Frequency and Phase Modulation

Frequency and Phase Modulation

To generate an angle modulated signal, the amplitude of the modulated carrier is held constant and either the phase or the time derivative of the phase is varied linearly with the message signal m(t).

The expression for an angle modulated signal is:

$$s(t) = A_c \cos\left(\omega_c t + \theta(t)\right),$$

ω_c is the modulated carrier frequency.

The instantaneous frequency of s(t) is :

$$f_i(t) = \frac{1}{2\pi} \times \frac{d}{dt}\left(\omega_c t + \theta(t)\right) = f_c + \frac{1}{2\pi} \times \frac{d\theta(t)}{dt}$$

For **phase modulation**, the phase is directly proportional to the modulating signal :

$\theta(t) = k_p m(t),$ k_p is the phase sensitivity measured in rad/volt.

The peak phase deviation is

$$\Delta\theta = k_p \times \max\left(m(t)\right).$$

For **frequency modulation**, the frequency deviation of the carrier is proportional to the modulating signal:

$$\frac{1}{2\pi} \times \frac{d\theta(t)}{dt} = k_f m(t) \Rightarrow f_i = f_c + k_f m(t).$$

The frequency deviation from the un-modulated carrier is

$$f_i(t) - f_c = \frac{1}{2\pi}\frac{d\theta}{dt}$$

The peak frequency deviation is

$$\Delta f = \max\{\frac{1}{2\pi} \times \frac{d\theta}{dt}\}.$$

The time domain representation of a phase modulated signal is :

$$s(t) = A_c \cos\left(\omega_c t + k_p m(t)\right).$$

The time domain representation of a frequency modulated signal is :

$$s(t) = A_c \cos\left(\omega_c t + 2\pi k_f \int_{-\infty}^{t} m(\alpha) d\alpha\right).$$

where $\theta(t) = 2\pi k_f \int_{-\infty}^{t} m(\alpha) d\alpha$

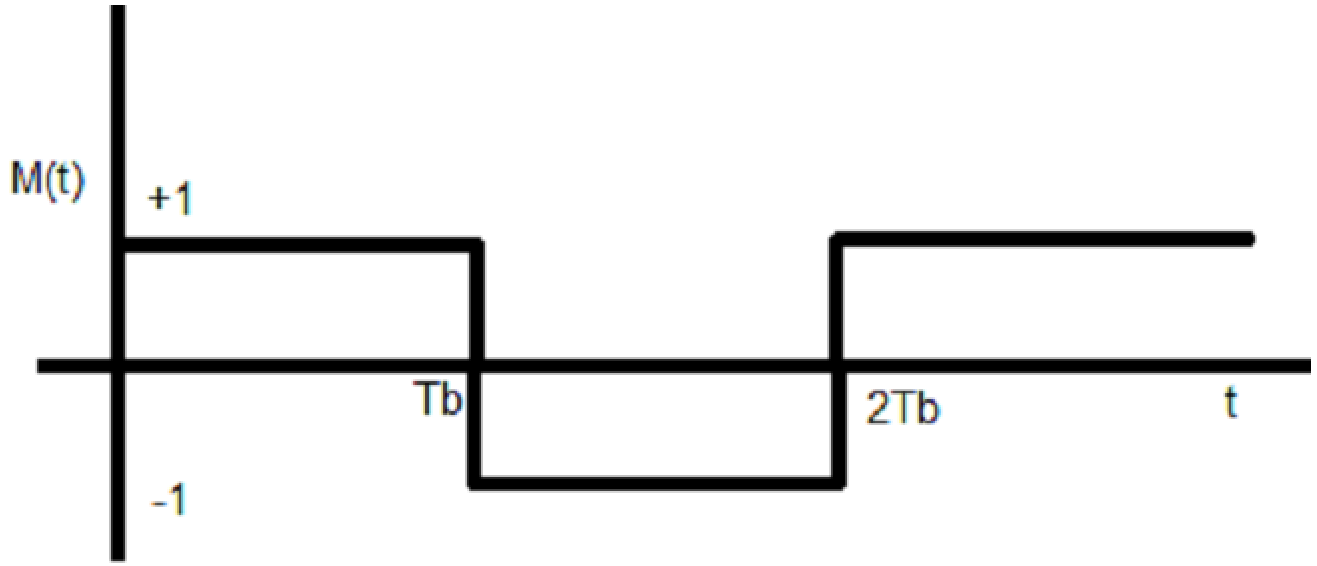
The average power in $s(t)$, for frequency modulation (FM) or phase modulation (PM) is:

$$p_{ava} = \frac{(A_c)^2}{2} = constant.$$

Example: Frequency Shift Keying.

The periodic square signal m(t), shown below, frequency modulates the carrier $c(t) = A_c \cos(2\pi 100t)$ to produce the signal $s(t) = A_c \cos\left((2\pi 100t) + 2\pi k_f \int m(\alpha)d\alpha\right)$ where k_f =10 HZ/V.

a. Find and plot the instantaneous frequency $f_i(t)$.
b. Find and sketch s(t).



Solution:

a) The instantaneous frequency is

$$f_i = f_c + k_f \times m(t)$$

f_i = { 100+10 =110 Hz when m(t) =+1.

f_i = { 100 -10 = 90Hz when m(t) =-1.

For $0 < t \leq T_b$, $f_i = 110$ Hz

$For\ T_b \leq t \leq 2T_b\ f_i = 90$ Hz

In digital transmission, we will see that a binary (1) may be represent by a signal of frequency f_1 for $0 \leq t \leq T_b$ and a binary (0) by a signal of frequency f_2 for $0 \leq t \leq T_b$.

b) The two signal possible to transmitted signal are :

$$s(t) = A\cos(2\pi(110)t), \qquad when\ m(t) = +1$$

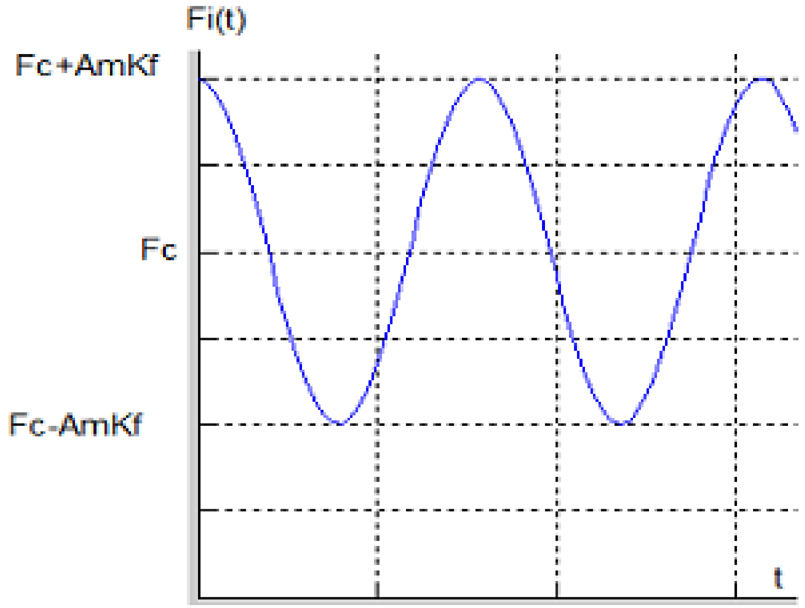
$$s(t) = A\cos(2\pi(90)t), \qquad when\ m(t) = -1$$

Single Tone Frequency Modulation:

Assume that the message $m(t) = A_m \cos \omega_m t$.

The instantaneous frequency is:

$$f_i = f_c + k_f m(t) = f_c + A_m k_f \cos 2\pi f_m t.$$



This frequency is plotted in the figure.

The peak frequency deviation (from the un-modulated carrier) is :

$$\Delta f = k_f A_m.$$

The FM signal is:

$$s(t) = A_c \cos\ (\omega_c t + \beta \sin 2\pi f_m t).$$

Where β is the FM modulation index:

$$\beta = \frac{k_f A_m}{f_m} = \frac{peak\ frequency\ deviation}{message\ bandwidth} = \frac{\Delta f}{f_m}$$

Spectrum of a Single-Tone FM Signal

The objective is to find a meaningful definition of the bandwidth of an FM signal:

Let $m(t) = A_m \cos 2\pi f_m t$ be the message signal, then the FM signal is:

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

Where $\beta = \frac{\Delta f}{f_m} = \frac{peak\ frequency\ deviation}{message\ bandwidth}$ is the modulation index.

$\Longrightarrow$ Recall that:

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

which can be rewritten as:

$$s(t) = Re\left\{e^{j(2\pi f_c t + \beta \sin 2\pi f_m t}\right\}$$

$$= Re\{e^{j(2\pi f_c t)} \times e^{j(\beta \sin 2\pi f_m t)}\}$$

Remember that: $e^{j\theta} = cos\theta + jsin\ \theta$ and that $cos\theta = Re\{e^{j\theta}\}$

The function [$\beta \sin 2\pi f_m t$] is "sinusoidal" and periodic with $T_m = \frac{1}{f_m}$. Therefore, $e^{j(\beta \sin 2\pi f_m t)}$ is also periodic with $T_m = \frac{1}{f_m}$.

As we know, a periodic function g(t) can be expanded into a complex Fourier series as:

$$g(t) = \sum_{-\infty}^{\infty} C_n \, e^{jn\omega_m t} \quad where \;\; C_n = \frac{1}{T_m}\int_0^{T_m} g(t)\, e^{-j\omega_m nt}\, dt \;\; .$$

$\Longrightarrow$ If we let $g(t) = e^{j(\beta \sin 2\pi f_m t)}$

It out that $\Longrightarrow$ $C_n = J_n(\beta).$

Where $J_n(\beta)$ is the Bessel function of the first kind of order n.

Hence, $$g(t) = \sum_{-\infty}^{\infty} J_n(\beta)\, e^{jn\omega_m t}$$

Substituting into $s(t)$, we get:

$$\Rightarrow \; s(t) = A_c \, Re\{e^{j(2\pi f_c t)} \times \sum_{-\infty}^{\infty} J_n(\beta) \, e^{jn\omega_m t}\}$$

$$= A_c \, Re\{\sum_{-\infty}^{\infty} J_n(\beta) \times e^{j2\pi(f_c+nf_m)t}\}$$

$$= A_c \sum_{-\infty}^{\infty} J_n(\beta) \times \cos(2\pi(f_c + nf_m)t)$$

Finally, the FM signal can be represented as

$$s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta) \times \cos(2\pi(f_c + nf_m)t)$$

Bessel Functions:

The Bessel equation of order n is:

$$x^2 \frac{dy^2}{dx_2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$$

This is a second order differential equation with variable coefficient. We can solve it by the power series method, for example:

Let $y = \sum_{n=0}^{\infty} C_n\, x^n \quad , \quad \frac{dy}{dx} = \sum_{n=1}^{\infty} n\, C_n\, x^{n-1} \quad , \quad \frac{dy^2}{dx_2} = \sum_{n=2}^{\infty} n(n-1)C_n\, x^{n-2}$.

