Pulse Modulation: Transition from Analog to Digital

What We will Learn

- *Lesson 1: Given a strictly band-limited message signal, the sampling theorem embodies the conditions for a uniformly sampled version of the signal to preserve its information content.*

  Lesson 2: Analog pulse-modulation systems rely on the sampling process to maintain continuous amplitude representation of the message signal. In contrast, digital pulse-modulation systems use not only the sampling process but also the quantization process, which is non-reversible. Quantization provides a representation of the message signal that is discrete in both time and amplitude. In so doing, digital pulse modulation makes it possible to exploit the full power of digital signal-processing techniques.

Sampling Process

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)$$

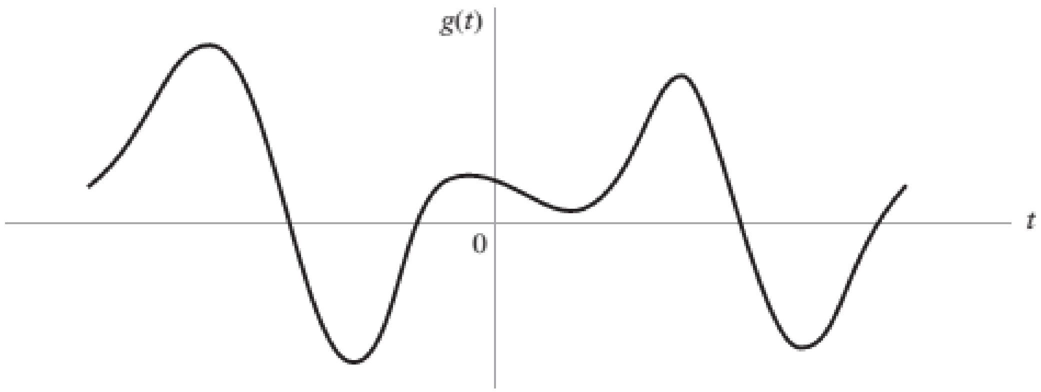
$$\sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) = \sum_{n=-\infty}^{\infty} g(nT_s)\exp(-j2\pi nT_s f) = G_\delta(f)$$

$$\sum_{m=-\infty}^{\infty} g(t - mT_0) \rightleftharpoons f_0 \sum_{n=-\infty}^{\infty} G(nf_0)\delta(f - nf_0)$$

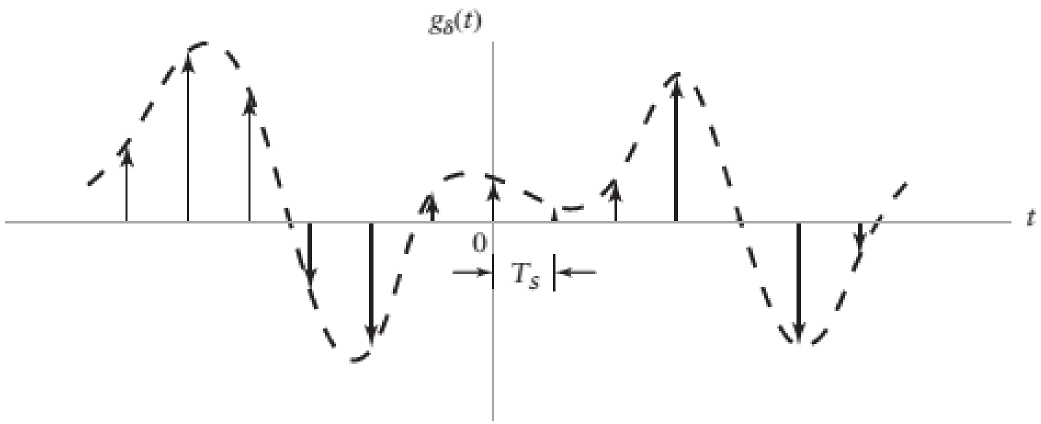
$$\sum_{m=-\infty}^{\infty} \delta(t - mT_0) = f_0 \sum_{n=-\infty}^{\infty} \exp(j2\pi nf_0 t)$$

$$\sum_{m=-\infty}^{\infty} \delta(t - mT_0) \rightleftharpoons f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0)$$

$$T_0 \sum_{m=-\infty}^{\infty} \exp(j2\pi mfT_0) = \sum_{n=-\infty}^{\infty} \delta(f - nf_0)$$



(a)



(b)

Time Frequency Sampling Duality Relationship

Ideal sampling in the frequency domain (Discrete spectrum); see Chapter 2	*Ideal sampling in the time domain (Discrete-time function); see this chapter*
Fundamental period $T_0 = 1/f_0$	Sampling rate $f_s = 1/T_s$
Delta function $\delta(f - mf_0)$, where $m = 0, \pm 1, \pm 2, \ldots$	Delta function $\delta(t - nT_s)$ where $n = 0, \pm 1, \pm 2, \ldots$
Periodicity in the time-domain	Periodicity in the frequency domain
Time-limited function	Band-limited spectrum
$T_0 \sum_{m=-\infty}^{\infty} g(t - mT_0) = \sum_{n=-\infty}^{\infty} G(nf_0) e^{j2\pi n f_0 t}$ $\rightleftharpoons$ $\sum_{n=-\infty}^{\infty} G(nf_0)\delta(f - nf_0)$	$\sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)$ $\rightleftharpoons$ $\sum_{n=-\infty}^{\infty} g(nT_s) e^{-j2\pi n T_s f} = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$

Sampling Theorem

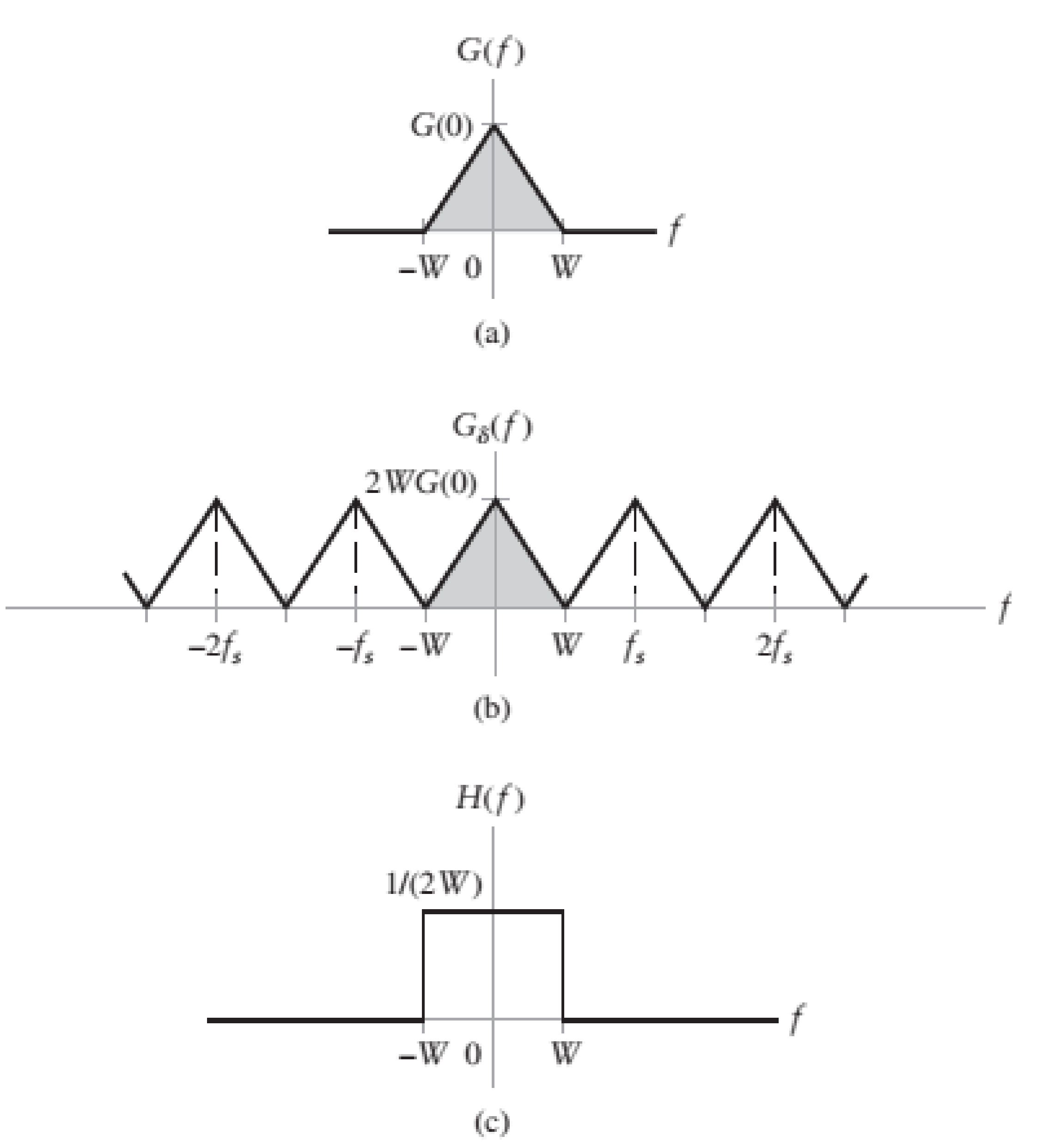
Suppose the signal is *strictly band-limited*, with no frequency components higher than W hertz.

We choose $T_s = 1/2W$

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right)$$

discrete-time Fourier transform

$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f - m f_s)$$



Sampling Theorem

1. $G(f) = 0$ for $|f| \geq W$
2. $f_s = 2W$

$$G(f) = \frac{1}{2W} G_\delta(f), \qquad -W < f < W$$

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right), \qquad -W < f < W$$

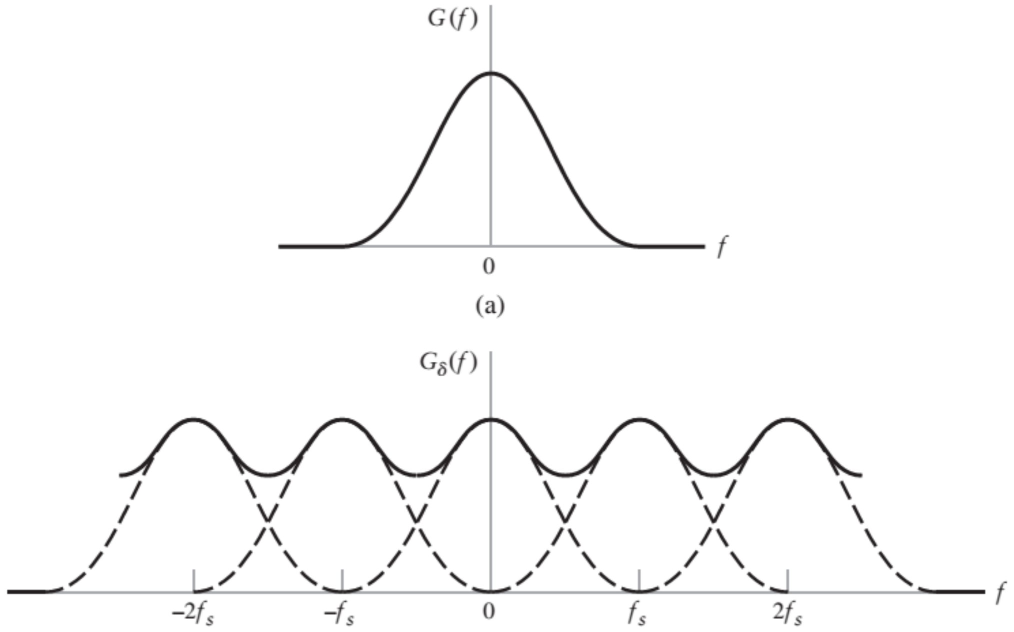
$$\begin{aligned} g(t) &= \int_{-\infty}^{\infty} G(f) \exp(j2\pi f t)\, df \\ &= \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right) \exp(j2\pi f t)\, df \end{aligned}$$

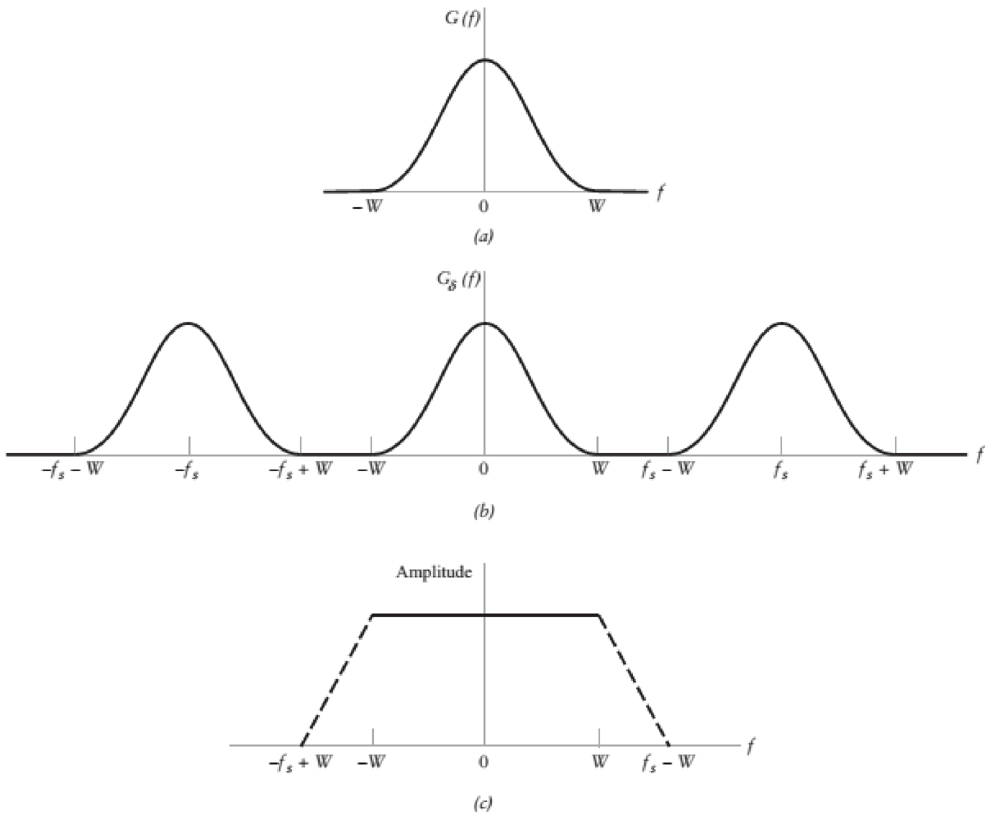
$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt - n), \qquad -\infty < t < \infty$$

Result….

The *interpolation formula* for reconstructing the original signal g(t) from the sequence of sample values $\{g(n/2W)\}$
with the sinc function sinc(2Wt) playing the
role of an *interpolation function*. Each sample is multiplied by a delayed version of the interpolation function, and all the resulting waveforms are added to obtain g(t)

Aliasing





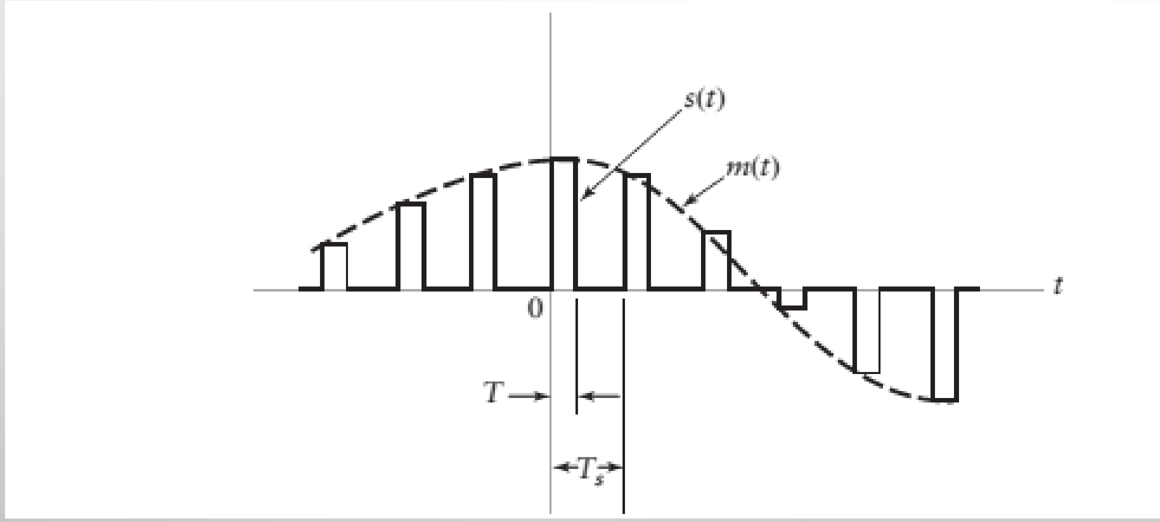
To combat the effects of aliasing in practice, we may use two *corrective measures*:

1. Prior to sampling, a low-pass *anti-alias filter* is used to attenuate those high-frequency components of a message signal that are not essential to the information being conveyed by the signal.

2. The filtered signal is sampled at a rate slightly higher than the Nyquist rate.

Pulse Amplitude Modulation (PAM)

pulse-amplitude modulation, which is the simplest and most basic form of analog pulse modulation techniques. In *pulse-amplitude modulation* (PAM), *the amplitudes of regularly spaced pulses are varied in proportion to the corresponding sample values of a continuous message signal*. The pulses can be of a rectangular form or some other appropriate shape, where the message signal is multiplied by a periodic train of rectangular pulses.

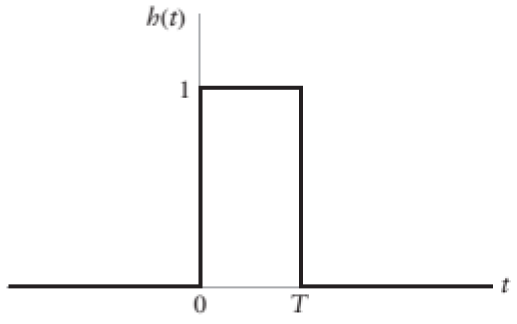


Sample and Hold

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

$$h(t) = \text{rect}\left(\frac{t - \frac{T}{2}}{T}\right) = \begin{cases} 1, & 0 < t < T \\ \frac{1}{2}, & t = 0, t = T \\ 0, & \text{otherwise} \end{cases}$$

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s)$$



(a)

$$\begin{aligned} m_\delta(t) \star h(t) &= \int_{-\infty}^{\infty} m_\delta(\tau)h(t - \tau)\, d\tau \\ &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} m(nT_s)\delta(\tau - nT_s)h(t - \tau)\, d\tau \\ &= \sum_{n=-\infty}^{\infty} m(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)h(t - \tau)\, d\tau \end{aligned}$$

$$\int_{-\infty}^{\infty} \delta(\tau - nT_s)h(t - \tau)\, d\tau = h(t - nT_s)$$

$$m_\delta(t) \star h(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

S/H Circuit

$$s(t) = m_\delta(t) \star h(t)$$

$$S(f) = M_\delta(f)H(f)$$

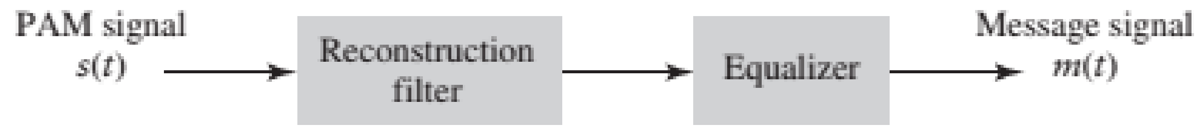
$$m_\delta(t) \star h(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

$$M_\delta(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)$$

$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)H(f)$$

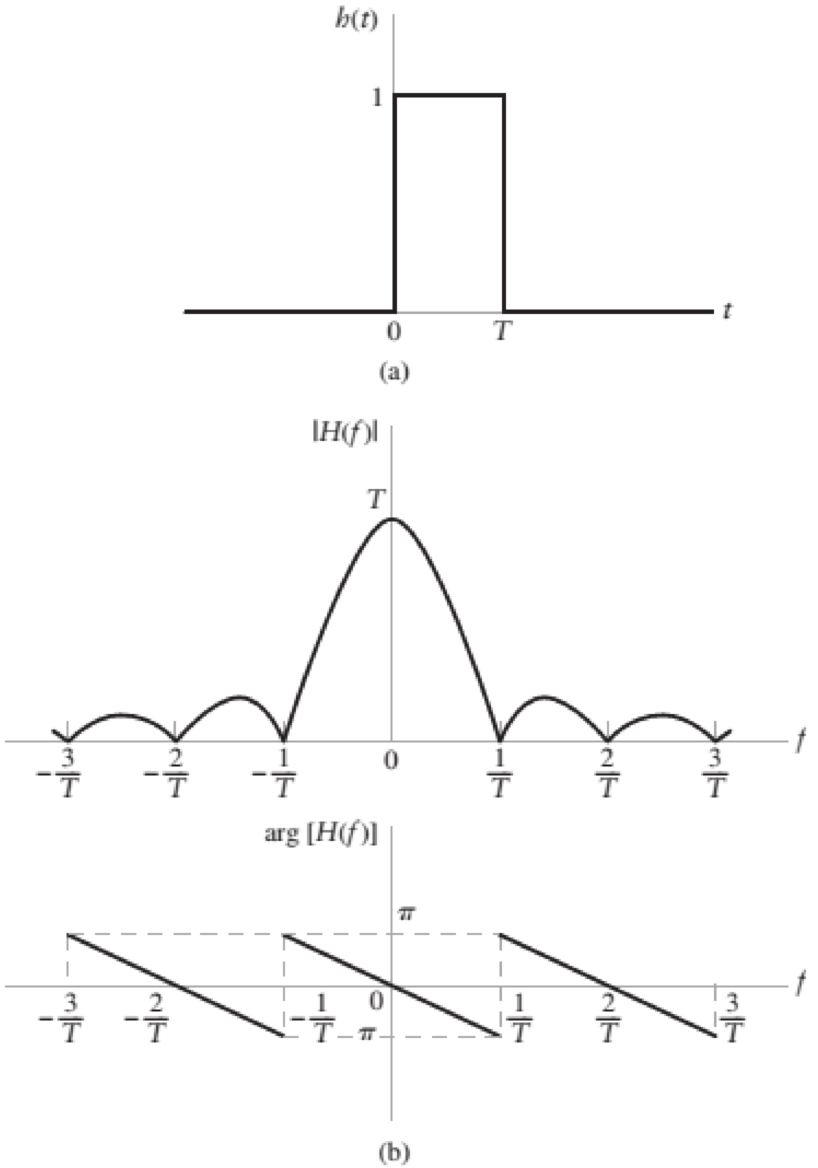
Aperture Effect

Amplitude distortion is resulted



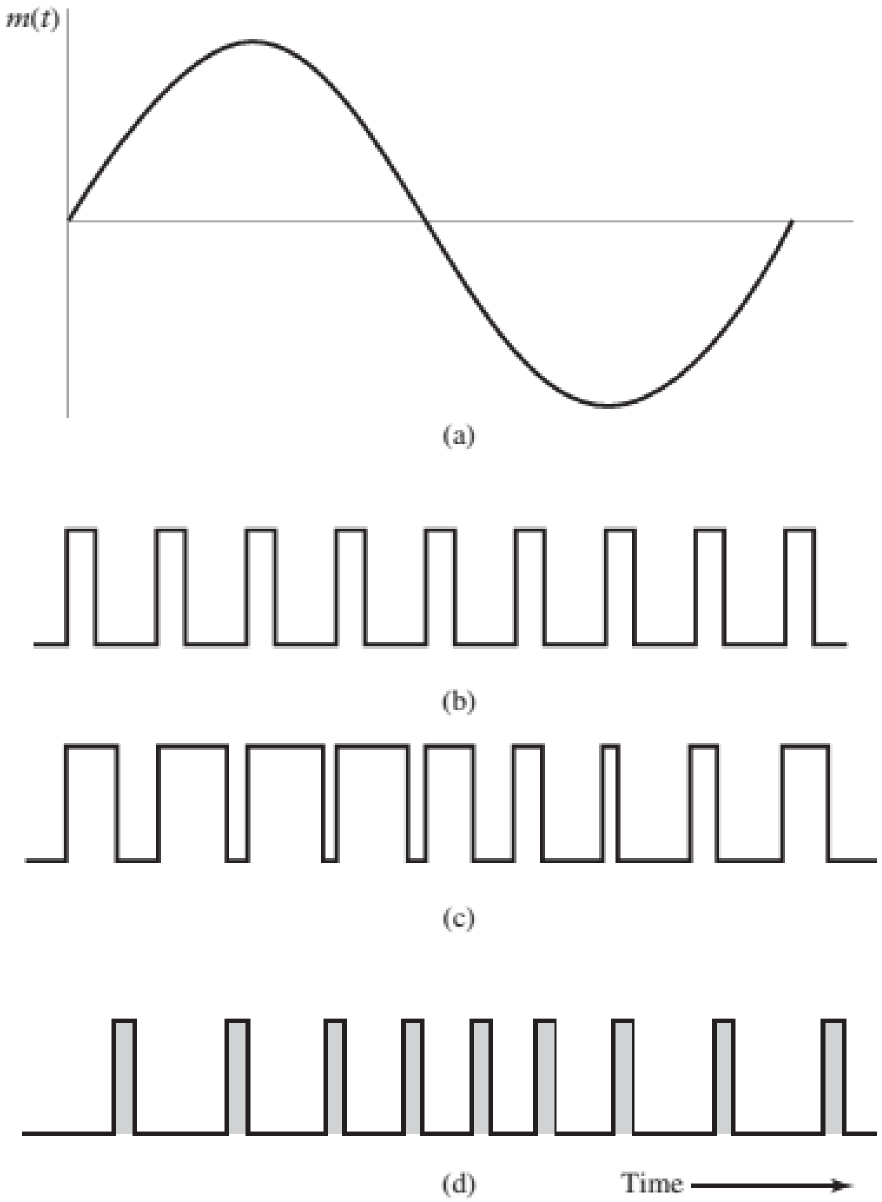
Equalizer:

$$\frac{1}{|H(f)|} = \frac{1}{T\,\mathrm{sinc}(fT)} = \frac{\pi f}{\sin(\pi f T)}$$



The amount of equalization needed in practice is usually small. Indeed, for a duty cycle $(T/T_s) \leq 0.1$, the amplitude distortion is less than 0.5 percent, in which case the need for equalization may be omitted altogether.

