos\omega_c t - A_c m_s(t)\sin\omega_c t$$

This signal is mixed with a version of the

transmitted carrier of the same phase and frequency.

$$v(t) = s(t)\acute{A}_c \cos 2\pi f_c t$$

$$= A_c\acute{A}_c[m(t)\cos\omega_c t - m_s(t)\sin\omega_c t]\cos\omega_c t$$

$$= A_c\acute{A}_c m(t)\cos^2\omega_c t - A_c\acute{A}_c m_s(t)\sin\omega_c t\cos\omega_c t$$

$$= \frac{A_c\acute{A}_c}{2}m(t) + \frac{A_c\acute{A}_c}{2}m(t)\cos 2\omega_c t - \frac{A_c\acute{A}_c}{2}m_s(t)\sin 2\omega_c t$$

The low pass filter admits only the low pass component, which is nothing but a scaled version of the message signal.

$$y(t) = \frac{A_c\acute{A}_c}{2}m(t)$$

Envelope Detection of VSB + Carrier :

This type of modulation takes the form :

$$s(t) = carrier + VSB$$

$$s(t) = A_c \cos \omega_c t + A_c \beta m(t) \cos \omega_c t \mp A_c \beta m_s(t) \sin \omega_c t$$

β is a scaling factor chosen to minimize envelope distortion. The addition of the carrier is meant to simplify the demodulation of the video signal in practical TV systems and avoids the complexity of coherent demodulation. It is also less expensive since a simple envelope detector, of the type described in demodulating a normal AM signal, can be used.

$$s(t) = \left(A_c + A_c\beta m(t)\right)\cos\omega_c t \mp A_c\beta m_s(t)\sin\omega_c t$$

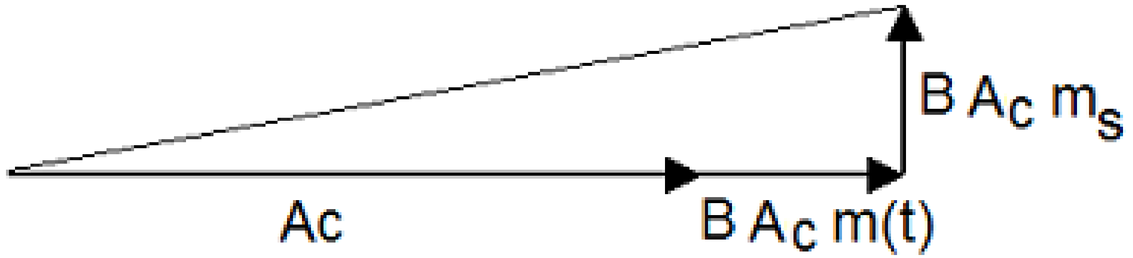
$$s(t) = \sqrt{\left(A_c + A_c\beta m(t)\right)^2 + {A_c}^2\beta^2 {m_s}^2(t)}\;\cos(\omega_c t + \emptyset)$$

If s(t) is applied to an envelope detector (which is insensitive to phase variations), the output is

