Introduction to Analog And Digital Communications

Second Edition

Simon Haykin, Michael Moher

Chapter 3 Amplitude Modulation

- Modulation
  - The process by which some characteristic of a carrier wave is varied in accordance with an information-bearing signal.
  - Continuous-wave modulation
    - *Amplitude modulation*
    - *Frequency modulation*
- AM modulation family
  - Amplitude modulation (AM)
  - Double sideband-suppressed carrier (DSB-SC)
  - Single sideband (SSB)
  - Vestigial sideband (VSB)

- Lesson 1 : Fourier analysis provides a powerful mathematical tool for developing mathematical as well as physical insight into the spectral characterization of linear modulation strategies
- Lesson 2 : The implementation of analog communication is significantly simplified by using AM, at the expense of transmitted power and channel bandwidth
- Lesson 3 : The utilization of transmitted power and channel bandwidth is improved through well-defined modifications of an amplitude-modulated wave’s spectral content at the expense of increased system complexity.

There is no free lunch in designing a communication system: for every gain that is made, there is a price to be paid.

- Theory
  - A sinusoidal carrier wave

$$c(t) = A_c \cos(2\pi f_c t) \quad (3.1)$$

  - An amplitude-modulated wave

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t) \quad (3.2)$$

  - The envelope of s(t) has essentially the same shape as the message signal m(t) provided that two conditions are satisfied :
    - *The amplitude of* $k_a m(t)$ *is always less than unity*

$$|k_a m(t)| < 1, \quad \text{for all } t \quad (3.3)$$

    - *The carrier frequency* f_c *is much greater than the highest frequency component W of the message signal*

$$f_c >> W \quad (3.4)$$

  - Envelope detector
    - *A device whose output traces the envelope of the AM wave acting as the input signal*

Fig. 3.1

Fig.3.1

Back
Next

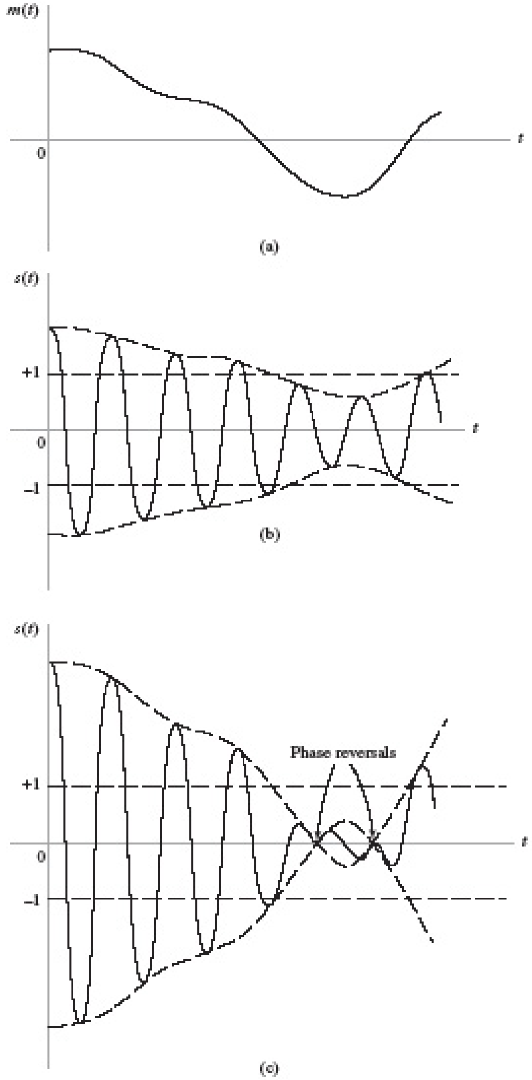


FIGURE 3.1 Illustration of the amplitude modulation process. (*a*) Message signal $m(t)$. (*b*) AM wave for $k_a m(t) < 1$ for all t. (*c*) AM wave for $|k_a m(t)| > 1$ for some t.

- The Fourier transform or spectrum of the AM wave *s(t)*

$$S(f)=\frac{A_c}{2}[\delta(f-f_c)+\delta(f+f_c)]+\frac{k_a A_c}{2}[M(f-f_c)+M(f+f_c)] \quad (3.5)$$

$$\cos(2\pi f_c t)=\frac{1}{2}[\exp(j2\pi f_c t)+\exp(-j2\pi f_c t)]$$

$$\exp(j2\pi f_c t)\Leftrightarrow \delta(f-f_c)$$

$$m(t)\exp(j2\pi f_c t)\Leftrightarrow M(f-f_c)$$

- From the spectrum of Fig. 3.2(b)
  1. *As a result of the modulation process, the spectrum of the message signal* m(t) *for negative frequencies extending from –W to 0 becomes completely visible for positive frequencies, provided that the carrier frequency satisfies the condition* $f_c>W$ *; wherein lies the importance of the idea of "negative" frequencies, which was emphasized in chapter 2.*
  2. *For positive frequencies, the portion of the spectrum of an AM wave lying above the carrier frequency* f_c *is referred to as the upper sideband, whereas the symmetric portion below* f_c *is referred to as the lower sideband.*

Fig. 3.2

3. *For positive frequencies, the highest frequency component of the AM wave equals* f_c+W*, and the lowest frequency component equals* f_c-W*. The difference between these two frequencies defines the transmission bandwidth* B_T *of the AM wave, which is exactly twice the message bandwidth* W*;*

$$B_T = 2W \quad (3.6)$$

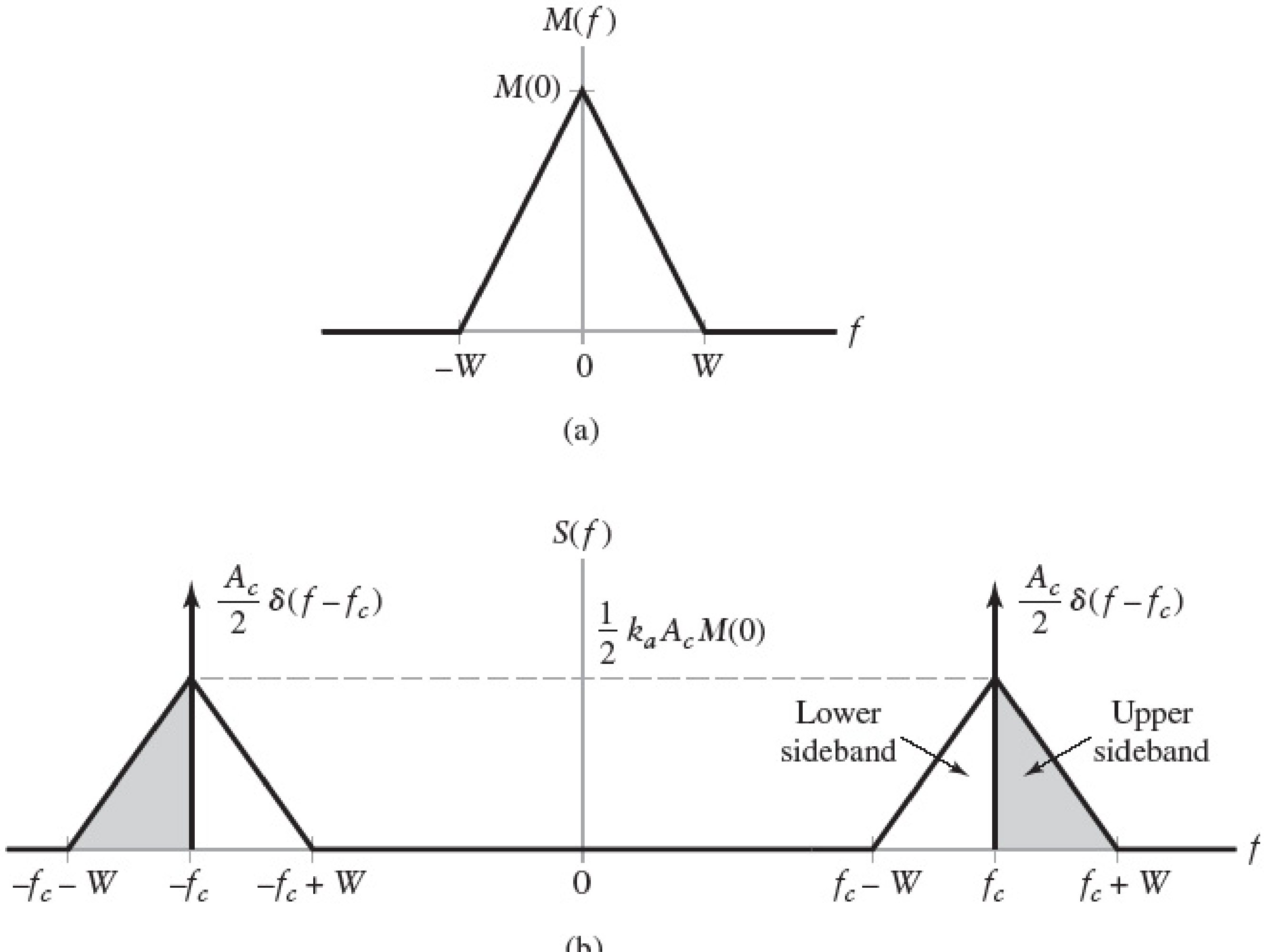


FIGURE 3.2 (*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of AM wave $s(t)$.

EXAMPLE 3.1 Single-Tone Modulation

Consider a modulating wave $m(t)$ that consists of a single tone or frequency component; that is,

$$m(t) = A_m \cos(2\pi f_m t)$$

where A_m is the amplitude of the sinusoidal modulating wave and f_m is its frequency (see Fig. 3.3(*a*)). The sinusoidal carrier wave has amplitude A_c and frequency f_c (see Fig. 3.3(*b*)). The corresponding AM wave is therefore given by

$$s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t) \tag{3.7}$$

where

$$\mu = k_a A_m$$

The dimensionless constant μ is called the *modulation factor*, or the *percentage modulation* when it is expressed numerically as a percentage. To avoid envelope distortion due to overmodulation, the modulation factor μ must be kept below unity, as explained previously.

Figure 3.3(c) shows a sketch of $s(t)$ for μ less than unity. Let A_{max} and A_{min} denote the maximum and minimum values of the envelope of the modulated wave, respectively. Then, from Eq. (3.7) we get

$$\frac{A_{max}}{A_{min}} = \frac{A_c(1 + \mu)}{A_c(1 - \mu)}$$

Rearranging this equation, we may express the modulation factor as

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

Expressing the product of the two cosines in Eq. (3.7) as the sum of two sinusoidal waves, one having frequency $f_c + f_m$ and the other having frequency $f_c - f_m$, we get

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2}\mu A_c \cos[2\pi(f_c - f_m)t]$$

The Fourier transform of $s(t)$ is therefore

$$\begin{aligned} S(f) = &\frac{1}{2}A_c[\delta(f - f_c) + \delta(f + f_c)] \\ &+ \frac{1}{4}\mu A_c[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)] \\ &+ \frac{1}{4}\mu A_c[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)] \end{aligned}$$

Fig. 3.3

Thus the spectrum of an AM wave, for the special case of sinusoidal modulation, consists of delta functions at $\pm f_c$, $f_c \pm f_m$, and $-f_c \pm f_m$, as shown in Fig. 3.3(c).

Fig.3.3

Back Next

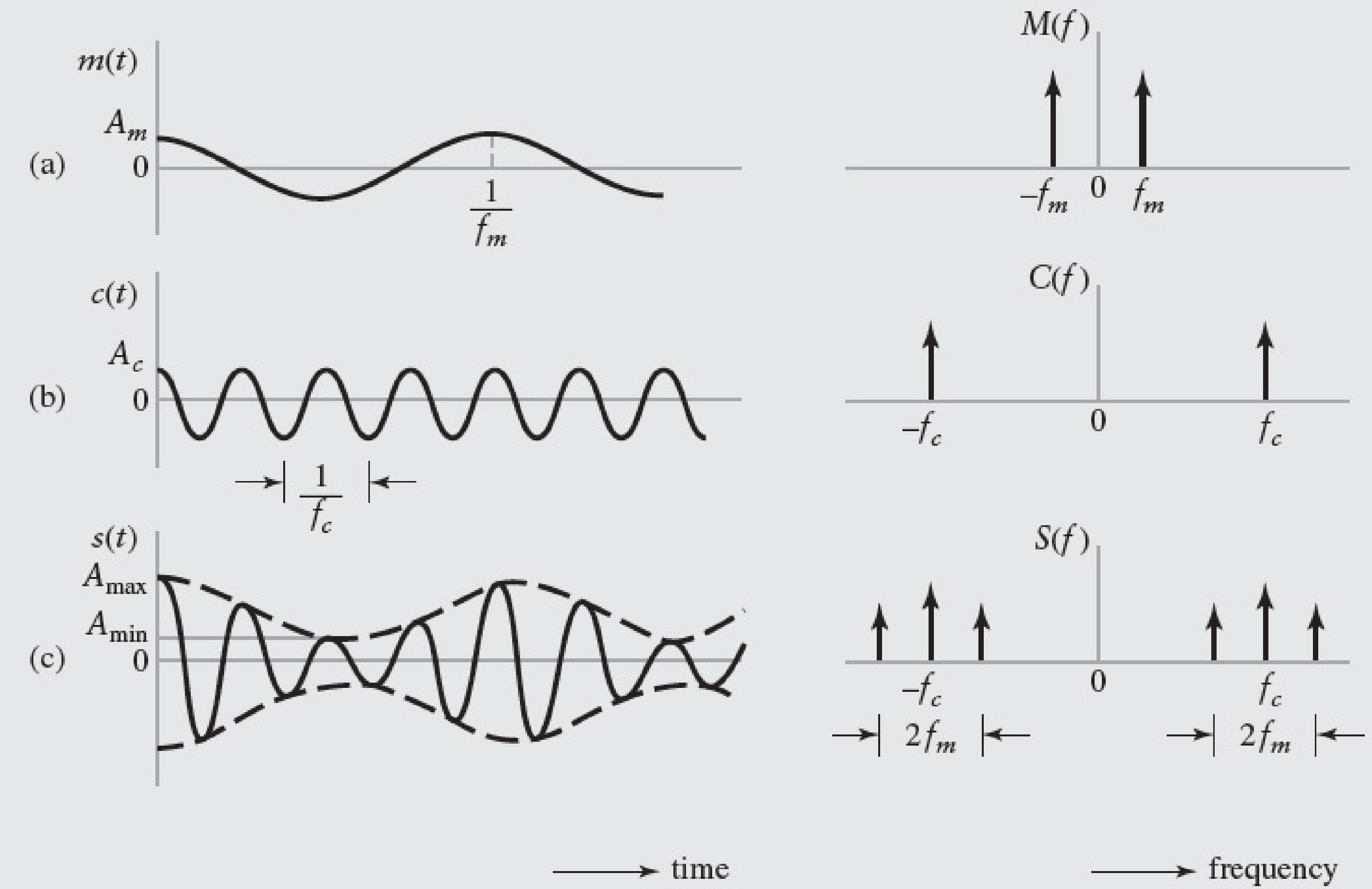


FIGURE 3.3 Illustration of the time-domain (on the left) and frequency-domain (on the right) characteristics of amplitude modulation produced by a single tone. (*a*) Modulating wave. (*b*) Carrier wave. (*c*) AM wave.

- The average power delivered to a 1-ohm resistor by s(t) is comprised of three components

$$\text{Carrier power} = \frac{1}{2} A_c^2$$

$$\text{Upper side-frequency power} = \frac{1}{8} \mu^2 A_c^2$$

$$\text{Lower side-frequency power} = \frac{1}{8} \mu^2 A_c^2$$

- Figure 3.4 shows the percentage of total power in both side frequencies and in the carrier plotted versus the percentage modulation.
- Notice that when the percentage modulation is less than 20 percent, the power in one side frequency is less than 1 percent of the total power in the AM wave.

