



Frequency Modulation

Part -2-

In general:

Remark:

We observe that the spectrum of an FM signal when $\beta \ll 1$ (called narrow band FM) is “similar” to the spectrum of a normal AM signal, in the sense that it consists of a carrier and two sidebands. The bandwidth of both signals is $2f_m$.

Carson's Rule

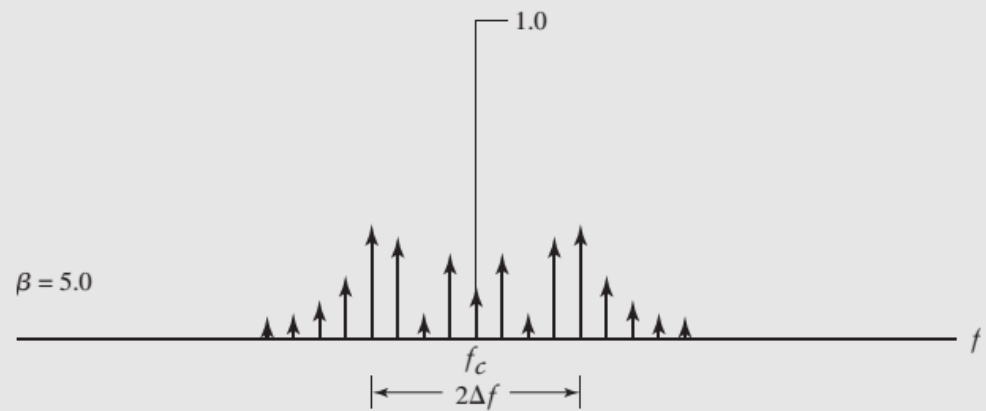
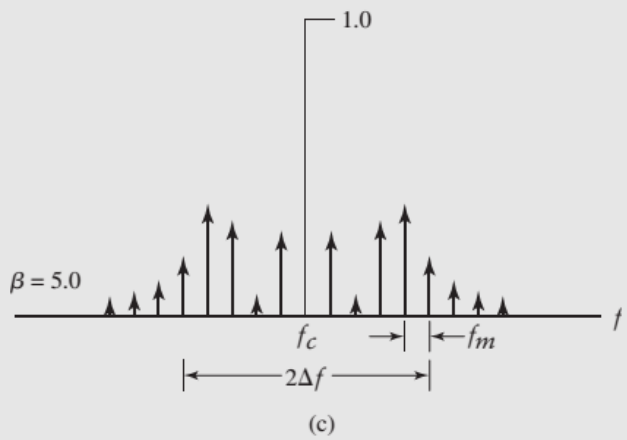
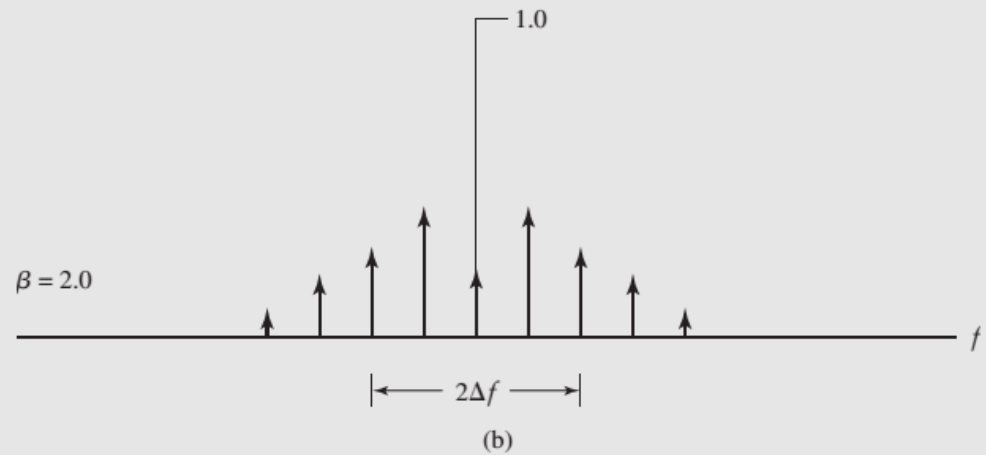
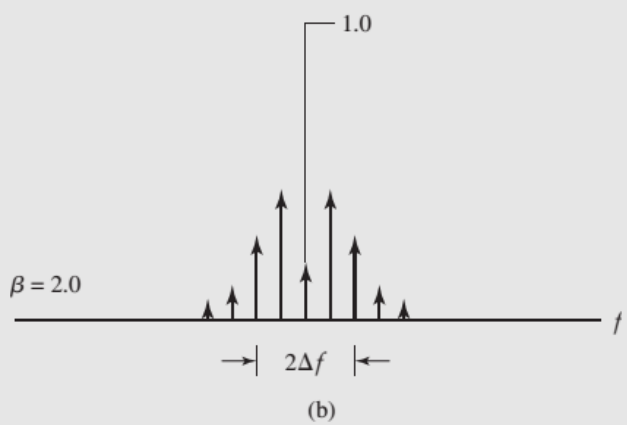
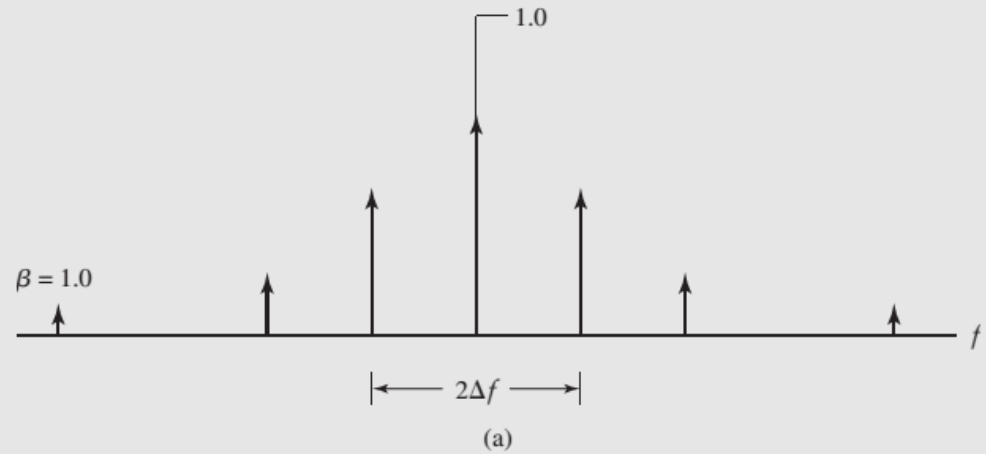
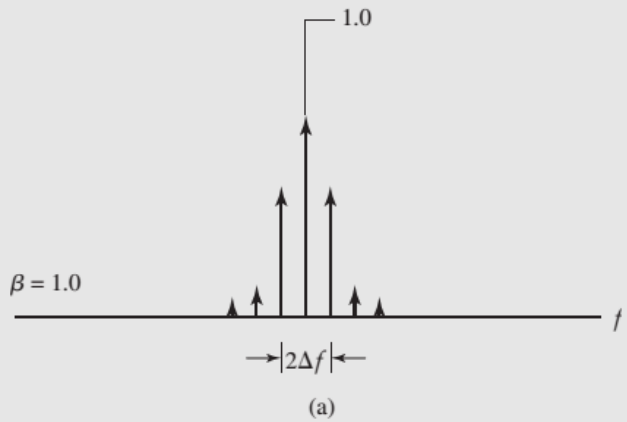
A 98% power B.W of an FM signal is estimated using Carson's rule:

$$B_T = 2(\beta + 1)f_m \quad B_T \approx 2\Delta f + 2f_m = 2\Delta f \left(1 + \frac{1}{\beta} \right)$$