Pulse Position Modulation



In pulse-amplitude modulation, pulse amplitude is the variable parameter. Pulse duration is the next logical parameter available for modulation. *In pulse-duration modulation (PDM),the samples of the message signal are used to vary the duration of the individual pulses*. This form of modulation is also referred to as *pulse-width modulation* or *pulse-length modulation*. The modulating signal may vary the time of occurrence of the leading edge, the trailing edge, or both edges of the pulse.

PDM is wasteful of power, If this unused power is subtracted from PDM, so that only time transitions are essentially preserved, we obtain a more efficient type of pulse modulation known as *pulse-position modulation* (PPM). In *PPM, the position of a pulse relative to its unmodulated time of occurrence is varied in accordance with he message signal*,

Transition from Analog to Digital

Advantages:

- *Performance*. In an analog communication system, the effects of signal distortion and channel noise are *cumulative*. These sources of impairments therefore tend to become progressively stronger, ultimately overwhelming the ability of the communication system to offer an acceptable level of performance from source to destination. Unfortunately, the use of repeaters in the form of amplifiers, placed at different points along the transmission path, offers little help because the message signal and noise are amplified to the same extent. In sharp contrast, digital pulse modulation permits the use of *regenerative repeaters*, which, when placed along the transmission path at short enough distances, can practically eliminate the degrading effects of channel noise and signal distortion.

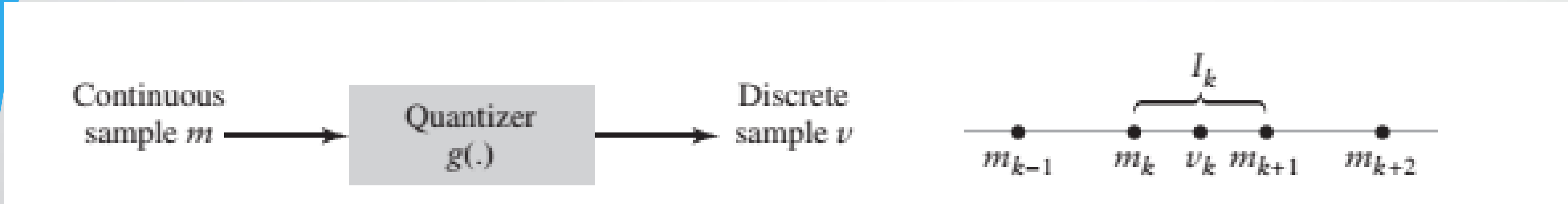
- *Ruggedness*. Unlike an analog communication system, a digital communication system can be designed to withstand the effects of channel noise and signal distortion, provided the noise and distortion are kept under certain limits.

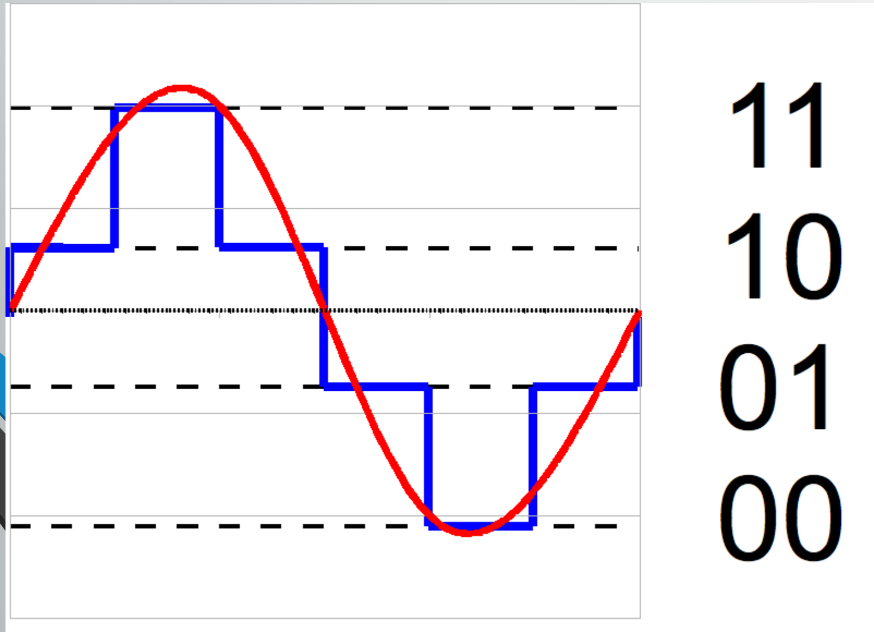
- *Security*. By the same token, digital communication systems can be made highly secure by exploiting powerful encryption algorithms that rely on digital processing for their implementation.

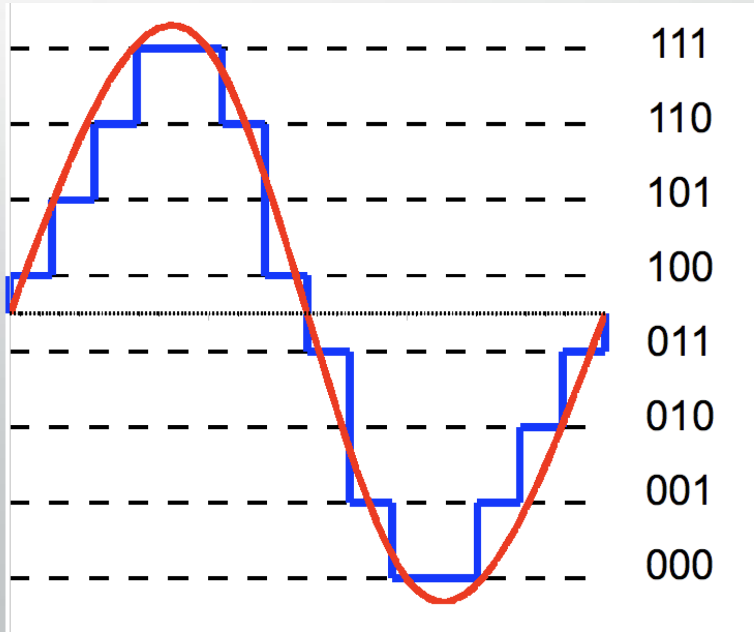
- *Efficiency*. Digital communication systems are inherently more efficient than analog communication systems in the tradeoff between transmission bandwidth and signal to-noise ratio.

- *System integration*. The use of digital communications makes it possible to integrate digitized analog signals (i.e., voice and video signals) with digital computer data, which is not possible with analog communications.

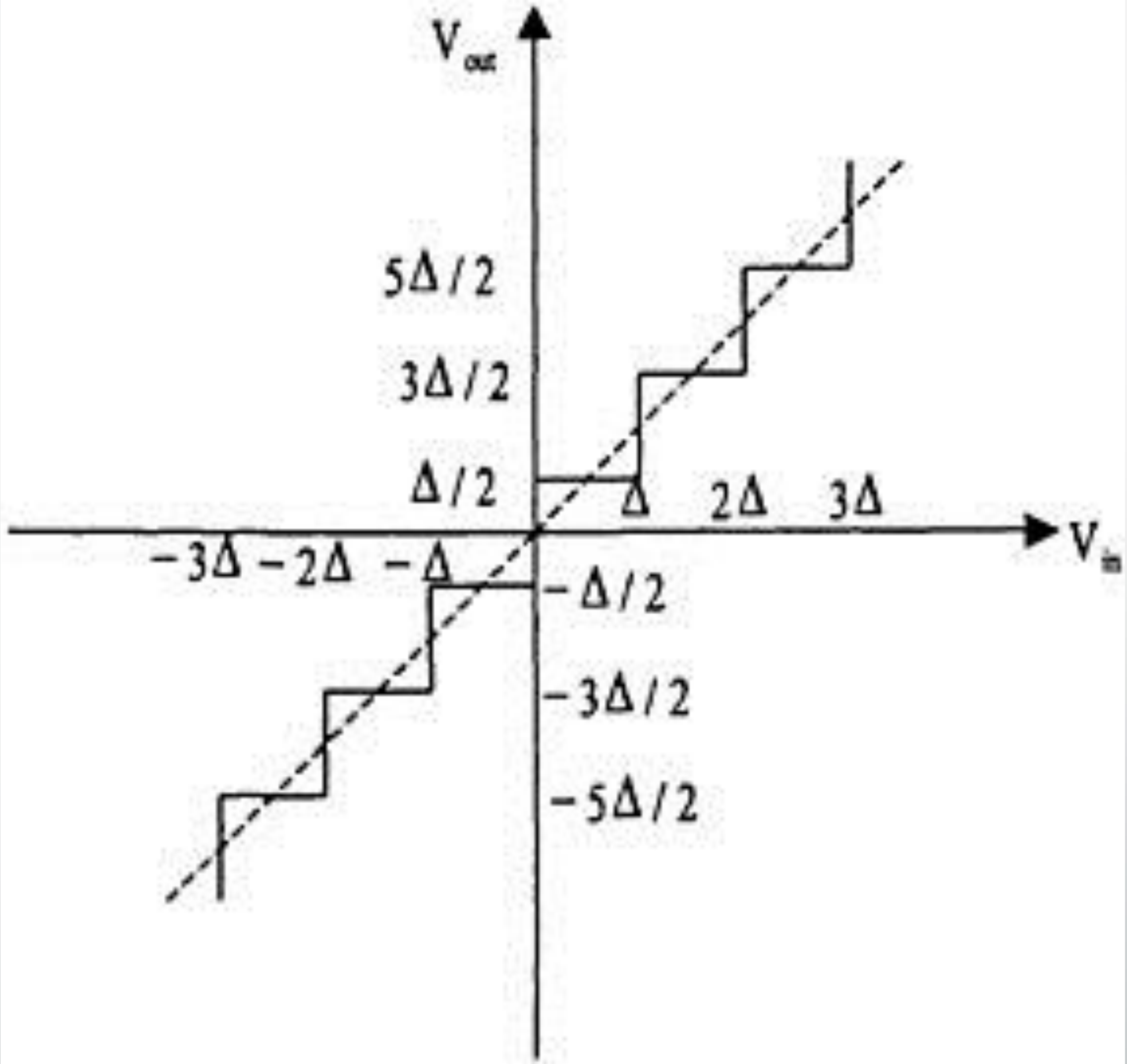
Quantization Process







Midtread Quantizer

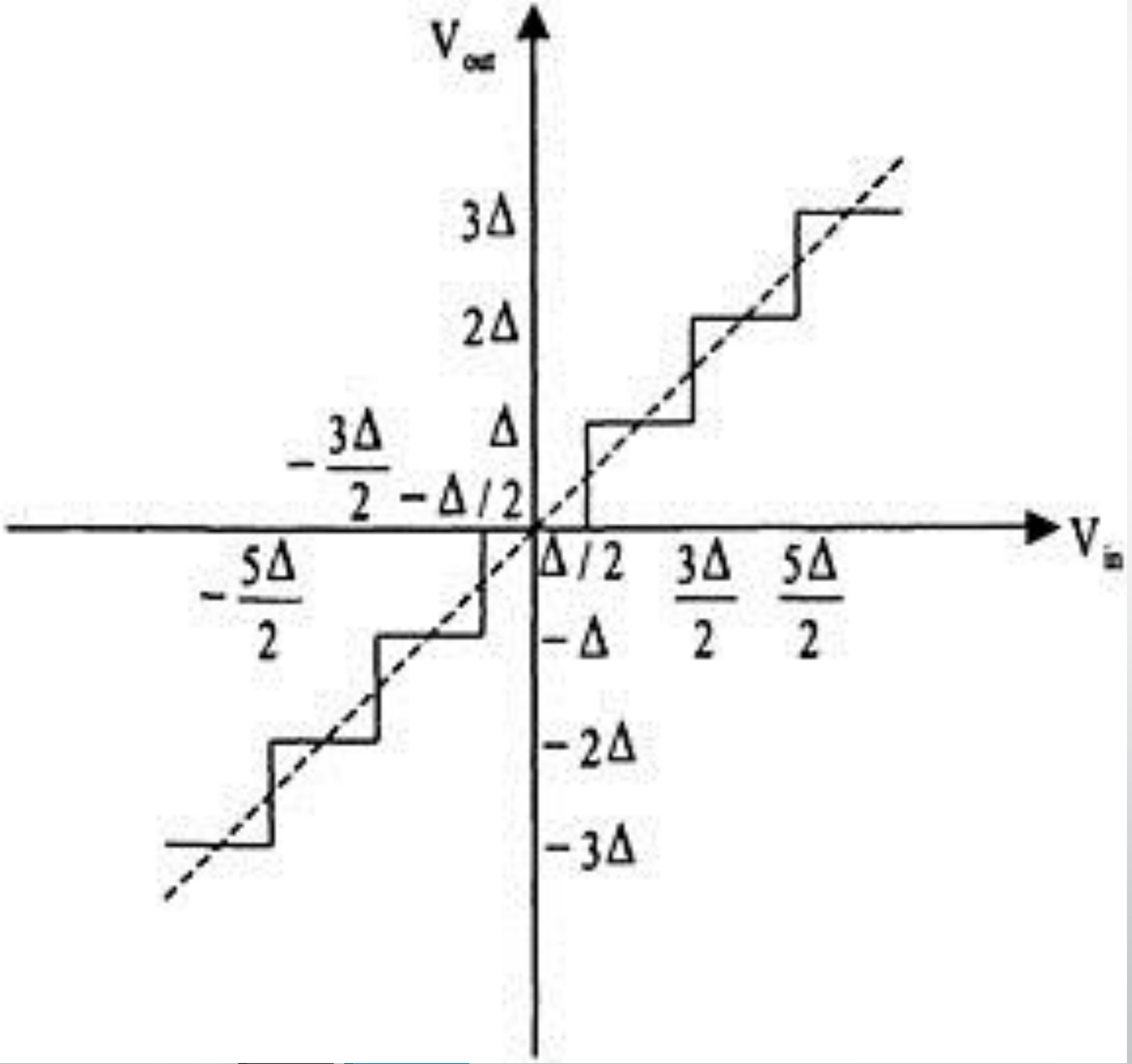


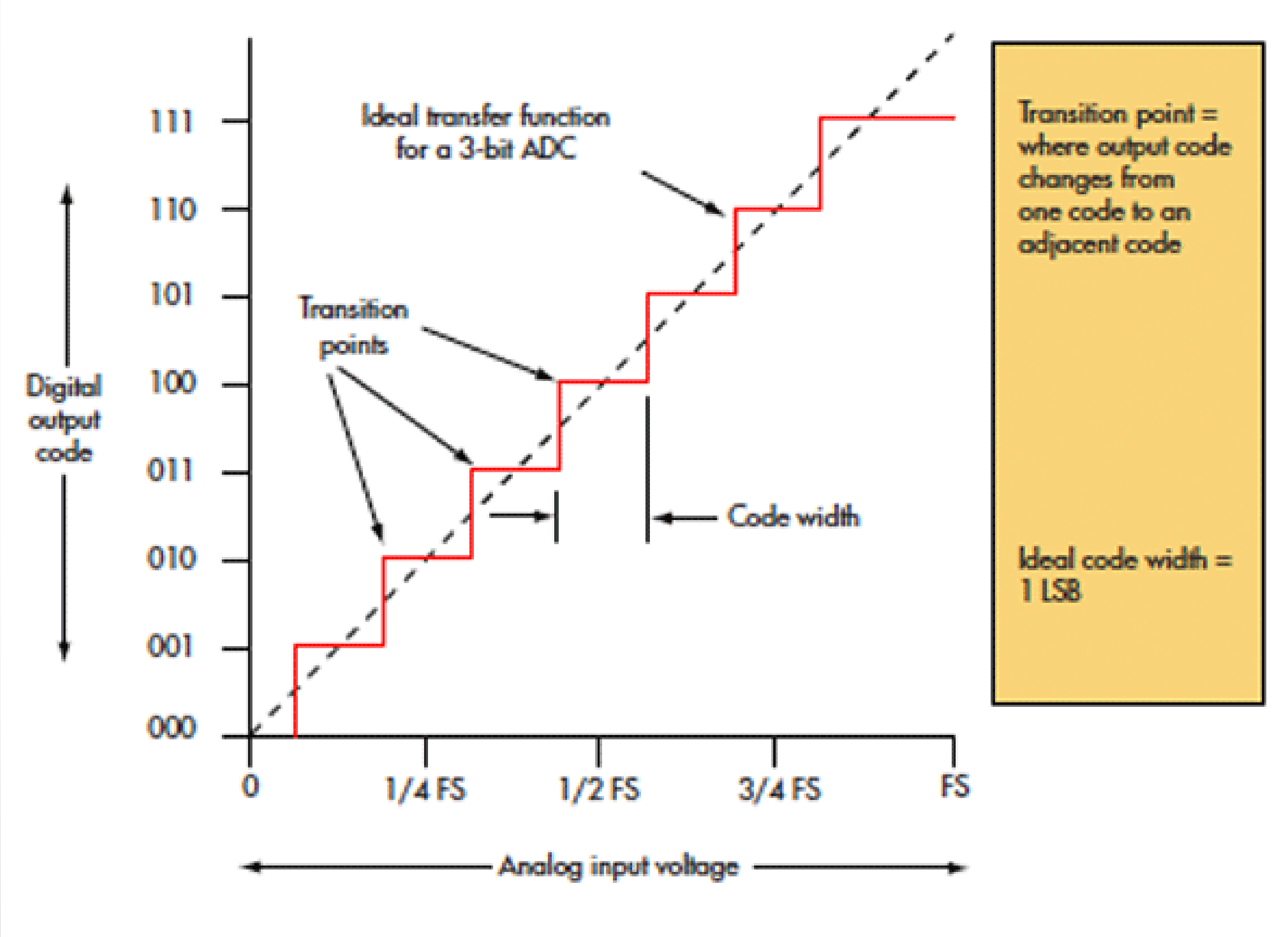
$Q = 2^n$

If $(m-1)\Delta < V_{in} < m\Delta \Longrightarrow V_{out} = (m - \frac{1}{2})\Delta$

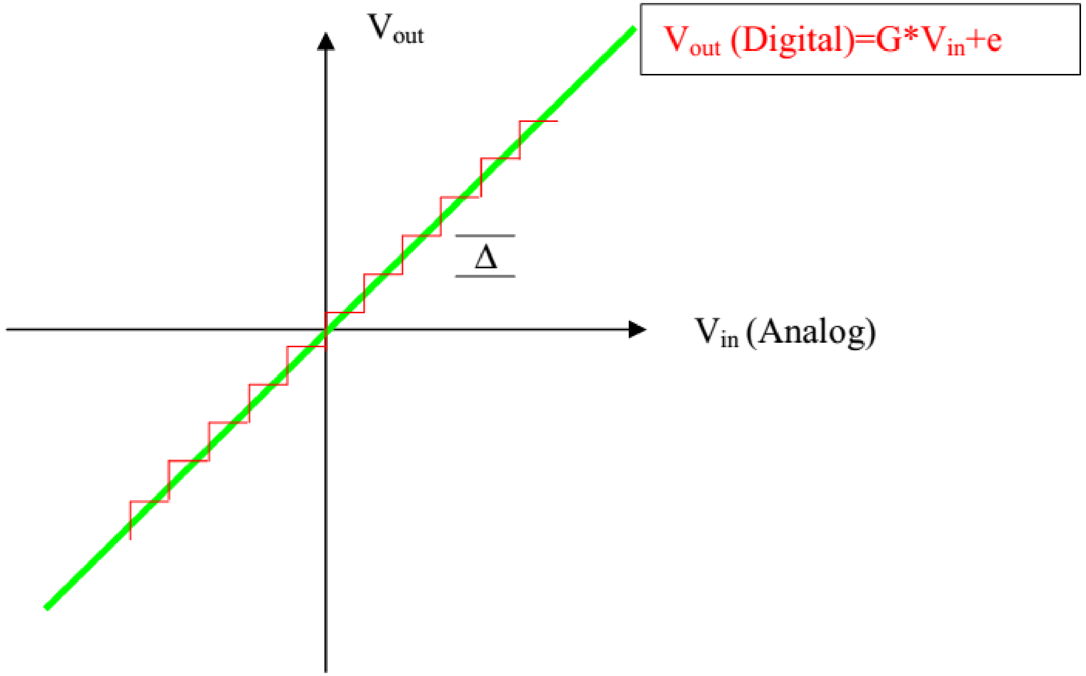
$m = -\frac{Q}{2} + 1, -\frac{Q}{2} + 2, \ldots, \frac{Q}{2} - 1, \frac{Q}{2}$