Fig. 3.4

Fig.3.4

Back Next

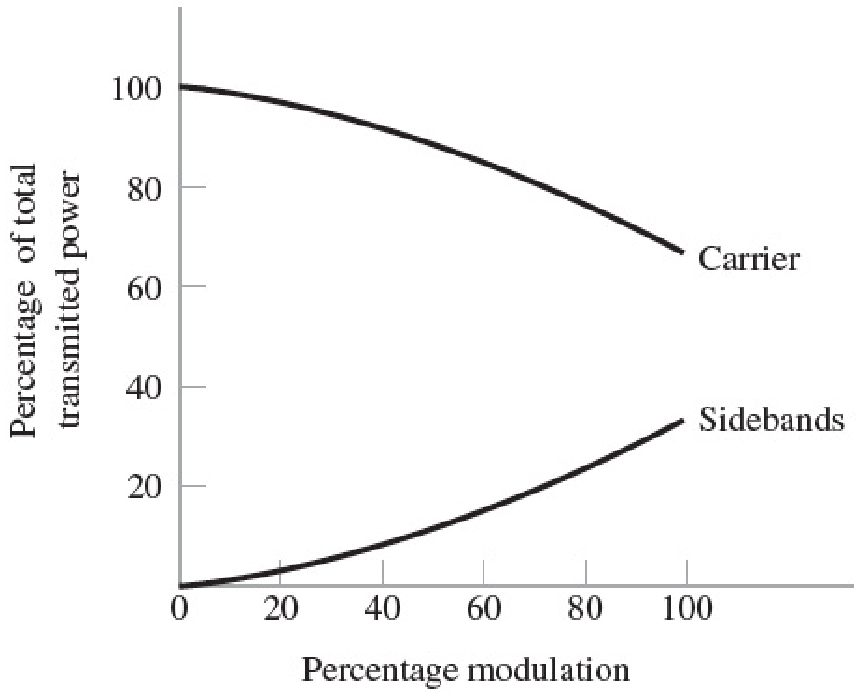


FIGURE 3.4 Variations of carrier power and total sideband power with percentage modulation in amplitude modulation.

- Computation experiment : AM
  - We will study sinusoidal modulation based on the following parameters

$$\text{Carrier amplitude,} \quad A_c = 1$$
$$\text{Carrier frequency,} \quad f_c = 0.4Hz$$
$$\text{Modulation frequency,} \quad f_m = 0.05Hz$$

  - It is recommended that the number of frequency samples satisfies the condition

$$M \geq \frac{f_s}{f_r} = \frac{10}{0.005} = 2000$$

  - The modulation factor μ

$$\mu = 0.5, \text{corresponding to undermodulation}$$
$$\mu = 1.0, \text{corresponding to 100 percent modulation}$$
$$\mu = 2.0, \text{corresponding to overmodulation}$$

- Modulation factor $\mu=0.5$
  - The lower side frequency, the carrier, and the upper side frequency are located at $(f_c-f_m)=\pm0.35$ Hz, $f_c=\pm0.4$ Hz, and $(f_c+f_m)=\pm0.45$ Hz.
  - The amplitude of both side frequencies is $(\mu/2)=0.25$ times the amplitude of the carrier

Fig. 3.5

- Modulation factor $\mu=1.0$

Fig. 3.6

- Modulation factor $\mu=2.0$

Fig. 3.7

Fig.3.5

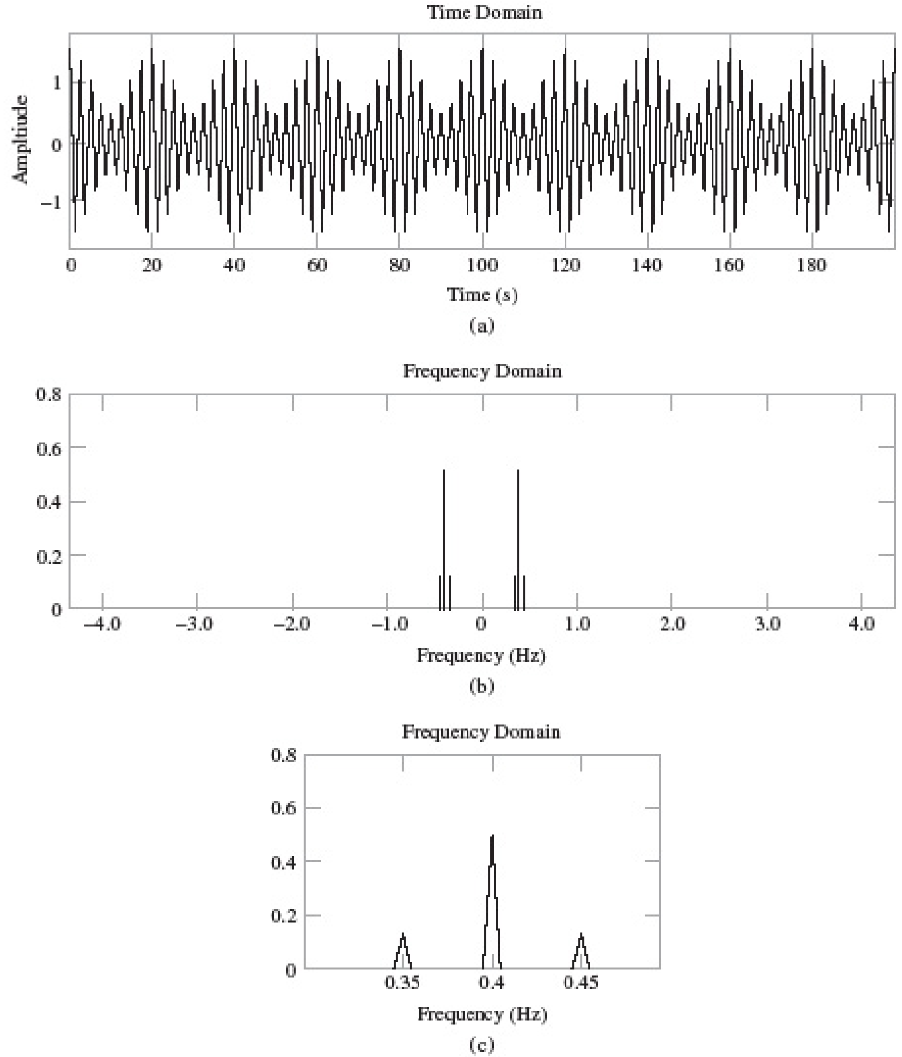


FIGURE 3.5 Amplitude modulation with 50 percent modulation: (*a*) AM wave, (*b*) magnitude spectrum of the AM wave, and (*c*) expanded spectrum around the carrier frequency.

Back Next

Fig.3.6

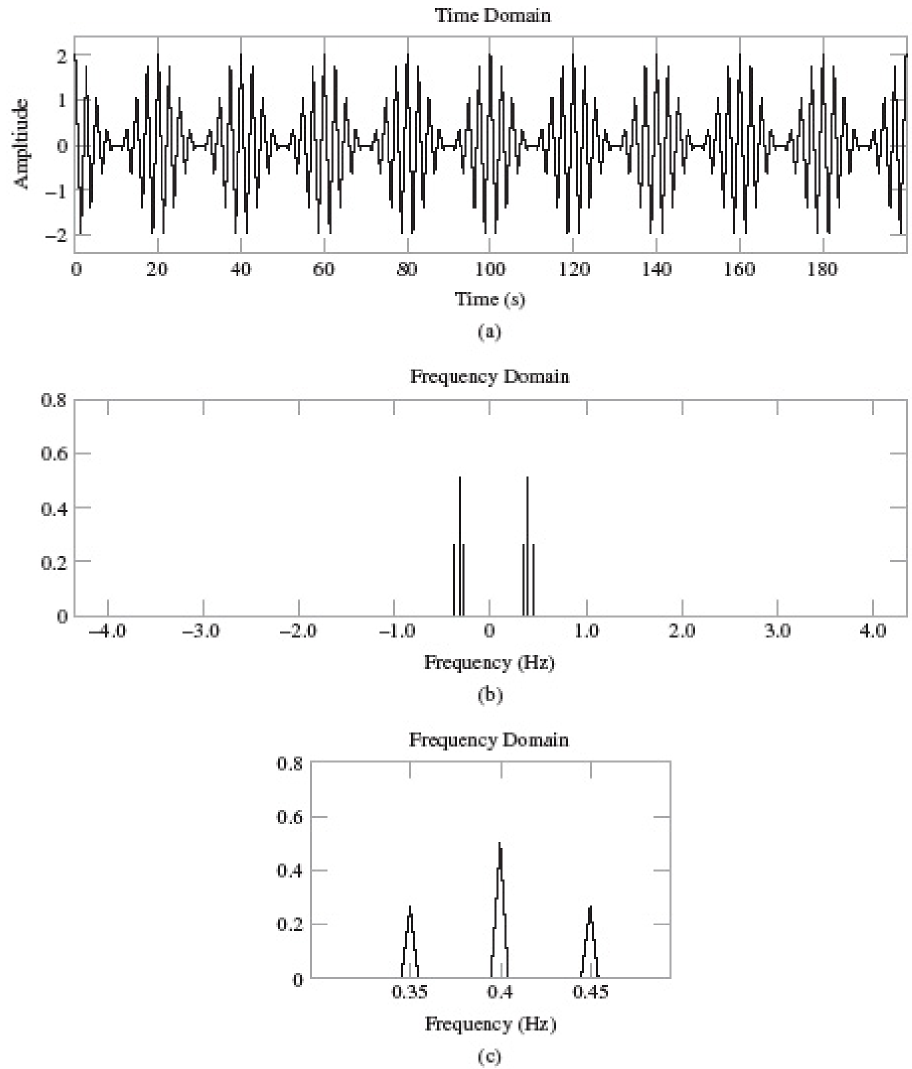


FIGURE 3.6 Amplitude modulation with 100 percent modulation: (*a*) AM wave, (*b*) magnitude spectrum of the AM wave, and (*c*) expanded spectrum around the carrier frequency.

Back Next

Fig.3.7

Back | Next

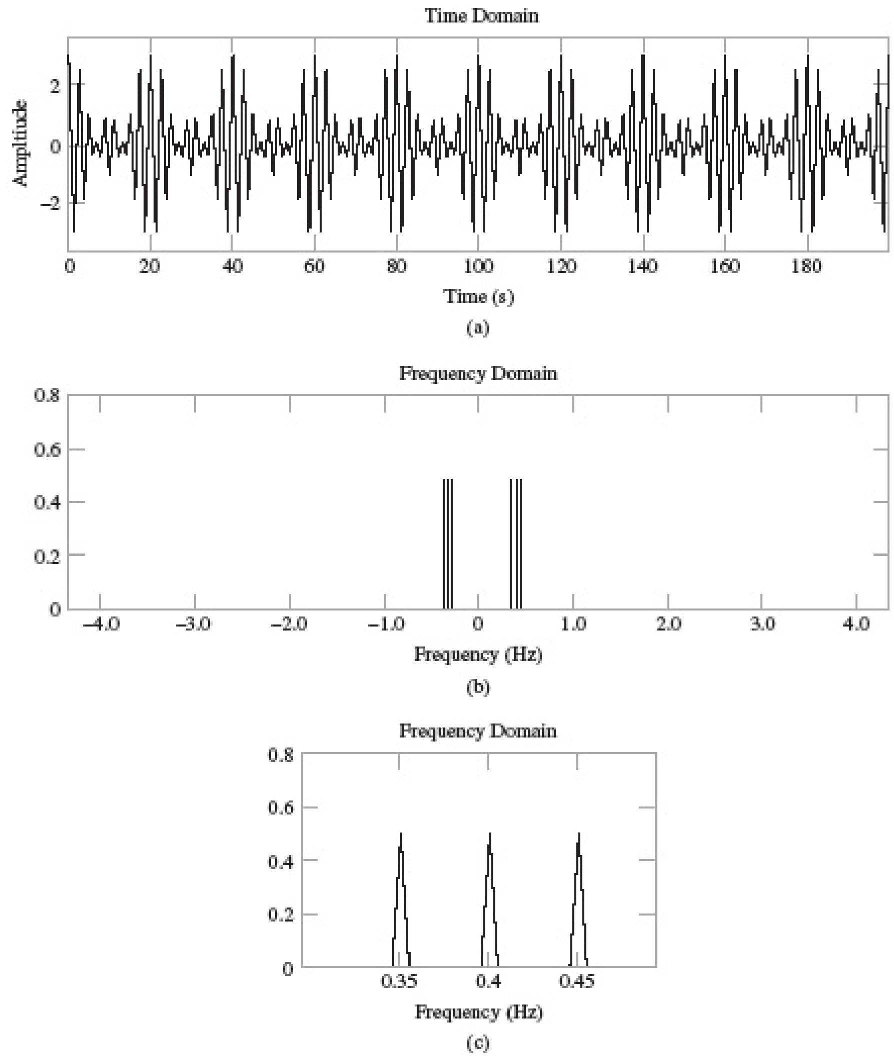


Figure 3.7 Amplitude modulation with 200 percent modulation: (*a*) AM wave, (*b*) magnitude spectrum of the AM wave, and (*c*) expanded spectrum around the carrier frequency.

- Enveloping detection
  - Enveloping detector
    - *The AM wave is narrowband, which means that the carrier frequency is large compared to the message bandwidth*
    - *The percentage modulation in the AM wave is less than 100 percent*
    - *So that the capacitor C charges rapidly and thereby follows the applied voltage up to the positive peak when the diodes is conducting .*

$$(r_f + R_s)C \ll \frac{1}{f_c}$$

$$\frac{1}{f_c} \ll R_l C \ll \frac{1}{W}$$

- Computer experiment :
  - Envelope detection for sinusoidal AM
    - *The envelope detector output is shown in Fig. 3.9(c).*
    - *The numerical values used in the computation of Fig. 3.9(c)*

Source resistance	$R_s = 75\ \Omega$
Forwared resistance	$r_f = 25\ \Omega$
Load resistance	$R_l = 10\ k\Omega$
Capacitance	$C = 0.01\ \mu F$
Message bandwidth	$W = 1\ kHz$
Carrier frequency	$f_c = 20\ kHz$

Fig. 3.9

Fig.3.9

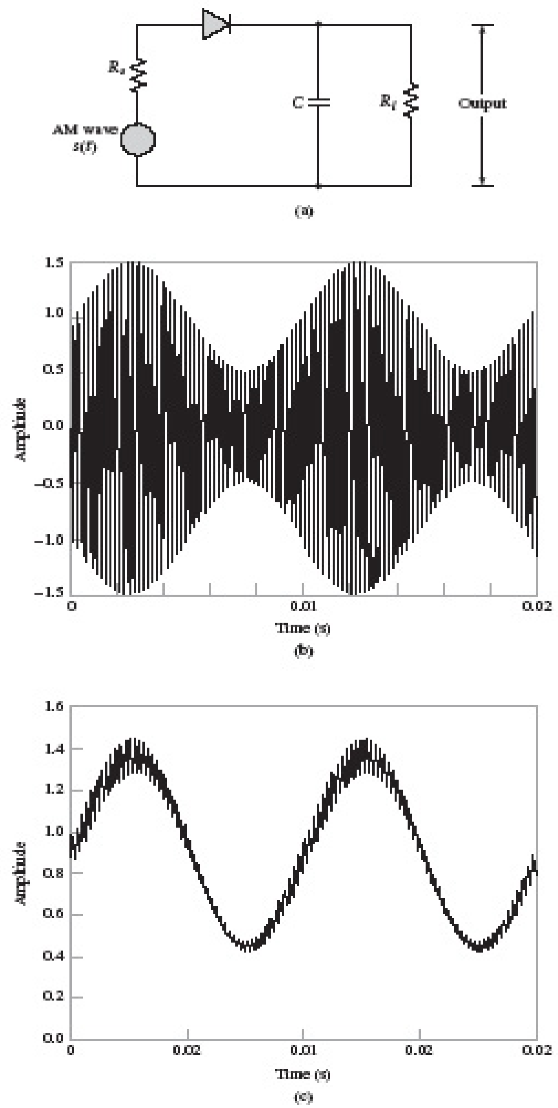


FIGURE 3.9 Envelope detector. (*a*) Circuit diagram. (*b*) AM wave input. (*c*) Envelope detector output

3.2 Virtues, Limitations, and Modifications of Amplitude Modulation

- Practical Limitation
  - Amplitude modulation is wasteful of transmitted power
    - *The transmission of the carrier wave therefore represents a waste of power*
  - Amplitude modulation is wasteful of channel bandwidth
    - *Insofar as the transmission of information is concerned, only one sideband is necessary, and the communication channel therefore needs to provide only the same bandwidth as the message signal.*
    - *It requires a transmission bandwidth equal to twice the message bandwidth*