Substituting $y, \frac{dy}{dx}$ *and* $\frac{dy^2}{dx_2}$ into the differential equation and equating terms of equal power results in:

$$y = \sum_{m=0}^{\infty} \frac{(-1)^m \times (\frac{1}{2}x)^{n+2m}}{m!\,(n+m)!}$$

The solution for each value of n (see the D.E where n appears) is $J_n(x)$, the Bessel function of the first kind of order n.

Some Properties of $J_n(x)$:

1- $J_n(x) = (-1)^n J_{-n}(x)$.

2- $J_n(x) = (-1)^n J_n(-x)$.

3- Recurrence formula $\Longrightarrow$ $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.

4- For small values of x: $\Longrightarrow$ $J_n(x) \cong \frac{x^n}{2^n\ n!}$

Therefore, $J_0(x) \cong 1$

$J_1(x) \cong \frac{x}{2}$

$J_n(x) \cong 0 \ \ for\ n > 1$.

5- For large value of x:

$J_n(x) \cong \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\pi}{4} - \frac{n\pi}{2})$, $J_n(x)$ behaves like a sine function with progressively decreasing amplitude.

6- For real x and fixed, $J_n(x) \longrightarrow 0\ as\ n \longrightarrow \infty$.

7- $\sum_{-\infty}^{\infty} (J_n(x))^2 = 1$, $for\ all\ x$.

The Bessel Functions Table for various values of β and n.

- Table 1, notice that $x = \beta$.

	$J_n(x)$								
n\x	0.5	1	2	3	4	6	8	10	12
0	0.9385	0.7652	0.2239	−0.2601	−0.3971	0.1506	0.1717	−0.2459	0.0477
1	0.2423	0.4401	0.5767	0.3391	−0.0660	−0.2767	0.2346	0.0435	−0.2234
2	0.0306	0.1149	0.3528	0.4861	0.3641	−0.2429	−0.1130	0.2546	−0.0849
3	0.0026	0.0196	0.1289	0.3091	0.4302	0.1148	−0.2911	0.0584	0.1951
4	0.0002	0.0025	0.0340	0.1320	0.2811	0.3576	−0.1054	−0.2196	0.1825
5	—	0.0002	0.0070	0.0430	0.1321	0.3621	0.1858	−0.2341	−0.0735
6		—	0.0012	0.0114	0.0491	0.2458	0.3376	−0.0145	−0.2437
7			0.0002	0.0025	0.0152	0.1296	0.3206	0.2167	−0.1703
8			—	0.0005	0.0040	0.0565	0.2235	0.3179	0.0451
9				0.0001	0.0009	0.0212	0.1263	0.2919	0.2304
10				—	0.0002	0.0070	0.0608	0.2075	0.3005
11					—	0.0020	0.0256	0.1231	0.2704
12						0.0005	0.0096	0.0634	0.1953
13						0.0001	0.0033	0.0290	0.1201
14						—	0.0010	0.0120	0.0650

n	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$	$\beta = 1$	$\beta = 2$	$\beta = 5$	$\beta = 8$	$\beta = 10$
0	0.997	0.990	0.938	0.765	0.224	−0.178	0.172	−0.246
1	0.050	0.100	0.242	0.440	0.577	−0.328	0.235	0.043
2	0.001	0.005	0.031	0.115	0.353	0.047	−0.113	0.255
3				0.020	0.129	0.365	−0.291	0.058
4				0.002	0.034	0.391	−0.105	−0.220
5					0.007	0.261	0.186	−0.234
6					0.001	0.131	0.338	−0.014
7						0.053	0.321	0.217
8						0.018	0.223	0.318
9						0.006	0.126	0.292
10						0.001	0.061	0.207
11							0.026	0.123
12							0.010	0.063
13							0.003	0.029
14							0.001	0.012
15								0.004
16								0.001

The FM Signal Series Representation

We saw earlier that a single tone FM signal can be represented in a Fourier series as :

$$s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta) \times \cos(2\pi(f_c + nf_m)t)$$

The first few terms in this expansion are:

$$s(t) = A_c \{ J_0(\beta)\cos(2\pi f_c t) + J_1(\beta)\cos 2\pi(f_c + f_m)t + J_{-1}(\beta)\cos 2\pi(f_c - f_m)t \; + J_2(\beta)\cos 2\pi(f_c + 2f_m)t \; + J_{-2}(\beta)\cos 2\pi(f_c - 2f_m)t + \cdots..$$

The FM signal consists of infinite number of spectral components concentrated around f_c. Therefore, the theoretical bandwidth of the signal is infinity. That is to say, if we need to recover the FM signal without any distortion, all spectral components must be accommodated. This means that a channel with infinite bandwidth is needed. This is of course not practical since the frequency spectrum is shared by many users.

In the following discussion we need to truncate the series so that say 99% of the total average power is contained within a certain bandwidth. But first let us find the total average power using the series approach.

The total average power in s(t)

Note that s(t) consists of an infinite number of Fourier terms, and the power in s(t) will be equal the power in the respective Fourier components .

Any term in s (t) takes the form: $A_c J_n(\beta)\cos(2\pi(f_c + nf_m)t)$

The average power in this term is: $\frac{(A_c)^2 (J_n(\beta))^2}{2}$

Hence the total power in s(t) is:

$$< S^2(t) > = \frac{A_C{}^2 J_0{}^2(\beta)}{2} + \frac{A_C{}^2 J_1{}^2(\beta)}{2} + \frac{A_C{}^2 J_{-1}{}^2(\beta)}{2} + \frac{A_C{}^2 J_2{}^2(\beta)}{2} + \frac{A_C{}^2 J_{-2}{}^2(\beta)}{2} + \cdots$$

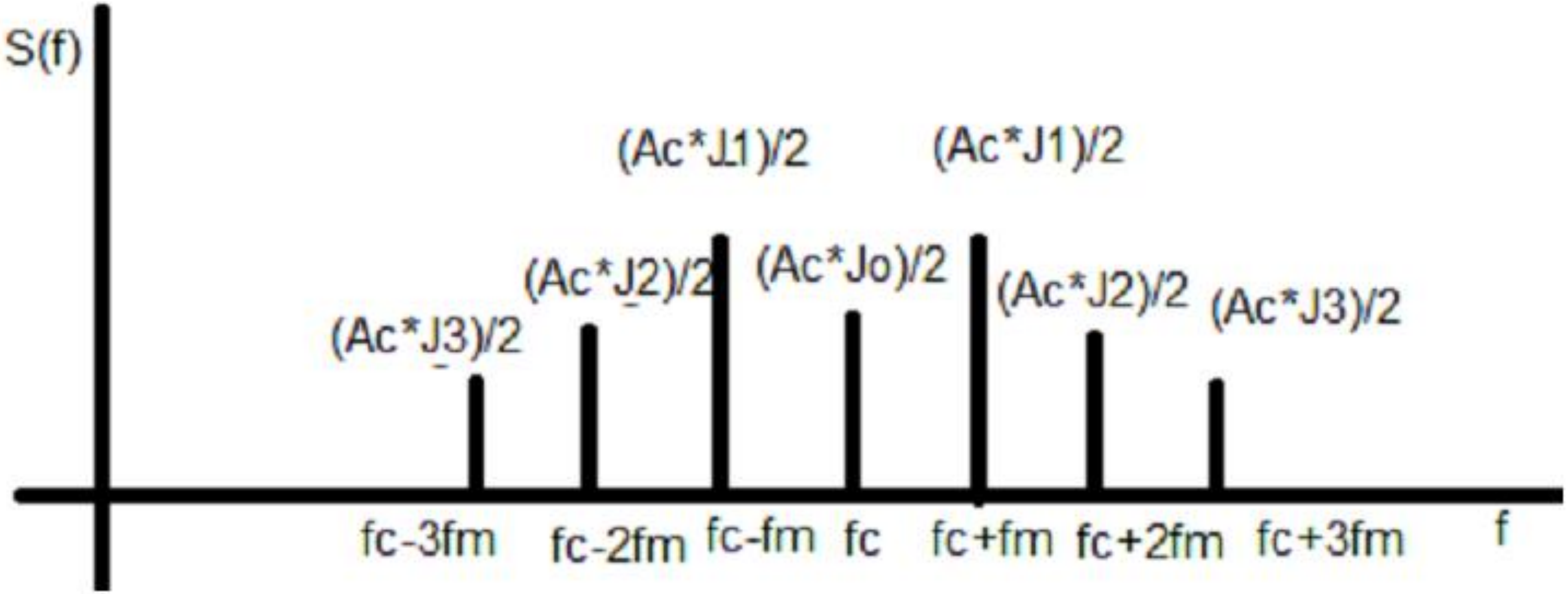
$$= \frac{A_C{}^2}{2}\{J_0{}^2(\beta) + J_1{}^2(\beta) + J_{-1}{}^2(\beta) + J_2{}^2(\beta) + J_{-2}{}^2(\beta) + \cdots..\}$$

$= \frac{A_C{}^2}{2}\{\sum_{n=-\infty}^{\infty} J_n{}^2(\beta)\}$, where $\sum_{n=-\infty}^{\infty} J_n{}^2(\beta) = 1$, (A property of Bessel Functions)