In general:

$$D = \frac{\Delta f}{W} \quad B_T = 2(\Delta f + W)$$

FM Spectrum for Varying Amplitude and Frequency of Sinusoidal Modulating Wave



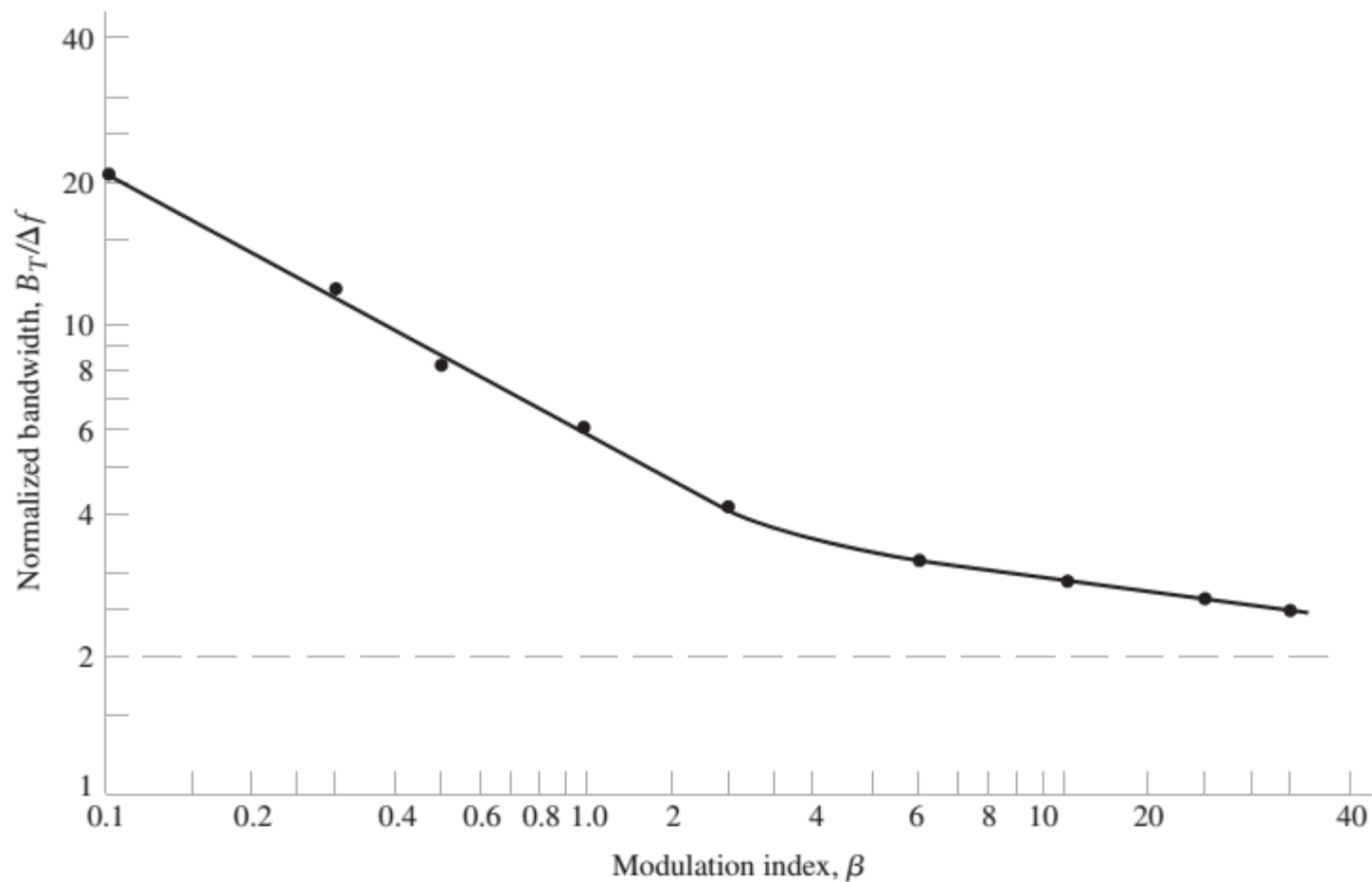


FIGURE 4.9 Universal curve for evaluating the one percent bandwidth of an FM wave.

We may thus define *the transmission bandwidth of an FM wave as the separation between the two frequencies beyond which none of the side frequencies is greater than one percent of the carrier amplitude obtained when the modulation is removed*

TABLE 4.2 Number of Significant Side-Frequencies of a Wide-Band FM Signal for Varying Modulation Index

Modulation Index β	Number of Significant Side-Frequencies $2n_{\max}$
0.1	2
0.3	4
0.5	4
1.0	6
2.0	8
5.0	16
10.0	28
20.0	50
30.0	70

$$|J_n(\beta)| > 0.01$$

EXAMPLE 4.3 Commercial FM Broadcasting

In North America, the maximum value of frequency deviation Δf is fixed at 75 kHz for commercial FM broadcasting by radio. If we take the modulation frequency $W = 15$ kHz, which is typically the “maximum” audio frequency of interest in FM transmission, we find that the corresponding value of the deviation ratio is [using Eq. (4.38)]

$$D = \frac{75}{15} = 5$$

Using the values $\Delta f = 75$ kHz and $D = 5$ in the generalized Carson rule of Eq. (4.39), we find that the approximate value of the transmission bandwidth of the FM signal is obtained as

$$B_T = 2(75 + 15) = 180 \text{ kHz}$$

On the other hand, use of the universal curve of Fig. 4.9 gives the transmission bandwidth of the FM signal to be

$$B_T = 3.2 \Delta f = 3.2 \times 75 = 240 \text{ kHz}$$

In this example, Carson’s rule underestimates the transmission bandwidth by 25 percent compared with the result of using the universal curve of Fig. 4.9.

Quadrature Carrier Multiplexing: Demodulation

Given $s(t)$, the objective is to recover $m_1(t)$ and $m_2(t)$ from $s(t)$. Consider first the in-phase channel

$$\begin{aligned}x_1(t) &= 2\cos 2\pi f_c t s(t) \\&= 2\cos 2\pi f_c t (A_c m_1(t)\cos 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t) \\&= 2A_c m_1(t)\cos^2 2\pi f_c t + 2A_c m_2(t)\sin\omega_c t \cos\omega_c t \\&= 2A_c m_1(t) \left(\frac{1+\cos 2\omega_c t}{2}\right) + A_c m_2(t)\sin 2\omega_c t \\&= A_c m_1(t) + A_c m_1(t)\cos 2\omega_c t + A_c m_2(t)\sin 2\omega_c t\end{aligned}$$

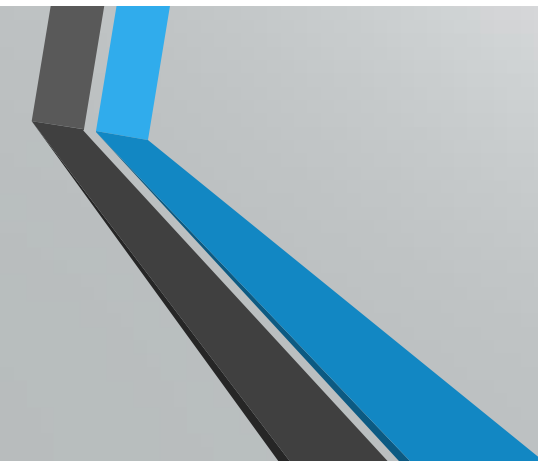
After low pass filtering, the output of the in-phase channel is

$$y_1(t) = A_c m_1(t).$$

Likewise, it can be shown that

$$y_2(t) = A_c m_2(t).$$

Note: Synchronization is a problem. That is to recover the message signals it is important that the two carrier signals (the sine and the cosine functions) at the receiver should have the same phase and frequency as the signals at the transmitting side. A phase error or a frequency error will result in an interference type of distortion. That is, A component of $m_2(t)$ will appear in the in-phase channel in addition to the desired signal $m_1(t)$ and a component of $m_1(t)$ will appear at the quadrature output.



Quadrature Carrier Multiplexing (QAM)

Quadrature Carrier Multiplexing: Modulation

This scheme enables two DSB-SC modulated signals to occupy the same transmission B.W and yet allows for the separation of the message signals at the receiver.

$m_1(t)$ and $m_2(t)$ are low pass signals each with a B.W = W Hz .

The composite signal is:

$$s(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t$$

$$s(t) = s_1(t) + s_2(t)$$

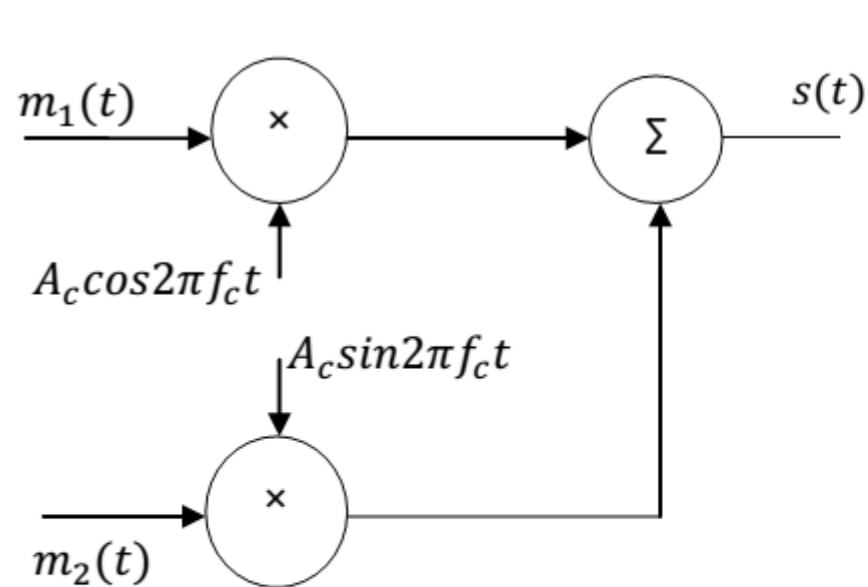
where $s_1(t)$ and $s_2(t)$ are both DSB-SC signals.

