NOISE IN ANALOG COMMUNICATIONS

Lessons to learn about Noise:

Lesson 1: Minimizing the effects of noise is a prime concern in analog communications, and consequently the ratio of signal power to noise power is an important metric for assessing analog communication quality.

Lesson 2: Amplitude modulation may be detected either coherently requiring the use of a synchronized oscillator or non-coherently by means of a simple envelope detector. However, there is a performance penalty to be paid for non-coherent detection.

Lesson 3: Frequency modulation is nonlinear and the output noise spectrum is parabolic when the input noise spectrum is flat. Frequency modulation has the advantage that it allows us to trade bandwidth for improved performance.

Lesson 4: Pre- and de-emphasis filtering is a method of reducing the output noise of an FM demodulator without distorting the signal. This technique may be used to significantly improve the performance of frequency modulation systems.

Properties of Noise

- *The mean of the random process*. For noise, the mean value corresponds to the *dc offset*. In most communication systems, dc offsets are removed by design since they require power and carry little information. Consequently, both noise and signal are generally assumed to have zero mean.

- *The autocorrelation of the random process*.

$$R_w(\tau) = \frac{N_0}{2}\delta(\tau)$$

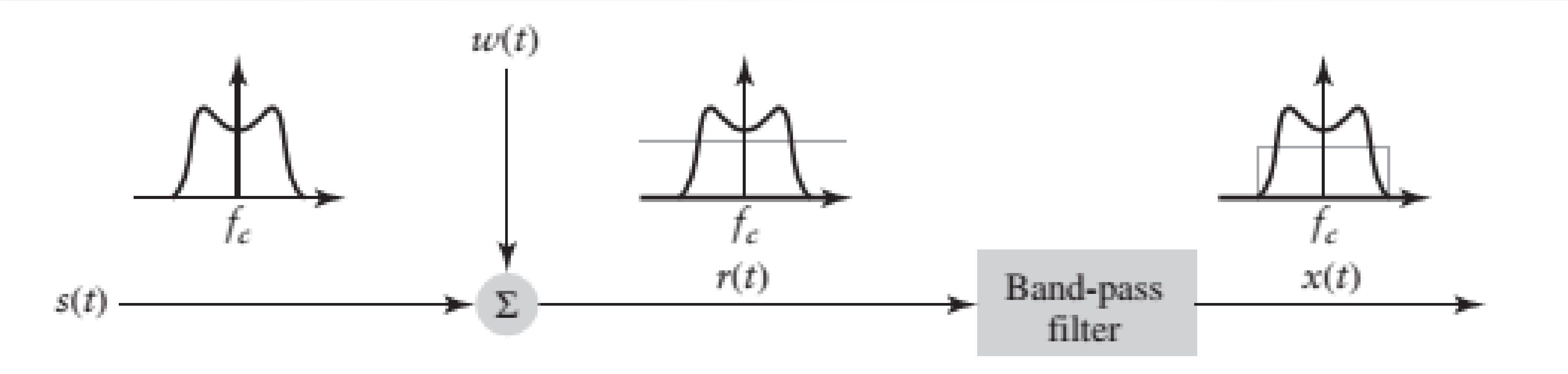
- *The spectrum of the random process*. For additive white Gaussian noise the spectrum is flat and defined as

$$S_w(f) = \frac{N_0}{2}$$

- The noise power at the output of a filter of equivalent-noise bandwidth is:

$$N = N_0 B_T$$

Block diagram of signal plus noise before and after filtering showing spectra at each point.



Noise in Communication Systems

-Given that communication deals with random signals,
how do we quantify the performance of a particular communication system?
we will focus on *signal-to-noise ratio* (SNR) as the measure of quality for analog systems;
-For zero-mean processes, a simple measure of the signal quality is the ratio of the variances of the desired and undesired signals.

$$\mathrm{SNR} = \frac{\mathbf{E}[s^2(t)]}{\mathbf{E}[n^2(t)]}$$

-The signal-to-noise ratio is often considered to be a ratio of the average signal power to the average noise power

Example: Sinusoidal Signal-to-Noise Ratio

Consider the case where the transmitted signal is $s(t) = A_c \cos(2\pi f_c t + \theta)$

where the phase is unknown at the receiver. The signal is received in the presence of additive AWGN noise, determine SNR?

$$
\begin{aligned}
E[s^2(t)] &= \frac{1}{T}\int_0^T (A_c \cos(2\pi f_c t + \theta))^2 \, dt \\
&= \frac{A_c^2}{2T}\int_0^T (1 + \cos(4\pi f_c t + 2\theta)) \, dt \\
&= \frac{A_c^2}{2T}\left[t + \frac{\sin(4\pi f_c t + 2\theta)}{4\pi f_c} \right]_0^T \\
&= \frac{A_c^2}{2}
\end{aligned}
$$