The average power becomes

$$< S^2(t) > = \frac{Ac^2}{2}.$$

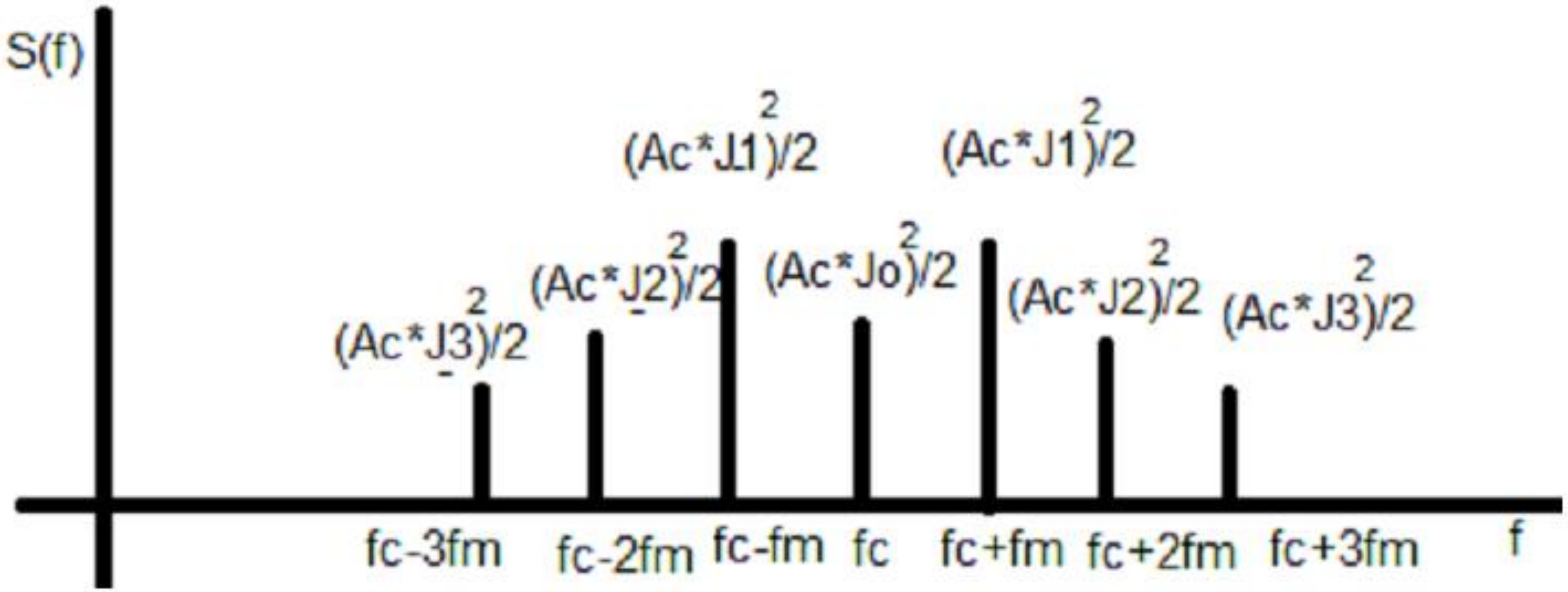
Spectrum of an Fm Signal



Fourier transform of $s(t) = A_c \cos\,(\omega_c t + \beta \sin 2\pi f_m t)$. (only + ve frequencies shown)

Note that in the figure above as f_m *decreases*, the spectral lines become closely concentrated about f_c .

The power spectral density, which is a plot of $|C_n|^2$ versus f, is shown below:



Example:

Plot the FM spectrum and find the 99% power bandwidth when $\beta = 1$ and $\beta = 0.2$

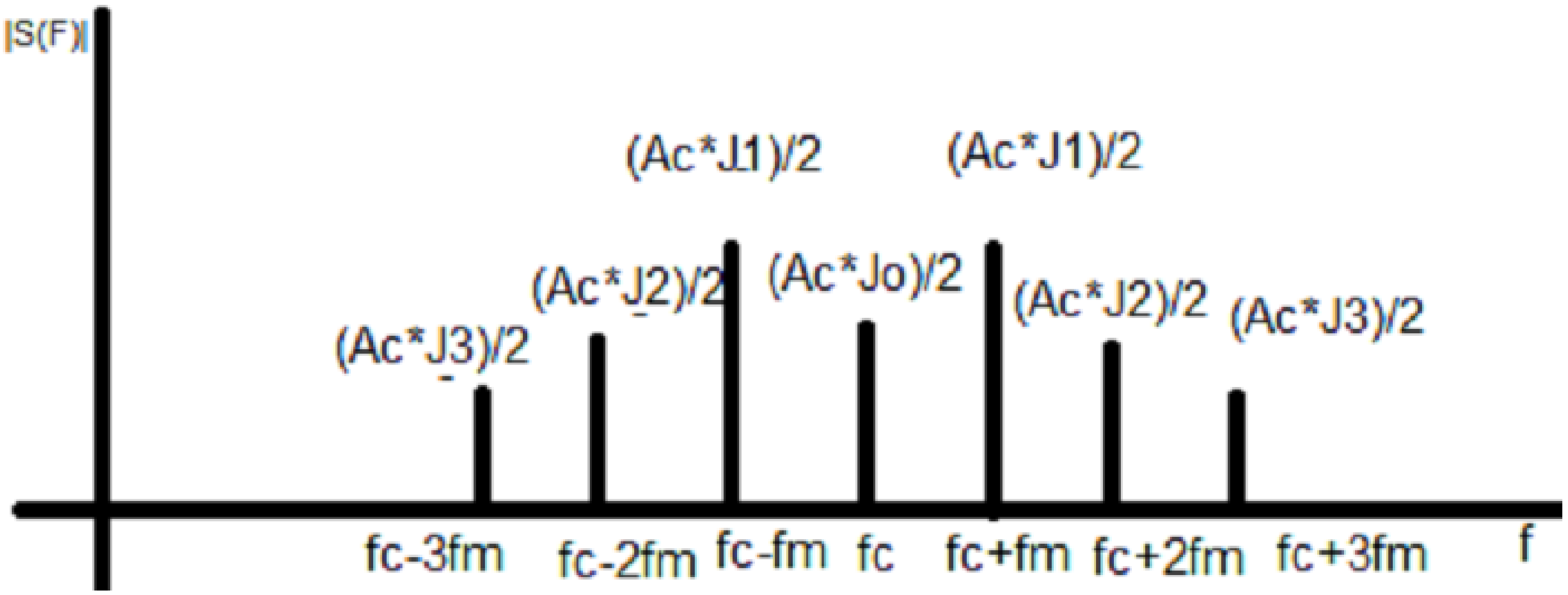
Solution:

$$s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + nf_m)t)$$

Case a: $\boldsymbol{\beta = 1}$

For $\beta = 1$, there are five significant Bessel terms (but we may not need all of them to achieve the 99% power bandwidth)

$J_0(1) = 0.7652$, $J_1(1) = 0.4401$, $J_2(1) = 0.1149$, $J_3(1) = 0.01956$, $J_4(1) = 0.002477$



The power in s(t) is $< S^2(t) > = \frac{A_C{}^2}{2}$

Let us try to find the average power in the terms at f_c , $f_c + f_m$, $f_c - f_m$, $f_c + 2f_m$, $f_c - 2f_m$

The average power in these five components can be calculated as:

1. f_c : $\frac{A_C{}^2 J_0{}^2(\beta)}{2}$
2. $f_c + f_m$: $\frac{A_C{}^2 J_1{}^2(\beta)}{2}$
3. $f_c - f_m$: $\frac{A_C{}^2 J_{-1}{}^2(\beta)}{2}$
4. $f_c + 2f_m$: $\frac{A_C{}^2 J_2{}^2(\beta)}{2}$
5. $f_c - 2f_m$: $\frac{A_C{}^2 J_{-2}{}^2(\beta)}{2}$

The average power in the five spectral components is the sum:

$$= \frac{A^2{}_c}{2}\left[J_0{}^2(1) + 2J_1{}^2(1) + 2J_2{}^2(1)\right]$$

$$= \frac{A^2{}_c}{2}\left[(0.7652)^2 + 2 * (0.4401)^2 + (0.1149)^2\right] = 0.9993\, \frac{A^2{}_c}{2}$$

So, these terms have 99.9 % of the total power.

Therefore, the 99.9 % power bandwidth is

$$\text{B.W} = (f_c + 2f_m) - (f_c - 2f_m) = 4f_m$$

Case b: $\beta = 0.2$

For $\beta = 0.2$, $J_0(0.2) = 0.99$, $J_1(0.2) = 0.0995$, $J_2(0.2) = 0.00498335$

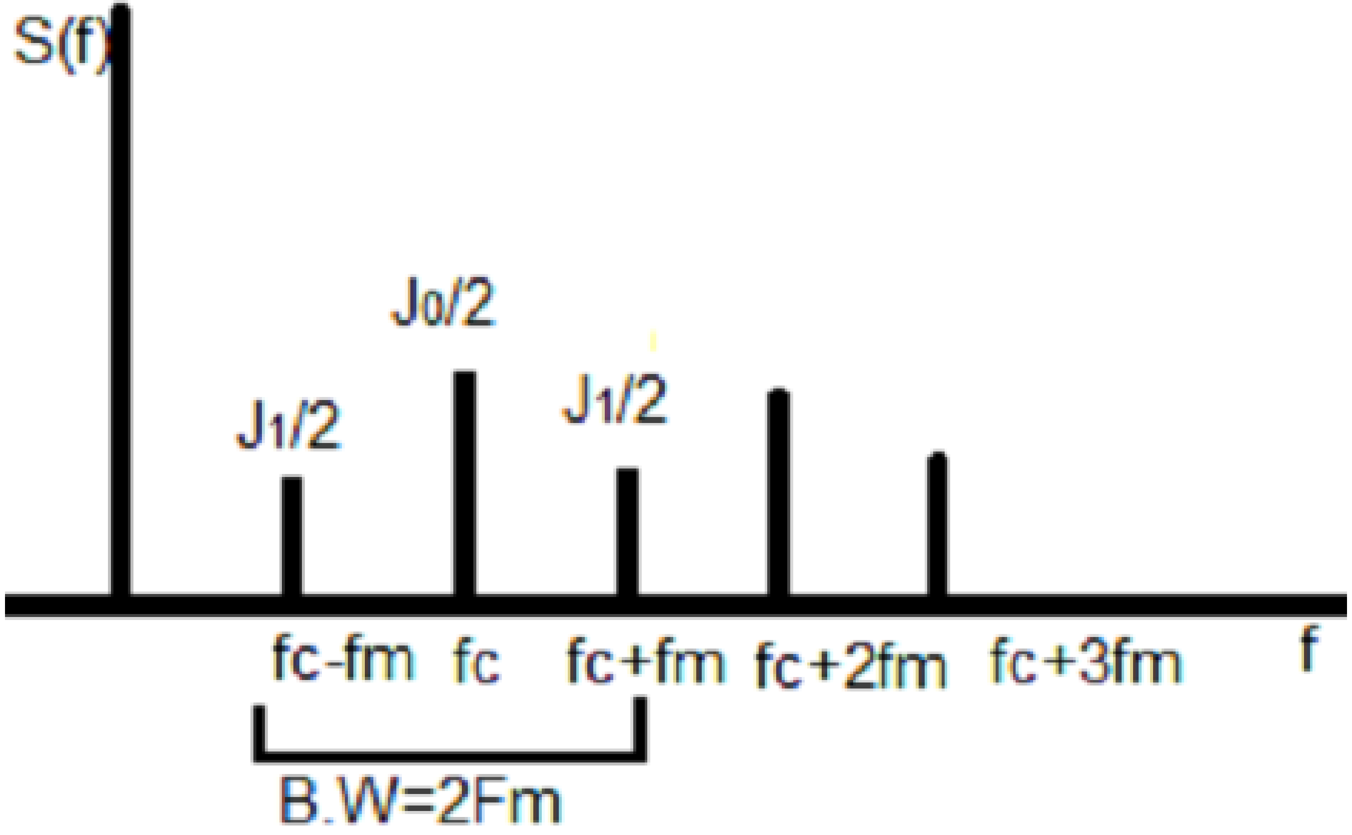
The power in the carrier and the two sidebands (at f_c , $f_c + f_m$, $f_c - f_m$) is

$$P = \frac{A^2{}_c}{2}\left[J_0{}^2(0.2) + 2J_1{}^2(0.2)\right]$$

$$P = \frac{A^2{}_c}{2}[0.9999]$$

Therefore,

$$99.99\% \text{ of the total power is found in the carrier and two side bands}$$



The 99% bandwidth is

$$\text{B.W} = (f_c + f_m) - (f_c - f_m) = 2f_m$$

Remark:

We observe that the spectrum of an FM signal when $\beta \ll 1$ (called narrow band FM) is "similar" to the spectrum of a normal AM signal, in the sense that it consists of a carrier and two sidebands. The bandwidth of both signals is $2f_m$.