B.W of $s_1(t) = 2W$

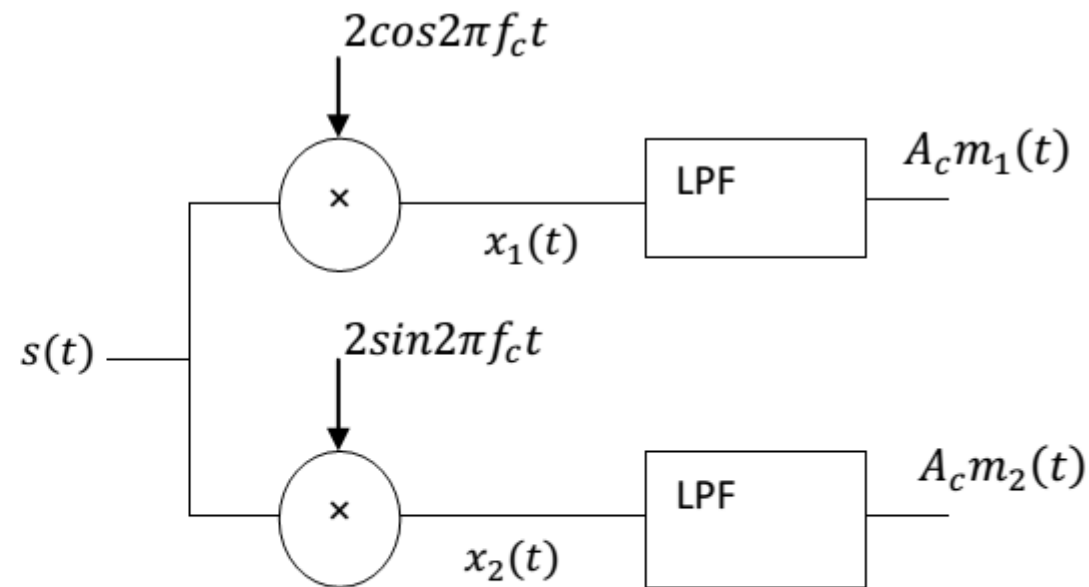
B.W of $s_2(t) = 2W$

B.W of $s(t) = 2W$

This method provides bandwidth conservation. That is, two DSB-SC signals are transmitted within the bandwidth of one DSB-SC signal. Therefore, this multiplexing technique provides bandwidth reduction by one half.



Modulation



Demodulation

Frequency Division Multiplexing:

A number of independent signals can be combined into a composite signal suitable for transmission over a common channel. The signals must be kept apart so that they do not interfere with each other and thus they can be separated at the receiving end.