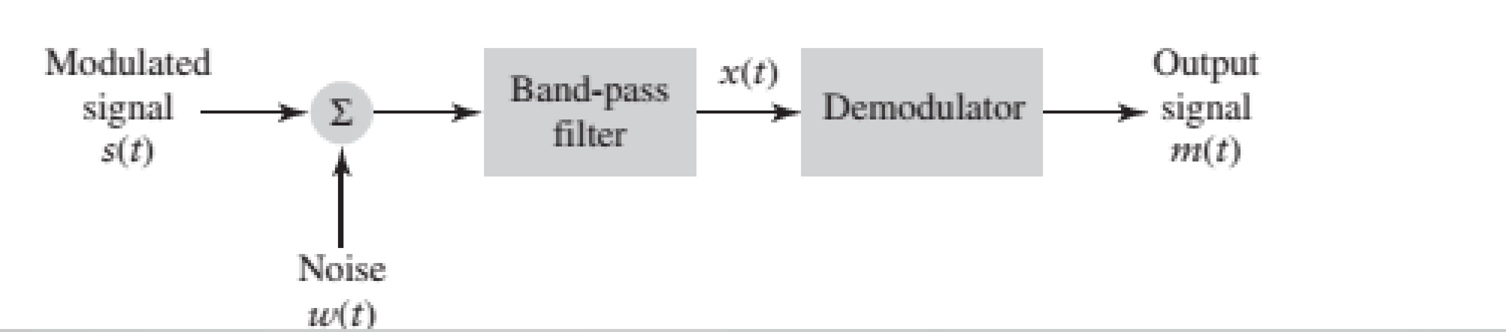
$$
\begin{aligned}
E[n^2(t)] &= N \\
&= N_0 B_T
\end{aligned}
$$

$$
\mathrm{SNR} = \frac{A_c^2}{2N_0 B_T}
$$

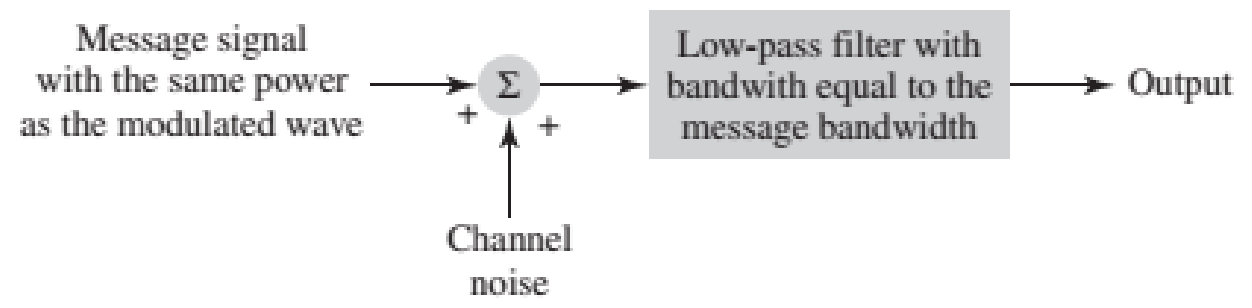
The signal-to-noise ratio is measured at the receiver, but there are several
points in the receiver where the measurement may be carried out. In fact, measurements at particular points in the receiver have their own particular importance and value.
For instance:

pre-detection signal-to-noise ratio: If the signal-to-noise ratio is measured at the front-end of the receiver, then it is usually a measure of the quality of the transmission link and the receiver front-end.

post-detection signal-to-noise ratio: If the signal-to-noise ratio is measured at the output of the receiver, it is a measure of
the quality of the recovered information-bearing signal whether it be audio, video, or otherwise.