Carson's Rule

A 98% power B.W of an FM signal is estimated using Carson's rule:

$$B_T = 2(\beta + 1)f_m$$

Generation of FM Signal

- Generation of narrow Band FM Signal

Consider an angel modulated signal:

$$s(t) = A_c \cos(2\pi f_c t + \theta(t))$$

When $s(t)$ is an FM signal, $\theta(t) = 2\pi k_f \int m(t)dt$

$s(t)$ can be expanded as:

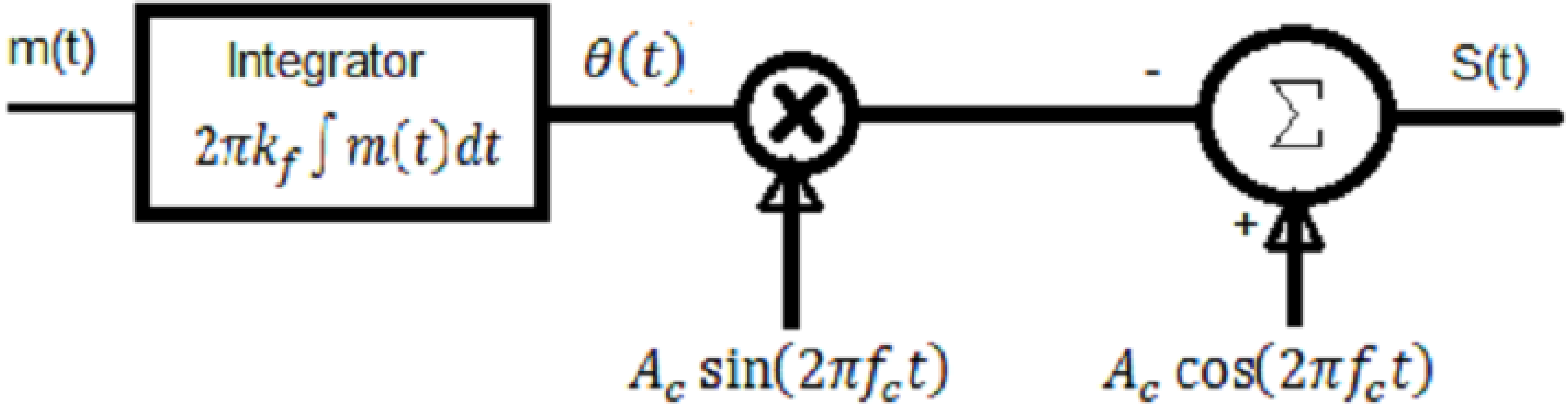
$$s(t) = A_c \cos(2\pi f_c t) \cos\big(\theta(t)\big) - A_c \sin(2\pi f_c t) \sin\big(\theta(t)\big)$$

When $|\theta(t)| \ll 1$, $cos\ \theta \cong 1$, $\sin(\theta) \cong \theta$ and $s(t)$, termed narrowband, can be approximated as:

$$s(t) \cong A_c \cos(2\pi f_c t) - A_c \theta \sin(2\pi f_c t)$$

This expression can serve as the basis for the generation of a narrowband FM or PM signals.

To Generate a narrowband FM, consider the block diagram below:



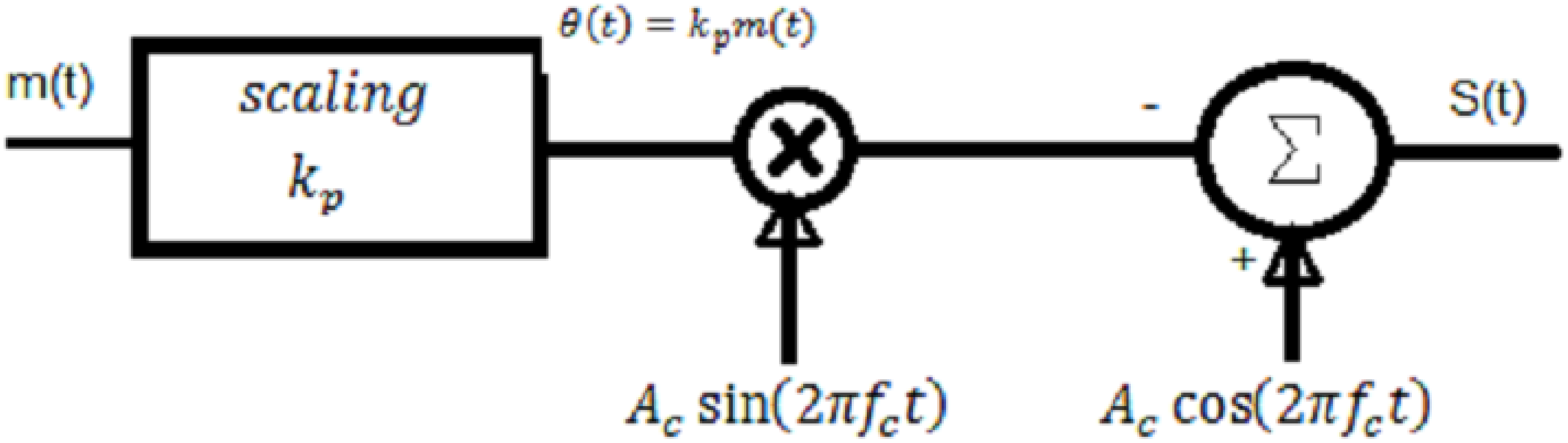
When $m(t) = A_m \cos(2\pi f_m t)$

$$\theta(t) = \beta \sin(2\pi f_m t)$$

And the modulated signal takes the form

$$s(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_m t) \sin(2\pi f_c t)$$

To generate a narrow band PM signal, we can use the scheme:



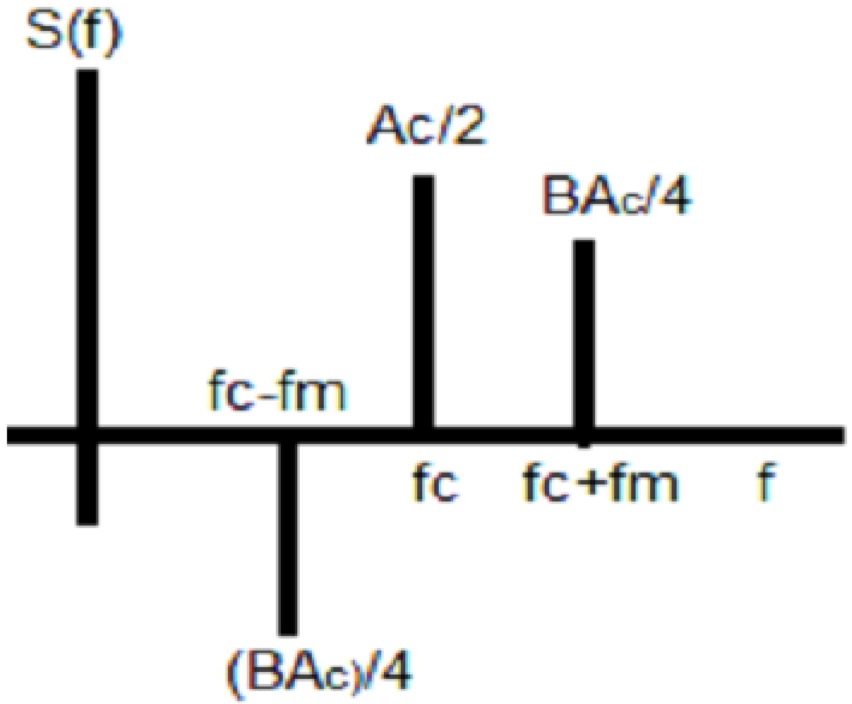
Spectrum of a single- tone NBFM:

For an FM signal, $\theta(t) = \beta \sin(2\pi f_m t)$

$$s(t) = A_c \cos(2\pi f_c t) - A_c\beta \sin(2\pi f_m t) \sin(2\pi f_c t)$$

$$s(t) = A_c \cos(2\pi f_c t) - \frac{A_c\beta}{2}[\cos(2\pi(f_c - f_m)t) - \cos(2\pi(f_c + f_m)t)]$$

The spectrum of $s(t)$ is shown below:



The spectrum consists of a component at the carrier frequency f_c , and at the two sidebands $f_c + f_m$ and $f_c - f_m$). Note the negative sign at the lower sideband. The bandwidth of this signal is $2f_m$.

Now consider the normal AM signal with sinusoidal modulation.

$$s(t)_{AM} = A_c \cos(2\pi f_c t) + A_c A_m \cos\,(2\pi f_m t) \cos(2\pi f_c t)$$

It can be represented as

$$s(t) = A_c \cos(2\pi f_c t) - \frac{A_c A_m}{2}[\cos(2\pi(f_c - f_m)t) + \cos(2\pi(f_c + f_m)t)]$$

As we recall this signal consists of a term at the carrier and two terms at $f_c + f_m$ and $f_c - f_m$.