- Three modifications of amplitude modulation
  - Double sideband-suppressed carrier (DSB-SC) modulation
    - *The transmitted wave consists of only the upper and lower sidebands*
    - *But the channel bandwidth requirement is the same as before*
  - Single sideband (SSB) modulation
    - *The modulated wave consists only of the upper sideband or the lower sideband*
    - *To translate the spectrum of the modulating signal to a new location in the frequency domain.*

- Vestigial sideband (VSB) modulation
  - *One sideband is passed almost completely and just a trace, of the other sideband is retained.*
  - *The required channel bandwidth is slightly in excess of the message bandwidth by an amount equal to the width of the vestigial sideband.*

- Theory
  - DSB-SC (product modulation) consists of the product of the message signal and the carrier wave,

$$s(t) = c(t)m(t)$$
$$= A_c \cos(2\pi f_c t)m(t) \quad (3.8)$$

  - Fourier transform of *s(t)*

$$S(f) = \frac{1}{2} A_c [M(f - f_c) + M(f + f_c)] \quad (3.9)$$

    - Its only advantage is saving transmitted power, which is important enough when the available transmitted power is at a premium

Fig. 3.10

Fig. 3.11

Fig.3.10

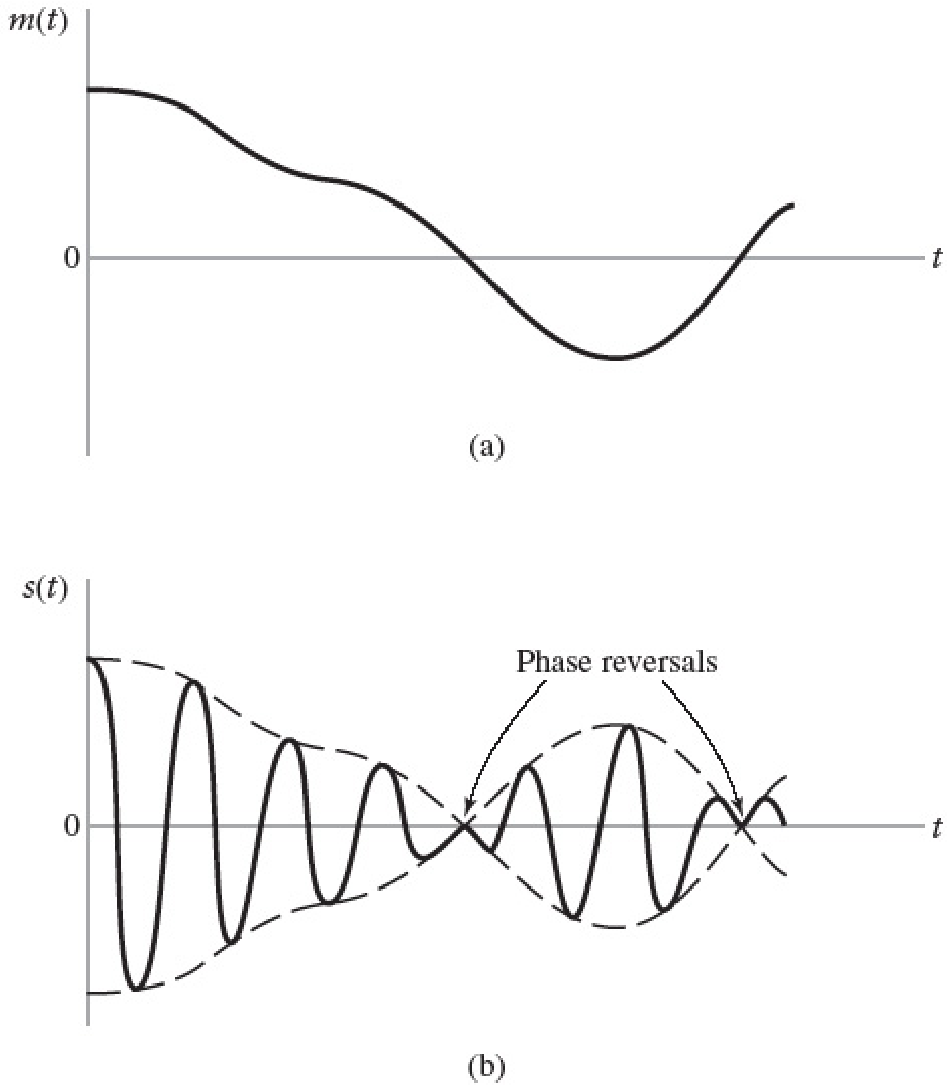


FIGURE 3.10 (*a*) Message signal $m(t)$. (*b*) DSB-SC modulated wave $s(t)$.

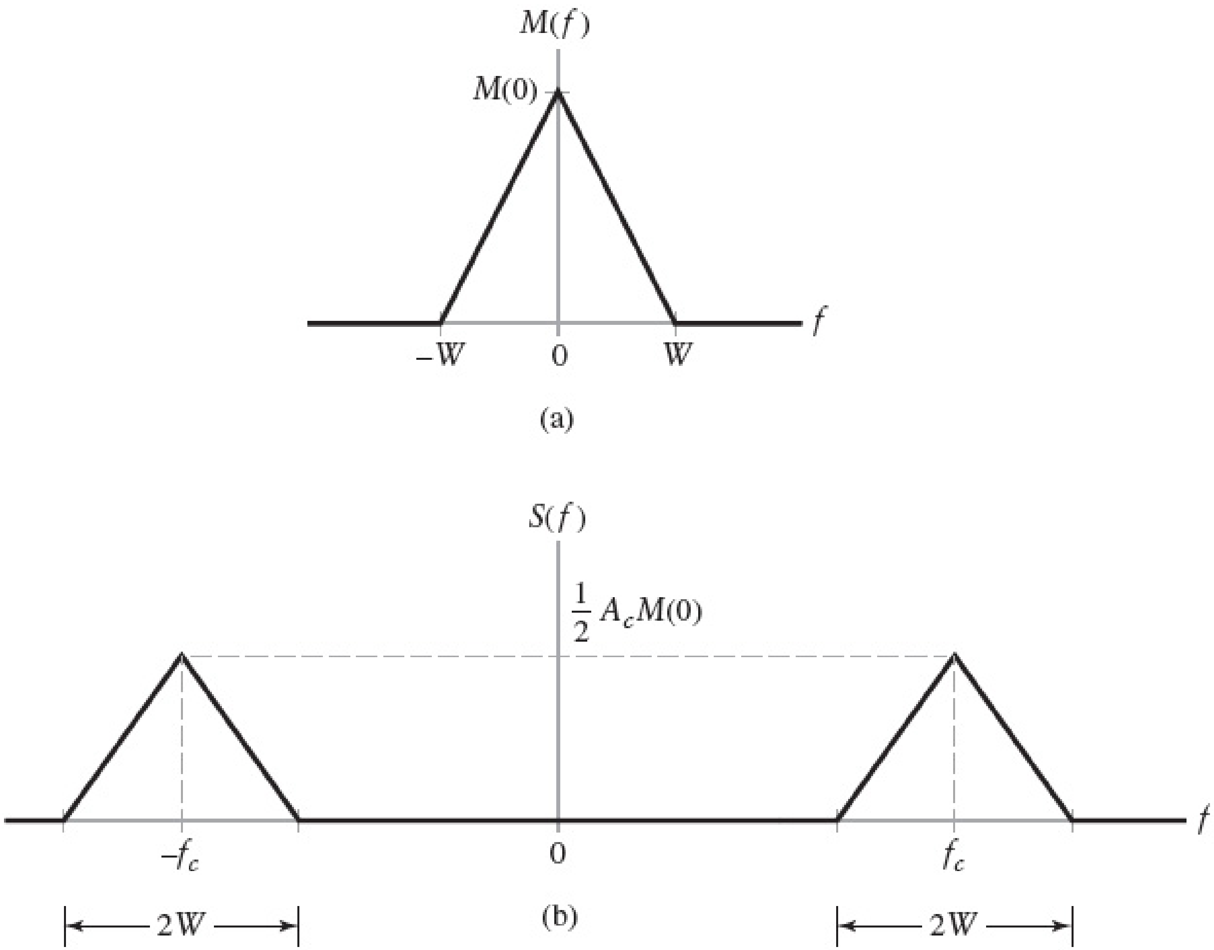


FIGURE 3.11 (*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of DSB-SC modulated wave $s(t)$.

Example 3.2 Sinusoidal DSB-SC spectrum

Consider DSB-SC modulation using a sinusoidal modulating wave of amplitude A_m and frequency f_m and operating on a carrier of amplitude A_c and frequency f_c. The message spectrum is

$$M(f) = \frac{1}{2}A_m\delta(f - f_m) + \frac{1}{2}A_m\delta(f + f_m)$$

Invoking Eq. (3.9), the shifted spectrum $\frac{1}{2}A_cM(f - f_c)$ defines the two side-frequencies for positive frequencies:

$$\frac{1}{4}A_cA_m\delta(f - (f_c + f_m)); \qquad \frac{1}{4}A_cA_m\delta(f - (f_c - f_m))$$

The other shifted spectrum of Eq. (3.9)—namely, $\frac{1}{2}A_cM(f + f_c)$,—defines the remaining two side-frequencies for negative frequencies:

$$\frac{1}{4}A_cA_m\delta(f + (f_c - f_m)); \qquad \frac{1}{4}A_cA_m\delta(f + (f_c + f_m))$$

which are the *images* of the first two side-frequencies with respect to the origin, in reverse order.

- Coherent detection (synchronous demodulation)
  - The recovery of the message signal *m(t)* can be accomplished by first multiplying *s(t)* with a locally generated sinusoidal wave and then low-pass filtering the product.

$$\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$

  - The product modulation output and the filter output are

$$v(t) = A_c' \cos(2\pi f_c t + \phi) s(t)$$

$$= A_c A_c' \cos(2\pi f_c t)\cos(2\pi f_c t + \phi) m(t)$$

$$= \frac{1}{2} A_c A_c' \cos(4\pi f_c t + \phi) m(t) + \frac{1}{2} A_c A_c' \cos(\phi) m(t) \quad (3.10)$$

$$\cos(\theta_1)\cos(\theta_2) = \frac{1}{2}\cos(\theta_1 + \theta_2) + \frac{1}{2}\cos(\theta_1 - \theta_2)$$

$$v_0(t) = \frac{1}{2} A_c A_c' \cos(\phi) m(t) \quad (3.11)$$

Fig. 3.12

Back Next

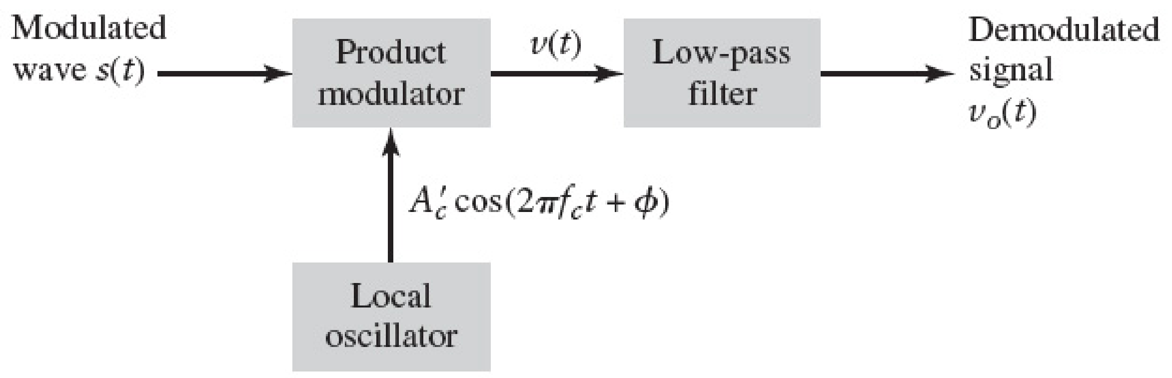


FIGURE 3.12 Block diagram of coherent detector, assuming that the local oscillator is out of phase by ϕ with respect to the sinusoidal carrier oscillator in the transmitter.

- The quadrature null effect
  - *The zero demodulated signal, when occurs for* $\Phi = \pm\pi/2$
  - *The phase error* Φ *in the local oscillator causes the detector output to be attenuated by a factor equal to* $\cos \Phi$

Fig. 3.13

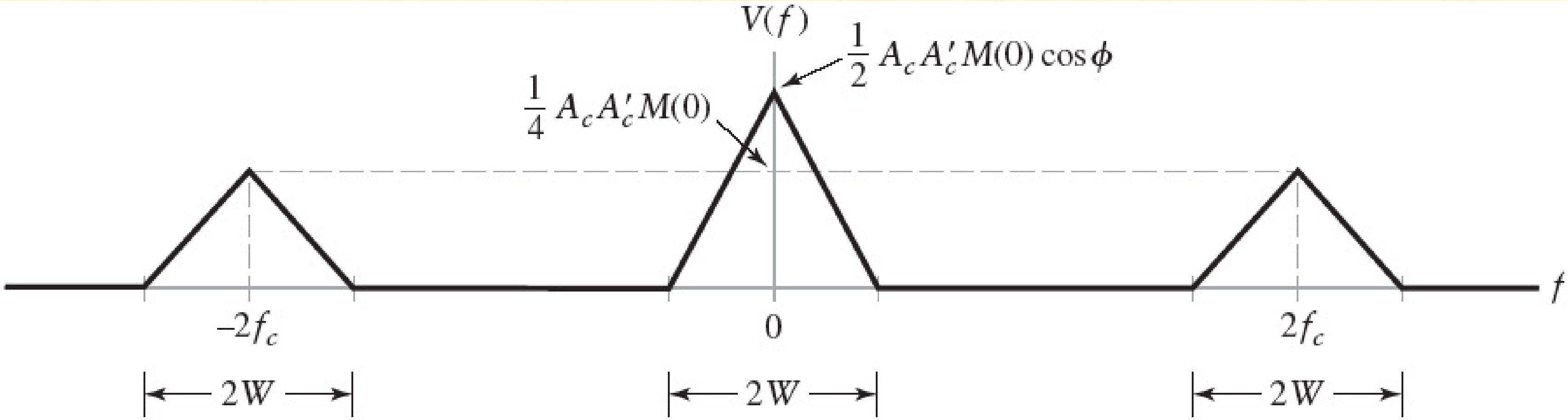


FIGURE 3.13 Illustration of the spectrum of product modulator output $v(t)$ in the coherent detector of Fig. 3.12, which is produced in response to a DSB-SC modulated wave as the detector input.