Reference transmission model for analog communications

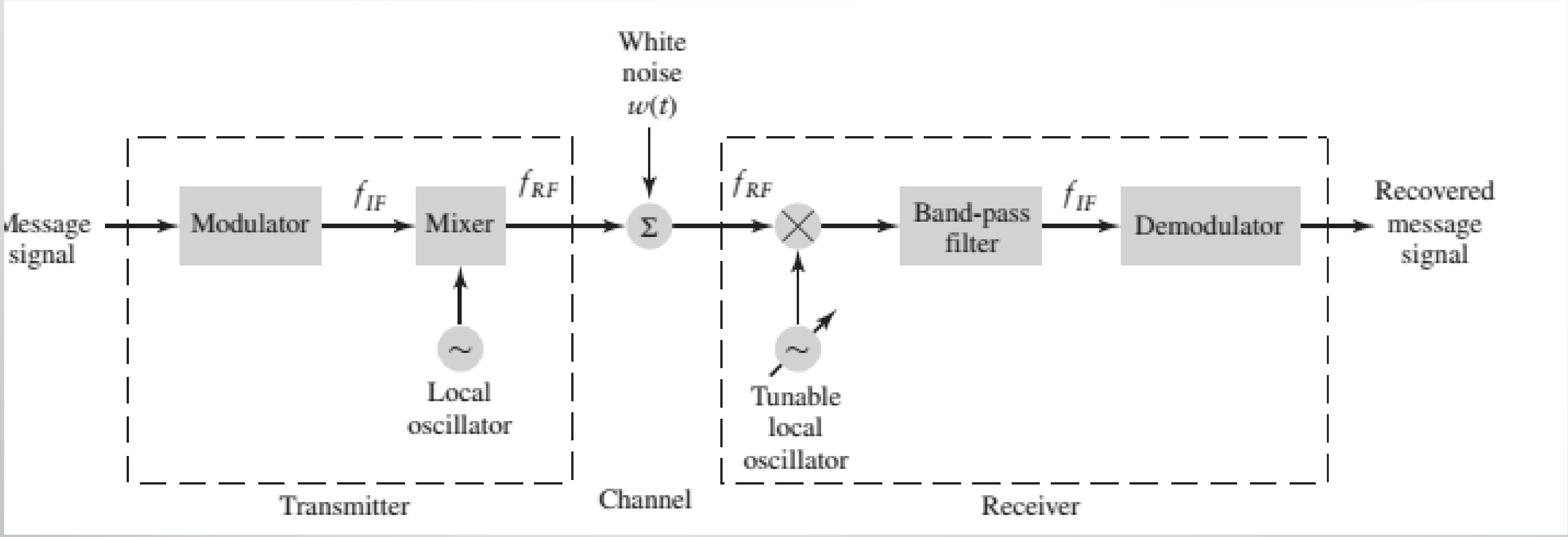


$$\text{Figure of merit} = \frac{\text{post}-\text{detection SNR}}{\text{reference SNR}}$$

$$\text{SNR}_{\text{ref}} = \frac{\text{average power of the modulated message signal}}{\text{average power of noise measured in the message bandwidth}}$$

- The pre-detection SNR is measured before the signal is demodulated.
- The post-detection SNR is measured after the signal is demodulated.
- The reference SNR is defined on the basis of a baseband transmission model.
- The figure of merit is a dimensionless metric for comparing different analog modulation–demodulation schemes and is defined as the ratio of the post-detection and reference SNRs.

Bandpass Receiver Structure:
Block diagram of band-pass transmission showing a superheterodyne receiver



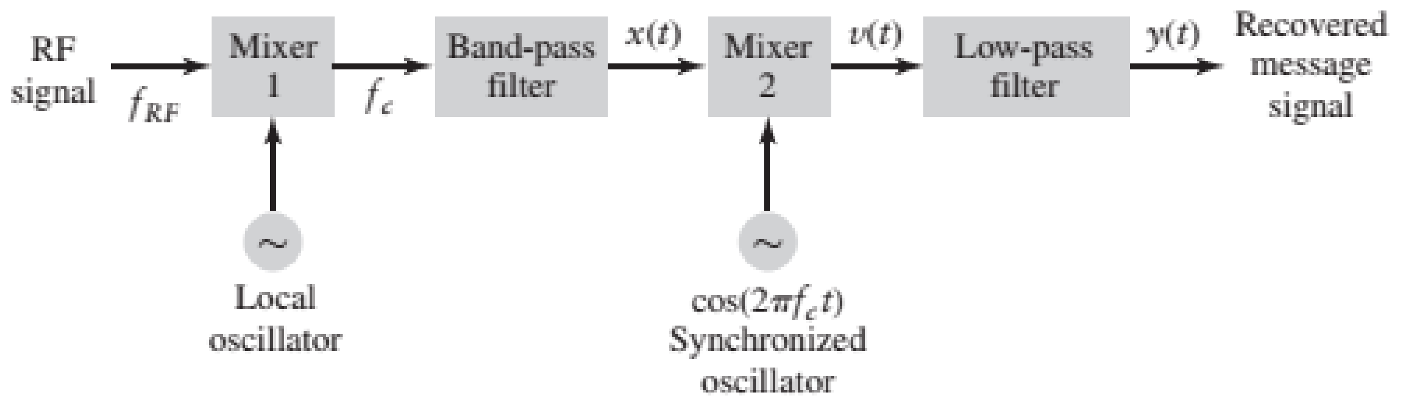
Common examples are AM radio transmissions, where the RF channels' frequencies lie in the range between 510 and 1600 kHz, and a common IF is 455 kHz; another example is FM radio, where the RF channels are in the range from 88 to 108 MHz and the
IF is typically 10.7 MHz