Frequency multiplier

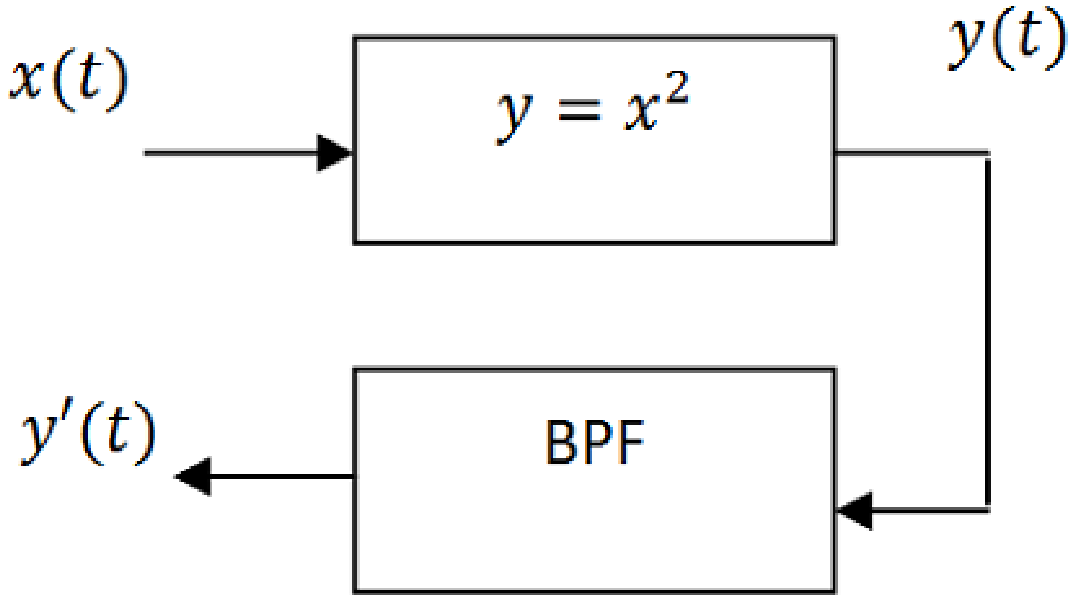
It is a device for which the frequency of the output signal is an integer multiple of the frequency of the input signal. It is primarily a nonlinear characteristic followed by a band pass filter. Now we illustrate the operation of this device.

The Square law device:

Let the input be an FM signal of the form:

$$x(t) = A_c \cos(2\pi f'_c + \beta' \sin 2\pi f_m t)$$

$$y(t) = A_c \cos(\phi)$$



The output of the square law characteristic is:

$$y(t) = x(t)^2 = A_c^2 cos^2(\phi) = \frac{A_c^2}{2}[1 + \cos(2\phi)] = \frac{A_c^2}{2} + \frac{A_c^2}{2}\cos(2\phi)$$

$$= \frac{A_c^2}{2} + \frac{A_c^2}{2} cos\left[2\pi(2f'_c) + 2\beta' \sin(2\pi f_m t)\right]$$

The bandpass filter

If $y(t)$ is passed through a BPF of center frequency $2f_c$, then the DC term will be suppressed and the filter output is:

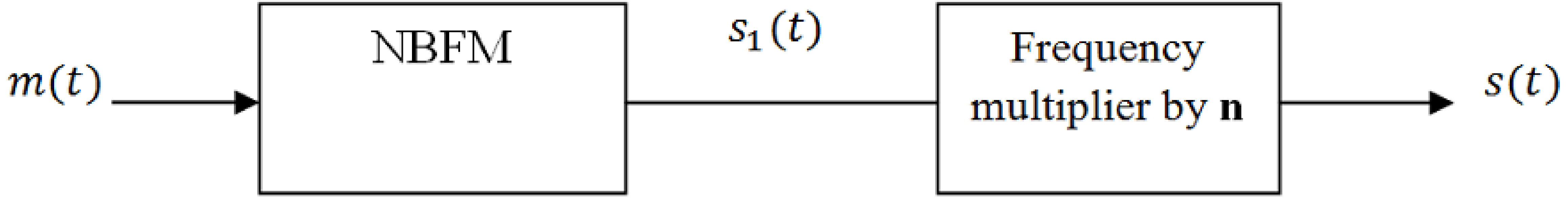
$$y'(t) = \frac{A_c^2}{2} cos\left[2\pi\left(2f'_c\right) + 2\beta' \sin(2\pi f_m t)\right]$$

$$y'(t) = \frac{A_c^2}{2} cos[2\pi(f_c) + \beta \sin(2\pi f_m t)]$$

As can be seen from this result, the output is a signal with twice the frequency of the input signal and a modulation index twice that of the input. To get frequency multiplication higher than two, a cascade of units, similar to what was described above, can be formed with the number of stages that achieve the desired frequency.

Indirect Method for Generating a Wideband FM:

A wideband FM can be generated indirectly using the block diagram below (Armstrong Method). First a narrowband FM is generated, and then the wideband FM is obtained by using frequency multiplication. Next, we analyze the operation of this modulator.



If $m(t) = A_m \cos 2\pi f_m t$ is the baseband signal, then

$$s_1(t) = A_c \cos(2\pi f_c' t + \beta' \sin 2\pi f_m t) \;;\; \beta' = \frac{k_f A_m}{f_m}$$

is a narrowband FM with $\beta' \ll 1$. The frequency of $s_1(t)$ is $f'_i = f_c' + k_f A_m \cos 2\pi f_m t$.

Multiplying f_i by n , we get the frequency of $s(t)$ as $f_i = nf_c^{'} + nk_f A_m cos2\pi f_m t$. This result in

$$s(t) = A_c \cos[2\pi(nf_c^{'})t + n\beta^{'} \sin 2\pi f_m t]$$

$$= A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

Where $\beta = n\beta'$ is the desired modulation index of WBFM

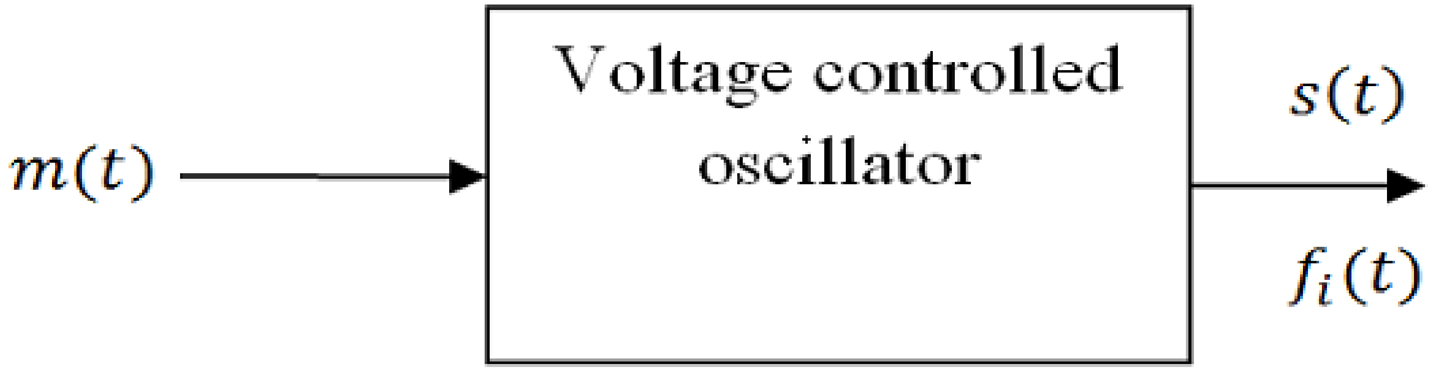
$f_c = nf_c^{'}$ is the desired carrier frequency of WBFM

Direct method for generating FM signal:

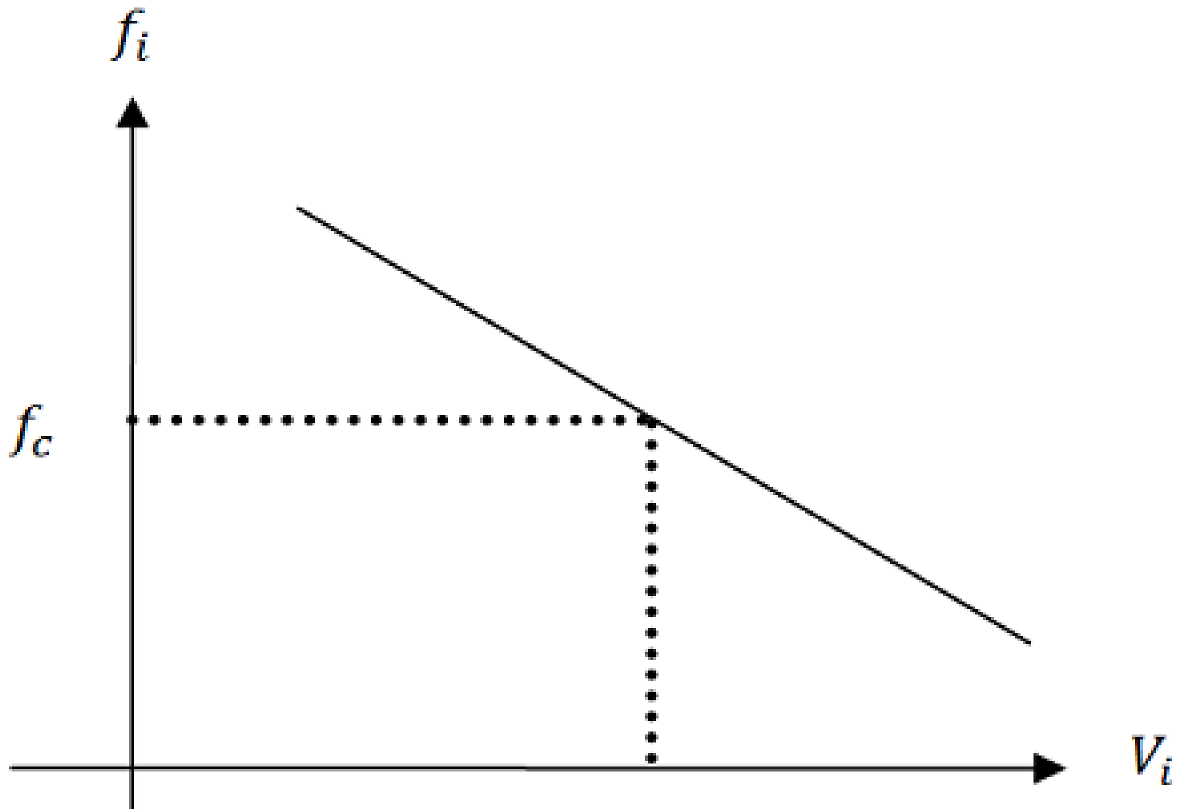
In a direct FM system, the instantaneous frequency of the carrier is varied in accordance with a message signal by means of a voltage controlled oscillator (VCO). The voltage – frequency characteristic of a VCO is given by

$$f_i = f_c - k\, m(t)$$

and is plotted in the figure below.



A realization of the CVO may be obtained by considering an oscillator (like the Hartley oscillator) shown below in which a varactor ((voltage variable capacitor) is used. The capacitance of the varactor varies in response to variations in the message signal. The variation is linear when the variation in the message is too small.