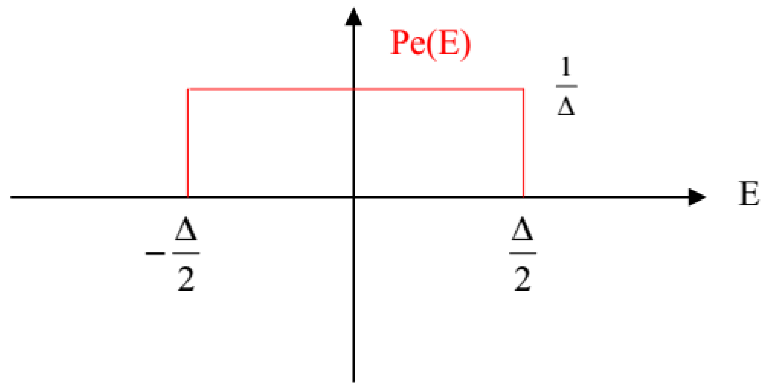
Midrise Quantizer



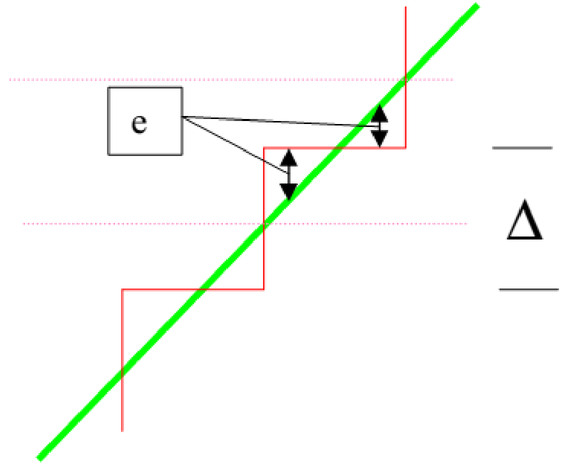


Quantization Signal to Noise Ration –QSNR-





PDF of Quantization Error



Zoom in of Staircase

Uniform versus Non-uniform Quantizer

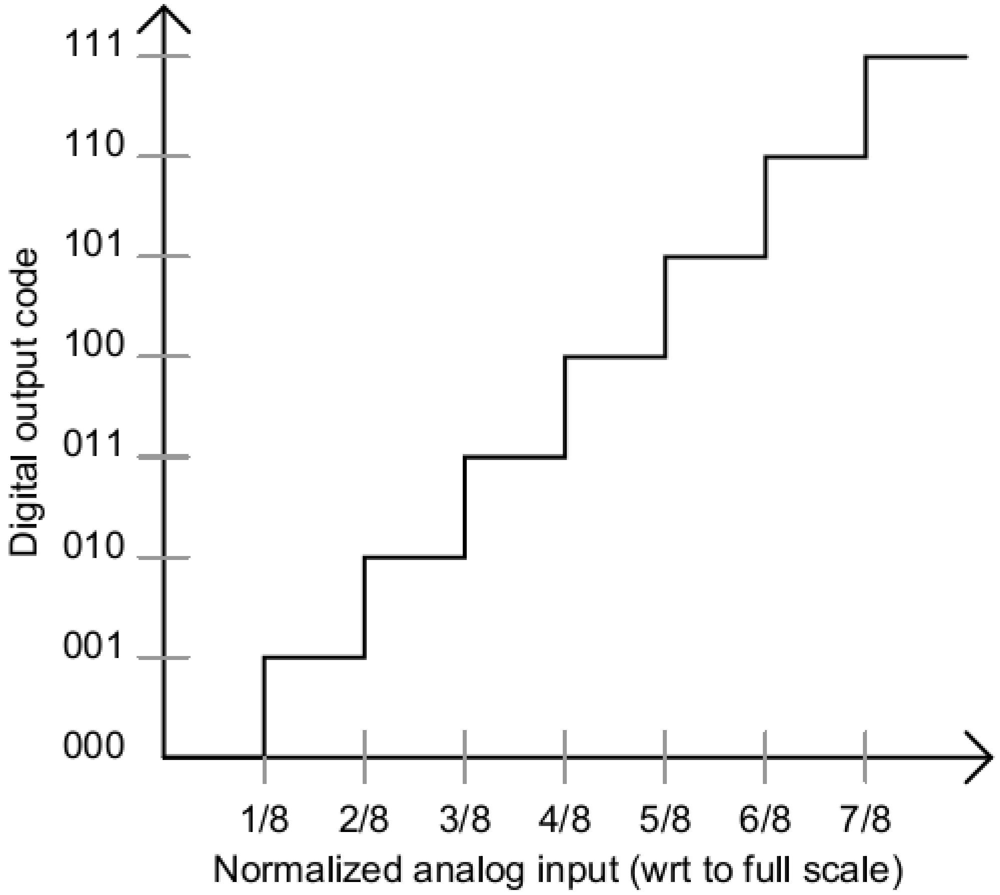


Fig : Uniform Quntization

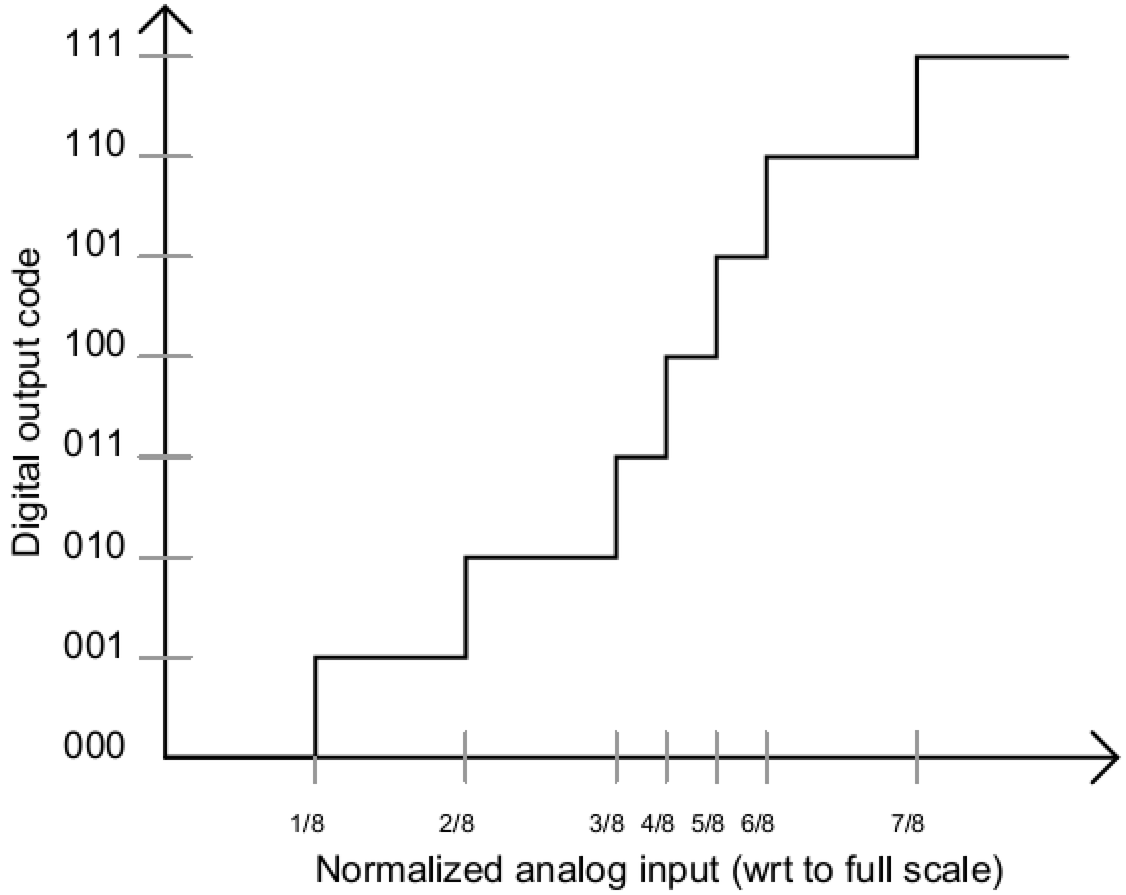


Fig : Non-uniform Quntization

- $$\sigma_{QNoise}^{2} = E(e^2) = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{1}{\Delta} e^2\, de = \frac{1}{\Delta} \frac{e^3}{3} \Big|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} = \frac{1}{\Delta}\left[\frac{\left(\frac{\Delta}{2}\right)^3}{3} - \frac{\left(-\frac{\Delta}{2}\right)^3}{3}\right] = \frac{1}{\Delta}\left[\frac{\Delta^3}{24} - \frac{-\Delta^3}{24}\right] = \frac{\Delta^2}{12}$$

- $$\sigma_{QNoise}^{2} = \text{Quantization Noise Power} = \frac{\Delta^2}{12}$$

- $$\sigma_{QNoise} = V_{QNoise\_rms} = \frac{\Delta}{\sqrt{12}}$$

- RMS value for a full scale sinusoidal input is

$$V_{MaxSignal\_rms} = \frac{\left(\frac{2^N}{2}\right)}{\sqrt{2}}\Delta$$

- $$\text{Max SNR} = 20\log\left(\frac{\frac{\left(\frac{2^N}{2}\right)}{\sqrt{2}}\Delta}{\frac{\Delta}{\sqrt{12}}}\right) = 20\log\left(\frac{\sqrt{6}}{2}2^N\right) = 20\log\left(\frac{\sqrt{6}}{2}\right) + 20N\log(2)$$

$$= 1.76 + 6.02\text{N}$$

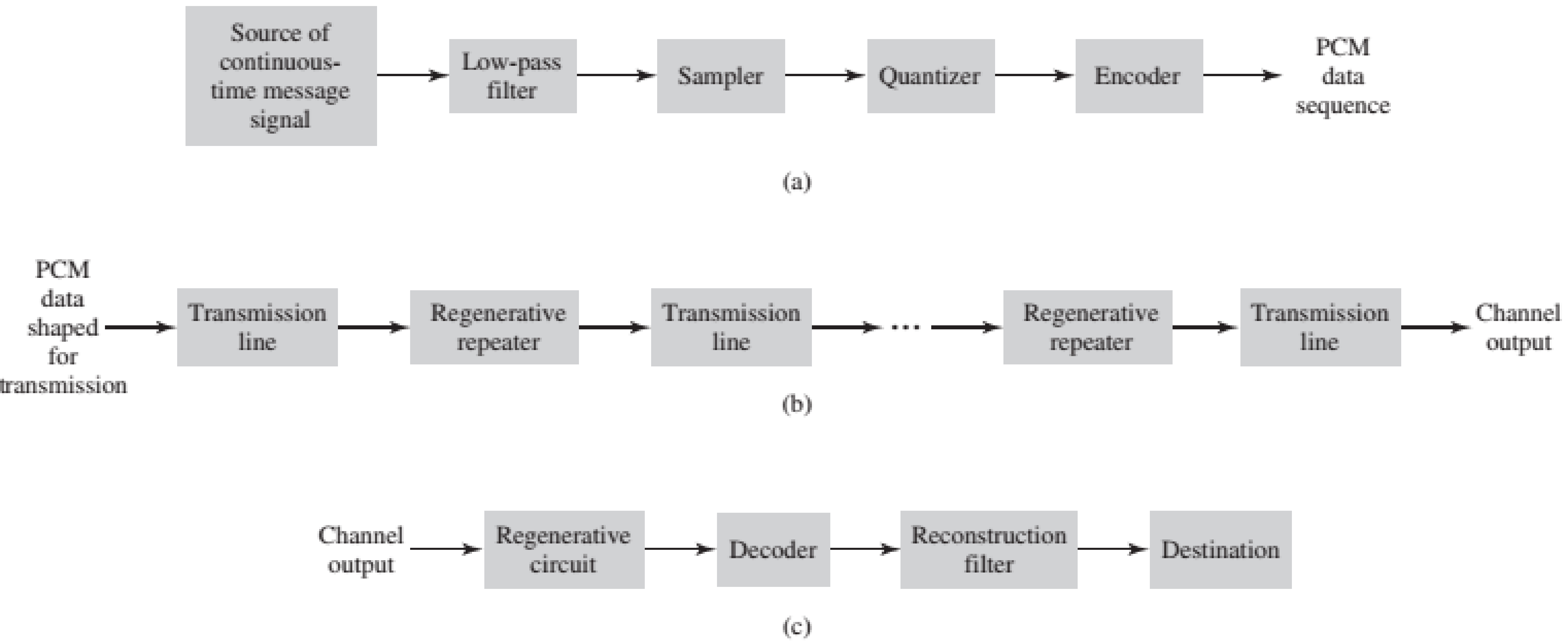
- $$\text{N} = \text{Effective\# of Bits} = \frac{\text{Max SNR - 1.76}}{6.02}$$

Exercise: Consider Vin =1/8 of the full scale. Determine QSNR

QSNR=-10.28+6.02n
Less 12 dB since input power is reduced by 12 dB.

Note: Range of input signal does not match the dynamic range of the quantizer

Pulse Code Modulation-PCM-



Pulse Code Modulation

Pulse Code Modulation is the most commonly used technique in the PAM family and uses a sampling rate of 8000 samples per second.

Each sample is an 8 bit sample resulting in a digital rate of 64,000 bps (8 x 8000).

Sampling Theorem: If a signal is sampled at a rate higher than twice the highest signal frequency, then the samples contain all the information of the original signal.

E.g.: For voice capped at 4Khz, can sample at 8000 times per second to regenerate the original signal.

Bit rate and bandwidth requirements of PCM

- The bit rate of a PCM signal can be calculated form the number of bits per sample x the sampling rate

$$\text{Bit rate} = n_b \times f_s$$

- The bandwidth required to transmit this signal depends on the type of line encoding used. Generally, BW is BW=(bit rate)/2
- A digitized signal will always need more bandwidth than the original analog signal. Price we pay for robustness and other features of digital transmission.

Nonuniform Quantizer

Pulse code modulation (PCM) is a common method of digitizing or quantizing an analog waveform. For any analog-to-digital conversion process, the quantization step produces an estimate of the waveform sample using a digital codeword. This digital estimate inherently contains some level of error due to the finite number of bits available. In practical terms, there is always tradeoff **between the amount of error and the size of the digital data samples**.

The goal in any system design is **quantizing the data in smallest number of bits** that results in a tolerable level of error. In the case of speech coding, linear quantization with 13 bits sampled at 8 KHz is the minimum required to accurately produce a digital representation of the full range of speech signals. For many transmission systems, wired or wireless, 13 bits sampled at 8 KHz is an **expensive proposition as far as bandwidth is concerned**. To address this constraint, a companding system is often employed. Companding is simply a system in which information is first compressed, transmitted through a bandwidthlimited channel, and expanded at the receiving end. It is frequently used to reduce the bandwidth requirements for transmitting telephone quality speech, by reducing the 13-bit codewords to 8-bit codewords. Two international standards for encoding signal data to 8-bit codes are A-law and m-law. Alaw is the accepted European standard, while m-law is the accepted standard in the United States and Japan.

Non-uniform Quantizer

- For a fixed uniform quantizer, an input signal with an amplitude less than the full dynamic load of the quantizer will have lower SQNR than a signal whose amplitude occupies the full range of the equalizer

- For speech signal, low amplitudes are more probable than larger amplitude

- For speech signals, a type nonuniform quantization was developed called logarithmic companding

- Low amplitude given more levels than higher amplitude because it is more probability of occurrence

- There are more quantization regions at lower amplitudes and less quantization regions at higher amplitudes

Non uniform Quantization

The use of a nonuniform quantizer is equivalent to passing the message signal through a *compressor* and then applying the compressed signal to a uniform quantizer. A particular form of compression law that is used in practice is the so called μ lay defined by:

$$|v| = \frac{\log(1 + \mu|m|)}{\log(1 + \mu)}$$

where the logarithm is the natural logarithm; *m* and v are respectively the normalized input and output voltages

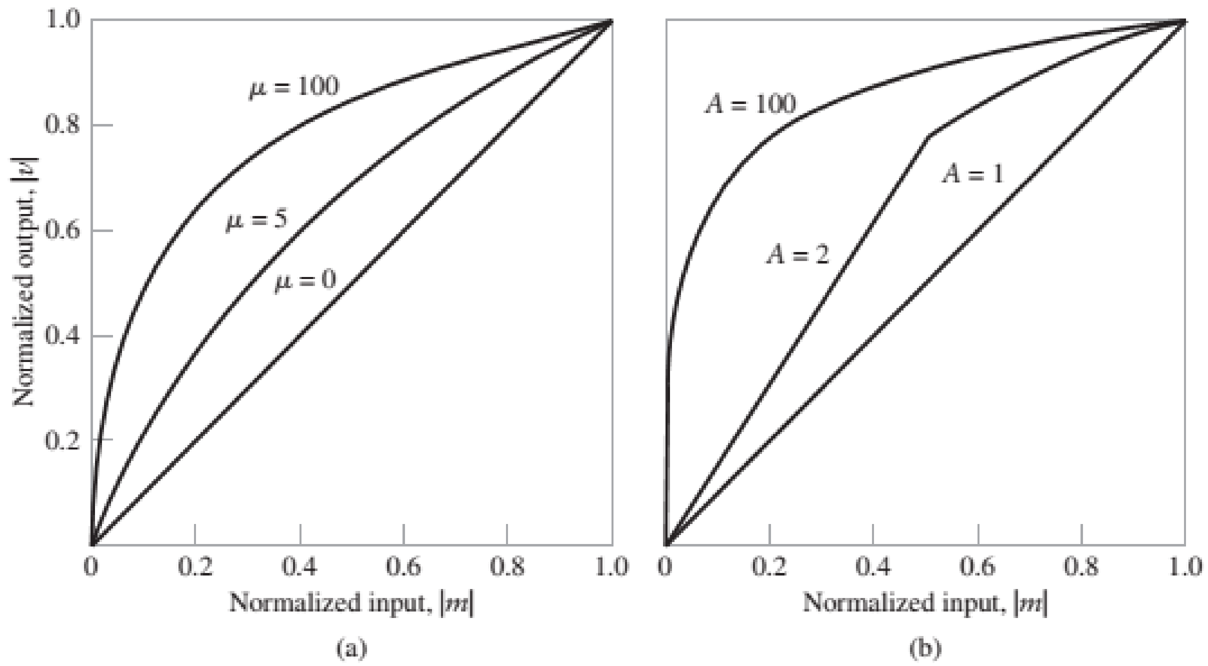
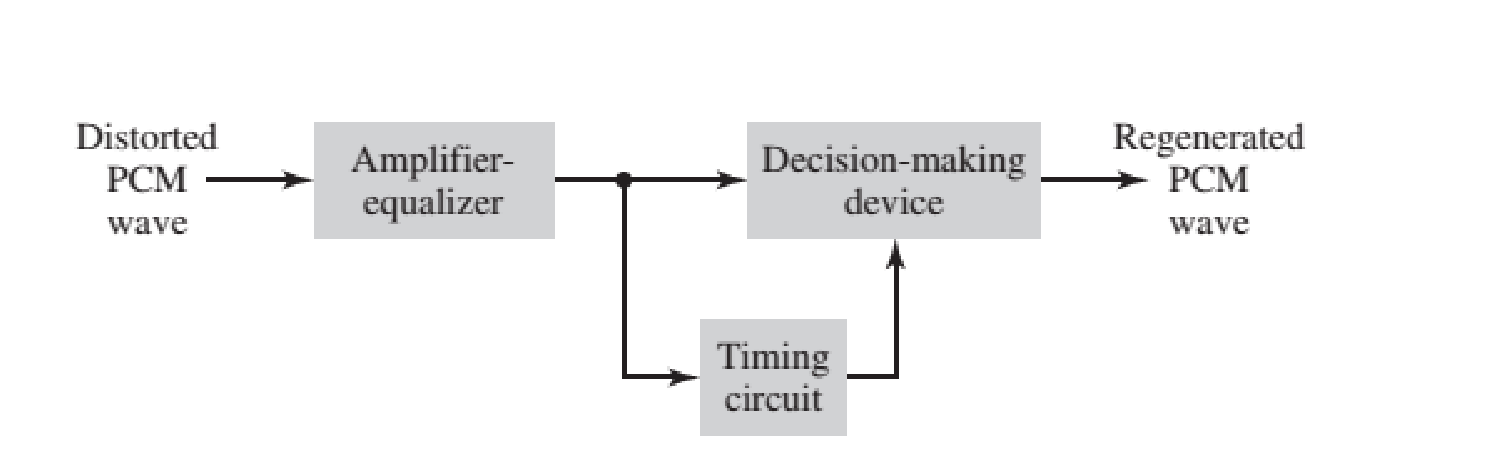


FIGURE 5.12 Compression laws. (*a*) μ-law. (*b*) *A*-law.

$$|v| = \begin{cases} \dfrac{A|m|}{1 + \log A}, & 0 \le |m| \le \dfrac{1}{A} \\ \dfrac{1 + \log(A|m|)}{1 + \log A}, & \dfrac{1}{A} \le |m| \le 1 \end{cases}$$

REGENERATION ALONG THE TRANSMISSION PATH

The most important feature of a PCM system lies in the ability to *control* the effects of distortion and noise produced by transmitting a PCM signal over a channel. This capability is accomplished by reconstructing the PCM signal by means of a chain of *regenerative repeaters* located at sufficiently close spacing along the transmission route.



equalization, timing, and *decision making*.

- The equalizer shapes the received pulses so as to compensate for the effects of amplitude and phase distortions produced by the transmission characteristics of the channel.

- The timing circuitry provides a periodic pulse train, derived from the received pulses; this is done for renewed sampling of the equalized pulses at the instants of time where the signal-to-noise ratio is a maximum.

- The sample so extracted is compared to a predetermined *threshold* in the decision-making device. In each bit interval, a decision is then made on whether the received symbol is a 1 or 0 on the basis of whether the threshold is exceeded or not

Operations in the Receiver

Decoding and Expanding

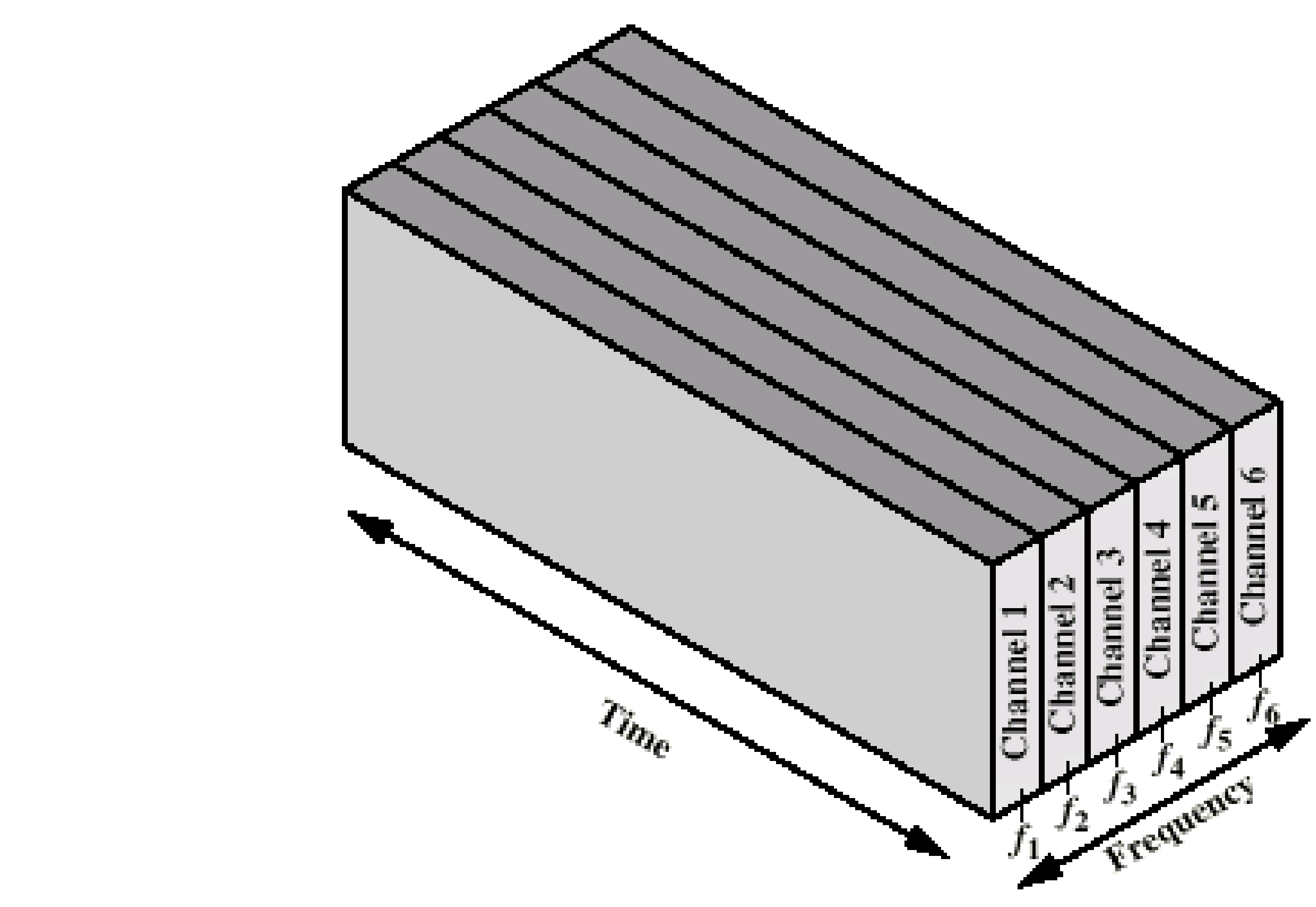
The first operation in the receiver is to *regenerate* (i.e., reshape and clean up) the received pulses one last time. These clean pulses are then regrouped into code words and decoded (i.e., mapped back) into a quantized PAM signal. The *decoding* process involves generating a pulse whose amplitude is the linear sum of all the pulses in the code word; each pulse is weighted by its place value in the code, where R is the number of bits per sample.

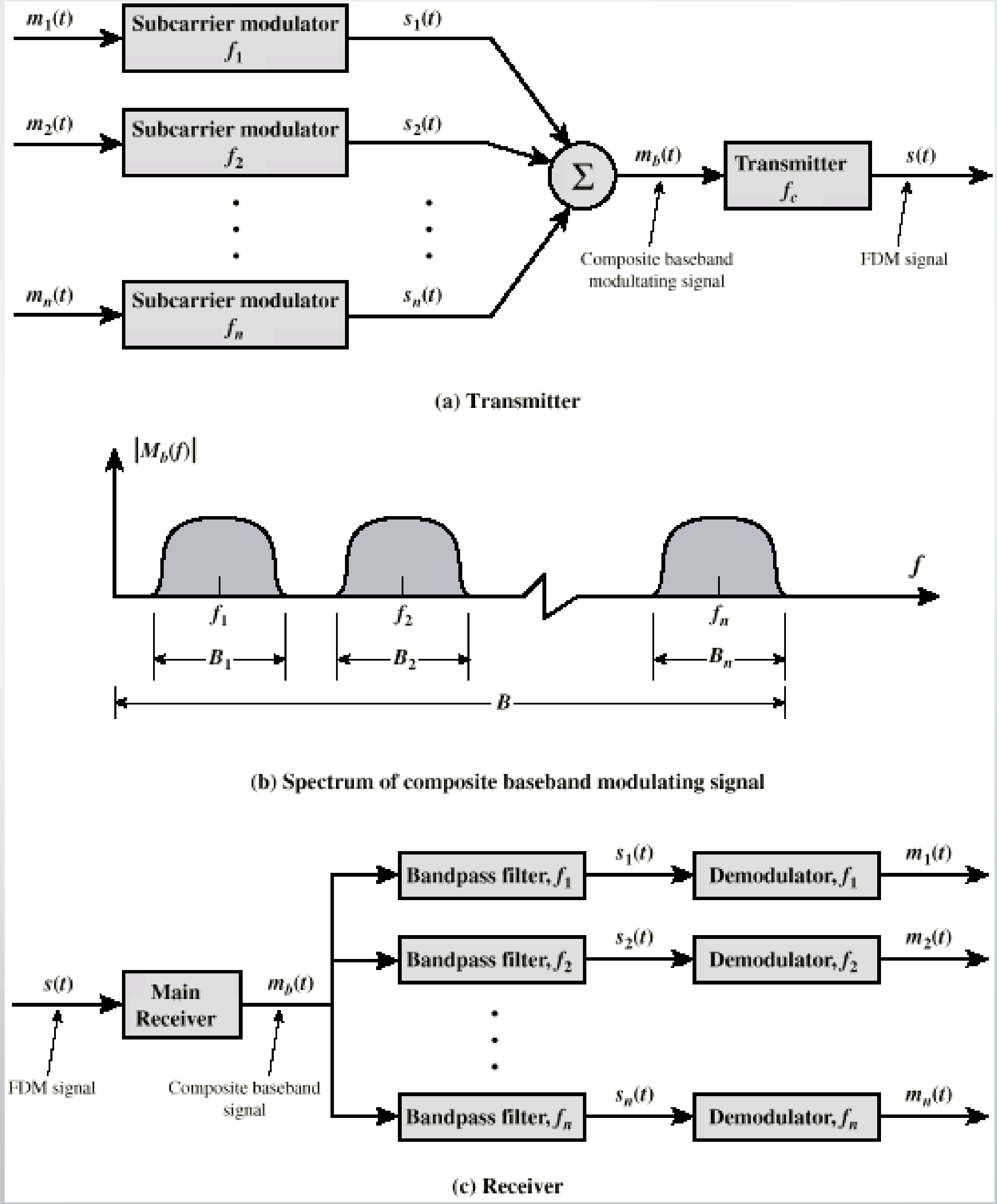
The sequence of decoded samples represents an *estimate* of the sequence of compressed samples produced by the quantizer in the transmitter. We use the term "estimate" here to emphasize the fact that there is no way for the receiver to compensate for the approximation introduced into the transmitted signal by the quantizer.