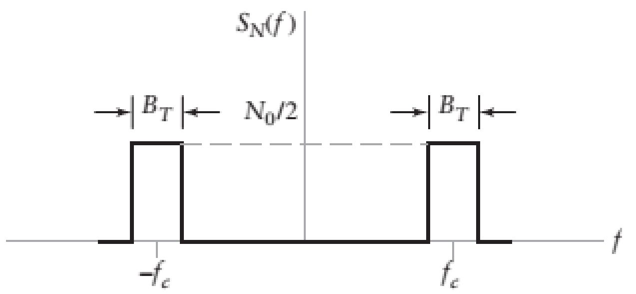
Noise in Linear Receivers Using Coherent Detection

Double sideband suppressed-carrier (DSB-SC) modulation, the modulated signal is represented as

$$s(t) = A_c m(t) \cos(2\pi f_c t + \theta)$$

$$x(t) = s(t) + n(t)$$





Power spectral density of band-pass noise

Pre-detection SNR for DSBSC

$$\mathbf{E}[s^2(t)] = \mathbf{E}[(A_c \cos(2\pi f_c t + \theta))^2]\mathbf{E}[m^2(t)]$$

If we let

$$P = \mathbf{E}[m^2(t)]$$

$$\mathbf{E}[s^2(t)] = \frac{A_c^2 P}{2}$$

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{DSB}} = \frac{A_c^2 P}{2 N_0 B_T}$$

POST-DETECTION SNR for DSBSC

$$x(t) = s(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$\begin{aligned} v(t) &= x(t)\cos(2\pi f_c t) \\ &= \tfrac{1}{2}(A_c m(t) + n_I(t)) \\ &+ \tfrac{1}{2}(A_c m(t) + n_I(t))\cos(4\pi f_c t) - \tfrac{1}{2} n_Q(t)\sin(4\pi f_c t) \end{aligned}$$

$$y(t) = \tfrac{1}{2}(A_c m(t) + n_I(t))$$

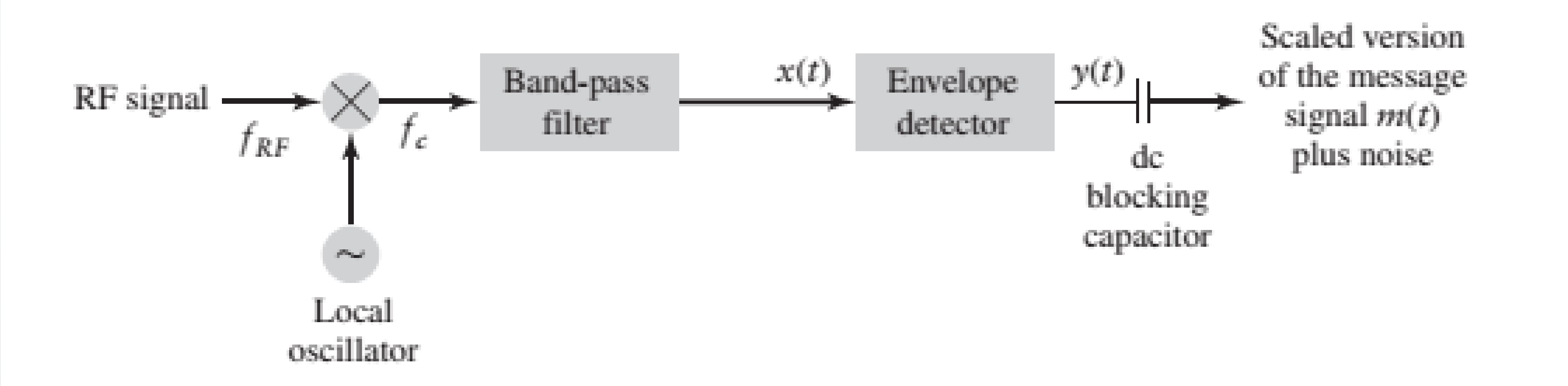
$$\begin{aligned} \mathbf{E}[n_I^2(t)] &= \int_{-W}^{W} N_0\, df \\ &= 2N_0 W \end{aligned}$$

$$\mathrm{SNR}_{\mathrm{ref}} = A_c^2 P/(2N_0 W$$

$$\begin{aligned} \mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}} &= \frac{\frac{1}{4}(A_c^2)P}{\frac{1}{4}(2N_0 W)} \\ &= \frac{A_c^2 P}{2N_0 W} \end{aligned}$$

$$\text{Figure of merit} = \frac{\mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}}}{\mathrm{SNR}_{\mathrm{ref}}} = 1$$