The frequency of the oscillator is

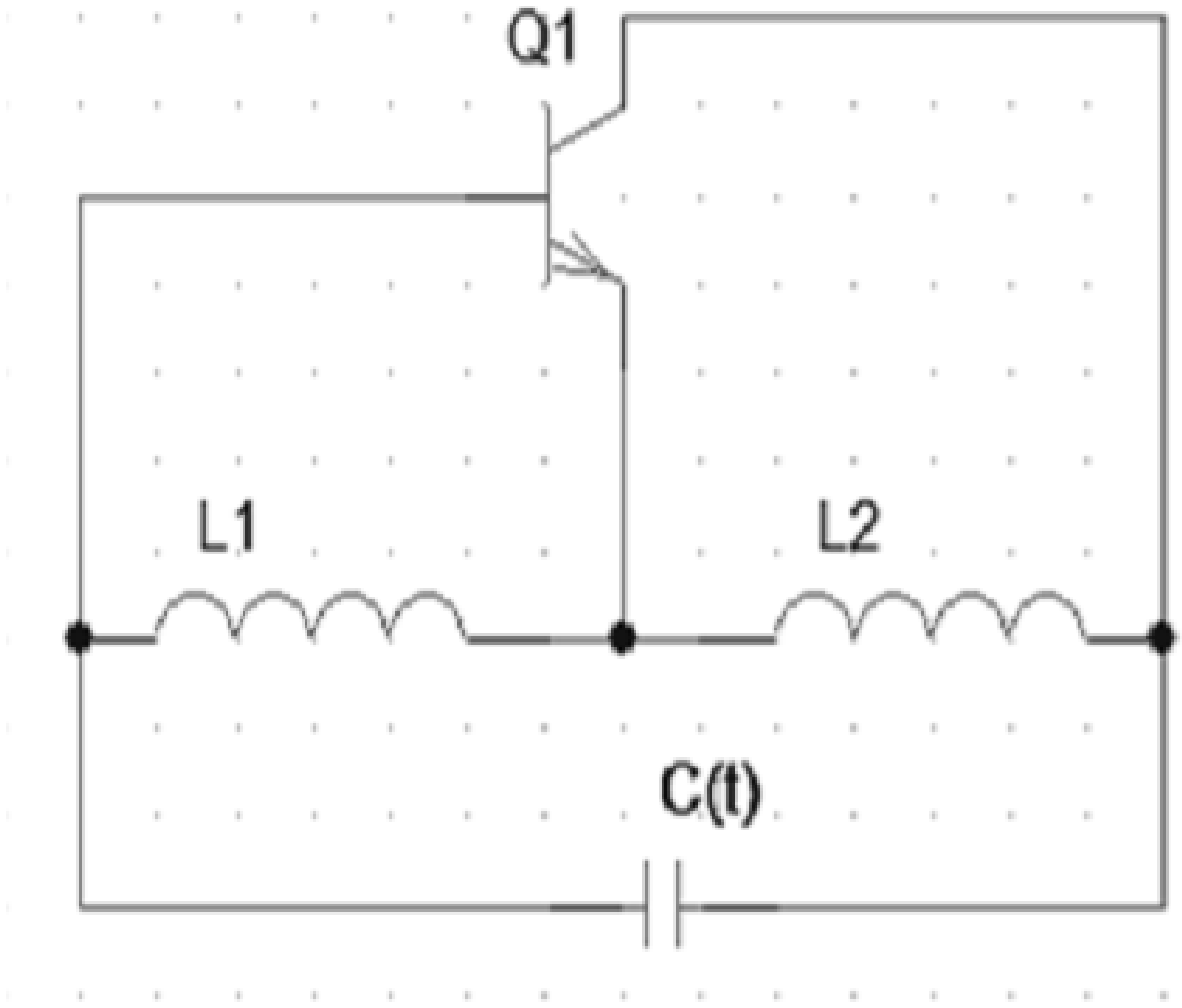
$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)C(t)}}$$

Let $C(t) = C_0 - k\,m(t)$

k: is a constant,

When m(t) = 0, $C(t) = C_0$, and the unmodulated frequency of oscillation is

$$f_c = \frac{1}{\sqrt{(L_1+L_2)C_0}}$$



Hartley Oscillator

When m(t) has a finite value, the frequency of oscillation is

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)(C_0 - k\, m(t))}}$$

$$= \frac{1}{2\pi} \frac{1}{\sqrt{(L_1 + L_2)C_0}} \frac{1}{\sqrt{(1 - \frac{k\, m(t)}{C_0})}}$$

$$= f_c\left(1 - \frac{k\, m(t)}{C_0}\right)^{-1/2} \quad , \quad [(1 + x)^n \cong 1 + nx]$$

When $\frac{k\, m(t)}{C_0} \ll 1$, we can make the approximation

$$f_i(t) = f_c\left(1 + \frac{k\, m(t)}{2C_0}\right) = f_c + C_f\, m(t)$$

Here it is clear that the instantaneous frequency varies linearly with the message signal.

Demodulation FM Signal

An FM signal may be demodulated by means of what is called a *discriminator*.

Let $s(t) = A_c \cos(\omega_c t + \theta(t))$ be an angle modulated signal. The output of an ideal discriminator is defined as:

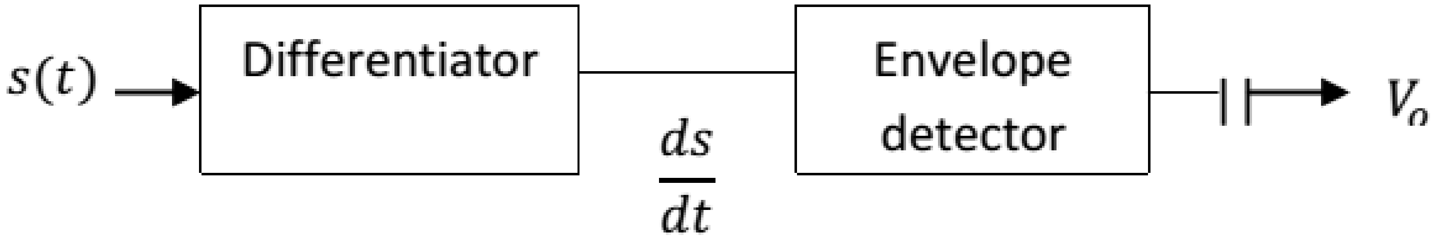
$$y(t) = \frac{1}{2\pi} k_D \frac{d\theta}{dt}$$

When $\theta = 2\pi k_f \int_{-\infty}^{t} m(\alpha)d\alpha$, then $\frac{d\theta}{dt} = 2\pi k_f\, m(t)$ and y(t) becomes

$$y(t) = k_D k_f\, m(t)$$

One practical realization of a discriminator is a differentiator followed by an envelope detector.

The operation of this discriminator can be explained as follows:



Let $s(t) = A_c \cos(\omega_c t + \theta(t))$

$$\frac{ds(t)}{dt} = -A_c \left(\omega_c + \frac{d\theta}{dt}\right) \sin(\omega_c t + \theta(t))$$

The output of the envelope detector is $A_c \left| \left(\omega_c + \frac{d\theta}{dt}\right) \right|$

The capacitor blocks the DC term and so output is:

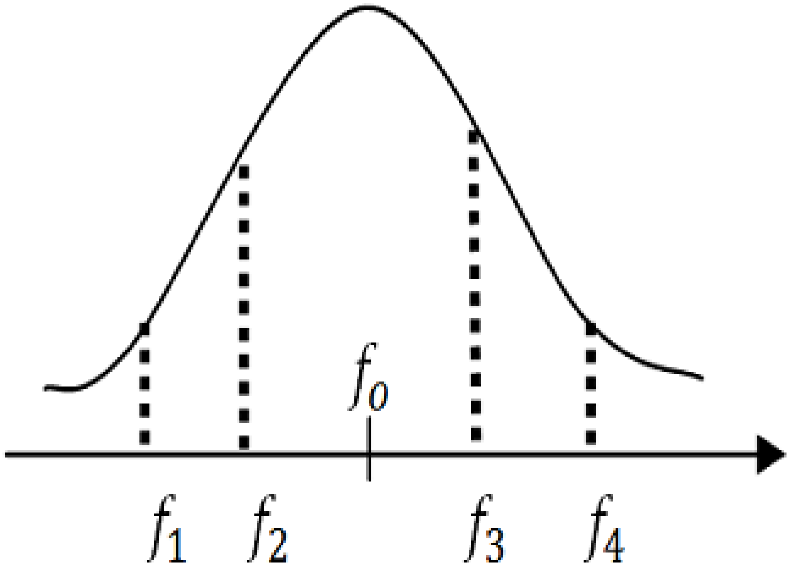
$$V_0 = A_c \frac{d\theta}{dt} = 2\pi k_f \, A_c m(t)$$

We know what an envelope detector is when we considered amplitude demodulation. Now we explain how differentiation is accomplished.

From the properties of Fourier transform we know that if $F\{g(t)\} = G(f)$, then

$$F\left\{\frac{dg(t)}{dt}\right\} = j2\pi f \; G(f)$$

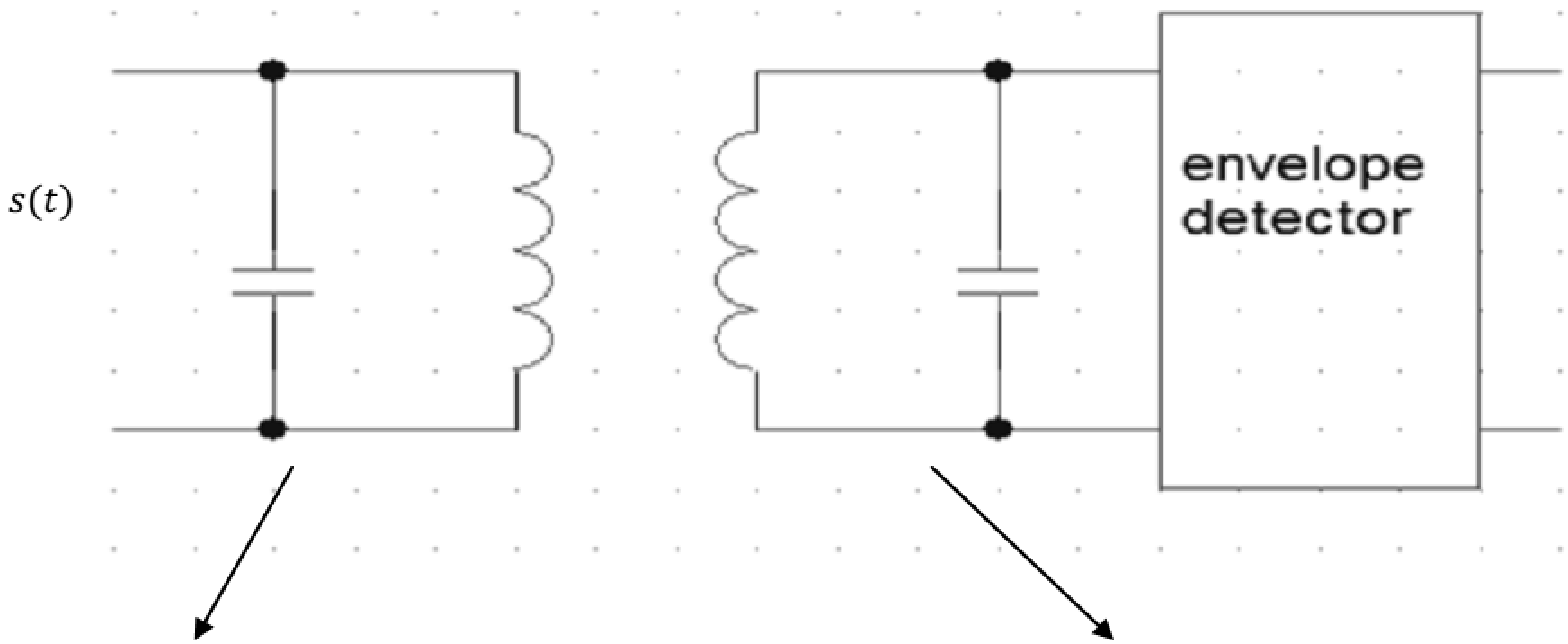
This means that multiplication by $j2\pi f$ in the frequency domain amounts to differentiating the signal in the time-domain. Hence, we need a circuit whose frequency response is linear in f to perform time differentiation. A circuit that performs this task is a tuned circuit, provided that the signal frequency falls within the linear part



of the characteristic, i.e., between either (f_1, f_2) *or* (f_3, f_4).