(ii) *Reconstruction*

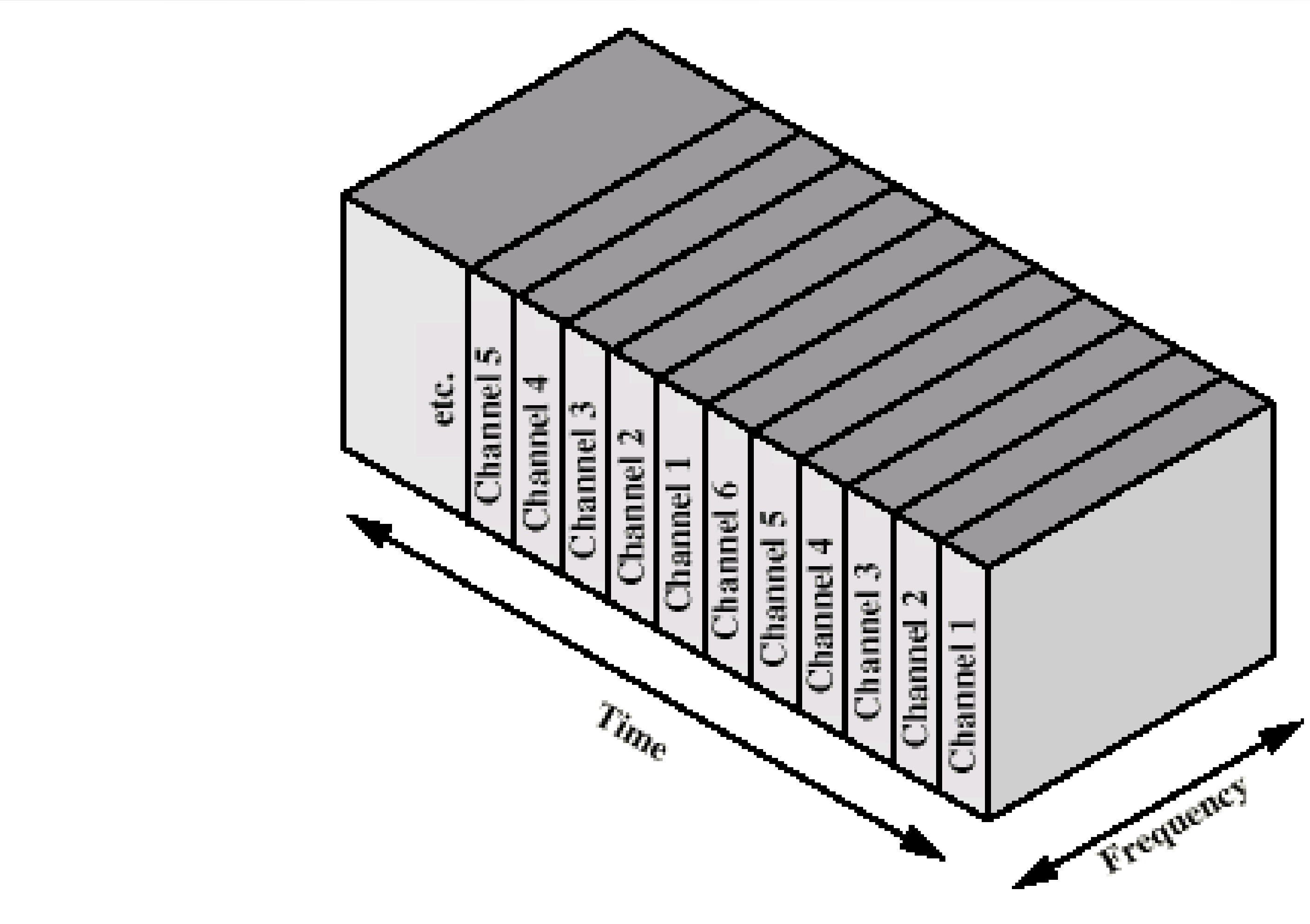
The final operation in the receiver is to recover the message signal. This operation is achieved by passing the expander output through a *low-pass reconstruction filter* whose cutoff frequency is equal to the message bandwidth. Recovery of the message signal is intended to signify estimation rather than exact reconstruction.

Frequency Division Multiplexing

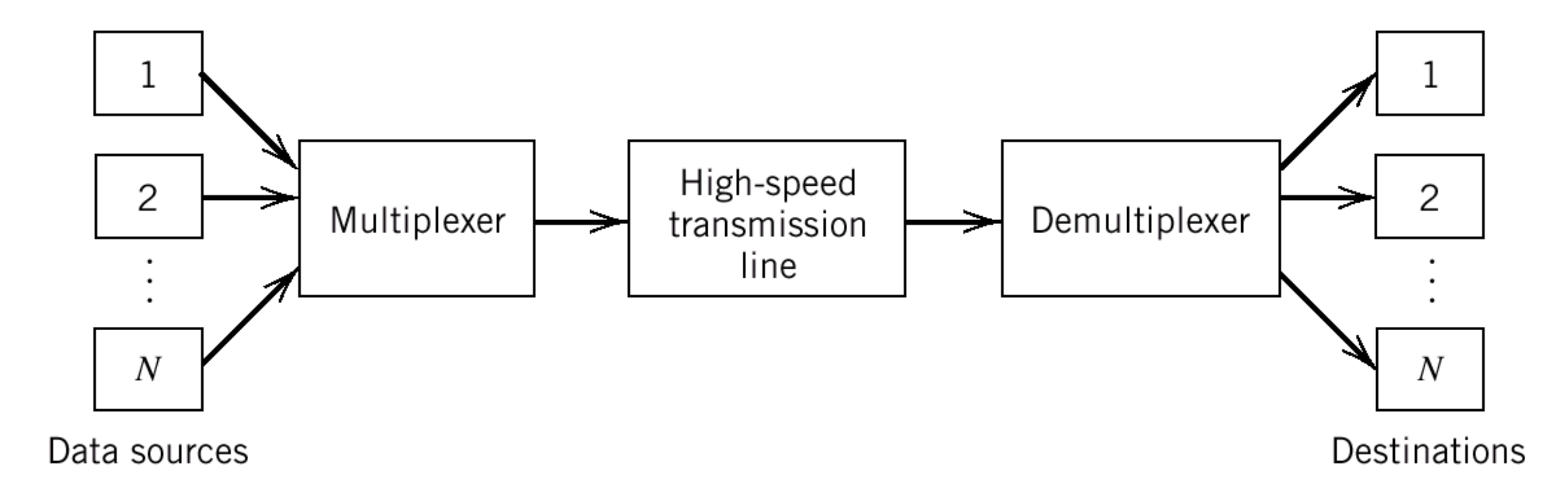



m1(t)
Subcarrier modulator f1
s1(t)
m2(t)
Subcarrier modulator f2
s2(t)
mn(t)
Subcarrier modulator fn
sn(t)
Σ
mb(t)
Transmitter fc
s(t)
Composite baseband modultating signal
FDM signal
(a) Transmitter
|Mb(f)|
f
f1
f2
fn
B1
B2
Bn
B
(b) Spectrum of composite baseband modulating signal
s(t)
Main Receiver
mb(t)
FDM signal
Composite baseband signal
Bandpass filter, f1
Bandpass filter, f2
Bandpass filter, fn
s1(t)
s2(t)
sn(t)
Demodulator, f1
Demodulator, f2
Demodulator, fn
m1(t)
m2(t)
mn(t)
(c) Receiver

Time Division Multiplexing



Time-Division Multiplexing



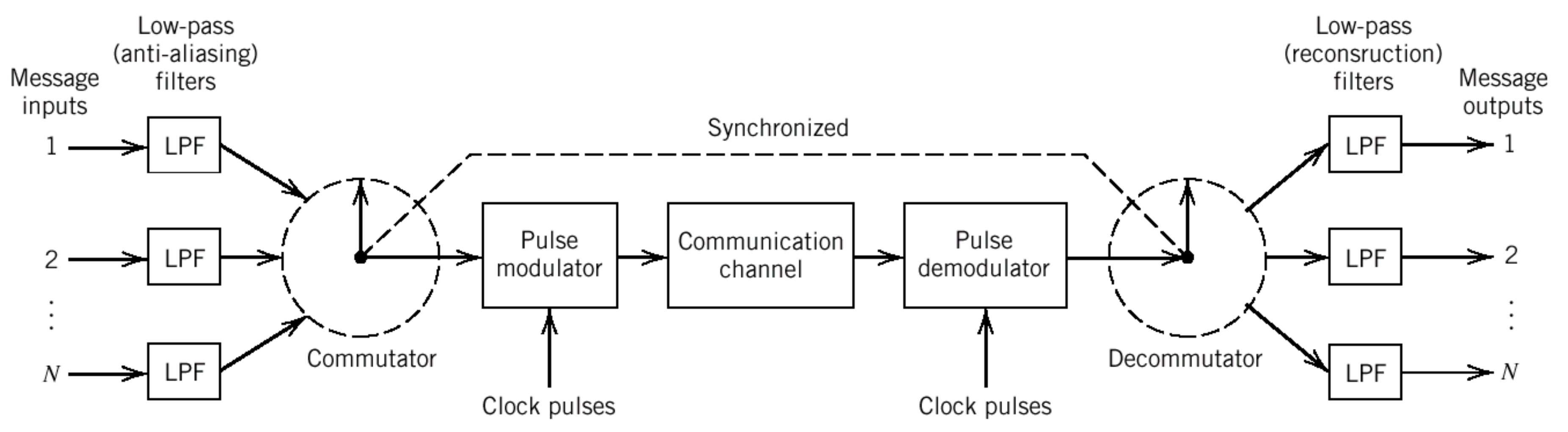
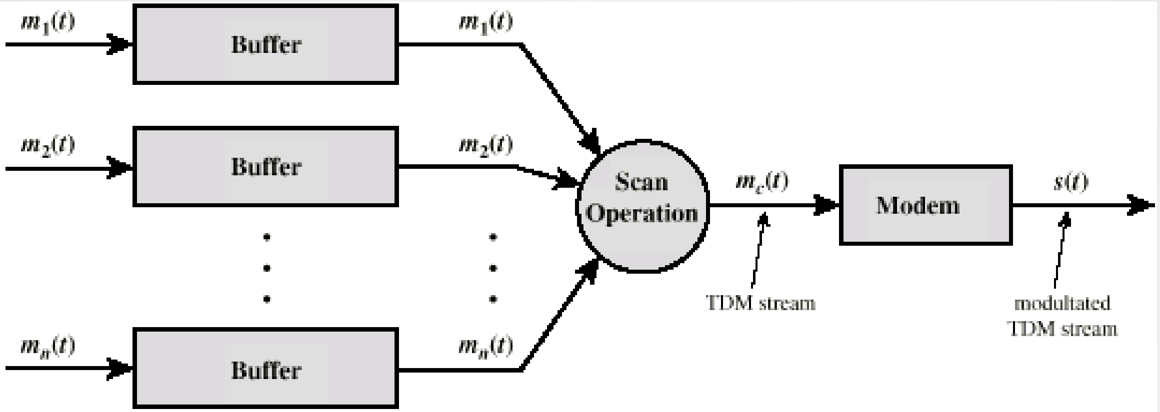
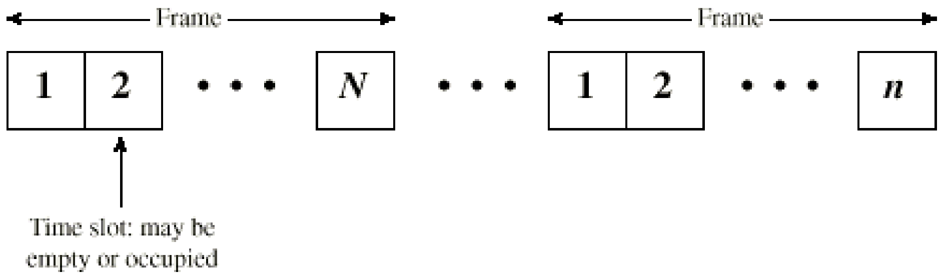


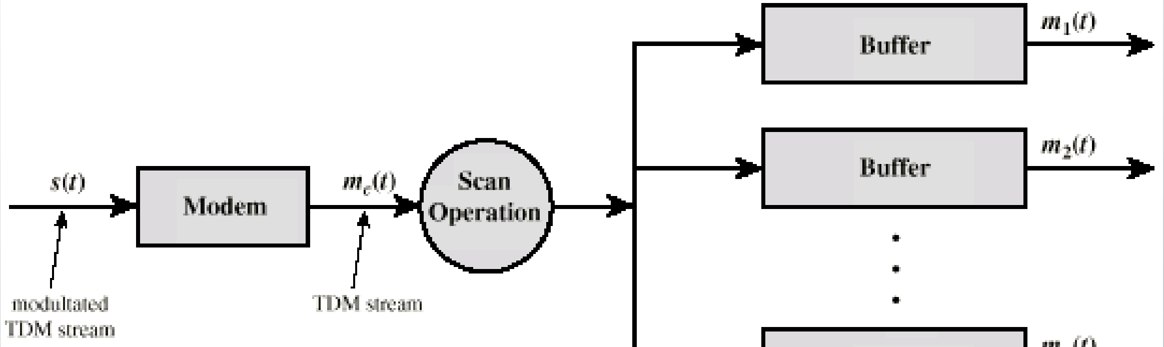
Figure Block diagram of TDM system.



(a) Transmitter



(b) TDM Frames



(c) Receiver

Digital Carrier Systems

- Hierarchy of TDM
- USA/Canada/Japan use one system
- ITU-T use a similar (but different) system
- US system based on DS-1 format
- Multiplexes 24 channels
- Each frame has 8 bits per channel plus one framing bit
- 193 bits per frame

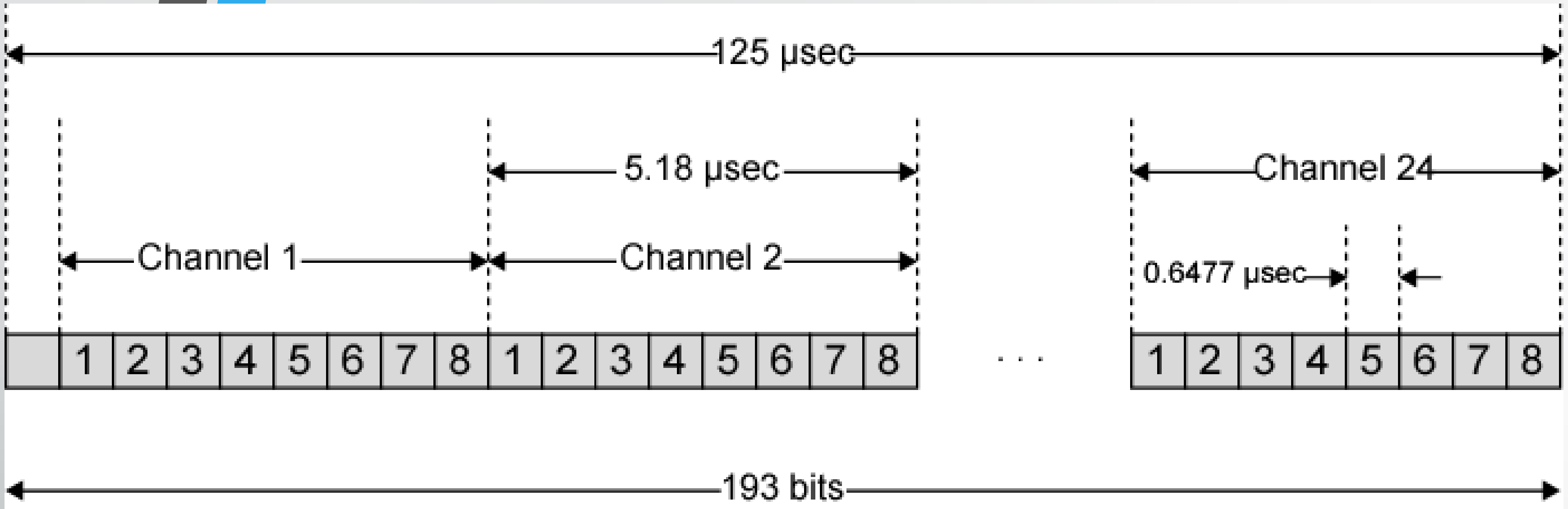
Digital Carrier Systems (2)

- For voice each channel contains one word of digitized data (PCM, 8000 samples per sec)
  - Data rate 8000x193 = 1.544Mbps
  - Five out of six frames have 8 bit PCM samples
  - Sixth frame is 7 bit PCM word plus signaling bit
  - Signaling bits form stream for each channel containing control and routing info
- Same format for digital data
  - 23 channels of data
    - 7 bits per frame plus indicator bit for data or systems control
  - 24th channel is sync

DS1/T1/E1

- **Digital signal 1** (**DS1**, also known as **T1**) is a T-carrier signaling scheme devised by Bell Labs. DS1 is a widely used standard in telecommunications in North America and Japan to transmit voice and data between devices. **E1** is used in place of **T1** outside of North America and Japan. Technically, DS1 is the transmission protocol used over a physical T1 line; however, the terms "DS1" and "T1" are often used interchangeably.
- A DS1 circuit is made up of twenty-four DS0
- DS1: (8 bits/channel * 24 channels/frame + 1 framing bit) * 8000 frames/s = 1.544 Mbit/s
- A E1 is made up of 32 DS0
- The line data rate is 2.048 Mbit/s which is split into 32 time slots, each being allocated 8 bits in turn. Thus each time slot sends and receives an 8-bit sample 8000 times per second (8 x 8000 x 32 = 2,048,000). 2.048Mbit/s

DS-1 Transmission Format



Notes:

1. The first bit is a framing bit, used for synchronization.
2. Voice channels:
   - 8-bit PCM used on five of six frames.
   - 7-bit PCM used on every sixth frame; bit 8 of each channel is a signaling bit.
3. Data channels:
   - Channel 24 is used for signaling only in some schemes.
   - Bits 1-7 used for 56 kbps service
   - Bits 2-7 used for 9.6, 4.8, and 2.4 kbps service.

Line Codes

1.On–off signaling, in which symbol 1 is represented by transmitting a pulse of constant amplitude for the duration of the symbol, and symbol 0 is represented by switching off the pulse, as in Fig. 5.20(*a*).

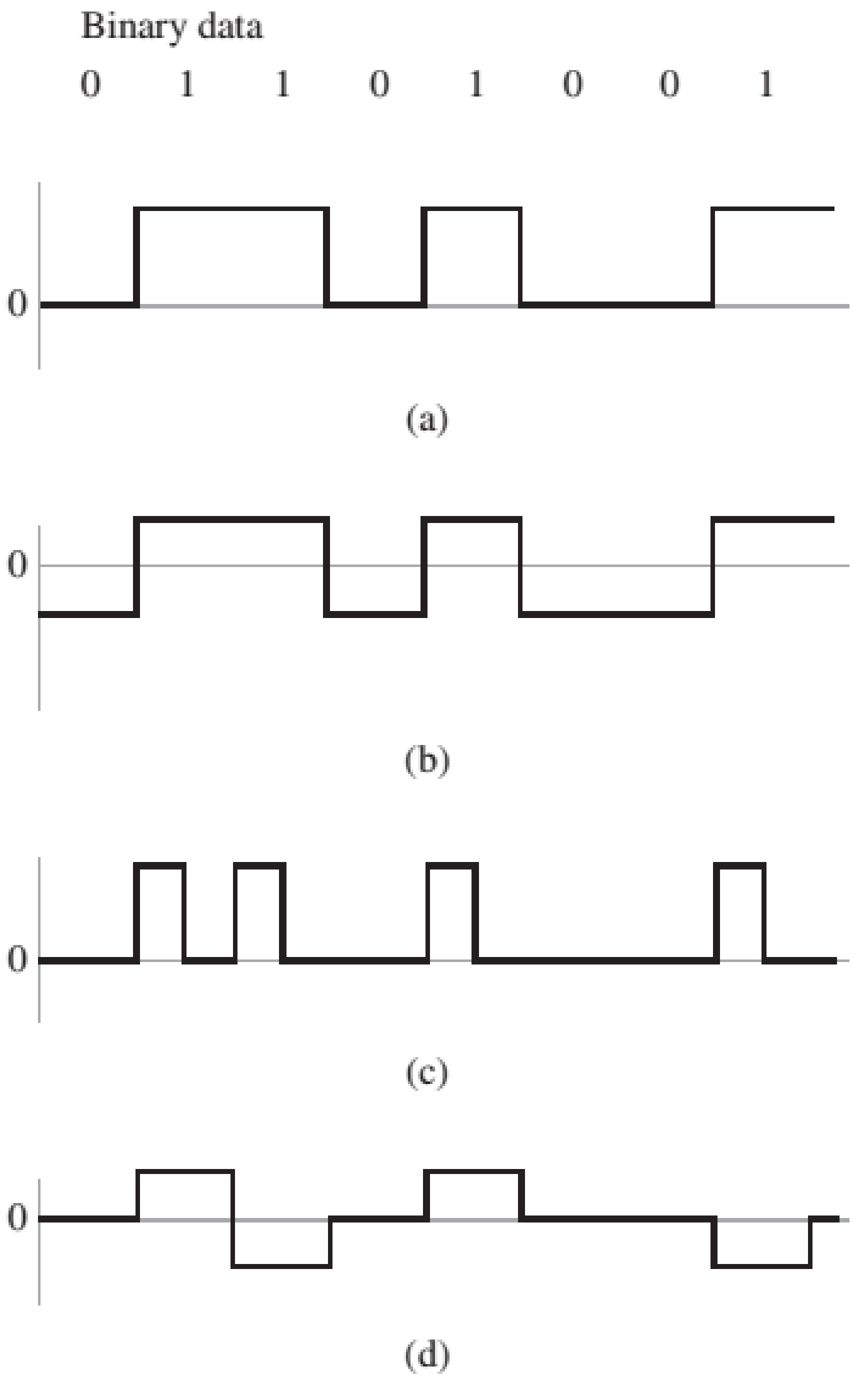
2. *Nonreturn-to-zero (NRZ) signaling*, in which symbols 1 and 0 are represented by pulses of equal positive and negative amplitudes, as illustrated in Fig. 5.20(*b*).

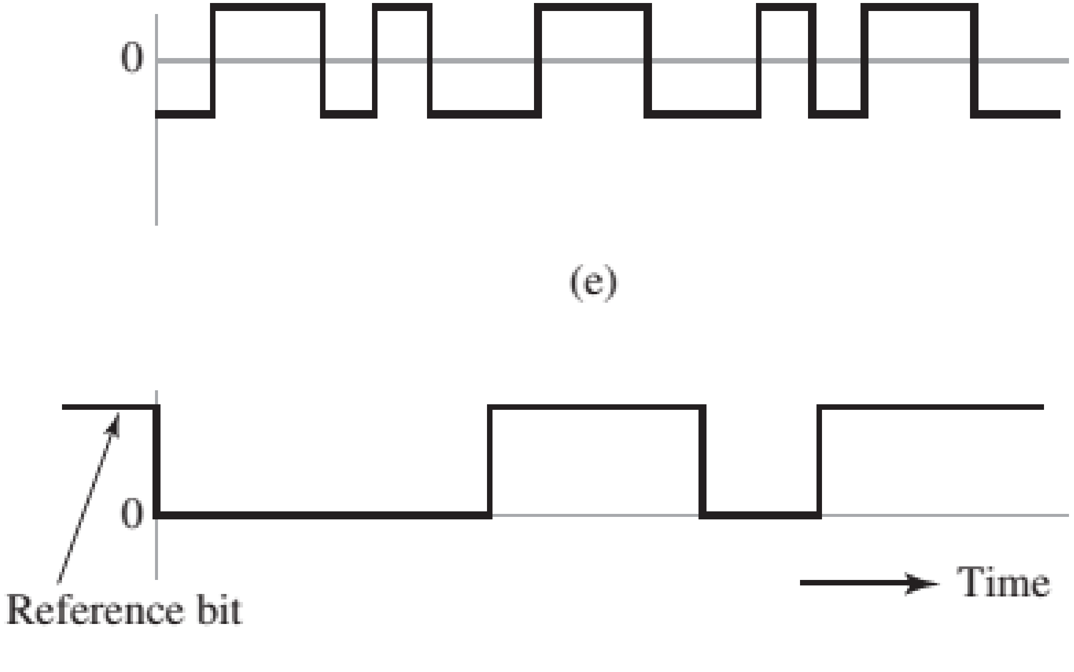
3. *Return-to-zero (RZ) signaling*, in which symbol 1 is represented by a positive rectangular pulse of half-symbol width, and symbol 0 is represented by transmitting *no* pulse, as illustrated in Fig. 5.20(*c*).

4. *Bipolar return-to-zero (BRZ) signaling*, which uses three amplitude levels as indicated in Fig. 5.20(*d*). Specifically, positive and negative pulses of equal amplitude are used alternately for symbol 1, and no pulse is always used for symbol 0. A useful property of BRZ signaling is that the power spectrum of the transmitted signal has no dc component and relatively insignificant low-frequency components for the case when symbols 1 and 0 occur with equal probability.

5. Split-phase (Manchester code), which is illustrated in Fig. 5.20(*e*). In this method of signaling, symbol 1 is represented by a positive pulse followed by a negative pulse, with both pulses being of equal amplitude and half-symbol width. For symbol 0, the polarities of these two pulses are reversed. The Manchester code suppresses the dc component and has relatively insignificant low-frequency components, regardless of the signal statistics.