Noise In AM Receivers Using Envelope Detection



$$s(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$$

PRE-DETECTION SNR

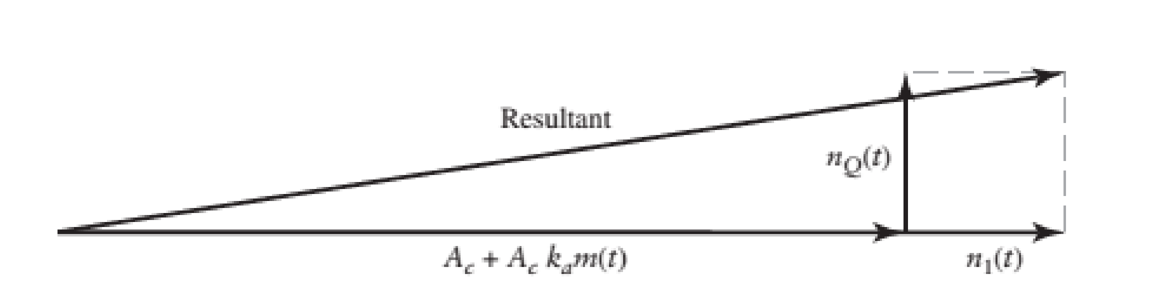
$$\begin{aligned} \mathbf{E}[(1 + k_a m(t))^2] &= \mathbf{E}[1 + 2k_a m(t) + k_a^2 m^2(t)] \\ &= 1 + 2k_a \mathbf{E}[m(t)] + k_a^2 \mathbf{E}[m^2(t)] \\ &= 1 + k_a^2 P \end{aligned}$$

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{AM}} = \frac{A_c^2(1 + k_a^2 P)}{2N_0 B_T}$$

Post-detection SNR

$$x(t) = s(t) + n(t)$$
$$= [A_c + A_c k_a m(t) + n_I(t)]\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$y(t) = \text{envelope of } x(t)$$
$$= \{[A_c(1 + k_a m(t)) + n_I(t)]^2 + n_Q^2(t)\}^{1/2}$$



$$y(t) \approx A_c + A_c k_a m(t) + n_I(t)$$

under high SNR conditions

$$\text{SNR}^{AM}_{post} = \frac{A_c^2 k_a^2 P}{2N_0 W}$$

Conditions:
-The SNR is high.
-*ka* is adjusted for 100% modulation or less, so there is no distortion of the signal envelope.

Figure of Merit

SNR is $A_c^2(1 + k_a^2 P)/(2N_0 W)$

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{AM}}}{\text{SNR}_{\text{ref}}} = \frac{k_a^2 P}{1 + k_a^2 P}$$

The figure of merit for this system is always less than 0.5. Hence, the noise performance of an envelope-detector receiver is always inferior to a DSB-SC receiver, the reason
is that at least half of the power is wasted transmitting the carrier as a component of the modulated (transmitted) signal

Noise in SSB
Do it…

$$s(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)$$

$$\text{SNR}_{\text{pre}}^{\text{SSB}} = \frac{A_c^2 P}{4 N_0 W}$$

$$y(t) = \frac{1}{2}\left(\frac{A_c}{2} m(t) + n_I(t)\right)$$

$$\text{SNR}_{\text{post}}^{\text{SSB}} = \frac{A_c^2 P}{4 N_o W}$$

the reference SNR is $A_c^2 P/(4 N_0 W$

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{SSB}}}{\text{SNR}_{\text{ref}}} = 1$$

Summery:

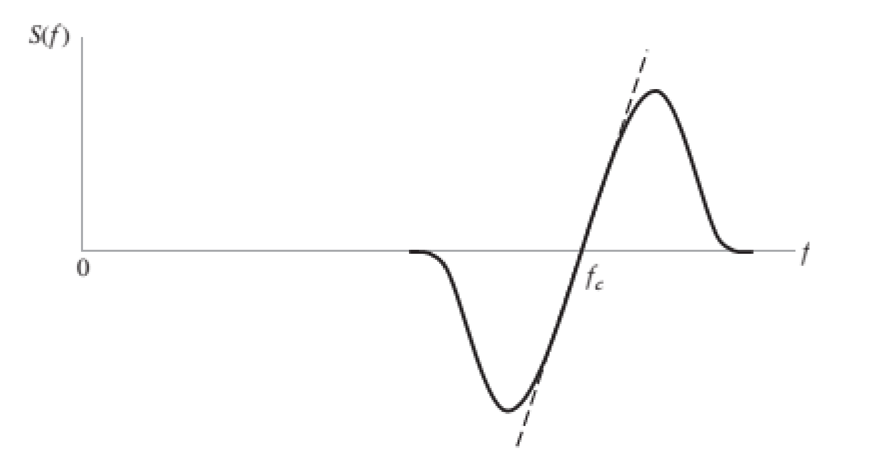
Comparing the results for the different amplitude modulation schemes, we find that there are a number of design tradeoffs. Double-sideband suppressed carrier modulation provides the same SNR performance as the baseband reference model but requires synchronization circuitry to perform coherent detection. Non-suppressed-carrier AM simplifies the receiver design significantly as it is implemented with an envelope detector.