Computer experiment : DSB-SC

1. Figure 3.14(a) displays 10 cycles of the DSB-SC modulated wave
2. To perform coherent detection,
   1. *The product modulator's otuput - Multiply the DSB-SC modulated wave by an exact replica of the carrier*
   2. *The waveform of the coherent detector's overall output - Pass the product through a low-pass filter*

Fig. 3.14

Fig. 3.15(a)

Fig. 3.15(b)

Fig.3.14

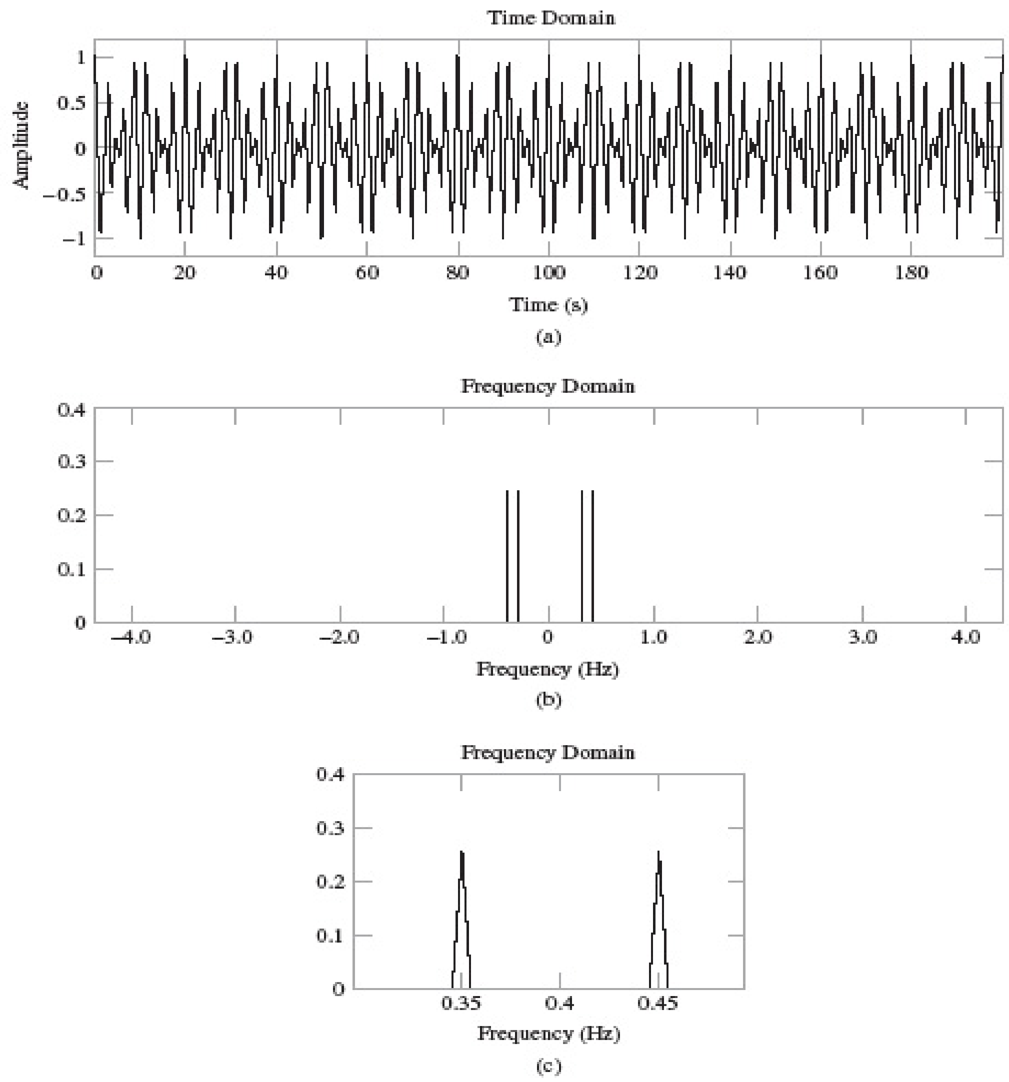
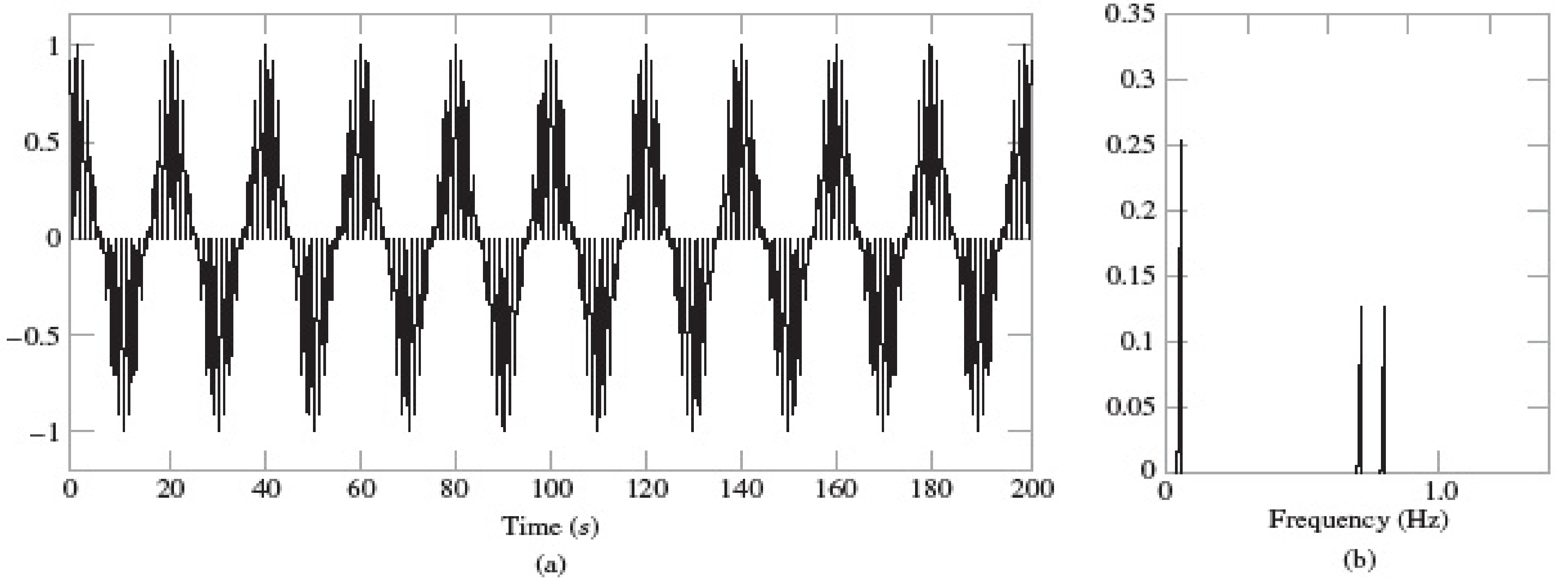


FIGURE 3.14 DSB-SC modulation: (*a*) DSB-SC modulated wave, (*b*) magnitude spectrum of the modulated wave, and (*c*) expanded spectrum around the carrier frequency.

Fig.3.15(a)

Back Next



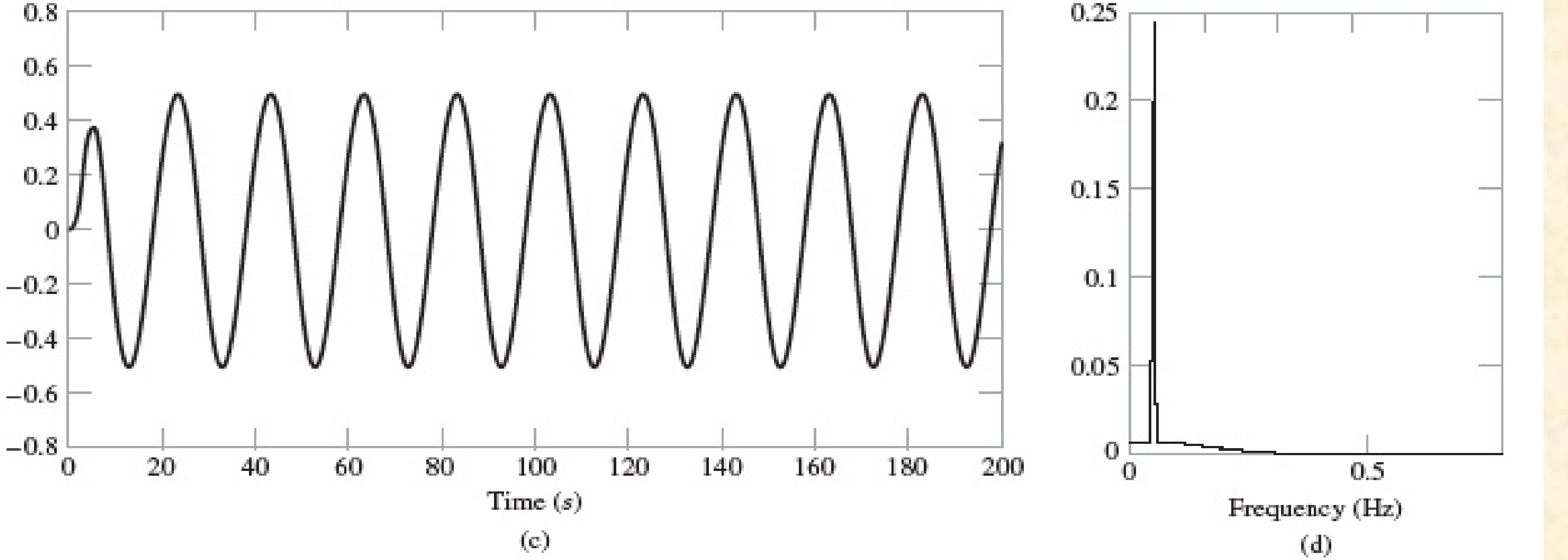


FIGURE 3.15 Coherent detection of DSB-SC modulated wave: (*a*) Waveform of signal produced at the output of product modulator, (*b*) amplitude spectrum of the signal in part (*a*); (*c*) waveform of low-pass filter output; and (*d*) amplitude spectrum of signal in part (*c*).

- Costas Receiver
  - Consists of two coherent detectors supplied with the same input signal
    - *Two local oscillator signals that are in phase quadrature with respect to each other*
    - *I-channel : in-phase coherent detector*
    - *Q-channel : quadrature-phase coherent detector*
  - Phase control in the Costas receiver ceases with modulation,
    - *Which means that phase-lock would have to be re-established with the reappearance of modulation*

Fig. 3.16

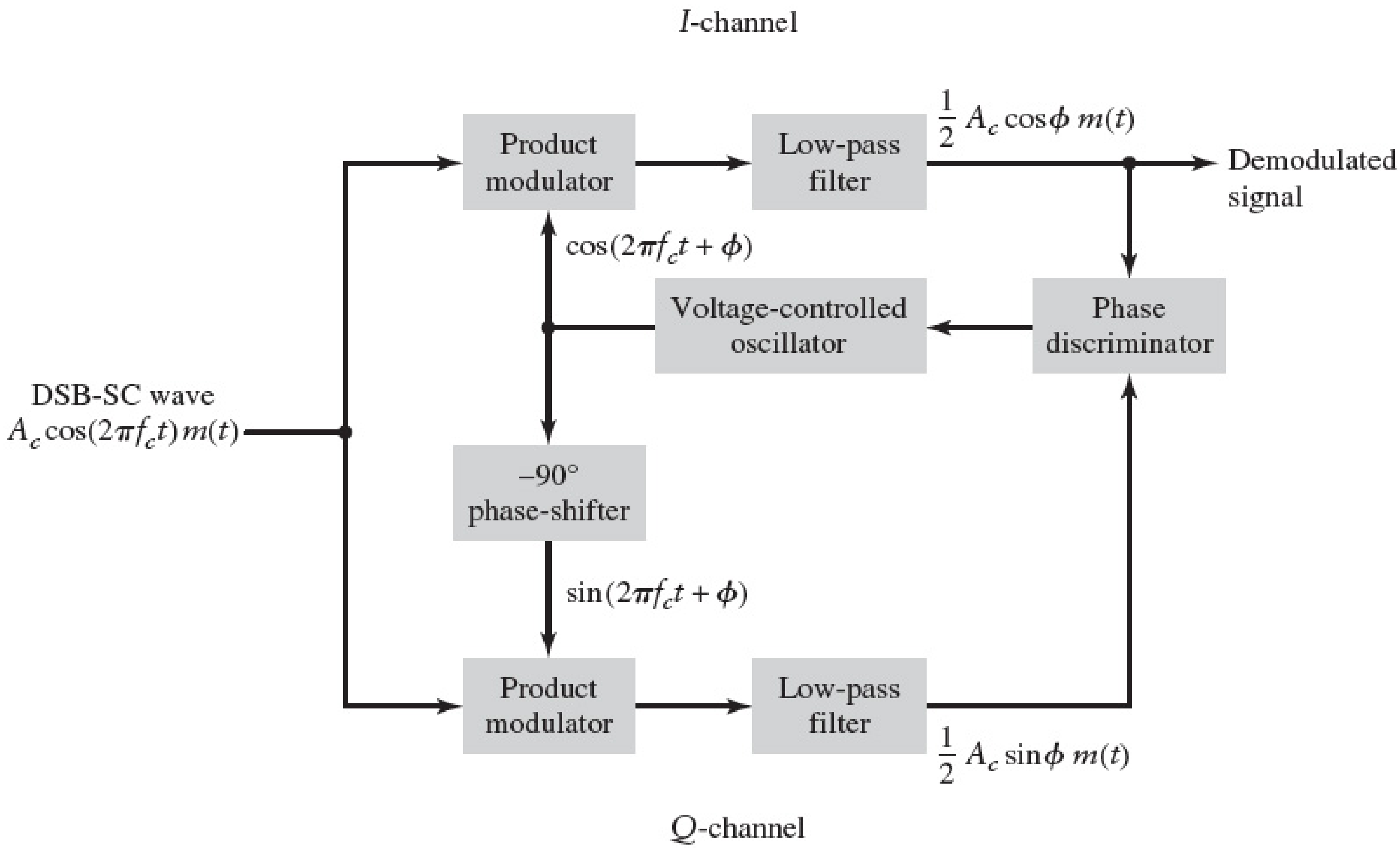


FIGURE 3.16 Costas receiver for the demodulation of a DSB-SC modulated wave.

3.5 Quadrature-Carrier Multiplexing

- Quadrature-Amplitude modulation (QAM)
  - This scheme enables two DSB-SC modulated waves to occupy the same channel bandwidth
  - Bandwidth-conversion system
  - This system send a pilot signal outside the passband of the modulated signal – to maintain the synchronization

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t) \quad (3.12)$$

Fig. 3.17

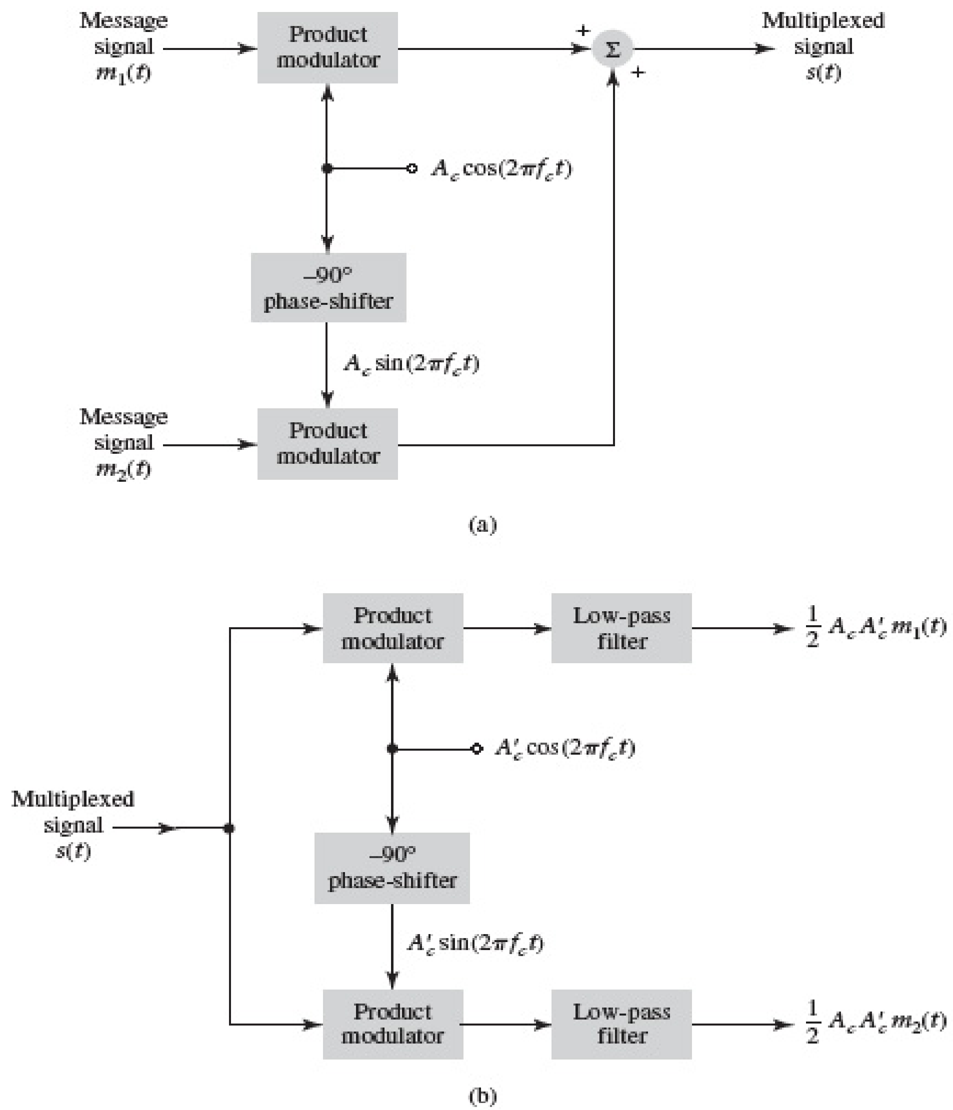


FIGURE 3.17 Quadrature-carrier multiplexing system: (*a*) Transmitter, (*b*) receiver.