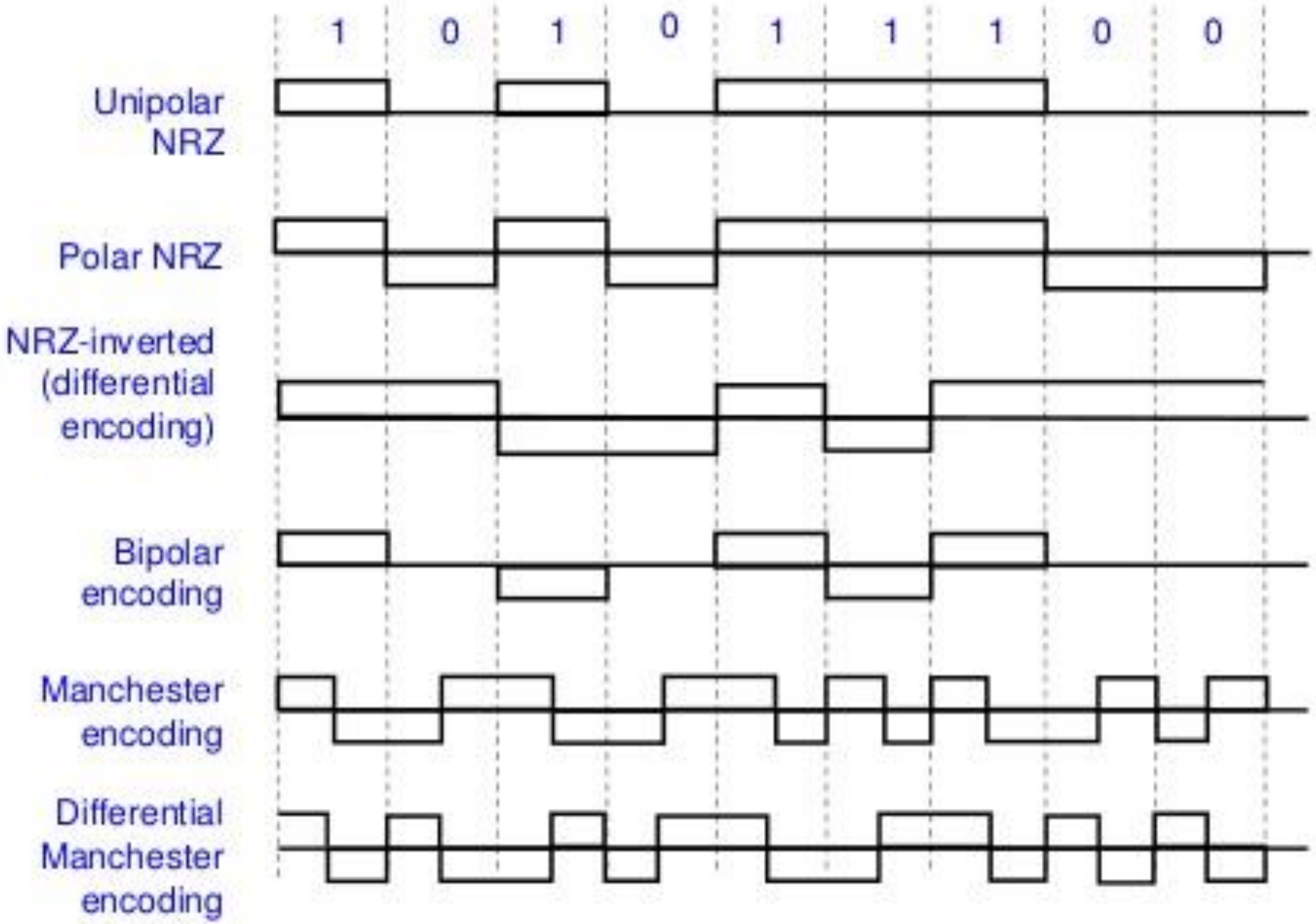
6. *Differential encoding*, in which the information is encoded in terms of signal transitions, as illustrated in Fig. 5.20(*f*). In the example of the binary PCM signal shown in the figure, a transition is used to designate symbol 0, whereas no transition is used to designate symbol 1. It is apparent that a differentially encoded signal may be inverted without affecting its interpretation. The original binary information is recovered by comparing the polarity of adjacent symbols to establish whether or not a transition has occurred. Note that differential encoding requires the use of a *reference bit*, as indicated in Fig. 5.20 (*f*).





Line codes. (*a*) On–off signaling. (*b*) Nonreturn-to-zero signaling. (*c*) Return-to-zero signaling. (*d*) Bipolar return-to-zero signaling. (*e*) Split-phase or Manchester encoding. (*f*) Differential encoding.

Line coding examples



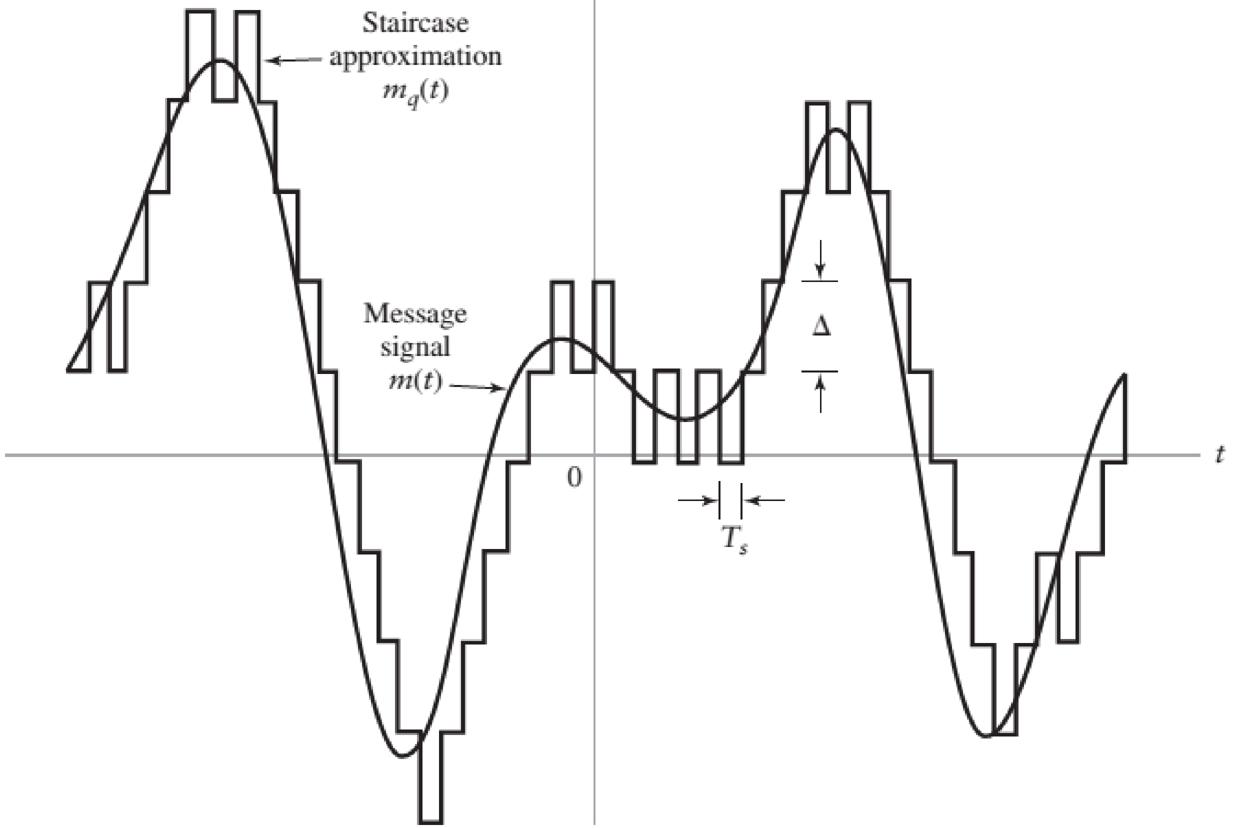
Delta Modulation

- In *delta modulation* (DM), an incoming message signal is oversampled (i.e., at a rate much higher than the Nyquist rate) to purposely increase the *correlation* between adjacent samples of the signal.

- The increased correlation is done so as to permit the use of a simple quantizing strategy for constructing the encoded signal

- DM provides a *staircase approximation* to the oversampled version of the message signal.

- If the approximation falls below the input signal at any sampling epoch, it is increased by Δ If, on the other hand, the approximation lies above the signal, it is diminished Δ .



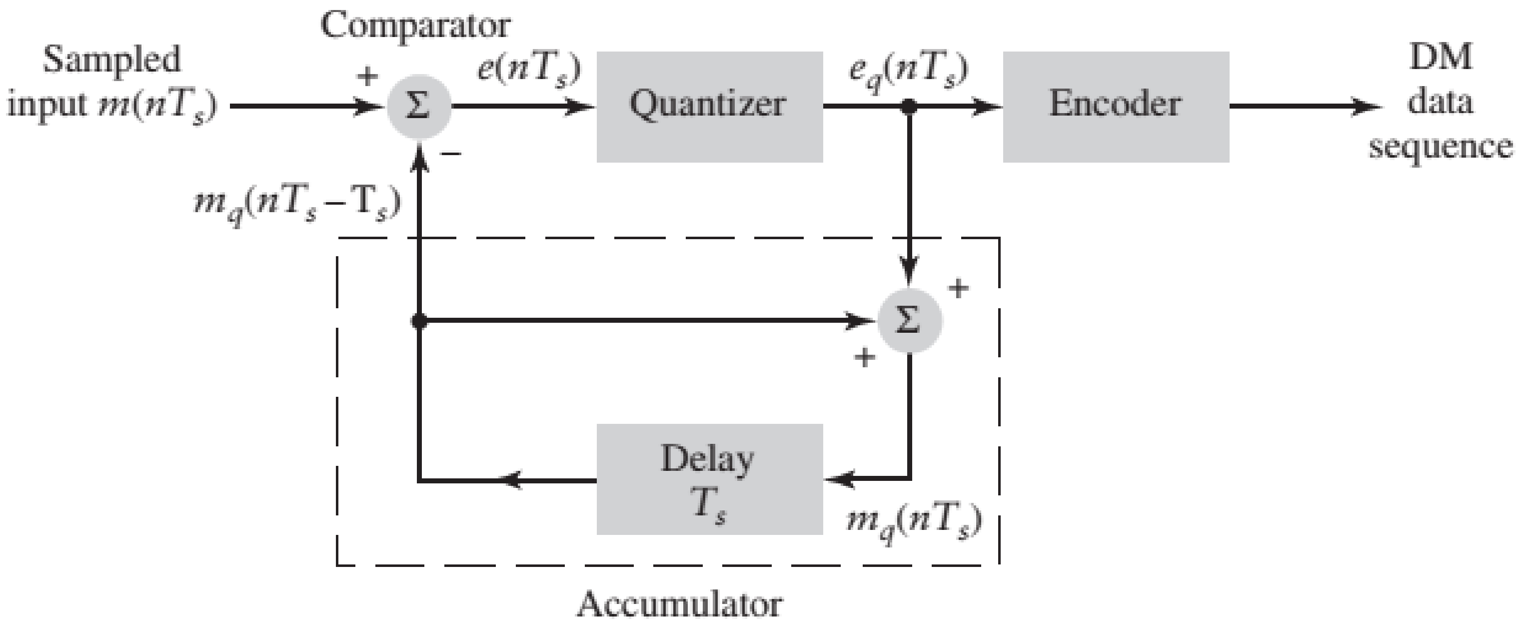
(a)

Binary
sequence
at modulator
output 1011110100000000011111101001010111101000000110111

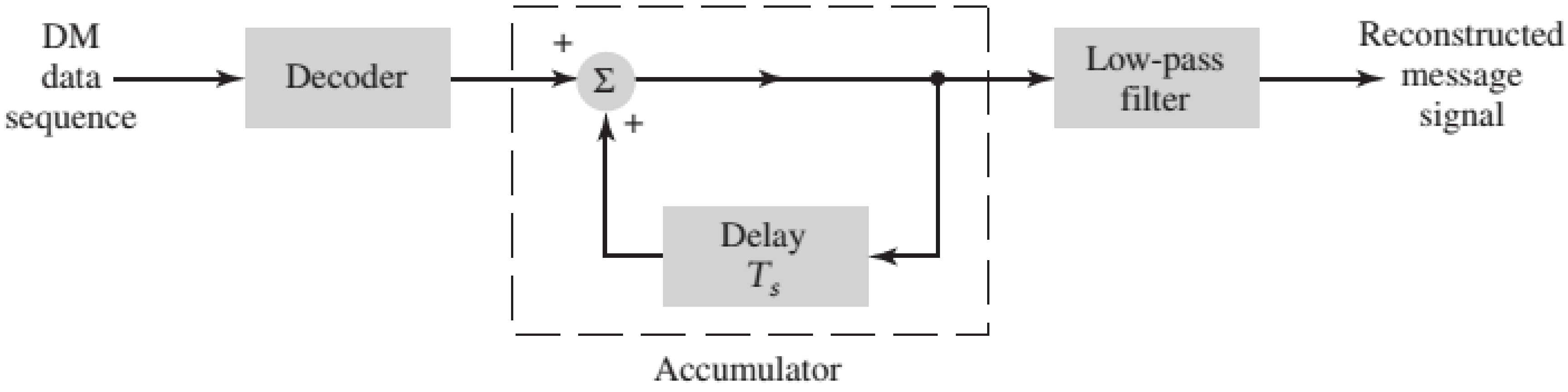
$$e(nT_s) = m(nT_s) - m_q(nT_s - T_s)$$
$$e_q(nT_s) = \Delta \operatorname{sgn}[e(nT_s)]$$
$$m_q(nT_s) = m_q(nT_s - T_s) + e_q(nT_s)$$

$$\begin{aligned} m_q(nT_s) &= m_q(nT_s - T_s) + e_q(nT_s) \\ &= m_q(nT_s - 2T_s) + e_q(nT_s - T_s) + e_q(nT_s) \\ &\;\;\vdots \\ &= \sum_{i=1}^{n} e_q(iT_s) \end{aligned}$$

System Details



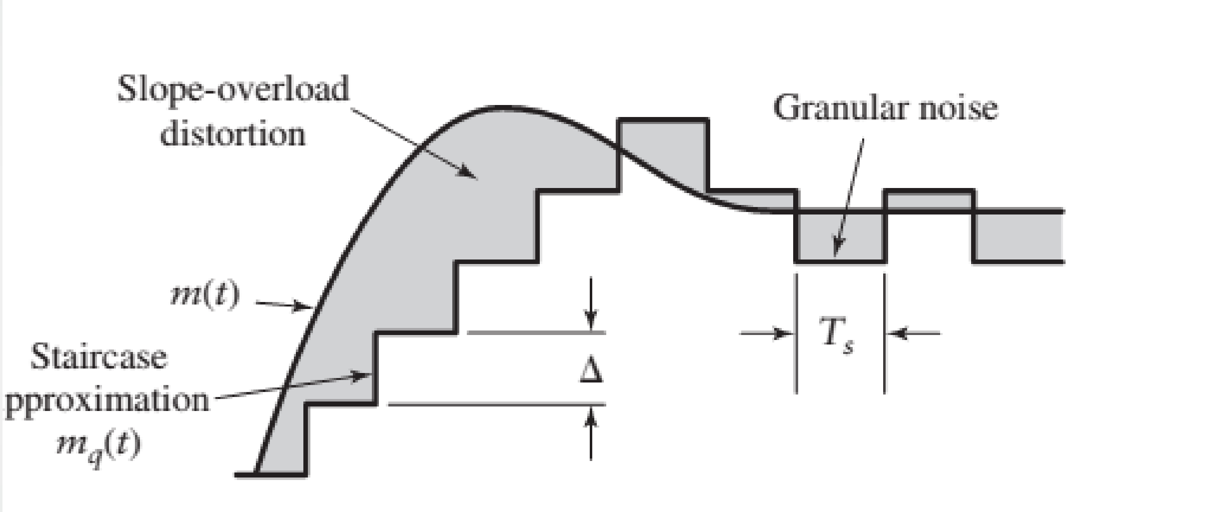
(a)



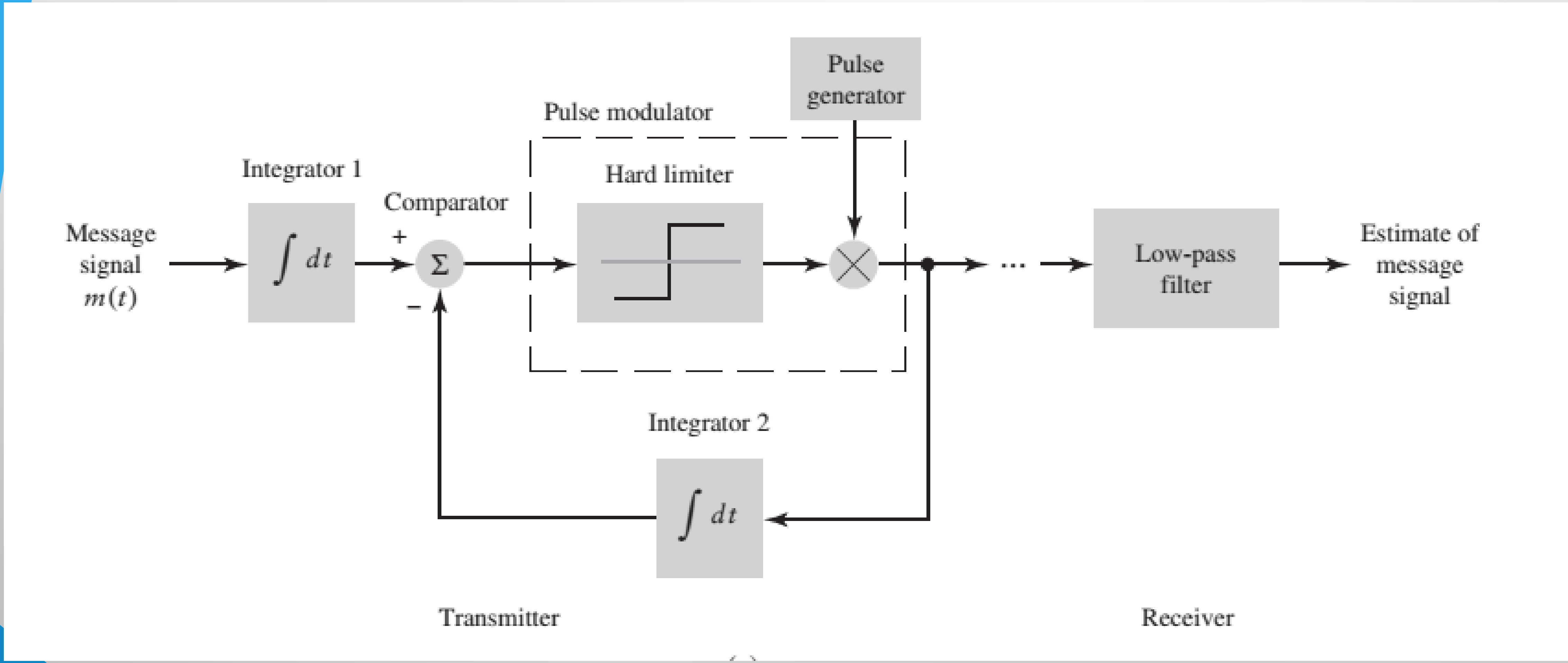
(b)

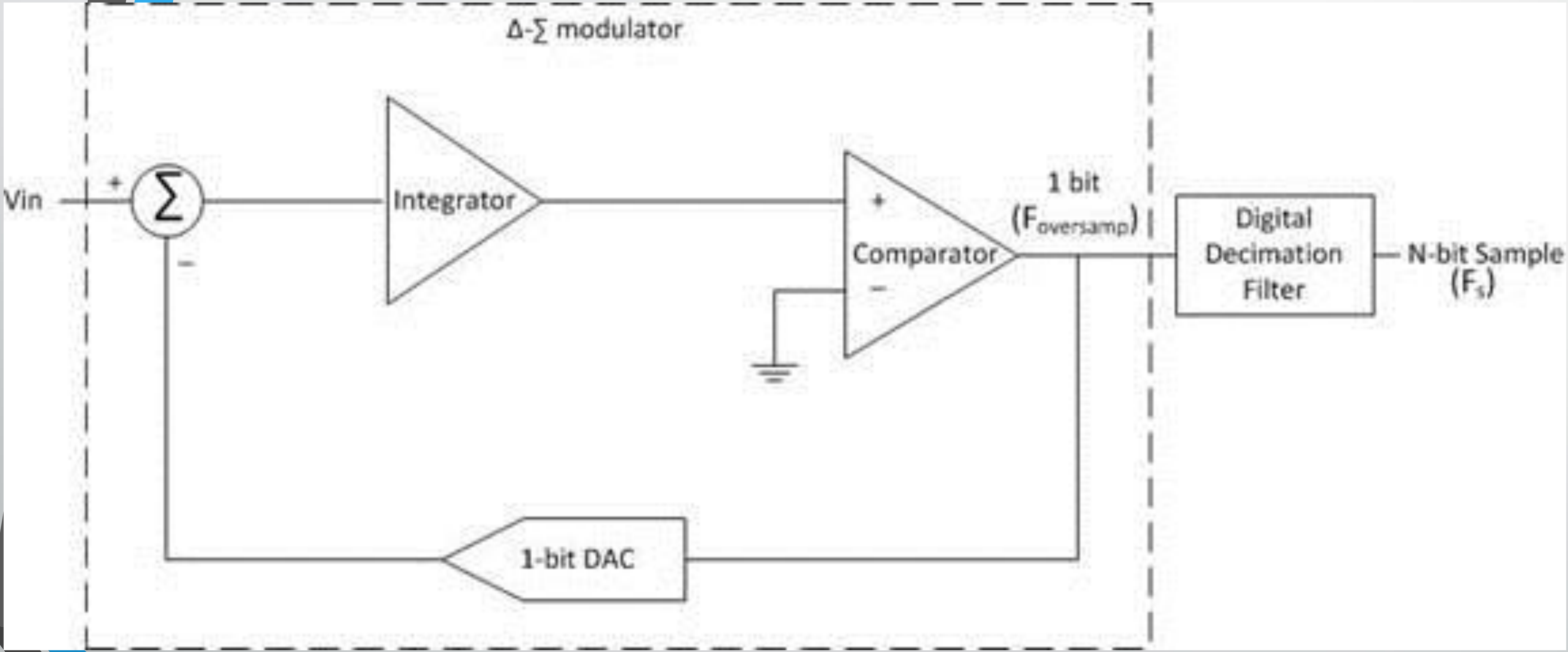
Quantization Error

$$\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$



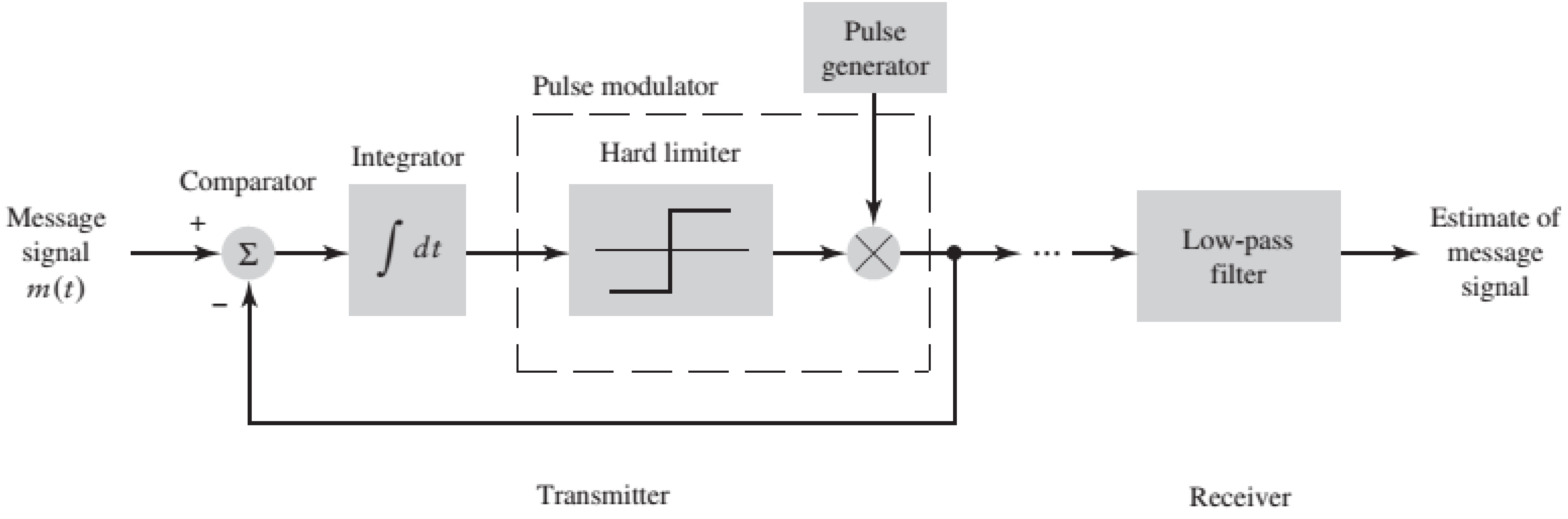
Delta-Sigma Modulation





Δ-∑ modulator
Vin
+
∑
−
Integrator
+
Comparator
−
1 bit
(F_oversamp)
Digital
Decimation
Filter
N-bit Sample
(F_s)
1-bit DAC

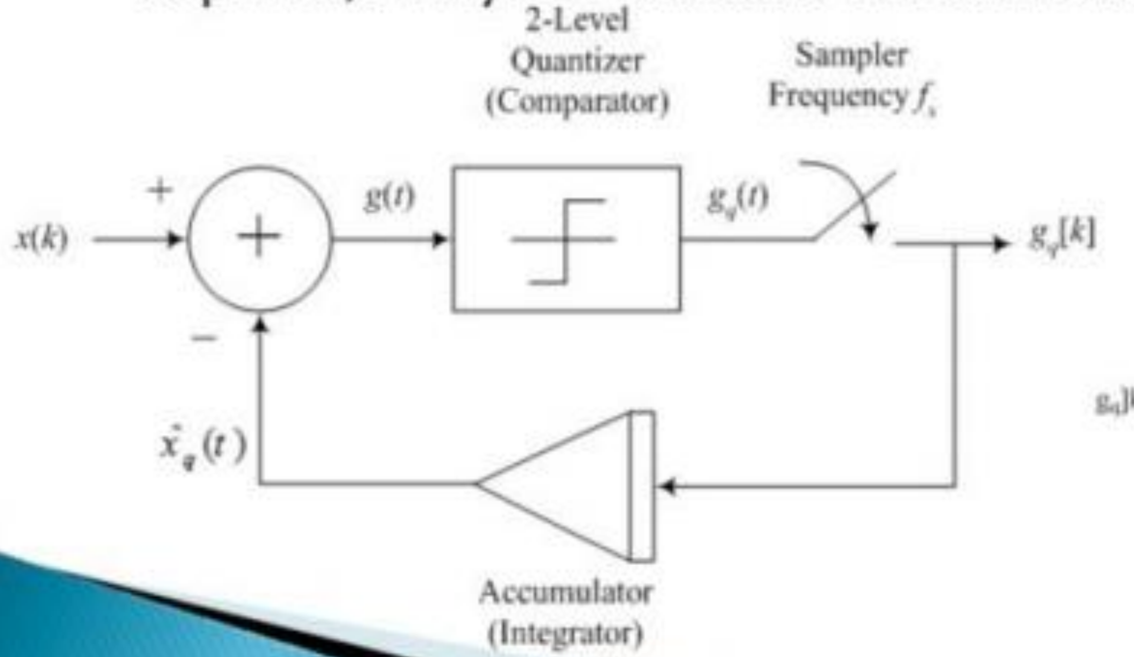
Delta-Sigma Modulation



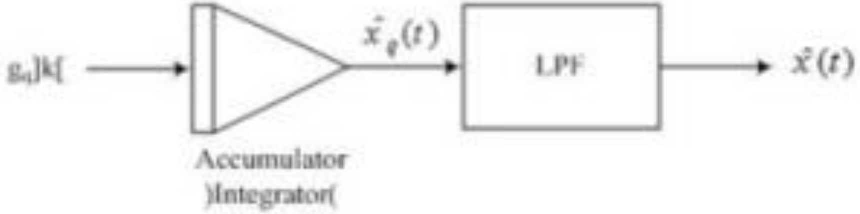
(b)

Delta Modulation

- A loss of **frame synchronization** in PCM destroys everything!
- Delta Modulator (DM) is special case of the DPCM which uses a 2-level quantizer. $d[k] = m[k] - m[k-1]$
- A lowpass filter after the accumulator will smooth the reconstructed signal.
- The sampling in the delta modulator must be performed at a very high rate (in many cases sampling at 30 to 40 times the Nyquist rate is required). Why? To increase the correlation.