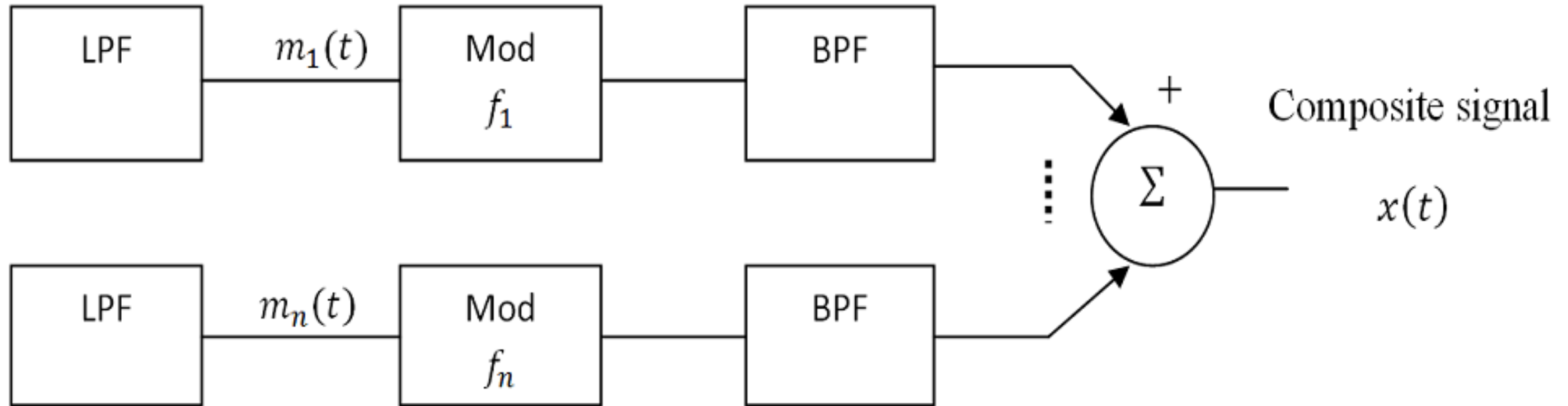


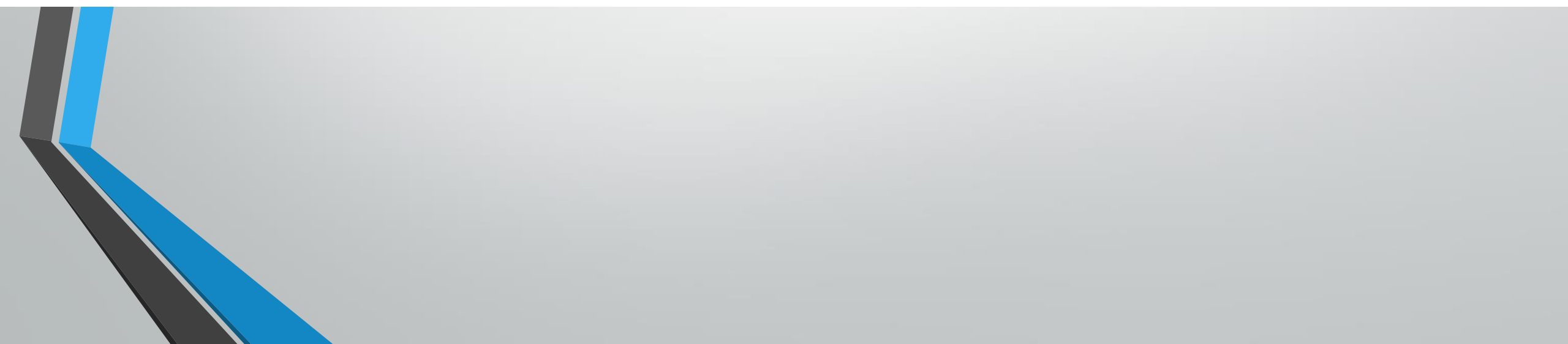
Illustration of FDM

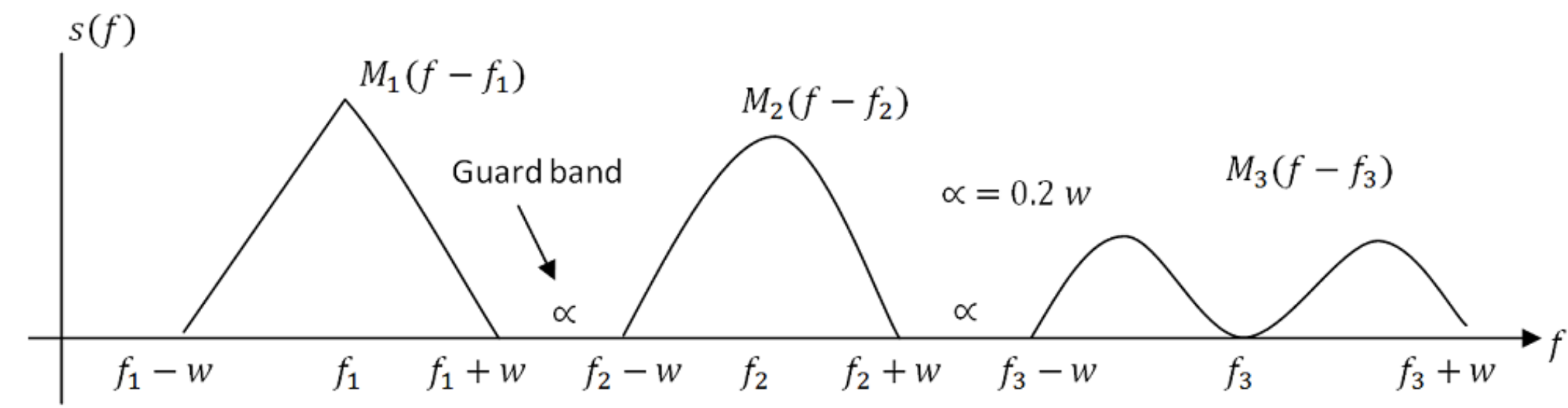
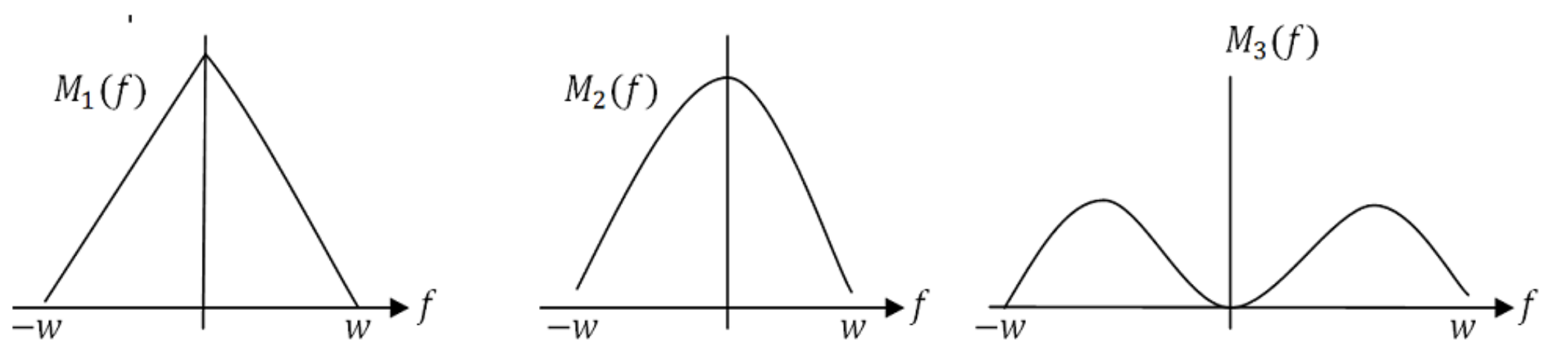
Let m_1, m_2 and m_3 be three baseband message signals each with a B.W = w .

The composite modulated signal $s(t)$ is

$$\begin{aligned} s(t) &= A_{c_1} m_1(t) \cos 2\pi f_1 t + A_{c_2} m_2(t) \cos 2\pi f_2 t + A_{c_3} m_3(t) \cos 2\pi f_3 t \\ &= s_1(t) + s_2(t) + s_3(t) \end{aligned}$$

s_1, s_2 and s_3 are DSB-SC signals with carrier frequencies f_1, f_2 and f_3 , respectively. If the spectrum of $m_1(t), m_2(t)$ and $m_3(t)$ are as shown, the spectrum of $s(t)$ can be found as shown below.



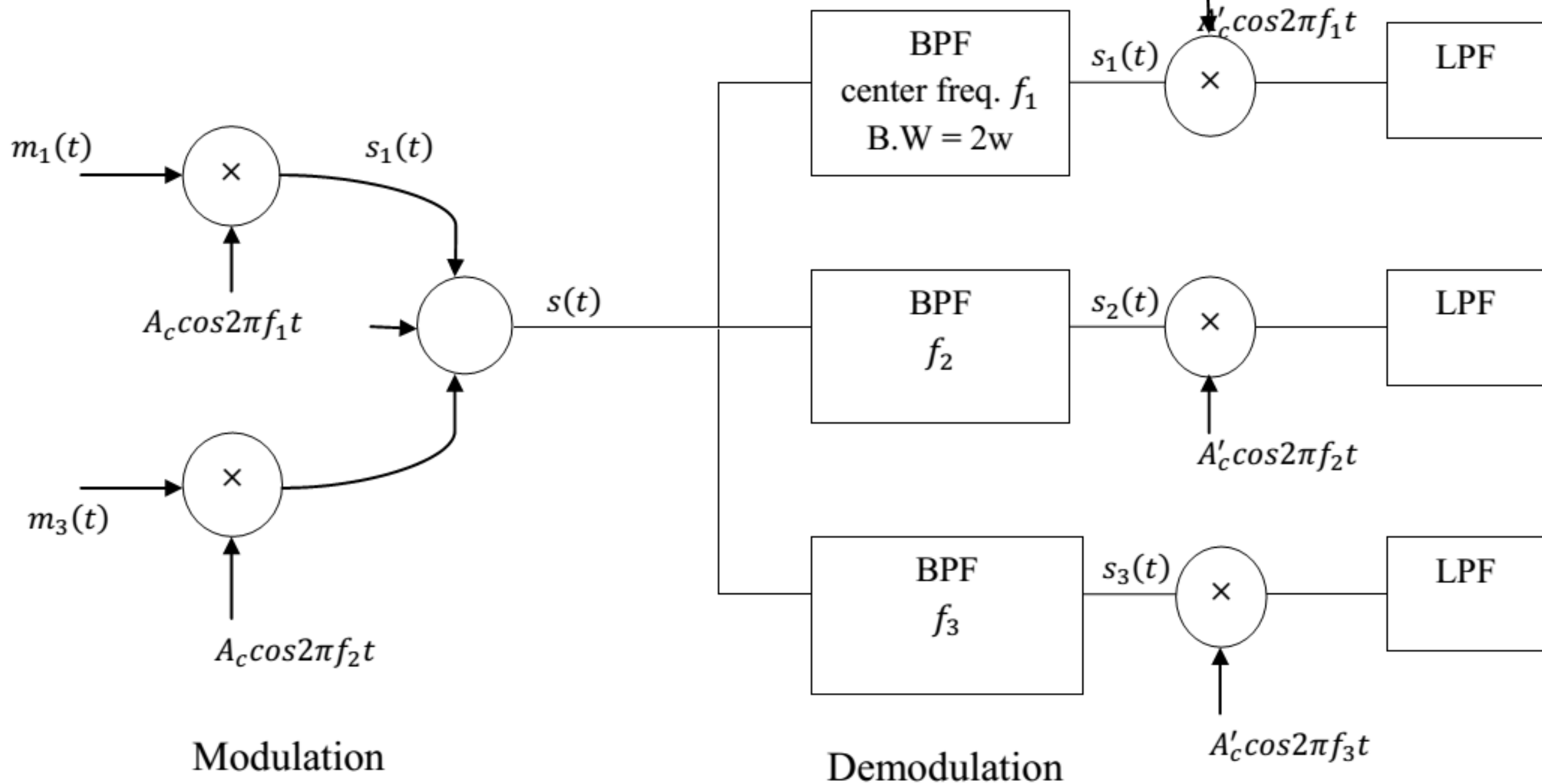


To prevent interference we demand that

$$f_2 - w \geq f_1 + w \text{ or } f_2 - f_1 \geq 2w$$

$$f_3 - w \geq f_2 + w \text{ or } f_3 - f_2 \geq 2w$$

The structure of the receiver is as follows:





AM/FM Radio

Communication Systems

- We have studied the basic blocks of any communication system
 - Modulator
 - Demodulator
- Modulation Schemes:
 - Linear Modulation (DSB, AM, SSB, VSB)
 - Angle Modulation (FM, PM)

AM/FM Radio System

- Principles:
 - Frequency Spectrum Sharing (many transmitters using one medium)
 - Demodulating desired signal and rejecting other signals transmitted at the same time

AM/FM Radio System

- The source signal is audio
- Different sources have different spectrum
 - Voice (speech)
 - Music
 - Hybrid signals (music, voice, singing)

AM/FM Radio System

- Different audio sources have different bandwidth "W"
 - Speech- 4kHz
 - High quality music- 15kHz
 - AM radio limits "baseband" bandwidth W to 5kHz
 - FM radio uses "baseband" bandwidth W to 15kHz

AM/FM Radio System

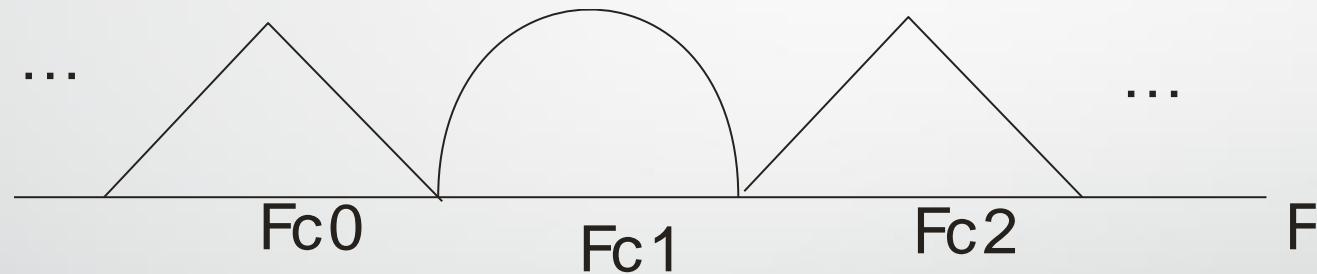
- Radio system should be able to receive any type of audio source simultaneously.
- Different stations with different sources transmit signals simultaneously.
- Different listeners tune to different stations simultaneously.