- Single-Sideband Modulation
  - Suppress one of the two sideband in the DSB-SC modulated wave
- Theory
  - A DSB-SC modulator using the sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

  - The resulting DSB-SC modulated wave is

$$\begin{aligned} S_{DSB}(t) &= c(t)m(t) \\ &= A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) \\ &= \frac{1}{2}A_c A_m \cos[2\pi(f_c + f_m)t] + \frac{1}{2}A_c A_m \cos[2\pi(f_c - f_m)t] \quad (3.13) \end{aligned}$$

  - Suppressing the second term in Eq. (3.13) the upper and lower SSB modulated wave are

$$S_{USSB}(t) = \frac{1}{2}A_c A_m \cos[2\pi(f_c + f_m)t] \quad (3.14)$$

$$S_{USSB}(t) = \frac{1}{2}A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) - \frac{1}{2}A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t) \quad (3.15)$$

$$S_{LSSB}(t) = \frac{1}{2}A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) + \frac{1}{2}A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t) \quad (3.16)$$

- A sinusoidal SSB modulated wave

$$S_{SSB}(t) = \frac{1}{2} A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) \mp \frac{1}{2} A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t) \quad (3.17)$$

- For a periodic message signal defined by the Fourier series, the SSB modulated wave is

$$m(t) = \sum_n a_n \cos(2\pi f_n t) \quad (3.18)$$

$$S_{SSB}(t) = \frac{1}{2} A_c \cos(2\pi f_c t) \sum_n a_n \cos(2\pi f_n t) \mp \frac{1}{2} A_c \sin(2\pi f_c t) \sum_n a_n \sin(2\pi f_n t) \quad (3.19)$$

- For another periodic signal, the SSB modulated wave is

$$\hat{m}(t) = \sum_n a_n \sin(2\pi f_n t) \quad (3.20)$$

$$S_{SSB}(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) \mp \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t) \quad (3.21)$$

1. Under appropriate conditions, the Fourier series representation of a periodic signal converges to the Fourier transform of a nonperiodic signal
2. A Hibert transformer is a wide-band phase-shifter whose frequency response is characterized in two parts as follows

$$H(f) = -j\,\mathrm{sgn}(f) \quad (3.22)$$

- *The magnitude response is unity for all frequencies, both positive and negative*
- *The phase response is +90° for positive frequencies.*

$$S(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2} \hat{m}(t)\sin(2\pi f_c t) \quad (3.23)$$

Fig. 3.18

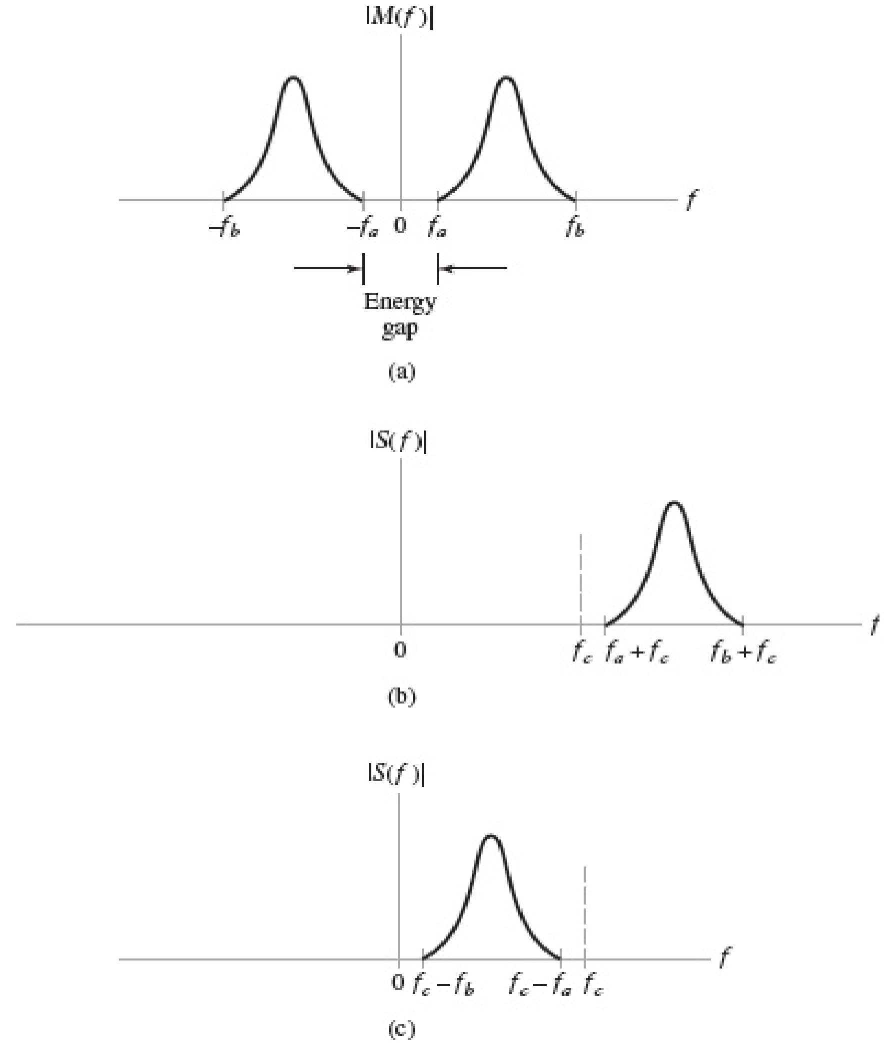


Figure 3.18 (*a*) Spectrum of a message signal $m(t)$ with energy gap centered around zero frequency. Corresponding spectra of SSB-modulated waves using (*b*) upper sideband, and (*c*) lower sideband. In parts (*b*) and (*c*), the spectra are only shown for positive frequencies.

- Modulators for SSB
  - Frequency Discrimination Method
    - *For the design of the band-pass filter to be practically feasible, there must be a certain separation between the two sidebands that is wide enough to accommodate the transition band of the band-pass filter.*

Fig. 3.19

  - Phase Discrimination Method
    - *Wide-band phase-shifter is designed to produce the Hilbert transform in response to the incoming message signal.*
    - *To interfere with the in-phase path so as to eliminate power in one of the two sidebands, depending on whether upper SSB or lower SSB is the requirement.*

Fig. 3.20

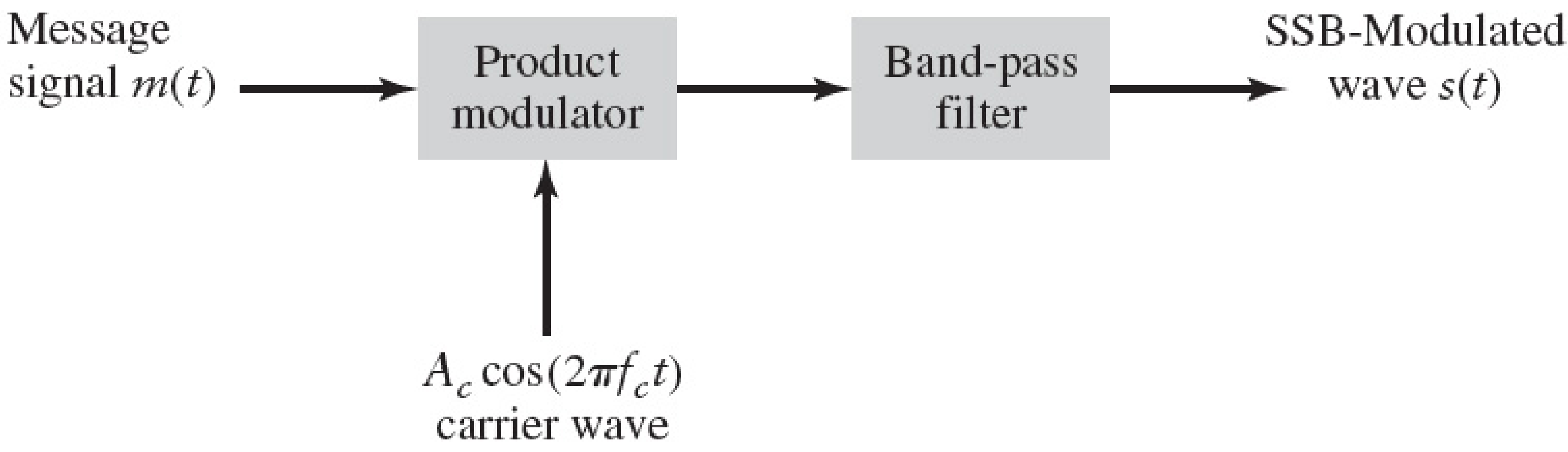


FIGURE 3.19 Frequency-discrimination scheme for the generation of a SSB modulated wave.

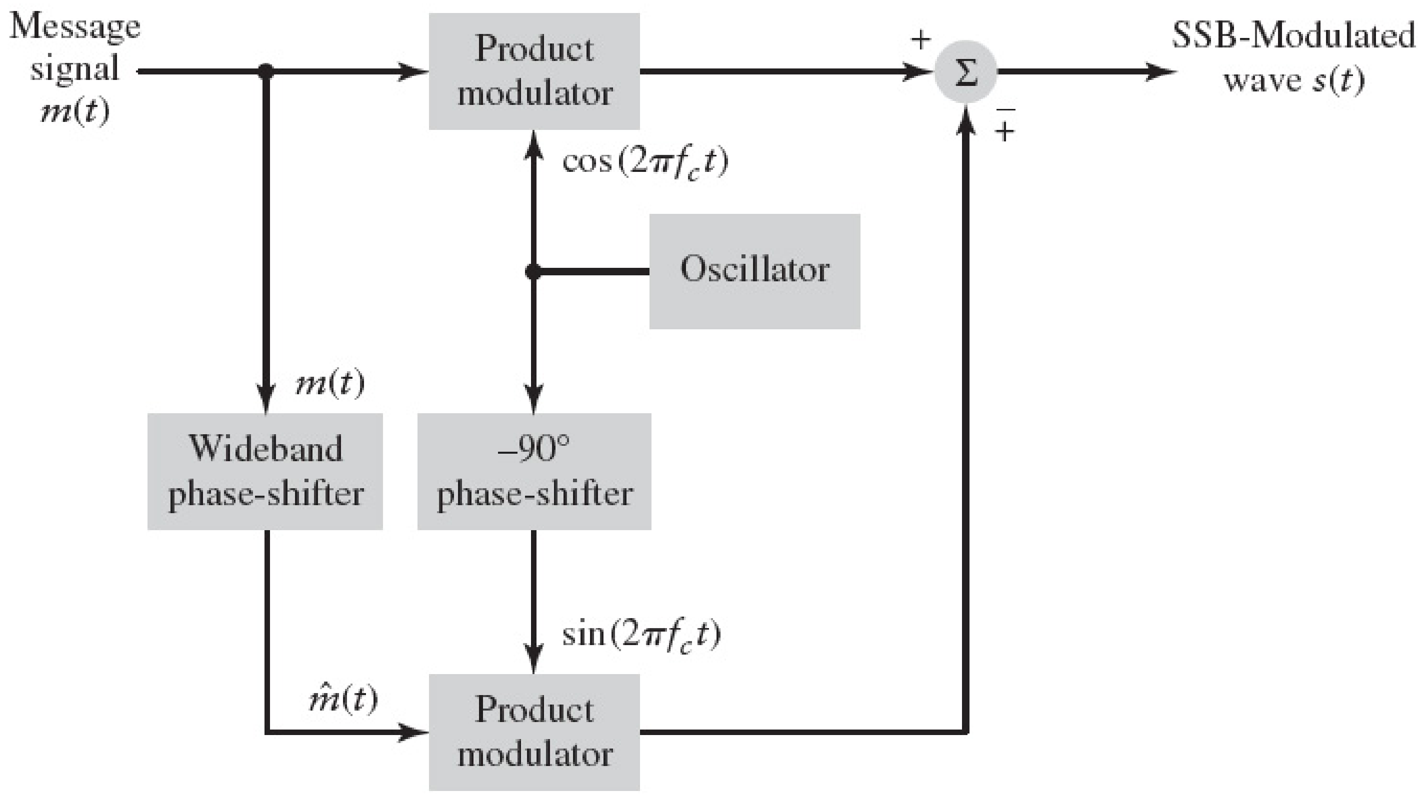


FIGURE 3.20 Phase discrimination method for generating a SSB-modulated wave. Note: The plus sign at the summing junction pertains to transmission of the lower sideband and the minus sign pertains to transmission of the upper sideband.

- Coherent Detection of SSB
  - Synchronization of a local oscillator in the receiver with the oscillator responsible for generating the carrier in the transmitter
  - The demodulation of SSB is further complicated by the additional suppression of the upper or lower sideband.

Fig. 3.12

- Frequency Translation
  - Single sideband modulation is in fact a form of frequency translation
    - *Frequency changing*
    - *Mixing*
    - *Heterodyning*

Fig. 3.21

  - Up conversion : the unshaded part of the spectrum in Fig. 3.22(b)

$$f_2 = f_1 + f_l \qquad f_l = f_2 - f_1$$

  - Down conversion : the shaded part of the spectrum in Fig. 3.22(b)

$$f_2 = f_1 - f_l \qquad f_l = f_1 - f_2$$

Fig. 3.22

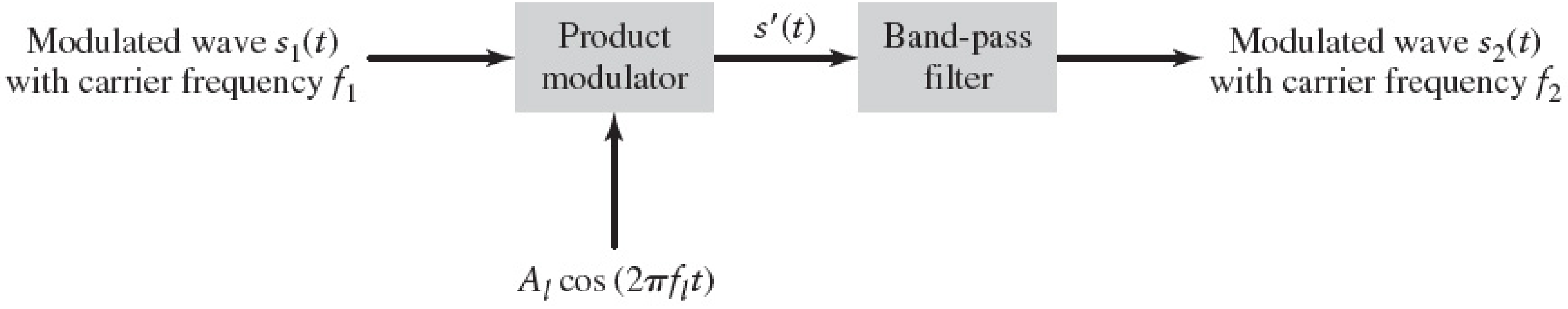


FIGURE 3.21 Block diagram of mixer.