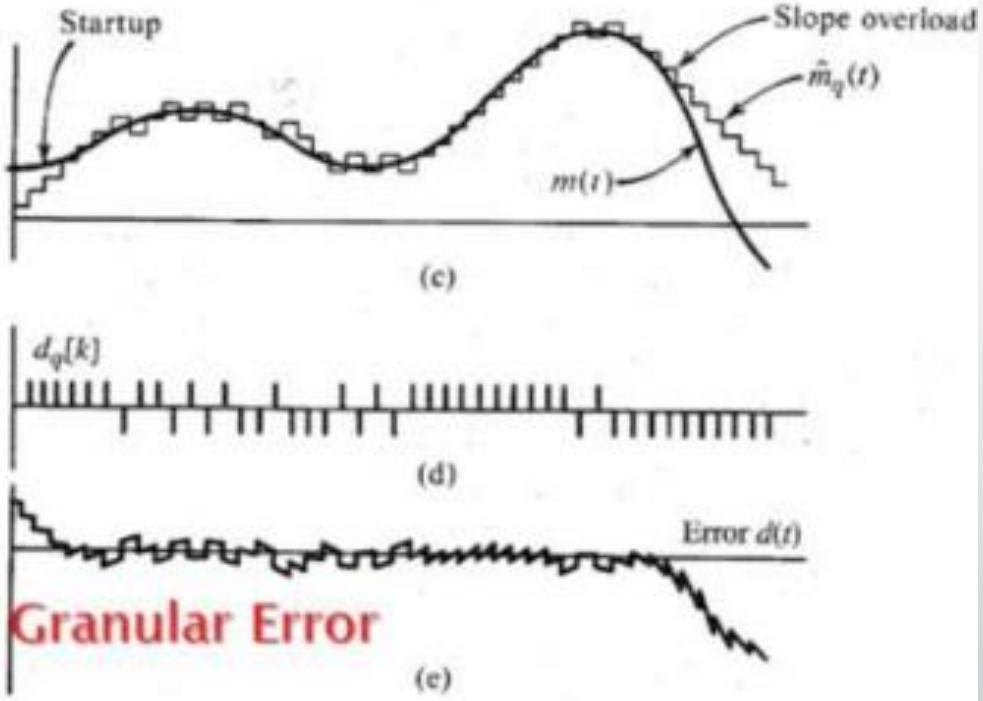


$$\hat{m}_q[k] = \sum_{m=0}^{k} d_q[k]$$



Slope overloading and Granular Errors

- $\hat{m}_q(t)$ is a stair case approximation of $m(t)$.
- + pulse $m(t) > \hat{m}_q(t)$.
- – pulse $m(t) < \hat{m}_q(t)$.
- Threshold of coding and overloading.
- Variation in $m(t)$ smaller than the step value (threshold of coding) are lost is DM (Granular Error)
- If $\dot{m}(t)$ is too high, $\hat{m}_q(t)$ cannot follow $m(t)$. (Slope overload)
- Adaptive Step , Adaptive DM (ADM)
- Detect the pattern and decide how to adapt accordingly
  - (++++, or ----) increase step
  - +-+-+-+ decrease step



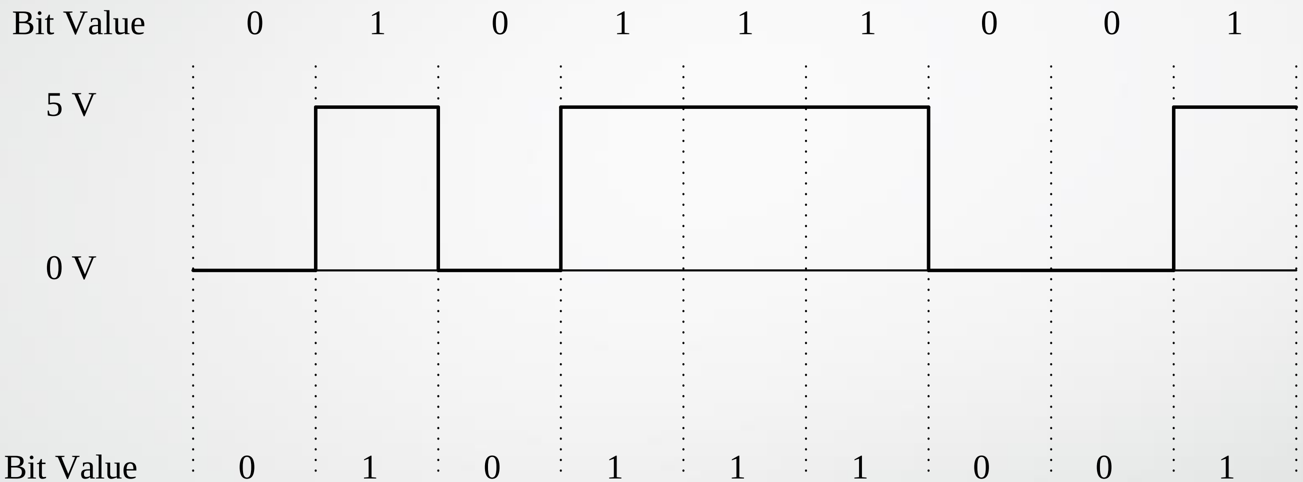
Compare DM with PCM

- In DM each digit has **equal importance.**
- In DM multiplexing each channel requires its own decoder while for PCM one decoder for all (On the other hand, permit flexibility and avoid cross talk (no need for stringent multiplexing design)
- DM outperforms at low SNR while PCM outperforms at high SNR.
- DM is very simple, inexpensive, and no need for framing bits (less overhead)
- DM is suitable for voice & TV signals.

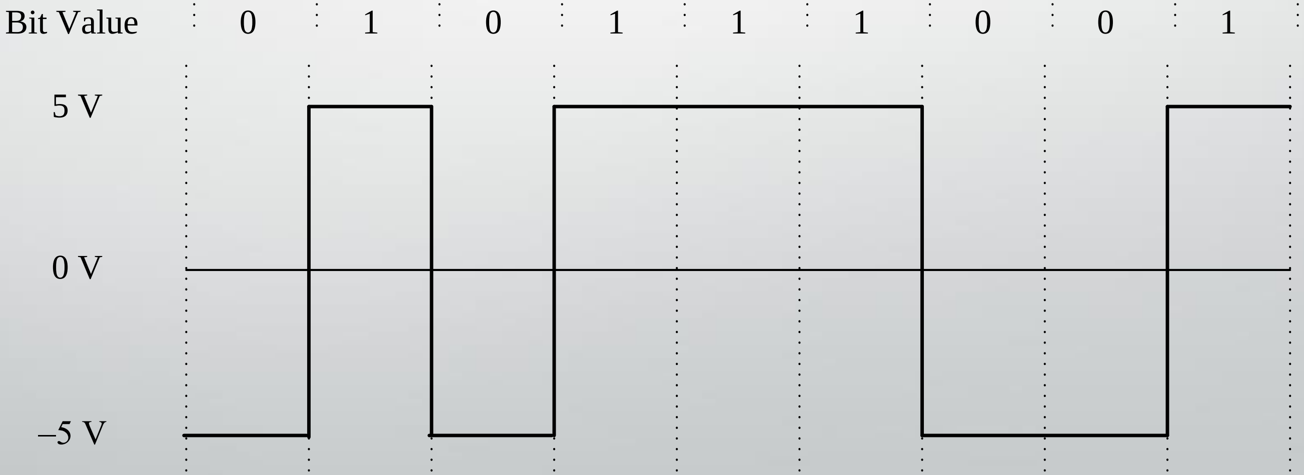
Fundamentals of Digital Transmission

Baseband Transmission (Line codes)

ON-OFF
or
Unipolar
(NRZ)
Non-Return-to-Zero



Polar
(NRZ)



Performance Criteria of Line Codes

- Zero DC value
- Inherent Bit-Synchronization
  - Rich in transitions
- Average Transmitted Power
  - For a given Bit Error Rate (BER)
- Spectral Efficiency (Bandwidth)
  - Inversely proportional to pulse width.

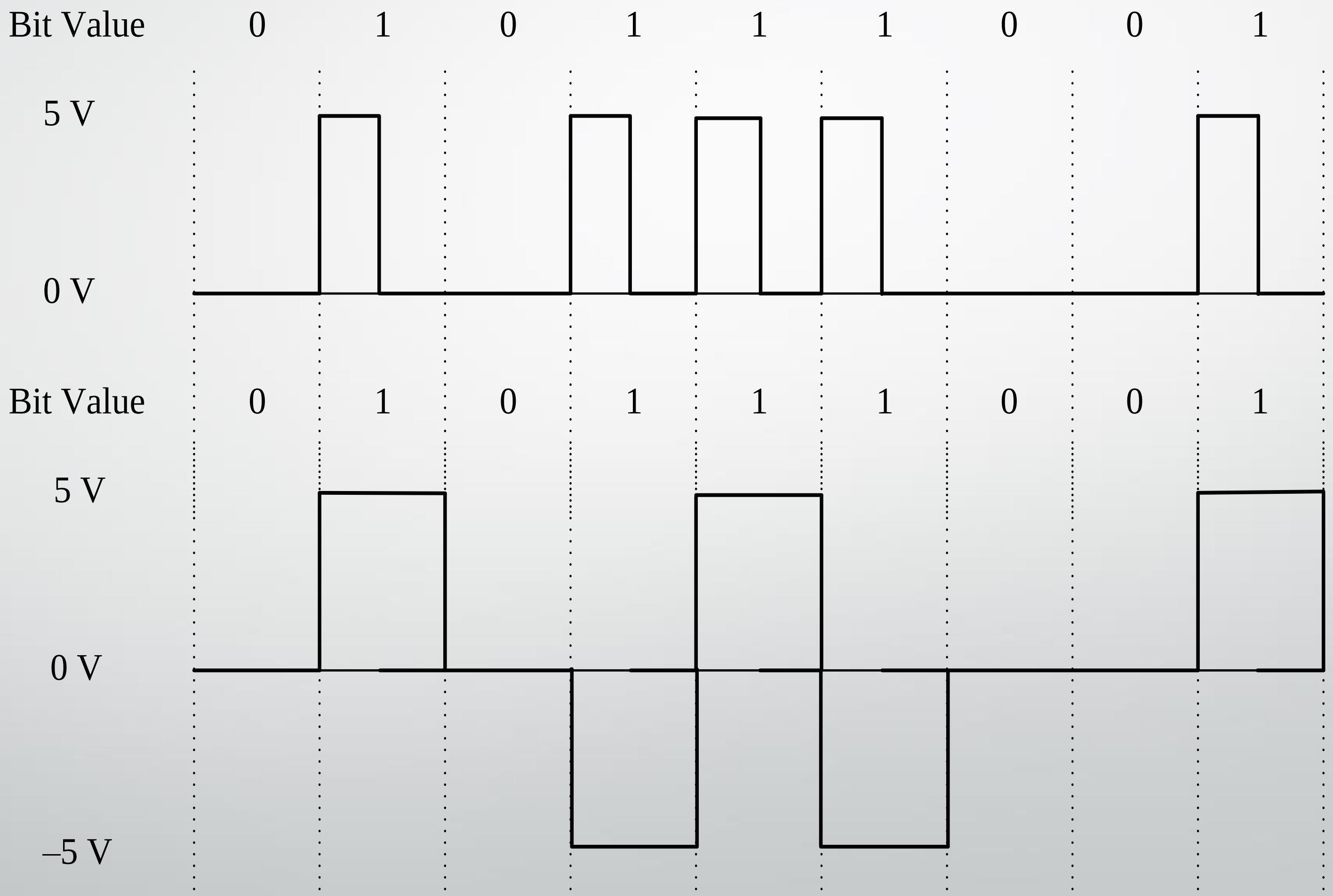
Comparison Between On-Off and Polar

- Zero DC value:
  - Polar is better.
- Bandwidth:
  - Comparable
- Power:
  - BER is proportional to the difference between the two levels
  - For the same difference between the two levels, Polar consumes half the power of on-off scheme.
- Bit Synchronization:
  - Both are poor (think of long sequence of same bit)

More Line Codes

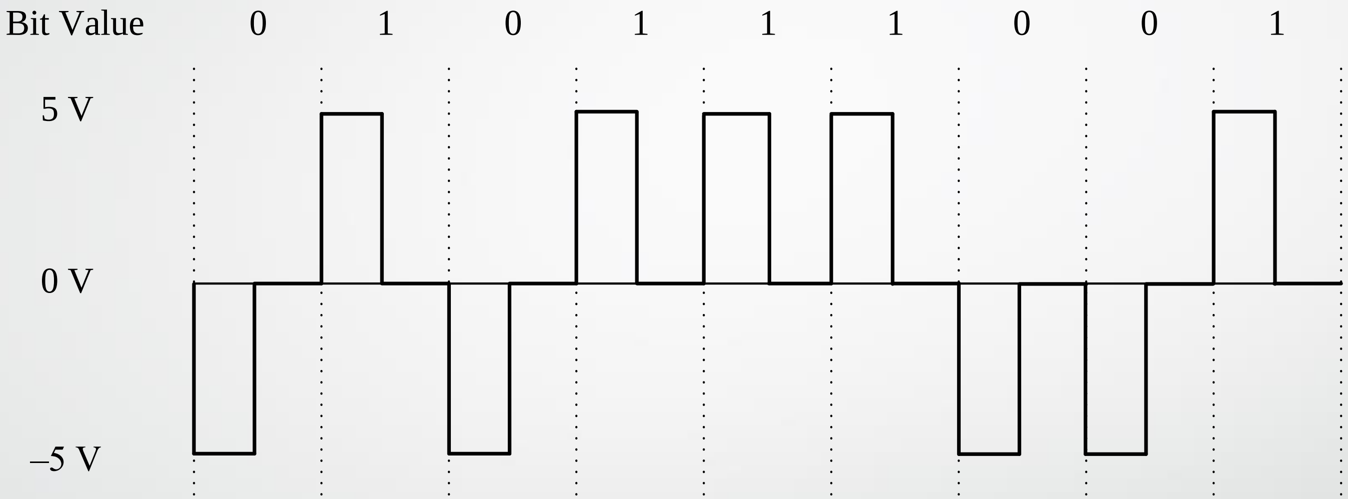
On-Off RZ
Better synch., at extra bandwidth

Bi-Polar
Better synch., at same bandwidth

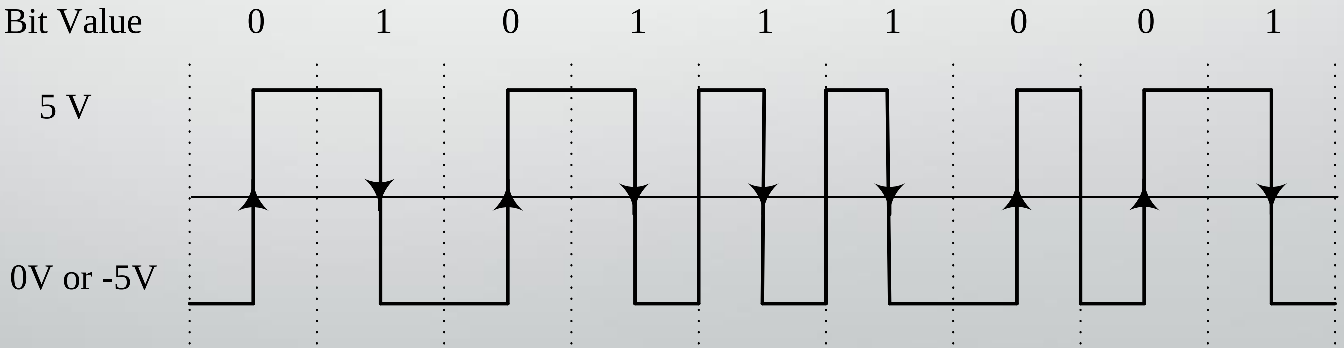


More Line Codes

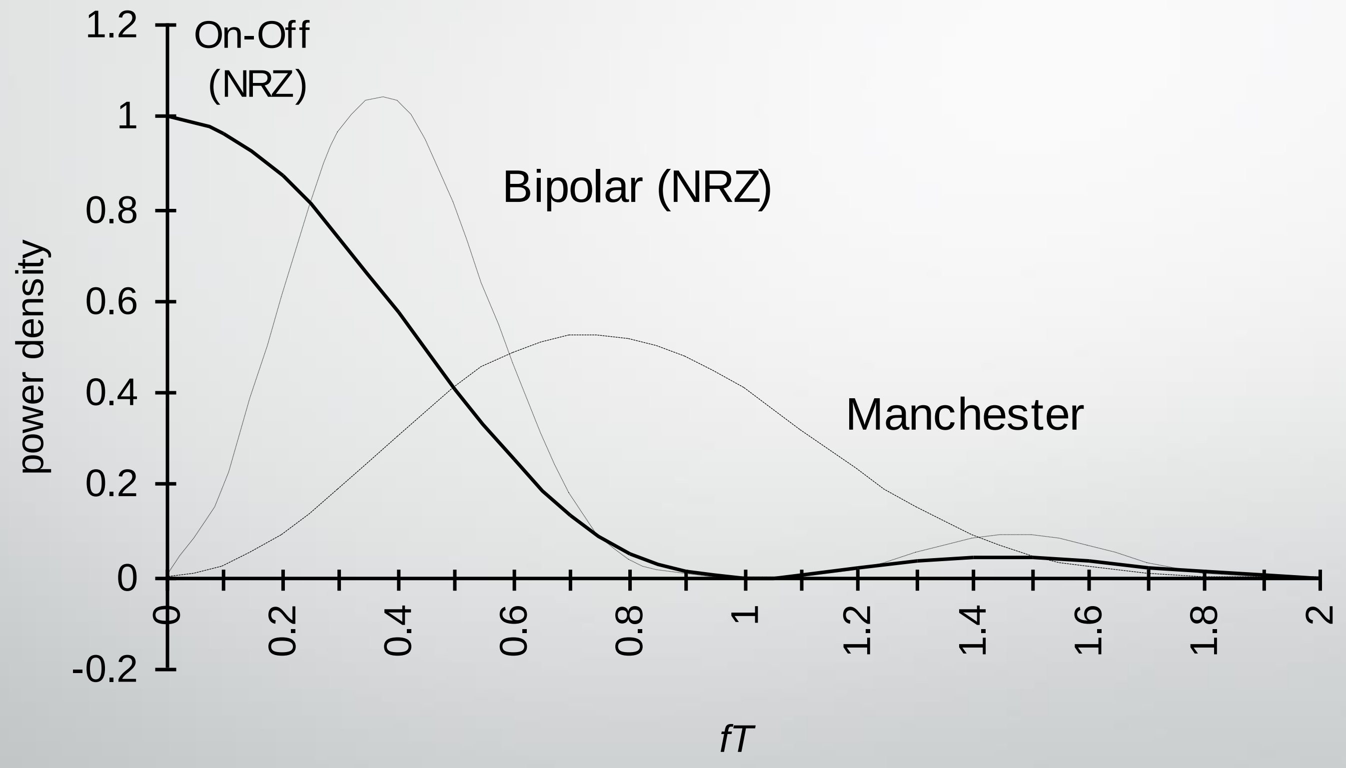
Polar RZ
Perfect synch
3 levels



Manchester (Bi-Phase)
Perfect Synch.
2 levels



Spectra of Some Line Codes



Pulse Shaping

- The line codes presented above have been demonstrated using (rectangular) pulses.
- There are two problems in transmitting such pulses:
  - They require infinite bandwidth.
  - When transmitted over bandlimited channels become time unlimited on the other side, and spread over adjacent symbols, resulting in Inter-Symbol-Interference (ISI).

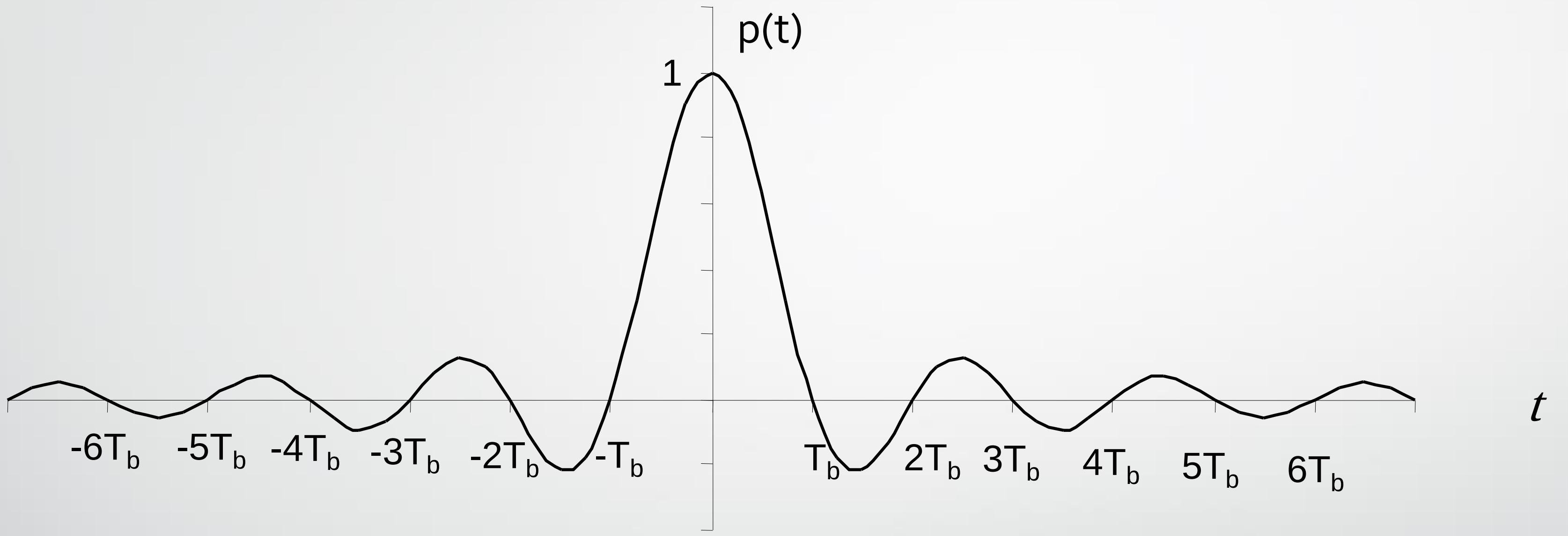
Nyquist-Criterion for Zero ISI

- Use a pulse that has the following characteristics

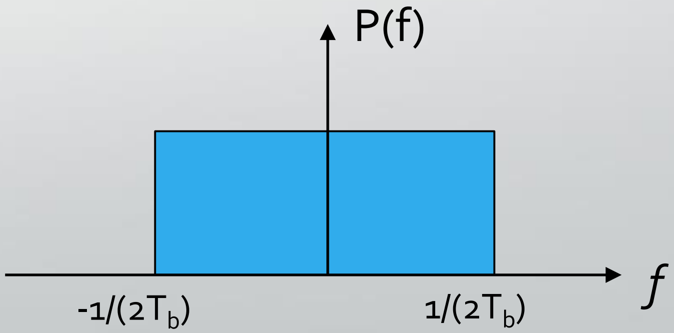
$$p(t) = \begin{cases} 1 & t = 0 \\ 0 & t = \pm T_b, \pm 2T_b, \pm 3T_b, \ldots \end{cases}$$

- One such pulse is the sinc function.

