Binary Phase Shift Keying (BPSK)

Binary Digital Bandpass Modulation

Here, the baseband data modulates a high frequency carrier to produce a modulated signal, whose spectrum is centered on the carrier frequency. We will consider four types of bandpass transmission schemes; Amplitude Shift Keying (ASK), Phase Shift Keying (PSK), Frequency Shift Keying (FSK), and Quadri-phase Shift Keying (QPSK). For each type, we consider the generation, optimum receiver, probability of error, power spectral density, and bandwidth.

Binary Phase Shift Keying: Signal Representation

Signal Representation:

Send: $s_1(t) = A\cos(2\pi f_c t)$ if the information bit is "1";

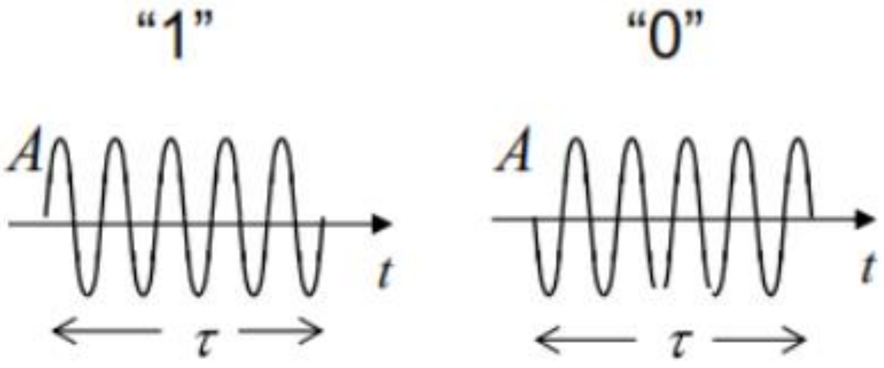
Send: $s_2(t) = A\cos(2\pi f_c t + \pi)$

$s_2(t) = -A\cos(2\pi f_c t)$ if the information bit is "0";

$\tau = nT_c$

τ: is the time allocated to transmit the binary digit.

$T_c = 1/f_c$ is the carrier period



$s_1(t) = A\cos(2\pi f_c t)$ $s_2(t) = A\cos(2\pi f_c t + \pi)$

$0 \le t \le \tau$ $0 \le t \le \tau$

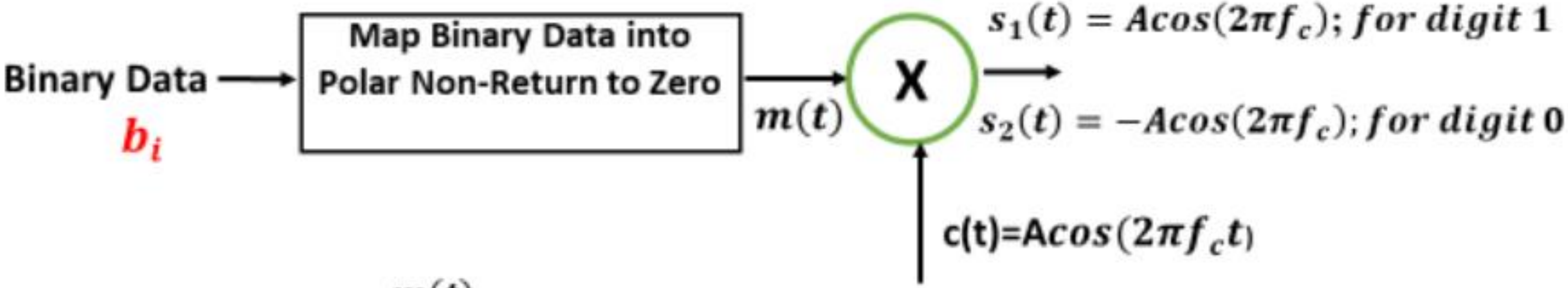
(τ is an integer number of $1/f_c$)