AM/FM Radio System

- The different radio stations share the frequency spectrum over the air through AM and FM modulation.
- Each radio station, within a certain geographical region, is designated a carrier frequency around which it has to transmit
- Sharing the AM/FM radio spectrum is achieved through Frequency Division Multiplexing (FDM)

Example of AM Radio Spectrum

- Different radio stations, different source signals



- Carrier spacing- 10kHz (AM)
- Bandwidth (3-5kHz)

AM/FM Radio System

- For AM radio, each station occupies a maximum bandwidth of 10 kHz
- Carrier spacing is 10 kHz
- For FM radio, each station occupies a bandwidth of 200 kHz, and therefore the carrier spacing is 200 kHz

AM/FM Radio System

- Transmission Bandwidth:
- is the bandwidth occupied by a message signal in the radio frequency spectrum
- is also the carrier spacing
- AM: $B_T = 2W$
- FM: $B_T = 2(D + 1)W$ (Carson's Rule)

AM/FM Radio Receiver

- Design of AM/FM radio receiver
- The radio receiver has to be cost effective
- Requirements:
 - Has to work with both AM and FM signals
 - Tune to and amplify desired radio station
 - Filter out all other stations
 - Demodulator has to work with all radio stations regardless of carrier frequency

AM/FM Radio Receiver

- For the demodulator to work with any radio signal, we “convert” the carrier frequency of any radio signal to Intermediate Frequency (IF)
- Radio receiver design can be optimized for that frequency
- IF filter and a demodulator for IF frequency

AM/FM Radio Spectrum

- Recall that AM and FM have different radio frequency (RF) spectrum ranges:
 - AM: 540 kHz – 1600 kHz
 - FM: 88 MHz – 108 MHz
- Therefore, two IF frequencies
 - AM: 455 kHz
 - FM: 10.7 MHz

AM/FM Radio Receiver

- A radio receiver consists of the following:
 - A Radio Frequency (RF) section
 - An RF-to-IF converter (mixer)
 - An Intermediate Frequency (IF) section
 - Demodulator
 - Audio amplifier



AM/FM Radio Receiver

- This is known as the “Superheterodyne” receiver
- Two stages: RF and IF
(filtering and amplification)
- The receiver was designed by Armstrong

AM/FM Radio Receiver

- RF Section
 - Tunes to the desired RF frequency, f_c
 - Includes RF bandpass filter centered around f_c
 - The bandwidth B_{RF}
 - Usually not narrowband, passes the desired radio station and adjacent stations

AM/FM Radio Receiver

- The minimum bandwidth of RF filter:

$$B_{RF} > B_T$$

- Passes the desired radio channel, and adjacent channels

AM/FM Radio Receiver

- RF-IF converter:
 - Converts carrier frequency to IF frequency
- How can we convert signals with different RF frequencies to the same IF frequency?

AM/FM Radio Receiver

- Local oscillator with a center frequency f_{LO}
- f_{LO} is a function of RF carrier frequency

$$f_{LO} = f_c + f_{IF}$$



AM/FM Radio Receiver

- RF-to-IF receiver includes:
 - An oscillator with a variable frequency f_{LO}
(varies with RF carrier frequency)
 - By tuning to the channel, you are tuning the local oscillator and RF tunable filter at the same time.

AM/FM Radio Receiver

- All stations are translated to a fixed carrier frequency for adequate selectivity.



AM/FM Radio Receiver

- Two frequencies are generated at the output of product modulator:

$$f_{LO} + f_c = 2f_c + f_{IF}$$

$$f_{LO} - f_c = f_{IF}$$

- The higher frequency component is eliminated through filtering
- We are left with IF frequency

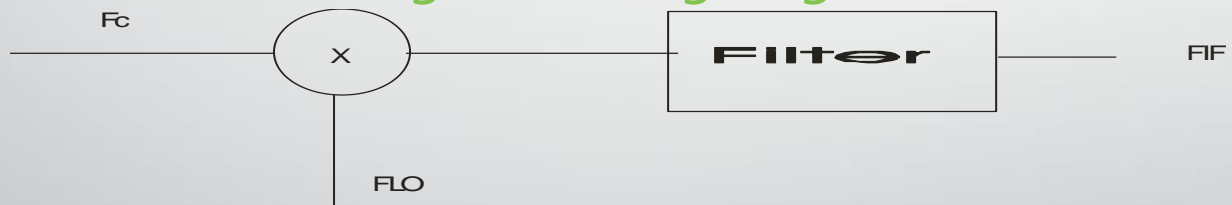
AM/FM Radio Receiver

- One problem with this receiver:
“Image Signal”
- Image signal has a center frequency:

$$f_i = f_c + 2 f_{IF}$$

AM/FM Radio Receiver

- If an “image signal” exists at the input of the “RF-to-IF” converter, then the output of the converter will include the desired signal + image signal



Local Oscillator

$$f_{LO} + f_i = (f_c + f_{IF}) + (f_c + 2f_{IF}) = 2f_c + 3f_{IF}$$

$$f_{LO} - f_i = (f_c + f_{IF}) - (f_c + 2f_{IF}) = -f_{IF}$$

AM/FM Radio Receiver

- Example: Incoming carrier frequency
1000 kHz,
- Local oscillator = $1000+455=1455$ kHz
- Consider another carrier at 1910 kHz
- If this is passed through the same oscillator, will have a
 $1910-1455=455$ kHz component
- Therefore, both carriers will be passed through RF-to-IF
converter

AM/FM Radio Receiver

- Therefore, RF filter should be designed to eliminate image signals
- The frequency difference between a carrier and its image signal is: $2f_{IF}$
- RF filter doesn't have to be selective for adjacent stations, have to be selective for image signals
- Therefore,