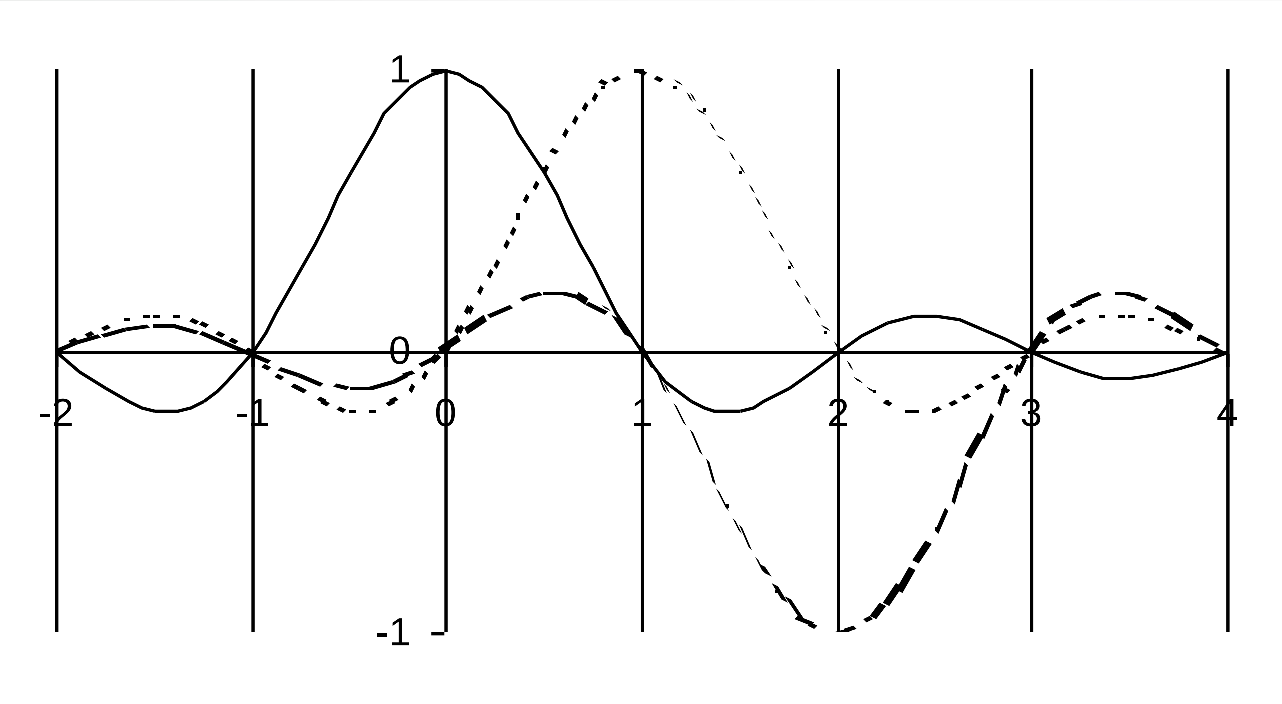
The Sinc Pulse



Note that such pulse has a bandwidth of $R_b/2$ Hz. Therefore, the minimum channel bandwidth required for transmitting pulses at a rate of R_b pulses/sec is $R_b/2$ Hz



Zero ISI



More on Pulse Shaping

- The sinc pulse has the minimum bandwidth among pulses satisfying Nyquist criterion.
- However, the sinc pulse is not fast decaying;
  - Misalignment in sampling results in significant ISI.
  - Requires long delays for realization.
- There is a set of pulses that satisfy the Nyquist criterion and decay at a faster rate. However, they require bandwidth more than $R_b/2$.

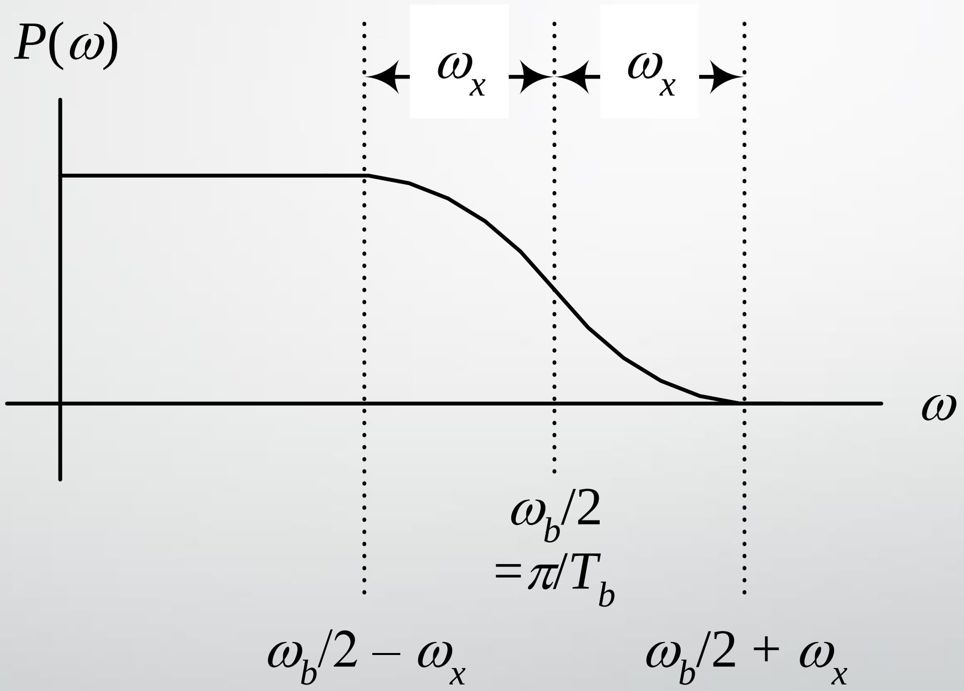
Raised-Cosine Pulses

$$P(\omega)=\begin{cases}\frac{1}{2}\left[1-\sin\left(\frac{\pi\{\omega-(\omega_b/2)\}}{2\omega_x}\right)\right] & \left|\omega-\frac{\omega_b}{2}\right|<\omega_x \\ 0 & |\omega|>\frac{\omega_b}{2}+\omega_x \\ 1 & |\omega|<\frac{\omega_b}{2}-\omega_x\end{cases}$$

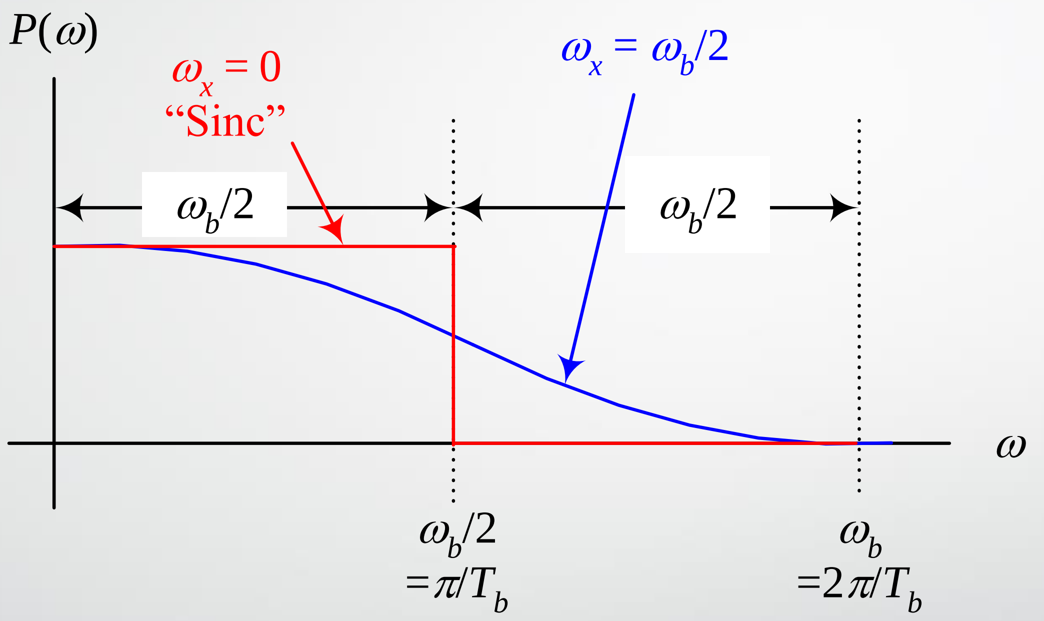
where ω_b is $2\pi R_b$ and ω_x is the *excess* bandwidth. It defines how much bandwidth required above the minimum bandwidth of a sinc pulse, where

$$0\le\omega_x\le\frac{\omega_b}{2}$$

Spectrum of Raised-Cosine Pulses

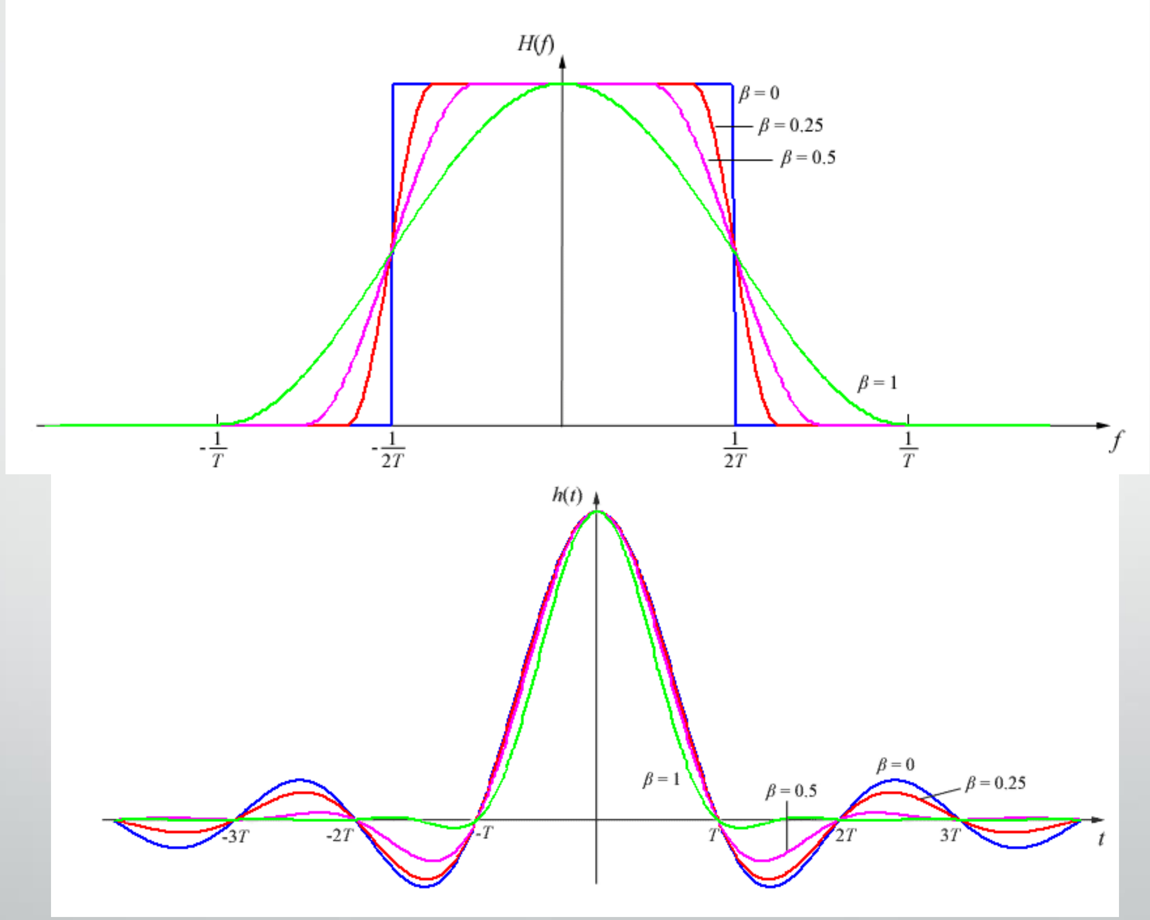


Extremes of Raised-Cosine Spectra



$$r = \frac{\text{Excess Bandwidth}}{\text{Minimum Bandwidth}} = \frac{\omega_x}{\omega_b / 2} = \frac{2\omega_x}{\omega_b}$$

Raised-Cosine Pulses

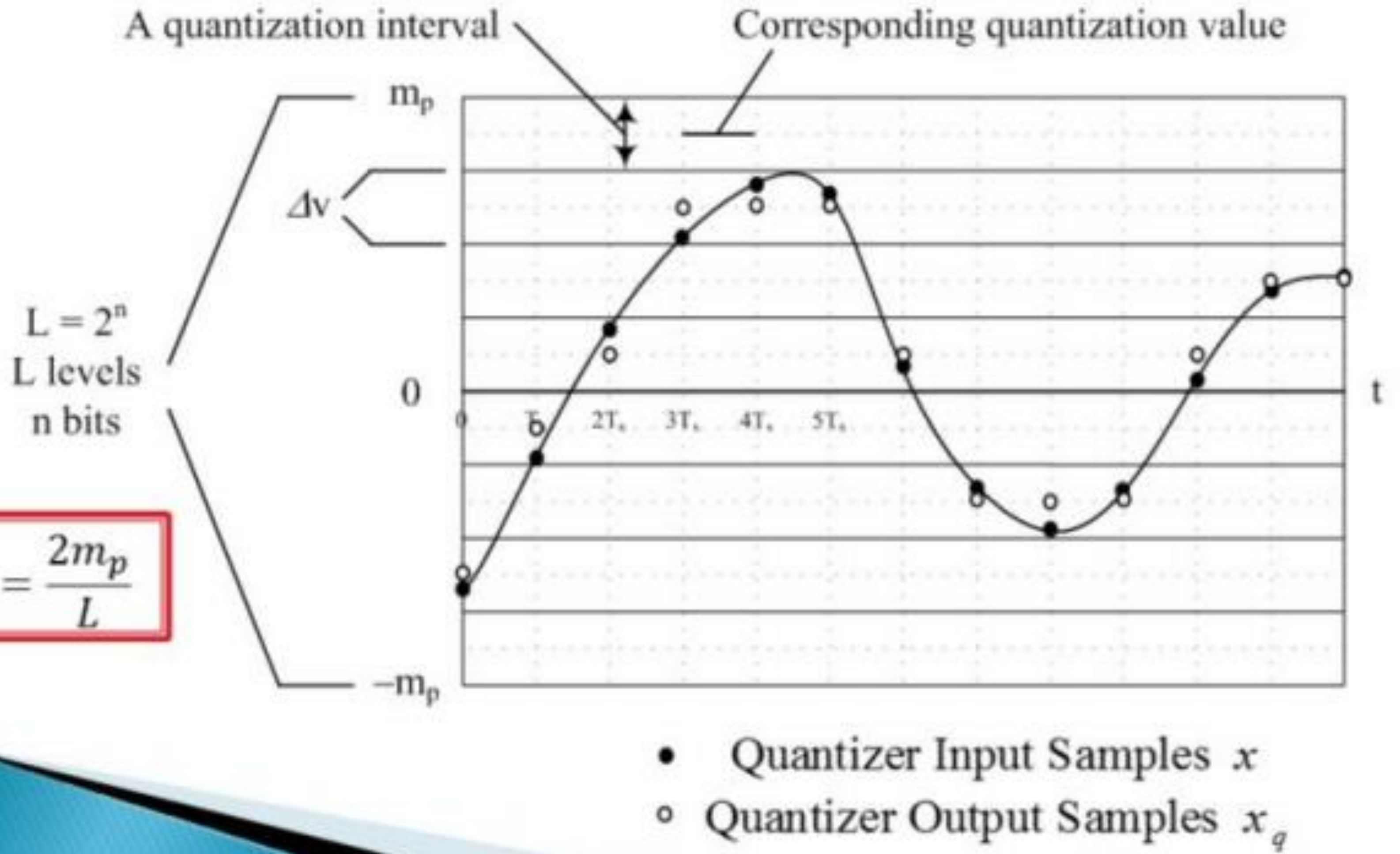


Quantization

Extra Material

Quantization

- In quantization, an analog sample with amplitude that may take value in a specific range is converted to a digital sample with amplitude that takes one of a specific pre-defined set of quantization values.



$$\Delta v = \frac{2m_p}{L}$$

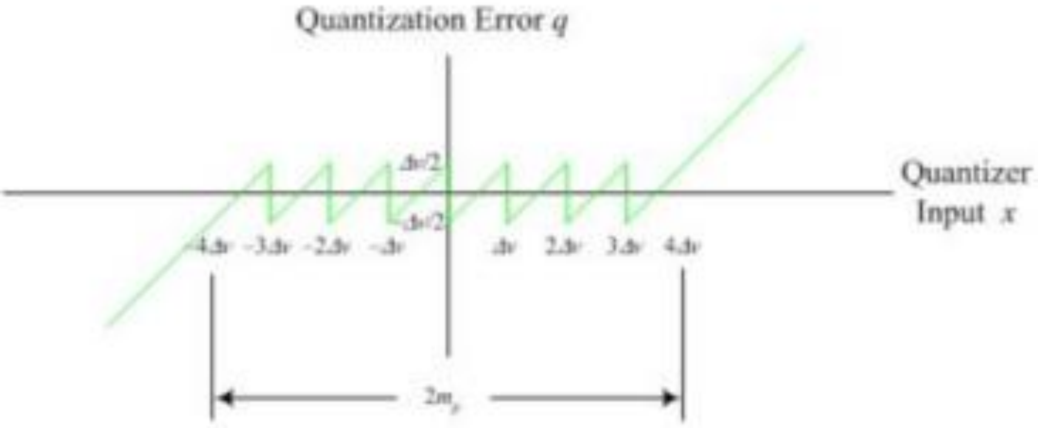
The process of quantizing a signal is the first part of converting a sequence of analog samples to a PCM code.

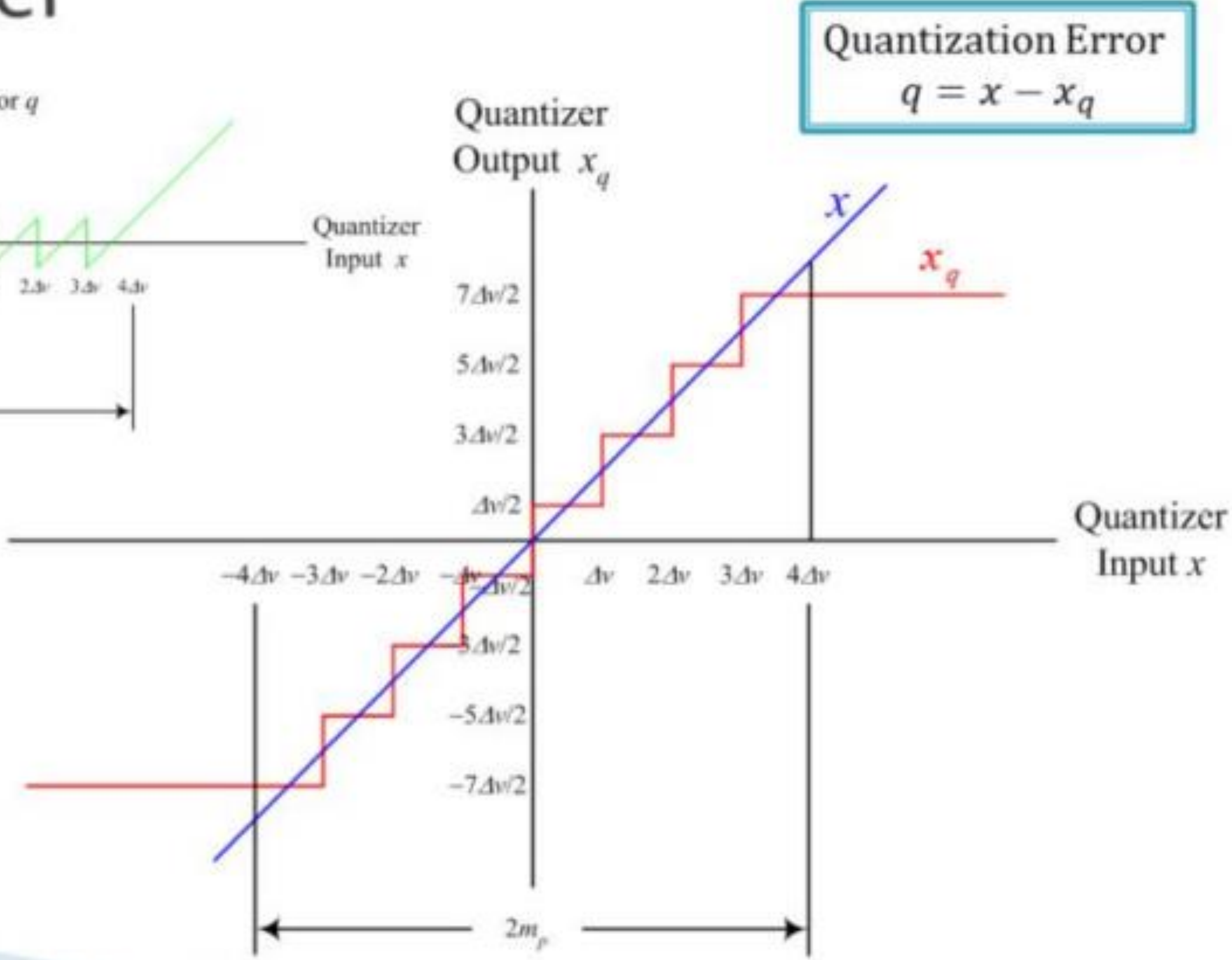
In quantization, an analog sample with amplitude that may take value in a specific range is converted to a digital sample with amplitude that takes one of a specific pre–defined set of quantization values. This is performed by dividing the range of possible values of the analog samples into L different levels, and assigning the center value of each level to any sample that falls in that quantization interval. The problem with this process is that it approximates the value of an analog sample with the nearest of the quantization values. So, for almost all samples, the quantized samples will differ from the original samples by a small amount. This amount is called the quantization error. To get some idea on the effect of this quantization error, quantizing audio signals results in a hissing noise similar to what you would hear when play a random signal. Assume that a signal with power P_s is to be quantized using a quantizer with $L = 2^n$ levels ranging in voltage from $-m_p$ to m_p as shown in the figure.

We can define the variable Δv to be the height of the each of the L levels of the quantizer as shown above. This gives a value of Δv equal to $\Delta v = \frac{2m_p}{L}$

Therefore, for a set of quantizers with the same m_p, the larger the number of levels of a quantizer, the smaller the size of each quantization interval, and for a set of quantizers with the same number of quantization intervals, the larger m_p is the larger the quantization interval length to accommodate all the quantization range.

Input output characteristics of the quantizer





the red line in the following figure. Note that as long as the input is within the quantization range of the quantizer, the output of the quantizer represented by the red line follows the input of the quantizer. When the input of the quantizer exceeds the range of $-m_p$ to m_p, the output of the quantizer starts to deviate from the input and the quantization error (difference between an input and the corresponding output sample) increases significantly.

Now let us define the quantization error represented by the difference between the input sample and the corresponding output sample to be q, or

$$q = x - x_q.$$

Plotting this quantization error versus the input signal of a quantizer is seen next. Notice that the plot of the quantization error is obtained by taking the difference between the blow and red lines in the above figure.

Quantization Noise Power

- Assuming that the input signal is restricted between $-m_p$ to $m_{p.}$
- Error q (quantization noise) will be a random process that is **uniformly** distributed between $-\Delta v/2$ and $\Delta v/2$ with a constant height of $1/$?v.
- The power of such a random process can be found by finding the average of the square of all noise values multiplied by probability of each of these values of the noise occurring.

- $P_q = \int_{-\frac{\Delta v}{2}}^{+\frac{\Delta v}{2}} q^2 \frac{1}{\Delta v} dq$
- $P_q = \frac{1}{\Delta v}\left[\frac{q^3}{3}\right]_{-\frac{\Delta v}{2}}^{+\frac{\Delta v}{2}} = \frac{1}{\Delta v}\left[\frac{\left(\frac{\Delta v}{2}\right)^3}{3} - \frac{\left(-\frac{\Delta v}{2}\right)^3}{3}\right] = \frac{1}{\Delta v}\left[\frac{(\Delta v)^3}{24} + \frac{(\Delta v)^3}{24}\right]$
- $P_q = \frac{(\Delta v)^2}{12}$

To understand the following, you will need to know something about probability theory. Assuming that the input signal is restricted between $-m_p$ to m_p, the resulting quantization error q (or we can call it quantization noise) will be a random process that is uniformly distributed between $-\Delta v/2$ and $\Delta v/2$ with a constant height of $1/\Delta v$. That is, all values of quantization error in the range $-\Delta v/2$ and $\Delta v/2$ are equally probable to happen. The power of such a random process can be easily found by finding the average of the square of all noise values multiplied by probability of each of these values of the noise occurring. So,

Now substituting for $\Delta v = \frac{2m_p}{L}$

in the above equation gives $P_q = \frac{\left(\frac{2m_p}{L}\right)^2}{12} = \frac{m_p^2}{3L^2}$

,

As predicted, the power of the noise decreases as the number of levels L increases, and increases as the edge of the quantization range m_p increases.