$$y(t) = \sqrt{\left(A_c(1 + \beta m(t)\right)^2 + {A_c}^2\beta^2 {m_s}^2(t)}$$

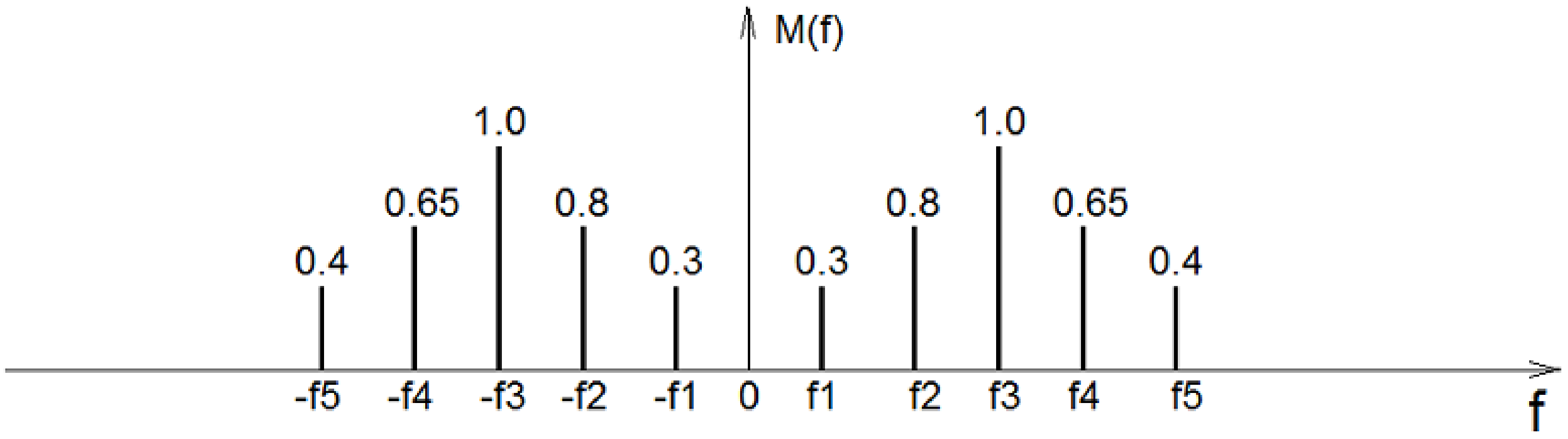
If $A_c \gg \beta m(t)$, then

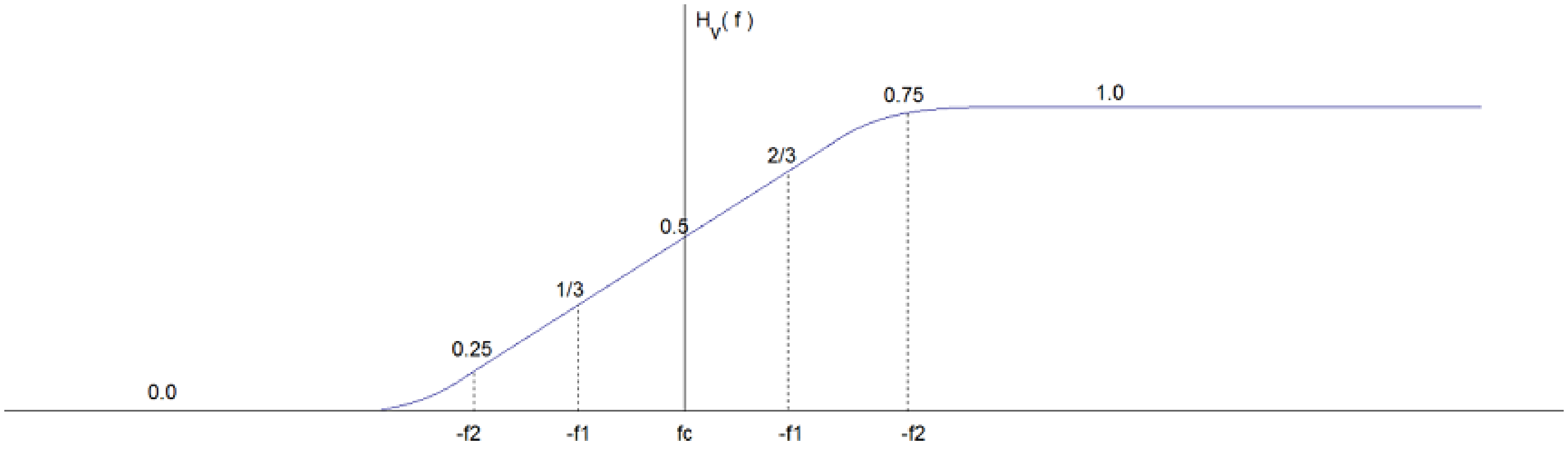
$$y(t) \cong A_c(1 + \beta m(t))$$

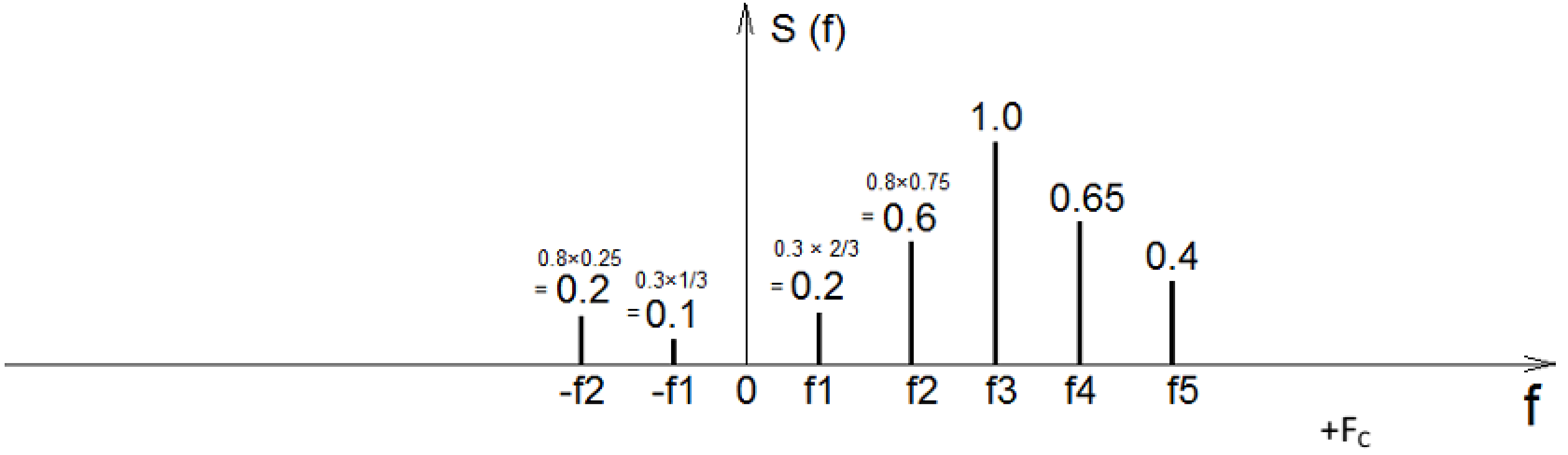


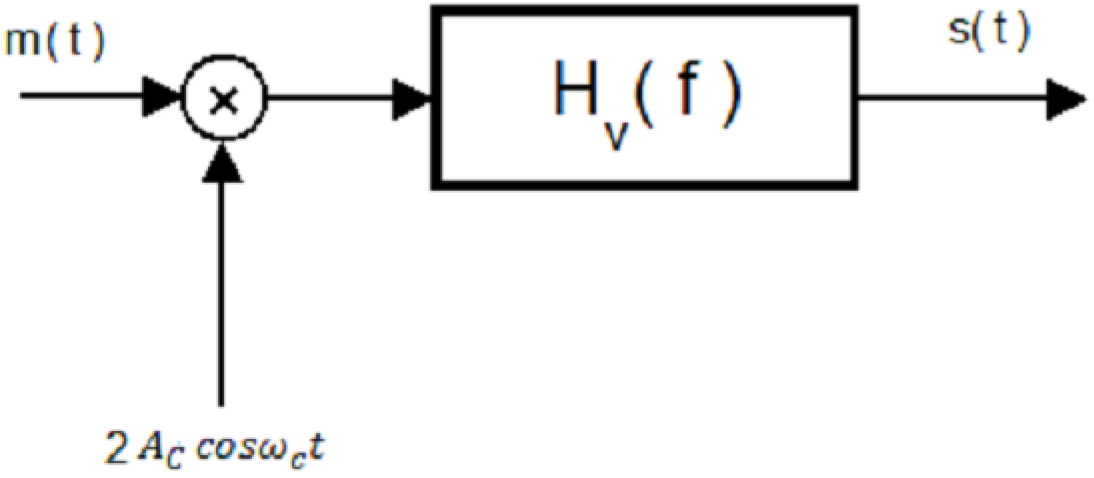
Hence , m(t) can be demodulated, almost without distortion, using simple envelope detection techniques if the above condition is satisfied .

Example: A VSB is generated from the DSB-SC signal $2m(t)\cos\omega_c t$. M(f) and $H_v(f)$ are shown below . Find the spectrum of the transmitted signal s(t).






S (f)
1.0
0.8×0.75
= 0.6
0.65
0.4
0.8×0.25
= 0.2
0.3×1/3
= 0.1
0.3 × 2/3
= 0.2
-f2
-f1
0
f1
f2
f3
f4
f5
+Fc
f


m(t)
×
Hv(f)
s(t)
2 Ac cosωct

Baseband Signal :

The input signal consists of five frequency components. It is represented as:

$$m(t) = 0.6\cos 2\pi f_1 t + 1.6\cos 2\pi f_2 t$$

$$+2\cos 2\pi f_3 t + 1.3\cos 2\pi f_4 t + 0.8\cos 2\pi f_5 t$$

Transmitted signal :

The spectrum of the transmitted signal is:

$$S(f) = H_v(f)M(f - f_c) + H_v(f)M(f + f_c)$$

If we perform the multiplication in the frequency domain and take the inverse Fourier transform, we get the time domain representation of the transmitted signal.

$$s(t) = 0.4\cos 2\pi(f_c - f_2)t + 0.2\cos 2\pi(f_c + f_1)t$$

$$+0.4\cos 2\pi(f_c + f_1)t + 1.2\cos 2\pi(f_c + f_2)t + 2\cos 2\pi(f_c + f_3)t$$

$$+ 1.3\cos 2\pi(f_c + f_4)t + 0.8\cos 2\pi(f_c + f_5)t$$