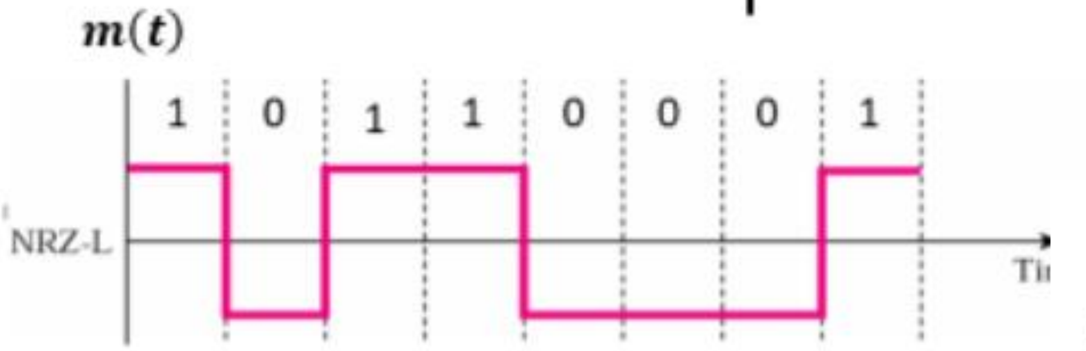
$s_2(t) = s_1(t+\pi) = -s_1(t)$

$\tau = nT_c$

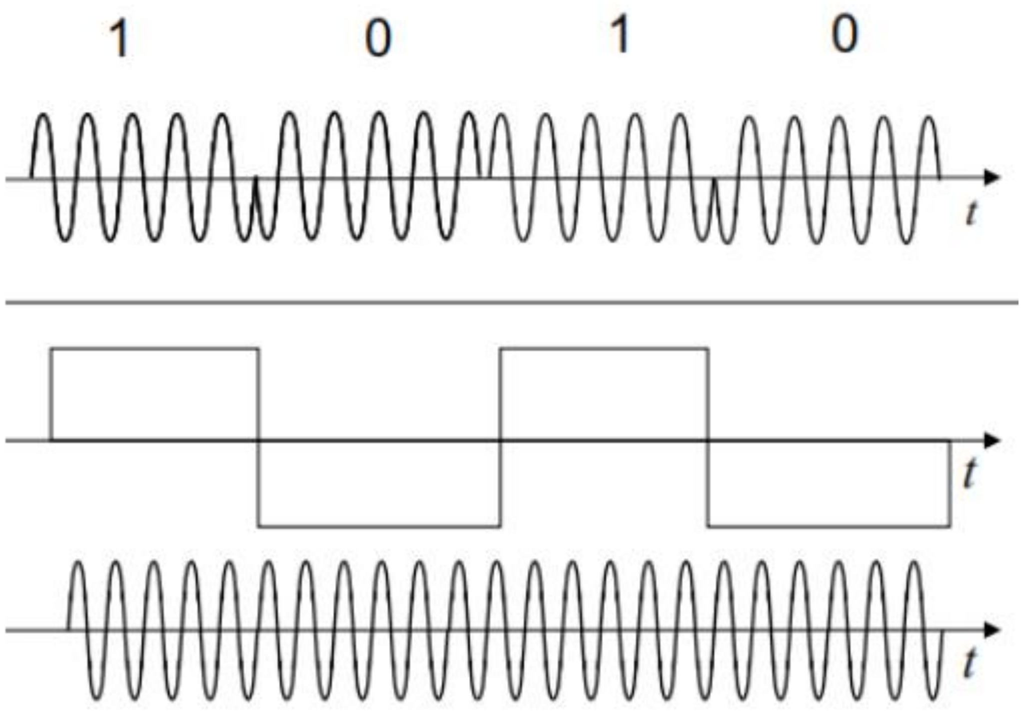
In this figure n=5

Binary Phase Shift Keying: Generation

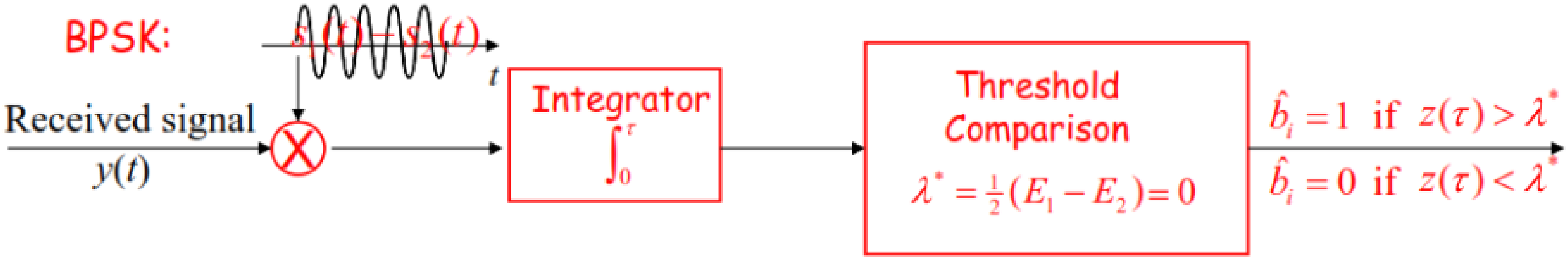




Polar Non-return to Zero



Binary Phase Shift Keying: The Optimum Receiver



Optimum Receiver Implemented as a Correlator Followed by a Threshold Detector

Probability of Error:

$$P_b^* = Q\left(\sqrt{\frac{\int_0^\tau (s_1(t) - s_2(t))^2 dt}{2N_0}}\right)$$

$$\text{Energy of } s_i(t) : E_1 = E_2 = \frac{1}{2}A^2\tau$$