Now let us define the Signal to Noise Ratio (SNR) as the ratio of the power of the input signal of the quantizer to the power of the noise introduced by the quantizer (note that the SNR has many other definitions used in communication systems depending on the applications)

Continue.... Quantization Noise Power

- Now substituting for $\Delta v = \frac{2m_p}{L}$ in the above equation $P_q = \frac{(\Delta v)^2}{12}$ gives

$$P_q = \frac{\left(\frac{2m_p}{L}\right)^2}{12} = \frac{m_p^2}{3L^2}$$

- Signal to Noise Ratio (SNR) is the ratio of the power of the input signal of the quantizer to the power of the noise introduced by the quantizer

$$SNR = \frac{Signal\ Power}{Noise\ Power} = \frac{S_{out}}{N_{out}} = \frac{S_{out}}{N_q} = \frac{P_s}{P_q} = \frac{\widetilde{m^2(t)}}{\widetilde{q^2(t)}} = \frac{3L^2}{m_p^2} P_s$$

SNR in dB scale

- In general the values of the SNR are either much greater than 1 or much less than 1. This suggests the use of dB scale.
- L of a quantizer is always a power of two or $L = 2^n$

$$SNR_{Linear} = \frac{3L^2}{m_p^2} P_s,$$

$$SNR_{dB} = 10 \cdot \log_{10}\left(\frac{3L^2}{m_p^2} P_s\right) = 10 \cdot \log_{10}\left(\frac{3}{m_p^2} P_s\right) + 10 \cdot \log_{10}\left(2^{2n}\right)$$

$$= \underbrace{10 \cdot \log_{10}\left(\frac{3}{m_p^2} P_s\right)}_{\alpha} + \underbrace{20n \cdot \log_{10}(2)}_{6n}$$

$$= \alpha + 6n \qquad \text{dB.}$$

In general the values of the SNR are either much greater than 1 or much less than 1. A more useful representation of the SNR can be obtained by using logarithmic scale or dB. We know that L of a quantizer is always a power of two or $L = 2^n$. Therefore,

Note that α shown in the above representation of the SNR is a constant when quantizing a specific signal with different quantizers as long as all of these quantizers have the same value of m_p.

Effect of Number of Bits (n)

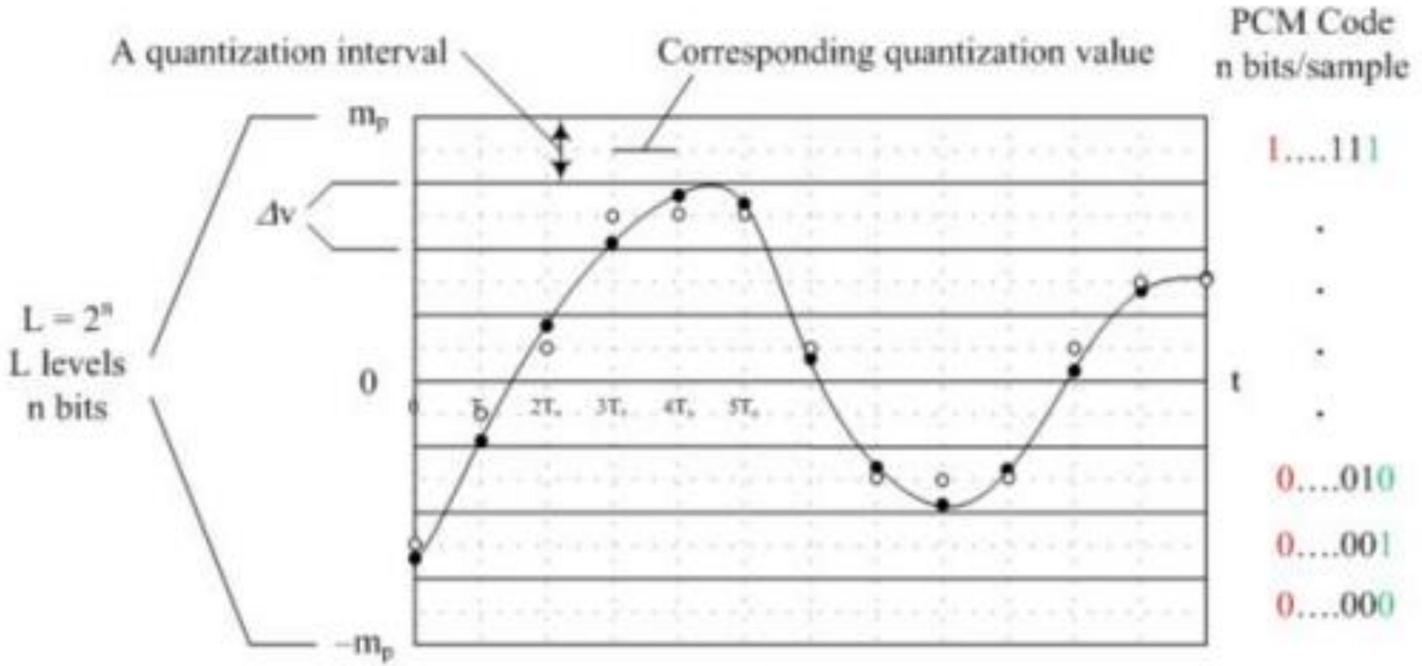
$$SNR_{dB} = \alpha + 6n \ dB$$

- The SNR of a quantizer in dB increases linearly by 6 dB as we increase the number of bits that the quantizer uses by 1 bit.
- The cost for increasing the SNR of a quantizer is that more bits are generated and therefore either a higher bandwidth or a longer time period is required to transmit the PCM signal.

It is clear that the SNR of a quantizer in dB increases linearly by 6 dB as we increase the number of bits that the quantizer uses by 1 bit. The cost for increasing the SNR of a quantizer is that more bits are generated and therefore either a higher bandwidth or a longer time period is required to transmit the PCM signal.

Generation of the PCM Signal

- Each of the levels of the quantizer is assigned a code from 000...000 for the lowest quantization interval to 111...111.
- A **digit-at-time** encoder makes n sequential comparisons to generate n-bits code word. The sample is compared with reference voltages 2^7,26,25, ..., 2^0

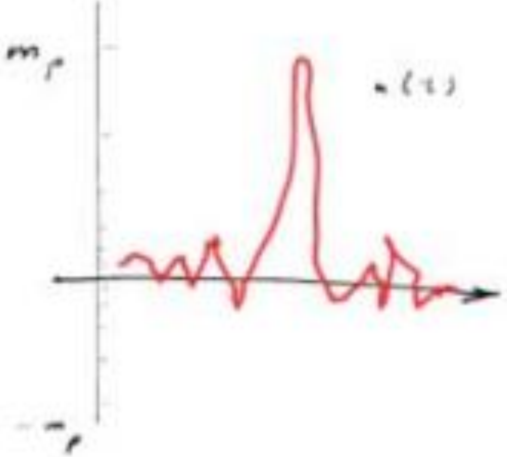


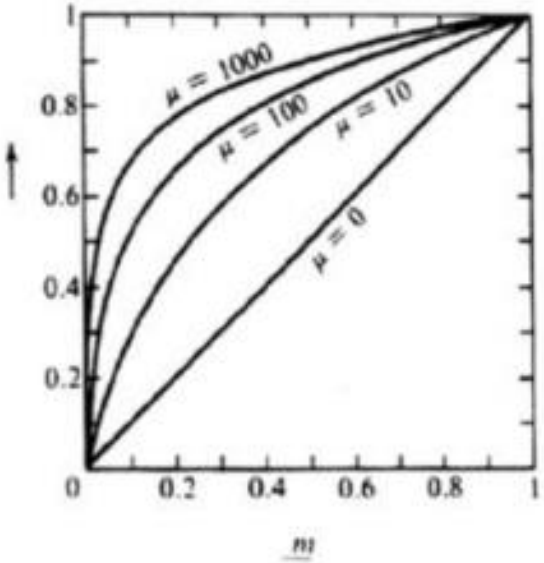
Now, once the signal has been quantized by the quantizer, the quantizer converts it to bits (1's and 0's) and outputs these bits. Looking at the figure in the previous lecture, which is shown here for convenience. We see that each of the levels of the quantizer is assigned a code from 000…000 for the lowest quantization interval to 111…111 for the highest quantization interval as shown in the column to the left of the figure. The PCM signal is obtained by outputting the bits of the different samples one bit after the other and one sample after the other.

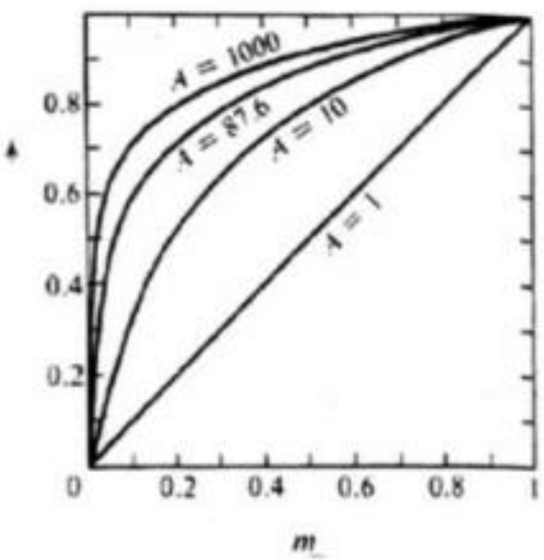
We can use a **digit-at-time** encoder which makes n sequential comparisons to generate n-bits code word. The sample is compared with reference voltages $2^7, 2^6, 2^5, \ldots, 2^0$.

Companding: Non-unifrom Quantization

- Ideally we want constant SNR for all values of the message.
- Signals (voices) varies as much as 40dB (10^4 power ratio). The variation could be different due to connection lengths.
- Statistically (for voice): most of the time the signal has small amplitudes (Low SNR most of the time).
- **For** uniform quantization $\Delta v = \frac{2m_p}{L}$ and $N_q = \frac{(\Delta v)^2}{12}$
- The error depend on the **step size.** The solution is to use small steps for small amplitudes and large steps for large amplitudes (**Progressive taxation**)
- This is equivalent to first compress the signal samples & then use uniform quantizer. (Later we will have to decompress).
- Since at the transmitter/receiver we do compress/expand, we call the compensation compander =Compresser +Expander.







An approximately logarithmic compression characteristic yields a quantization noise nearly proportional to the signal power. The SNR becomes practically independent of the signal power over a large dynamic range. (Loud talks and stronger signals are penalized more than soft ones)

Two standards are accepted by the CCITT

1) μ-law (North America & Japan)

2) A-law (Europe & the rest of the world & international routes)

A and μ determines the degree of compression (compression parameter).

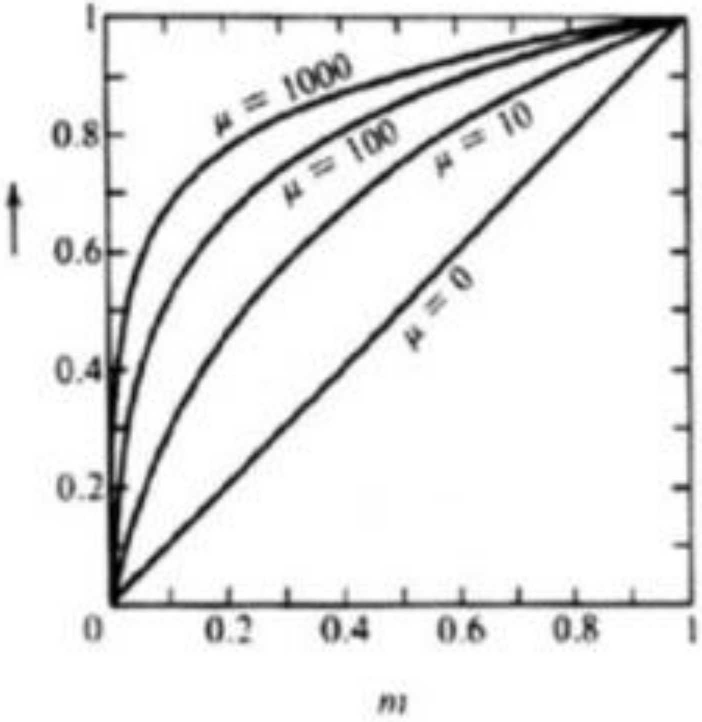
CCITT (*Comité Consultatif International Téléphonique etTélégraphique*, an organization that sets internationalcommunications standards. CCITT, now known as ITU (the parent organization))

μ–law

$$F(x) = \text{sgn}(x)\frac{\ln(1+\mu|x|)}{\ln(1+\mu)} \quad -1 \le x \le 1$$

$$F^{-1}(y) = \text{sgn}(y)(1/\mu)((1+\mu)^{|y|} - 1) \quad -1 \le y \le 1$$

- For input variation greater than $40dB$, $\mu > 100$.
- For practical telephone systems
  - $\mu = 100$ for 7bit (128 levels)
  - $\mu = 255$ for 8bit (256 levels)
- The compander with logarithmic compression can be realized by a semiconductor diode. We can also use piecewise approximation with small end-to-end inferiority.
- See Wikipedia for "μ–law algorithm"

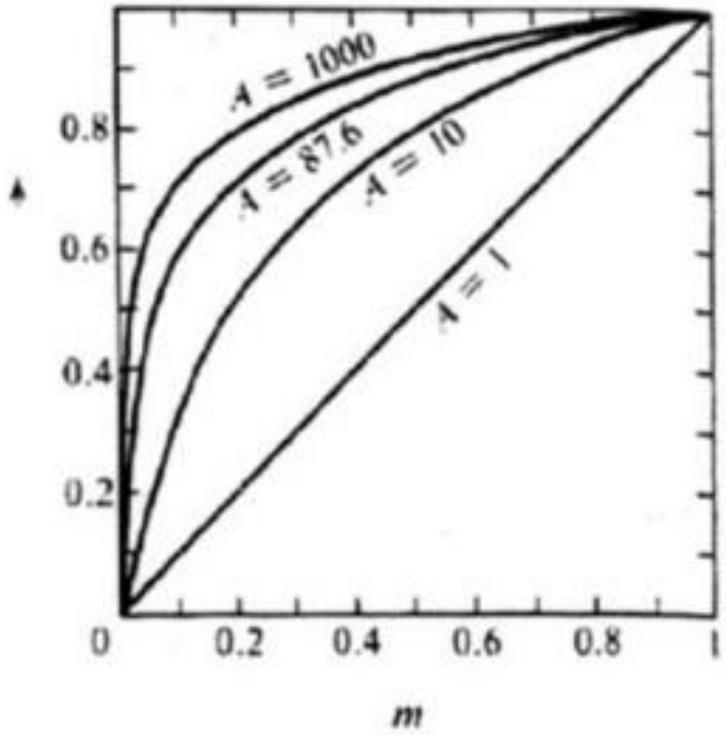


A-law

$$F(x) = \mathrm{sgn}(x)\begin{cases} \frac{A|x|}{1+\ln(A)}, & |x| < \frac{1}{A} \\ \frac{1+\ln(A|x|)}{1+\ln(A)}, & \frac{1}{A} \le |x| \le 1, \end{cases}$$

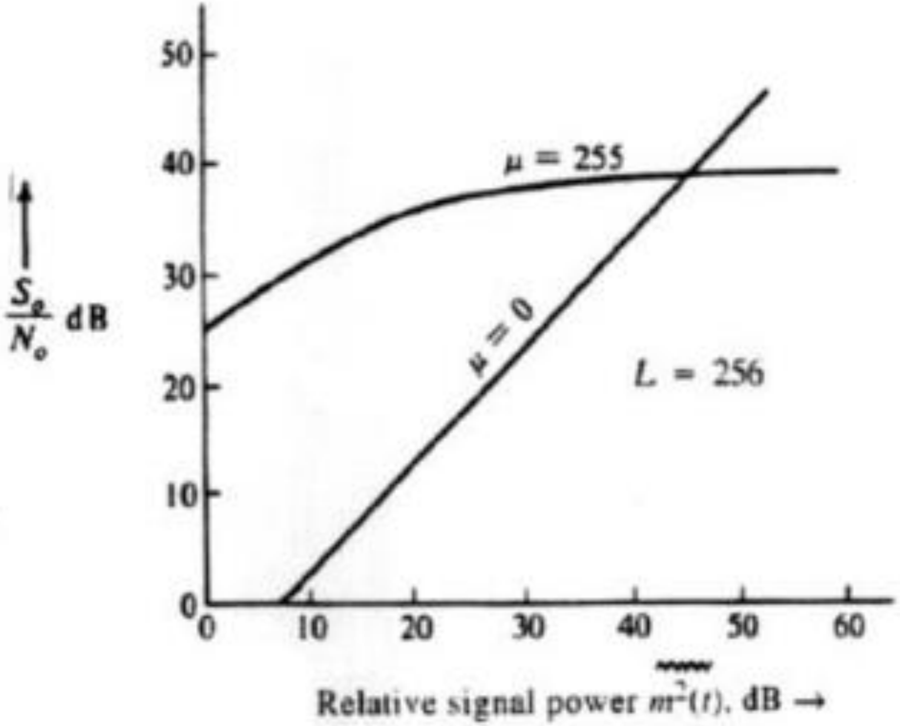
- For a given input x, the equation for A-law encoding is as $F(x)$
- A-law expansion is given by the inverse function, $F^{-1}(y)$
- In Europe, $A = 87.7$; the value 87.6 is also used.
- **Example**: how many levels will be used to represent the lowest 20% of the signal level for the case of $A = 1$ and $A = 10$?

$$F^{-1}(y) = \mathrm{sgn}(y)\begin{cases} \frac{|y|(1+\ln(A))}{A}, & |y| < \frac{1}{1+\ln(A)} \\ \frac{\exp(|y|(1+\ln(A))-1)}{A}, & \frac{1}{1+\ln(A)} \le |y| < 1. \end{cases}$$



SNR impact

- When μ–law is used:
- $\frac{S_0}{N_0} = \frac{3L^2}{[\ln(1+n)]^2}$ for $\mu^2 \gg \frac{m_p^2}{\widetilde{m^2(t)}}$
- The output SNR is almost independent from the input SNR.
- Note the scale above is in dB.
- Are you familiar with the dB scale?



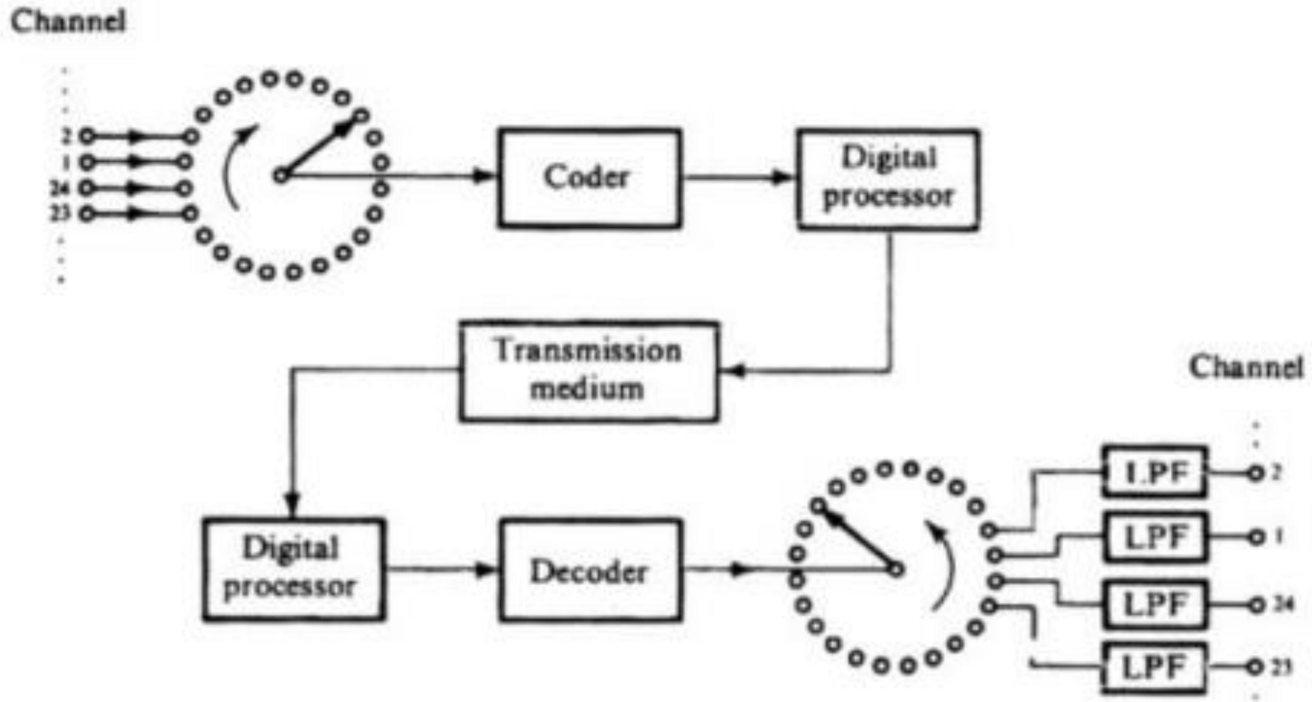
Signal-to-Quantization-noise ratio in PCM with and without compression

- In a certain digital voice communication system, the error in sample amplitudes cannot be greater than 3% of the peak amplitude.
- Determine the number of bits for the quantizer, n.

- $Ans. n = 6$

The T1 Carrier System

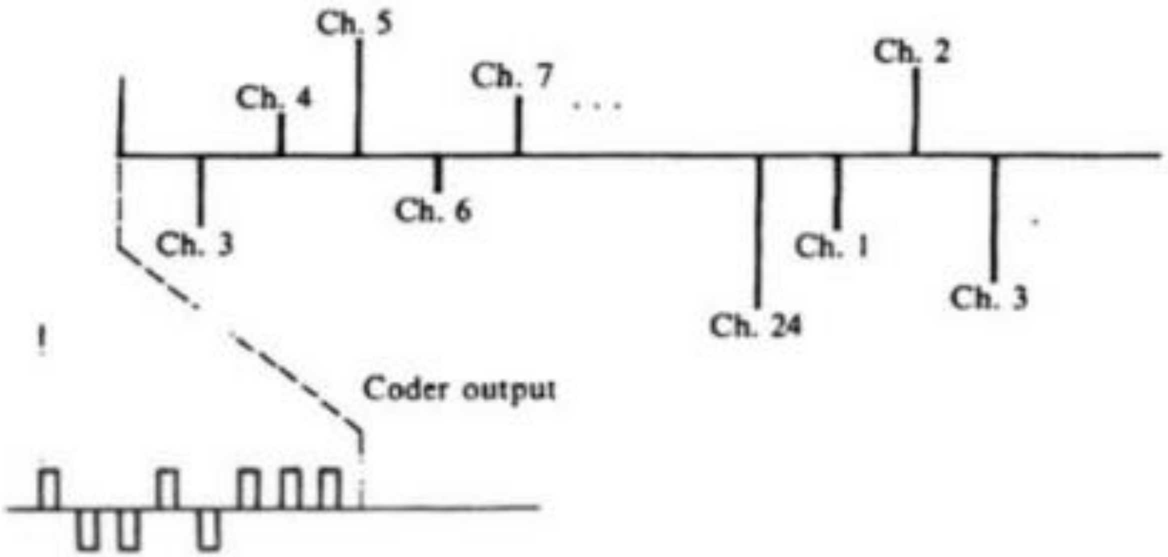
- 24 Channels (TDM-PAM).
- Encoder (Quantize and Encode) samples ,8 binary pulses (Binary Codeword).
- Regenerative repeaters /6000 feet
- @ the receiver: decode binary pulses into samples, demultiplex, LPF.



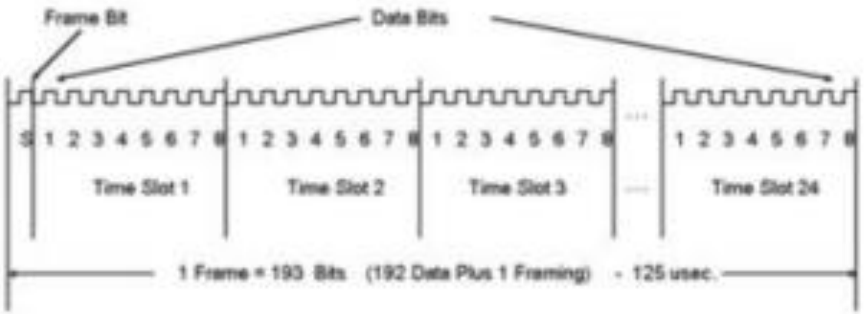
Multiplexing is not mechanical

More about T1

- The $1.544\ Mbit/s$ of the T1 system is called Digital Signal Level 1 (DS1)….DS2, DS3,DS4 also exist.
- T1(N. America & Japan)
- By CCITT a different system of 30 Channels PCM ($2.048\ Mbits/s$) is used in Europe and others



Frame Synchronization



- **Frame:** A segment containing one code-word (corresponding to one sample) from each of the 24 channels.
  - $\# \, of \, bits = 24 \times 8 = 192 +$ a framing bit (in order to separate information bits correctly)=193 bits/frame
  - $8000 \frac{samples}{s} => \frac{1}{8000} = 125 \, \mu s$
  - $193 \frac{bits}{frame} \div 125 \frac{\mu s}{frame} = \mathbf{1.544} \, \boldsymbol{Mbit/s}$
- **Framing bits:** Chosen so that a sequence of framing bits, one at the beginning of each frame, forms a special pattern that is unlikely to be formed in speech signal.
- 0.4 to $0.6ms$ to detect synchronization loss, $50 \, ms$ to reframe.

T1 System Signaling Format

- Signaling: bits corresponding to dialing pulses & telephone on-hook/off-hook.