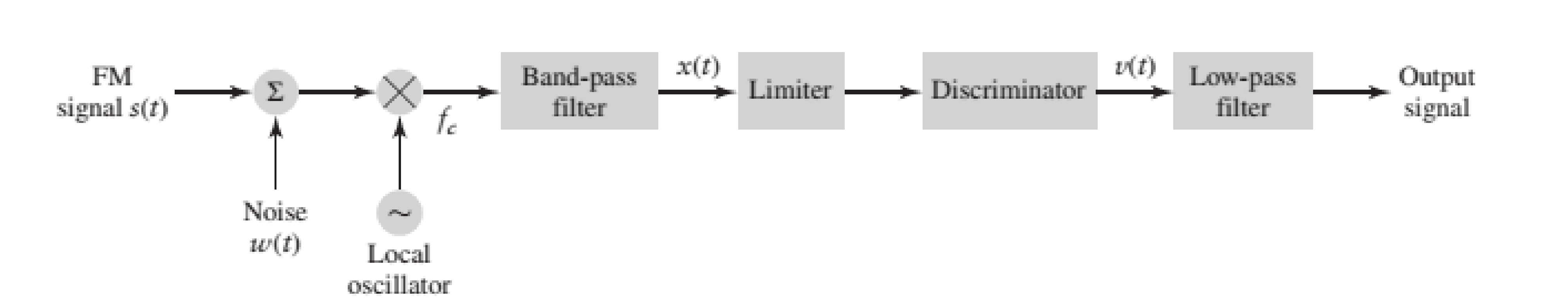
However, non-suppressed-carrier AM requires significantly more transmitter power to obtain the same SNR performance as the baseband reference model. Single-sideband modulation achieves the same SNR performance as the baseband reference model but only requires half the transmission bandwidth of the DSC-SC system. On the other hand, SSB requires more transmitter processing. *These observations are our first indication that communication system design involves a tradeoff between power, bandwidth, and processing complexity.*

Detection of Frequency Modulation (FM)

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

$$\text{SNR}_{\text{pre}}^{\text{FM}} = \frac{A_c^2}{2N_0 B_T}$$





Post Detection SNR

$$x(t) = s(t) + n(t)$$

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$n(t) = r(t)\cos[2\pi f_c t + \phi_n(t)]$$

$$r(t) = [n_I^2(t) + n_Q^2(t)]^{1/2}$$

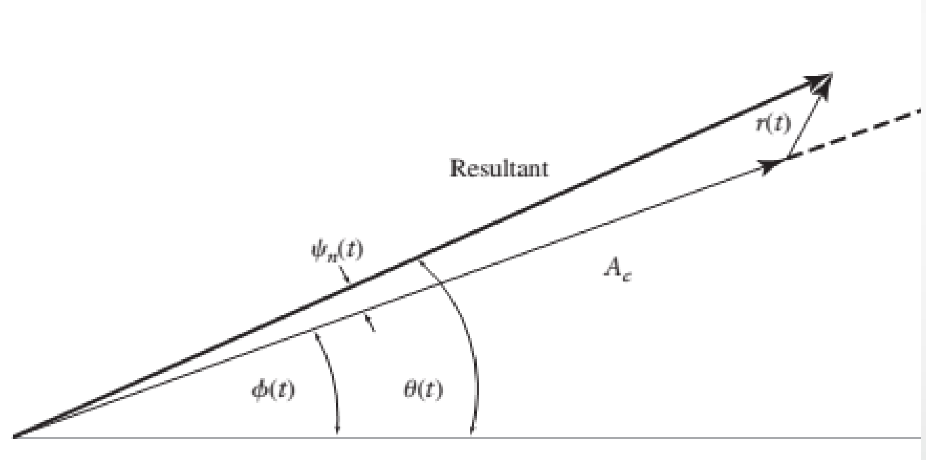
$$\phi_n(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right)$$

$$\phi(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

$$\begin{aligned} x(t) &= s(t) + n(t) \\ &= A_c\cos[2\pi f_c t + \phi(t)] + r(t)\cos[2\pi f_c t + \phi_n(t)] \end{aligned}$$