$$B_T < B_{RF} < 2f_{IF}$$

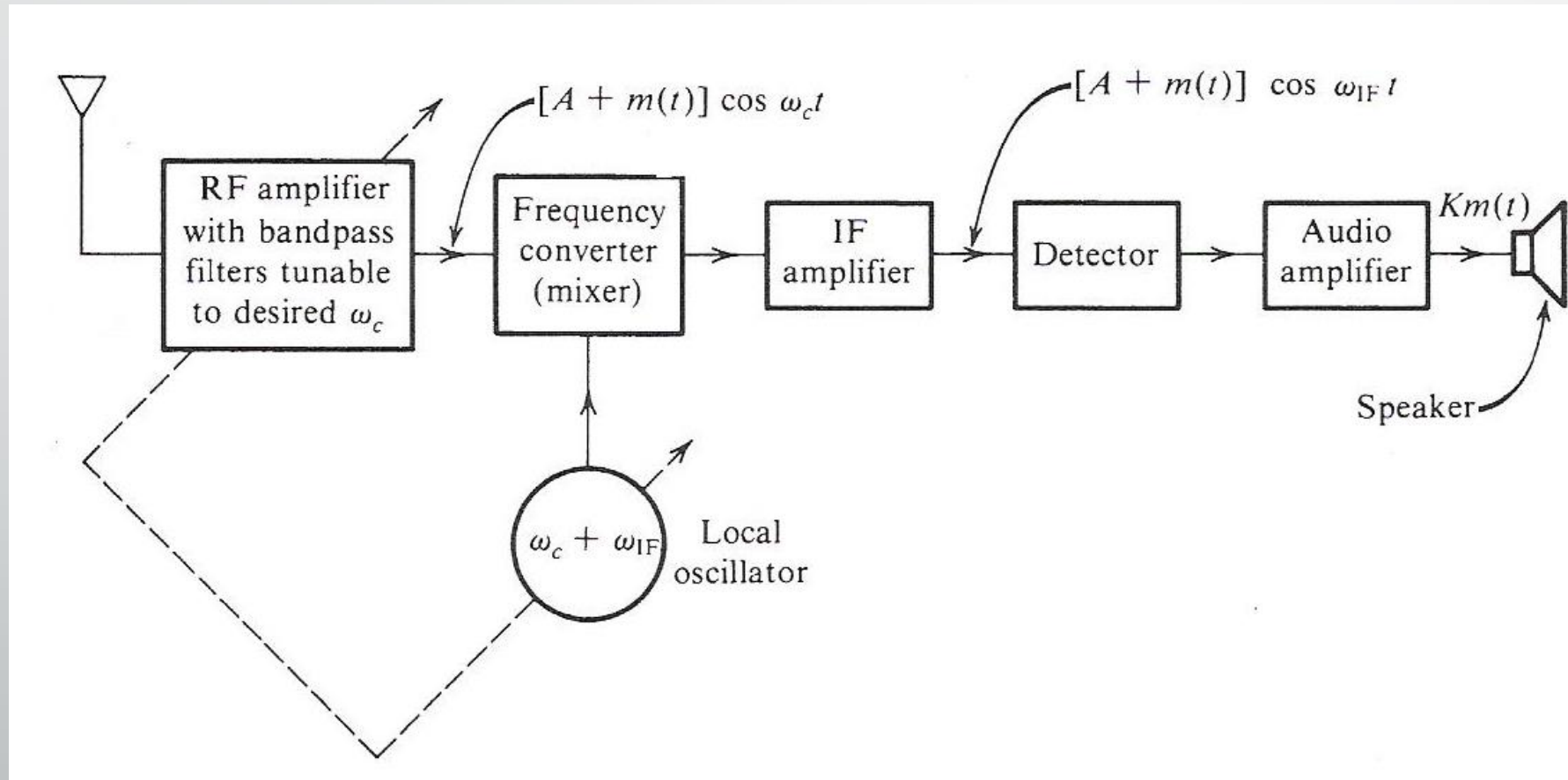
AM/FM Radio Receiver

- IF filter:
 - Center frequency f_{IF}
 - Bandwidth approximately same as transmission bandwidth, B_T
 - For AM: $B_T = 2W$
 - For FM: $B_T = 2(D+1)W$

AM/FM Radio Receiver

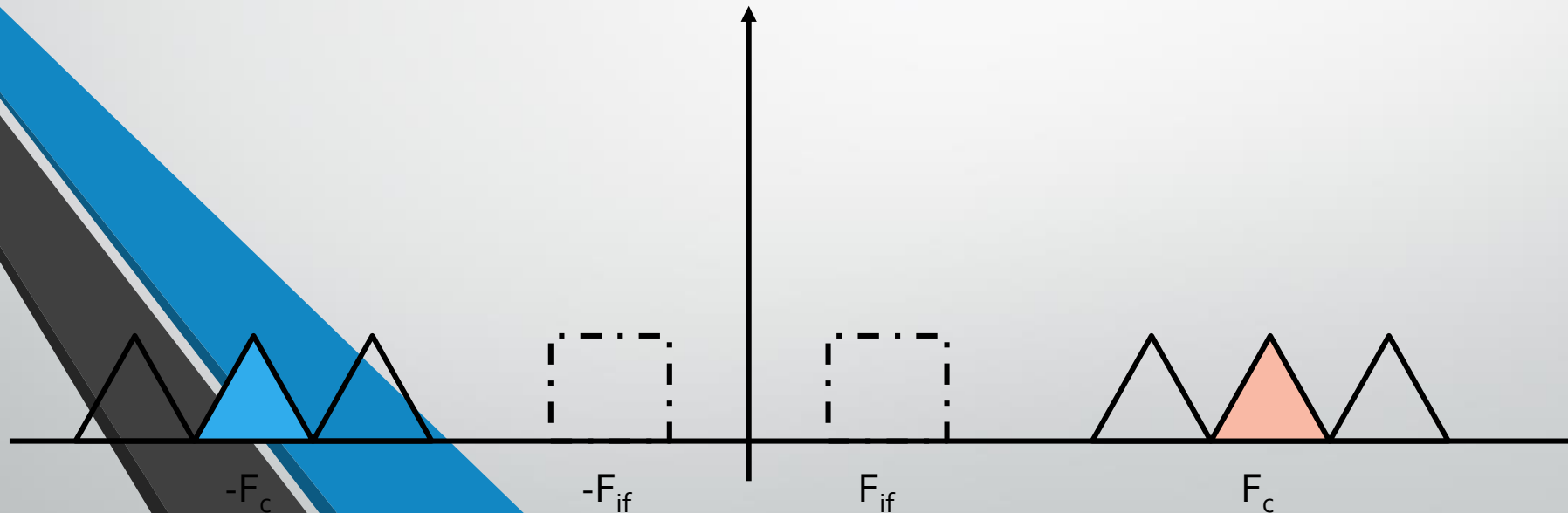
- Depending on the type of the received signal, the output of “IF filter” is demodulated using AM or FM demodulators.
- For AM: envelope detector
- For FM: frequency discriminator

Super heterodyne receiver



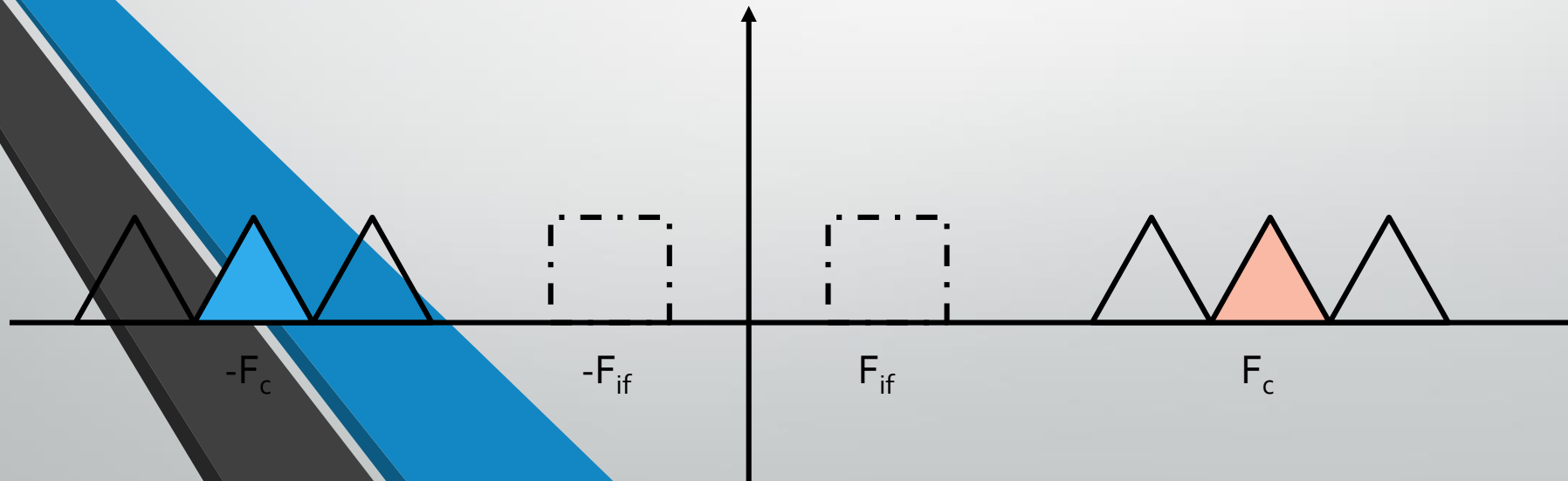
What is the intermediate frequency f_{if} ?

- It is fixed frequency located at 455 kHz
- The IF filter is band-pass with center frequency of 455 kHz and bandwidth equal to the bandwidth of one AM channel approximately =10 kHz.