A balanced FM detector called *balanced discriminator* is such a circuit.

Tuned circuit demodulator



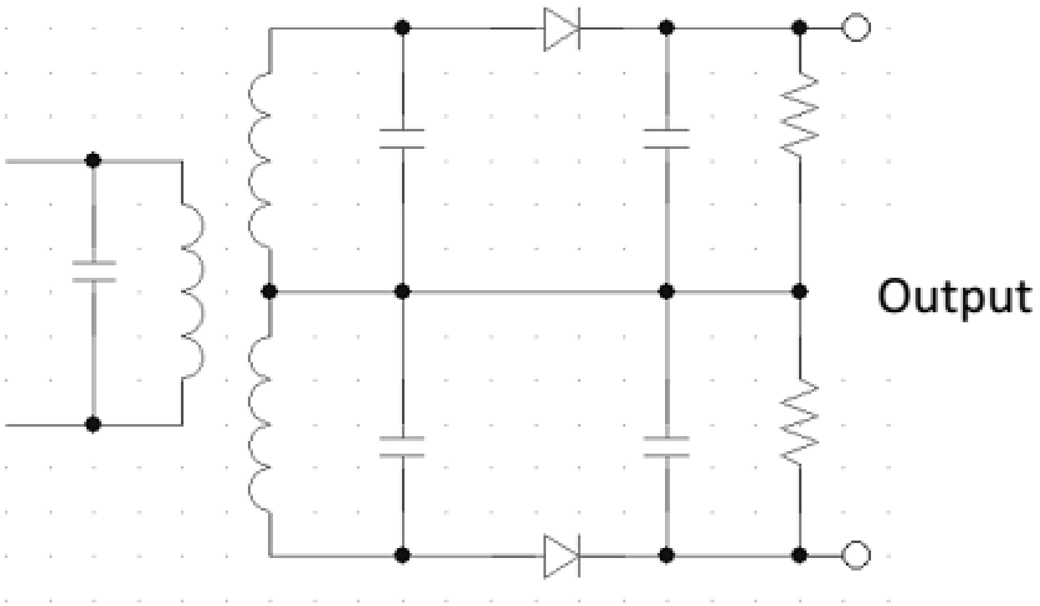
Primary circuit tuned to carrier freq. f_c

Secondary circuit tuned to $f_o > f_c$

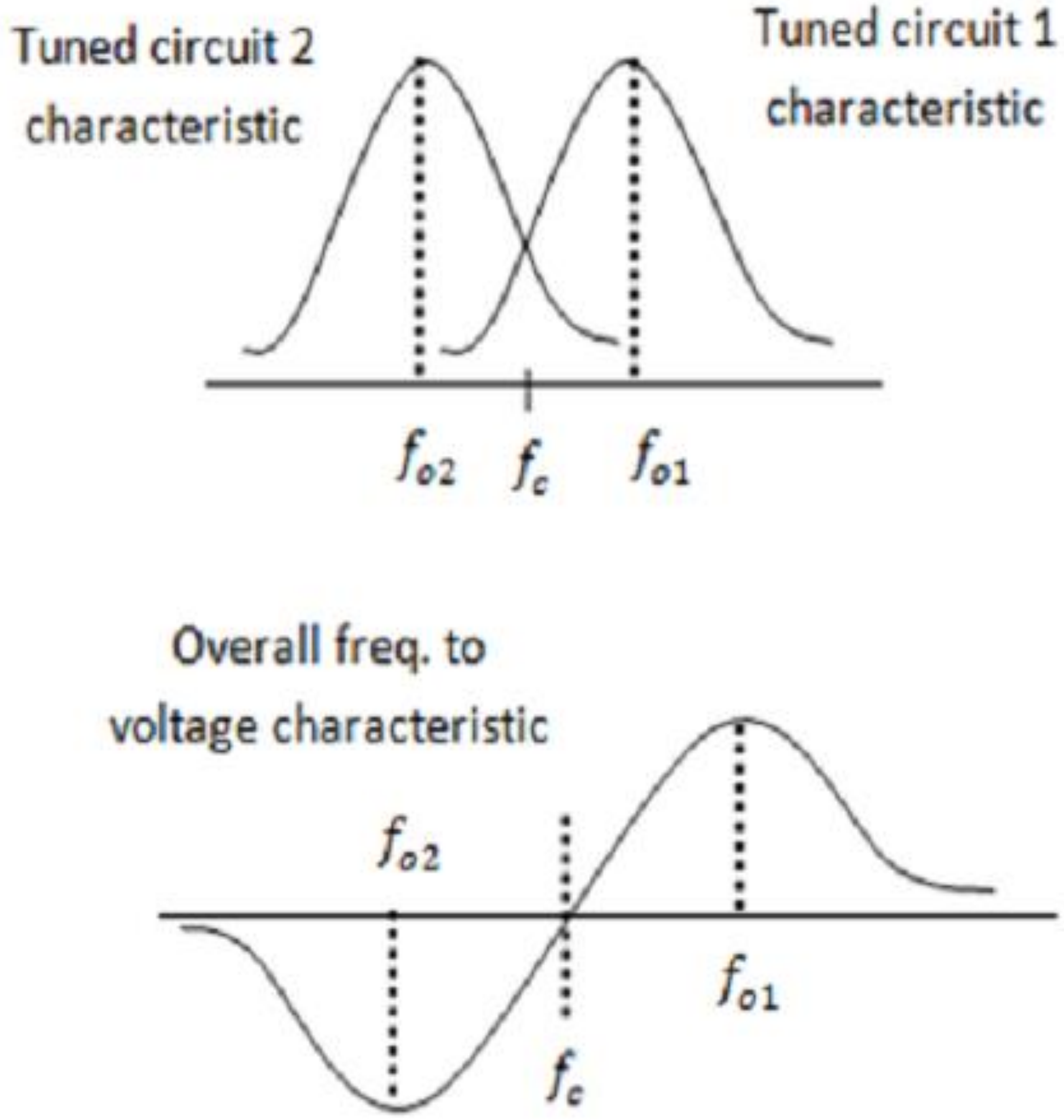
To extend the dynamic range of the differentiating circuit, two tuned circuits with center frequencies f_{o1} and f_{o2} are used as will illustrated next.

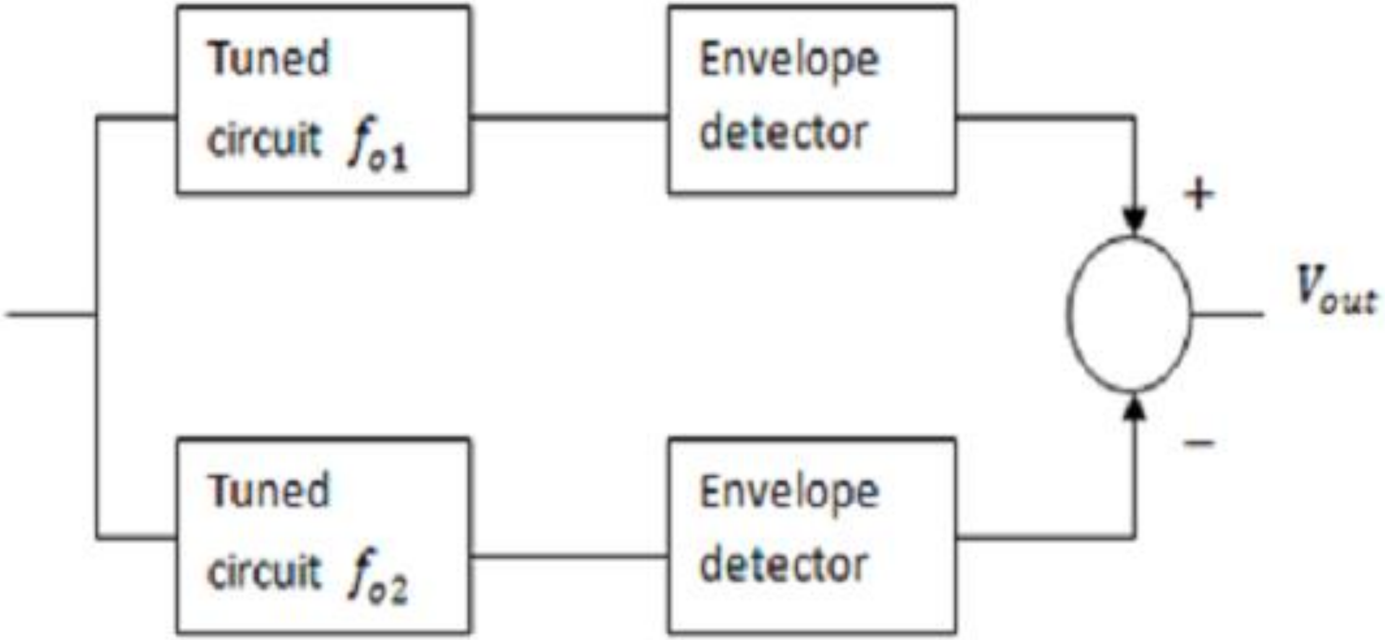
Balanced slope detector:

Two tuned circuits are tuned to two different frequencies $f_{o_1} > f_{o_2}$. The primary circuit is tuned to f_c .