$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin(\psi_n(t))}{A_c + r(t)\cos(\psi_n(t))}\right\}$$

$$\psi_n(t) = \phi_n(t) - \phi(t)$$



$$\theta(t) = \phi(t) + \frac{r(t)}{A_c}\sin[\psi_n(t)]$$

$$\tan^{-1}\xi \approx \xi \text{ since } \xi \ll 1$$

$$\theta(t) = \phi(t) + \frac{n_Q(t)}{A_c}$$

$$n_Q(t) = r(t)\sin[\phi_n(t)]$$

$$\theta(t) \approx 2\pi k_f \int_0^t m(\tau)\,d\tau + \frac{n_Q(t)}{A_c}$$

$$v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = k_f m(t) + n_d(t)$$

$$n_d(t) = \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}$$

$$G(f) = \frac{j2\pi f}{2\pi A_c} = \frac{jf}{A_c}$$

$$S_{N_d}(f) = |G(f)|^2 S_{N_Q}(f) = \frac{f^2}{A_c^2} S_{N_Q}(f)$$

$$S_{N_d}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| < \frac{B_T}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$S_{N_o}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| < W \\ 0, & \text{otherwise} \end{cases}$$

$$\text{Average post-detection noise power} = \frac{N_0}{A_c^2}\int_{-W}^{W} f^2\, df = \frac{2N_0 W^3}{3A_c^2}$$

$$\text{SNR}_{\text{post}}^{\text{FM}} = \frac{3A_c^2 k_f^2 P}{2N_0 W^3}$$

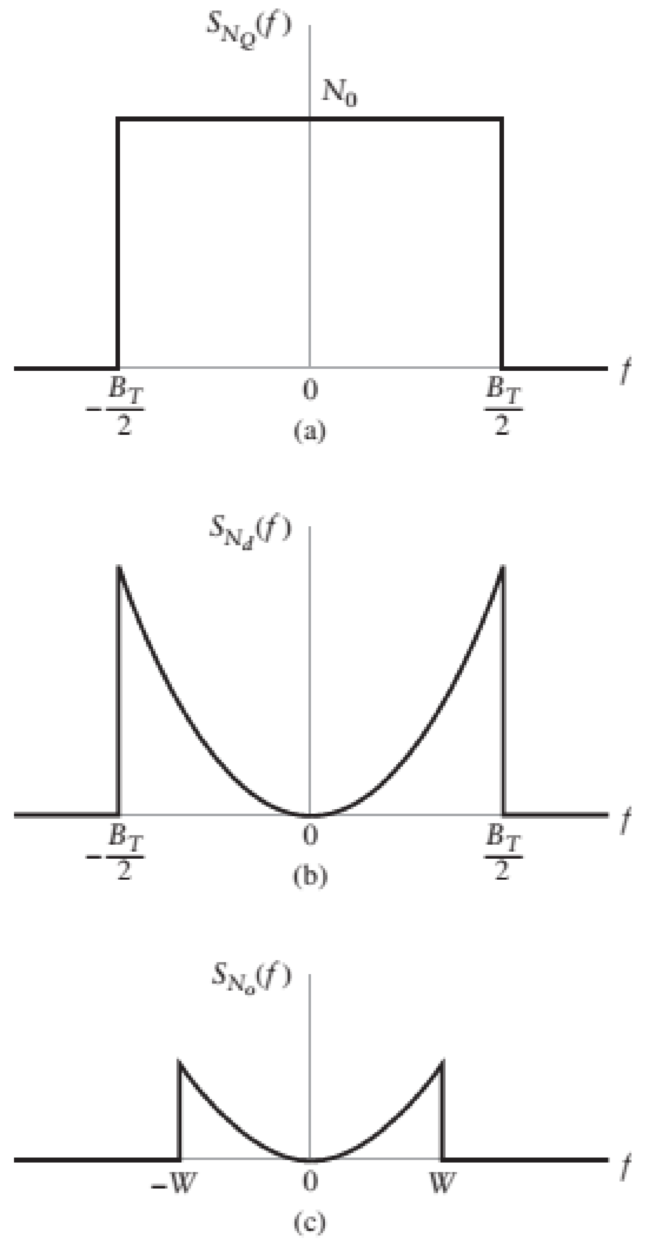


Figure of Merit

With FM modulation, the modulated signal power is simply $A_c^2/2$, hence the reference SNR is $A_c^2/(2N_0W)$. Consequently, the figure of merit for this system is given by