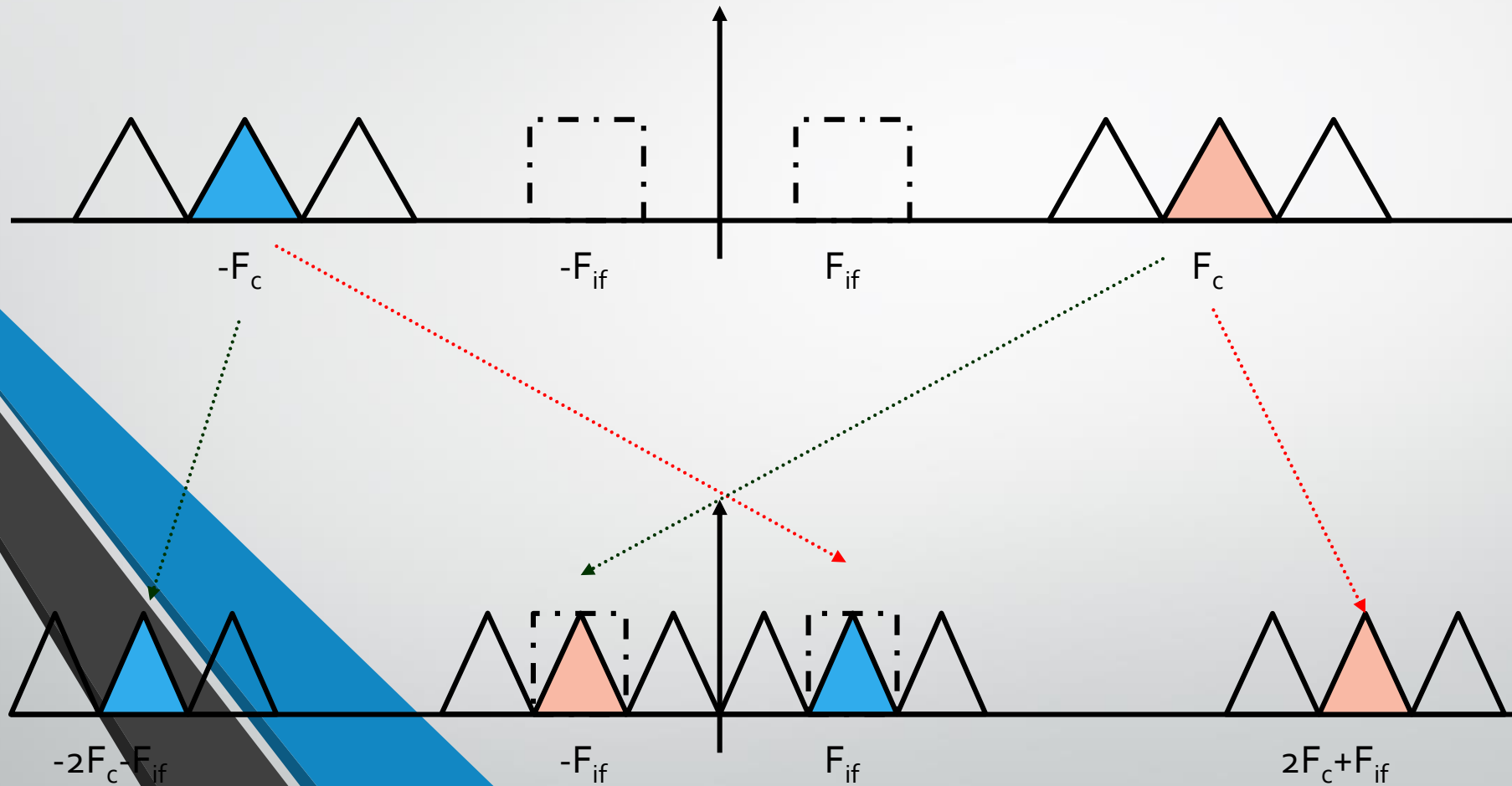


Why do we need the IF Stage?

- It is too difficult to design a tunable and sharp filter. So we design sharp & fixed filter.
- The channel to be filtered out should first be frequency shifted to the IF frequency by a frequency converter as shown in the super heterodyne Figure



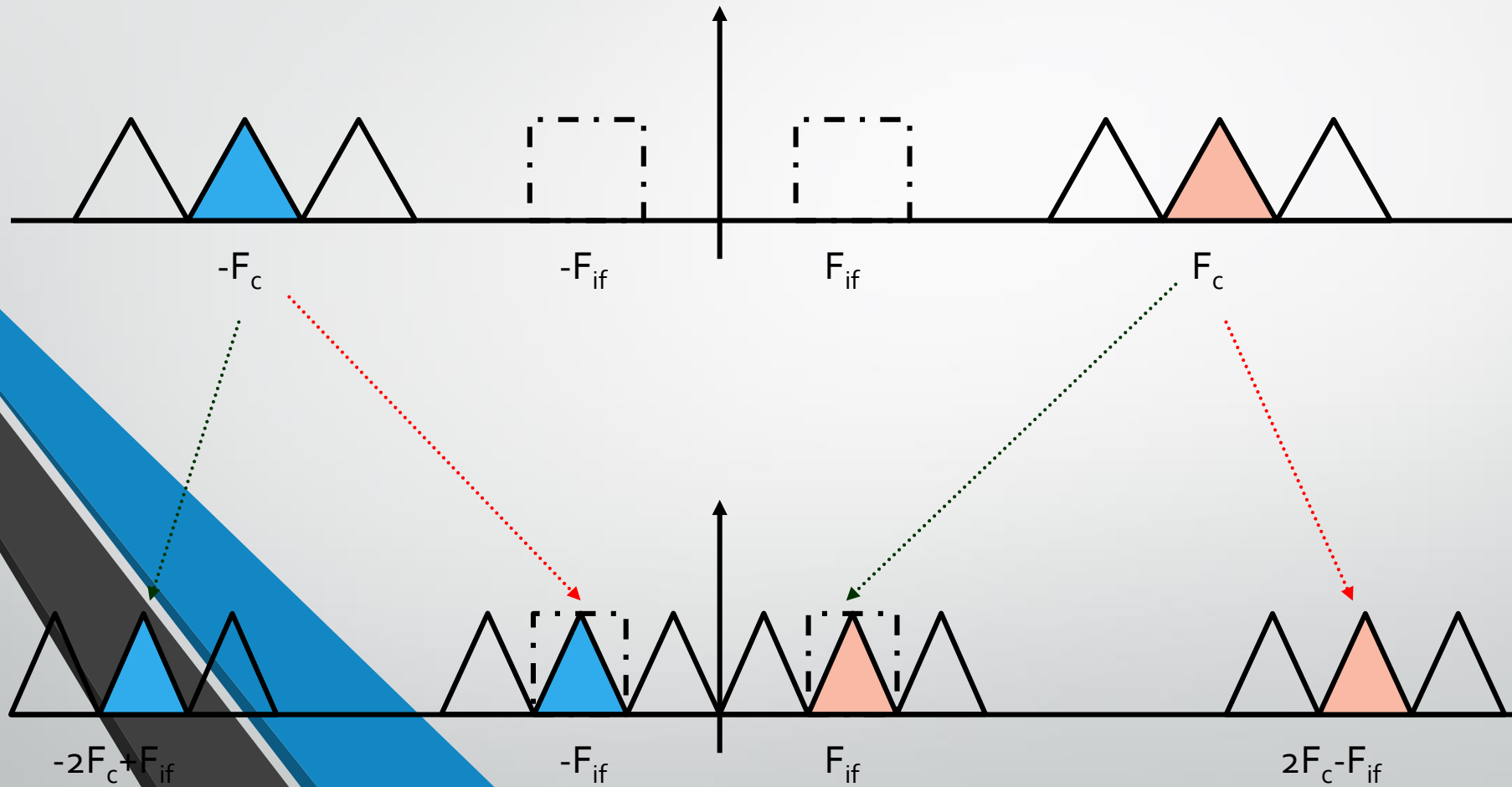
Up conversion $F_c + F_{if}$



Second Image
Subtract $F_c + F_{if}$

First Image
Add $F_c + F_{if}$

Down conversion $F_c - F_{if}$



Second Image
Subtract $F_c - F_{if}$

First Image
Add $F_c - F_{if}$

Why up conversion is better than down conversion?