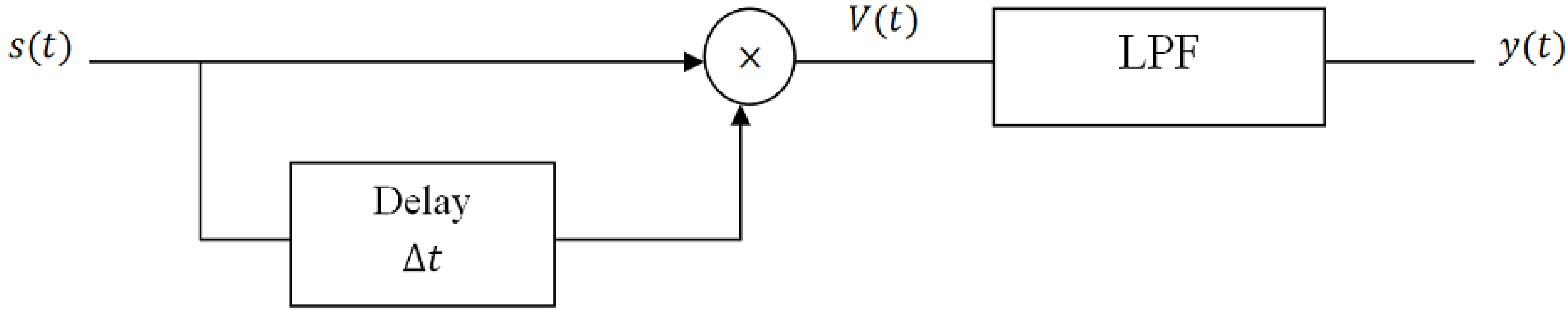
- This circuit has wider width of linear frequency response.
- No DC blocking is necessary.


Tuned circuit 2 characteristic
Tuned circuit 1 characteristic
f_{o2} f_c f_{o1}
Overall freq. to voltage characteristic
f_{o2}
f_{o1}
f_c


Tuned circuit f_{o1}
Envelope detector
Tuned circuit f_{o2}
Envelope detector
+
−
V_{out}

Phase shift discriminator:

The quadrature detector: This demodulator converts frequency variations into phase variation and detecting the phase changes. The block diagram of the demodulator is shown below



Let $s(t) = A_c \cos\big(2\pi f_c t + \varphi(t)\big)$; $\varphi(t) = 2\pi K_f \int_0^t m(\alpha)\, d\alpha$

$$s(t-\Delta t) = A_c \cos[2\pi f_c (t-\Delta t) + \varphi(t-\Delta t)]$$

$$= A_c \cos[2\pi f_c t - 2\pi f_c \Delta t + \varphi(t-\Delta t)]$$

The delay Δt is chosen such that $2\pi f_c \Delta t = {}^{\pi}/_{2}$

Hence,

$$s(t-\Delta t) = A_c \cos\left[2\pi f_c t - \frac{\pi}{2} + \varphi(t-\Delta t)\right]$$

$$= A_c \sin[2\pi f_c t + \varphi(t-\Delta t)]$$

$$V(t) = s(t)s(t-\Delta t)$$

$$= A_c^2 \sin[2\pi f_c t + \varphi(t-\Delta t)]\, cos[2\pi f_c t + \varphi(t)]$$

$$= \frac{A_c^2}{2} \sin[2\pi(2f_c)t + \varphi(t) + \varphi(t-\Delta t)] + \frac{A_c^2}{2} \sin[\varphi(t) - \varphi(t-\Delta t)]$$

The high frequency component is suppressed by the LPF. What remains is the second term

$$\frac{A_c^2}{2}\sin[\varphi(t) - \varphi(t - \Delta t)] \cong \frac{A_c^2}{2}[\varphi(t) - \varphi(t - \Delta t)]$$

Where Δt is small to justify the approximation $\sin(x) \cong x$

Hence,

$$y(t) = \frac{A_c^2}{2}[\varphi(t) - \varphi(t - \Delta t)]$$

$$y(t) = \frac{A_c^2}{2}\ .\Delta t.\ \ \frac{\varphi(t)-\varphi(t-\Delta t)}{\Delta t}$$

The second term is the derivative $\dfrac{d\varphi(t)}{dt}$. The output then becomes

$$y(t) = \frac{A_c^2}{2} \;.\Delta t.\; \frac{d\varphi}{dt}$$

But $\varphi(t) = 2\pi K_f \int_0^t m(\alpha)\, d\alpha$ and $\frac{d}{dt}\varphi(t) = 2\pi K_f m(t)$

$$y(t) = \frac{A_c^2}{2}\,\Delta t\, .2\pi K_f\, m(t)$$

$$y(t) = K\, m(t)$$

$y(t)$ is proportional to $m(t)$. It performs demodulation.

Transfer function of the delay:

From Fourier transform properties

$$g(t) \rightarrow G(f)$$

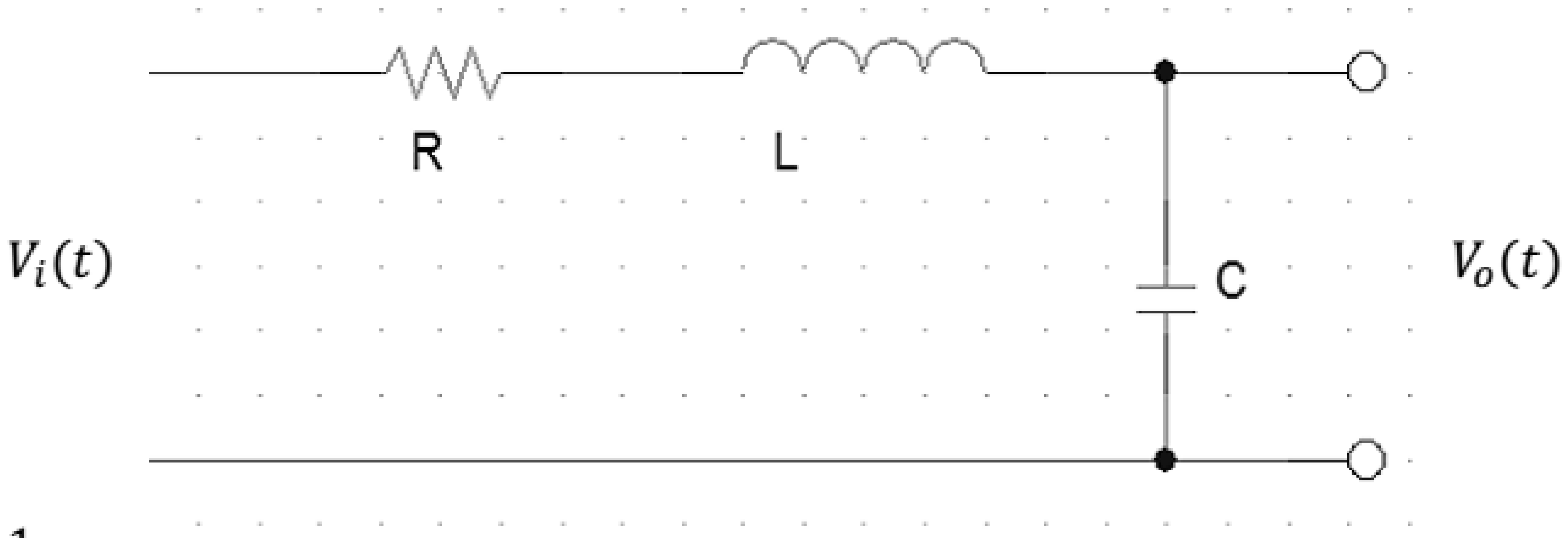
$$g(t - \Delta t) \rightarrow G(f)e^{-j2\pi f \Delta t}$$

The transfer function of the time delay is

$$H(f) = e^{-j2\pi f \Delta t}$$

Therefore, a circuit whose phase characteristic is linear in f can provide time delay of the type that we need.

A circuit with linear phase characteristic is the network shown



If $f_o = \frac{1}{2\pi\sqrt{LC}}$, $f_b = \frac{R}{2\pi L}$

then it can be shown that arg $(H(f))$ for this circuit is

$$\arg(H(f)) = -\frac{\pi}{2} - \frac{2Q}{f_o}(f - f_c) \;,\; Q = \frac{f_o}{f_b}$$

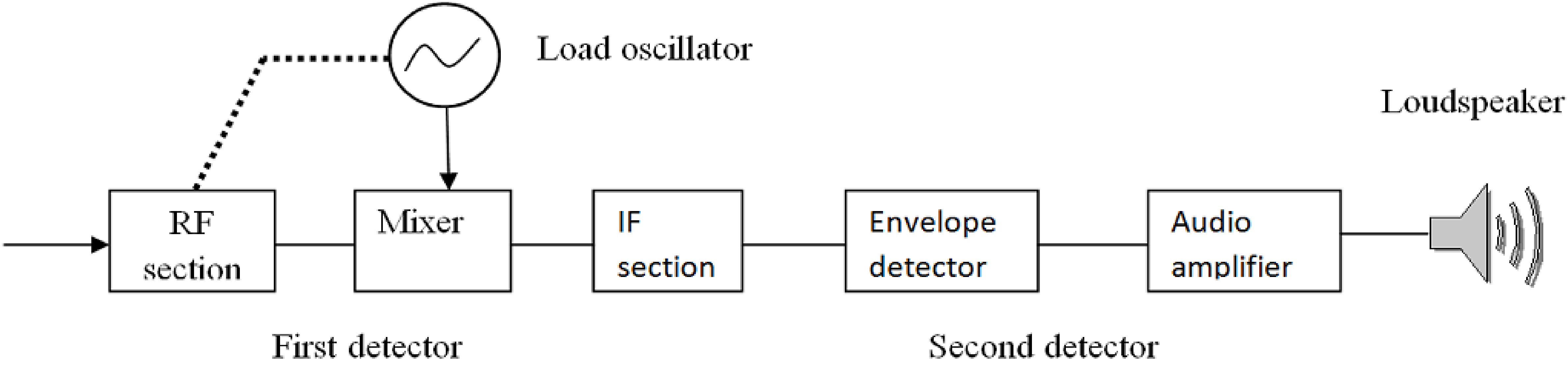
Remark: To perform time differentiation, we searched for a circuit whose amplitude spectrum varies linearly with frequency, while to perform time delay, we searched for a circuit with a linear phase spectrum.

The Super heterodyne Receiver:

Practically, all radio and TV receivers are made of the super heterodyne type. The receiver performs the following functions :

- Carrier frequency tuning: The purpose of which is to select the desired signal.
- Filtering: the desired signal is to be separated from other modulated signals.
- Amplification: to compensate for the loss of signal power incurred in the course of transmission.

The description of the receiver is summarized as follows:



- The incoming signal is picked up by the antenna and amplified in the RF section that is tuned to the carrier frequency of the incoming signal.
- The incoming RF section is down converted to a fixed intermediate frequency (IF). $f_{IF} = f_{lO} - f_{RF}$
- The IF section provides most of the amplification and selectivity in the receiver. The IF bandwidth corresponds to that required for the particular type of modulation.
- The IF output is applied to a demodulator, the purpose of which is to recover the baseband signal.
- The final operation in the receiver is the power amplification of the recovered signal.
- The basic difference between AM and FM super heterodyne lies in the use of an FM demodulator such as a discriminator (differentiator followed envelope detector)