$$\text{Average Energy per bit: } E_b = \frac{1}{2}(E_1 + E_2) = \frac{1}{2}A^2\tau$$

$$\boldsymbol{E = \int_0^\tau (s(t))^2 dt}$$

With $\boldsymbol{\tau = nT_c}$

$$\boldsymbol{E = A^2\tau/2}$$

Verify this result

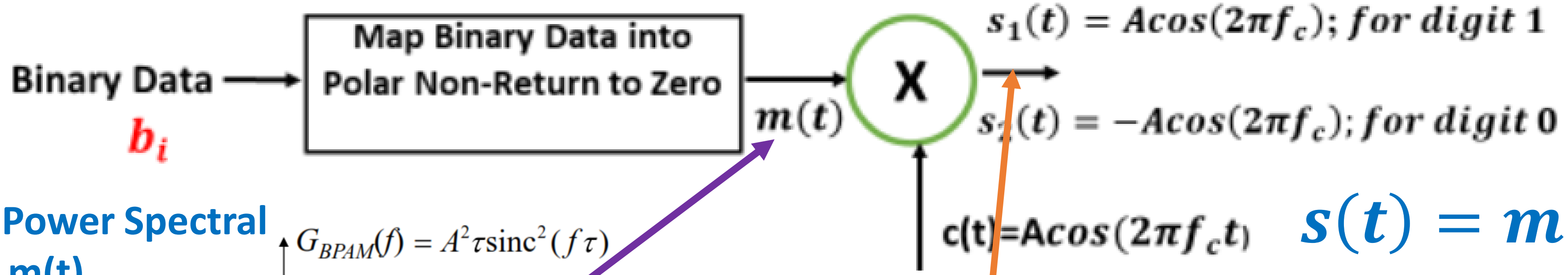
$$s_1(t) = A\cos(2\pi f_c t)$$

$$s_2(t) = -A\cos(2\pi f_c t)$$

Optimal BER:

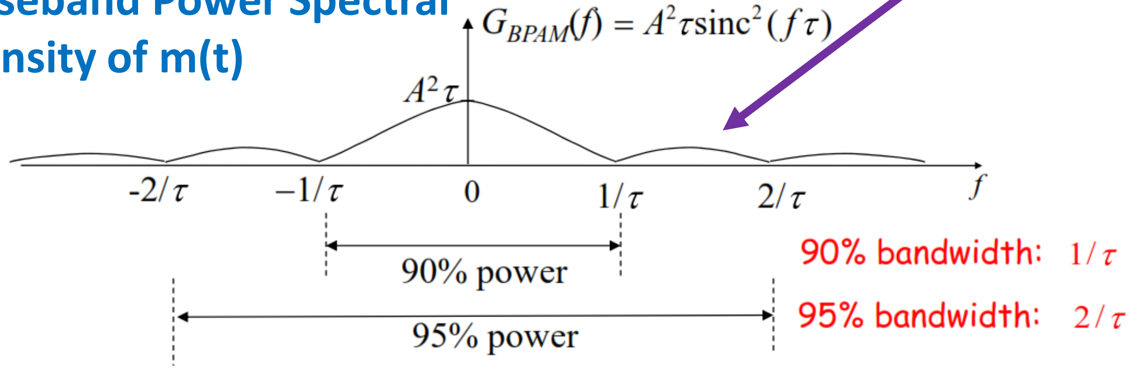
$$P_b^* = Q\left(\sqrt{\frac{A^2\tau}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Binary Phase Shift Keying: Power Spectral Density and Bandwidth