The range of radio station on AM is: 550kHz→1600kHz

Up ($F_c + F_{if}$): 1005kHz→2055kHz

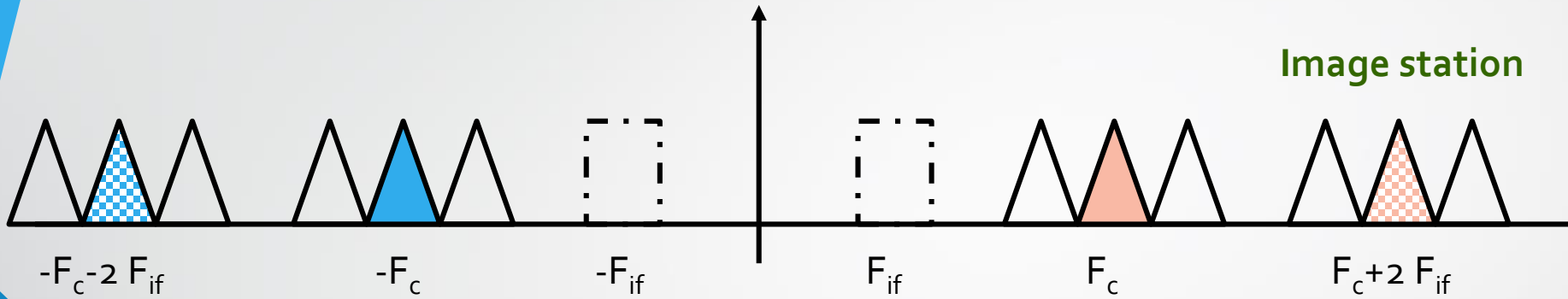
ratio frequency is 1:2

down ($F_c - F_{if}$): 95kHz→1155kHz

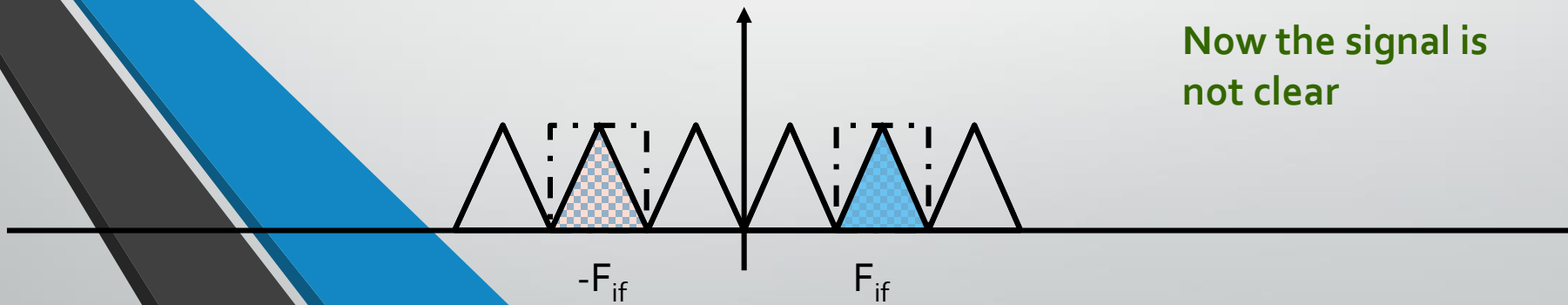
ratio frequency is 1:12

We see the ratio frequency in up conversion is smaller than in down conversion which means it is easier to design.

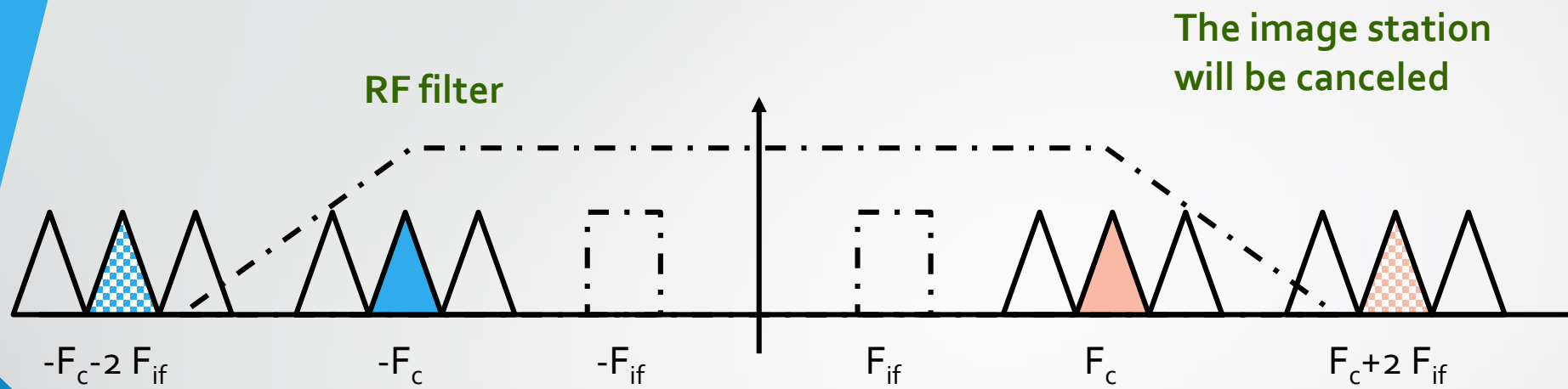
Why we filter at RF stage?



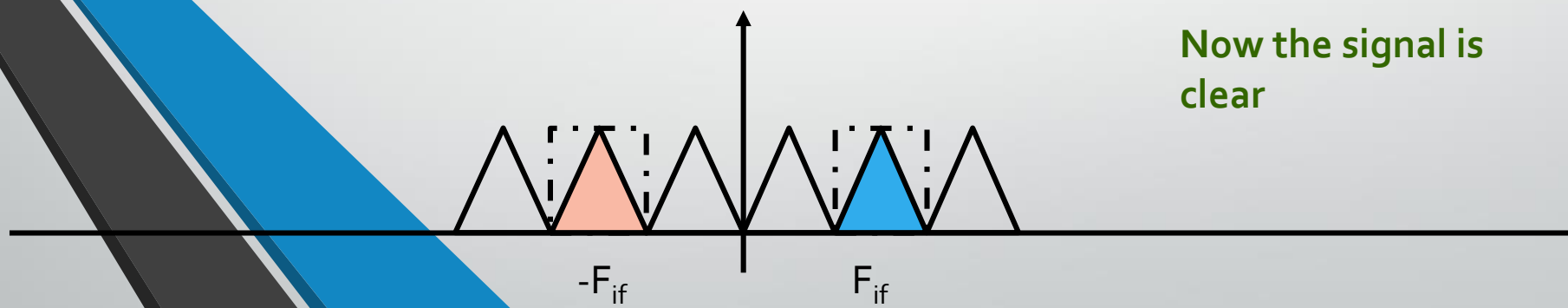
After up conversion



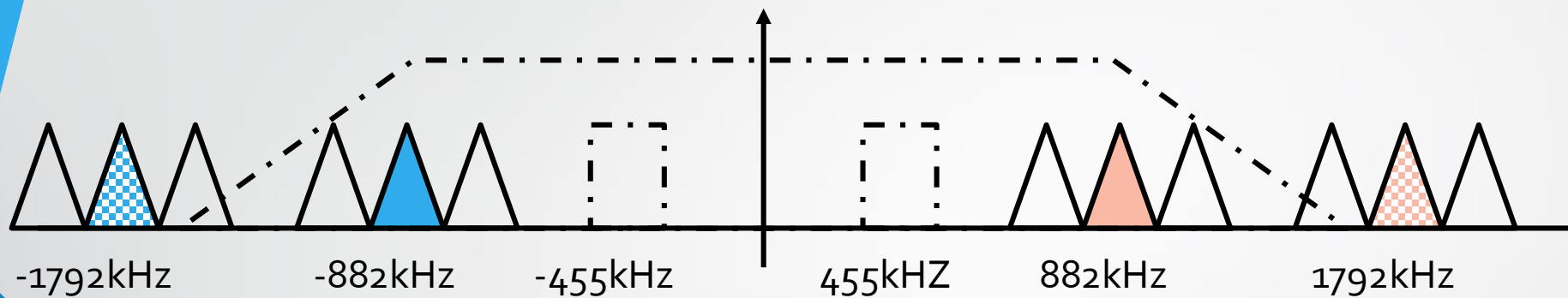
The image station is a station that is spaced by $2 * F_{if}$ from the desired station as shown in the figure



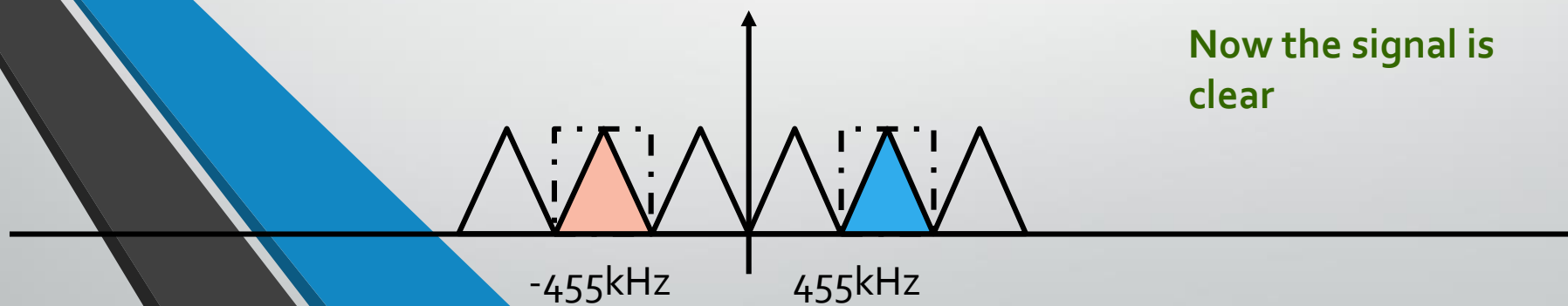
After up conversion



For example: we take $F_c=882\text{kHz}$ (the Holy Quran station)



After up conversion



Now the signal is clear