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{FM}}}{\text{SNR}_{\text{ref}}} = \frac{\dfrac{3A_c^2k_f^2P}{2N_0W^3}}{\dfrac{A_c^2}{2N_0W}}$$

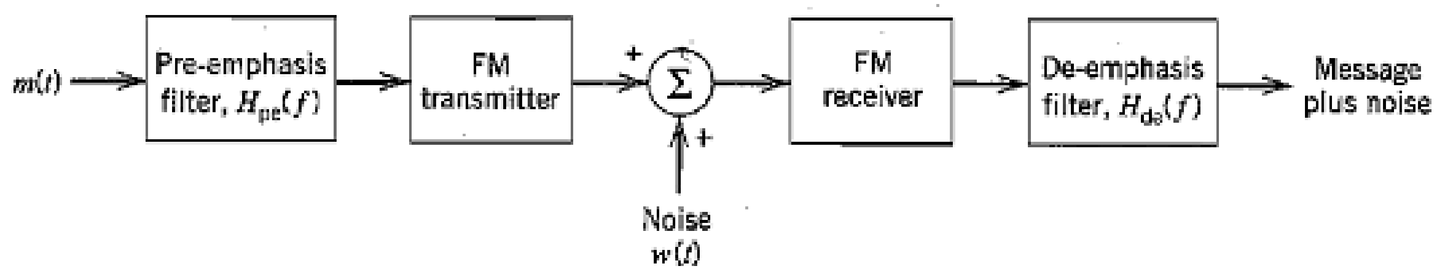
$$= 3\left(\frac{k_f^2P}{W^2}\right)$$

$$= 3D^2$$

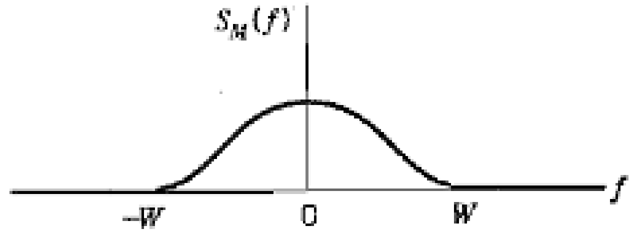
where, we have introduced the definition D as the deviation ratio . Recall from generalized Carson rule yields the transmission bandwidth $B_T = 2(k_fP^{1/2} + W) \approx 2k_fP^{1/2}$ for an FM signal. So, substituting $B_T/2$ for $k_fP^{1/2}$ for in the definition of *D*, the figure of merit for an FM system is approximately given by

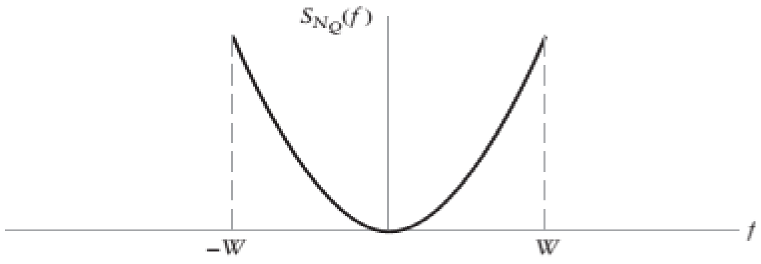
$$\text{Figure of merit} \approx \frac{3}{4}\left(\frac{B_T}{W}\right)^2$$

FM Pre-emphasis and De-emphasis



$$H_{pre}(f) = \frac{1}{H_{de}(f)} \qquad |f| < W$$





$$H_{de}(f) = \frac{1}{1 + j\dfrac{f}{f_{3dB}}}$$

$$H_{pre}(f) = 1 + j\frac{f}{f_{3dB}}$$

$$|H_{de}(f)|^2 S_{N_d}(f) = \begin{cases} \dfrac{N_0 f^2}{A_c^2}|H_{de}(f)|^2, & |f| \leq \dfrac{B_T}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$I = \frac{\text{average output noise power without pre-emphasis and de-emphasis}}{\text{average output noise power with pre-emphasis and de-emphasis}}$$

$$\begin{pmatrix}\text{Average output noise} \\ \text{power with de-emphasis}\end{pmatrix} = \frac{N_0}{A_c^2}\int_{-W}^{W} f^2 |H_{de}(f)|^2 \, df$$

$$I = \frac{2W^3}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 \, df}$$

In commercial FM broadcasting, we typically have $f_{3dB} = 2.1$ kHz, and we may reasonably assume $W = 15$ kHz. This set of values yields $I = 22$, which corresponds to an improvement of 13 dB in the post-detection signal-to-noise ratio of the receiver. This example illustrates that a significant improvement in the noise performance of an FM system may be achieved by using pre-emphasis and de-emphasis filters made up of simple RC circuits.