$$s(t) = m(t)\cos(2\pi f_c t)$$

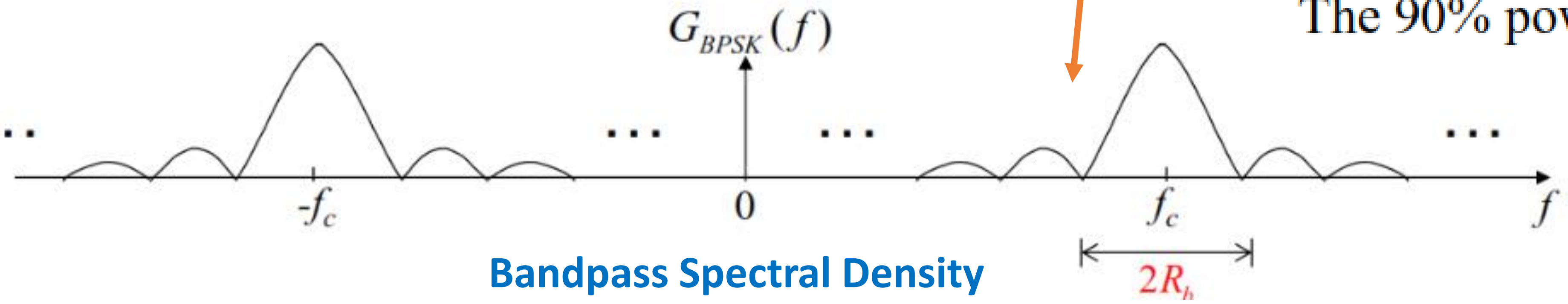
Baseband Power Spectral Density of m(t)



Bandwidth of BPSK s(t)

Power Spectral Density

$$G_{BPSK}(f) = \frac{1}{4}[G_{BPAM}(f - f_c) + G_{BPAM}(f + f_c)]$$



The 90% power bandwidth $= \frac{2}{\tau} = 2R_b$

(twice the data rate); Same as that of BASK

Bandpass Spectral Density

Extra Material on the Power Spectral Density

The Wiener –Khintchine Thorem:

The power spectral density $G_X(f)$ and the autocorrelation function $R_X(\tau)$ of a stationary random process $X(t)$ form a Fourier transform pairs:

$G_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f\tau} d\tau$ (Fourier Transform)

$R_X(\tau) = \int_{-\infty}^{\infty} G_X(f) e^{j2\pi f\tau} df$ (Inverse Fourier Transform)

Example: Mixing of a random process with a sinusoidal signal.

- A random process $X(t)$ with an autocorrelation function $R_X(\tau)$ and a power spectral density $G_X(f)$ is mixed with a sinusoidal function $\cos(2\pi f_c t + \theta)$; θ is a r.v uniformly distribution over $(0, 2\pi)$ to form a new process

  $Y(t) = X(t)\cos(2\pi f_c t + \theta)$. Find $R_Y(\tau)$ and $G_Y(f)$
- **Solution**: We first find $R_Y(\tau)$
- $R_Y(\tau) = E\{ Y(t)Y(t+\tau)\}$
- $= E\{X(t)\cos(2\pi f_c t + \theta) \cdot X(t+\tau) \cos(2\pi f_c t + 2\pi f_c \tau + \theta)\}$

  When $X(t)$ and θ are independent, then
- $= E\{X(t)\, X(t+T)\}E\{\cos(2\pi f_c t + \theta) \cdot \cos(2\pi f_c t + 2\pi f_c \tau + \theta)\}$
- $= R_X(\tau)E\{\frac{\cos(4\pi f_c t + 2\pi f_c \tau + 2\theta) + \cos 2\pi f_c \tau}{2}\}$
- $\boldsymbol{R_Y(\tau) = \frac{R_X(\tau)}{2} \cdot \cos 2\pi f_c \tau}$;
- The power spectral density is
- $\boldsymbol{S_Y(f) = \frac{1}{4}\{G_X(f - f_c) + G_X(f + f_c)\}}$
- Which is quite similar to the modulation property of the Fourier transform.