Fig.3.22

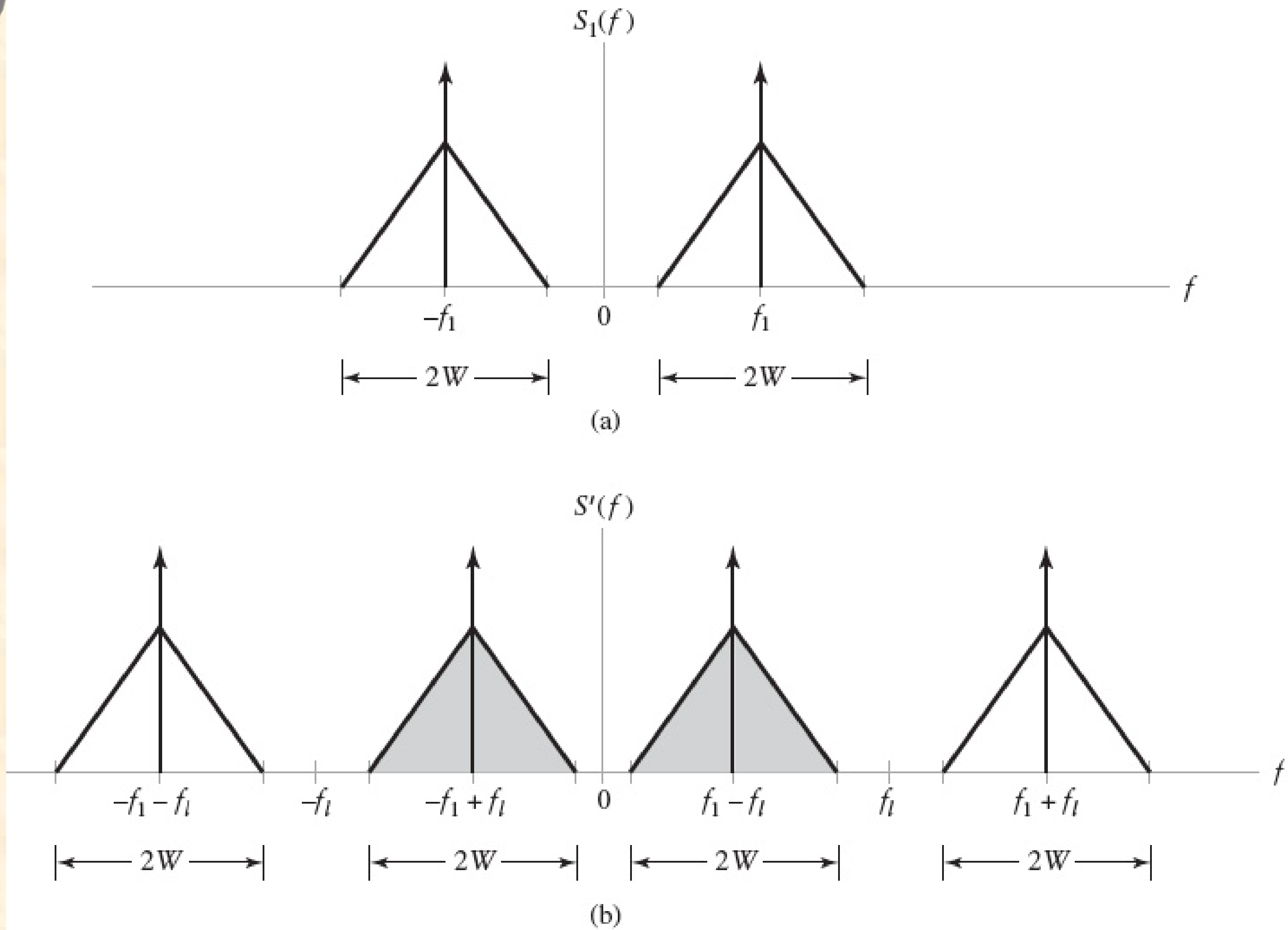


FIGURE 3.22 (*a*) Spectrum of modulated signal $s_1(t)$ at the mixer input. (*b*) Spectrum of the corresponding signal $s'(t)$ at the output of the product modulator in the mixer.

3.7 Vestigial Sideband Modulation

- For the spectrally efficient transmission of wideband signals
  - Typically, the spectra of wideband signals contain significant low frequencies, which make it impractical to use SSB modulation.
  - The spectral characteristics of wideband data befit the use of DSB-SC. However, DSB-SC requires a transmission bandwidth equal to twice the message bandwidth, which violates the bandwidth conservation requirement.

- Vestigial sideband (VSB) modulation
  - Instead of completely removing a sideband, a trace of vestige of that sideband is transmitted, the name *"vestigial sideband"*
  - Instead of transmitting the other sideband in full, almost the whole of this second band is also transmitted.

$$B_T = f_v + W$$

- Sideband Shaping Filter
  - The band-pass filter is referred to as a sideband shaping filter
  - The transmitted vestige compensates for the spectral portion missing from the other sideband.
  - The sideband shaping filter must itself satisfy the following condition.

$$H(f+f_c)+H(f-f_c)=1, \quad \text{for } -W \le f \le W \quad (3.26)$$

  - Two properties of the sideband shaping filter
    1. *The transfer function of the sideband shaping filter exhibits odd symmetry about the carrier frequency*

$$H(f)=u(f-f_c)-H_v(f-f_c), \quad \text{for } f_c-f_v<|f|<f_c+W \quad (3.27)$$

$$u(f)=\begin{cases}1, & \text{for } f>0 \\ 0, & \text{for } f<0\end{cases} \quad (3.28)$$

$$H_v(-f)=-H_v(f) \quad (3.29)$$

2. *The transfer function $H_v(f)$ is required to satisfy the condition of Eq. (3.26) only for the frequency interval $-W \le f \le W$*

Fig. 3.23

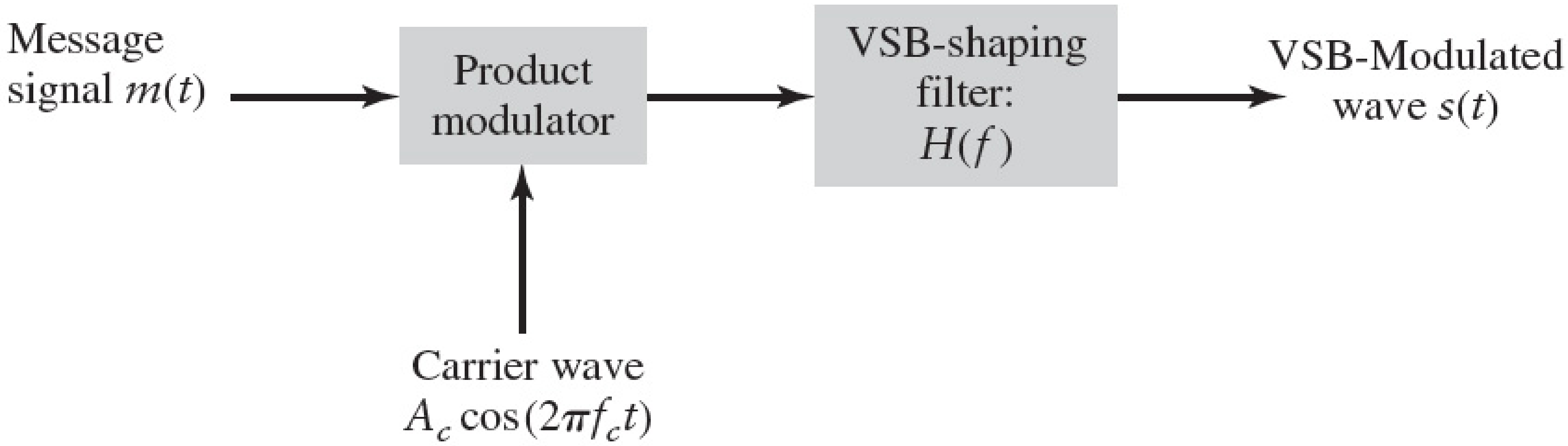


FIGURE 3.23 VSB modulator using frequency discrimination.

Fig. 3.24

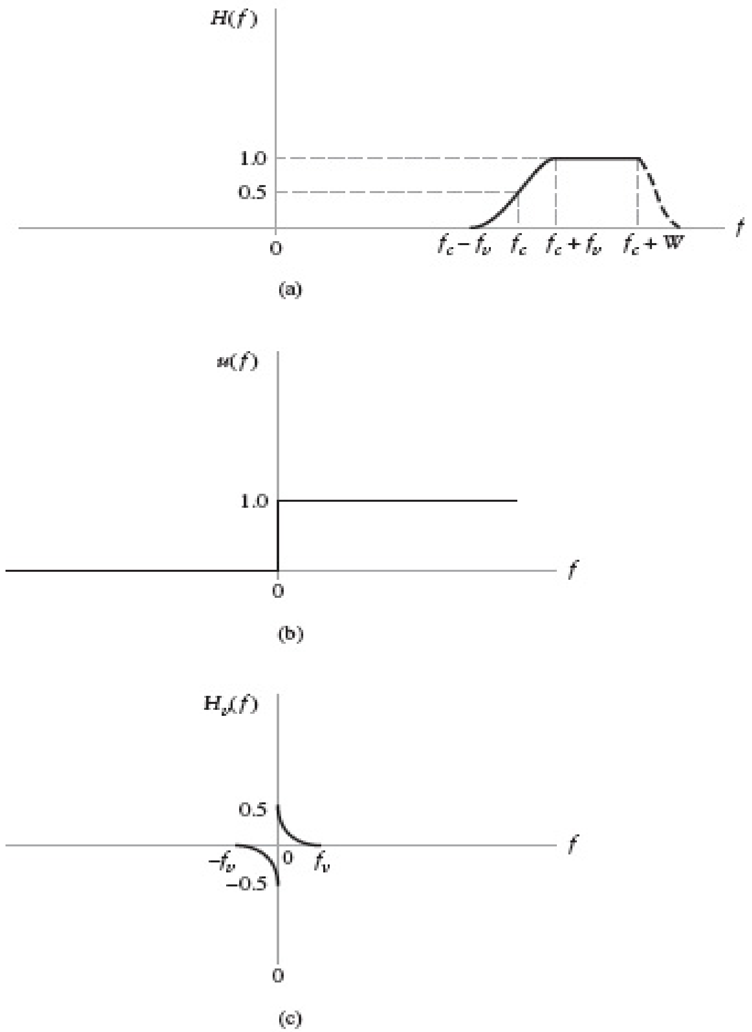


FIGURE 3.24 (*a*) Amplitude response of sideband-shaping filter; only the positive-frequency portion is shown, the dashed part of the amplitude response is arbitrary. (*b*) Unit-step function defined in the frequency domain. (*c*) Low-pass transfer function $H_v(f)$.

Example 3.3 Sinusoidal VSB

Consider the simple example of sinusoidal VSB modulation produced by the sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

and carrier wave

$$c(t) = A_c \cos(2\pi f_c t)$$

Let the upper side-frequency at $f_c + f_m$ as well as its image at $-(f_c + f_m)$ be attenuated by the factor k. To satisfy the condition of Eq. (3.26), the lower side-frequency at $f_c - f_m$ and its image $-(f_c - f_m)$ must be attenuated by the factor $(1 - k)$. The VSB spectrum is therefore

$$\begin{aligned} S(f) = {} & \frac{1}{4} k A_c A_m [\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))] \\ & + \frac{1}{4}(1 - k) A_c A_m [\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))] \end{aligned}$$

Correspondingly, the sinusoidal VSB modulated wave is defined by

$$s(t) = \frac{1}{4}kA_cA_m[\exp(j2\pi(f_c + f_m)t) + \exp(-j2\pi(f_c + f_m)t)]$$
$$+\frac{1}{4}(1 - k)A_cA_m[\exp(j2\pi(f_c - f_m)t) + \exp(-j2\pi(f_c - f_m)t)]$$
$$= \frac{1}{2}kA_cA_m \cos(2\pi(f_c + f_m)t) + \frac{1}{2}(1 - k)A_cA_m \cos(2\pi(f_c - f_m)t) \qquad (3.30)$$

Using well-known trigonometric identities to expand the cosine terms $\cos(2\pi(f_c + f_m)t)$ and $\cos(2\pi(f_c - f_m)t)$, we may reformulate Eq. (3.30) as the linear combination of two sinusoidal DSB-SC modulated waves.

$$s(t) = \frac{1}{2}A_cA_m \cos(2\pi f_c t) \cos(2\pi f_m t)$$
$$+\frac{1}{2}A_cA_m(1 - 2k) \sin((2\pi f_c t) \sin(2\pi f_m t)) \qquad (3.31)$$

where the first term on the right-hand side is the in-phase component of $s(t)$ and the second term is the quadrature component.

- Coherent Detection of VSB
  - The demodulation of VSB consists of multiplying *s(t)* with a locally generated sinusoid and then low-pass filtering the resulting product signal *v(t)*
  - Fourier transform of the product signal is

$$v(t) = A_c' s(t)\cos(2\pi f_c t)$$

$$V(f) = \frac{1}{2} A_c' [S(f - f_c) + S(f + f_c)] \quad (3.32)$$

$$s(t) \Leftrightarrow S(f)$$

$$S(f) = \frac{1}{2} A_c [M(f - f_c) + M(f + f_c)] H(f) \quad (3.33)$$

  - Shifting the VSB spectrum to the right and left

$$S(f - f_c) = \frac{1}{2} A_c [M(f - 2f_c) + M(f)] H(f - f_c) \quad (3.34)$$

$$S(f + f_c) = \frac{1}{2} A_c [M(f) + M(f + 2f_c)] H(f + f_c) \quad (3.35)$$

$$V(f) = \frac{1}{4} A_c A_c^{'} M(f)[H(f - f_c) + H(f + f_c)]$$

$$+ \frac{1}{4} A_c A_c^{'} [M(f - 2f_c) H(f - f_c) + M(f + 2f_c) H(f + f_c)]$$

$$V(f) = \frac{1}{4} A_c A_c^{'} M(f)$$

$$+ \frac{1}{4} A_c A_c^{'} [M(f - 2f_c) H(f - f_c) + M(f + 2f_c) H(f + f_c)] \quad (3.36)$$

- The low-pass filter in the coherent detector has a cutoff frequency just slightly greater than the message bandwidth
- The result demodulated signal is a scaled version of the desired message signal.

Example 3.4 Coherent detection of sinusoidal VSB

Recall from Eq. (3.31) of Example 3.3, that the sinusoidal VSB modulated signal is defined by

$$s(t) = \frac{1}{2}A_c A_m \cos(2\pi f_m t)\cos(2\pi f_c t) + \frac{1}{2}A_c A_m(1-2k)\sin(2\pi f_c t)\sin(2\pi f_m t)$$

Multiplying $s(t)$ by $A_c' \cos(2\pi f_c t)$ in accordance with perfect coherent detection yields the product signal

$$\begin{aligned} v(t) &= A_c' s(t)\cos(2\pi f_c t) \\ &= \frac{1}{2}A_c A_c' A_m \cos(2\pi f_m t)\cos^2(2\pi f_c t) \\ &\quad + \frac{1}{2}A_c A_c' A_m(1-2k)\sin(2\pi f_m t)\sin(2\pi f_c t)\cos(2\pi f_c t) \end{aligned}$$

Next, using the trigonometric identities

$$\cos^2(2\pi f_c t) = \frac{1}{2}[1 + \cos(4\pi f_c t)]$$

and

$$\sin(2\pi f_c t)\cos(2\pi f_c t) = \frac{1}{2}\sin(4\pi f_c t)$$

we may redefine $\nu(t)$ as

$$\nu(t) = \frac{1}{4}A_c A_c' A_m \cos(2\pi f_m t) + \frac{1}{4}A_c A_c' A_m[\cos(2\pi f_m t)\cos(4\pi f_c t) + (1 - 2k)\sin(2\pi f_m t)\sin(4\pi f_c t)] \quad (3.37)$$

The first term on the right-hand side of Eq. (3.37) is a scaled version of the message signal $A_m \cos(2\pi f_m t)$. The second term of the equation is a new sinusoidal VSB wave modulated onto a carrier of frequency $2f_c$, which represents the high-frequency components of $\nu(t)$. This second term is removed by the low-pass filter in the detector of Fig. 3.12, provided that the cut-off frequency of the filter is just slightly greater than the message frequency f_m.

Example 3.5 Envelope detection of VSB plus carrier

The coherent detection of VSB requires synchronism of the receiver to the transmitter, which increases system complexity. To simplify the demodulation process, we may purposely add the carrier to the VSB signal (scaled by the factor k_a) prior to transmission and then use envelope detection in the receiver.[3] Assuming sinusoidal modulation, the "VSB-plus-carrier" signal is defined [see Eq. (3.31) of Example 3.3) as

$$
\begin{aligned}
s_{\mathrm{VSB+C}}(t) &= A_c \cos(2\pi f_c t) + k_a s(t), \qquad k_a = \text{amplitude sensitivity factor} \\
&= A_c \cos(2\pi f_c t) + \frac{k_a}{2} A_c A_m \cos(2\pi f_m t)\cos(2\pi f_c t) \\
&\quad + \frac{k_a}{2} A_c A_m (1 - 2k) \sin(2\pi f_m t)\sin(2\pi f_c t) \\
&= A_c\left[1 + \frac{k_a}{2} A_m \cos(2\pi f_m t)\right]\cos(2\pi f_c t) \\
&\quad + \frac{k_a}{2} A_c A_m (1 - 2k) \sin(2\pi f_m t)\sin(2\pi f_c t)
\end{aligned}
$$

The envelope of $s_{\mathrm{VSB+C}}(t)$ is therefore

$$a(t) = \left\{A_c^2\left[1 + \frac{k_a}{2}A_m\cos(2\pi f_m t)\right]^2 + A_c^2\left[\frac{k_a}{2}A_m(1-2k)\sin(2\pi f_m t)\right]^2\right\}^{1/2}$$

$$= A_c\left[1 + \frac{k_a}{2}A_m\cos(2\pi f_m t)\right]\left\{1 + \left[\frac{\frac{k_a}{2}A_m(1-2k)\sin(2\pi f_m t)}{1 + \frac{k_a}{2}A_m\cos(2\pi f_m t)}\right]^2\right\}^{1/2} \quad (3.38)$$

Equation (3.38) shows that *distortion* in the envelope detection performed on the envelope $a(t)$ is contributed by the quadrature component of the sinusoidal VSB signal. This distortion can be reduced by using a combination of two methods:

- The amplitude sensitivity factor k_a is reduced, which has the effect of reducing the percentage modulation.
- The width of the vestigial sideband is reduced, which has the effect of reducing the factor $(1 - 2k)$.

Both of these methods are intuitively satisfying in light of what we see inside the square brackets in Eq. (3.38).

3.8 Baseband Representation of Modulated Waves and Band-Pass Filters

- Baseband
  - Is used to designate the band of frequencies representing the original signal as delivered by a source of information

- Baseband Representation of Modulation Waves
  - A linear modulated wave

$$s(t) = s_1(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \quad (3.39)$$

$$c(t) = \cos(2\pi f_c t)$$

$$\hat{c}(t) = \sin(2\pi f_c t)$$

  - The modulated wave in the compact form – canonical representation of linear modulated waves

$$s(t) = s_I(t)c(t) - s_Q(t)\hat{c}(t) \quad (3.40)$$

- The complex envelope of the modulated wave is

$$\tilde{s}(t) = s_I(t) + js_Q(t) \quad (3.41)$$

- The complex carrier wave and the modulated wave is

$$\begin{aligned} \tilde{c}(t) &= c(t) + j\hat{c}(t) \\ &= \cos(2\pi f_c t) + j\sin(2\pi f_c t) \\ &= \exp(j2\pi f_c t) \qquad (3.42) \end{aligned}$$

$$\begin{aligned} s(t) &= \mathrm{Re}\left[\tilde{s}(t)\tilde{c}(t)\right] \\ &= \mathrm{Re}\left[\tilde{s}(t)\exp(j2\pi f_c t)\right] \quad (3.43) \end{aligned}$$

Table. 3.1

- The practical advantage of the complex envelope
  - *The highest frequency component of s(t) may be as large as f_c+W, where f_c is the carrier frequency and W is the message bandwidth*
  - *On the other hand, the highest frequency component of $\hat{s}$ (t) is considerably smaller, being limited by the message bandwidth W.*

Back Next

Table 3.1

TABLE 3.1 *Different Forms of Linear Modulation as Special Cases of Eq. (3.39), assuming unit carrier amplitude*

Type of modulation	*In-phase component* $s_I(t)$	*Quadrature component* $s_Q(t)$	*Comments*
AM	$1 + k_a m(t)$	0	k_a = amplitude sensitivity $m(t)$ = message signal
DSB-SC	$m(t)$	0	
SSB:			
(a) Upper sideband transmitted	$\frac{1}{2}m(t)$	$\frac{1}{2}\hat{m}(t)$	$\hat{m}(t)$ = Hilbert transform of $m(t)$ (see part (i) of footnote 4)
(b) Lower sideband transmitted	$\frac{1}{2}m(t)$	$-\frac{1}{2}\hat{m}(t)$	
VSB:			
(a) Vestige of lower sideband transmitted	$\frac{1}{2}m(t)$	$\frac{1}{2}m'(t)$	$m'(t)$ = response of filter with transfer function $H_Q(f)$ due to message signal $m(t)$. The $H_Q(f)$ is defined by the formula (see part (ii) of footnote 4)
(b) Vestige of upper sideband transmitted	$\frac{1}{2}m(t)$	$-\frac{1}{2}m'(t)$	$H_Q(f) = -j[H(f + f_c) - H(f - f_c)]$ where $H(f)$ is the transfer function of the VSB sideband shaping filter.

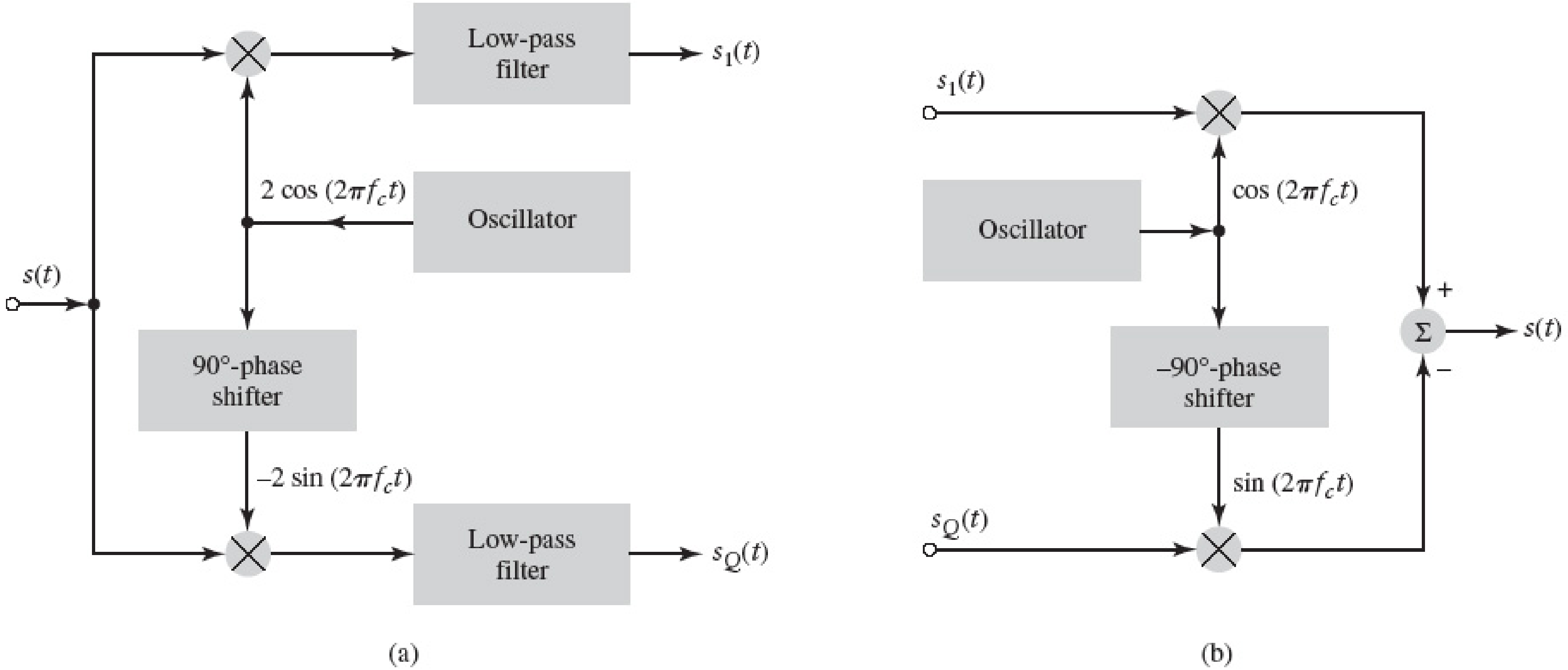


FIGURE 3.25 (*a*) Scheme for deriving the in-phase and quadrature components of a linearly modulated (i.e., band-pass) signal. (*b*) Scheme for reconstructing the modulated signal from its in-phase and quadrature components.

- Baseband Representation of Band-pass Filters
  - The desire to develop the corresponding representation for band-pass filters, including band-pass communication channels

$$\tilde{H}(f - f_c) = 2H(f), \quad \text{for } f > 0 \quad (3.44)$$

  - We may determine $\hat{H}(f)$ by proceeding as follows
    - *Given the transfer function H(f) of a band-pass filter, which is defined for both positive and negative frequencies, keep the part of H(f) that corresponds to positive frequencies; let $H_+(f)$ denote this part.*
    - *Shift $H_+(f)$ to the left along the frequency axis by an amount equal to f_c, and scale it by the factor 2. The result so obtained defines the desired $\hat{H}(f)$.*
  - Actual output *y(t)* is determined from the formula

$$y(t) = \mathrm{Re}\left[\tilde{y}(t)\exp(j2\pi f_c t)\right] \quad (3.45)$$

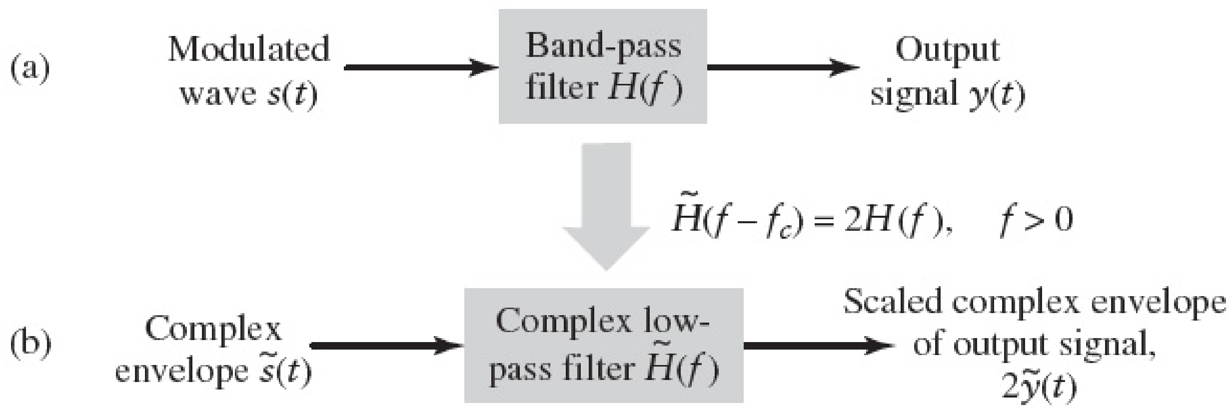


FIGURE 3.26 Band-pass filter to complex low-pass system transformation: (*a*) Real-valued band-pass configuration, and (*b*) corresponding complex-valued low-pass configuration.

- Superheterodyne Receiver (supersht)
  - Carrier-frequency tunning, the purpose of which is to select the desired signal
  - Filtering, which is required to separate the desired signal from other modulated signals that may be picked up along the way.
  - Amplification, which is intended to compensate for the loss of signal power incurred in the course of transmission.
  - It overcomes the difficulty of having to build a tunable highly frequency-selective and variable filter

$$f_{IF} = f_{RF} - f_{LO} \quad (3.46)$$

Table. 3.2

  - Intermediate frequency (IF)
    - *Because the signal is neither at the original input frequency nor at the final baseband frequency*

Fig. 3.27

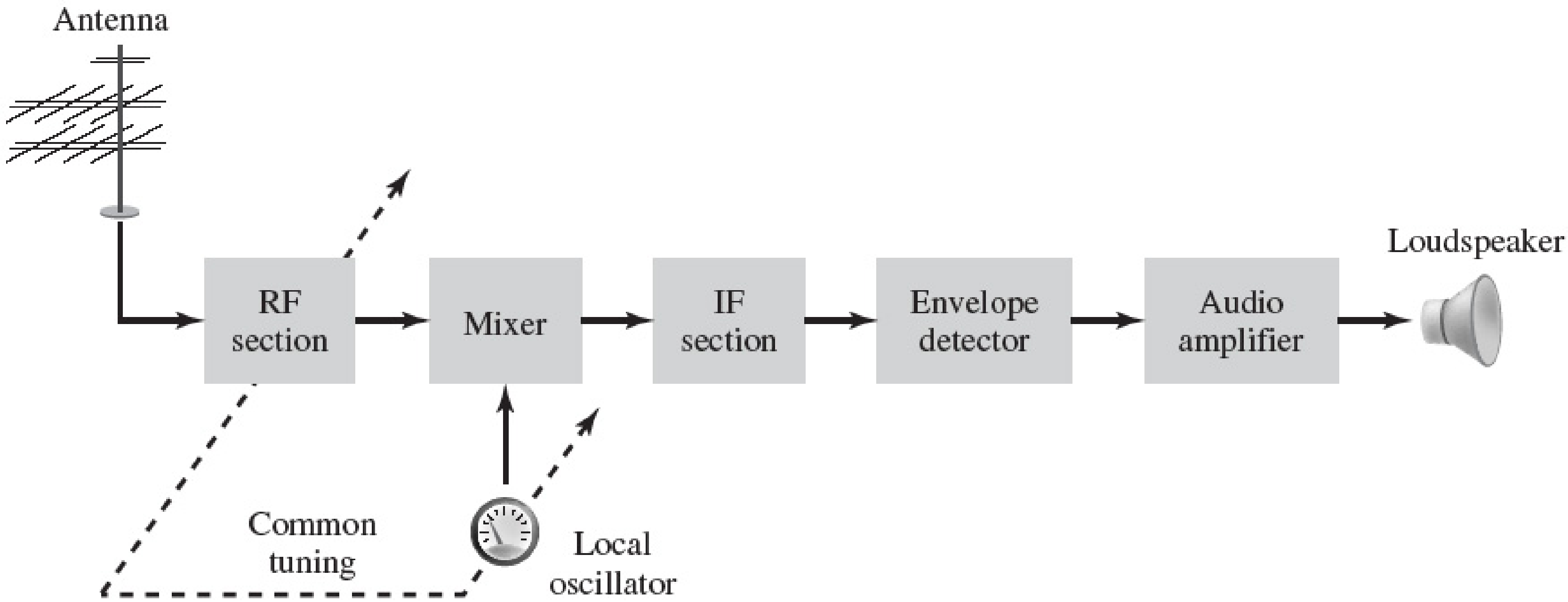


Figure 3.27 Basic elements of an AM radio receiver of the superheterodyne type.

Back Next

TABLE 3.2 *Typical Frequency Parameters of AM and FM Radio Receivers*

	AM Radio	*FM Radio*
RF carrier range	0.535–1.605 MHz	88–108 MHz
Mid-band frequency of IF section	0.455 MHz	10.7 MHz
IF bandwidth	10 kHz	200 kHz

- Television Signals
  1. The video signal exhibits a large bandwidth and significant low-frequency content, which suggest the use of vestigial sideband modulation.
  2. The circuitry used for demodulation in the receiver should be simple and therefore inexpensive. This suggest the use of envelope detection, which requires the addition of a carrier to the VSB modulated wave.

Back Next

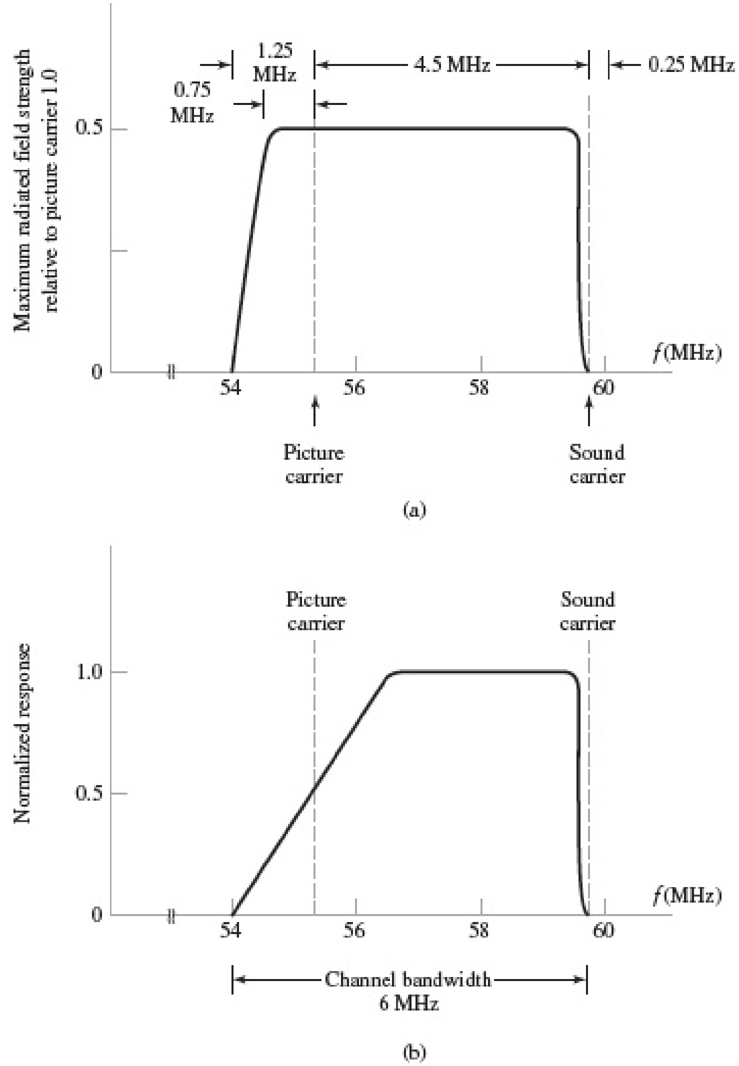


FIGURE 3.28 (*a*) Idealized amplitude spectrum of a transmitted TV signal. (*b*) Amplitude response of a VSB shaping filter in the receiver.

- Frequency-Division Multiplexing
  - To transmit a number of these signals over the same channel, the signals must be kept apart so that they do not interfere with each other, and thus they can be separated at the receiving end.
  - Frequency-division multiplexing (FDM)
  - Time-division multiplexing (TDM)

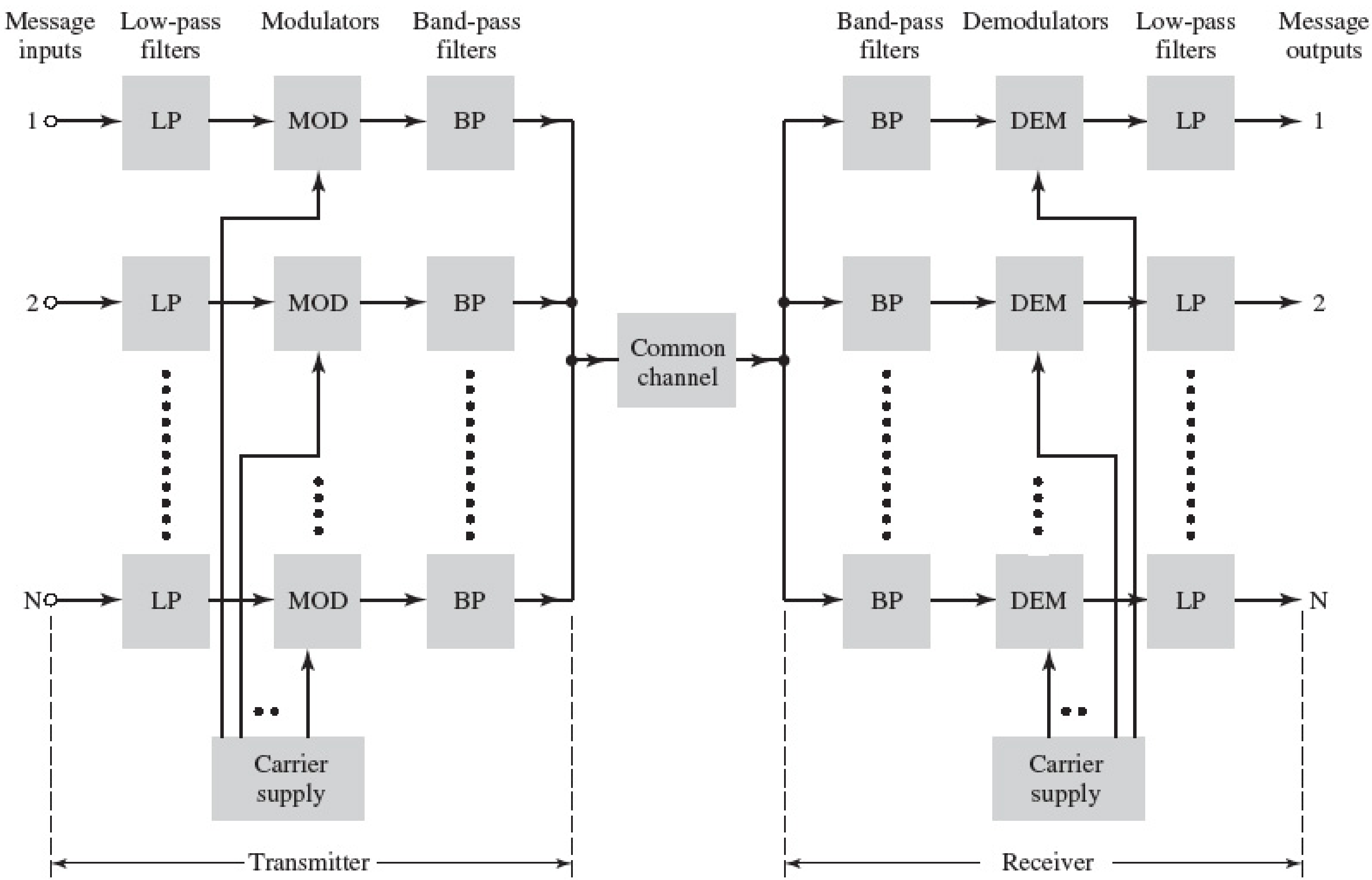


FIGURE 3.29 Block diagram of frequency-division multiplexing (FDM) system.

EXAMPLE 3.6 Modulation steps in a 60-channel FDM system

The practical implementation of an FDM system usually involves many steps of modulation and demodulation, as illustrated in Fig. 3.30. The first multiplexing step combines 12 voice inputs into a *basic group*, which is formed by having the nth input modulate a carrier at frequency $f_c = 60 + 4n$ kHz, where $n = 1, 2, \ldots, 12$. The lower sidebands are then selected by band-pass filtering and combined to form a group of 12 lower sidebands (one for each voice input). Thus the basic group occupies the frequency band 60–108 kHz. The next step in the FDM hierarchy involves the combination of five basic groups into a *supergroup*. This is accomplished by using the nth group to modulate a carrier of frequency $f_c = 372 + 48n$ kHz, where $n = 1, 2, \ldots, 5$. Here again the lower sidebands are selected by filtering and then combined to form a supergroup occupying the band 312–552 kHz. Thus, a supergroup is designed to accommodate 60 independent voice inputs. The reason for forming the supergroup in this manner is that economical filters of the required characteristics are available only over a limited frequency range. In a similar manner, supergroups are combined into *mastergroups*, and mastergroups are combined into *very large groups*.

Fig. 3.30

Fig.3.30

Back

Next

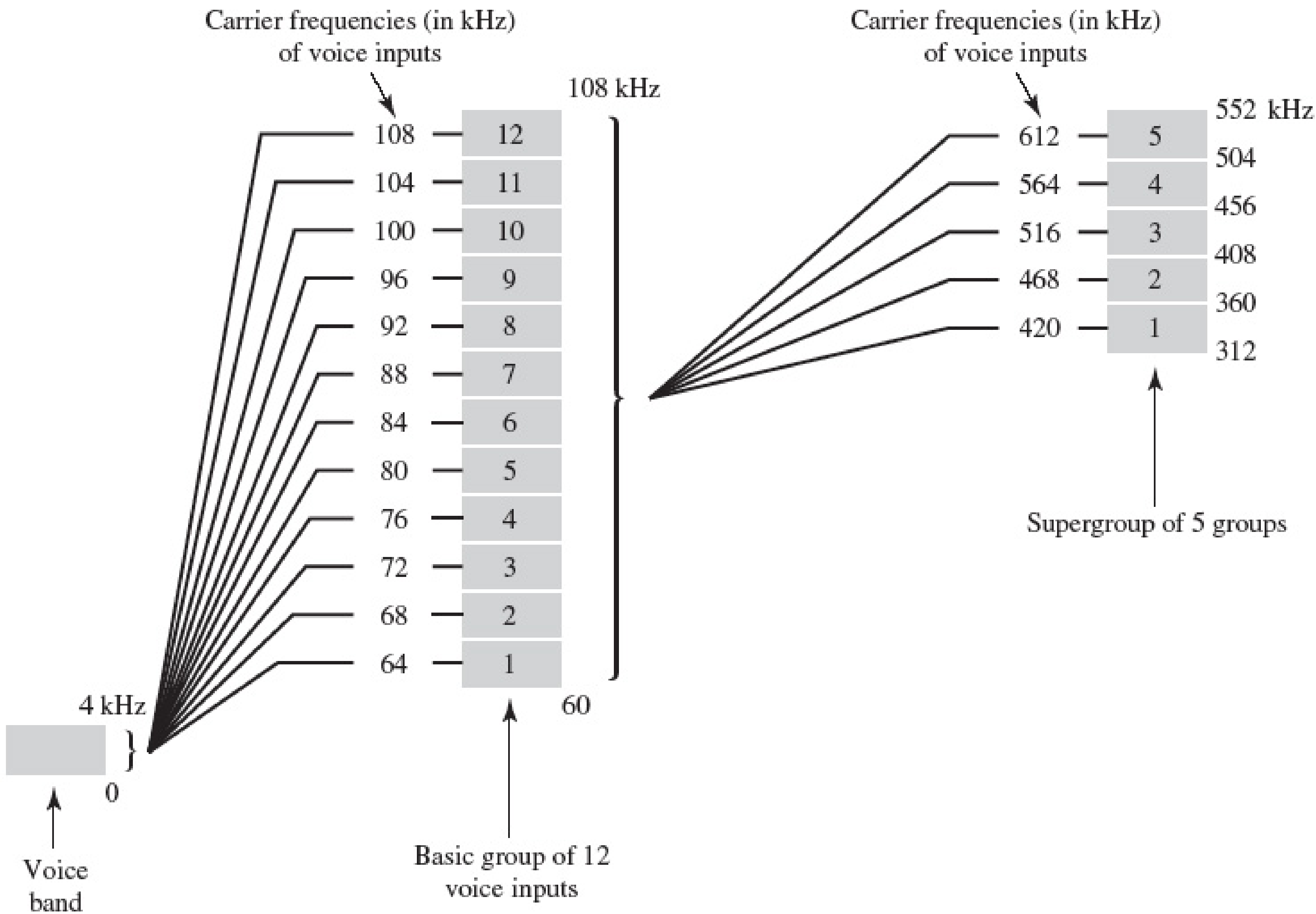


FIGURE 3.30 Illustration of the modulation steps in an FDM system.

- The example modulated wave is

$$s(t) = A_c m(t) \cos(2\pi f_c t) \quad (3.47)$$

1. Amplitude modulation (AM), in which the upper and lower sidebands are transmitted in full, accompanied by the carrier wave
   - *Demodulation of the AM wave is accomplished equally simply in the receiver by using an envelope detector*
2. Double sideband-suppressed carrier (DSB-SC) modulation, in which only the upper and lower sidebands are transmitted.
   - *This advantage of DSB-SC modulation over AM is, attained at the expense of increased receiver complexity.*
3. Single sideband (SSB) modulation, in which only the upper sideband or lower sideband is transmitted.
   - *It requires the minimum transmitted power and the minimum channel bandwidth for conveying a message signal from one point to another.*
4. Vestigial sideband modulation, in which "almost" the whole of one sideband and a "vestige" of the other sideband are transmitted in a prescribed complementary fashion
   - *VSB modulation requires an channel bandwidth that is intermediate between that required for SSB and DSB-SC systems, and the saving in bandwidth can be significant if modulating signals with large bandwidths are being handled.*

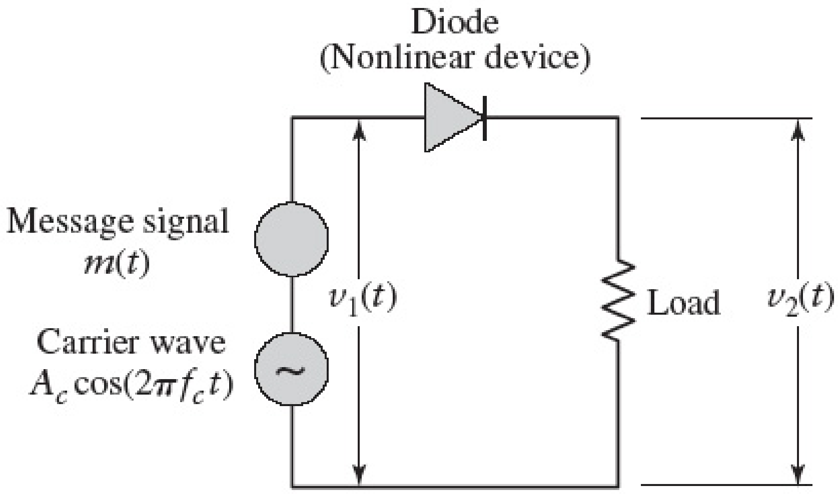


FIGURE 3.8 Nonlinear circuit using a diode.

Fig.3.31

Back Next

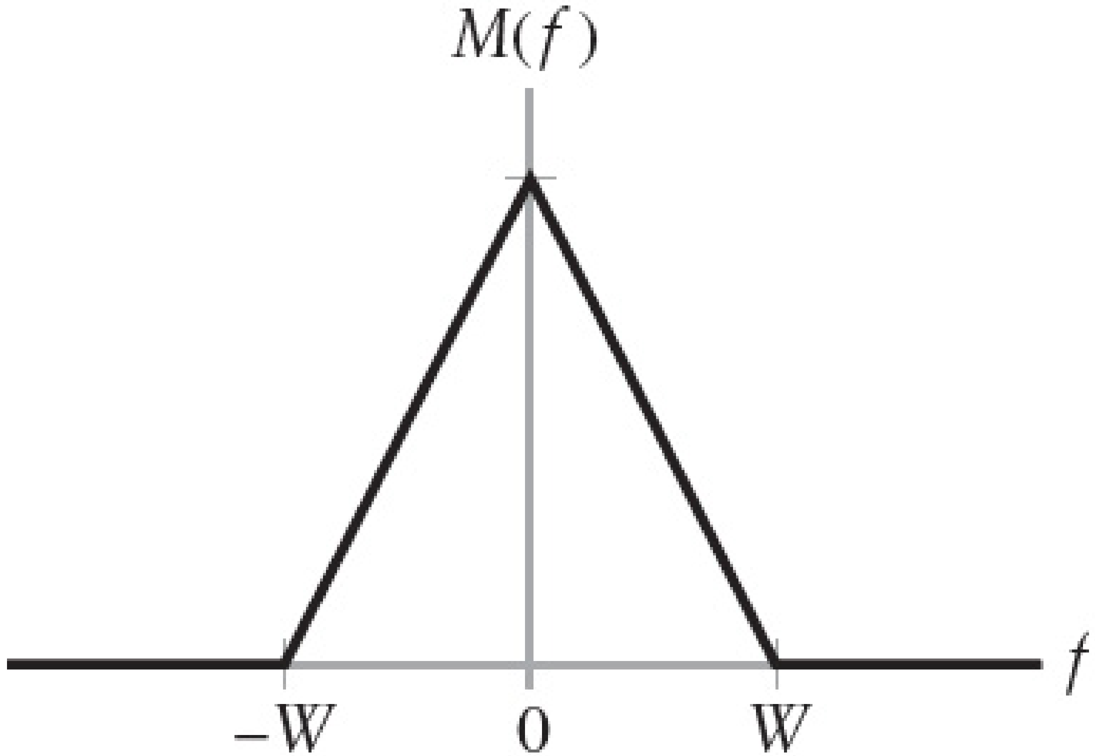


FIGURE 3.31